Development of a mathematical model to determine the temperature distribution in the metal layer and hearth of an electrical resistance smelter

Kurt B. Carlson

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DEVELOPMENT OF A MATHEMATICAL MODEL TO DETERMINE
THE TEMPERATURE DISTRIBUTION IN THE METAL LAYER
AND HEARTH OF AN ELECTRICAL RESISTANCE SMELTER

BY

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A THESIS SUBMITTED
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ABSTRACT

A steady-state finite-difference heat transfer mathematical model was derived to determine the temperature distribution in the metal layer and the hearth of a cylindrical electrical-resistance smelter. The temperature distribution is required for the redesign of the refractory materials and their positions in the hearth to insure that the metal layer is kept molten at normal smelting temperatures and that mechanical and structural integrity of the hearth is maintained. An extensive literature search revealed that no previously defined model of this type existed and that consideration of a three-dimensional model was beyond the scope of this work. The literature search also verified that the metal layer and the hearth could be modeled independently of the slag layer. The finite-difference heat transfer model was then developed by defining nine different types of nodes in the model and deriving steady-state heat transfer equations for each type of node. An algorithm was then developed for the solution of the non-linear dependent set of simultaneous equations. Beyond the scope of this work, ninety percent of a Fortran
A computer program was written and compiled employing the algorithm. It is recommended that the computer program be finished, debugged, and that various combinations of refractory material types and positions be tried to determine the optimum design of the refractory hearth. Proper design of the hearth has the potential to improve the quality of the metal poured from the smelter, reduce operating costs, and increase the capacity of the smelter.
NOMENCLATURE

$\text{Ar}$ - Area of heat transfer resistance [in$^2$]

$\text{As}$ - Area of heat transfer resistance for a partial node [in$^2$]

$h_1, h_2$ - Height of the face of a partial node [in]

$I$ - Unit vector in the radial direction

$J$ - Unit vector in the axial direction

$k$ - Thermal conductivity [BTU-in/hr-ft$^2$-°F]

$m$ - Equivalent slope of the bottom shell of the smelter

$Q$ - Heat flow from a typical node [BTU/hr-ft$^2$-°F]

$Q_c$ - Convective heat flow from a node to the air surrounding the smelter [BTU/hr-ft$^2$-°F]

$Q_r$ - Radiative heat flow from a node to surfaces surrounding the smelter [BTU/hr-ft$^2$-°F]

$R$ - Radial direction

$r$ - Length of node in the radial direction [in]

$R_o$ - Outside radius of the shell [in]

$T$ - Temperature of a node [°F]

$T_{air}$ - Temperature of the air surrounding the shell [°F]

$T_s$ - Temperature of the surfaces surrounding the shell [°F]

$Z$ - Axial direction

$Z_h$ - Maximum thickness of the refractory hearth [in]

$Z_s$ - Thickness of the silver layer [in]

$\epsilon$ - Emissivity of the shell

$\sigma$ - Stephan-Boltzman constant 
[0.1714E-08 BTU/hr-ft$^2$-°R$^4$]

$\Theta_D$ - Debye temperature [°K]
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1.0 INTRODUCTION

The materials needed in today's highly technical world must be of high quality. Gone are the days when man could take materials from the earth and, with a minimum of effort and technology, fashion the tools for his world. To provide these high quality materials, the metallurgical industry has had to devise processes to take the materials from their native state in the earth and to refine them or modify them to the desired purity levels. To do this there are basically three refining processes: hydrometallurgy, pyrometallurgy, and electrolytic processes.

Hydrometallurgical processes involve the separation of the desired metal from gangue or impurities in an aqueous or liquid phase by virtue of solubility or density differences. Pyrometallurgical processes involve the separation of the desired metal from the gangue by the application of thermal energy to either volatilize off impurities or products, or else to melt and/or solubilize the materials and to separate them by solubility or density differences. Electrolytic processes involve the
solubilization or ionization of metals or impurities from partially refined metals and subsequent separation by virtue of solubility and/or electromotive differentials. Typical industrial refining operations might involve from one to all three of these processes in varying combinations and orders.

One of the pyrometallurgical processes, also called fire refining, is smelting. Smelting usually involves melting the charge materials in a refractory lined crucible and either separating the materials by volatilizing off (called fuming) the metals or impurities, or by separating the materials in the liquid phase by virtue of solubility or density differences. Fluxing and reacting chemicals are added to reduce or oxidize the metals and/or impurities and to form a slag bath. The slag bath acts as a matte or sponge to absorb the impurities or product and is separated from the metal by virtue of solubility and density differences.

There are four types of smelters which are distinguished by the manner in which the thermal energy is supplied to melt or volatilize the materials. These types are:
1. Reverbaratory - the energy is supplied by a fossil fuel burner and the heat is transferred by radiation and convection.

2. Induction - the energy is supplied by electrical induction into a charge inside of a crucible lined by induction coils.

3. Electrical Arc - the energy is supplied by an electrical arc induced by a high voltage differential between the metal layer and an electrode immersed into the charge.

4. Electrical Resistance - the energy is supplied by the resistance of the molten slag layer of the charge.

The smelter under consideration in this thesis is an electrical resistance silver smelter at Kodak Park (Eastman Kodak Company) in Rochester, N.Y. (U.S.A.). Silver rich ash derived from waste photographic products is smelted to separate the silver and impurities. The silver halide in the ash is reduced to elemental silver, which precipitates to the bottom of the smelter by virtue of its greater density. The metal and the metallic oxides in the ash are solubilized in a fluid slag consisting of excess reducing chemicals and fluxing chemicals. The charge is heated by three graphite electrodes that are immersed into the slag bath.
A schematic of the smelter is shown in figure 1.1. The smelter is a cylindrical steel vessel lined with refractory in the hearth and on the upper sidewalls. The unit is covered by a refractory brick cover through which the three electrodes are placed, the feed is introduced, and the exhaust gases are drawn out. The three graphite electrodes are lowered into the slag layer and the slag is heated by virtue of its resistance to the flow of electrical current through it. The silver, at the bottom of the slag layer, is heated by conductive heat transfer from the slag layer.

The smelting process is a continuous operation where premixed silver rich ash and fluxing and reacting chemicals are added through the top of the smelter. When the smelter becomes full, the slag is superheated to complete the reduction reaction and to thin the viscosity of the slag in order to allow as much silver to precipitate to the bottom of the smelter as possible. A refractory plug is removed from the slag taphole and the slag is poured out of the smelter into an indexing casting machine. Enough slag is left in the smelter to allow the electrodes to be immersed into the bath and the feeding process is resumed. This process is repeated a number of times until enough silver
TWO OF THREE GRAPHITE ELECTRODES CONCENTRIC TO SMELTER CENTERLINE

REFRACTORY LINED COVER

STEEL SHELL

REFRACTORY SIDEWALL

SLAG TAP HOLE AND SPOUT

REFRACTORY LINED HEARTH

FIGURE 1.1
CROSS SECTION VIEW OF ELECTRICAL RESISTANCE SMELTER
has accumulated at the bottom of the smelter to justify a silver pour. After the last slag pour, the remaining slag is superheated to bring the silver to a temperature where it is fluid enough to allow it to be underpoured from beneath the slag and not solidify in the ladle on the casting system. The smelter is tilted up to pool the silver in the corner of the refractory crucible, a refractory plug is removed, and the pour is started. At the end of the silver pour, the feeding cycle is restarted and the process is repeated.

The principle area of concern of this thesis is the temperature profile of the silver at the bottom of the smelter. The temperature of the silver layer is of primary concern due to:

1. If the silver is molten and fluid then mass transfer of impurities can take place out of the silver into the slag layer across the slag/silver interface. If the silver is solid or crusted over then mass transfer cannot take place.

2. When the slag is superheated, before a silver pour, the following occurs:

   A. the solubility of impurities in the slag and silver layers is altered potentially resulting in impurities precipitating from the slag layer into the silver layer.
B. the slag viscosity is reduced allowing for greater penetration of the slag into the refractory and subsequent greater solubilization of the refractory into the slag.

C. smelter capacity is lost as the unit is not fed during this time.

D. additional costs are incurred for the electrical energy.

3. Overheating of the slag and silver layers can result in damage to the refractory sidewalls, the hearth and the molds on the casting machine.

Presently, no method exists to give a direct indication of the temperature of the silver layer. When superheating the slag layer prior to a silver pour the operators must use their judgement based upon the following factors; the amount of residual slag in the smelter, the thickness of the refractory sidewalls, history of the smelt (downtime, power and feed levels, etc.), exhaust temperature, and slag top temperature. This often ends in abortive attempts to pour the silver which is not fluid or molten or too hot to be properly handled in the casting equipment.
Heat is transferred from the slag to the silver layer, which loses heat through the refractory hearth to the steel shell, where it is lost to the ambient environment by radiation and convection. The amount of heat lost by the silver is greatest at the junction of the refractory sidewalls and the hearth where the conductive path to the steel shell is shortest. If enough heat is lost from the silver at this point a skull of solidified material may exist at the outer radius of the silver layer that prevents it from being poured out of the smelter. This skull may exist even though the silver inside of the skull may be molten.

The layout of the refractory at the bottom of the smelter is shown in figure 1.2. The hearth is composed of three different materials:

1. Cured refractory brick - the brick is laid in a stadium design where the length of the brick is shortened at the outer radius of the dished head to match the curvature of the head.

2. Plastic-fill refractory - this formable refractory material is used to fill in the space between the brick and shell.

3. Chrome-fill refractory - this powdered refractory will allow for thermal expansion of the refractory brick and plastic.
FIGURE 1.2
DETAIL OF REFRAC TORY HEARTH OF ELECTRICAL RESISTANCE SMELTER
The type of refractory materials, their thickness, and their relative positions in the hearth can be varied to modify the temperature of the silver layer. Increasing the thickness of the materials will reduce the heat loss but will also reduce the capacity of the smelter. Installing refractory materials with lower thermal conductivities will decrease the heat loss from the shell to the environment. This will result in greater penetration of the molten silver into the pores and joints between the bricks. Insulating materials below the brick layer may drive the temperature profiles to the point where the silver may become molten at the bottom of the refractory bricks. If this happens the silver will pool, float the lower density refractory bricks, and a catastrophic failure of the hearth will result. The benefits of decreasing the heat losses therefore have to be weighed versus the effect of the problems listed above.

The intent of this thesis is to derive a model to mathematically analyze the temperature profile of the silver layer and the refractory hearth to determine the temperature profile of the silver in the present hearth design. The effect of varying the thickness of different refractories, with different thermal properties, in varying geometries could then be fed into the mathematical model to determine the optimum materials and arrangements to be employed to in order to keep the silver molten and fluid at normal smelting temperatures.
2.0 DETAILED PROBLEM DESCRIPTION

The details of the layout of materials in the hearth in the electrical resistance smelter are shown in figure 2.1. The hearth is composed of refractory materials. The one inch thick carbon steel smelter shell is lined with a chromium oxide granular refractory that serves as a filler layer and as a layer to accommodate thermal expansion of the other refractory materials in the hearth. Just above the chrome-fill refractory is a layer of 70% aluminum oxide plastic refractory. The plastic is installed in a wet clay form that is rammed into place with pneumatic hammers and is formed to receive the rectangular cured refractory bricks that sit above it. The purpose of the plastic refractory is to fill the space between this dished shaped shell and the rectangular bricks. The cured bricks are made of 85% aluminum oxide and 10% chromium oxide and are fitted into the hearth in a stadium course design to match the decreasing profile of the shell bottom and to form a flat horizontal surface on which the refractory sidewalls are built up and on which the melt is held.
FIGURE 2.1
DETAIL OF REFRACTORY HEARTH OF ELECTRICAL RESISTANCE SMELTER
The problem that arises is that at the normal smelting temperatures of 1800-2200 degrees Fahrenheit (slag top temperature) the silver is not molten or fluid. It is desirable to have the silver molten because mass transfer of impurities can take place between the silver and slag layers. Heavy metals such as antimony, lead, copper, and metallic sulfides can exist in a layer between the slag and the silver layer. Due to their density being less than the silver they may float on the silver and not be solubilized to any large extent. These metals and metallic sulfides can become trapped in the solid or semi-solid silver layer as silver precipitates from the slag. During the short heatup time before a silver pour enough time may not elapse to allow the impurities to separate from the molten silver. If the silver were always kept molten then equilibrium would be reached.

Also, the slag must be superheated to drive thermal energy down into the silver layer to completely melt it. Superheating the slag results in higher operating costs for electrical energy needed to melt the silver and a loss in capacity while the slag is superheated. In addition, superheating the slag results in a modification of the solubilities in the slag and silver layers potentially
providing for a decrease in the quality of the silver by allowing impurities to be desolubilized in the slag layer and either precipitate into or become solubilized in the silver layer. Superheating the slag layer also reduces the viscosity of the slag thus allowing greater penetration of the slag into the refractory sidewalls. This results in greater solubilization of the refractory into the slag and reduces the life of the refractory lining.

Due to the heat loss from the sidewalls and the smaller thickness of the refractory hearth at the outer radius of the smelter, the potential exists that the silver is molten at the center and solidified near the sidewalls. When the operator attempts to tap the smelter the silver cannot be poured out from the smelter due to this skull or membrane at the periphery of the silver layer. This causes abortive attempts to tap the smelter which results in a loss of capacity. Mechanical means of mixing the silver layer, such as gas injection, to melt this skull are not feasible due to the spatial constraints around the smelter.

There are two methods of keeping the silver molten at normal operating temperatures. One is to reformulate the composition of the slag so that it has similar viscosity and solubility parameters to those of the existing slag, at a temperature high enough to keep the silver at the bottom
of the smelter molten. The other is to provide enough thermal insulation in the hearth and lower sidewalls to raise the temperature of the silver above its melting point. The second method is comparatively easy to do and does not involve considerable expense while the first method would encompass a detailed, expensive program.

Modifying the hearth design does, however, involve making sure that the integrity of the hearth is maintained. Catastrophic results can occur if the hearth fails and allows the hot metal and slag to come into contact with the steel shell. The steel shell would lose its strength if heated to normal smelting temperatures and could mechanically fail. This could present a serious problem to personnel and adjoining equipment.

In a hearth such as the one under consideration the 85% aluminum oxide (alumina)- 10% chromium oxide (chromia) fire cured brick is employed because it is relatively insoluble to the slag and the silver and will provide a mechanically sound hearth. The problem, however, arises at the joints of the bricks where a mortar is used to join and fasten the bricks. These joints are the weakest portion of the hearth and will allow much easier penetration and
solubilization of the refractory by the slag and silver. At normal smelting temperatures silver and slag (primarily silver because it occupies the hearth the bulk of the time) penetrate the semi-porous firebrick and the joints until they reach the solidification point. This point is where the temperature of the refractory material is the same as the melting temperature of the silver or slag. The highly conductive silver reduces the net thermal conductivity of the brick further lowering the solidification position.

In the design of a refractory hearth it is of the utmost importance to control the position of the solidification point. In the case of the hearth under consideration, the solidification point must be above the firebrick/plastic refractory interface. If the solidification point is below the interface then molten silver will exist at the interface. Due to the density of the silver being so much greater than the density of the refractory (650 versus 180 pounds per cubic foot, respectively) the molten silver can float the refractory brick out of the hearth. The full weight and temperature of the silver layer will then be against the plastic refractory. The thermal resistivity of the plastic and chrome-fill refractories will allow the steel shell to rise to a temperature where it will soften, lose its strength, and catastrophically fail.
There are four potential design techniques for the hearth under consideration. The first is to minimize the thermal resistivity of the materials underneath the bricks so that the solidification point is above the bottom of the bricks. While this technique provides for the maximum safety in the hearth it also has the disadvantage of maximizing heat loss through the bottom of the hearth by decreasing the total thermal resistivity of the hearth.

The second method is to install materials whose thermal conductivities are lower than the existing bricks at the top of the hearth to raise the solidification point and minimize heat loss. This will minimize the flow of thermal energy from the silver, but can lead to a catastrophic failure. The refractory bricks that are used to hold the slag and silver at the top of the hearth are high density bricks that are relatively non-porous and prevent the silver from penetrating through the brick. The high density brick, however, has a relatively high thermal conductivity that maximizes heat loss. If an insulating refractory brick were used at the top of the hearth it would lead to a quick failure of the hearth. The insulating refractory materials have a relatively low density and high porosity. The silver and slag can easily
penetrate the insulating brick and either rapidly solubilize it or greatly increase its net thermal conductivity and quickly bring the solidification point to the refractory materials interface.

The third technique is to minimize heat losses from the shell to the environment by providing an insulating layer below the brick layer. This, however, will cause the solidification point to move downwards towards the interface. In the event that the solidification point descends below the bricks and the molten silver is allowed to come into contact with the insulating material it will quickly penetrate it for the reasons listed in the preceding paragraph. This can lead to a catastrophic failure of the hearth. In the preceding explanation of the potential failure of the existing hearth when the higher thermal conductivity refractory brick failed there was still a fair amount of plastic refractory left in the hearth that would keep the molten silver away from the steel shell. The failure of the steel shell would take some period of time and the smelter could be potentially shutdown and cooled before the refractory failed. In this example the silver would penetrate the insulating material so quickly that the failure of the steel shell
would occur before the smelter could be cooled down. For this reason, a backup safety lining is installed below the insulating layer that is of sufficient strength and thermal properties that in the event of a failure of the brick and insulating layers the backup lining will protect the steel shell against the slag and silver until the smelter can be shutdown. Thermocouples can be buried in the backup lining that would detect the rise in temperature of the backup lining, when the primary lining fails, and warn the operator that the smelter must be immediately shut down. This technique has the disadvantage of increasing the hearth thickness which reduces the capacity of the smelter. The advantage is that the loss of energy from the bottom of the hearth is minimized.

The fourth technique is to install a plastic monolithic lining at the bottom of the hearth. The formable plastic refractory is pneumatically rammed into the bottom of the hearth and then cured in the smelter. The resultant monolithic hearth then has no joints. Insulating materials and a backup lining can be added below the plastic refractory depending upon the thickness of the plastic and the location of the solidification point. The disadvantage of this design is the capital cost for the equipment to cure
the plastic in the hearth. Also, if the plastic is not cured correctly then moisture will remain in the plastic hearth. Due to the extremely high vapor pressure of water at normal smelting temperatures, moisture poses a serious personnel safety and equipment availability concern if it exists below the slag and silver. The advantage is a mechanically integral hearth with no joints for easy slag or silver penetration.

The potential therefore exists, as can be seen from the previous discussion, to redesign the hearth of the electric smelter in order to keep the silver layer molten and fluid at normal smelting temperatures while maintaining the integrity of the hearth.
3.0 LITERATURE SEARCH

3.0 Introduction to the Chapter

An extensive literature search was conducted to determine if any previously defined models could be employed for the smelter under consideration. No useable models were found but similar models revealed important information for the development of the new model. A three-dimensional model of a rectangular glass melting furnace, which is simpler than the smelter under consideration, revealed that the complexity of a three-dimensional model was well beyond the scope of this work. The model took over seventy minutes to run on a CDC 6500 computer. Five two-dimensional models were found in the literature of electrical resistance smelters or melting processes. The models revealed that a simulation of the electromagnetic and convective induced flow currents in the slag and metal layer was also well beyond the scope of this work. Due to the presence of the three concentric electrodes in the smelter under consideration it is not felt that a two-dimensional model of these parameters would be accurate.
The two-dimensional models, however, also revealed that, in the radial direction of the cylindrical smelter, the temperature of the slag/metal interface is essentially constant. This allows the slag and metal layers to be modeled independently of each other. Since this work is not concerned with modeling the slag layer, the silver layer and the refractory hearth will be modeled assuming the slag/silver interface to be at a constant temperature throughout.

3.1 Introduction to Mathematical Models

The mathematical models are of the temperature and the convective and electromagnetically induced flow profiles in rectangular and cylindrical smelters. One of the models is of a three-dimensional rectangular glass melter and from the complexity of the model it was determined that a three-dimensional model of the non-axisymmetric smelter under consideration would be far beyond the scope of this thesis. The two-dimensional axisymmetric models revealed that the temperature of the slag/silver interface is constant in the radial direction. This allows the silver and slag layers to be modeled independently.
3.1.1 Three Dimensional Model Including the Effect of Convective and Electromagnetic Fields

A. Abstract

Chen and Goodson\(^1\) formulated a mathematical model of the three-dimensional temperature and convective flow profiles of an electric resistance glass furnace. The model involves the numerical solution of the energy, momentum transfer and the heat generation equations. The article gives the assumptions, the defining equations and boundary conditions for the cubic furnace. Results showing plots of the temperature and convective flow profiles in the furnace are shown. The authors conclude that the results agree quantitatively with characteristics of an operating furnace, that the three dimensional model is absolutely needed, and that the major limitation on the model is the large amount of computer time needed for a solution. The major limiting assumption appears to be that the viscosity was assumed to be constant with respect to temperature.
B. **Furnace Description**

The furnace considered in the model is an electric glass melting furnace as shown in figure 3.1.1.1.

![Diagram of Electric Glass Melting Furnace](image)

**Figure 3.1.1.1 - Electric Glass Melting Furnace**

The melting end, where the raw materials are fed, is the primary concern of this model. The melting box is a 2.12 meter cube where heat is generated by the flow of electricity from electrodes, in the refractory lined walls, through the glass.

C. **Model Assumptions**

The following major assumptions are considered in the model:

1. Glass is a Newtonian fluid (experimentally verified).
2. Flow is laminar.
3. Viscous heat dissipation can be neglected.
4. The Boussineq approximation is valid (the maximum temperature variation is small compared to the reciprocal of the cubic thermal expansion coefficient).
5. The electrical resistivity of the glass is assumed to be constant.
6. Glass is optically thick; therefore, there is no radiative heat transfer in the glass.
7. All fluid properties are constant except that the buoyancy effect due to density difference is accounted for.
8. The furnace is at steady state with no glass throughput.
9. The complications caused by melting and varying input batch blanket thickness at the top surface has been neglected.

D. Equational Setup and Method of Solution

The conservation equations of mass, energy, momentum, and electric charge were formulated in terms of the system variables. The equations were then recast in vorticity transport form and expressed in terms of the nondimensional Prandtl, Grashof, and Nusselt numbers and the furnace aspect ratio as the only physical parameters. The method
is given by Aziz and Hellums\textsuperscript{2}. The resulting form yields differential and integral equations of the following form in three dimensional coordinates: voltage, energy, vorticity, hydrodynamic vector potential, velocity and power. The boundary and initial conditions for the equations are given by the authors.

The method of solution is an alternating direction implicit finite-difference method\textsuperscript{3} that is applied to the voltage, energy, vorticity and hydrodynamic vector potential equations. An approximate solution is obtained over an 11 point grid system in the glass in a finite time. The voltage equation is first solved to determine the energy input to the glass. The assumption of constant electrical resistivity allows the voltage equation to be uncoupled from the energy equation. Knowing the voltage distribution, the power equation is solved to yield the power intensity. An algorithm is solved over a finite time differential until steady state is reached. The values of the temperatures, vorticities, hydrodynamic vector potential, and the velocities are fed in at their initial values. The algorithm solves the energy equation for the temperature field and the vorticity transport equation for the velocities at all interior grid points, the
hydrodynamic vector potentials are solved knowing the vorticities, and the velocities and boundary vorticities are then calculated. The boundary and initial conditions are given in the Appendix at the back of the article.

