Vibration analysis of a thin moving web and its finite element implementation

Gavin Chunye Liu

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VIBRATION ANALYSIS OF A THIN MOVING WEB
AND ITS
FINITE ELEMENT IMPLEMENTATION

by

GAVIN CHUNYE LIU

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VIBRATION ANALYSIS OF A THIN MOVING WEB
AND ITS
FINITE ELEMENT IMPLEMENTATION

GAVIN CHUNYE LIU

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ABSTRACT

Equations of motion (lateral and axial) of an axially moving web are developed based on the Newton’s Second Law and the Euler-Bernoulli thin beam theory. The equations of motion in the axial direction are solved by using the fourth-order Runge-Kutta Method. The fourth-order, partial differential equation for the lateral motion is solved using Galerkin’s finite element method and the Three-point Recurrence Scheme. Effects of the flexibility of the end-supports, the weight of the web, the axial web speeds, the eccentricities of the rollers, and the applied torque to the web-roller system are studied.
# TABLE OF CONTENTS

NOTATION ........................................................................ vi
SYMBOLS ......................................................................... viii
LIST OF FIGURES ............................................................ ix

1 INTRODUCTION ............................................................ 1

2 THEORY and DERIVATION ............................................. 4
  2.1 Governing Differential Equation ................................. 4
  2.2 Supplementary Differential Equation ....................... 11

3 FINITE ELEMENT IMPLEMENTATION ............................ 14
  3.1 Spatial Approximation .............................................. 14
  3.2 Time Approximation ................................................ 18

4 RESULTS and DISCUSSION .......................................... 20
  4.1 Free Vibration .......................................................... 25
  4.2 Vibration of Constant Web Speeds ............................. 34
    4.2.1 Effect of The Web Speed ..................................... 34
    4.2.2 Effect of Phase Shift Between
       The Two Eccentricities of The Rollers ..................... 39
  4.3 Vibration Due To The
       Sudden Changing in The Applied Torque ................. 44

5 CONCLUSION .............................................................. 50

REFERENCE ..................................................................... 52
APPENDIX A ..................................................................... 55
APPENDIX B ..................................................................... 63
APPENDIX C ..................................................................... 72
NOTATION

A
a
B₁,B₂
b
B
B*
b*,i
c
[C]
[C]*
D₁,D₂
d
E
e
F_y
F
F_
F*
f
f*,i
g
h
I
J₁,J₂
K,K₁,K₂
k
k_f
[K]
[K]*

cross-section area of the web
lateral acceleration of element dx
viscous damping at rollers
width of the web
the global boundary condition vector
the element boundary condition vector
boundary condition entries
kinematic damping
the global damping matrix
the element damping matrix
diameters of rollers
thickness of the web
modulus of elasticity of the web material
eccentricity of unbalance rollers
the resultant force of element dx in the lateral direction
force vector that includes the F and B
the global external force vector on the web in lateral direction
the distributed external force on the web
the element external force vector
entries of external force matrix
gravitational acceleration (≈ 9.81 m/s²)
length of an element
area moment of inertia of the web
polar moment of inertia of the web
stiffness of the end-supports
axial stiffness of the web
flexural stiffness of the web
the global stiffness matrix
the element stiffness matrix
length of the web
the global mass matrix
the element mass matrix
mass per unit length of the web
unbalanced mass of the rollers
tension of the web
shear force
residual of the approximation
applied torque
frictional torque
axial velocity of a material point on the web
axial velocity of a material point at \( x/l = 1 \)
axial velocity of a material point at \( x/l = 0 \)
displacement velocity in axial direction
displacement in lateral direction
the element lateral displacement as a function of time only
the global lateral displacement as a function of time only
fixed coordinates
moving coordinates
Galerkin's coefficient \( (\beta = \frac{1}{3}) \)
Galerkin's coefficient \( (\gamma = \frac{3}{2}) \)
a constant of a stiffness matrix
density of the web material
strain
equivalent axial force
angular accelerations of the rollers
angular velocities of the rollers
angular displacements of the rollers
angular displacements of the unbalanced mass
deflection angle of element \( dx \)
Hermite's interpolation functions (shape functions)
the weight functions
SYMBOLS

δ  infinitesimal increment
Δ  small increment
[ ]  matrix
Σ  summation
∥∥  determinant of a matrix
( )′  derivative with respect to space
<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A simplified model of the web-roller system</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>An element of the moving web</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Free body diagrams for end conditions</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>Free body diagrams for derivation of supplementary equations of axial motion</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>Model of &quot;rigid-flexible&quot; setup</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>Model of &quot;flexible-flexible&quot; setup</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>Initial displacement for &quot;rigid-flexible&quot; setup</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>Initial displacement for &quot;flexible-flexible&quot; set-up</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>Response of free vibration for the &quot;rigid-flexible&quot; setup at x/l = 1.0</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>Response of free vibration for the &quot;rigid-flexible&quot; setup at x/l = 0.8</td>
<td>27</td>
</tr>
<tr>
<td>11</td>
<td>Response of free vibration for the &quot;flexible-flexible&quot; setup at x/l = 1.0</td>
<td>28</td>
</tr>
<tr>
<td>12</td>
<td>Response of free vibration for the &quot;flexible-flexible&quot; setup at x/l = 0.8</td>
<td>29</td>
</tr>
<tr>
<td>13</td>
<td>Response of free vibration for the &quot;flexible-flexible&quot; setup at x/l = 0.0</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>Response of system for the &quot;flexible-flexible&quot; setup due to the weight of the web at x/l = 0.8</td>
<td>32</td>
</tr>
<tr>
<td>15</td>
<td>Displacement profile of the web with and without the weight of the web at t = .501 sec</td>
<td>33</td>
</tr>
<tr>
<td>16</td>
<td>Response of the system at for the &quot;rigid-flexible&quot; setup with ( \omega = 20 \text{ rad/sec} (V = 5 \text{ m/s}) ) at x/l = 0.8</td>
<td>36</td>
</tr>
<tr>
<td>17</td>
<td>Response of the system at for the &quot;rigid-flexible&quot; setup with ( \omega = 40 \text{ rad/sec} (V = 10 \text{ m/s}) ) at x/l = 0.8</td>
<td>37</td>
</tr>
<tr>
<td>18</td>
<td>Response of the system at for the &quot;rigid-flexible&quot; setup with ( \omega = 60 \text{ rad/sec} (V = 15 \text{ m/s}) ) at x/l = 0.8</td>
<td>38</td>
</tr>
<tr>
<td>19</td>
<td>Response of the system for &quot;flexible-flexible&quot; setup with ( V = 10 \text{ m/s} ) and phase shift of 0° at x/l = 0.8</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>Response of the system for &quot;flexible-flexible&quot; setup with ( V = 10 \text{ m/s} ) and phase shift of 90° at x/l = 0.8</td>
<td>41</td>
</tr>
<tr>
<td>21</td>
<td>Response of the system for &quot;flexible-flexible&quot; setup with ( V = 10 \text{ m/s} ) and phase shift of 180° at x/l = 0.8</td>
<td>42</td>
</tr>
<tr>
<td>Page</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Displacement profile of the web for three different phase angles at three instants (t = 0.302, 1.201, 1.801 sec)</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Plot of the changing torque for the transient vibration analysis</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Response of the web tension due the changing torque</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Response of the angular velocities of rollers 1 &amp; 2 due the changing torque</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Response of the system for the &quot;flexible-flexible&quot; setup due the changing torque at x/l = 1.0</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Response of the system for the &quot;flexible-flexible&quot; setup due the changing torque at x/l = 0.8</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Response of the system for the &quot;flexible-flexible&quot; setup due the changing torque at x/l = 0.0</td>
<td></td>
</tr>
<tr>
<td>A-1</td>
<td>Diagram for the derivation of equation (2.13)</td>
<td></td>
</tr>
<tr>
<td>A-2</td>
<td>Approximate function for the first and the last nodes</td>
<td></td>
</tr>
<tr>
<td>B-1</td>
<td>Model for calculation of the natural frequencies of the roller-supports</td>
<td></td>
</tr>
<tr>
<td>B-2</td>
<td>Model for calculation of the natural frequencies of the web</td>
<td></td>
</tr>
<tr>
<td>B-3</td>
<td>Model for calculation of the natural frequencies of the web under the assumption of rigid supports</td>
<td></td>
</tr>
</tbody>
</table>
1 INTRODUCTION

Vibration control on moving continua, such as moving magnetic tapes, paper tapes, band-saw blades, pulley belts, coating film, and other applications, is an active research area for many manufacturing industries. Naguleswaran and Williams [1] studied the vibration and stability of a moving string excited parametrically by the periodic vibration of tension in the string. Ulsoy and Mote [2] studied the vibration of wide band-saw blades by modeling the blades as moving plates. Mote and Naguleswaran [3] did research on the linear free vibration of an axially moving thin beam using an exact solution, Galerkin solution, and flexible band solution. Chonan [4] investigated the steady state response of an axially moving strip under a transverse point load. Effects of translational velocity on the natural frequencies and modes of an axially moving beam between fixed ends were investigated by Simpson [5] using the Euler beam theory. Wickert and Mote [6] analyzed the response of an axially moving continua between two fixed point using a second-order model and a fourth-order model. Manor and Adams [7] analyzed the response of an elastic beam moving with constant speed across a drop-out at a smooth rigid foundation using the Euler-Bernoulli beam theory. Analysis of the vibration due to the tension variation of moving continua together with effects of friction between the moving continua and rollers were done by Whitworth and Harrison [8]. Gottlieb [9] studied the non-linear vibration of a constant-tension string. Daly did some studies on the effect of air pressure on a moving web [10]. Brandenburg studied the effect of the frictional force between the web and the roller by assuming the web is stretchable and obeys the Hooke's law [11]. Most of these previous studies were done under the assumption of rigid supports.
Figure 1 shows the model of the web-roller system to be investigated in this study. The moving continuum is a thin web, and it is modeled as a continuous elastic beam. The driving and the driven mechanisms are simplified as rollers which are supported by linear springs. Note that the deflection of the roller-supports in the horizontal direction are not taken into consideration in this investigation.

Due to many factors such as the flexibility of supports, the velocity of the web, the imperfection (eccentricities of the rollers) of the machinery, the non-linear material properties of the moving web, the frictional force between the moving web and the stationary machine parts, and the
aerodynamics effects (for high speed only), vibration control of the moving web is a very complicated process. For the sake of simplicity, the effects of the non-linear material properties, the frictional force, and the aerodynamics are not taken into account in this work. Studies of the web are done only on the effects of the flexibility of supports, the velocity of the web, and the imperfection of the machinery (the unbalance on the rollers). Newton’s second Law and Euler-Bernoulli beam theory are applied in the formulation of the flexural vibration of the continuous beam. The Fourth-order Runge-Kutta method is employed to solve the equations of motion in the axial direction. The Galerkin’s finite element method and the Three-point Recurrence Scheme[12] (the Newmark Recurrence Scheme [13]) are used for the solution of the flexural vibration.
2. THEORY and DERIVATION

The development of the basic differential equations (governing and supplementary equations) in this study is based on Newton’s Second Law and Euler-Bernoulli theory (the thin beam theory) [14]. The governing differential equation for the flexural vibration of the beam (in this case, the beam is a flexible web) is developed first. Then supplementary differential equations are derived for the axial motion of the web.

2.1 GOVERNING DIFFERENTIAL EQUATION

An element of length \(dx\) of the web is shown in Figure 2 together with all forces and moments.

![Diagram of an element of the moving web](image)

Figure 2. An element of the moving web.
Applying Newton's Second Law \( \sum F_y = ma \) to the free body diagram in Figure 2 gives

\[
-p\sin\theta + Q + f dx - (Q + \frac{\delta Q}{\delta x} dx)
+ (p + \frac{\delta p}{\delta x} dx)\sin(\theta + \frac{\delta \theta}{\delta x} dx) = ma \quad (2.1)
\]

Under the assumption of small \( \theta \), the following relationships are valid.

\[
\sin\theta \approx \tan\theta = \frac{\delta w}{\delta x} \quad (2.2)
\]

\[
\sin(\theta + \frac{\delta \theta}{\delta x} dx) \approx \theta + \frac{\delta \theta}{\delta x} dx

\approx \tan\theta + \frac{\delta \theta}{\delta x} dx

= \frac{\delta w}{\delta x} + \frac{\delta^2 w}{\delta x^2} dx \quad (2.3)
\]

Substituting equations (2.2)-(2.3) into equation (2.1), taking into consideration that \( m = \rho A dx \), and \( a = \frac{D^2 w}{Dt^2} \), and neglecting the second order terms yield

\[
f - \frac{\delta Q}{\delta x} + p \frac{\delta^2 w}{\delta x^2} = \rho A \frac{D^2 w}{Dt^2} \quad (2.4)
\]

where \( p \) is assumed to be time but not space dependent for simplicity.

Summing the moment about point 0 (Figure 2), and assuming the moment of inertia of the element to be negligible gives

\[
\sum M_0 = -M + f dx \frac{dx}{2} + (M + \frac{\delta M}{\delta x} dx) - (Q + \frac{\delta Q}{\delta x} dx) dx = 0 \quad (2.5)
\]
Once again, neglecting the second order terms, equation (2.5) reduces to

$$\frac{\delta M}{\delta x} - Q dx = 0$$

or

$$Q = \frac{\delta M}{\delta x}$$  \hspace{1cm} (2.6)

Substituting equation (2.6) into equation (2.4) results in

$$f - \frac{\delta}{\delta x} \left( \frac{\delta M}{\delta x} \right) + p \frac{\delta^2 w}{\delta x^2} = \rho A \frac{D^2 w}{Dt^2}$$  \hspace{1cm} (2.7)

Applying the Euler - Bernoulli beam theory (the thin beam theory) [14], \(M(x,t) = EI(x)\frac{\delta^2 w(x,t)}{\delta x^2}\), to equation (2.7) gives

$$f - \frac{\delta^2}{\delta x^2} \left[ EI(x)\frac{\delta^2 w(x,t)}{\delta x^2} \right] + p \frac{\delta}{\delta x} \left( \frac{\delta w}{\delta x} \right) = \rho A \frac{D^2 w}{Dt^2}$$  \hspace{1cm} (2.8)

Assuming the cross-sectional area of the web is constant along the web, the moment of inertia \(I(x)\) is equal \(I\), and equation (2.8) can be rewritten as

$$- EI \frac{\delta^4 w(x,t)}{\delta x^4} + p \frac{\delta^2 w}{\delta x^2} + f = \rho A \frac{D^2 w}{Dt^2}$$  \hspace{1cm} (2.9)

Where

- \(w\) ...the lateral displacement of the centroid of element \(dx\),
- \(E\) ...the modulus of elasticity of the web material,
- \(I\) ...the cross sectional area moment of inertia,
- \(p\) ...the axial force (tension) in the web,
- \(f\) ...the lateral distributed force per unit length,
- \(\rho\) ...the density of the web material,
- \(A\) ...the cross sectional area of web.
The term \( \frac{D^2w}{Dt^2} \) in equation (2.9) is the material derivative of \( w \) with respect to time [15], and it can be expressed as:

\[
\frac{D^2w}{Dt^2} = \frac{D}{Dt} \left( \frac{Dw}{Dt} \right) \tag{2.10}
\]

where

\[
\frac{Dw}{Dt} = \frac{\delta w}{\delta t} + \frac{\delta w}{\delta x} \frac{\delta x}{\delta t}
\]

or

\[
\frac{Dw}{Dt} = \frac{\delta w}{\delta t} + \frac{\delta w}{\delta x} V \tag{2.11}
\]

where \( V \) is the axial velocity of a material point on the web, and it is a function of both time and position, \( x \). It is defined, at any given instant, as

\[
V = V_2 + (V_1 - V_2) \frac{x}{l} \tag{2.12}
\]

where \( V_1 \) and \( V_2 \) are the web velocity at the right end \((x/l = 1.0)\) and the left end \((x/l = 0.0)\), respectively, and they are defined as

\[
V_1 = \frac{D_1}{2} \omega_1 \tag{2.13}
\]

and

\[
V_2 = \frac{D_2}{2} \omega_2 \tag{2.14}
\]

where \( D_1 \) and \( D_2 \) are the diameters of the rollers \#1 and \#2, respectively, \( \omega_1 \) and \( \omega_2 \) are the angular velocities of the rollers \#1 and \#2, respectively, and they can be found from the supplementary differential equations.

Substitution from equation (2.11) into equation (2.10) with further expansion yields
\[
\frac{D^2 w}{Dt^2} = \frac{\delta^2 w}{\delta t^2} + 2V \frac{\delta^2 w}{\delta t \delta x} + \frac{\delta w \delta V}{\delta x \delta t} + \frac{\delta^2 w}{\delta x^2} + \frac{\delta w \delta V}{\delta x} \]  

(2.15)

for small deformation

\[
\frac{\delta V}{\delta x} = \frac{\delta}{\delta t}(\epsilon) 
\]

(2.16)

where \( \epsilon \), which is the strain [16] in the web due the web tension \( p \), is defined as

\[
\epsilon = \frac{P}{EA} 
\]

(2.17)

Derivation of equation (2.16) is shown in Appendix A.

Substituting equations (2.15)-(2.17) into equation (2.9), and rearranging terms yields

\[
m \frac{\delta^2 w}{\delta t^2} + 2c \frac{\delta^2 w}{\delta t \delta x} + k \frac{\delta^4 w}{\delta x^4} + \tau \frac{\delta^2 w}{\delta x^2} + \lambda \frac{\delta w}{\delta x} = f 
\]

(2.18)

where

\[
m = \rho A 
\]

(2.19)

\[
c = \rho VA 
\]

(2.20)

\[
k = EI 
\]

(2.21)

\[
\tau = \rho AV^2 - p 
\]

(2.22)

and

\[
\lambda = \rho A \left( \frac{\delta V}{\delta t} + \frac{V \delta P}{EA \delta t} \right) 
\]

(2.23)

The term \( \frac{\delta V}{\delta t} \) can be defined as

\[
\frac{\delta V}{\delta t} = \frac{D_2}{2} \frac{\delta \omega_2}{\delta t} + \left( \frac{D_1}{2} \frac{\delta \omega_1}{\delta t} - \frac{D_2}{2} \frac{\delta \omega_2}{\delta t} \right) \times \frac{1}{I} 
\]

(2.24)

where \( \frac{\delta \omega_1}{\delta t} \) and \( \frac{\delta \omega_2}{\delta t} \) can be found from the supplementary differential equations.
The boundary conditions of equation (2.18) are derived from free body diagrams in Figure 3. Figure 3 shows that the continuous beam is postulated to be simply supported at each end. For simplicity, two rollers are assumed to be identical with unbalanced \( m_0e \), and are supported by two identical elastic springs with stiffness \( K \) as shown in Figure 3. The unbalance \( m_0e \) in each roller is assumed to be identical. It should be pointed out that the phase angle between the two eccentricities is zero in Figure 3. However, the phase angle could be any value.

By summing all forces in the lateral direction at each end of the free body diagram one at a time, the boundary conditions [14] are found to be

1) At the left end:

\[
Q_{(0,t)} = E\!I\! \frac{\delta^3 w(x,t)}{\delta x^3}\bigg|_{(0,t)}
\]

\[
= M\dot{w}(0,t) + Kw(0,t) + m_0e\omega_2^2\sin\phi - m_0e\alpha_2\cos\phi
\]

(2.25)

where \( K = K_1 = K_2 \) in this study.

2) At the right end:

\[
Q_{(l,t)} = E\!I\! \frac{\delta^3 w(x,t)}{\delta x^3}\bigg|_{(l,t)}
\]

\[
= - M\dot{w}(l,t) - Kw(l,t) - m_0e\omega_1^2\sin\phi + m_0e\alpha_1\cos\phi
\]

(2.26)

Where \( m_0 \) is the unbalanced mass.
For simply-supported beams, the ends of the beams do not take any moment at any time, i.e.

\[ M(0,t) = EI \frac{\delta^2 w(x,t)}{\delta x^2} |_{0} = 0 \]  \hspace{1cm} (2.27)

\[ M(l,t) = EI \frac{\delta^2 w(x,t)}{\delta x^2} |_{l} = 0 \]  \hspace{1cm} (2.28)

Equation (2.18) is the equation of motion of the element dx in the lateral direction with the boundary conditions as defined by equations (2.25)-(2.28). The equations of motion in the axial direction for providing the values of V, p, \( \frac{\delta V}{\delta t} \), and \( \frac{dp}{dt} \) are to be derived in Section 2.2.
2.2 SUPPLEMENTARY DIFFERENTIAL EQUATIONS

In order to solve the fourth order partial differential equation (equation (2.18)), variables $V$, $p$, $\frac{\delta V}{\delta t}$, and $\frac{dp}{dt}$ in the axial motion have to be found first. Figure 4 shows the free body diagrams of the two rollers of the web-roller system.