E. Results of the Program

The outputs of the 11x11x11 grid point simulation are given in terms of graphic plots of the power intensity, hydrodynamic vector potential, velocity, and temperature fields at differing planes throughout the melter. Figure 3.1.1.2 shows the contour plot of power intensity at one of the electrode surfaces that are at two opposite walls. The pattern of intense power input at the edge of the electrodes agrees well with wear patterns on industrial smelters.

![Contour Plot of Power Intensity](image)

Figure 3.1.1.2 - Contour Plot of Power Intensity at the Vertical Surface
Figure 3.1.1.3 shows equipotential lines of the hydrodynamic vector potential at the plane 0.1 normalized units distance from and parallel to the electrodes. The core or roll is expected due to the electric current taking the path of least resistance through a central core.

Figure 3.1.1.4 shows a plot of the vector sum of the $y$ and $z$ component velocities at the plane 0.1 normalized units distance from and parallel to the electrodes. A picture of the flow profile can be obtained from this plot.
Figure 3.1.1.4 - Flow Velocity in the Y-Z Plane

Figure 3.1.1.5 shows a plot of the isotherms in the plane 0.1 normalized units from and parallel to the electrode surface.
Figure 3.1.1.5 - Isothermal Plot in the X=0.1 Plane

The authors state that the verification of the output data was not made due to a lack of data on operational furnaces but the gross quantities such as temperatures and power consumption agree well with industrial estimates and measurements.

The authors list the factors that affect the accuracy of the results. The first is the estimation of parameters and physical properties such as constant viscosity, no system throughput, and no insulating layer at the top of the melt due to unmelted glass are deemed to be the most
critical assumptions. Chen⁴ has done work on the effect of the physical properties on free convection but no work has been done to date on industrial furnaces. Data is available for the effect of temperature on the heat capacity, thermal conductivity, and viscosity of the slag employed in the smelter under consideration in this thesis. No data is available on the effect of temperature on the electrical resistivity of the slag. The second factor affecting the accuracy is the assumption that the formulation of the conservation equations and the boundary conditions were assumed to be less critical than the previous assumptions. The third is that the spatial discretion error in the numerical solution, due to the complexity of the convective flow profile, is assumed to be small compared to the first assumption.

F. Conclusions

The conclusions of Chen and Goodson are:

1. The results of the model for flow and temperature profiles agree qualitatively with operating industrial furnaces.
2. For a model with simple electrode placement, such as the melter, the three dimensional model is justified. They state that for a complex electrode placement the method offers even more justification. The smelter under consideration in this thesis offers a complex arrangement of three cylindrical electrodes placed symmetrically in a cylindrical crucible.

3. The large amount of computer time (approximately 70 minutes on a CDC 6500 computer) needed is caused by the large Prandtl numbers seen in slow viscous flow (1 to 10 poise). Chen and Goodson give a detailed explanation of the problem in the Appendix of their article. The Prandtl number for the glass slags investigated by Chen and Goodson was 1000. Since this is within the Prandtl number range for the smelter under consideration in this thesis the problem would be encountered if the slag layer was modelled.

From this article it was determined that a three-dimensional model of the smelter under consideration would be well beyond the scope of this thesis project. The smelter is much more complex than the smelter in this article because of the three cylindrical electrodes placed symmetrically into the cylindrical vessel. The problems that Chen and Goodson had with large computation times would occur due to the large Prandtl number for the slag under consideration.
3.1.2 Two-Dimensional Axisymmetric Models Considering the Effect of Convective and/or Electromagnetic Fields

A. Mathematical Model of an Electroslag Refining Unit

Dilawari and Szekely\textsuperscript{5} developed a mathematical formulation to represent the electromagnetic force field, fluid flow and heat transfer in an electroslag refining unit. The output includes plots of velocity and temperature fields in a cylindrical axisymmetric system.

An electroslag refining unit is used to melt or dissolve an electrode made of a mixture of slag and metal to separate the two phases. The model used is shown in figure 3.1.2.1.
The major differences between this model and the smelter under consideration are first that this model is axisymmetric. The ESR unit is also water cooled which will set up higher temperature and lower flow gradients than the smelter under consideration in this thesis which has a low thermal conductivity refractory liner cooled by the ambient air. The feed is introduced as ash feed and the power is supplied by graphite electrodes. The smelter power is conducted between the three electrodes and heats the slag bath by the resistance of the slag layer.
The mathematical formulation includes the following conservation equations set up in the vorticity transport form:

1. Fluid flow - the equation of continuity and the equation of motion with the effect of buoyancy and electromagnetic force fields included

2. Electromagnetic force field - Maxwell's equation

3. Heat transfer - the convective heat balance with allowance for eddy transfer, heat generation, and transport of thermal energy by droplets descending through the slag layer is solved.

The authors present the detailed formulation of the equations and the boundary equations with references to validate the assumptions.

The results are derived in terms of the flow patterns obtained for the system considering electromagnetic effects only and also considering electromagnetic and buoyancy effects together, temperature profile and isotherms, slag velocity versus current level, and overall process parameters such as melting rates.
Figures 3.1.2.2 and 3.1.2.3 show the computed streamline patterns for electromagnetic and electromagnetic and buoyant effects, respectively, with a current level of 45 KA (rms).

**Figure 3.1.2.2 - Streamline Patterns**
Electromagnetic Effects only

**Figure 3.1.2.3 - Streamline Patterns of**
Electromagnetic and Buoyancy Effects
It can be seen that considering the effect of buoyancy in figure 3.1.2.3 shows the formation of a secondary flow loop. However, the magnitude of the flow caused by the buoyancy effect is approximately one-tenth of the magnitude of the electromagnetically induced flow loop in figure 3.1.2.2.

Figure 3.1.2.4 shows the computed isotherms for the case considering only electromagnetic effects with a current level of 45 KA (rms).

Figure 3.1.2.4 - Isotherms in the Slag and Metal Layers

The large temperature differential at the outer wall is caused by the water cooled jacket on the ESR unit. It is expected that this differential would be less in the refractory lined smelter. Of interest is the fairly
constant temperature of the slag-metal interface in the slag region. The gradient at the wall would not be as high for the smelter under consideration for the reason listed above. For the model, an assumption of constant slag temperature at the slag-metal interface would be justified by this result.

The authors also present additional plots that show:

1. With decreasing current level the dominance of the electromagnetically driven fluid flow over the bouyancy induced flow is reduced

2. The ratio of effective thermal conductivity to molecular conductivity is much greater than one and the flow is turbulent

3. The effect of droplet formation in the slag pool is also demonstrated. This is not of interest, due to the graphite electrodes used in the smelter, but could be of potential interest if the ash and chemical feed to the smelter is ever modeled.

Figure 3.1.2.5 shows the effect of current on the velocity in the slag layer.
Figure 3.1.2.5 - Effect of Current Level on the Linear Velocity in the Slag

In the event that the power input to the slag layer is ever modeled, this paper brings up some useful observations. Of primary importance would be the temperature and flow profile in the delta region between the electrodes. A tremendous amount of power is fed into this small region.

B. Model of the Electroslag Welding Process

Dilawari, Szekely, and Eager\(^5\) took the computer model developed for the electroslag refining unit in the previous article\(^6\) and adapted it to the case of electroslag welding. The authors set forth the two dimensional
A mathematical model employing Maxwell's equations, the turbulent Navier-Stokes equations, and the convective heat balance equation.

Electroslag welding, which is a similar process to electroslag remelting, is used to produce relatively defect-free welding joints at fast deposition rates. The authors set forth to understand the process in terms of the relationship of the size of the weld pool, the thickness of the weld joint, the system geometry, current, voltage, and flux conductivity to obtain fundamental understanding of the process. A model of the electroslag welding process is as shown in figure 3.1.2.6.
The derivation of the governing equations are given in both cylindrical and rectangular coordinates. The equations are formulated separately for the slag pool, the metal pool, electrode, cooling jacket, and the solidified weld. The critical assumptions are: steady state conditions, all physical properties are independent of temperature (except density), flow is laminar, and the process is axisymmetric. The primary objective was to assess the effect of the electromagnetic and buoyancy force fields on the fluid flow and temperature distribution.

This analysis is only of the cylindrical case and the results for the rectangular system will not be discussed. The primary difference between this model and the model in the previous article is the distance from the electrode to the container wall. The article by Dilawari and Szekely used an electrode to crucible diameter ratio of 5 to 7, while this article has only a ratio of 1 to 10. This study would be thought to more closely simulate the delta region between the electrodes where the electrodes are close to one another while this article would simulate the space between the electrode and the refractory crucible.

Maps of the computed streamline patterns of the fluid flow and isotherms for the cylindrical system are shown in figures 3.1.2.7 and 3.1.2.8.
As in the previous article, the authors conclude that the electromagnetic forces are the dominant force for fluid flow. The lower velocities in this model can be attributed to the smaller electrode diameter to crucible diameter ratio and the lower current applied to the system (376 amps versus 45,000 amps, respectively). Figure 3.1.2.7 would show the maximum velocity of the slag to be in the range of 0.02 kg/sec.
Of interest in the given streamlined flow pattern figure is the depth of electrode penetration in the slag and how close it is to the metal layer. In this model, the electrode bottom is two-thirds of the way through the slag layer, and its minimal effect on the flow in the metal layer can be seen. The maximum velocity in the metal layer is one-fifth of that in the slag layer. With the water cooled casing it is felt that the flow in the metal layer is controlled more by buoyancy forces than by electromagnetic forces due to the higher thermal conductivity of the metal. The location of the maximum flow streamline being next to the container wall would tend to support this.

In the smelter under consideration the electrode penetrates only one-fourth to one-half the way through the slag layer and where in the model the electrode sits only 0.5 cm above the metal layer it sits 18 to 36 inches above the metal layer in the smelter. For this reason it would be thought that the effect of electromagnetic forces on fluid flow in the silver layer in the smelter would be minimal. Also, since the smelter investigated in this thesis is not water jacketed but is jacketed by a low thermal conductivity refractory the buoyancy induced currents would be expected to be orders of magnitude lower than those shown in the model.
Of interest in the isothermal plot is the temperature profile of the slag and metal layers in the vicinity of the slag/metal interface. Again the plots show fairly isothermal behavior along the interface. The non-isothermal behavior under the electrode and at the wall would not be thought to exist in the smelter for the reasons noted in the previous paragraph.

The authors state that the vast majority of the heat flow to the metal layer is by the descending metal droplets from the consumable electrodes. In the smelter under consideration droplets of silver would be precipitating from the feed ash but the electrodes are made of graphite. During the heatup time for slag and silver pours there is no silver rich ash being fed to the smelter so the flow of thermal energy to the silver layer is by conduction and convection only. This would drastically reduce the rate of heat flow to the silver layer by this theory.

C. Fluid Flow in the Electroslag Remelting Unit

This paper is concerned with the effect of fluid flow and droplet formation in the Electroslag Remelting Process (ESR). The process is the same as in article (5) in this
section. The paper is based upon visual observations of a transparent model fabricated by the author as shown in figure 3.1.2.9.

![Diagram of ESR Unit]

**Figure 3.1.2.9 - ESR Unit**

The bulk of the work is concerned with the effect of droplet formation on the ESR process. Since the smelter under consideration employs non-metallic electrodes, that part of the work will not be considered in this analysis.
If the feed of silver rich ash to the smelter is modeled, the effect of silver droplets on heat transfer and fluid flow needs to be reviewed in this article.

The author experimented with consumable and non-consumable electrodes and found the results differed sharply. The observation with consumable electrodes had agreed qualitatively with observations by Dilawari. et. al. The results from the non-consumable graphite electrodes showed that a hot spot was formed around the electrodes. In the ESR process the heat around the electrode is dissipated by the melting of the electrode. The hot spot around the graphite electrode results in the formation of convection currents in opposition to the electromagnetic currents. A turbulent region existed around the electrodes while the remaining slag layer remained largely quiescent.

The author proposes equation 3.1.2.1 to determine a qualitative value of the degree of mixing in the slag layer. The equation is derived from a pressure balance of a point just below the mixed region and in the cooler quiescent region.
\[ X = k \times (I^3) \times (1/A_1 - 1/A_2)^2 \]  

Equation 3.1.2.1

where:

\( X \) = depth of the stirred region  
\( k \) = thermal conductivity of the slag  
\( A_1 \) = electrode area  
\( A_2 \) = area of the metal layer at the bottom of the crucible  
\( I \) = current

if \( X \) is less than the diameter of the electrode, the turbulent region is localized around the electrode; and if \( X \) is equal to or greater than the diameter of the electrode, then the slag layer is essentially uniform throughout the volume.

The smelter under consideration is somewhat different than this model, with three electrodes concentrically mounted in the crucible. A mean electrode diameter formulation of the three electrodes could allow this formulation to be used.

Due to the metal layer being opaque, the author was not able to make observations of flow and temperature profiles in the metal layer. He does, however, make several observations on the entry of the metal droplets into the pool that would be of interest if the slag and liquid metal pool were simulated.
The author notes the existence of a cool boundary layer at the slag/metal interface and that the boundary seems to exist as a rigid boundary. The slag and metal layers appeared to move independently of each other. The presence of this boundary layer would support the assumption of the interface in the smelter being considered isothermal in the radial direction.

D. Temperature and Flow Profiles in an ESR Unit

This paper presents the modelling of temperature and velocity fields for an ESR unit. The Navier-Stokes equations and heat transfer equations are solved for the cylindrical axisymmetric model. A detailed derivation of the equations and boundaries are given in the article.

The calculated flow lines show the effect of electrode penetration and current levels on the system. These outputs are shown in figures 3.1.2.10 and 3.1.2.11.
Figure 3.1.2.10 - Flow Profiles With Current Level Increased by 10 and 100 Times

Figure 3.1.2.11 - Flow Profiles With Current Level Increased by 10 and 100 Times
From figure 3.1.2.10 the effect of raising the current levels by a factor of ten for the second plot and a factor of 100 for the third plot is shown. The effect of raising the currents level is to compress and accelerate the flow core. Figure 3.1.2.11 shows the effect of increasing the electrode penetration from 9 to 40 mm. It can be seen that, compared to figure 3.1.2.10, the effect is to compress the flow regime and accelerate the flow core. The flow core is also shifted to the right towards the container wall as the current level and electrode penetration are increased.

The smelter under consideration allows for variable penetration of the electrodes into the slag bath at variable voltage and current levels. If the heat transfer in the slag layer is ever simulated this article would be useful.

E. Metal Layer Growth in an ESR Unit

This paper is concerned with the growth of the metal ingot in the bottom of the ESR unit. Liquid metal is introduced into the slag layer and the solidification and
segregation of the metal in the ingot are studied to determine the effect of process parameters on metal ingot quality. This article would not be of direct use in this simulation, but would be useful if the effect of the silver precipitating through the slag layer was studied.

3.1.3 Models Considering All Layers as Solids and Considering No Effect of Convective and Electromagnetic Fields

This mathematical model is of a three electrode circular furnace. This model by Kaiser and Downing predicts the hearth temperature distribution resulting from different operations. A sketch of the hearth arrangement is shown in figure 3.1.3.1.
Figure 3.1.3.1 - Hearth Arrangement

For this model the temperature of the metal/hearth interface is assumed to be constant in the radial direction. The effect of the heat loss from the bottom of the hearth by radiation and convection is considered in the model, which sets up a gridwork in the hearth and uses a forward-differencing technique to achieve the solution. The model starts out by assuming values for the temperature distribution and iterates the solution of the equations until steady-state is achieved to yield the correct values.
The effect of temperature perturbations is then simulated by making the appropriate changes in the variables and plotting the transient temperature distribution as a function of time. One important parameter was the effect of losing the underhearth cooling fan. If the fan is lost, the steelwork (grillage) under the furnace can overheat and potentially undergo permanent or catastrophic damage. A plot of the effect of losing the fan and leaving the furnace energized is shown below. The plot reveals that the steel temperature rises above the maximum permissible temperature.

![Graph showing temperature history](image)

**Figure 3.1.3.2 - Temperature History Of Hearth**
Plots are given for different variable perturbations with different materials lining the hearth.

3.2.0. Physical Properties of Metallurgical Slags

This section of the thesis will summarize results of physical properties of metallurgical slags similar to the slags employed in the smelter under consideration.

A. Thermal Conductivity and Heat Capacity

Kishimoto, et. al. measured the thermal conductivity and specific heat of metallurgical slags by the laser flash method at temperatures from room temperature to 1750 degrees Kelvin. The slags were calcium oxide-silica-alumina and sodium oxide-silica base materials.

The data for thermal conductivity is shown in table 3.2.1. It can be seen that at low temperatures the thermal conductivity increases linearly with increasing temperature while at high temperatures the thermal conductivity
### TABLE 3.2.1 - THERMAL CONDUCTIVITY OF SLAGS SIMILAR TO THOSE USED IN THE ELECTRICAL RESISTANCE SMELTER

<table>
<thead>
<tr>
<th>Composition</th>
<th>(θ&lt;sub&gt;D&lt;/sub&gt;)</th>
<th>Thermal Conductivity (W/m·°K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T&lt;θ&lt;sub&gt;D&lt;/sub&gt;</td>
<td>T&gt;θ&lt;sub&gt;D&lt;/sub&gt;</td>
</tr>
<tr>
<td>40% Calcium Oxide</td>
<td>800</td>
<td>0.883+2.87*10&lt;sup&gt;-4&lt;/sup&gt;*T</td>
</tr>
<tr>
<td>40% Silica</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40% Alumina</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45% Calcium Oxide</td>
<td>800</td>
<td>0.828+3.03*10&lt;sup&gt;-4&lt;/sup&gt;*T</td>
</tr>
<tr>
<td>40% Silica</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% Alumina</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50% Calcium Oxide</td>
<td>830</td>
<td>0.757+2.74*10&lt;sup&gt;-4&lt;/sup&gt;*T</td>
</tr>
<tr>
<td>35% Silica</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% Alumina</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29.0% Sodium Oxide</td>
<td>650</td>
<td>1.11+1.0*10&lt;sup&gt;-4&lt;/sup&gt;*T</td>
</tr>
<tr>
<td>71.0% Silica</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.6% Sodium Oxide</td>
<td>650</td>
<td>1.08+1.0*10&lt;sup&gt;-4&lt;/sup&gt;*T</td>
</tr>
<tr>
<td>61.4% Silica</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
decreases proportionally with temperature. The thermal conductivity also increases with decreasing basicity.

The authors also reference three articles which contain the density of the slags in the liquid state by Kammel\textsuperscript{12}, Bockris\textsuperscript{13}, and Haggins\textsuperscript{14}. The author of this thesis was not able to obtain any of these articles.

B. Thermal Conductivities\textsuperscript{15}

The thermal conductivities of sodium oxide and calcium oxide based slags were measured by the hot wire method from room temperature to 1500 degrees centigrade. The results are given in graphical form in figures 3.2.1, 3.2.2, and 3.2.3.

![Figure 3.2.1 - Thermal Conductivities of Sodium Oxide-Silicate Slags](image-url)
Figure 3.2.2 Thermal Conductivity of Calcium Oxide-Aluminum Oxide-Silicate Slags

Figure 3.2.3 - Thermal Conductivity of Sodium Oxide-Silicate Slags
4.0 THEORETICAL ANALYSIS

A finite-difference analysis model was formulated to determine the flow of thermal energy through the hearth of the electrical resistance smelter. The goal of the model was to develop the temperature distribution in the hearth and the total heat loss from the smelter to the environment employing varying refractory materials and geometries in the hearth. The optimum selection of refractory materials and placement could then be determined to keep the silver layer at its highest temperature and reduce heat loss while maintaining mechanical and structural integrity in the hearth.

In order to reduce the complexity of the model, it was decided that the slag layer would not be modeled. The model would therefore consist of the silver layer, the lower refractory sidewalls, and the refractory hearth. The assumption of not modeling the slag layer is felt to be valid due to the work of Dilawari and Szekely\textsuperscript{5,7} and Dilawari, et. al.\textsuperscript{6}, referred to in the literature search that shows the metal/slag interface temperature to be constant from the axial centerline to the outer radius of a
cylindrical smelter. The effect of the large radial and axial temperature differentials in the slag layer therefore does not have an effect at the metal/slag interface. This interface temperature could then be fed into the model as a variable, but the temperature will not be derived from the effect of heating the slag layer with the electrodes.

The work of Chen and Goodson\textsuperscript{1} showed that a three-dimensional model of a simple cubic glass furnace was well beyond the scope of this thesis. The smelter under consideration in this thesis is more complex than the glass furnace that was modeled due to the presence of the three non-concentric electrodes and the need to analyze the heat loss through the refractory. The previously mentioned assumption of the slag/silver interface being a constant temperature coupled with the radial symmetry in the hearth made the decision to use a two-dimensional model valid.

It was also assumed that due to the thermal conductivity of the silver being so much greater than that of the refractory (4.0 versus 0.4 W/cm·°K, respectively), the convective heat transfer in the silver layer would be negligible. This allows the silver layer to be treated as a solid layer.
The model generated is therefore a two-dimensional heat flow analysis with the refractory and silver being considered as solids. The next step was to analyze the heat inputs and outputs to the model as seen in figure 4.1. The only heat input is from the slag/silver interface, which is at a constant temperature. The sole heat loss is from the shell to the ambient environment by convection and radiation. Axial heat transfer in the refractory sidewalls at the extension of the slag/silver interface line is assumed to be negligible in order to simplify the model.

The desired inputs and outputs to and from the model were then determined. The desired inputs are:

1. the temperature of the slag/silver interface
2. the depth of the silver layer
3. the diameter of the shell
4. the slope of the bottom head of the shell
5. the thickness of the lower refractory sidewalls
6. the position of various types of refractories in the lower sidewalls and the hearth
7. the thermal conductivity, as a function of temperature, of the various refractory materials and the silver
Macroscopic Heat Balance on the Model

- Refractory Sidewall
- Slag Layer
- Constant Temperature
- Silver Layer
- Refractory Hearth
- Slag/Silver Interface Line

Area inside dotted line is area of the model.

No heat flow in the refractory sidewalls at this line.

Heat loss to environment by radiation and convection from the shell.

Figure 4.1
8. the temperature of the air surrounding the shell
9. the temperature of the surfaces that receive radiant energy from the shell
10. the emissivity of the shell

Employing these variable inputs would allow the model to be used for the solution of the heat flow problem for various shell geometries, various depths of the silver layer, and various types, thicknesses and orientations of the refractory materials in the lower sidewalls and the hearth. The only input that will be a function of another variable is the thermal conductivity of the silver and the refractory which will be fed in as functions of the temperatures of the materials. This is necessary due to the large variation of the thermal conductivity of the refractory materials with respect to temperature. If, for instance, a solid plastic refractory were used in the hearth, then if the top of the hearth were at 2400 degrees Fahrenheit, then the bottom of the hearth would probably be in the 300 to 400 degrees Fahrenheit range. For a 90 percent alumina plastic refractory, the thermal conductivity increases from 17 to 23 BTU-inch/square foot-hr-degrees Fahrenheit between 2400 and 400 degrees fahrenheit, respectively.
The desired outputs from the program were two. The first is a description of the temperature field in the hearth, and the second is the heat flow from the shell to the environment by convection and radiation. The temperature distribution in the hearth is needed to determine where the freeze point for the metal exists.