![Free body diagrams of rollers](image)

Figure 4 Free body diagram of rollers for derivation of supplementary differential equations

Summing the moment about the center of roller #1, $O_1$, gives

$$J_1 \dot{\omega}_1 = T - B_1 \omega_1 - \frac{D_1}{2}$$

or

$$\frac{\delta \omega_1}{\delta t} = \frac{1}{J_1} \left( T - B_1 \omega_1 - \frac{D_1}{2} \right)$$  \hspace{1cm} (2.29)

where $J_1$ ... the polar moment of inertia of the roller #1,
B_1 \ldots \text{the viscous damping at the roller \#1,}
\omega_1 \ldots \text{the angular velocity of the roller \#1,}
D_1 \ldots \text{the diameter of the roller \#1,}
T \ldots \text{the applied torque at the driving roller.}

Summing the moment about the center of roller \#2, \theta_2, gives

\[ J_2 \dot{\omega}_2 = \frac{D_2^2}{2} - T_f - B_2 \omega_2 \]

or

\[ \frac{\delta \omega_2}{\delta t} = \frac{1}{J_2} \left( \frac{D_2^2}{2} - T_f - B_2 \omega_2 \right) \] (2.30)

where

\[ J_2 \ldots \text{the polar moment of inertia of the roller \#2,} \]
\[ B_2 \ldots \text{the viscous damping at the roller \#2,} \]
\[ \omega_2 \ldots \text{the angular velocity of the roller \#2,} \]
\[ D_2 \ldots \text{the diameter of the roller \#2,} \]
\[ T_f \ldots \text{the fictional torque at the driven roller.} \]

Assuming that

\[ p = k \Delta l \] (2.31)

where \( k \) is the axial stiffness of the web in this study and \( \Delta l \) is the change in the length of the web due to the different angular displacement of the two rollers. Under the assumption of no slipping between the rollers and the web, \( \Delta l \) is defined as

\[ \Delta l = \frac{D_1}{2} \Theta_1 - \frac{D_2}{2} \Theta_2 \] (2.32)

where \( \Theta_1, \Theta_2 \) are the angular displacements of rollers.

Substitution from equation (2.31) into equation (2.32) yields

\[ p = k \left( \frac{D_1}{2} \Theta_1 - \frac{D_2}{2} \Theta_2 \right) \] (2.33)

Taking the derivative of equation (2.33) with respect to
time gives

\[ \frac{dp}{dt} = \frac{k}{2}(D_1 \omega_1 - D_2 \omega_2) \quad (2.34) \]

Equations (2.29), (2.30), and (2.34) represent the supplementary differential equations that are required to evaluate the values of \( V, p, \frac{\delta V}{\delta t}, \) and \( \frac{dp}{dt} \) in the axial direction of the web. Initial conditions for equations (2.29), (2.30) and (2.34) are to be defined in the chapter of Results and Discussion in this paper.

Solution of equation (2.18) is to be approximated by applying the finite element method with the boundary conditions that are established in equations (2.25)-(2.28). The supplementary differential equations are to be solved by the fourth-order Runge-Kutta method. The finite element formulation of equation (2.18) is to be presented in the next chapter.
3. **FINITE ELEMENT IMPLEMENTATION**

Solution of the partial differential equation (equation (2.18)) is accomplished in two stages: (1) Spatial approximation which is achieved by using the Galerkin's method (the Weighted Residual Method). (2) Time approximation which is approached by using the Three-point Recurrence Schemes [12] (the Newmark recurrence scheme[13]). The first stage approximates the partial differential equation (2.18) by a set of simultaneous, ordinary differential equations, and the second stage approximates the set of ordinary differential equations by a set of linear algebraic equations, which can be solved at each time step. Such a procedure, which finds the spatial approximation first, then the time approximation, is known as a semi-discrete approximation [17].

3.1 **SPATIAL APPROXIMATION**

The Galerkin's method is used for the approximation of the partial differential equation.

Assuming the solution of equation (2.18) for each element, \( w^e(x,t) \), to be

\[
    w^e(x,t) = \sum_{j=1}^{N} \psi(x)_j \ W^e(t)_j, \quad N = 4 \quad (3.1)
\]

where \( \psi(x)_j \) is the shape function within each element, \( W^e(t)_j \) is the nodal lateral displacement of the element, and \( N \) is the order of the interpolation function. For simplicity, \( \psi(x)_j \) and
$W^e(t)_j$ are to be written as $\psi_j$ and $W^e_j$, respectively, in the later description.

Substituting equation (3.1) and its derivative with respect to time and space into equation (2.18) gives

$$\begin{align*}
m\sum_{j=1}^{N}\psi_j \dot{W}^e_j + 2c\sum_{j=1}^{N}\psi_j' \dot{\dot{W}}^e_j + k\sum_{j=1}^{N}\psi_j''' \dot{W}^e_j & \\
+ \tau\sum_{j=1}^{N}\psi_j'' W^e_j + \lambda\sum_{j=1}^{N}\psi_j' W^e_j - f = R \neq 0 \quad (3.2).
\end{align*}$$

Note that expression (3.2) is no longer exactly equal to zero since the approximate representation for $w^e(x,t)$ is used. Hence $R$ is called the residual of its original equation.

Setting the integral of a weighted residual of the approximation over the whole domain of each element to zero results in

$$\int_0^h \phi_i R \, dX = 0 \quad (3.3)$$

where $\phi_i$ are the weight functions (which, in general, are not the same as the shape function $\psi_j$).

Substituting equation (3.2) into equation (3.3), and after some manipulation, gives

$$\begin{align*}
(m\sum_{j=1}^{N} \int_0^h \psi_j \phi_i \, dX) \dot{W}^e_j + (2c\sum_{j=1}^{N} \int_0^h \psi_j' \phi_i \, dX) \dot{\dot{W}}^e_j & \\
+ (k\sum_{j=1}^{N} \int_0^h \phi_i \psi_j''' \, dX) \dot{W}^e_j + (\tau\sum_{j=1}^{N} \int_0^h \phi_i \psi_j'' \, dX) W^e_j & \\
+ (\lambda\sum_{j=1}^{N} \int_0^h \psi_j' \phi_i \, dX) W^e_j - \int_0^h \phi_i f^e \, dX &= 0 \quad (3.4)
\end{align*}$$

15
Integrating the third term of equation (3.4) by parts, it becomes

\[(k\sum_{j=1}^{N} \int_{0}^{h} \phi_j \psi_j''' \, dX) \dot{W}_j^e = (k\sum_{j=1}^{N} \int_{0}^{h} \psi_j'' \phi_i' \, dX) \dot{W}_j^e - [\phi_i' \ M - \phi_i \ Q]_0^h \] (3.5)

where \( M = k \sum_{j=1}^{N} \psi_j'' \ W_j^e \) is the bending moment of each element, and \( Q = k \sum_{j=1}^{N} \psi_j''' \ W_j^e \) is the shear force of each element.

Substituting equation (3.5) into equation (3.4), gives

\[M_{ij}^e \dot{W}_j^e + C_{ij}^e \dot{W}_j^e + K_{ij}^e \ W_j^e = f^e_i + b^e_i \] (3.6)

where

\[K_{ij}^e = K_{0ij}^e + K_{1ij}^e + K_{2ij}^e \] (3.7)

and

\[M_{ij}^e = \int_{0}^{h} \psi_j \ m \ \phi_i \ dX \] (3.8)

\[C_{ij}^e = \int_{0}^{h} \psi_j' \ c \ \phi_i \ dX \] (3.9)

\[K_{0ij}^e = \int_{0}^{h} \psi_j'' \ k \ \phi_i'' \ dX \] (3.10)

\[K_{1ij}^e = \int_{0}^{h} \psi_j'' \ \tau \ \phi_i \ dX \] (3.11)

\[K_{2ij}^e = \int_{0}^{h} \psi_j' \ \lambda \ \phi_i \ dX \] (3.12)

\[f^e_i = \int_{0}^{h} f \ \phi_i \ dX \] (3.13)

and

\[b^e_i = [M \ \phi_i' - Q \ \phi_i]_0^h \] (3.14)

In this study, the Hermite cubic interpolation functions (shown in Appendix A) are adopted for the shape function.
Applying the Galerkin's method, the weight functions $\phi_i$ are the same as the shape functions $\psi_j$. The corresponding element matrices, the external force vector, and the boundary condition vector are evaluated based on the Hermite cubic interpolation functions in Appendix A.

The global equations (a set of ordinary differential equations) are obtained via the standard assembly method [18], i.e.

$$\sum [M]^e \vec{W}^e + \sum [C]^e \vec{W}^e + \sum [K]^e \vec{W}^e = \sum \vec{F}^e + \sum \vec{b}^e$$

or

$$[M] \vec{W} + [C] \vec{W} + [K] \vec{W} = \vec{F} + \vec{b} \quad (3.15)$$

where $n$ is the number of elements, $\vec{W}$ is the global lateral displacement vector, and $[M]$, $[C]$, $[K]$, $\vec{F}$, and $\vec{b}$ are the global mass matrix, damping matrix, stiffness matrix, external force vector, and boundary condition vector, respectively.

It should be pointed out that the global boundary condition vector, $\vec{b}$, has zero components except for those at the end nodes (the first and the last nodes), i.e.

$$\vec{b} = ( Q(0), - M(0), 0, 0, \ldots, 0, 0, -Q(l), M(l) ) \quad (3.16)$$

where the shear forces of the first and the last nodes are to be determined from equations (2.25) and (2.26), and the moments of the first and the last nodes are zero according to equations (2.27) and (2.28).

Equation (3.15) is the global ordinary differential equation which needs further approximation in the time domain.
3.2 TIME APPROXIMATION

Equation (3.15) is exactly the same as equation (21.1) in "The Finite Element Method" by O. C. Zienkiewicz [12]. The Three-point Recurrence Scheme (the Newmark Recurrence Scheme) was used by Zienkiewicz for the time approximation of equation (21.1). Following Zienkiewicz's derivation procedure, the time approximation of equation (3.15) takes the form of

\[
\begin{align*}
\left[ [M] + \gamma \Delta t [C] + \beta \Delta t^2 [K] \right] \bar{w}_{t+1} \\
+ \left[ -2[M] + (1-2\gamma)\Delta t [C] + (\frac{1}{2}+\beta-\gamma)\Delta t^2 [K] \right] \bar{w}_t \\
+ \left[ [M] - (1-\gamma)\Delta t [C] + (\frac{1}{2}+\beta-\gamma)\Delta t^2 [K] \right] \bar{w}_{t-1} + \bar{F}\Delta t^2 = 0 \ (3.17)
\end{align*}
\]

where \([M]\), \([C]\), and \([K]\) are defined as in equation (3.15), \(\bar{F}\) is the force vector that includes both the global force vector and the boundary condition vector, \(\beta\) and \(\gamma\) are the Galerkin's coefficients for time approximation, and the subscripts \(t+1\), \(t\), and \(t-1\) indicate three consecutive instants. The values of \(\beta\) and \(\gamma\) are \(\frac{4}{5}\) and \(\frac{3}{4}\), respectively, which are evaluated by Zienkiewicz.

According to Zienkiewicz's derivation, the \(\bar{F}\) is interpolated as

\[
\bar{F} = \bar{F}_{t+1}\beta + \bar{F}_t(\frac{1}{2}-2\beta+\gamma) + \bar{F}_{t-1}(\frac{1}{2}+\beta-\gamma) \quad (3.18)
\]

Equations (3.17) and (3.18) are the final forms for the solution of equation (2.18), and they are used in the finite
element program. The values of $\bar{W}_{t-1}$, $\bar{W}_t$, $F_{t-1}$, $F_t$, and $F_{t+1}$, which are used to start the program, are described in the program in Appendix C. This chapter described the whole process of solving equation (2.18) starting from implementing the element matrices, to assembling the global matrices, and then to finding the final form of approximation. The numerical results of equation (3.17) for this study are shown in the next chapter.
4. RESULTS and DISCUSSION

All results are generated by three computer programs: AXIAL, WEB1, and CONVER. The AXIAL program, which was written by Dr. H. Ghoneim, utilizes the fourth-order Runge-Kutta method to solve the supplementary differential equations of the axial motion, and it has been updated to include the investigation of different web speeds and different phase angles between the two unbalanced rollers. The AXIAL program has to be run first to provide the WEB1 program with the necessary values of \( \frac{\delta V}{\delta t} \), \( p \), and \( \frac{dp}{dt} \) at each time step. The WEB1 program is a finite element program, which was originally written by Dr. H. Ghoneim in FORTRAN code. The WEB1 program utilizes the final approximation equations (equations (3.17) and (3.18)) with the data of \( V \), \( \frac{\delta V}{\delta t} \), \( p \), and \( \frac{dp}{dt} \) from the AXIAL program to solve the partial differential equation (equation (2.18)) for the flexural vibration of the web-roller system. The WEB1 program has been updated to incorporate the gravitational force of the web. The CONVER program is used to convert the time domain response from the WEB1 program into the frequency domain. The CONVER program calls a sub-program, FFTRF, from the IMSL library of the RITVAX system [19]. The FFTRF employs the Fast Fourier Transform algorithm. All programs are listed in the Appendix C.

The web-roller system is investigated for two different setups (Figure 5 and 6):

1) A "rigid-flexible" setup (Figure 5), where one roller-support is treated as a rigid support and the other is
Figure 5  A simplified model of the "rigid-flexible" web-roller system.

Figure 6  A simplified model of the "flexible-flexible" web-roller system.
Studies are conducted for the following cases:

1. Free vibration of the web when it is not axially moving \( (V = 0) \) for both "rigid-flexible" and "flexible-flexible" setups. This is done to verify the validity of the computer programs and to study the effect of the flexibility of the roller-supports. Effect of the weight is also considered for the "flexible-flexible" setup when the web is not moving.

2. Vibration of the web when it is moving with constant speeds. Two factors are investigated:
   a. The effects of different web speeds for the "flexible-rigid" setup.
   b. The effects of the phase shift between the eccentricities of the rollers for the "flexible-flexible" setup.

3. Vibration of the web under the effect of a sudden change of the applied torque at the driving roller.

The following values of the system parameters are adopted for the investigation.

\[
\begin{align*}
p &= 10^3 \text{ kg/m}^3, & D_1 = D_2 &= 0.5 \text{ m}, \\
E &= 4 \times 10^9 \text{ pa}, & J_1 = J_2 &= 8.283 \text{ kgm}^2, \\
d &= 1.25 \times 10^{-4} \text{ m}, & B_1 = B_2 &= 0.5 \text{ Nms}, \\
b &= 1.35 \text{ m}, & K_1 = K_2 &= 7.36 \times 10^9 \text{ N/m}, \\
l &= 1.52 \text{ m}, & M_1 = M_2 &= 8.27 \times 10^3 \text{ kg},
\end{align*}
\]
\[ b = 1.35 \text{ m}, \quad K_1 = K_2 = 7.36 \times 10^9 \text{ N/m}, \]

\[ l = 1.52 \text{ m}, \quad M_1 = M_2 = 8.27 \times 10^3 \text{ kg}, \]

\[ k = 4.44 \times 10^5 \text{ N/m}, \quad m_0 e = 10 \text{ kgm}. \]

where \( d, b, l, \) and \( k \) are the thickness, width, length, and axial stiffness of the web, respectively.

Other parameters, that are needed for obtaining the results of this paper from the WEB1 and AXIAL program, are listed as the following:

\begin{align*}
\text{start time} & = 0.0 \text{ second}, \\
\text{end time} & = 1.93 \text{ second}, \\
\text{time increment} & = 0.001 \text{ second}, \\
\text{number of elements} & = 21 \text{ elements}.
\end{align*}

However, the values of the above parameters can be changed to different values.

Results are displayed in Figures 9-22, and 24-28. Note that for Figures 9-14, 16-21, and 26-28, each figure is composed of three graphs: a, b, and c. Part a of each figure represents the temporal response of the system in the time domain, while parts b and c show the corresponding response of the system in the frequency domain for the linear and logarithmic amplitudes respectively. Logarithmic plot for all frequency responses of the system are displayed to give better visibility of the peaks of the frequency spectrum.
Figure 7  A simplified model of the "rigid-flexible" web-roller system.

Figure 8  A simplified model of the "flexible-flexible" web-roller system.
4.1 FREE VIBRATION

The following parameters are considered for the free vibration studies of the web-roller system when the web is not in motion \((\omega_1 = \omega_2 = 0 \text{ rad/sec})\), and the pre-tension, \(p = 200 \text{ N}\). The system is excited from rest via a linear, initial displacement in the lateral direction as shown in Figure 7 and 8. The values of the displacement at the left and the right ends are \(w(0,0) = 0 \text{ m}\), and \(w(l,0) = 4\times10^{-6} \text{ m}\), respectively.

Samples of the free vibration results of the web-roller system are shown in Figures 9 and 10 for the "rigid-flexible" setup, and in Figures 11, 12, and 13 for the "flexible-flexible" setup. Figure 9 shows the response of the system at the position of \(x/l = 1.0\). Figures 9b, and 9c show the natural frequency of the roller-support is about 140 Hz, which crudely agrees with the natural frequency of the roller support. Analytical results (shown in Appendix B) of the natural frequencies of the roller-supports and the web are found to verify the numerical solution obtained from the program. Figures 10b and 10c show the numerical values of the natural frequencies of the web from the programs. Those numerical values are in excellent agreement with the analytical results in the Appendix B. Figures 10b and c also indicate very clearly that the vibration of the web-roller system is dominated by the natural frequency of the roller-supports.

Comparison of Figure 11, 12 with Figure 9, 10 respectively indicates that the flexibility of the left end has very little effect on the responses of the web-roller system. Figure 13 shows the responses of the system at \(x/l = 0.0\), and it indicates that the natural frequency of the left roller-support to be 140 Hz. That is because the two roller-supports are
Figure 9  Free vibration response of the web-roller system at $x/l = 1.0$ for the "rigid-flexible" setup.
Figure 10 Free vibration response of the web-roller system at $x/l = 0.8$ for the "rigid-flexible" setup.

Figure a Temporal resp. of free vibration at $x/l = 0.8$
for one end fixed

Figure b Temporal resp. of free vibration at $x/l = 0.8$
for one end fixed

Figure c Frequency spectrum of free vibration at $x/l = 0.8$
for one end fixed
Figure 11  Free vibration response of the web-roller system at $x/l = 1.0$ for the "flexible-flexible" setup.
Figure 12  Free vibration response of the web-roller system at $x/l = 0.8$ for the "flexible-flexible" setup.

(b)  Freq. spect. of free vibration at $x/l = 0.8$

for both ends not fixed

(a)  Temp. resp. of free vibration at $x/l = 0.8$

for both ends not fixed

(c)  Freq. spect. of free vibration at $x/l = 0.8$

for both ends not fixed
Figure 13

Free vibration response of the web-roller system at x/L = 0.0 for the "flexible-flexible" setup.
exactly the same for the "flexible-flexible" setup. An interesting phenomenon is observed in Figure 13a. The left roller support, which is initially at rest, is gradually picking up vibration that is transferring from the right roller through the web. This is happening because the web-roller system is acting as a predominantly two degrees of freedom system with the two rollers linked together by a very soft spring (the web) as shown in Figure 8. Since the stiffness of the web, in this analysis, is far smaller than those of the roller supports (calculations of stiffness for support and web are shown in Appendix B), the energy transfer from the right roller to the left roller is happening in an extremely slow manner. (Note that after 2 sec, the left roller picked up a vibrating amplitude of $10^{-8}$ m which is about 450 times smaller than the vibrating amplitude at the right roller.) The significant difference between the stiffness of the roller-supports and the flexural stiffness of the web is the reason why most previous studies in this field were done under the assumption of rigid supports.

When the web is not in motion ($V = 0$ m/s), the damping of the system is equal to zero as represented by $c$ in equation (2.17). The beating phenomena (Figure 10a, and 12a) occur when the system with very little damping (in this case the system has no damping) is subjected to the excitation frequency (the natural frequency of the support) that is very close to the thirteenth mode natural frequency of the web [20] (analytical solution for the natural frequencies of the roller-supports and the web are shown in Appendix B).

Responses at the position of $x/l = 0.8$ due to the weight of the web are shown in Figure 14. Due to the downward direction of gravitational force, the temporal response (Figure 14a) of the web shifts toward the negative side. Comparing the plots in Figures 14b and 14c with the plots in Figure 12b and 12c, reveals that the vibrating amplitude is bigger for the web
Figure 14  Response of the web-roller system at $x/l = 0.8$ for the "flexible-flexible" setup due to the effect of the weight of the web.
with the weight than for the web without the weight. The weight of the web tends to promote the lower frequencies.