The silver, lower sidewalls, and the hearth were then broken down into a finite-difference analysis. It was found by analyzing the model that nine distinctive types of nodes existed in the model. These various types of nodes are defined and shown in figure 4.2. A steady-state heat balance was performed on each type of node to derive the descriptive steady-state heat transfer equation for each node in the model. The form of the steady-state heat balance equation is as shown in equation 4.1.

\[
\text{Sum of the Heat Inputs} - \text{Sum of the Heat Outputs} = 0
\]

\(\quad\text{Into the Node} \quad \text{Out of the Node}\)

\[\text{Equation 4.1}\]

Where the heat flow into and out of the nodes is defined by Fourier's equation:

\[Q = -k \ast Ar \ast \frac{dT}{dR}\]

\[\text{Equation 4.2}\]
Finite Element Analysis of the Flow of Energy Through the Hearth

I. Breakdown of Types of Nodes in the System

1. Typical nodes not at an exterior boundary
2. Corner node on the centerline at the slag/silver interface
3. Nodes at the slag/silver interface not at centerline
4. Nodes on the slag/silver interface line inside of the monolithic sidewall
5. Nodes on the centerline between the slag/silver interface and the hemispherical shell
6. Node at the intersection of the centerline and the hemispherical shell
7. Nodes on the hemispherical shell between the centerline and the cylindrical shell
8. Node at the intersection of the slag/silver interface line and the cylindrical shell
9. Nodes on the cylindrical shell between the slag/silver interface line and the hemispherical shell

Figure 4.2 - Location of Nodes
The derivation of the equations for the nine nodes can be found in the Appendix of this thesis. The summary listing of all the equations for the nodes can be found on figure 4.3.

The model described by the equation is for a rectangular node of variable size where the node dimensions would be fed into the model. The major assumptions that were used in the derivation were that each node can only be made of one distinctive material and that all nodes must be of the same size.

The descriptive simultaneous equations for all of the nodes form a rectangular matrix equation of the form shown in equation 4.3

\[
\begin{array}{ccc}
\text{: COEFFICIENT : x : NODAL TERMS : = : TEMPERATURES TERMS : } \\
\text{: TERMS : TEMPERATURES TERMS : } \\
\end{array}
\]

Equation 4.3
### Summation of the Nodal Equations

<table>
<thead>
<tr>
<th>Type of Node(s)</th>
<th>$T(i+1,j)$</th>
<th>$T(i-1,j)$</th>
<th>$T(i,j+1)$</th>
<th>$T(i,j-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Nodes</td>
<td>$A_3(1+\frac{1}{2}m)K_0$</td>
<td>$A_3(1-\frac{1}{2}m)K_0$</td>
<td>$\Delta r^2K_0$</td>
<td>$\Delta r^2K_0$</td>
</tr>
<tr>
<td>Corner Node at &amp; Slag/Silver Inter.</td>
<td>$8\alpha_3K_0$</td>
<td>0</td>
<td>$\Delta r^2K_0$</td>
<td>0</td>
</tr>
<tr>
<td>Nodes at Slag/Silver Not at &amp; Slag/Silver Line</td>
<td>$A_3(1+\frac{1}{2}m)K_0$</td>
<td>$A_3(1-\frac{1}{2}m)K_0$</td>
<td>$\Delta r^2K_0$</td>
<td>0</td>
</tr>
<tr>
<td>Nodes at Slag/Silver Line</td>
<td>$8\alpha_3K_0$</td>
<td>0</td>
<td>$\Delta r^2K_0$</td>
<td>$\Delta r^2K_0$</td>
</tr>
<tr>
<td>Nodes on &amp; Between Shell and Slag/Silver Line</td>
<td>$2\alpha_3K_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Nodes on &amp; at Intersection with Bottom Shell</td>
<td>$2\alpha_3K_0$</td>
<td>0</td>
<td>0</td>
<td>$\frac{\Delta r}{2} + \Delta r^2K_0$</td>
</tr>
<tr>
<td>Nodes on Bottom Shell</td>
<td>$2\pi m b(1+\frac{1}{2}m)K_0$</td>
<td>$2\pi m a(1-\frac{1}{2}m)K_0$</td>
<td>0</td>
<td>$2\pi m \Delta r^2K_0$</td>
</tr>
<tr>
<td>Case I - Radial Face Intersection</td>
<td>0</td>
<td>$2\pi m a(1-\frac{1}{2}m)K_0$</td>
<td>$A_6 K_0$</td>
<td>$2\pi m \Delta r^2K_0$</td>
</tr>
<tr>
<td>Case II - Axial Face Intersection</td>
<td>$2\pi m b(1+\frac{1}{2}m)K_0$</td>
<td>$2\pi m a(1-\frac{1}{2}m)K_0$</td>
<td>$A_6 K_0$</td>
<td>$2\pi m \Delta r^2K_0$</td>
</tr>
<tr>
<td>Bottom Node</td>
<td>0</td>
<td>$2\pi m a(1-\frac{1}{2}m)K_0$</td>
<td>$A_6 K_0$</td>
<td>$2\pi m \Delta r^2K_0$</td>
</tr>
<tr>
<td>Top Node</td>
<td>0</td>
<td>$2\pi m a(1-\frac{1}{2}m)K_0$</td>
<td>$A_6 K_0$</td>
<td>$2\pi m \Delta r^2K_0$</td>
</tr>
<tr>
<td>Node at Intersection of Sidewall and Side Shell</td>
<td>0</td>
<td>0</td>
<td>$A_6 K_0$</td>
<td>$2\pi m \Delta r^2K_0$</td>
</tr>
<tr>
<td>Nodes on Sidewall</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Equations**

- $T(i,j) = -(A_3(1+\frac{1}{2}m)K_0 + (1-\frac{1}{2}m)K_0) + \Delta r^2(K_0 + K_0)$
- $T(i,j)^4 = 0$ (Constant)

**Figure 4.3**

**Where:**

- $K_0 = K(i+\frac{1}{2},j)$
- $K_a = K(i+\frac{1}{2},j)$
- $K = K(i,j)$
- $A_3, \ldots, A_{36}$ and $a, b, m$ are previously defined.
The only unknowns are the temperature of each node and the thermal conductivity, as a function of the node temperature, of the material in the node. The solution is also non-linear due to the presence of the temperature terms raised to the fourth power that is derived from the flow of radiant energy from the shell to the environment.

A non-linear solution of the equations would be difficult because the derivative of each equation would have to be taken. It was therefore decided to translate the solution to a linear form by employing the following algorithm. This algorithm converts the solution to linear form and uncouples the dependency between the temperature and the thermal conductivity.

1. Input the desired values of node size, smelter geometry, refractory material properties and position, depth of the silver layer, and the temperature of the air and the emissivity and the temperature of the ambient surroundings around the smelter.

2. Formulate an initial estimate of the temperature in the nodes by assuming a linear temperature drop from the slag/silver interface to the steel shell, which is assumed to be at 350 degrees Fahrenheit. The steel shell on a typical smelter will usually be in the 300 to 400 degrees Fahrenheit range.
3. Calculate the coefficient terms in the temperature matrix and the remainder terms using the initial estimate of the temperature fields in the nodes to calculate the thermal conductivity of the material in the nodes. Use the initial estimate of the temperature field to calculate the values of the coefficients having fourth powered temperature terms. This transforms the fourth powered terms into constant remainder terms. The equations are now in linear form and dependent only upon temperature.

4. Using a linear solution method, such as Newton's method, solve for the temperature field now that the coefficient and remainder terms are known.

5. Compare the values of the temperature field derived to the initial estimated values and if the difference is not less than a predetermined tolerance criterion then the algorithm needs to be reiterated from steps 2 to 6 using the last calculated values of the temperature field in place of the initially estimated values.

6. The algorithm is terminated either by the tolerance criterion being satisfied for all of the nodes or by a predetermined convergence criterion not being satisfied.

One problem that may cause the solution to diverge during the algorithm is the presence of the nodal temperatures raised to the fourth power. The value of the
remainder term associated with this quantity may be so adversely large as to affect the solution. This may present a real problem in the first few iterations, when the value used for this temperature term may be significantly different than the true value. In order to check to see if this is happening the solution should first be obtained negating the effect of radiation from the shell. This could be done by inputting a value of zero for the emissivity of the shell. The solution could then be attempted with the actual value of the emissivity to see if convergence could still be obtained. If it cannot, then the solution could be repeatedly iterated slowly increasing the value of the emissivity to its full value.
5.0 PROGRAM STRUCTURE

Using the algorithm for the solution of the temperature field in the hearth that was derived in the previous section a Fortran 77 program was written. The program contains the desired inputs and outputs listed in the previous section. The program is composed of a master program that accesses a network of 11 interwoven subroutines. This was done outside the scope of the thesis and is included as extra work.

The structure of the program is shown on figure 5.1 on the next page. An explanation of the program and subroutines is listed below:

**PROGRAM SMELTER** - This master program controls the entire program flow. It accesses the **INPUT** and **OUTPUT** subroutines and subroutine **CONTROL**, which controls the solution of the temperature distribution. Subroutine **CALC** is also called to insure that the input data is in the proper format.

**SUBROUTINE INPUT** - This subroutine reads all of the data in from a **DATA** file and stores it in common block form. The subroutine also prints out the data fed in.
FIGURE 5.1 - PROGRAM STRUCTURE

SUBROUTINE ←----------------------------→ PROGRAM ←----------------------------→ SUBROUTINE
INPUT                     ←----------------------------→ SMELTER                     ←----------------------------→ OUTPUT

INPUT DATA

V ←----------------------------→ SUBROUTINE ←----------------------------→ CALC

V ←----------------------------→ CONTROL ←----------------------------→ V

V ←----------------------------→ TINT ←----------------------------→ LIBRARY ←----------------------------→ V

V ←----------------------------→ SUBROUTINE ←----------------------------→ COEFMAT ←----------------------------→ V

V ←----------------------------→ SUBROUTINE ←----------------------------→ REFPPOS

V ←----------------------------→ COEFCAL ←----------------------------→ INTERCEPT

V ←----------------------------→ THERCON
**SUBROUTINE CALC** - This subroutine analyzes the data fed in to make sure it is in the proper format and that it satisfies certain fundamental criteria established by assumptions used in the development of the model.

**SUBROUTINE OUTPUT** - The output of this subroutine is the derived hearth temperature field and heat losses by convection and radiation.

**SUBROUTINE CONTROL** - This subroutine controls the flow of data to the subroutines that calculate:

1. the initial temperature assumption - TINT
2. the refractory material in each node - REFPOS
3. the value of the coefficient and remainder matrices - COEFMAT
4. a library subroutine that solves for the value of the temperature field - LIBRARY

Subroutine CONTROL also controls whether the solution has been achieved to a tolerance, a convergence criterion has been exceeded, or if the algorithm for the determination of the temperature field must be reiterated.
SUBROUTINE TINT - This subroutine calculates the initial value of the temperature field based on an assumption that the shell temperature is at 350 degrees Fahrenheit and that there is a linear temperature drop between the shell and the slag/silver interface.

SUBROUTINE REPPOS - This subroutine assigns a refractory or metal material to each finite difference node. This program transduces the refractory and metal position data that was fed in into a nodal form.

SUBROUTINE LIBRARY - This subroutine employs a library subroutine to calculate the value of the temperature field from the derived values of the coefficient matrix and a remainder matrix. The library subroutine is a linear solution type.

SUBROUTINE COEFMAT - This subroutine is employed to control the calculation of the coefficient and remainder matrices. This subroutine and subroutine INCEPT (which it calls) are used to identify each node as one of the nine distinctive types of nodes. Subroutine COEFCAL is then called to calculate the actual values of the coefficient and remainder terms in the heat balance equation for that type of node. Once called from subroutine CONTROL, this
subroutines controls the calculation of all the coefficient and remainder terms for the model before returning to subroutine CONTROL.

**Subroutine INTCEPT** - This subroutine calculates the intercept points of the nodal system and the bottom shell or head of the smelter.

**SUBROUTINE COEFCAL** - This subroutine calculates the value of the coefficient and remainder terms in the descriptive heat transfer equation for each type of node. The subroutine receives a node identifier from subroutine COEFMAT that guides the subroutine to the correct calculation for the node type under consideration. Subroutine THERCON is called to calculate the value of the net thermal conductivity of this node and the adjacent nodes. These values are needed for the calculation of the coefficient and remainder terms. As each node is identified in subroutine COEFMAT, this subroutine is called to determine the terms for that node and then returns to subroutine COEFMAT. This subroutine also calculates the value of the convective and radiative heat transfer from the nodes on the shell boundary to the environment. The total heat flows are summed up in subroutine COEFMAT.
SUBROUTINE THERCON - This subroutine calculates the values of the thermal conductivity, as a function of node temperature, for the node under consideration and for that of the the adjacent nodes. The net thermal conductivity of each pair of adjacent nodes is then calculated and transferred back to subroutine COEFCAL. The thermal conductivity is calculated from a n\text{th} order polynomial equation of whose coefficients were fed into subroutine INPUT.

All of the subroutines except CALC and OUTPUT have been written, typed into the computer and compiled. A listing of each of the programs and subroutines is in the Appendix. Subroutines Input and Tint have been run and correct output values obtained. These are listed with the subroutines in the Appendix.

Each of the programs or subroutines is written in a structured programming format including the following:

1. Program name
2. Author and date written
3. Purpose and function
4. Form of call statement
5. Explanation of input variables
6. Explanation of derived output variables
7. Call statement
8. Implicit, real, integer, and common statements
9. Body of program, including common statements at every distinct point

10. Write statements to print out internal calculations - the signals for these write statements are fed in through the input file.

The work remaining on the program is as follows:

1. Write subroutines CALC and OUTPUT
2. Link and debug the program
3. Run the program employing variations of the input data
From the work completed to date on deriving the nodal equations and writing and compiling the bulk of the program, it appears that solution of the problem by the model is feasible if the program converges. It is therefore recommended that another student be given the charge to complete the model by finishing the computer program, debugging the program, and running the program with various combinations of input variables. The benefits of improved silver quality, increased smelter throughput, and decreased operating costs would appear to far outweigh the cost of completing this thesis.

In addition, if financial and personnel resources allow, it may be advantageous to extend the mathematical analysis to include the slag layer. From the information gained in the literature search, an evaluation considering the effect of convective currents in the slag layer induced by density and electromotive differentials would be feasible. This analysis would, however, be extremely time consuming and costly and the justifications for it would have to be derived.
7.0 REFERENCES


2. Aziz, K and Hellums, J.D. Physics Fluids, 1967, 10 (2), 314


4. Chen, T.S. PhD Thesis - Purdue University, 1971


8.0 APPENDIX

8.1.0 Derivation of Nodal Equations

8.2.0 Thermal Conductivity of Silver

8.3.0 Listing of Computer Programs

8.3.1 Program Smelter
8.3.2 Subroutine Input
8.3.3 Subroutine Tint
8.3.4 Subroutine Refpos
8.3.5 Subroutine Control
8.3.6 Subroutine Coefmat
8.3.7 Subroutine Intcept
8.3.8 Subroutine Coefcal
8.3.9 Subroutine Thercon
8.1.0 Derivation of Nodal Equations
Finite Element Analysis of the Flow of Energy Through the Hearth

I. Breakdown of Types of Nodes in the System

1. Typical nodes not at an exterior boundary
2. Corner node on the centerline at the slag/silver interface
3. Nodes at the slag/silver interface not at centerline
4. Nodes on the slag/silver interface line inside of the monolithic sidewall
5. Nodes on the centerline between the slag/silver interface and the hemispherical shell
6. Node at the intersection of the centerline and the hemispherical shell
7. Nodes on the hemispherical shell between the centerline and the cylindrical shell
8. Node at the intersection of the slag/silver interface line and the cylindrical shell
9. Nodes on the cylindrical shell between the slag/silver interface line and the hemispherical shell

Figure 1 - Location of Nodes
II. CRITICAL ASSUMPTIONS USED IN THE MODEL

1. The arrangement of the hearth is axisymmetric so the flow of energy in the circumferential direction is neglected.

2. The silver and the refractory are optically black so the effect of radiative conductivity is neglected.

3. Due to the high thermal conductivity of the silver, the effect of convection in the silver layer is neglected.

4. The temperature of the slag/silver interface is considered to be constant in the radial direction.

5. There is no energy transfer to the monolithic sidewall at the slag/silver interface line.

6. A single node can only be composed of one distinct material.

7. All of the nodes in the system are of constant size $\Delta r$ by $\Delta z$. 


II. Explanation of the Nodal System

Due to circumferential symmetry the region under consideration is broken up into circular disks of size $\Delta r$ by $\Delta \phi$. The first disk is centered around the smelter centerline. The other disks radiate outward concentric around the centerline.

The coordinate $(i,j)$ refers to the position of the centerline of the disk with respect to the axial and radial axis, as shown below:

![Cut-out section of the disk](image)

Certain assumptions are made to simplify the model complexity:

1. \( \frac{x_s}{\Delta r} \), \( \frac{R_0}{\Delta r} \), \( \frac{\phi_s}{\Delta \phi} \), \( \frac{\phi_h}{\Delta \phi} \) must be whole numbers.

2. \( M \leq \frac{2\Delta r}{\Delta \phi} \) This will be explained later.

Figure 2 - Nodal System
IV. Finite Element Analysis of Each Distinct Type of Node

1. Typical Nodes not at an Exterior Boundary

For steady state heat conduction a heat balance on the node would be:

\[ \sum \text{Heat Inflow} + \sum \text{Heat Outflow} = 0 \]

or

\[ Q(i,j) + Q(i+1,j) + Q(i-1,j) + Q(i,j) + Q(i,j+1) + Q(i,j-1) = 0 \]

Note: For ease of visual analysis only a portion of the disk is shown.
NOW DESCRIBING THE FLOW OF HEAT IN THE TYPICAL ELEMENT - REFER BACK TO FIGURE 3 ON THE PAGE BEFORE LAST.

A. HEAT FLOWS IN THE RADIAL DIRECTION

From Fourier's Equation

\[ Q = -kA_r \frac{dT}{dr} \]

Considering the area of a ring and integrating the differential equation results in

\[ Q = -kA_r \frac{dT}{dr} \]

WHERE IN THIS CASE THE AREA IS EQUAL TO

\[ A_r = \frac{2\pi (m \Delta r + \frac{1}{2} \Delta r) \Delta z}{\Delta r} = 2\pi m (1 + \frac{1}{2m}) \Delta z \]

THEREFORE

\[ Q(i \pm 1, j) \to (i, j) = k(i \pm \frac{1}{2}, j) 2\pi m \Delta z (1 + \frac{1}{2m}) (T(i \pm 1, j) - T(i, j)) \]
Before describing each of the heat flows in detail the conventions used in the model are:

1. Node Identifiers - The position of the node in the radial and azimuthal directions are described as \((i,j)\) where \(i\) is the coordinate in the radial direction and \(j\) the coordinate in the azimuthal direction.

2. Heat Flow - The flow of heat is shown as \(Q(A,B) \rightarrow (C,D)\) where \((A,B)\) is the node where the heat is flowing from and \((C,D)\) is the coordinates of the node where the heat is flowing to.

3. Thermal Conductivity - The net thermal conductivity describing the two nodes transferring heat to/from each other will be described as \(K\left(i \pm \frac{1}{2}, j \pm \frac{1}{2}\right)\). If the two nodes under consideration are the \((i+1,j)\) and \((i,j)\) node then the convention used would be \(K(i+1/2,j)\) and so on for the other nodes. The net thermal conductivity will be the series conductivity of the two materials in the two nodes and will be:

\[
\frac{1}{K(i \pm \frac{1}{2}, j \pm \frac{1}{2})} = \frac{K \text{ of node } (i,j) + K \text{ of node } (i \pm 1, j \pm 1)}{(K \text{ of node } (i,j))(K \text{ of node } (i \pm 1, j \pm 1))}
\]

4. Temperature - \(T(A,B)\) describes the temperature of the node at coordinate \((A,B)\).

5. Node Location - The distance from the axis will be defined by \(N(AR)\) and \(M(AR)\) for the radial and axial directions where \(N\) and \(M\) are counters from 0 to the maximum number of node spacings.
B. Heat Flows in the Axial Direction

From Fourier's Equation as Before

\[ Q = -kA_T \frac{dT}{dr} \]

But in this case

\[ A_T = \pi \left[ \left( m \Delta r + \frac{1}{2} \Delta r \right)^2 - \left( m \Delta r - \frac{1}{2} \Delta r \right)^2 \right] \]
\[ A_T = \pi \left[ m^2 \Delta r^2 + m \Delta r^2 + \frac{1}{4} \Delta r^2 - m^2 \Delta r^2 + m \Delta r^2 - \frac{1}{4} \Delta r^2 \right] \]
\[ A_T = 2\pi m \Delta r^2 \]

Therefore

\[ Q(i,j \pm 1) \to (i,j) = k(i,j \pm 1/2) 2\pi m \Delta r^2 \left( T(i,j \pm 1) - T(i,j) \right) \]

C. Heat Flows in the Circumferential Direction (\( \phi \))

\[ Q_{\phi_1} = Q_{\phi_2} = 0 \] Due to the assumption that the smelter is axisymmetric
Now summing up the heat flows for the typical nodes

Heat input in the radial direction + heat input in the axial direction
+ heat input in the circumferential direction = 0

\[ Q(i-1,j) \rightarrow (i,j) + Q(i+1,j) \rightarrow (i,j) + Q(i,j-1) \rightarrow (i,j) + Q(i,j+1) \rightarrow (i,j) + Q\phi_1 + Q\phi_2 = 0 \]

\[ K(i-\frac{1}{2},j) 2\pi h A_3 (1-\frac{1}{2m})(T(i-1,j)-T(i,j)) + K(i+\frac{1}{2},j) 2\pi h A_3 (1+\frac{1}{2m})(T(i+1,j)-T(i,j)) \]
\[ + K(i,j-\frac{1}{2}) 2\pi h A^2 (T(i,j-1)-T(i,j)) + K(i,j+\frac{1}{2}) 2\pi h A^2 (T(i,j+1)-T(i,j)) \]
\[ + 0 + 0 = 0 \]

Dividing through by \(2\pi h\) and rearranging the coefficient terms with respect to the five temperature terms yields:

\[ A_3 (1-\frac{1}{2m}) K(i-\frac{1}{2},j) T(i-1,j) + A_3 (1+\frac{1}{2m}) K(i+\frac{1}{2},j) T(i+1,j) + \Delta r^2 K(i,j-\frac{1}{2}) T(i,j-1) + \Delta r^2 K(i,j+\frac{1}{2}) T(i,j+1) \]
\[ - (A_3 (1-\frac{1}{2m}) K(i-\frac{1}{2},j) + (1+\frac{1}{2m}) K(i+\frac{1}{2},j)) + \Delta r^2 (K(i,j-\frac{1}{2}) + K(i,j+\frac{1}{2})) T(i,j) = 0 \]

Which is the equation describing the heat flow into the typical nodes at position \((i,j)\)
Note - For the remaining 8 types of nodes the equation derivation will not be given due to their being similar to the typical nodes.