Figure 15 shows the displacement profile of the web, with and without the weight of the web, at the instance of $t = 0.501$ second after the right end has been released from the initial displacement of $1E-4$ meter. The lower curve is the displacement profile of the web with the weight of the web, and the other curve is the displacement profile of the web excluding the weight.

![Figure 15: Displacement profile of the web with and without the weight of the web at $t = .501$ sec](image)

**Figure 15** Displacement profile of the web with and without the weight of the web at $t = .501$ sec

Upper curve: without the effect of the weight.

Lower curve: with the effect of the weight.
4.2 VIBRATION OF CONSTANT WEB SPEEDS

The web speeds, that are going to be used in this section, are actually the axial velocity of a material point on the web (equations (2.12)-(2.14)) since the angular velocities of the rollers are the same.

4.2.1 Effects of the Web Speed

The system is studied, when the initial displacements at the left and the right ends are \( w(0,0) = 0 \) m, and \( w(l,0) = 4E-6 \) m, respectively, for the following cases:

1) \( V = 5 \text{ m/s} \ (\omega_1 = \omega_2 = 20 \text{ rad/s}), \text{ and } p = 160 \text{ N (constant)}, \)

2) \( V = 10 \text{ m/s} \ (\omega_1 = \omega_2 = 40 \text{ rad/s}), \text{ and } p = 200 \text{ N (constant)}, \)

3) \( V = 15 \text{ m/s} \ (\omega_1 = \omega_2 = 60 \text{ rad/s}), \text{ and } p = 240 \text{ N (constant)}. \)

Figure 16, 17, and 18 show the responses of the system for the "flexible-flexible" setup with the web speeds of 5 m/sec, 10 m/sec, and 15 m/sec at \( x/l = 0.8 \), respectively. A closer look at the Figures (b, and c of Figures 16, 17, and 18) reveals that the natural frequencies of the web decrease as the web speed increases. That is because the stiffness of the web decreases as the web moves faster in the axial direction. Three factors contribute to the stiffness of the web-roller system as described by equations (3.7). These factors are: 1) the flexural stiffness of the web, \( k(EI) \) in equation (3.10), 2) the axial velocity and tension of the web, as represented by \( \tau \) in equation (3.11), and 3) the time rate of change of the
equation (3.12). For constant web axial speed and tension, the third factor drops out \((\lambda = 0)\). As illustrated in Appendix B, the value of \(\tau\) decreases with increasing web axial speed, and consequently, the total stiffness of the web decreases as well.

It should be pointed out that the axial speed introduces a "kinematic" damping to the system through the parameter \(c\) in equation (2.18). As the speed increases, the damping factor, \(c\), (equation (2.20)) increases. It appears, however, from the figures that the change in \(c\) with the speed is too small to introduce any significant effects.

Again, the beating phenomenon occurs in all the three cases as one of the higher natural frequencies (the 13\(^{th}\), the 14\(^{th}\), and the 16\(^{th}\) mode for \(V = 5\) m/sec, 10 m/sec, and 15 m/sec respectively) modulates with the natural frequency of the rotor support.
Figure 16: Response of the web-roller system at $x/l = 0.8$ for the "rigid-flexible" setup with $\omega = 20 \text{ rad/sec}$. 

a. 

b. 

c. 

d.
Figure 17  Response of the web-roller system at $x/l = 0.8$ for the "rigid-flexible" setup with $\omega = 40$ rad/sec.
Figure 18  Response of the web-roller system at x/l = 0.8 for the "rigid-flexible" setup with \( \omega = 60 \) rad/sec.
4.2.2 Effects of Phase Shift Between The Two Eccentricities of The Rollers

Investigation on the effects of 0°, 90°, and 180° phase shift between the two eccentricities of the rollers is done when the web speed is 10 m/sec (ω₁ = ω₂ = 40 rad/s), and the tension of the web is 200 N.

For the "flexible-flexible" system, the phase angle of the eccentricities of the rollers plays an important roll in the vibrating modes of the moving continua [1]. Figures 19, 20, and 21 show the responses of the system with phase angles of 0°, 90°, and 180°, respectively. Comparison of the results of the three cases shows that the 0° phase angle-difference case produces the smoothest vibration as shown in Figure 22a. For this case, since both end-rollers move together, only the lower modes are excited, and the higher modes are relatively suppressed as shown in Figure 19b. That also explains why the amplitude of vibration of the first peak is slightly higher for the 0° case than the other two cases (90° and 180°). For the 90° and 180° phase difference, the opposite movement of the two rollers excites the higher modes and triggers the natural frequencies of the roller supports as demonstrated in Figures 20b, 20c, 21b, and 21c. It appears that there is no significant difference between the responses for the 90° and the 180° phase shift case. These suggest that beyond a certain phase shift the degree of shift does not have a significant influence on the dominant vibrating frequencies.

Figure 22 shows the displacement profiles of the web for the three different phase angles (0°, 90°, and 180°) at three different instants (t = 0.302 sec, t = 1.201 sec, and t = 1.801 sec). Clearly, for the 0° phase angle case, the 1" mode dominates the rest (Figure 22a) as a result of the synchronized motion of the end rollers, while in the other two cases (Figure 22b, 22c) higher frequencies are well excited.
Figure 19  Response of the web-roller system at $x/l = 0.8$ for the "flexible-flexible" setup with $\omega = 40$ rad/sec and phase shift of $0^\circ$. 

(a) Response at $x/l = 0.8$ for constant $W_1, W_2, P$

(b) Freq. resp. at $x/l = 0.8$ for constant $W_1, W_2, P$

- on unbal. mass $\theta$ each end, & $\omega = 40$ r/sec

(c) Freq. resp. at $x/l = 0.8$ for constant $W_1, W_2, P$

- on unbal. mass $\theta$ each end, & $\omega = 40$ r/sec
Figure 20: Response of the web-roller system at $x/L = 0.8$ for the "flexible-flexible" setup with $\omega = 40$ rad/sec and phase shift of 90°.
Figure 21  Response of the web-roller system at $x/l = 0.8$ for the "flexible-flexible" setup with $\omega = 40$ rad/sec and phase shift of 180°.
Figure 22

DISPLACEMENT PROFILE OF WEB W/ 90° PHASE ANGLE
- a) t = 0.302, b) t = 1.201, c) t = 1.801 sec

DISPLACEMENT PROFILE OF WEB W/ 0° PHASE ANGLE
- a) t = 0.302, b) t = 1.201, c) t = 1.801 sec

DISPLACEMENT PROFILE OF WEB W/ 180° OUT PHASE
- a) t = 0.302, b) t = 1.201, c) t = 1.801 sec
4.3 VIBRATION DUE TO THE SUDDEN CHANGE IN THE APPLIED TORQUE

Studies of the effect of a sudden change in the applied torque from 70 NM to 110 NM (Figure 23) are done when the web is initially moving at 10 m/sec ($\omega_1 = \omega_2 = 40$ rad/s) and has an initial tension of 200 N. The phase angle in this case is $0^\circ$.

![Torque vs Time](image.png)

**Figure 23**

Figures 24-25 show the responses of tension, and the angular velocities of rollers due to the sudden change of the applied torque. As the applied torque is suddenly increased, the tension of the web follows almost instantaneously and increases to an average steady state value of 280 N (Figure 24). Because of the assumption of an elastic web, the steady
state tension in the web is oscillatory. As for the angular velocities of the end rollers, they follow "sluggishly" and increase very slowly (Figure 25). This is attributed to the high inertia of the rollers which resists any sudden change in speed and delays the response.

Figures 26, 27 and 28 show the responses of the system, for the "flexible-flexible" setup due to the sudden change of the applied torque at three points on the web. Figures 26a and 28a demonstrate that the amplitudes of vibration of the roller-supports are gradually increasing. This is because the angular velocities of both rollers are gradually increasing and consequently the unbalanced forces are also gradually increasing. The same, but less obvious, can be observed about the web vibration in Figure 27a. Plots b and c of Figures 26, 27, and 28 clearly show that the response of the system is dominated by the excitation frequency of the rotating unbalanced rotors.
Figure 24

Response of tension due to the sudden change of torque with the initial value of 200 N

Figure 25

a. Response of W2 due to the sudden change of torque with the initial value of 40 rad/sec

b. Response of W1 due to the sudden change of torque with the initial value of 40 rad/sec
Response of the web-roller system at x/l = 1.0 for the "flexible-flexible" setup due to the sudden change of the applied torque, with phase shift of 0°, and under the following initial conditions:

\[ T = 70.0 \text{ N-M} \]
\[ p = 200 \text{ N} \]
Response of the web-roller system at $x/l = 0.8$ for the "flexible-flexible" setup due to the sudden change of the applied torque, with phase shift of $0^\circ$, and under the following initial conditions:

- $T = 70.0 \text{ N-M}$
- $p = 200 \text{ N}$
Figure 28  Response of the web-roller system at \( x/l = 0.0 \) for the "flexible-flexible" setup due to the sudden change of the applied torque, with phase shift of 0°, and under the following initial conditions:

\[ T = 70.0 \text{ N-M} \]
\[ p = 200 \text{ N} \]
5. CONCLUSION

From the foregoing analysis, it can be concluded that the web-roller system is affected by the flexibility of the roller supports, the weight of the web, the axial speed of the web, the phase shift between the unbalance masses, and the sudden change of the applied torque on the driving roller.

1) Flexibility of the driven roller-supports has very little effect on the free vibration of the system for a short period of time. However, for a long run, the free vibration indicates that the energy transfer phenomenon (beating phenomenon) between the two roller-supports can occur.

2) Increasing the speed of the moving web increases the "kinematic damping", which reduces the vibration. However, it turns out that the increased damping is too small to be significant.

3) Increasing the speed of the moving web reduces the overall lateral stiffness of the system and consequently decreases the values of the natural frequencies of the web. This indicates that there is a higher possibility for one of the natural frequencies of the web to match the angular velocities of the rollers (excitation frequencies).

4) Increasing the speed of the web (which means increasing the angular velocity of the rollers) amplifies the unbalanced forces at the rollers and consequently introduces more vibration to the system.
5) The phase shift between the unbalanced forces of the rollers excites the higher modes of vibration and aggravates the vibration of the system.

6) A sudden increase in the applied torque introduces fluctuation in the web tension.

This work can be extended to study the effects of other factors such as the viscoelasticity of the web, the change in the dimensions of the web and the material properties, the aerodynamic forces on the web, and the frictional force between the web and the rollers.
REFERENCES


APPENDIX A

A1 DERIVATION OF EQUATION (2.13) IN CHAPTER 2

A2 SOME DETAILS OF THE FINITE ELEMENT IMPLEMENTATION
Figure A-1 shows a section of undeformed web, O'o, and a section of deformed web oo', with a moving coordinate system xy in the space of a fixed coordinate system XY. Note that section O'o has been straightened as section Oo for better understanding of the diagram.

At time equal to zero, point A is measured from system XY with Z, so Z is time independent. After a period of time, \( \Delta t \), the moving system xy moves from \( O \), the origin of fixed system XY, to \( o \), and point A moves to \( A' \). For the sake of simplicity, the motion of the web is considered in the \( x \)-direction only. At any time, the following relationship holds:

\[
Z + u = r + x \quad \text{(A1.1)}
\]
where \( Z \) is the material coordinate of point \( A \), \( r \) is the distance traveled by the origin of the moving system, \( u \) is the displacement of point \( A \), and \( x \) is the coordinate of point \( A' \).

Taking the time derivative of equation (A1.1) gives

\[
\frac{dZ}{dt} + \frac{\delta u}{\delta t} = \frac{\delta r}{\delta t} + \frac{\delta x}{\delta t} \tag{A1.2}
\]

where the first term \( \frac{dZ}{dt} \) is zero. Hence, equation (A1.2) can be rewritten as

\[
\frac{\delta x}{\delta t} = \frac{\delta u}{\delta t} - \frac{\delta r}{\delta t} \tag{A1.3}
\]

Equation (A1.3) gives the velocity of a material point.

Considering the left hand side of equation (2.13),

\[
\frac{\delta V}{\delta x} = \frac{\delta}{\delta x} \left( \frac{\delta x}{\delta t} \right) \tag{A1.4}
\]

Under the assumption of small deformation, substitution of equation (A1.3) into equation (A1.4) gives

\[
\frac{\delta V}{\delta x} = \frac{\delta}{\delta x} \left( \frac{\delta u}{\delta t} - \frac{\delta r}{\delta t} \right) \\
= \frac{\delta}{\delta x} \left( \frac{\delta u}{\delta t} \right) - \frac{\delta}{\delta x} \left( \frac{\delta r}{\delta t} \right) \\
= \frac{\delta}{\delta x} \left( \frac{\delta u}{\delta t} \right) \\
= \frac{\delta}{\delta t} \left( \frac{\delta u}{\delta x} \right) \\
= \frac{\delta}{\delta t} (\epsilon)
\]

The term \( \frac{\delta}{\delta x} \left( \frac{\delta r}{\delta t} \right) \) is zero because \( r \) is time dependent only.
A2 SOME DETAILS OF THE FINITE ELEMENT IMPLEMENTATION

A2-1 The Hermite interpolation function and the element matrices

Hermite cubic interpolation functions are used to interpolate the spatial approximation, and they take the form as:

\[
\psi_1 = 1 - 3\left(\frac{x}{h}\right)^2 + 2\left(\frac{x}{h}\right)^3 \tag{A2.1}
\]

\[
\psi_2 = x - \frac{x^2}{h} + \frac{x^3}{h^2} \tag{A2.2}
\]

\[
\psi_3 = 3\left(\frac{x}{h}\right)^2 - 2\left(\frac{x}{h}\right)^3 \tag{A2.3}
\]

\[
\psi_4 = -\frac{x^2}{h} + \frac{x^3}{h^2} \tag{A2.4}
\]

where \(h\) is the length of each element.

Substitution of equation (A2.1)-(A2.4) and their derivatives into equation (3.8)-(3.13), all element matrices are defined as the following:

\[
[M] = \frac{m h}{420} \begin{bmatrix}
156 & 22h & 54 & -13h \\
22h & 4h^2 & 13h & -3h^2 \\
54 & 13h & 156 & -22h \\
-13h & -3h^2 & -22h & 4h^2 \\
\end{bmatrix} \tag{A2.5}
\]
\[ [C]^e = 2c \begin{bmatrix} -0.5 & 0.1 & 0.5 & -0.1 \\ -0.1 & 0 & 0.1 & -\frac{h^2}{60} \\ -0.5 & -0.1 & 0.5 & 0.1 \\ 0.1 & -\frac{h^2}{60} & -0.1 & 0 \end{bmatrix} \]  
(A2.6)

\[ [K_0]^e = \frac{k}{h^3} \begin{bmatrix} 12 & 6h & -12 & 6h \\ 6h & 4h^2 & -6h & 2h^2 \\ -12 & -6h & 12 & -6h \\ 6h & 2h^2 & -6h & 4h^2 \end{bmatrix} \]  
(A2.7)

\[ [K_1]^e = \frac{\tau}{30h} \begin{bmatrix} 36 & 3h & -36 & 3h \\ 3h & 4h^2 & -3h & -h^2 \\ -36 & -3h & 36 & -3h \\ 3h & -h^2 & -3h & 4h^2 \end{bmatrix} \]  
(A2.8)

\[ [K_2]^e = \frac{\lambda}{30h} \begin{bmatrix} -36 & -33h & 36 & -3h \\ -3h & -4h^2 & 3h & h^2 \\ 36 & 3h & -36 & 33h \\ -3h & h^2 & 3h & -4h^2 \end{bmatrix} \]  
(A2.9)
where $f^e$ in equation (A2.10) is the distributive, external force along an element of the web. In this study, the only external force, that is taken into account for one case, is the gravitational force of the web. As a result, the external force acting on an element is

$$f^e = \rho Ag$$  \hspace{1cm} (A2.11)

where $\rho$ is the density of web, $A$ is the cross-sectional area of the web, and $g$ is the acceleration due to gravity.

With equation (A2.11), the external element force matrix is expressed as

$$[f]^e = \frac{\rho Ag h}{2} \begin{bmatrix} 1 \\ \frac{h}{6} \\ 1 \\ -\frac{h}{6} \end{bmatrix}$$  \hspace{1cm} (A2.12)
A2-2 The end conditions and its approximated function

The only boundary conditions that needed to be evaluated are those at the first and the last nodes, because those end conditions between elements will cancel each other out when the global matrices are being assembled.

Figure A-2 shows the approximate functions for the solutions at both ends, and those approximate function and their derivatives take the values of

\[
\begin{align*}
\phi_1 &= 1, \quad \phi_1' = 0 \quad \text{at } x = 0 \quad (A2.13) \\
\phi_2 &= 0, \quad \phi_2' = 1 \quad \text{at } x = 0 \quad (A2.14) \\
\phi_{n-1} &= 1, \quad \phi_{n-1}' = 0 \quad \text{at } x = 1 \quad (A2.15) \\
\phi_n &= 0, \quad \phi_n' = 1 \quad \text{at } x = 1 \quad (A2.16)
\end{align*}
\]

![Figure A-2 Approximate function of first and last nodes.](image)

Applying the approximate function equations (A2.13)-(A2.16)
to equation (3.14)

\[ b_i = [M \phi'_i - Q \phi_i]^h \quad (3.14) \]

for the first and the last elements at the first and the last nodes gives

\[ b_1 = Q(0) \]
\[ b_2 = - M(0) \]
\[ b_{n-1} = - Q(i) \]
\[ b_n = M(i) \]

The shear forces \( Q \) and the moments \( M \) can be found by using equations (2.21)-(2.24).
APPENDIX B


B2  Analytical solution for the natural frequencies of web

B3  Calculation of \( \tau \) with \( \omega = 20, 40, 60 \) rad/sec.
Analytical calculations of the natural frequencies of the supports, the lateral stiffness of the web, the natural frequencies of the web, and the value of \( \tau \) with different angular velocities are carried out in this appendix.

**B1. Calculations of natural frequencies of the roller-supports and the flexural stiffness of the web.**

The system is modeled as shown in Figure B1.

![Figure B1](image)

Figure B1  Model for calculation of natural frequencies of roller-supports.

The energy method [15] is used for the formulation of the equations.

\[
\begin{pmatrix}
M & 0 \\
0 & M
\end{pmatrix}
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix}
+ \begin{bmatrix}
K + k_f - k_f \\
-k_f & K + k_f
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= 0
\]  \hspace{1cm} (B.1)

or
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\ddot{y}_1 \\
\ddot{y}_2
\end{bmatrix}
+ \begin{bmatrix}
r + s & -s \\
-s & r + s
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= 0
\] (B.2)

and

\[
[K]_{eq} = \begin{bmatrix}
r + s & -s \\
-s & r + s
\end{bmatrix}
\] (B.3)

where

\[r = \frac{K}{M} \quad \text{and} \quad s = \frac{k_f}{M}\] (B.4)

Finding the value of \(k_f\) (the flexural stiffness of the web)

\[E\] (the modulus of elasticity of the web) is 4E9 Pa,

\[I\] (the area moment of inertia of the web)

\[I = \frac{(1.35) (1.25E-4)^3}{12} = 2.1973E-13 \text{ m}^4\]

\[k_f = \frac{3EI}{l^3} = \frac{3(4E9)(2.1973E-13)}{1.52^3} = 7.5082E-4 \text{ N/m}\]

The \([A]\) matrix is

\[\begin{bmatrix}
r + s & -s \\
-s & r + s
\end{bmatrix}
\] (B.5)

the eigenvalue \(\Lambda\) of \([A]\) will give the natural frequencies of the system. The natural frequency of the system is

\[\omega_n = \sqrt{\Lambda}\] (B.6)

\[
\begin{vmatrix}
r + s - \Lambda & -s \\
-s & r + s - \Lambda
\end{vmatrix}
= 0
\] (B.7)

or

\[(r + s - \Lambda)^2 - s^2 = 0\] (B.8)

Solve equation (B.7) for \(\Lambda\)
\[ A_1 = r \]
\[ A_2 = r + 2s \]
so
\[ \omega_1 = \sqrt{r} = \sqrt{\frac{K}{M}} = 943.38 \text{ rad/sec} \]
or
\[ n_{f1} = 150 \text{ Hz} \]

where \( n_f \) means natural frequency.

\[ \omega_2 = \sqrt{r + 2s} = \sqrt{889963.724 + 2(9.0788E-8)} \approx 943.38 \text{ rad/sec} \]
or
\[ n_{f2} = 150 \text{ Hz} \]

**B2 Analytical solution for the natural frequencies of the web**

Consider the problem described in Figure B2 with tension \( p \) in the beam, the governing differential equation of the beam is

\[
EI \frac{\delta^4 w}{\delta x^4} - p \frac{\delta^2 w}{\delta x^2} + \rho A \frac{\delta^2 w}{\delta t^2} = 0 \quad (B.9)
\]

![Figure B2](image-url)  
*Figure B2 Model for calculation of natural frequencies of the web.*

66
Equation (B.9) is the same as equation (2.15) when $V$ is zero.

Because of the significant difference between the stiffness of the end-supports and the flexural stiffness of the web ($K = 7.36E9$ N/m, $k_I = 7.5082E-4$ N/m), the system of Figure B2 can be model as the system in Figure B3.