2. Corner Node on the Centerline at the Slag/Silver Interface

This is the node at location \((i, j)\)

A. Heat Flows in the Radial \(Q(i \pm 1, j) \rightarrow (i, j)\)

Because the first node is concentric with the smelter centerline

\(Q(i-1, j) \rightarrow (i, j)\) does not exist

\(Q(i+1, j) \rightarrow (i, j) = -KA_T \frac{dT}{dr}\)

Where \(A_T = 2\pi \Delta r \Delta z\)

Therefore

\(Q(i+1, j) \rightarrow (i, j) = K(i+\frac{1}{2}, j) 2\pi \Delta r \Delta z (T(i+1, j) - T(i, j))\)
B. Heat Flows in the Axial Direction

\[ Q(i,j+1) \rightarrow (i,j) \]

\[ Q(i,j+1) \rightarrow (i,j) = k(i,j+\frac{1}{2}) \pi (\frac{\Delta r}{2})^2 (T(i,j+1) - T(i,j)) \]

For the flow of heat from above the disk downward the following assumptions are taken:

1. The slag/silver interface temperature is \( T_s \)

2. The net thermal conductivity between the slag and this node is assumed to be the value of solely of the node material

Therefore:

\[ Q(i,j-1) \rightarrow (i,j) = k(i,j) \pi (\frac{\Delta r}{2})^2 (T_s - T(i,j)) \]

Summing the heat flows, equating to zero, dividing by \( \Delta r \), and rearranging the equation yields:

\[ Bk(i+\frac{1}{2},j) \Delta r A^2 \left( T(i+1,j) + k(i,j) \Delta r^2 T_s + k(i,j+\frac{1}{2}) \Delta r^2 T(i,j+1) \right) \]

\[ - (Bk(i+\frac{1}{2},j) \Delta r A^2 + \Delta r^2 (k(i,j) + k(i,j+\frac{1}{2}))) T(i,j)) = 0 \]
3. NODES AT THE SLAG/SILVER INTERFACE NOT AT CENTERLINE

USING PREVIOUSLY DEFINED EQUATIONS

\[ Q(i \pm 1,j) \rightarrow (i,j) = K(i \pm \frac{1}{2},j) 2\pi m \Delta z (1 \pm \frac{1}{2m}) (T(i \pm 1,j) - T(i,j)) \]

\[ Q(i,j-1) \rightarrow (i,j) = K(i,j) 2\pi m \Delta r^2 (T_2 - T(i,j)) \]

\[ Q(i,j+1) \rightarrow (i,j) = K(i,j + \frac{1}{2}) 2\pi m \Delta r^2 (T(i,j+1) - T(i,j)) \]

SUMMING THE HEAT FLOWS, EQUATING TO ZERO, DIVIDING BY \(2\pi m\) AND REARRANGING THE EQUATION YIELDS:

\[ K(i-\frac{1}{2},j) \Delta z (1-\frac{1}{2m}) T(i-1,j) + K(i+\frac{1}{2},j) \Delta z (1+\frac{1}{2m}) T(i+1,j) + K(i,j) \Delta r^2 T_2 + K(i,j + \frac{1}{2}) \Delta r^2 T(i,j+1) \]

\[-(\Delta z (K(i-\frac{1}{2},j)(1-\frac{1}{2m}) + K(i+\frac{1}{2},j)(1+\frac{1}{2m})) + \Delta r^2 (K(i,j) + K(i,j + \frac{1}{2})) T(i,j)) \]

\[ = 0 \]
4. NODES ON THE SLAG/SILVER INTERFACE LINE INSIDE OF THE MONOLITHIC SIDEWALL

USING PREVIOUSLY DEFINED EQUATIONS

\[ Q(i \pm 1,j) \Rightarrow (i,j) = K(i \pm 1,j) \Delta y \Delta z (1 \pm \frac{1}{2m})(T(i \pm 1,j) - T(i,j)) \]

\[ Q(i,j+1) \Rightarrow (i,j) = K(i,j + 1) \Delta y \Delta z \Delta r^2 (T(i,j+1) - T(i,j)) \]

\[ Q(i,j-1) \Rightarrow (i,j) \] IT IS ASSUMED THAT THERE IS NO AXIAL TRANSFER OF HEAT IN THE REFRACTORY SIDEWALLS AT THE SLAG/SILVER INTERFACE AND THEREFORE THIS QUANTITY IS EQUAL TO ZERO.

SUMMING THE HEAT FLOWS, EQUATING TO ZERO, DIVIDING BY \(2\pi m\) AND REARRANGING YIELDS:

\[
K(i-\frac{1}{2},j) \Delta z (1 - \frac{1}{2m}) T(i-\frac{1}{2},j) + K(i+\frac{1}{2},j) \Delta z (1 + \frac{1}{2m}) T(i+\frac{1}{2},j) \\
+ K(i,j+\frac{1}{2}) \Delta r^2 T(i,j+1) \\
- (\Delta z (K(i-\frac{1}{2},j)(1 - \frac{1}{2m})+K(i+\frac{1}{2},j)(1 + \frac{1}{2m})+K(i,j+\frac{1}{2})\Delta r^2) T(i,j) = 0
\]
NODES ON THE CENTERLINE BETWEEN THE SLAG/SILVER INTERFACE AND THE HEMISPHERICAL SHELL

USING PREVIOUSLY DEFINED EQUATIONS

\[ Q(i+1,j) \Rightarrow (i,j) = k(i+\frac{1}{2},j)2\pi r_3 \Delta z (T(i+1,j) - T(i,j)) \]
\[ Q(i-1,j) \Rightarrow (i,j) \text{ DOES NOT EXIST} \]
\[ Q(i,j+1) \Rightarrow (i,j) = k(i,j+\frac{1}{2}) \pi (\Delta r)^2 (T(i,j+1) - T(i,j)) \]

SUMMING THE HEAT FLOWS, EQUATING TO ZERO, DIVIDING BY \( \Delta t \) YIELDS:

\[ 8k(i+\frac{1}{2},j)\pi r_3 T(i+1,j) + k(i,j-\frac{1}{2})\Delta r^2 T(i,j-1) \]
\[ + k(i,j+\frac{1}{2})\Delta r^2 T(i,j+1) \]
\[ - (8k(i+\frac{1}{2},j) + \Delta r^2 (k(i,j-\frac{1}{2}) + k(i,j+\frac{1}{2}))) T(i,j) = 0 \]
Nodes 6, 7, 8, 9 all lie along the outer shell of the smelter. The balance of heat flows to/from these nodes involves the flow of heat to the surrounding environment by convection and conduction. For this equation derivation the fundamental relationships shown below will be used:

**CONVECTION**

\[ Q = (h_{c_s} \text{ or } h_{c_c}) A_s (T_{\text{node}} - T_{\text{air}}) \]

**WHERE**

- \( h_{c_s} \) = convective heat transfer coefficient from a sphere
- \( h_{c_c} \) = convective heat transfer coefficient from a cylinder
- \( A_s \) = area of the node exposed to the heat flow
- \( T_{\text{air}} \) = temperature of the surrounding air

**RADIATION**

\[ Q = \varepsilon \sigma A_s ((T_{\text{node}})^4 - (T_{\text{sur}})^4) \]

**WHERE**

- \( \varepsilon \) = emissivity of the node
- \( \sigma \) = Stephan-Boltzmann constant
- \( A_s \) = area of the node exposed to the heat flow
- \( T_{\text{sur}} \) = temperature of the surfaces to be radiated to

The correlations for \( h_{c_s}, h_{c_c} \), and \( \varepsilon \) will not be given here.
NOTE: NODES 6 AND 7 ARE ALONG THE BOTTOM HEAD OF THE SMELTER. THIS MODEL ASSUMES THAT THE BOTTOM HEAD IS OF PYRAMIDAL SHAPE AND NOT HEMISPHERICAL SHAPE.

NOTE: IT IS ASSUMED THAT THE AFFECT OF HEAT TRANSFER ACROSS THE SHELL IS NEGLIGIBLE AND THEREFORE THE PRESENCE OF THE SHELL IS IGNORED

6. NODE AT THE INTERSECTION OF THE CENTERLINE AND THE HEMISPHERICAL HEAD

1. $Q_R$ AND $Q_c$ ARE THE RADIATIVE AND CONVECTIVE HEAT FLOWS TO THE ENVIRONMENT

2. THE SLOPE OF THE STEEL SHELL IS LIMITED, FOR SAKE OF SIMPLICITY IN THE MODEL, SO THAT THE SHELL DOES NOT INTERSECT THE TOP OF THE NODE. THEREFORE IF THE SLOPE IS $M$

   $$M \leq \frac{\Delta z}{\Delta r} \text{ OR } M \leq \frac{2\Delta z}{\Delta r}$$

3. IN ORDER FOR THIS NODE TO BE OF AXIAL LENGTH $\Delta z$ AT THE CENTERLINE THEN THE HEIGHT OF THE SILVER LAYER PLUS THE HEIGHT OF SLAG LAYER DIVIDED BY $\Delta z$ MUST BE A WHOLE NUMBER.

   $Q(i+1,j) \rightarrow (i,j)$
Using previously defined equations

\[ Q(i+1,j) \Rightarrow (i,j) = k(i + \frac{1}{2}, j) 2\pi \Delta r \Delta z \left( T(i+1,j) - T(i,j) \right) \]

\[ Q(i,j-1) \Rightarrow (i,j) = k(i,j - \frac{1}{2}) \pi \left( \frac{\Delta r}{2} \right)^2 \left( T(i,j-1) - T(i,j) \right) \]

\[ Q_R = \varepsilon \sigma A_S_1 \left( T(i,j)^4 - T_{\text{sur}}^4 \right) \]

\[ Q_c = h_c A_S_1 \left( T(i,j) - T_{\text{air}} \right) \]

Summing the equations and equating to zero yields:

\[ k(i + \frac{1}{2}, j) 2\pi \Delta r \Delta z T(i+1,j) + k(i,j - \frac{1}{2}) \pi \Delta r^2 T(i,j-1) \]

\[ + \varepsilon \sigma A_S_1 T_{\text{sur}}^4 + h_c A_S_1 T_{\text{air}} \]

\[ - \left( k(i + \frac{1}{2}, j) 2\pi \Delta r \Delta z + k(i,j - \frac{1}{2}) \pi \Delta r^2 + h_c A_S_1 \right) T(i,j) \]

\[ - \varepsilon \sigma A_S_1 T(i,j)^4 = 0 \]

Now to derive the value of \( A_S_1 \) (area of the node exposed to convective and radiative heat flow)
The surface area of a right circular cone (A) is:

\[ A = \pi R \sqrt{R^2 + h^2} \]

where:

\[ h = \Delta z \]
\[ R = \frac{\Delta r}{2} \]

therefore:

\[ A = \frac{\pi \Delta r}{2} \sqrt{\frac{\Delta r^2}{4} + \Delta z^2} \]

7. Nodes on the hemispherical shell between the centerline and the cylindrical sidewall.

Note: There are two cases of these nodes. The first is when the shell intersects the outer axial face of the node and the second is where the shell intersects the top radial face of the node. These two cases will be separately analyzed.
CASE I - THE SHELL INTERSECTS THE OUTSIDE AXIAL FACE OF THE NODE

USING PREVIOUSLY DEFINED EQUATIONS

\[ Q(i,j) - Q(i, j-1) = K(i, j, j-\frac{1}{2}) 2\pi m \Delta r^2 (T(i, j) - T(i, j-1)) \]

\[ Q(i+1, j) - Q(i, j) = K(i+\frac{1}{2}, j) 2\pi m b (1 + \frac{1}{2m}) (T(i+1, j) - T(i, j)) \]

\[ Q(i, j-1) - Q(i, j) = K(i-\frac{1}{2}, j) 2\pi m a (1 - \frac{1}{2m}) (T(i, j-1) - T(i, j)) \]

\[ Q_R = \varepsilon G A_{S_2} (T(i, j)^4 - T_{sur}^4) \]

\[ Q_C = h_{cs} A_{S_2} (T(i, j) - T_{Air}) \]

SUMMING THE EQUATIONS AND EQUATING TO ZERO YIELDS

\[ K(i, j, j-\frac{1}{2}) 2\pi m \Delta r^2 T(i, j-1) + K(i+\frac{1}{2}, j) 2\pi m b (1 + \frac{1}{2m}) T(i+1, j) + K(i-\frac{1}{2}, j) 2\pi m a (1 - \frac{1}{2m}) T(i, j-1) + \varepsilon G A_{S_2} T_{sur}^4 + h_{cs} A_{S_2} T_{Air} \]

\[ - (2\pi m (K(i, j, j-\frac{1}{2}) \Delta r^2 + K(i+\frac{1}{2}, j) b (1 + \frac{1}{2m}) + K(i-\frac{1}{2}, j) a (1 - \frac{1}{2m})) + h_{cs} A_{S_2}) T(i, j) = \varepsilon G A_{S_2} T(i, j)^4 = 0 \]

WHERE \[ A_{S_2} = \pi R_1 \sqrt{R_1^2 + h_1^2} - \pi R_2 \sqrt{R_2^2 + h_2^2} \]

WHERE \[ R_1 = m \Delta r + \frac{1}{2} \Delta r \]

\[ R_2 = m \Delta r - \frac{1}{2} \Delta r \]

\[ h_1, h_2 = \text{AXIAL HEIGHT FROM THE BOTTOM CENTERLINE OF THE SHELF TO THE OUTER AND INNER EDGES OF THE BOTTOM RADIAL FACE OF THE NODE} \]
CASE II - THE SHELL INTERSECTS THE TOP RADIAL FACE OF THE NODE

AS CAN BE SEEN FROM THE FIGURE TO THE RIGHT
THERE ARE TWO SEPARATE NODES THAT MUST BE ANALYZED
AND EQUATIONS DERIVED FOR THE HEAT FLOWS. THESE EQUATIONS
WILL BE DERIVED ON THE FOLLOWING PAGES.

THE ASSUMPTION USED FOR NODE NO. 6 - THAT
\[ M \leq \frac{2\Delta z}{\Delta r} \]
LIMITS THE STEEL SHELL FROM BEING
ABLE TO INTERSECT THE TOP RADIAL FACE OF
TWO NODES STACKED ON TOP OF EACH OTHER.
FROM PREVIOUSLY DEFINED EQUATIONS

\[ Q(i-1,j) \rightarrow (i,j) = k(i-\frac{1}{2},j) \times 2\pi ma_i (1-\frac{1}{2}m) \times (T(i-1,j) - T(i,j)) \]

\[ Q_r = \varepsilon \sigma A_3 \left[ T(i,j)^4 - T_{sur}^4 \right] \]

\[ Q_c = h_c A_3 (T(i,j) - T_{air}) \]

\[ Q(i,j-1) \rightarrow (i,j) = k(i,j-\frac{1}{2})A_4 \times (T(i,j-1) - T(i,j)) \]

**SUMMING THE EQUATIONS AND EQUATING TO ZERO YIELDS**

\[ k(i-\frac{1}{2},j) \times 2\pi ma_i (1-\frac{1}{2}m) \times (T(i,j)) + k(i,j-\frac{1}{2})A_4 \times (T(i,j-1)) + \varepsilon \sigma A_3 \times T_{sur}^4 + h_c A_3 \times T_{air} \times 4 \]

\[ - (k(i-\frac{1}{2},j) \times 2\pi ma_i (1-\frac{1}{2}m) + k(i,j-\frac{1}{2})A_4 \times (h_c A_3)) \times (T(i,j)) - \varepsilon \sigma A_3 \times T(i,j)^4 = 0 \]

WHERE

\[ A_3 = \pi R_1 \sqrt{R_1^2 + h_1^2} - \pi R_2 \sqrt{R_2^2 + h_2^2} \]

\[ R_1 = m \Delta R - \frac{\Delta r}{2} + c \]

\[ R_2 = m \Delta R - \frac{1}{2} \Delta r \]

\[ h_1, h_2 = \text{AXIAL HEIGHT FROM THE BOTTOM CENTERLINE OF THE SMELTER TO THE OUTER AND INNER EDGES OF THE BOTTOM RADIAL FACE OF THE NODE} \]

\[ A_4 = \pi \left( (m \Delta r - \frac{\Delta r}{2} + c)^2 - (m \Delta r - \frac{1}{2} \Delta r)^2 \right) \]
TOP NODE

FROM PREVIOUSLY DEFINED EQUATIONS

\[ \begin{align*}
Q(i+1,j) \rightarrow (i,j) &= K(i,i,j)2\pi m b_n(l+\frac{1}{2}m)\left(T(i+1,j)-T(i,j)\right) \\
Q(i-1,j) \rightarrow (i,j) &= K(i,i,j)2\pi m a_3(l-\frac{1}{2}m)\left(T(i+1,j)-T(i,j)\right) \\
Q(i,j+1) \rightarrow (i,j) &= K(i,j+1)A_3\left(T(i,j+1)-T(i,j)\right) \\
Q(i,j-1) \rightarrow (i,j) &= K(i,j-1)2\pi m a_4\left(T(i,j+1)-T(i,j)\right) \\
Q_R &= E\sigma A_3\left(T(i,j) - T_{air}\right) \\
Q_C &= h c_s A_5\left(T(i,j) - T_{air}\right)
\end{align*} \]

SUMMING THE EQUATIONS AND EQUATING TO ZERO YIELDS:

\[ \begin{align*}
K(i,i,j)2\pi mb_n(l+\frac{1}{2}m)\left(T(i+1,j)+K(i,i;j)2\pi m a_3(l-\frac{1}{2}m)\right) T(i-1,j) \\
&+ K(i,j+1)A_3\left(T(i,j+1) + K(i,i;j)2\pi m a_4\right) T(i,j-1) \\
&+ E\sigma A_3\left(T_{air} - T_{air}\right) \\
&\quad - \left(K(i+1,j)2\pi m b_n(l+\frac{1}{2}m) + K(i-1,j)2\pi m a_3(l-\frac{1}{2}m) + K(i,j+1)A_3 + K(i,j-1)2\pi m a_4 + h c_s A_5\right) T(i,j) = E\sigma A_3 T(i,j) = 0
\end{align*} \]

WHERE

\[ \begin{align*}
A_3 &= \pi \left((ma - \frac{1}{2}m a)^2 - (ma - \frac{1}{2}ma)^2\right) \\
A_5 &= \pi R_1 \sqrt{R_1^2 + h_1^2} - \pi R_2 \sqrt{R_2^2 + h_2^2} \\
R_1 &= ma + \frac{1}{2}ma + c \\
R_2 &= ma + \frac{1}{2}ma - c \\
h_1, h_2 &= AXIAL HEIGHT FROM THE BOTTOM CENTERLINE OF THE SMELTER TO THE OUTER AND INNER EDGES OF THE FACE
\end{align*} \]
8. NODE AT THE INTERSECTION OF THE SLUG/SILVER INTERFACE
LINE AND THE CYLINDRICAL SHELL

USING PREVIOUSLY DEFINED EQUATIONS

\[ Q(i-1,j) \rightarrow (i,j) = \frac{K(i-\frac{1}{2},j)}{2\pi m} \Delta \Omega \left( 1 - \frac{1}{2m} \right) (T(i-1,j) - T(i,j)) \]
\[ Q(i,j+1) \rightarrow (i,j) = \frac{K(i,j+\frac{1}{2})}{2\pi m} \Delta \Omega \left( T(i,j+1) - T(i,j) \right) \]
\[ Q(i,j-1) \rightarrow (i,j) \]

IT IS ASSUMED THAT THERE IS NO AXIAL
TRANSFER OF HEAT IN THE REFRACTORY SIDEWALL AT THE
SLUG/SILVER INTERFACE AND THEREFORE THIS QUANTITY
IS EQUAL TO ZERO

\[ Q_r = EGA_{s6} (T(i,j) - T_{sur}) \]
\[ Q_c = h_c A_{s6} (T(i,j) - T_{air}) \]

SUMMING THE HEAT FLOWS AND EQUATING TO ZERO

\[ K(i-\frac{1}{2},j) \frac{2\pi m \Delta \Omega}{2} \left( 1 - \frac{1}{2m} \right) T(i-1,j) + K(i,j+\frac{1}{2}) \frac{2\pi m \Delta \Omega}{2} T(i,j+1) \]
\[ + EGA_{s6} T_{sur} \Delta \Omega + EGA_{s6} T_{air} \]
\[ - \left( 2\pi m \Delta \Omega K(i-\frac{1}{2},j) \frac{1}{2m} + K(i,j+\frac{1}{2}) \frac{2\pi m \Delta \Omega}{2} + h_c A_{s6} \right) T(i,j) \]
\[ - EGA_{s6} T(i,j) \Delta \Omega = 0 \]

WHERE \[ A_{s6} = 2\pi R_0 \Delta \Omega \]
WHERE \[ R_0 = \text{smelter outside diameter} \]

IN ORDER FOR THIS NODE TO BE OF RADIAL LENGTH \( \Delta r \) THEN THE OUTSIDE DIAMETER DIVIDED BY
\( \Delta r \) MUST BE A WHOLE NUMBER.
Nodes on the cylindrical shell between the slag/silver interface line and the hemispherical shell

Using previously defined equations

\[ Q(i-1,j) \Rightarrow (i,j) = K(i-\frac{1}{2},j) \Delta z \left( \frac{1}{2\pi} \right) (T(i-1,j) - T(i,j)) \]
\[ Q(i,j-1) \Rightarrow (i,j) = K(i,j-\frac{1}{2}) 2\pi m \Delta r^2 \left( T(i,j-1) - T(i,j) \right) \]
\[ Q(i,j+1) \Rightarrow (i,j) = K(i,j+\frac{1}{2}) 2\pi m \Delta r^2 \left( T(i,j+1) - T(i,j) \right) \]

\[ Q_r = EGA_{s6} (T(i,j)^4 - T_{air}^4) \]
\[ Q_c = h_c A_{s6} (T(i,j) - T_{air}) \]

Summing the heat flows and equating to zero yields:

\[ K(i-\frac{1}{2},j) 2\pi m \Delta z \left( \frac{1}{2\pi} \right) T(i-1,j) + K(i,j-\frac{1}{2}) 2\pi m \Delta r^2 T(i,j+1) \]
\[ + K(i,j+\frac{1}{2}) 2\pi m \Delta r^2 T(i,j) = EGA_{s6} T_{air}^4 \]
\[ + h_c A_{s6} T_{air} \]
\[ - (K(i-\frac{1}{2},j) 2\pi m \Delta z \left( \frac{1}{2\pi} \right) + 2\pi m \Delta r^2 (K(i,j-\frac{1}{2}) + K(i,j+\frac{1}{2})) + h_c A_{s6}) T(i,j) \]
\[ - EGA_{s6} T(i,j)^4 = 0 \]

where \[ A_{s6} = 2\pi R_0 \Delta z \]

The equations for the nine distinct types of nodes are summarized in matrix form on the next page.
<table>
<thead>
<tr>
<th>Spe of Node(s)</th>
<th>T(i+1,j)</th>
<th>T(i-1,j)</th>
<th>T(i,j+1)</th>
<th>T(i,j-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPICAL NODES</td>
<td>a_k(i+1/2)K_0</td>
<td>a_k(i-1/2)K_i</td>
<td>a_2^2K_0</td>
<td>a_2^2K_u</td>
</tr>
<tr>
<td>CORNER NODE AT E</td>
<td>Baraz K_0</td>
<td>0</td>
<td>a_2^2K_D</td>
<td>0</td>
</tr>
<tr>
<td>NODES AT SLAG/SILVER LINE</td>
<td>a_k(i+1/2)K_0</td>
<td>a_k(i-1/2)K_i</td>
<td>a_2^2K_0</td>
<td>0</td>
</tr>
<tr>
<td>NODES IN SIDIWALL AT SLAG/SILVER LINE</td>
<td>a_k(i+1/2)K_0</td>
<td>a_k(i-1/2)K_i</td>
<td>a_2^2K_0</td>
<td>0</td>
</tr>
<tr>
<td>5. NODES ON E BETWEEN SHELL AND SLAG/SILVER LINE</td>
<td>Baraz K_0</td>
<td>0</td>
<td>a_2^2K_D</td>
<td>0</td>
</tr>
<tr>
<td>6. NODE ON E AT INTERSECTION WITH BOTTOM SHELL</td>
<td>2Baraz K_0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7 NODES ON BOTTOM SHELL</td>
<td>2Ti mb (i+1/2)K_0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CASE I - RADIAL FACE INTERSECTION</td>
<td>2Ti mb (i+1/2)K_0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CASE II - AXIAL FACE INTERSECTION</td>
<td>0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>0</td>
<td>A_2^2K_D</td>
</tr>
<tr>
<td>BOTTOM NODE</td>
<td>2Ti mb (i+1/2)K_0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>A_2^2K_0</td>
<td>2Ti ma^2K_u</td>
</tr>
<tr>
<td>TOP NODE</td>
<td>2Ti mb (i+1/2)K_0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>A_2^2K_0</td>
<td>0</td>
</tr>
<tr>
<td>8. NODE AT INTERSECTION OF SIDIWALL AND SIDE SHELL AT SLAG/SILVER LINE</td>
<td>0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>2Ti ma^2K_D</td>
<td>0</td>
</tr>
<tr>
<td>9. NODES ON SIDEWALL</td>
<td>0</td>
<td>2Ti ma (i-1/2)K_i</td>
<td>2Ti ma^2K_D</td>
<td>2Ti ma^2K_u</td>
</tr>
</tbody>
</table>

Where:
- $K_0 = k(i+1/2)$
- $K_u = k(i,j-1/2)$
- $K_i = k(i-1/2)$
- $K_D = k(i,j+1/2)$
- $A_{a1} ... A_{a6}$ and $a,b,m$ are previously defined.

**Coefficients**

- $T(i,j)$
  - $-(a_k(i+1/2)K_0 + (1-\frac{1}{2})K_i) + a_2^2(K_{a0} + K_u)$
  - $-(a_k(i+1/2)K_0 + (1-\frac{1}{2})K_i)$
  - $+(1-\frac{1}{2})K_{a0}$
  - $+(1-\frac{1}{2})K_{a0}$
  - $+(1-\frac{1}{2})K_{a0}$
  - $+(1-\frac{1}{2})K_{a0}$

**Figure 4.3**

<table>
<thead>
<tr>
<th>Constants</th>
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<tbody>
<tr>
<td>$E_G A_{a1} + T_{a1}$</td>
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<tr>
<td>$E_G A_{a2} + T_{a2}$</td>
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<tr>
<td>$E_G A_{a3} + T_{a3}$</td>
</tr>
<tr>
<td>$E_G A_{a4} + T_{a4}$</td>
</tr>
<tr>
<td>$E_G A_{a5} + T_{a5}$</td>
</tr>
<tr>
<td>$E_G A_{a6} + T_{a6}$</td>
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</tbody>
</table>
8.2.0 Thermal Conductivity of Silver
Table 1

Thermal Conductivity of Silver

(Temperature, $T$, K; Thermal Conductivity, $k$, W cm$^{-1}$ K$^{-1}$)

<table>
<thead>
<tr>
<th>Solid</th>
<th>Liquid</th>
</tr>
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<tr>
<td>$T$</td>
<td>$k$</td>
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<td>0</td>
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<tr>
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<td>100</td>
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†The values are for well-annealed high-purity silver, and those below 150 K are applicable only to a specimen having residual electrical resistivity of 0.000621 μΩ cm. The values for molten silver are provisional.

*Extrapolated or estimated.

8.3.0 Listing of Computer Programs

8.3.1 Program Smelter
8.3.2 Subroutine Input
8.3.3 Subroutine Tint
8.3.4 Subroutine Reffpos
8.3.5 Subroutine Control
8.3.6 Subroutine Coefmat
8.3.7 Subroutine Intcept
8.3.8 Subroutine Coefcal
8.3.9 Subroutine Thercon
8.3.1 Program Smelter
PROGRAM SMELTER

WRITTEN BY KURT B. CARLSON
MAY 22, 1986

PURPOSE AND FUNCTION: THIS PROGRAM CALCULATES THE FLOW OF HEAT THROUGH
A REFRACTORY LINED HEARTH OF A SMELTER. THE PROGRAM USES A FINITE ELEMENT
METHOD TO CALCULATE THE TEMPERATURE DISTRIBUTION IN THE HEARTH AND THE
TOTAL FLOW OF HEAT FROM THE HEARTH BY RADIATION AND CONVECTION.

THE CONFIGURATION OF THE HEARTH IS AS LISTED BELOW:

\[ \text{SLAG LAYER} \]
\[ \text{REFRACTORY} \]
\[ \text{SIDEWALL} \]
\[ \text{OUTER SHELL} \]
\[ \text{SILVER LAYER} \]
\[ \text{CENTERLINE OF THE HEARTH} \]
\[ \text{HEARTH} \]
\[ \text{HEMISPHERICAL HEAD} \]

THE OPERATOR OF THE PROGRAM INPUTS INTO THE PROGRAM:

1. THE PHYSICAL DIMENSIONS OF THE SMELTER
2. THE TEMPERATURE OF THE SLAG/SILVER INTERFACE
3. THE TEMPERATURE OF THE AIR AN SURFACES SURROUNDING THE SMELTER
4. THE POSITION OF THE SILVER AND VARIOUS REFRACTORY MATERIALS IN THE HEARTH
5. THE THERMAL CONDUCTIVITIES AS FUNCTIONS OF TEMPERATURE FOR THE REFRACTORY MATERIALS AND THE SILVER

THE PROGRAM AUTOMATICALLY BREAKS THE HEARTH UP INTO A DEFINED NODAL SYSTEM AND IDENTIFIES EACH NODE WITH A DISTINCTIVE REFRACTORY MATERIAL. EACH NODE IS IDENTIFIED AS ONE OF ELEVEN DISTINCTIVE NODES IN THE SYSTEM AND A COEFFICIENT AND REMAINDER MATRIX IS GENERATED FROM HEAT BALANCE.
AROUND EACH OF THE NODES USING CYLINDRICAL COORDINATES. THE TEMPERATURE
OF EACH NODE IS THEN DETERMINED BY SOLVING THE LINEAR EQUATIONS DERIVED.

THE MAJOR ASSUMPTIONS USED IN THE PROGRAM ARE:

1. ALL MATERIALS ARE SOLIDS (I.E. THERE ARE NO BOUYANCY EFFECT
2. THE SYSTEM IS AXISYMMETRIC
3. THE TEMPERATURE OF THE SLAG/SILVER LAYER IS CONSTANT IN THE RADIAL DIRECTION

LISTING OF PROGRAM STRUCTURE

SUBROUTINE (--------- PROGRAM -------) SUBROUTINE
0080 CC INPUT SMELTER OUTPUT
0081 CC ------- ------- ------- ------- -------
0082 CC |
0083 CC |
0084 CC |
0085 CC |
0086 CC |
0087 CC |
0088 CC |
0089 CC |
0090 CC |
0091 CC |
0092 CC |
0093 CC |
0094 CC |
0095 CC |
0096 CC |
0097 CC |
0098 CC |
0099 CC |
0100 CC |
0101 CC |
0102 CC |
0103 CC |
0104 CC |
0105 CC |
0106 CC |
0107 CC |
0108 CC |
0109 CC |
0110 CC |
0111 CC |
0112 CC |
0113 CC |
0114 CC
DESCRIPTION OF EACH SUBROUTINE

INPUT - READS IN THE VALUES DESIRED BY THE PROGRAM OPERATOR.
PRINTS THE VALUES OUT, AND TRANSFERS THEM TO THE MAIN
PROGRAM SMELTER

OUTPUT - PRINTS OUT THE DERIVED VALUES

CALC - CHECKS THE INPUTTED VALUES TO ENSURE THAT FUNDAMENTAL
CRITERION HAVE NOT BEEN VIOLATED AND CALCULATES VALUES
USED IN THE OTHER SUBROUTINES

CONTROL - CONTROLS THE FLOW AND TRANSFER OF DATA BETWEEN THE
SUBROUTINES IN ORDER TO CALCULATE THE TEMPERATURE
FIELD

TINT - CALCULATES AN INITIAL ASSUMPTION OF THE TEMPERATURE
FIELD

IMSL - AN IMSL SUBROUTINE CALLED TO SOLVE THE SYSTEM OF N TIMES
M LINEAR EQUATIONS

REFPOS - ASSIGNS EACH NODE WITH A DISTINCTIVE REFRACTORY OR SILVER
MATERIAL

COEFMAT - IDENTIFIES EACH NODE AS ONE OF ELEVEN DISTINCTIVE TYPES
OF NODES USED AND DIRECTS THE CALCULATION OF THE VALUES
OF THE COEFFICIENT AND REMAINDER MATRICES

INTCEPT - CALCULATES THE INTERCEPT POINT OF THE RADIAL NODAL SYSTEM
AND THE HEMISPHERICAL STEEL SHELL

COEFS - CALCULATES THE COEFFICIENT AND REMAINDERS TERMS FOR EACH
DISTINCTIVE TYPE OF NODE

THERCON - CALCULATES THE VALUES OF THE THERMAL CONDUCTIVITIES OF
THE NODES

LISTING OF VARIABLES USED IN THE PROGRAM

A - THE MATRIX CONTAINING THE NAMES OF THE SILVER AND REFRACTORY
MATERIALS

B1 - THE HEIGHT OF THE INWARD SIDE OF THE RADIAL BAND

B2 - THE HEIGHT OF THE OUTWARD SIDE OF THE RADIAL BAND
C - THE INTERCEPT POINT OF THE NEXT TO THE LAST NODE WITH THE
HEMISPHERICAL STEEL SHELL
CM - THE CALCULATED VALUES OF THE COEFFICIENT AND REMAINDER
MATRICES
DH - THE DEPTH OF THE REFRACTORY HEARTH - (INPUT) - [INCHES]
DR - THE DIMENSION OF THE NODE IN THE RADIAL DIRECTION - (INPUT) -
[LINCHES]
DS - THE DEPTH OF THE SILVER LAYER - (INPUT) - [INCHES]
DTM - CONVERGENCE CRITERION OF CONSECUTIVE SOLUTIONS - (INPUT) -
[DEG F]
DZ - THE DIMENSION OF THE NODE IN THE AXIAL DIRECTION - (INPUT) -
[LINCHES]
E - THE EMISSIVITY OF THE STEEL SHELL - (INPUT)
FM - THE NUMBER OF THE REFRACTORY MATERIAL THAT FILLS UP THE VOID
BETWEEN THE DISKS AND THE HEMISPHERICAL SHELL - (INPUT)
I - THE RADIAL COORDINATE OF THE NODE UNDER CONSIDERATION
ITER - THE NUMBER OF TRIALS USED FOR THE SOLUTIONS
J - THE AXIAL COORDINATE OF THE NODE UNDER CONSIDERATION
KC - THE MATRIX THAT CONTAINS THE VALUES OF THE COEFFICIENTS FOR
THE N ORDER POLYNOMIAL EQUATION THAT DESCRIBES THE THERMAL
CONDUCTIVITY OF THE REFRACTORY AND SILVER MATERIALS - (INPUT) -
KD - THE NET THERMAL CONDUCTIVITY OF THE NODE UNDER CONSIDERATION
AND THE NODE UNDERNEATH IT - [BTU-IN/Ft**2-HR-DEG F]
KE - THE THERMAL CONDUCTIVITY OF THE NODE UNDER CONSIDERATION
AND NODE INSIDE OF IT - [BTU-IN/Ft**2-HR-DEG F]
KI - THE NET THERMAL CONDUCTIVITY OF THE NODE UNDER CONSIDERATION
AND NODE OUTSIDE OF IT - [BTU-IN/Ft**2-HR-DEG F]
KU - THE NET THERMAL CONDUCTIVITY OF THE NODE UNDER CONSIDERATION
AND THE NODE ON TOP OF IT - [BTU-IN/Ft**2-HR-DEG F]
L - THE NET LENGTH OF THE HEMISPHERICAL SHELL - [INCHES]
M - THE NUMBER OF NODES IN THE AXIAL DIRECTION
N - THE NUMBER OF NODES IN THE RADIAL DIRECTION
ND - THE NODE IDENTIFIER THAT IDENTIFIES EACH NODE AS ONE OF THE
ELEVEN DISTINCTIVE TYPES OF NODES USED
NI - THE ORDER OF THE POLYNOMIAL EQUATION DESCRIBING THE THERMAL
CONDUCTIVITY OF THE REFRACTORY AND SILVER MATERIALS - (INPUT)
NIP - THE MAXIMUM NUMBER OF PROGRAM ITERATIONS ALLOWED TO SOLVE FOR THE
TEMPERATURE FIELD S - (INPUT)
NN1 - THE NUMBER OF NODES IN THE (1-1) RADIAL BAND
NN2 - THE NUMBERS OF NODES IN THE (I) RADIAL BAND
NRD - THE NUMBER OF REFRACTORY DISKS - (INPUT)
NRM - THE NUMBER OF REFRACTORY MATERIALS USED - (INPUT)
PCALC - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE CALC - (INPUT)
PCOEFCAL - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE COEFCAL - (INPUT)
PCOEFFMAT - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE COEFFMAT - (INPUT)
PCONTROL - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE CONTROL - (INPUT)
PINTCEPT - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE INTCEPT - (INPUT)
PREFPOS - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE REFP - (INPUT)
PIHERCON - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM
SUBROUTINE THERCON - (INPUT)

PTINT - THE SWITCH TO PRINT OUT THE CALCULATED VALUES FROM

SUBROUTINE TINT - (INPUT)

QTOT - THE CALCULATED TOTAL HEAT LOSS FROM THE STEEL SHELL - [BTU/HR]

QC - THE HEAT LOSS BY CONVECTION FROM THE NODE UNDER CONSIDERATION - [BTU/HR]

QR - THE HEAT LOSS BY RADIATION FROM THE NODE UNDER CONSIDERATION - [BTU/HR]

QTOTC - THE TOTAL HEAT LOSS FROM THE SHELL BY CONVECTION - [BTU/HR]

QTOTR - THE TOTAL HEAT LOSS FROM THE SHELL BY RADIATION - [BTU/HR]

RNI - THE N BY M MATRIX THAT IDENTIFIES EACH NODE WITH A SILVER OR

REFRACTORY MATERIAL

RO - THE OUTSIDE RADIUS OF THE SHELL - (INPUT) - [INCHES]

RFM - THE POSITION OF EACH RECTANGULAR CROSS SECTION REFRACTORY DISK

AND THE TYPE OF REFRACTORY MATERIAL IN EACH DISK - (INPUT)

SA - THE NET SLOPE OF THE HEMISPHERICAL STEEL SHELL

T - THE CALCULATED VALUES OF THE TEMPERATURE FIELD - [DEG F]

TA - THE TEMPERATURE OF THE AMBIENT AIR SURROUNDING THE SMELTER -

[INPUT] - [INCHES]

TNEW - THE LAST CALCULATED VALUE OF THE TEMPERATURE FIELD - [DEG F]

TRS - THE THICKNESS OF THE REFRACTORY SIDEWALL - (INPUT) - [INCHES]

TR - THE TEMPERATURE OF THE SURFACES SURROUNDING THE SHELL - (INPUT) -

[DEG F]

TS - THE TEMPERATURE OF THE SLAG/SILVER INTERFACE - (INPUT) -

[DEG F]

IMPLICIT, CHARACTER, REAL, INTEGER, AND COMMON STATEMENTS

IMPLICIT NONE

INTEGER FM, I, ITER, J, M, N, ND, NNI, NN2, NR2, RNI(10,10), NI

INTEGER NRM, NIP, FCALC, PCON, PCALC, PCOEFCAL, PCOEFMAT, PTINT, PREFPOS

INTEGER PTCINT, PHERCON, II, IA, AM, IDOT, X, NA, IER, JJ, LL, L2

INTEGER L3, L4, L5, L6, KKK

REAL B1, B2, C, CM(10,10), DH, DR, DS, UTM, DZ, E, KO, KE, K1, KO, KU, L, QTOT

REAL QC, QR, QTOTC, QTOTR, RO, BPM(5,5), SA, TA, TRS, TR, TS, T(10,10)

REAL TNEW(10,10), R, B(5), KAREA, X, Y, A1, A2, A3, A4, H5, H6, H7, INT

REAL SUML, Z, KC(5,5)

CHARACTER A(5)

COMMON/DI MM/RO, DS, DH, TRS, SA

COMMON/TMP/T, TA, TR, E

COMMON/REFRAC/RND, FM, NI, NRM

COMMON/XWRITE/PTHERCON, PCOEFCAL, PINTCEPT, PCOEFMAT, PREFPOS, PTINT,

1 FCALC, PCON

COMMON/FINAL/N, M, DR, DZ

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C
SMELTER$MAIN

0286 C
0287 CC  CALL SUBROUTINE INPUT TO BRING THE INPUTTED VALUES INTO THE PROGRAM
0288 C
0289 C
0290 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
0291 C
0292 C
0293 C
0294 C
0295 C
0296 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
0297 C
0298 C
0299 CC  CALL TINT SUBROUTINE TO CALCULATE THE INITIAL VALUES OF THE TEMPERATURE FIELD
0300 CC
0301 C
0302 C
0303 CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
0304 C
0305 C
0306 N=BO/DR
0307 M=(DS+DH)/DZ
0308 C
0309 STOP
0310 C
0311 END

PROGRAM SECTIONS

Name     Bytes          Attributes
0   $CODE            77   PIC CON REL LCL   SHR   EXE   RD   NOWRT   LONG
2   $LOCAL           1872  PIC CON REL LCL NO SHR NO EXE   RD   WRT   LONG
3   DIMEN            20    PIC OVR REL GBL   SHR NO EXE   RD   WRT   LONG
4   TEMP            16    PIC OVR REL GBL   SHR NO EXE   RD   WRT   LONG
5   REFRAC           16    PIC OVR REL GBL   SHR NO EXE   RD   WRT   LONG
6   WRITE           32    PIC OVR REL GBL   SHR NO EXE   RD   WRT   LONG
7   FINEL           16    PIC OVR REL GBL   SHR NO EXE   RD   WRT   LONG

Total Space Allocated  2049

ENTRY POINTS

Address   Type   Name
0-00000000   SMELTER$MAIN
8.3.2 Subroutine Input
SUBROUTINE INPUT

WRITTEN BY KURT B. CARLSON
DATE WRITTEN MARCH 21, 1986

PURPOSE: THE PURPOSE OF THIS SUBROUTINE IS TO CONTAIN THE INPUTTED VALUES
AND VARIABLES NEEDED FOR THE SOLUTION OF THE HEAT TRANSFER IN THE
HEARTH OF THE ELECTRICAL SMELTER

THIS SUBROUTINE TRANSFER THE INPUTTED VALUES AND VARIABLES BACK TO THE
MASTER PROGRAM SMELTER WHICH DIRECTS THE PROGRAM FLOW. THE INPUTTED DATA
IS READ FROM A DATA FILE DATA.

THE BULK OF THE INPUTTED DATA IS PLACED INTO COMMON BLOCKS THAT EFFICIENTLY
TRANSFER THE VALUES TO THE VARIOUS SUBROUTINES.