![Diagram](image)

**Figure B3** Model for calculation of natural frequencies of the web under the assumption of rigid supports.

The simple support boundary conditions for the system in Figure B3 are

1) at the left hand side:

$$w(0,t) = 0, \quad \text{and} \quad EI \frac{\partial^2 w}{\partial x^2}(0,t) = 0.$$
2) at the right hand side:

\[ w(l,t) = 0, \quad \text{and} \quad EI \frac{\partial^2 w}{\partial x^2}(l,t) = 0. \]

Assuming the solution of equation (B.9) to be

\[ w(x,t) = W(x)(A_1 \sin \omega t + B_1 \cos \omega t) \quad (B.10) \]

where \( W(x) \) is a function of space only, \( A_1 \) and \( B_1 \) are constants which can be found from the initial conditions, and the \( \omega \) is the natural frequencies of the web in rad/sec. Substituting equation (B.10) and its time derivative into equation (B.9) gives

\[ EI \frac{d^4 W}{dx^4} - \rho \frac{d^2 W}{dx^2} - \rho A_1^2 W = 0 \quad (B.11) \]

By assuming the solution of \( W \) to be

\[ W(x) = Ce^{\xi x} \quad (B.12) \]

the auxiliary equation of equation (B.11) can be obtained:

\[ \xi^4 - \frac{P}{EI} \xi^2 - \frac{\rho A_1^2}{EI} = 0 \quad (B.13) \]

Solving equation (B.13) for \( \xi \) gives

\[ \xi^2, \xi^2 = \frac{P}{2EI} \pm \sqrt{\frac{P^2}{4EI^2} + \frac{\rho A_1^2}{EI}} \quad (B.14) \]

Equation (B.12) can be rewritten as (with absolute value of \( \xi_2 \))

\[ W(x) = C_1 \cos \xi_2 x + C_2 \sin \xi_2 x + C_3 \cosh \xi_1 x + C_4 \sinh \xi_1 x \quad (B.15) \]
where $C_1, C_2, C_3$ and $C_4$ are to be determined by the boundary conditions.

Applying the left hand side boundary conditions to equation (B.15), constant $C_1 = C_3 = 0$. Hence equation (B.15) becomes

$$ W_0 = C_2 \sin \xi_2 x + C_4 \sinh \xi_1 x \quad \text{(B.16)} $$

Applying the right hand side boundary conditions to equation (B.16) gives

$$ C_2 \sin \xi_2 l + C_4 \sinh \xi_1 l = 0 \quad \text{(B.17)} $$

and

$$ -C_2 \xi_2^2 \sin \xi_2 l + C_4 \xi_1^2 \sinh \xi_1 l = 0 \quad \text{(B.18)} $$

Equations (B.17) and (B.18) are a set of simultaneous equations for unknowns $C_2$ and $C_4$, and they can be put into the matrix form as

$$
\begin{bmatrix}
\sin \xi_2 l & \sinh \xi_1 l \\
-\xi_2^2 \sin \xi_2 l & \xi_1^2 \sinh \xi_1 l
\end{bmatrix}
\begin{bmatrix}
C_2 \\
C_4
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

$\xi_1$ and $\xi_2$ can be found by setting the determinant of the coefficient matrix equal to zero.

$$ (\sinh \xi_1 l)(\sin \xi_2 l) = 0 \quad \text{(B.19)} $$

Since $\sinh \xi_1 l > 0$ for all values of $\xi_1 l \neq 0$, the only roots for equation (B.19) are

$$ \xi_2 l = n\pi, \quad n = 0, 1, 2, \ldots \quad \text{(B.20)} $$
Thus the natural frequencies of the system in Figure B3 (using equations (B.14) and (B.20) after algebraic manipulation) are

\[ \omega_n = \frac{\pi^2 EI}{l^2 \rho A} \left( n^4 + \frac{n^2 \rho l^2}{\pi^2 EI} \right) \quad (B.21) \]

The natural frequencies for the first few modes are found when \( \rho = 1000 \text{ kg/m}^3 \), \( l = 1.52 \text{ m} \), \( E = 4E9 \text{ Pa} \), \( I = 2.1273E-13 \text{ m}^4 \) (calculation is shown previously in this Appendix), \( p = 200 \text{ N} \), and \( A = 1.6875E-4 \text{ m}^2 \), and listed on Table B1.

<table>
<thead>
<tr>
<th>mode ( n )</th>
<th>Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>11.32</td>
</tr>
<tr>
<td>2</td>
<td>22.65</td>
</tr>
<tr>
<td>3</td>
<td>34.0</td>
</tr>
<tr>
<td>4</td>
<td>45.3</td>
</tr>
<tr>
<td>5</td>
<td>56.64</td>
</tr>
<tr>
<td>6</td>
<td>68.0</td>
</tr>
<tr>
<td>7</td>
<td>79.31</td>
</tr>
<tr>
<td>8</td>
<td>90.65</td>
</tr>
<tr>
<td>9</td>
<td>102.0</td>
</tr>
<tr>
<td>10</td>
<td>113.35</td>
</tr>
<tr>
<td>11</td>
<td>124.71</td>
</tr>
<tr>
<td>12</td>
<td>136.08</td>
</tr>
<tr>
<td>13</td>
<td>147.45</td>
</tr>
</tbody>
</table>
Calculation of $r$ with $\omega = 20, 40, 60$ rad/sec.

The relationship of $\omega_1$, $\omega_2$ and $p$ is described in equation (2.25), (2.26), and (2.30) in Chapter 2. If the values of $\omega$s are changed, the value of $p$ would also be changed accordingly.

Assuming $\omega_1$ and $\omega_2$ are the same, equation (2.30) is zero.

Inputting 20, 40, and 60 rad/sec into equation (2.25) for $\omega_2$, the values of $p$ are 160, 200, and 240 N, respectively.

Converting the values of angular velocities of 20, 40, and 60 rad/sec into linear velocities, the linear velocities are 5, 10, and 15 m/sec, respectively.

Using equation (2.24)

$$r = pAV^2 - p$$  \hspace{1cm} (2.24)

for $V = 5$ m/sec ($\omega = 20$ rad/sec), and $p = 160$

$$r = 10^3 (1.6875E-4)(5^2) - 160 = -155 \text{ N};$$

for $V = 5$ m/sec ($\omega = 20$ rad/sec), and $p = 160$

$$r = 10^3 (1.6875E-4)(10^2) - 160 = -183 \text{ N};$$

for $V = 5$ m/sec ($\omega = 20$ rad/sec), and $p = 160$

$$r = 10^3 (1.6875E-4)(15^2) - 240 = -202 \text{ N}. $$
APPENDIX C

All programs, which are used for this work, are listed in this appendix.

C1  PROGRAM WEB1
C2  PROGRAM AXIAL
C3  PROGRAM CONVER
PROGRAM WEB1
THIS PROGRAM IS USED TO CALCULATE THE ELEMENT STIFFNESS MATRIX, DAMPING MATRIX, AND MASS MATRIX OF THE WEB, TO CALCULATE THE GLOBAL STIFFNESS MATRIX, DAMPING MATRIX, AND MASS MATRIX OF THE WEB BASED ON THE ELEMENT MATRICES, AND IT IS ALSO USED TO TRANSFORM A SYSTEM OF DIFFERENTIAL EQUATIONS IN A FORM OF:

\[
\begin{bmatrix} \text{GM} \end{bmatrix} \cdot \ddot{Y} + \begin{bmatrix} \text{GC} \end{bmatrix} \cdot \dot{Y} + \begin{bmatrix} \text{GK} \end{bmatrix} \cdot Y = BD
\]

INTO A SYSTEM OF LINEAR EQUATIONS IN A FORM OF:

\[
\begin{bmatrix} A \end{bmatrix} \cdot Y + \begin{bmatrix} B \end{bmatrix} \cdot \dot{Y} + \begin{bmatrix} D \end{bmatrix} \cdot Y_0 = R
\]

SUB-PROGRAM DESCRIPTION:

ELEMENT...PROVIDES ELEMENT MATRICES
GLOBFX....ASSEMBLES FIXED GLOBAL MATRICES
GLOBVR....ASSEMBLES VARIABLE GLOBAL MATRICES
NUMARK....CALCULATES THE NEWMARK'S COEFFICIENT [A], [B], [D]
MULT......PREPARES ARRAYS FOR ITERATION.
SEIDEL OR GUAS....FINDS THE APPROXIMATION.
OUTPUT....WRITES DATA FOR PLOTS.

VARIABLES DESCRIPTION:

HT......THE THICKNESS OF THE WEB
WD......THE WIDTH OF THE WEB
LENGTH......THE LENGTH OF THE WEB
RO......THE DENSITY OF THE WEB
EE......MODULUS OF ELASTICITY OF THE WEB
DY......INITIAL DISPLACEMENT
G......GRAVITY
DT......TIME INCREMENT
BETA......GALERKIN'S COEFFICIENT
GAMMA......GALERKIN'S COEFFICIENT
IX......MASS MOMENT OF INERTIA
H......LENGTH OF ELEMENT
SLOPE......SLOPE OF THE LINEAR INITIAL DISPLACEMENT
VI......AXIAL VELOCITY OF WEB
Y(I)......Y-DIRECTION DISPLACEMENT ARRAY
M......ELEMENT MASS MATRIX
C......ELEMENT DAMPING MATRIX
KL,K2,K3......ELEMENT STIFFNESS MATRICES
EF......ELEMENT FORCE MATRIX
GM.......GLOBAL MASS MATRIX
GC.......GLOBAL DAMPING MATRIX
GK1,GK..GLOBAL STIFFNESS MATRICES
GF.......GLOBAL FORCE MATRIX
V1,V2...LINEAR VELOCITIES OF ROLLER 1 & 2
V1D,V2D..LINEAR ACCELERATION OF ROLLER 1 & 2
Vin.....CONSTANT INPUT VELOCITY
P.......TENSION OF WEB (CONSTANT OR VARIABLE)
P0......PRE-TENSION (FOR FREE VIBRATION ANALYSIS ONLY)