THE COMMON BLOCKS ARE:

DIMENT - THIS BLOCK CONTAINS THE VALUES THAT DESCRIBE THE DIMENSIONS
OF THE SMELTER
TEMP - THIS BLOCK CONTAINS THE INPUTTED TEMPERATURE VALUES
REFRAC - THIS BLOCK CONTAINS THE VALUES THAT DESCRIBE THE NUMBER AND
LOCATIONS OF THE REFRACTORY MATERIALS AND THEIR THERMAL
CONDUCTIVITY

SUBROUTINE INPUT(DR,DZ,NIP,DIM,RFM,KC)

IMPLICIT, INTEGER, REAL, AND COMMON BLOCK STATEMENTS

IMPLICIT NONE
INTEGER NRD,FM,NI,NRM,NIP,I,J,N
INTEGER PHERCON,POEFCAL,PINTCEPT,POEWMAT,PREFPOS,PTINT,PCalc
INTEGER PCNTROL
REAL RO,DS,DH,TR,S,TI,TR,E,RFM(5,5),DIM,DR,DZ,KC(5,5)
CHARACTER*6 A(5)
COMMON/DIEN/RO,DS,DH,TR,S
COMMON/TEMP/TI,TR,E
COMMON/REFRAC/NRD,FM,NI,NRM
COMMON/XWRITE/PHERCON,POEFCAL,PINTCEPT,POEWMAT,PREFPOS,PTINT,
& PCalc,PCNTROL

SMELTER DIMENSIONAL DATA IN COMMON BLOCK STATEMENT DIEN

DS = DEPTH OF THE SILVER LAYER (INCHES)
DH = DEPTH OF THE REFRAC TORY HEARTH (INCHES)
TR = THICKNESS OF THE REFRAC TORY SIDEMALL (INCHES)
OR = OUTER RADIUS OF THE STEEL SHELL (INCHES)
SA = THE SLOE OF THE STEEL SHELL

READ (5,*)DS,DH,TR,S,RO,SA
TEMPERATURE DATA IN COMMON BLOCK STATEMENT TEMP

TS = THE TEMPERATURE OF THE SLAG/SILVER INTERFACE (DEG F)
TA = AMBIENT AIR TEMPERATURE (DEG F)
TR = TEMPERATURE OF THE SURFACES TO WHICH THE STEEL SHELL IS
RADIATING HEAT (DEG F)
E = EMISSIVITY OF THE STEEL SHELL

READ (5,*)TS,TA,TR,E

DATA DESCRIBING THE REFRATORY MATERIALS IN COMMON BLOCK STATEMENT REFRAC
NRM = THE NUMBER OF DIFFERENT TYPES OF REFRATORY MATERIALS USED
NI = THE ORDER OF THE POLYNOMIAL DESCRIBING THE THERMAL CONDUCTIVITY
OF THE REFRATORY MATERIALS
KC = THE NI+1 COEFFICIENTS IN THE NI ORDER POLYNOMIAL FOR THE THERMAL
CONDUCTIVITY FOR THE NM REFRATORY MATERIALS
(BTU-FT**2/HR-DEG F-IN)
NRD = THE NUMBER OF DIFFERENT RECTANGULAR CROSS SECTION BLOCKS
RPM = THE REFRATORY POSITION MATRIX WHICH IS A NRD+1 BY 5 MATRIX.
The SETUP OF THE MATRIX IS AS FOLLOWS:
RPM(Y,X,R1,R2,Z1,Z2)

WHERE:
Y = THE DISK NUMBER
X = THE REFRATORY MATERIAL NUMBER FROM 1 TO NRM+1
THE NUMBER 1 IS RESERVED FOR THE SILVER LAYER.
R1 = THE INNER RADIUS OF THE RECTANGULAR CROSS-SECTION
DISK (INCHES)
R2 = THE OUTER RADIUS OF THE RECTANGULAR CROSS-SECTION
DISK (INCHES)
Z1 = THE DISTANCE FROM THE SLAG/SILVER INTERFACE TO THE
TOP OF THE DISK (INCHES)
Z2 = THE DISTANCE FROM THE SLAG SILVER INTERFACE TO THE
BOTTOM OF THE DISK (INCHES)
FM = THE NUMBER OF THE REFRATORY MATERIAL THAT FILLS UP THE VOID
BETWEEN THE DISKS AND THE HEMISPHERICAL SHELL
A = THE NAME OF THE REFRATORY MATERIAL (SIX LETTERS)

READ (5,*)NRM,NI,NRD
DO 2 I=1,NRM+1
READ (5,*) (KC(I,J),J=1,NI+2)
2 CONTINUE
DO 3 I=1,NRD
READ (5,*) (RPM(I,J),J=1,5)
3 CONTINUE
READ (5,100) (A(I),I=1,NRM+1)
100 FORMAT ((NRM+1)(A6,1X))
DATA DESCRIBING THE PARAMETERS FOR CONTROLLING THE DEGREE OF ACCURACY
OF THE SOLUTION AND THE RUNNING TIME OF THE PROGRAM WHERE:

NIP = MAXIMUM NUMBER OF ITERATIONS OF THE SOLUTION OF THE
TEMPERATURE FIELD ALLOWED
DYM = CONVERGENCE CRITERION OF CONSECUTIVE SOLUTIONS (DEG F)

READ (5,*) NIP, DYM

DATA DESCRIBING THE SIZE OF THE NODE TO BE USED IN THE AXIAL AND
RADIAL DIRECTIONS WHERE:

DR = THE SIZE OF THE NODE IN THE RADIAL DIRECTION (INCHES)
DZ = THE SIZE OF THE NODE IN THE AXIAL DIRECTION (INCHES)

READ (5,*) DR, DZ

READ IN THE SWITCH STATEMENTS FOR THE SUBROUTINE WHERE IF THE
SWITCH = 1 THE CALCULATED VALUES IN EACH SUBROUTINE ARE AUTOMATICALLY
PRINTED OUT AND IF THE SWITCH = 0 THEN THEY ARE NOT

READ (5,*) DTM, CATM, PCOEFCAL, PINTCEPT, PPTPOS, PTTINT,
1 PCALC, PCONTROL

OUTPUT THE VALUES READ INTO THE PROGRAM

WRITE (6,11)
11 FORMAT(///,9X,50(1H*),/,,10X,'OUTPUT OF THE VARIABLES INPUTTED','
1 INTO THE PROGRAM',///,9X,50(1H*),/,,)/)
WRITE (6,12) TS, TA, TR, E
12 FORMAT(10X,'TEMPERATURE AND EMMISIVITY VALUES',/,,10X,34(1H*),
160 1 ///,10X,'TEMPERATURE OF THE SLAG/SILVER INTERFACE',
161 2 '(DEG F) = ',T80,F6.1///,10X,
162 3 'TEMPERATURE OF THE AMBIENT AIR (DEG F) = '
163 4 T80,F6.1///,10X,'TEMPERATURE OF THE SURROUNDING',
164 5 RADIATIVE SURFACES (DEG F) = ',T80,
165 6 F6.1///,10X,'EMMISIVITY OF THE STEEL SHELL = ',T80,F5.2///)
WRITE (6,13) DS, DH, RO, TRS, SA
13 FORMAT(10X,'PHYSICAL DIMENSIONS OF THE SMELTER',/,,10X,35(1H*),/,,
168 1 10X,'DEPTH OF THE SILVER LAYER (INCHES) = ',T80,F4.1///,
169 2 10X,'DEPTH OF THE REFRATORY HEARTH AT THE CENTERLINE',
170 3 ' (INCHES) = ',T80,F4.1///,10X,'OUTSIDE RADIUS OF'
171 4 ' THE CYLINDRICAL SHELL (INCHES) = ',T80,F4.1///,10X.
0172 5 'THICKNESS OF THE REFRACTORY SIDEWALLS (INCHES) = ',T80,
0173 6 F4.1,/,10X,'SLOPE OF THE HEMISPERICAL SHELL (INCHES) = ',
0174 7 T80,F5.2,/,)
0175 WRITE (6,14) DR,ZE
0176 14 FORMAT(10X,'FINITE ELEMENT SIZE',/,10X,19(1H-),/,,
0177 1 10X,'INCREMENTAL SIZE IN THE RADIAL DIRECTION (INCHES) = ',
0178 2 T80,F5.2,/,10X,
0179 3 'INCREMENTAL SIZE IN THE AXIAL DIRECTION (INCHES) = ',
0180 4 T80,F5.2,/,)
0181 WRITE (6,15) NIP,DIMP
0182 15 FORMAT(10X,'PROGRAM CONTROLLERS',/,10X,20(1H-),/,,
0183 1 10X,'MAXIMUM NUMBER OF ITERATIONS = ',T80,I3,/
0184 2 10X,'CONVERGENCE CRITERION (DEG F) = ',T80,F5.2)
0185 N=N+1
0186 WRITE (6,16) (I,I=1,N1+1)
0187 16 FORMAT(1H,,///,10X,'REFRACTORY AND SILVER PROPERTIES AND POSITION',
0188 1 ///,10X,45(1H-),///,10X,'MATERIAL PROPERTIES',/,10X,19(1H-),///,
0189 2 34X,'COEFFICIENTS IN THE POLYNOMIAL EQUATION',/,34X,
0190 3 'DESCRIBING THE THERMAL CONDUCTIVITY',/,10X,'MATERIAL',/5X,
0191 4 'NUMBER',/5X,(N)'(A',/11.7X)/,10X,8(1H-),5X,6(1H-),
0192 5 5X,(N)'(B1(1H-)),///)
0193 DO 21 I=1,NRM+1
0194 WRITE (6,17) (A(I),I,(KC(I,J)),J=1,N1+1))
0195 17 FORMAT(12X,A6,5X,I2,5X,(N1+1)(3X,F5.3),/) 196 21 CONTINUE
0197 WRITE (6,18)
0198 18 FORMAT(///,10X,'REFRACTORY MATERIAL POSITIONS',/,10X,28(1H-),
0199 1 ///,34X,'POSITION OF EACH REFRACTORY DISK (INCHES)',/,10X,
0200 2 'MATERIAL',/5X,'NUMBER',/5X,'INNER',/5X,'OUTER',/4X,'TOP AXIAL',
0201 3 'LOWER AXIAL',/34X,'RADIUS',/4X,'RADIUS',/3X,'DISTANCE',
0202 4 '3X,DISTANCE',/,10X,8(1H-),5X,6(1H-),4X,6(1H-),4X,6(1H-),
0203 5 3X,8(1H-),3X,8(1H-),///)
0204 DO 22 I=1,NRM+1
0205 WRITE (6,19) (A(I),I,(RPM(I,J)),J=2,5))
0206 19 FORMAT(12X,A6,6X,I2,4X,45X,F5.3),/) 207 22 CONTINUE
0208 WRITE (6,20) PTHERCON,PCOEFCAL,PINTCEPT,PCEOFMAT,PREP,FPTN
0209 20 FORMAT (///,10X,'PARAMETERS FOR WRITING VALUES CALCULATED ',
0210 1 ///,'INTERNALLY IN THE SUBROUTINES',/,10X,'PARAMETER',/20X,
0211 2 'VALUE',/10X,9(1H-),20X,5(1H-),/10X,'PTHERCON',/23X,I1,
0212 3 ///,10X,'PCOEFCAL',/23X,I1,///,10X,'PINTCEPT',/23X,I1,///,
0213 4 'PCEOFMAT',/23X,I1,///,10X,'PREP',/24X,I1,///,10X,'PFTN
0214 5 26X,I1,///,10X,'PCALC',/26X,I1,///,10X,'PCONTROL',/23X,I1,///)
0215 C
0216 CC
0217 C
0218 C
0219 RETURN
0220 END
### OUTPUT OF THE VARIABLES INPUTTED INTO THE PROGRAM

**TEMPERATURE AND EMISSIVITY VALUES**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature of the slag/silver interface (deg F)</td>
<td>2500.0</td>
</tr>
<tr>
<td>Temperature of the ambient air (deg F)</td>
<td>100.0</td>
</tr>
<tr>
<td>Temperature of the surrounding radiative surfaces (deg F)</td>
<td>200.0</td>
</tr>
<tr>
<td>Emissivity of the steel shell</td>
<td>0.80</td>
</tr>
</tbody>
</table>

**PHYSICAL DIMENSIONS OF THE SHELTER**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of the silver layer (inches)</td>
<td>10.0</td>
</tr>
<tr>
<td>Depth of the refractory hearth at the centerline (inches)</td>
<td>30.0</td>
</tr>
<tr>
<td>Outside radius of the cylindrical shell (inches)</td>
<td>50.0</td>
</tr>
<tr>
<td>Thickness of the refractory sidewalls (inches)</td>
<td>20.0</td>
</tr>
<tr>
<td>Slope of the hemispherical shell (inches)</td>
<td>-5.00</td>
</tr>
</tbody>
</table>

**FINITE ELEMENT SIZE**

<table>
<thead>
<tr>
<th>Size</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incremental size in the radial direction (inches)</td>
<td>5.00</td>
</tr>
<tr>
<td>Incremental size in the axial direction (inches)</td>
<td>5.00</td>
</tr>
</tbody>
</table>

**PROGRAM CONTROLLERS**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum number of iterations</td>
<td>2</td>
</tr>
<tr>
<td>Convergence criterion (deg F)</td>
<td>5.00</td>
</tr>
</tbody>
</table>
# Refractory and Silver Properties and Position

## Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Number</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>RubyXX</td>
<td>2</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>KORXDX</td>
<td>3</td>
<td>3.000</td>
<td>3.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>

## Refractory Material Positions

<table>
<thead>
<tr>
<th>Material</th>
<th>Number</th>
<th>Inner Radius</th>
<th>Outer Radius</th>
<th>Top Axial Distance</th>
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## Signal Parameters for Writing Values Calculated Internally in the Subroutines

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<tr>
<td>GORECAL</td>
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</tr>
<tr>
<td>FINTCEPT</td>
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<tr>
<td>GOREFMAT</td>
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8.3.3 Subroutine Tint
SUBROUTINE TINT

WRITTEN BY KURT B. CARLSON
APRIL 14, 1986

PURPOSE: DETERMINE THE FIRST ASSUMPTION OF THE TEMPERATURE FIELD IN THE HEARTH

FORM: TINT(T)

INPUT (FROM CALL STATEMENT OR COMMON STATEMENTS):

TS = TEMPERATURE OF THE SLAG/SILVER INTERFACE
TRS = THICKNESS OF THE REFRACTORY SIDEWALL
RO = THE OUTSIDE RADIUS OF THE STEEL SHELL
DS = THE DEPTH OF THE SILVER LAYER
DM = THE DEPTH OF THE REFRACTORY HEARTH
DZ = THE AXIAL NODE INCREMENT
DR = THE RADIAL NODE INCREMENT
M = THE NUMBER OF NODES IN THE AXIAL DIRECTION
N = THE NUMBER OF NODES IN THE RADIAL DIRECTION

OUTPUT:
T = THE CALCULATED FIRST ESTIMATE OF THE TEMPERATURE FIELD

SUBROUTINE TINT(T)

IMPLICIT, REAL, INTEGER, AND COMMON STATEMENTS

IMPLICIT NONE

INTEGER II, JJ, N, M, I, J
INTEGER PRHCON, PCOEFCAL, PINTCEPT, PCOEFMAT, PIFP08, PINT, PCALC
INTEGER PCONTROL
REAL TI(10,10), X, Y, RO, DS, DH, TRS, SA, TS, TA, TR, E, DR, DZ
COMMON/DIMEN/RO, DS, DH, TRS, SA
COMMON/TEMP/TI, TS, TA, TR, E
COMMON/FINEL/N, M, DR, DZ
COMMON/XWRITE/PRHCON, PCOEFCAL, PINTCEPT, PCOEFMAT, PIFP08, PINT,

1 PCALC, PCONTROL

ASSUME THE INITIAL SHELL TEMPERATURE AT ALL PLACES IS 350 DEGREES
FAHRENHEIT
ASSUME THE ENTIRE SILVER LAYER IS AT THE TEMPERATURE OF THE SLAG SILVER
INTERFACE
II=(RO-TRS)/DR
JJ=DS/DZ
DO 10 I=1,II
DO 10 J=1,JJ
10 T(I,J)=TS
FOR THE MATERIAL UNDER THE SILVER LAYER CALCULATE A LINEAR TEMPERATURE DROP BETWEEN THE SILVER LAYER AND THE STEEL SHELL.

THE CALCULATION DOES NOT STOP AT THE STEEL SHELL BUT PROCEEDS TO THE BOTTOM OF THE MATRIX.

\[
X = 1.0
\]

DO 20 I=1,N

DO 20 J=J+1,M

T(I,J)=TS-(TS-350.0)*(X*DZ/DH)

X=X+1.0

IF (J.EQ,M) X=1.0

20 CONTINUE

FOR THE MONOLITHIC SIDEWALL ASSUME A LINEAR TEMPERATURE TEMPERATURE DROP FROM THE SILVER TO THE SIDEWALL. FOR THE MATERIAL DIRECTLY UNDER THE SIDEWALL USE THE AVERAGE OF THE TWO TECHNIQUES LISTED ABOVE.

Y=1.0

DO 30 I=II+1,N

DO 30 J=1,M

IF(J.GT.JJ) GO TO 40

T(I,J)=TS-(TS-350)\AST(Y*DR/TRS)

GO TO 30

40 T(I,J)=T(I,J)+(TS-350)\AST(Y*DR/TRS))/5.0

60 CONTINUE

Y=Y+1.0

30 CONTINUE

WRITE THE CALCULATED VALUES IF PTINT = 1

IF (PTINT.EQ,0) GO TO 50

WRITE (6,2) II,JJ

2 FORMAT (1HL,10X,'VALUES WRITTEN FROM SUBROUTINE TINT',10X,10X)

WRITE (6,3) II,JJ

3 FORMAT (10X,'CALCULATED VALUES OF THE TEMPERATURE FIELD',10X)

DO 4 J=1,M

WRITE (6,5) (TII,J),I=1,N

4 CONTINUE

DO 5 FORMAT (10X,10X)

5 CONTINUE

RETURN

END
VALUES WRITTEN FROM SUBROUTINE TINT

II COUNTER = 6
JJ COUNTER = 2

CALCULATED VALUES OF THE TEMPERATURE FIELD

<p>| | | | | | | | |</p>
<table>
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8.3.4 Subroutine Refpos
SUBROUTINE REFPOS

WRITTEN BY KURT B. CARLSON
APRIL 14, 1986

PURPOSE: IDENTIFY EACH REFRACTORY NODE IN THE MODEL WITH A DISTINCTIVE REFRACTORY MATERIAL USING THE INPUT FILE WHICH CLASSIFIES THE REFRACTORY MATERIALS INTO RECTANGULAR CROSS SECTION DISKS.

FORM: REFPOS(KC,RPM,RNI)

INPUT (THROUGH THE SUBROUTINE OR COMMON STATEMENTS)

N = THE NUMBER OF NODES IN THE RADIAL DIRECTION
M = THE NUMBER OF NODES IN THE AXIAL DIRECTION
RPM = THE SIZE OF THE RECTANGULAR CROSS SECTION REFRACTORY DISKS OF EACH TYPE OF MATERIAL AND THE TYPE OF MATERIAL IN EACH DISK
NRD = THE NUMBER OF REFRACTORY DISKS
FM = THE NUMBER OF THE REFRACTORY MATERIAL AT THE BOTTOM OF THE SHELL
SA = THE SLOPE OF THE HEMISPHERICAL HEAD
DS = THE DEPTH OF THE SILVER LAYER
DH = THE DEPTH OF THE REFRACTORY HEARTH
RO = THE OUTSIDE RADIUS OF THE HORIZONTAL CYLINDRICAL SHELL
TRG = THE THICKNESS OF THE REFRACTORY SIDEWALL
RZ = THE RADIAL NODE INCREMENT
DZ = THE AXIAL NODE INCREMENT

OUTPUT:

RNI = THE N BY M MATRIX THAT IDENTIFIES EACH OF THE NODES WITH A REFRACTORY MATERIAL

SUBROUTINE REFPOS(KC,RPM,RNI)

IMPLICIT, REAL, INTEGER, AND COMMON STATEMENTS

IMPLICIT NONE

INTEGER N,M,NRD,FM,J,I,RNI(10,10),II,KK,K,NI,NRM
INTEGER PTHRCON,PCOEFCAL,PINTCEPT,PCEOMAT,REFPOS,PTINT,PCALC
INTEGER PCONTR
REAL SA,DS,DH,RO,TRS,DR,DZ,KC(5,5),SUM1,RPM(5,5),X,ZINT,Z
COMMON/DIMEN/RO,DS,DH,TRS,SA
COMMON/REFRAC/NRD,FM,NI,NRM
COMMON/FINEL/N,M,DR,DE
COMMON/WRITR/PTHCON,PCEOMAT,PINTCEPT,PCEOMAT,REFPOS,PTINT,
1 PCALC,PCONTR
IN THIS ANALYSIS THE FOLLOWING DEFINITIONS ARE USED:

RADIAL BAND REFERS TO A CIRCULAR DISK OF D2 HEIGHT
AXIAL BAND REFERS TO A HORIZONTAL COLUMN OF SOME RECTANGULAR PROFILE
NUMBER INCREMENTS OF DR

INITIALIZE THE REFRACTORY NODE INDICATION MATRIX

DO 5 I=1,N
  DO 5 J=1,M
  5 RN1(I,J)=0.0

INITIALIZE THE SEARCH MATRIX

J=1
Z=DZ

SCAN ACROSS THE FIRST RADIAL BAND TO FIND OUT WHERE THE FIRST REFRACTORY DISK STARTS IN THIS BAND. INITIALIZE ALL COUNTERS
SUM1 IS A CUMULATIVE INDEX OF THE RADIAL DISTANCE FROM THE CENTERLINE.

10 I=1
  SUM1=0.0
  20 II=1

STARTING AT SUM1 DOES THE SCAN REVEAL THAT ANY DISK STARTS A
THIS POINT. IF THEY DON'T GO TO LINE 30. WHEN ALL NRD+1
ARE TRIED AND NONE ARE IN THIS NODE THEN THE FILLER MATERIAL
IS IN THIS NODE.

IF(II.EQ.NRD+2) GO TO 30

RPM(II,3) IS THE RADIAL STARTING POINT OF EACH OF THE DISKS.
IF IT IS EQUAL TO SUM1 THEN THAT DISK STARTS AT SUM1.

IF(RPM(II,3).GT.SUM1) GO TO 40

IF A DISK HAS BEEN IDENTIFIED AS BEING IN THE AXIAL BAND
MAKE SURE THAT THE DISK IS IN THE CORRECT RADIAL BAND.
RPM(II,5) AND RPM(II,6) CONTAI N THE AXIAL STARTING AND
TERMINATION POINT OF EACH REFRACTORY DISK, RESPECTIVELY.

IF(RPM(II,5).LT.2) GO TO 50
IF(RPM(II,6).GT.2) GO TO 50
GO TO 40

IF THE LAST ANALYZED DISK DOES NOT START AT THE GIVEN
NODE THEN TRY THE NEXT NODE

50 II=II+1
GO TO 20

THE II NODE STARTS AT SUM1, THE PROGRAM NOW FINDS THE END OF
DISK IN THIS RADIAL BAND

IF SUM1=0.0 THEN THE DISK STARTS AT THE CENTERLINE. IF SUM1 IS
0115 CC  GREATER THAN 0.0 THEN IT STARTS OUTBOARD OF THE CENTERLINE.
0116 c  
0117 c  40 IF(SUM1.GT.0.0) GO TO 70
0118 C  
0119 CC  IF SUM1=0.0 THEN IS THE DISK LARGER THAN THE FIRST NODE OF
0120 CC  DR/2.0 WIDTH
0121 C  
0122 CC  IF(RPM(II,4).GT.DR/2.0) GO TO 80
0123 C  
0124 CC  THE DISK IS ONLY DR/2.0 WIDTH. IDENTIFY THIS DISK WITH THE II
0125 CC  REFRACTORY MATERIAL AND INCREMENT SUM1 BY DR/2.0 TO SCAN THE NEXT NODE
0126 C  
0127 CC  SUM1=SUM1+DR/2.0
0128 C  
0129 RNI(I,J)=RPM(II,2)
0130 I=I+1
0131 G0 TO 90
0132 C  
0133 CC  THE DISK STARTS AT THE CENTERLINE BUT ITS WIDTH IS GREATER THAN
0134 CC  DR/2.0. CALCULATE HOW MANY NODES ARE IN THIS DISK (IN THIS
0135 CC  RADIAL BAND) AND IDENTIFY EACH WITH THE II REFRACTORY MATERIAL.
0136 C  
0137 CC  INCREMENT SUM1 TO RPM(II,4) AND SCAN THE NEXT NODE.
0138 C  
0139 80 SUM1=RPM(II,4)
0140 X=((RPM(II,4)-RPM(II,3)-DR/2.0)/DR)+1.0
0141 KKK=X
0142 DO 100 K=I,I+KKK
0143 100 RNI(K,J)=RPM(II,2)
0144 I=K
0145 G0 TO 90
0146 C  
0147 CC  THE DISK UNDER CONSIDERATION DOES NOT START AT THE CENTERLINE.
0148 C  
0149 CC  CALCULATE HOW MANY NODES ARE COVERED BY THIS DISK AND IDENTIFY
0150 CC  EACH WITH THE II REFRACTORY MATERIAL. INCREMENT SUM1 TO RPM(II,4)
0151 CC  AND SCAN THE NEXT NODE.
0152 C  
0153 70 SUM1=RPM(II,4)
0154 X=(RPM(II,4)-RPM(II,3))/DR
0155 KKK=X
0156 DO 110 K=I,I+KKK
0157 110 RNI(K,J)=RPM(II,2)
0158 I=K
0159 G0 TO 90
0160 C  
0161 CC  NONE OF THE DISKS START AT SUM1. INCREASE SUM1 BY ONE NODE INCREMENT.
0162 CC  IF SUM1=0.0 THEN INCREMENT BY DR/2.0. INCREMENT ALL OTHER NODES BY DR.
0163 C  
0164 30 IF(SUM1.GT.0.0) GO TO 120
0165 120 SUM1=SUM1+DR
0166 G0 TO 90
0167 C  
0168 CC  CHECK TO SEE IF THE SCAN HAS INTERSECTED THE HEMISPHERICAL
0169 CC  STEEL SHELL
0170 C  
0171 90 ZINT=RO*SA+(DS+DH)
0172 C  
0173 130 IF(Z.GT.ZINT) GO TO 130
0172 CC  CHECK TO INSURE IF THE OUTER CYLINDER IS REACHED AS THE RADIAL BAND IS
0173 CC  CONTAINED WITHIN THE CYLINDER.
0174 C
0175 IF(SUM1.LT.R0) GO TO 140
0176  GO TO 150
0177 C
0178 CC  CHECK TO SEE IF THE HEMISPHERICAL HEAD IS REACHED AS THE RADIAL BAND
0179 CC  IS CONTAINED WITHIN THE HEAD.
0180 C
0181  130 ZINT=(Z-((D3+DH))/SA
0182 C
0183 C
0184 C
0185 CC  THE HEMISPHERICAL HEAD OR THE OUTER CYLINDER HAS NOT BEEN REACHED.
0186 CC  SEARCH THE NEXT REFRATORY DISK TO SEE IF IT STARTS AT SUM1.
0187 C
0188  140 GO TO 20
0189 C
0190 C
0191 CC  THE HEMISPHERICAL HEAD OR THE CYLINDER HAS BEEN REACHED. DETERMINE
0192 CC  IF THE RADIAL BAND UNDER CONSIDERATION IS THE LAST BAND. IF NOT START
0193 CC  SEARCHING IN THE NEXT RADIAL BAND
0194 C
0195  150 IF(J.EQ.M+1) GO TO 160
0196  J=J+1
0197  Z=Z+DZ
0198 C
0199 C
0200 CC  THE LAST RADIAL BAND HAS BEEN REACHED. FILL IN ALL OF THE NODES
0201 C
0202  160 DO 170 I=1,N
0203   DO 170 J=1,M
0204   IF(RNI(I,J).GT.0.0) GO TO 170
0205   RNI(I,J)=FM
0206   170 CONTINUE
0207 C
0208 CC  PRINT OUT THE CALCUALTED VALUES IF PREFP0S = 1
0209 C
0210 C
0211 IF (PREFP0S.EQ.0) GO TO 200
0212 WRITE (6,2)((RNI(I,J),J=1,M),I=1,N)
0213  2 FORMAT (///,'CALCULATED VALUES FROM SUBROUTINE REFP0S',///,10X,
0214   1 'RNI = ',///,(N),(M),(I3,5X),//0,A,///)
0215 C
0216 C
0217 C
0218 RETURN
0219 C
0220 END
8.3.5 Subroutine Control
SUBROUTINE CONTROL

Written by Kurt B. Carlson

April 15, 1986

Purpose: Control the flow of subroutines called to calculate the temperature field in the refractory hearth and determine if the program must be reiterated to calculate a more accurate solution, to check if the solution has been obtained, or to verify that the program is not converging.

Called by Master Program Shelter

Subroutines called:

TINT
COEFMAT
PREPPOS
IMSL - LEQT2F

Form: CONTROL (NIP,DTM,KC,RPM,T,QTOTR,QTOTC,ITER)

Input (in call statement or in common blocks):

NIP = maximum number of iterations of the solution of the temperature
DIT = field allowed
DM = convergence criterion of consecutive solutions (deg F)

Output:

T = calculated value of the temperature field in the refractory hearth
QTOTR = the total radiative heat loss from the steel shell
QTOTC = the total convective heat loss from the steel shell
ITER = the number of trials used to achieve the solution

Subroutine control (NIP,DTM,KC,RPM,T,QTOTR,QTOTC,ITER)

Implicit, real, integer, and common statements

Implicit None

Integer RNI(10,10),M,N,KC(5,5),NRM,FMT,ITER,ICH,NA,NM,IA,K,DGST,NIP

Integer NRM,ITER,IER

Integer PTHRCN,POEFCAL,PINTCEPT,POEFMAT,PREPPOS,PINT,TCEL,CPPM

Integer PCHECK

Real CM(10,10),B(5),WAREA,THEN(10,10),DTM,TRS

Real CM(10,10),B(5),WAREA,THEN(10,10),DTM,TRS

Common/DIMENSION,RO,DS,DR,DT,DZ,T(10,10),TS,TA,KT,E

Common/TEMP,TS,TA,TR,E

Common/REFRAC/RM,FM,N,DIR

Common/FINE/RO,IS,TR,DZ

Common/XWRITE/PTHRCN,POEFCAL,PINTCEPT,POEFMAT,PREPPOS,PINT,
CONTROL

0058 1  PCALC,PCONTROL

0059 C  CALCULATE THE INITIAL VALUES OF THE TEMPERATURE FIELD

0060 C  CALL TINT (T)

0061 C  ASSIGN EACH NODE A DISTINCTIVE REFRACTORY MATERIAL

0062 C  CALL REPPOS (KC,RPM,RNI)

0063 C  START THE ITERATION OF THE PROGRAM. ITER IS THE NUMBER OF ITERATIONS

0064 CC  USED IN THE PROGRAM

0065 C  ITER=0

0066 C  CALCULATE THE VALUES OF THE COEFFICIENT MATRIX

0067 C  140 CALL COEFMAT (RNI,T,KC,RPM,CM,L,QTOIR,QTOTC)

0068 C  CALCULATE THE VALUE OF THE TEMPERATURE FIELD USING IMSL SUBROUTINE

0069 C  LENT2P WHICH PERFORMS GAUSSIAN ELIMINATION (CRUT ALGORITHM) WITH

0070 C  EQUILIBRATION, PARTIAL PIVOTING, AND ITERATIVE IMPROVEMENT AS REQUIRED.

0071 C  SET UP A DUMMY MATRIX A OF JUST COEFFICIENTS AND B AS THE REMAINDER TERMS

0072 C  DO 10 I=1,M\(A\)

0073 C  DO 10 J=1,M\(A\)

0074 C  10 A(I,J)=CM(I,J)

0075 C  DO 20 I=1,M\(A\)

0076 C  20 B(I)=CM(M\(A\)+I,1)

0077 CC  THE INPUT AND OUTPUT OF THE PROGRAM IS AS FOLLOWS:

0078 C  INPUT:

0079 C  A = THE COEFFICIENT MATRIX OF DIMENSION M\(A\)*M\(A\)

0080 C  NN = 1 THE NUMBER OF REMAINDER COLUMNS

0081 C  NA = M\(A\) THE ORDER OF MATRIX A

0082 C  IA = ROW DIMENSION OF A AND B AS SPECIFIED IN THE CALLING SUBROUTINE

0083 C  B = THE REMAINDER MATRIX OF DIMENSION 1 BY M\(A\)

0084 C  IDG1 = THE NUMBER OF DECIMAL POINTS OF ACCURACY REQUIRED

0085 C  N\(A\) = WORK ARE OF DIMENSION EQUAL TO (M\(A\)2)+3*M\(A\)

0086 C  OUTPUT:

0087 C  B = THE TEMPERATURE FIELD OF DIMENSION 1 BY M\(A\)

0088 C  IER = ERROR MESSAGES

0089 C  IF = 34 ACCURACY TEST FAILED

0090 C  129 MATRIX IS ALGORITHMICALLY SINGULAR

0091 C  131 MATRIX IS ILL-CONDITIONED

0092 C

0093 CC  N\(A\) = M\(A\)

0094 CC  M\(A\) = 1

0095 CC  IA = K
CALL THE IMSL SUBROUTINE

CALL LQRT2F(A,N,M,NA,IA,B,IBOT,IMNRIA,AIE)

GENERATE ANY ERROR MESSAGES AND IF THE ERROR MESSAGES ARE GENERATED
THEN EXIT TO THE MAIN PROGRAM

IF (IER.EQ.34) GO TO 30
IF (IER.EQ.129) GO TO 40
IF (IER.EQ.131) GO TO 50
GO TO 60

WRITE (6,2)

2 FORMAT(/,'ACCURACY TEST FAILED IN THE IMSL SUBROUTINE - IER = 34',/)

WRITE (6,3)

3 FORMAT(/,'MATRIX IS ALGORITHMICALLY SINGULAR IN THE IMSL.',/)

WRITE (6,4)

4 FORMAT(/,'MATRIX IS ILL-CONDITIONED TO PASS THE ACCURACY TEST IN
THE IMSL SUBROUTINE, IER = 129',/)

GO TO 70

GO TO 70

WRITE (6,4)

GO TO 70

COMPARE THE CALCULATED VALUES OF B TO THE LAST CALCULATED VALUES OF T
TO SEE IF THERE ARE ANY VALUES OUTSIDE THE DESIRED CONVERGENCE CRITERION
IF THERE ARE THEN REITERATE THE PROGRAM, IF NOT THEN OUTPUT THE
PROGRAM WITH THE CORRECT VALUES.

CHANGE MATRIX B INTO THE FORM OF THE TEMPERATURE MATRIX T

DO 80 I=1,N
DO 80 J=1,M
80 T(I,J)=B((I-1)*J+J)
DO 90 I=1,N
DO 90 J=1,M
90 CONTINUE
GO TO 110

UPDATE THE PROGRAM USING THE VALUES OF TNEW AS THE VALUES OF T

100 DO 120 I=1,N
120 T(I,J)=TNEW(I,J)

UPDATE THE ITERATION COUNTER ITER

ITER=ITER+1

Determine if the iteration counter is greater than the maximum number
of iterations allowed NIP. If it is exit the program and create an
ERROR MESSAGE. If not re-iterate the solution method.

IF (ITER.GT.NIP) GO TO 130
GO TO 140
CONTROL

0172   130 WRITE (6,5)
0173   5 FORMAT(//,'MAXIMUM NUMBER OF SOLUTION ITERATIONS EXCEEDED',///)
0174      CC GENERATE GENERAL ERROR MESSAGE FOR ALL OF THE ERROR SIGNALS GENERATED
0175      CC
0176      70 WRITE(6,6)
0177      6 FORMAT(//,'SOLUTION WAS NOT ACHIEVED THE LAST CALCULATED VALUES',
0178            'A WERE:',///)
0179      GO TO 150
0180      CC
0181      CC GENERATE MESSAGE THAT THE SOLUTION METHOD CONVERGED TO A SOLUTION
0182      CC
0183      110 WRITE (6,7)
0184      7 FORMAT(///,'SOLUTION CONVERGED TO THE ANSWERS AS SHOWN BELOW:',///)
0185      150 CONTINUE
0186      CC
0187      CC WRITE THE CALCULATED VALUES IF PCONTROL = 1
0188      CC
0189      IF (PCONTROL.EQ.0) GO TO 200
0190      CC
0191      WRITE (6,8) (TNEW(I,J),J=1,M),I=1,N)
0192      8 FORMAT (///,'CALCULATED VALUES FROM SUBROUTINE CONTROL',///)
0193      1 10X,'CALCULATED VALUES OF TNEW',<N>(10X,<M>(F7.3),///),///)
0194      200 CONTINUE
0195      CC
0196      CC
0197      RETURN
0198      END
8.3.6 Subroutine Coefmat
SUBROUTINE COEFMAT

WRITTEN BY KURT B. CARLSON

APRIL 12, 1986

PURPOSE: DIRECT THE CALCULATION OF THE VALUES IN THE COEFFICIENT MATRIX.

THIS SUBROUTINE IDENTIFIES WHAT TYPE EACH NODE IS AND CALLS

SUBROUTINE DIRECT AND EQUATION FOR

THAT NODE. SUBROUTINE INTERCEPT IS ALSO CALLED TO INTERMESH THE

GRID SYSTEM WITH THE STEEL SHELL.

FORM: COEFMAT(RNI,T,KC,RPM,CM,L,OTOTK,OTOTC)

INPUT(THROUGH THE CALL STATEMENT OR COMMON BLOCKS):

SA = THE SLOPE OF THE HEMISPHERICAL HEAD

DB = THE DEPTH OF THE SILVER LAYER (INCHES)

DH = THE DEPTH OF THE REFRACTORY HEARTH (INCHES)

DR = THE RADIUS OF THE STEEL CYLINDER (INCHES)

DR = THE RADIAL NODE INCREMENT (INCHES)

DZ = THE AXIAL NODE INCREMENT (INCHES)

M = THE NUMBER OF NODES IN THE AXIAL DIRECTION

N = THE NUMBER OF NODES IN THE RADIAL DIRECTION

RNI = THE MATRIX THAT IDENTIFIES EACH NODE WITH A DISTINCT

REFRACTORY MATERIAL

KC = THE MATRIX THAT CONTAINS THE VALUES OF THE COEFFICIENTS FOR

THE POLYNOMIAL EQUATION DESCRIBING THE THERMAL CONDUCTIVITY

OF EACH REFRACTORY MATERIAL

NI = THE ORDER OF THE POLYNOMIAL EQUATION

NRM = THE NUMBER OF REFRACTORY MATERIALS USED

L = THE LAST CALCULATED VALUE OF THE TEMPERATURE FIELD (DEG F)

TS = THE TEMPERATURE OF THE SLAG/SILVER INTERFACE (DEG F)

TA = THE TEMPERATURE OF THE AMBIENT AIR SURROUNDING THE SMELTER

TR = THE TEMPERATURE OF THE SURFACES SURROUNDING THE SMELTER

E = THE EMISIVITY OF THE SMELTER AND THE SURROUNDING SURFACES

L = THE LENGTH OF THE HEMISPHERICAL SHELL (INCHES)

OUTPUT:

CM = THE CALCULATED VALUES OF THE COEFFICIENT MATRIX

OTRK = THE HEAT LOST FROM THE SHELL BY RADIATION (BTU/HR)

OTOTK = THE HEAT LOST FROM THE SHELL BY CONVECTION (BTU/HR)

SUBROUTINE COEFMAT(RNI,T,KC,RPM,CM,L,OTOTK,OTOTC)

IMPLICIT, COMMON, REAL, AND INTEGER STATEMENTS

IMPLICIT NONE

INTEGER RNI(10,10),K,N,KC(5),NRM,EM,I,J,MO,NN1,NN2,II,NI,NMR

INTEGER PTH,CON,POGFCAL,PINTCPT,POGFCMAT,PREPPOS,PRINT,PCALC,

PCONTROL
0058  REAL CM(10,10),LT,GTOR,GTDC,T,SA,DS,DN,RO,DR,DZ,T(10,10),TS,TA,TE
0059  REAL RI,RJ,RS,RT,TS,RS(5)
0060  COMMON/DIMEN/RO,DS,DN,TR,TS,SA
0061  COMMON/TERM/TS,TA,TR,TE
0062  COMMON/REFER/NO,FM,NI,NRM
0063  COMMON/FIXEL/N,M,DR,DZ
0064  COMMON/XWRITE/PRINT,,PCOEFCAL,PCOEFCAL,PINTEP,PCOEFCAL,PREFPOS,PINT,
0065  1 PCALC,PCONTROL
0066  C
0067  CC  INITIALIZE THE COEFFICIENT MATRIX AT 0.0
0068  C
0069  DO 10 I=1,N
0070  DO 10 J=1,M
0071  10 CM(I,J)=0.0
0072  C
0073  CC  INITIALIZE THE COUNTERS FOR THE POSITIONS IN THE COEFFICIENT MATRIX
0074  CC  I = THE ROW INDICATOR
0075  CC  J = THE COLUMN INDICATOR
0076  C
0077  I=1
0078  J=1
0079  C
0080  CC  AT I=1, J=1 THE NODE INDICATOR (ND) THAT IS USED TO DIRECT THE PROGRAM
0081  CC  TO THE CORRECT HEAT BALANCE EQUATION IN SUBROUTINE COEFCAL IS EQUAL TO 2
0082  C
0083  ND=2
0084  CALL COEFCAL(RNI,I,J,ND,NC,CM,1,B1,B2,C,GTOR,GTDC)
0085  C
0086  CC  IDENTIFY ALL OF THE NODES ON THE AXIAL BAND FOR I=1
0087  C
0088  C
0089  CC  NN1 IDENTIFIES HOW MANY NODES THERE ARE IN THIS BAND.  SUBROUTINE PINTEP
0090  CC  CALCULATES THE POINT WHERE THE OUTER EDGE OF THIS BAND INTERSECTS THE STEEL
0091  C
0092  R=DR/2.0
0093  CALL PINTEP(1,1,NN1,B1,B2,NN2,C)
0094  C
0095  CC  IDENTIFY THE NODES IN THIS TYPE OF BAND AS:
0096  C
0097  CC  ND = 3 CENTERLINE NODE BETWEEN (1,1) AND (1,M)
0098  CC  ND = 4 CENTERLINE NODE AT THE BOTTOM
0099  C
0100  J=2
0101  20 IF(J.EQ.NN2) GO TO 30
0102  ND=3
0103  CALL COEFCAL(RNI,I,J,ND,NC,CM,1,B1,B2,C,GTOR,GTDC)
0104  J=J+1
0105  GO TO 20
0106  30 ND=4
0107  CALL COEFCAL(RNI,I,J,ND,NC,CM,1,B1,B2,C,GTOR,GTDC)
0108  C
0109  CC  GO TO THE NEXT AXIAL BAND AND IDENTIFY EACH OF THE NODES FROM THE
0110  CC  SLAG/SILVER INTERFACE TO THE SHELL.
0111  C
0112  40 I=I+1
0113  R=R+DR
0114  IF(I.EQ.N) GO TO 50
11*2


J=1

CALL COEFCAL(RNI, I, J, ND, KC, RPM, CM, L, B1, B2, C, OTOTR, OTOTC)

GO TO 70


60 J=1

CALL COEFCAL(RNI, I, J, ND, KC, RPM, CM, L, B1, B2, C, OTOTR, OTOTC)

FOR THE CASE OF ND EQUAL TO 5 AND 9 CALCULATE THE INTERCEPT POINT OF THE AXIAL BAND AND THE HEMISPHERICAL SHELL.

70 CALL INCEPT(I, J, NN1, B1, B2, NN2, C)

CALCULATE THE VALUES OF THE COEFFICIENT MATRIX FOR J=2 TO NN2-1 WHICH REPRESENTS THE TYPICAL NODE

IF(I.EQ.NN2) GO TO 80

J=J+1

GO TO 90

CALCULATE THE VALUES OF THE COEFFICIENT MATRIX FOR THE NODES THAT INTERSECT THE HEMISPHERICAL STEEL SHELL.

Determine if the steel shell intersects one or two nodes

80 IF(NNI.EQ.NN2) GO TO 100

THE SHELL INTERSECTS THE AXIAL BAND AT TWO NODES

FIRST CONSIDERING THE UPPER NODE

J=NN1

CALL COEFCAL(RNI, I, J, ND, KC, RPM, CM, L, B1, B2, C, OTOTR, OTOTC)

CONSIDERING THE BOTTOM NODE

J=NN2

CALL COEFCAL(RNI, I, J, ND, KC, RPM, CM, L, B1, B2, C, OTOTR, OTOTC)

GO TO 110

THE SHELL INTERSECTS THE AXIAL BAND AT ONLY ONE PLACE

COEFCAL
100  J=NN2
173  ND=6
174  CALL COEFCAL(RNI,T,I,J,ND,KC,RPM,CM,L,B1,B2,C,OTOTK,OTOTC)
176  PROCEED TO THE NEXT AXIAL BAND
177  
178  110  B1=B2
179  NN1=NN2
180  GO TO 40
181  
182  THIS AXIAL BAND IS THE BAND THAT INTERSECTS OR HAS FOR ITS OUTER
183  BOUNDARY THE CYLINDRICAL STEEL SHELL
184  
185  THE FIRST NODE IS AT THE TOP OF THE SIDEWALL WHERE ND =10
186  
187  50  J=1
188  ND=10
189  CALL COEFCAL(RNI,T,I,J,ND,KC,RPM,CM,L,B1,B2,C,OTOTK,OTOTC)
190  
191  CALCULATE THE INTERCEPT POINT OF THIS AXIAL BAND AND THE HEMISPHERICAL
192  STEEL SHELL
193  
194  CALL INCEPT(I,J,NN1,B1,B2,NN2,C)
195  
196  CALCULATE THE VALUES OF THE COEFFICIENT MATRIX FOR J EQUAL TO 2 TO NN2-1
197  WHERE ND = 11.
198  
199  120  IF(J.EQ.NN2) GO TO 130
200  J=J+1
201  ND=ND
202  CALL COEFCAL(RNI,T,I,J,ND,KC,RPM,CM,L,B1,B2,C,OTOTK,OTOTC)
203  
204  CALCULATE THE VALUES OF THE COEFFICIENT MATRIX FOR THE NODE AT THE
205  INTERSECTION OF THE CYLINDER AND THE HEMISPHERICAL HEAD. IF THE
206  DISTANCE FROM THE TOP OF THE NODE TO THE INTERSECTION POINT IS GREATER
207  THAN HALF THE SIZE OF THE TOTAL HEIGHT OF THE NODE THEN ND = 11.
208  
209  IF NOT THEN ND =6.
210  
211  130  IF(R2.GT.B1/2.0) GO TO 140
212  ND=6
213  CALL COEFCAL(RNI,T,I,J,ND,KC,RPM,CM,L,B1,B2,C,OTOTK,OTOTC)
214  GO TO 150
215  
216  140  ND=11
217  CALL COEFCAL(RNI,T,I,J,ND,KC,RPM,CM,L,B1,B2,C,OTOTK,OTOTC)
218  
219  150  CONTINUE
220  
221  WRITE OUT THE CALCULATED VALUES IF PCOEFCM=1
222  
223  IF (PCOEFCM.EQ.0) GO TO 160
224  WRITE (6,2) (CM(I,J),J=1,M),I=1,N),OTOTK,OTOTC
225   2 FORMAT (/,'CALCULATED VALUES FROM COEFCAL SUBROUTINE',
226       'CALCULATED VALUES OF THE COEFFICIENT MATRIX',/,
227       'M(10X,M (/5.3,5X))//,10X,OTOTK = ',/5.3,/,)
228  
229  160  CONTINUE
COEPMAT 9-Jun-1986 18:26:11 VAX FORTRAN V4.4-177
Page 5
9-Jun-1986 17:19:10 USER6:CKBC86@9ICOEPMAT.FOR;11

0229 CC
0230 C
0231 RETURN
0232 END

PROGRAM SECTIONS

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ENTRY POINTS

Address Type Name
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8.3.7 Subroutine Intcept
SUBROUTINE INCEPT

WRITE BY KURT B. CARLSON
APRIL 9, 1986

PURPOSE: CALCULATE THE INTERCEPT POINTS OF THE HOMAL SYSTEM AND THE
HEMISPHERICAL STEEL SHELL WHOSE SLOPE IS:

Z(INTERCEPT)=SAR + (DS+DH)

FORM: SUBROUTINE INCEPT (I,J,NN1,B1,B2,NN2,C)

INPUT (IN THE CALL OR COMMON STATEMENTS):

I = THE RADIAL COORDINATE OF THE NODE UNDER CONSIDERATION
J = THE AXIAL COORDINATE OF THE NODE UNDERT CONSIDERATION
SA = THE SLOPE OF THE HEMISPHERICAL STEEL SHELL
DS = THE DEPTH OF THE SILVER LAYER
DH = THE DEPTH OF THE REFRACTORY HEARTH
DZ = THE AXIAL NODE INCREMENT
DR = THE RADIAL NODE INCREMENT

NN1 = THE NUMBER OF AXIAL NODES IN THE PREVIOUS AXIAL BAND
B1 = THE HEIGHT OF THE INWARD SIDE OF THE LAST NODE IN THIS BAND
WHICH WAS PREVIOUSLY CALCULATED AS THE OUTWARD SIDE HEIGHT
OF THE LAST NODE IN THE PREVIOUS BAND

OUTPUT:

NN2 = THE NUMBER OF AXIAL NODES IN THIS RADIAL BAND
B2 = THE HEIGHT OF THE OUTWARD SIDE OF THIS NODE
C = THE RADIAL INTERCEPT POINT OF THE SHELL WITH THE NEXT TO LAST

SUBROUTINE INCEPT(I,J,NN1,B1,B2,NN2,C)

IMPLICIT, REAL, INTEGER, AND COMMON STATEMENTS

IMPLICIT NONE

INTEGER I,J,NN1,NN2,II,JJ,M
INTEGER PHIENC,PHELICAL,PINTERP,PDEP,PINT,PCALC
INTEGER PCTRL
REAL B1,B2,C,ZINT,A,SUM1,RO,DS,DH,TRS,SA,DR,DZ
COMMON/DIMEN/RO,DS,DH,TRSA
COMMON/PHIENC/NN1,NN2,M,DR,DZ
COMMON/XWRITE/PHELICAL,PDEP,PINTERP,PDEP,PINT,
PCALC,PCTRL

CALCULATE THE INTERCEPT POINT AT R + DR

A=0.0
B=5.0
C=1.0
D=A-1.0
INTCEPT

ZINT=SAA((AMDR)+(DR/2.0))+((DS+DH)

0050  C
0059  C
0060  C
0061  C
0062  C
0063  C
0064  C
0065  C
0066  C
0067  C
0068  C
0069  C
0070  C
0071  C
0072  C
0073  C
0074  C
0075  C
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0107  C
0108  C
0109  C
0110  C
0111  C
0112  C
0113  C
8.3.8 Subroutine Coefcal
SUBROUTINE COEFCAL

APRIL 10, 1986

PURPOSE: THIS SUBROUTINE CALCULATES THE VALUES OF THE COEFFICIENT MATRIX
FOR THE PROGRAM. THIS SUBROUTINE CALLS THERCON TO CALCULATE
THE VALUES OF THE THERMAL CONDUCTIVITIES. THIS SUBROUTINE
IS COMPOSED OF THE DESCRIPTIVE EQUATIONS FOR THE HEAT BALANCE
AROUND THE ELEVEN DIFFERENT DISTINCTIVE TYPES OF NODES IN THE
MODEL.


INPUT (FROM THE CALL OR COMMON STATEMENTS):
N = THE NUMBER OF NODES IN THE RADIAL DIRECTION
M = THE NUMBER OF NODES IN THE AXIAL DIRECTION
T = THE LAST CALCULATED VALUES OF THE TEMPERATURE FIELD
I = THE RADIAL COORDINATE OF THE NODE UNDER CONSIDERATION
J = THE AXIAL COORDINATE OF THE NODE UNDER CONSIDERATION
ND = THE NODE IDENTIFIER TRANSFERRED FROM SUBROUTINE COEFMAT THAT
IDENTIFIES THE NODE UNDER CONSIDERATION AS ONE OF THE ELEVEN
DISTINCTIVE TYPES OF NODE IN THE SIMULATION
DZ = THE AXIAL NODE INCREMENT (INCHES)
DR = THE RADIAL NODE INCREMENT (INCHES)
TS = THE TEMPERATURE OF THE SLAG/SILVER INTERFACE (DEG F)
TA = THE AMBIENT AIR TEMPERATURE AROUND THE SHELL (DEG F)
TR = THE TEMPERATURE OF THE RADIATIVE SURFACES AROUND THE SHELL
(TOT C) (DEG F)
E = THE EMISSIVITY OF THE STEEL SHELL
CM = THE PREVIOUSLY CALCULATED VALUES OF THE COEFFICIENT MATRIX
QTOTR = THE PREVIOUSLY CALCULATED RADIATIVE HEAT LOSS FROM THE
STEEL SHELL (BTU/HR)
QTOTC = THE PREVIOUSLY CALCULATED CONVECTIVE HEAT LOSS FROM THE
STEEL SHELL (BTU/HR)

OUTPUT:
CM = THE UPDATED VALUES IN THE COEFFICIENT MATRIX
QTOTC = UPDATED VALUE OF THE CONVECTIVE HEAT LOSS FROM THE STEEL SHELL
QTOTR = UPDATED VALUE OF THE RADIATIVE HEAT LOSS FROM THE STEEL SHELL

SUBROUTINE COEFCAL(RNI,T,I,J,ND,KC,RPM,CM,L,B1,B2,C,QTOTR,QTOTC)

IMPLICIT, REAL, INTEGER, AND COMMON STATEMENTS

IMPLICIT NONE
INTEGER ND,L1,L2,L3,L4,L5,L6,N,M,I,J,RPM(10,10),RNI(10,10),FM,II
INTEGER NRK,NK,NRM
REAL PI,SB,A,B,MM,MP,CM(10,10),L,QR,QC,HC,HS,QTOTR,QTOTC,A1,A2,A3
REAL T(10,10),KU,KE,KD,KN,KO,B2,B1,C,A4,DZ,TR,TS,TA,E,DS,RO
REAL DR,DM,TR,SA,KC(5,5)
COMMON/DIMEN/R0,DS,DR,TR,SA
CALL SUBROUTINE THERCON TO CALCULATE THE NET THERMAL CONDUCTIVITIES OF THE NODE UNDER CONSIDERATION AND ITS FOUR SURROUNDING NEIGHBORS

CALL THERCON(RHI,T,J,KE,KO,KI,KD,KU)

CALCULATE THE VALUES OF THE CONSTANTS USED IN THE MATRIX COEFFICIENT CALCULATIONS

SB = STEPHEN-BOLTZMAN CONSTANT (BTU/HR-FT**2-(DEG R)**4

PI=3.14159
SB=0.1714E-8
IF(1.EQ.1) GO TO 10
A=0.0
DO 5 II=1,1
5 A=A+1.0
MM=(1.0-1.0/(2.0A))
MP=(1.0-1.0/(2.0A))

USING THE VALUE OF KD TO STEER THE SUBROUTINE TO THE PROPER TYPE
OF NODAL CALCULATIONS THE VALUES OF THE COEFFICIENT MATRIX IS CALCULATED. THE COEFFICIENT MATRIX IS OF THE SIZE M*N+1 BY M*N WHERE THE LAST COLUMN IS FILLED WITH THE REMAINDER TERMS. THE COLUMNS ARE SET AS FOLLOWS:

COLUMN = (I-1)*J + 1
ROW = (I-1)*J + 1

SET UP THE MATRIX POSITION LOCATORS (L1 TO L6) THAT INPUT THE VALUES OF THE COEFFICIENT MATRIX INTO THE PROPER POSITION

IF ND = 1 THEN THE NODE IS A TYPICAL NODE WITHIN ALL OF THE GIVEN BOUNDARIES

IF(ND.GT.1) GO TO 20
CM(L1,L1)=-DZ*(K0+MP+KM+MM)-(DR**2.0)*(KD*KU)
CM(L1,L2)=K0+MP+DZ
CM(L1,L3)=K1+MM+DZ
CM(L1,L4)=KD*(DR**2.0)
CM(L1,L5)=KU*(DR**2.0)
CM(L1,L6)=0.0
GO TO 200

IF ND = 2 THEN THE NODE IS A CORNER NODE AT (1,1)

20 IF(ND.GT.2) GO TO 30
CM(L1,L1)=-K0+(1.0/ALOG((1.0+1.0+DR)/((1.0+2.0)*DR)))*DZ-2.0*PI*A
COEFCAL

0115  ^  *(DRAA2.0)*(KD+KE)
0116  CM(L1,L2)=KO*(1.0/(ALOG(((A+1.0)*DR)/((A+2.0)*DR))))*DZ
0117  CM(L1,L3)=0.0
0118  CM(L1,L4)=2.0*PI*A*KDA*AA*(DRAA2.0)
0119  CM(L1,L5)=0.0
0120  CM(L1,L6)=0.0
0121  GO TO 200
0122  C
0123  C
0124  CC IF ND = 3 THEN THE NODE IS A CENTERLINE NODE BETWEEN THE TOP AND BOTTOM
0125  C
0126  30 IF(ND.GT.3) GO TO 40
0127  CM(L1,L1)=KO*(1.0/(ALOG(((A+1.0)*DR)/((A+2.0)*DR))))*DZ-2.0*PI*
0128  ^  *(A+1.0)*(DRAA2.0)+(KD+KE)
0129  CM(L1,L2)=KO*(1.0/(ALOG(((A+1.0)*DR)/((A+2.0)*DR))))*DZ
0130  CM(L1,L3)=0.0
0131  CM(L1,L4)=2.0*PI*(A+1.0)*KDA*(DRAA2.0)
0132  CM(L1,L5)=0.0
0133  CM(L1,L6)=0.0
0134  GO TO 200
0135  C
0136  CC IF ND = 5 THEN THE NODE IS AT THE SLAG/SILVER INTERFACE
0137  C
0138  40 IF(ND.EQ.5) GO TO 50
0139  GO TO 60
0140  50 CM(L1,L1)=DZ*(KO+MP+KMM)*(DRAA2.0)*(KD+KE)
0141  CM(L1,L2)=KO+MP+DZ
0142  CM(L1,L3)=KMM+DZ
0143  CM(L1,L4)=KDA*(DRAA2.0)
0144  CM(L1,L5)=0.0
0145  CM(L1,L6)=KE*(DRAA2.0)*TS
0146  GO TO 200
0147  C
0148  CC IF ND = 9 THEN THE NODE IS AT THE UPPERFACE OF THE REFRATORY SIDEWALL
0149  CC EXCEPT AT (N,1)
0150  C
0151  60 IF(ND.EQ.9) GO TO 70
0152  GO TO 80
0153  70 CM(L1,L1)=DZ*(KO+MP+KMM)-(DRAA2.0)*KD
0154  CM(L1,L2)=KO+MP+DZ
0155  CM(L1,L3)=KMM+DZ
0156  CM(L1,L4)=KDA*(DRAA2.0)
0157  CM(L1,L5)=0.0
0158  CM(L1,L6)=0.0
0159  GO TO 200
0160  C
0161  CC CALCULATE THE VALUE OF THE HEAT TRANSFER COEFFICIENT FOR THE HEMISPHERICAL
0162  CC STEEL SHELL FOR NODES 4,6,7,8
0163  C
0164  80 IF(ND.GT.9) GO TO 90
0165  HS=1.0
0166  C
0167  CC IF ND = 4 THEN THE NODE IS A CENTERLINE NODE AT (1,M)
0168  C
0169  IF (ND.GT.4) GO TO 100
0170  A1=((LAA2.0)-(ROAA2.0))/(ROAL)*PI*DR=(SQRT((DRAA2.0)-
0171  ^(DZ-B2)**2.0)))
COEFCAL

20-May-1986 18:50:31

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0172 CM(LI,LI)=-K0*(1.0/(ALOG(((A+1.0)*DR)/(((A+2.0)*DR)))))*B2-2.0*

0173 *PI*(A+1.0)*(DR+2.01)KU-HS*A1

0174 CM(LI,LI2)=K0*(1.0/(ALOG(((A+1.0)*DR)/(((A+2.0)*DR)))))*B2

0175 CM(LI,L1)=0.0

0176 CM(LI,L4)=0.0

0177 CM(LI,L5)=2.0*PI*(A+1.0)*(DR+2.0)KU

0178 QR=SB*E*A1*(TS+646.0)**4.0-(TI, J)+466.0)**4.0)

0179 QC=HS*A1*TA

0180 CM(LI,L6)=QR+QC

0181 CM(LI,L6)=QR+QC

0182 CM(LI,L6)=QR+QC

0183 IF ND = 6 THEN THE NODE IS ON THE HEMISPHERICAL STEEL SHELL WHERE NN1=NN2

0184

0185 100 IF (ND.GT.6) GO TO 110

0186 A2=((1.0+2.0)~(ROA+2.0))/((ROA)~PI*DR*(SQRT((DR+2.0)+

0187 ((B2-B1)**2.0)))

0188 CM(LI,L1)=-2.0*PI*A*B2*KONMF+BI*KI**MM+(DR+2.0)KU-HS*A2

0189 CM(LI,L2)=2.0*PI*A*B2*KONMF

0190 CM(LI,L3)=2.0*PI*A*B1*KI**MM

0191 CM(LI,L4)=0.0

0192 CM(LI,L5)=2.0*PI*A***(DR+2.0)KU

0193 QR=SB*E*A2*(TS+646.0)**4.0-(TI, J)+466.0)**4.0))

0194 Q=HS*A2*TA

0195 CM(LI,L6)=QR+QC

0196 CM(LI,L6)=QR+QC

0197 CM(LI,L6)=QR+QC

0198 IF ND = 7 THEN THE NODE IS ON THE HEMISPHERICAL STEEL SHELL WHERE

0199 NN1=NN2 AND THIS IS THE TOP NODE

0200

0201 110 IF (ND.GT.7) GO TO 120

0202 A2=((1.0+2.0)~(ROA+2.0))/((ROA)~PI*DR*(SQRT((DR+2.0)+

0203 ((B2-B1)**2.0)))

0204 CM(LI,L1)=-2.0*PI*A*(B2*KONMF+DG*KI**MM)-KU*HSA*(DR+2.0)*PI*CA*(DR+2.0)**

0205 **-1.0***(C))**KD**HSA**A4

0206 CM(LI,L2)=2.0*PI*A*B2*KOMF

0207 CM(LI,L3)=2.0*PI*A*D2*KI**MM

0208 CM(LI,L4)=KD*PI*CA*(DR+2.0)**A1**C)

0209 CM(LI,L5)=KU*2.0*PI*A***(DR+2.0)

0210 QR=SB*E*A4*(TS+646.0)**4.0-(TI, J)+466.0)**4.0))

0211 Q=HS*A4*TA

0212 CM(LI,L6)=QR+QC

0213 CM(LI,L6)=QR+QC

0214 CM(LI,L6)=QR+QC

0215 IF ND = 8 THEN THE NODE IS ON THE HEMISPHERICAL STEEL SHELL WHERE

0216 NN1=NN2 AND THIS IS THE BOTTOM NODE

0217

0218 120 A3=((1.0+2.0)~(ROA+2.0))/((ROA)~PI*(DR+2.0)**A1**C)**

0219 *(SQRT((C**2.0+2.0)**B2**2.0)))

0220 CM(LI,L1)=2.0*PI*A*B1*KMM*KI**PI*L*(DR+2.0)**A1**C-HS*A3

0221 CM(LI,L2)=0.0

0222 CM(LI,L3)=2.0*PI*A*B1**KMM

0223 CM(LI,L4)=0.0

0224 CM(LI,L5)=K*I**PI*L*(DR+2.0)**A1**C)

0225 QR=SB*E*A3*(TS+646.0)**4.0-(TI, J)+466.0)**4.0))

0226 Q=HS*A3)*TA

0227 CM(LI,L6)=QR+QC

0228 GO TO 200
0229 C
0230 CC  CALCULATE THE VALUE OF THE CONVECTIVE HEAT TRANSFER COEFFICIENT FOR
0231 CC  THE CYLINDRICAL SIDEWALL FOR NODES 10 AND 11
0232 C
0233   20  HC=1.0
0234   20  B=J
0235 C
0236 C  IF ND = 10 THEN THIS NODE IS AT THE SIDEWALL UPPERFACE AT (N,1)
0237 C
0238   20  IF(ND GT 10) GO TO 130
0239   20  CM(L1,L1)=-BA(MM+K1+DZ-KD*(DR+2.0))-HC*RO+DZ
0240   20  CM(L1,L2)=0.0
0241   20  CM(L1,L3)=BAMM+K1+DZ
0242   20  CM(L1,L4)=BKA*(DR+2.0)
0243   20  CM(L1,L5)=0.0
0244   20  QC=HC*RO+DZ+TA
0245   20  GO TO 200
0246 C
0247 C  IF ND = 11 THEN THIS NODE IS ON THE CYLINDRICAL OUTER WALL
0248 C
0249 C
0250 C
0251   20  CM(L1,L1)=-BA(MM+K1+DZ-(DR+2.0)*(KD+K1+DZ)+HC*RO+DZ
0252   20  CM(L1,L2)=0.0
0253   20  CM(L1,L3)=BAMM+K1+DZ
0254   20  CM(L1,L4)=BKA*(DR+2.0)
0255   20  CM(L1,L5)=BKA*(DR+2.0)
0256   20  QC=HC*RO+DZ+TA
0257   20  CM(L1,L6)=QR+QC
0258 C
0259 C  UPDATE THE COUNTERS FOR THE HEAT LOSS FROM THE SHELL BY RADIATION AND
0260 C  CONVECTION
0261 C
0262 C
0263   20  QTDR=QTDR+QR
0264 QTDC=QTDC+QC
0265 C
0266 C
0267 C
0268 RETURN
0269 END
8.3.9 Subroutine Thercon
SUBROUTINE THERCON

WRITTEN BY KURT B. CARLSON
APRIL 8, 1986

PURPOSE: THIS SUBROUTINE CALCULATES THE VALUE OF THE THERMAL CONDUCTIVITY OF THE NODE UNDER CONSIDERATION AND ITS FOUR SURROUNDING NODES. THE NET THERMAL CONDUCTIVITY BETWEEN THE NODE AND ITS NEIGHBORS IS THEN CALCULATED. THE THERMAL CONDUCTIVITY EQUATION IS IN THE FORM OF A 2ND ORDER POLYNOMIAL (K = A + B*T + C*T**2 + ....).

FORM: THERCON(RNI,T,I,J,KC,KE,KO,KI,KD,KU)

INPUT (IN THE CALL STATEMENT OR IN COMMON BLOCKS):
RNI = THE POSITION MATRIX INDICATING THE TYPE OF REFRACTORY MATERIAL IN EACH OF THE NODES
NRM = NUMBER OF DIFFERENT REFRACTORY MATERIALS USED
NI = THE ORDER OF THE POLYNOMIAL EQUATION
T = THE LAST CALCULATED VALUES OF THE TEMPERATURE FIELD
I = THE ROW NUMBER OF THE NODE UNDER CONSIDERATION
J = THE COLUMN NUMBER OF THE NODE UNDER CONSIDERATION

OUTPUT: (NOTE: ALL THERMAL CONDUCTIVIES IN BTU-IN/HR-FT**2-DEG F)
KE = THE THERMAL CONDUCTIVITY OF THE NODE UNDER CONSIDERATION
KO = THE NET THERMAL CONDUCTIVITY BETWEEN KE AND THE NEXT NODE OUTWARDS IN THE RADIAL DIRECTION
KI = THE NET THERMAL CONDUCTIVITY BETWEEN KE AND THE PREVIOUS NODE INWARDS IN THE RADIAL DIRECTION
KD = THE NET THERMAL CONDUCTIVITY BETWEEN KE AND THE NEXT NODE IN THE AXIAL DIRECTION
KU = THE NET THERMAL CONDUCTIVITY BETWEEN KE AND THE PREVIOUS NODE IN THE AXIAL DIRECTION

INTERNAL:
K1 = THE THERMAL CONDUCTIVITY OF THE (1+1,J) NODE
K2 = THE THERMAL CONDUCTIVITY OF THE (1-1,J) NODE
K3 = THE THERMAL CONDUCTIVITY OF THE (1,J+1) NODE
K4 = THE THERMAL CONDUCTIVITY OF THE (1,J-1) NODE

SUBROUTINE THERCON (RNI,T,I,J,KC,KE,KO,KI,KD,KU)

IMPLICIT, REAL, INTEGER, AND COMMON STATEMENTS

IMPLICIT NONE
INTEGER NRM,NI,I,J,LL,L,K,KK,RPM(5,5),II,IRD,FM
INTEGER PMERCON,PCDEFCAL,PINCPECT,PCOEPMAT,PREFPOS,PTINT,PCALC
INTEGER PCONTROL
REAL KC(5,5), RNI(10,10), T(10,10), KE, K0, K1, KD, KU, TT, SUM1
REAL K1, K2, K3, K4
COMMON/REFRAC/RND, FM, NI, NR
COMMON/XWRITE/THERCON, PCOEFCAL, PCOEFPAT, PPREP0S, PTINT,
          1 PCALC, PCONTROL
C
SET SUBROUTINE COUNTERS TO ZERO AND TRANSFER I AND J TO L AND K,
RESPECTIVELY
L=1
K=J
LL=0
CALCULATE THE VALUE OF THE THERMAL CONDUCTIVITY, SUM1
DO 20 II=2, NI+2
     20 SUM1=SUM1+KC(KK,II)*TT**2(II-2)
CALCULATE THE VALUE OF THE THERMAL CONDUCTIVITY FOR THE FOUR
NEIGHBORING NODES
THE COUNTER LL SIGNIFIES THE FOLLOWING:
LL = 1 SUM1 = THE (I,J) ELEMENT
LL = 2 SUM1 = THE (I+1,J) ELEMENT
LL = 3 SUM1 = THE (I-1,J) ELEMENT
LL = 4 SUM1 = THE (I,J+1) ELEMENT
LL = 5 SUM1 = THE (I,J-1) ELEMENT
IF(LL.GT.1) GO TO 30
KE=SUM1
L=I+1
GOTO 10
30 IF(LL.GT.2) GO TO 40
     40 IF(LL.GT.3) GO TO 50
     50 IF(LL.GT.4) GO TO 60
     60 K4=SUM1
     70 K3=SUM1
     80 K2=SUM1
     90 K1=SUM1
     100 L=I
     110 K=J+1
     120 GO TO 10
     130 GO TO 10
     140 GO TO 10
     150 GO TO 10
     160 GO TO 10
     170 GO TO 10
     180 GO TO 10
     190 GO TO 10
     200 GO TO 10
     210 GO TO 10
     220 GO TO 10
     230 GO TO 10
     240 GO TO 10
     250 GO TO 10
     260 GO TO 10
     270 GO TO 10
     280 GO TO 10
     290 GO TO 10
     300 GO TO 10
     310 GO TO 10
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     400 GO TO 10
     410 GO TO 10
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     950 GO TO 10
     960 GO TO 10
     970 GO TO 10
     980 GO TO 10
     990 GO TO 10

CALCULATE THE NET THERMAL CONDUCTIVITY BETWEEN KE AND ITS SURROUNDING NODES
KO=(1.0/(1.0/KE+(1.0/K1)))
KI=(1.0/(1.0/KE+(1.0/K2)))