PARAMETER (NB=6, N=4, NN=168)
INTEGER CONTROL
REAL*4 A(NN,NB), B(NN,NB), D(NN,NB)
REAL*4 GM(NN,NB), GC(NN,NB), GK(NN,NB), GK1(NN,NB), GF(NN)
REAL*4 M(N,N), C(N,N), K1(N,N), K2(N,N), K3(N,N), EF(N)
REAL*4 R(NN), R0(NN), R1(NN)
REAL*4 Y(NN), Y0(NN), Y1(NN), V0(NN), SIG(NN/2)
REAL*4 ZER(NN), LENGTH, IX, P0, Vin
DIMENSION IBD(4), FBD(NN), FF(3,4), EN(3,NN)

COMMON/MATDIM/RO,EE,AREA,IX,HT,P,G,P0,Vin
COMMON/NEWMK/BETA,GAMMA
COMMON/DELTA/DT,H
COMMON/SEIDEL/ERRCRT,MAXITR
COMMON/CONTR/NFRQ,IWRT,CONTROL

TITLE
WRITE(6,600)
600 FORMAT('VIBRATION OF A MOVING WEB',
+ '------------------------')

INITIALIZATION
DO 10 I=1,NN
   R(I)=0.0
   R0(I)=0.0
   R1(I)=0.0
   ZER(I)=0.0
DO 10 J=1,NB
   A(I,J)=0.0
   B(I,J)=0.0
   D(I,J)=0.0
   GM(I,J)=0.0
   GC(I,J)=0.0
   GK(I,J)=0.0
10 END DO

AREA=HT*WD
IX=WD*HT**3/12.0
NE=NN/2-1
H = LNGTH / NE
N4 = NN - 4
T = DT
NBOUND = 4
KOUNT = 2
IFRQ = 1
STOP = -1.0
ZERO = 0.0

*.....NEWMARK'S METHOD FORMULATION ON P584 OF THE FEM
*.....BY O. C. ZIENKIEWICZ.

DT2 = DT * DT
A2 = 0.5 - 2.0 * BETA + GAMMA
A1 = 0.5 + BETA - GAMMA

*.....WRITE DATA
WRITE (6, 610)
610 FORMAT (/5X, 'INPUT DATA', /5X, '---------')
WRITE (6, 620) RO, EE
620 FORMAT (/5X, 'MATERIAL:',
1 /5X, 'DENSITY = ', 1PE12.3,
2 /5X, 'MODULUS = ', 1PE12.3)
WRITE (6, 630) HT, WD, LNGTH, IX, AREA
630 FORMAT (/5X, 'GEOMETRY:',
1 /5X, 'HEIGHT = ', 1PE12.3,
2 /5X, 'WIDTH = ', 1PE12.3,
3 /5X, 'LENGTH = ', 1PE12.3,
4 /5X, 'IXX = ', 1PE12.3,
5 /5X, 'AREA = ', 1PE12.3)
WRITE (6, 640) GAMMA, BETA, ERRCRT, MAXITR
640 FORMAT (/5X, 'GUASS-SEIDEL:',
1 /5X, 'GAMMA = ', 1PE12.3,
1 /5X, 'BETA = ', 1PE12.3,
1 /5X, 'ERRCRT = ', 1PE12.3,
2 /5X, 'MAXITR = ', I5)
WRITE (6, 650) DT, TERM, H, NE
650 FORMAT (/5X, 'OTHER INFORMATIONS:',
1 /5X, 'TIME INCREMENT = ', 1PE12.3,
1 /5X, 'TERMINATION T = ', 1PE12.3,
2 /5X, 'ELEMENT LENGTH = ', 1PE12.3,
3 /5X, 'NUMMER OF ELEMENTS = ', I5)
IF (IWRT.EQ.0) WRITE (6, 654) NFRQ
654 FORMAT (/5X, 'FREQUENCY OF PRINT OUT IS ', I4)
IF (IWRT.EQ.1) WRITE (6, 656)
656 FORMAT (/5X, 'JUST WRITE EVERY THING !')
IF (KODBND.EQ.0) WRITE (6, 660)
660 FORMAT (/5X, 'PRESCRIBED BOUNDARIES')
IF (KODBND.EQ.1) WRITE (6, 670)
670 FORMAT (/5X, 'VIBRATORY BOUNDARIES')
IF (KODLIN.EQ.0) WRITE (6, 674)
674 FORMAT (/5X, 'LINEAR PROBLEM')
IF (KODLIN.EQ.1) WRITE (6, 676)
676 FORMAT (/5X, 'NONLINEAR PROBLEM')

*.....IBD=BOUNDARY CODE (0 FIXED, 1 FREE)
READ (5, *) (IBD(I), I=1,NBOUND)
WRITE (6, 710) (IBD(I), I=1,NBOUND)
710 FORMAT (/5X, 'BOUNDARY CONDITIONS:', /5X, 'IBD(I): ', I4)
READ (5, *) EQM1, EQC1, EQK1, EQMN, EQCN, EQKN
WRITE (6, 730) EQM1, EQC1, EQK1, EQMN, EQCN, EQKN
FORMAT(//2X,'EQUIVALENT MASS, DAMPING, STIFFNESS:\','
1       /5X,'AT FIRST (LHS) END:\',1P3E12.3,
2       /5X,'AT LAST (RHS) END:\',1P3E12.3)

IF(KODINT.EQ.0) WRITE(6,678)
IF(KODINT.EQ.1) WRITE(6,679)

FORMAT(//5X,'INITIAL DISPLACEMENT AND VELOCITIES')

WRITE(6,678)
WRITE(6,679)

IF(KODINT.EQ.O)
 THEN
WRITE(6,700)

FORMAT(///5X,'INITIAL CONDITIONS')
READ(5,*),(Y0(I),I=1,NN)
WRITE(6,*),(Y0(I),I=1,NN)

DO 20 I=1,NN
 20       Y1(I)=DT*V0(I)+Y0(I)
END IF

DO 25 I=1,NN
 25       Y(I)=Y1(I)
ENDDO

WRITE THE FIRST TWO STEPS TO OUTPUT FILE

WRITE(12,*),ZERO,Y1(1)
WRITE(12,*),DT,Y1(1)
WRITE(33,*),ZERO,Y1(33)
WRITE(33,*),DT,Y1(33)
WRITE(41,*),ZERO,Y1(41)
WRITE(41,*),DT,Y1(41)

*.....FIXED MATRICES, [GM] AND [GK1]

CALL ELEMENT(N,H,M,C,K1,K2,K3,EF)
CALL GLOBFX(N,NE,NN,NB,M,K1,EF,GM,GK1,GF)

*.....INSERT B.C. (KOD=1)

*.....INITIAL FORCED B. C.

L=NN-1
K=NB-1

IF (CONTROL.EQ.3 .OR. CONTROL.EQ.4) THEN
READ(8,*),((FF(I,J),J=1,4),I=1,2)
ELSE
ENDDO

DO 101 I=1,2
 101       EN(I,1)=FF(I,1)+GF(1)
 102       EN(I,2)=FF(I,2)+GF(2)
 103       EN(I,3)=FF(I,3)+GF(3)
 104       EN(I,4)=FF(I,4)+GF(4)

DO 102 J=3,NN-2
 102       EN(I,J)=GF(J)
ENDDO

77
BOUNDARY CONDITIONS

IF (KODBND.EQ.1) THEN
  GM(1,1) = GM(1,1) + EQM1
  GM(L,K) = GM(L,K) + EQMN
END IF

TIME MARCHING

DO 100 WHILE (T.LE.TERM)
  T = T + DT
  KOUNT = KOUNT + 1
  IFRQ = IFRQ + 1
  CALL GLOBVR (N, NE, NN, NB, LNGTH, C, K2, K3, GK1, GC, GK)

INSERT B.C. (KOD=1)

IF (KODBND.EQ.1) THEN
  GC(1,1) = GC(1,1) + EQC1
  GK(1,1) = GK(1,1) + EQK1
  GC(L,K) = GC(L,K) + EQCN
  GK(L,K) = GK(L,K) + EQKN
END IF

CALL NUMARK (N, NB, NN, GM, GC, GK, A, B, D)
CALL MULT (NN, NB, NE, D, Y0, R0)
CALL MULT (NN, NB, NE, B, Y1, R1)
DO 30 I = 1, NN
  R(I) = -(R1(I) + R0(I))
  IF (IWR.T.EQ.1) THEN
    WRITE (6, 930)
  WRITE (6, 940) (R(I), I = 1, NN)
  WRITE (6, 940) (R(I), I = 1, NN)
  WRITE (6, 940) (R(I), I = 1, NN)
  END IF

INSERT B.C. (KOD=0)

FBD = FORCED BOUNDARIES (Q0, -M0, -QN, MN)

IF (CONTROL.EQ.3 .OR. CONTROL.EQ.4) THEN
  READ (8, *) (FF(3, I), I = 1, NBOUND)
ELSE
  DO I = 1, NBOUND, 1
    FF(3, I) = 0.0
  END DO
END IF

EN(3, 1) = FF(3, 1) + GF(1)
EN(3, 2) = FF(3, 2) + GF(2)
EN(3, L) = FF(3, L) + GF(L)
EN(3, NN) = FF(3, NN) + GF(NN)
DO 103 J = 3, NN - 2
  EN(J) = GF(J)
103 ENDDO

FBD(I) = -DT2*(BETA*EN(3, I) + A2*EN(2, I) + A1*EN(1, I))
DO 104 I = 1, NN
  R(I) = R(I) + FBD(I)
104 ENDDO

R(L) = R(L) + FBD(3)
R(NN) = R(NN) + FBD(4)
N4 = NN - N
DO 60 I = 1, NBOUND
60 IF (IBD(I).EQ.0) THEN
IF (I .LE. 2) THEN
   DO 40 J=1,N
   A(I,J)=0.0
   A(J,I)=0.0
   END DO
   A(I,I)=1.0
   R(I)=0.0
ELSE
   I2=N4+I
   IP2=I+2
   DO 50 J=1,N
   J2=N4+J
   JP2=J+2
   A(I2,JP2)=0.0
   A(J2,IP2)=0.0
   END DO
   A(I2,IP2)=1.0
   R(I2)=0.0
END IF
END IF
60 END DO
IF (IWRT.EQ.1) THEN
   WRITE(6,910)
   WRITE(6,940) ((A(I,J),J=1,NB),I=1,NN)
   WRITE(6,920)
   WRITE(6,940) (R(I),I=1,NN)
910 FORMAT(/5X,'[A]')
920 FORMAT(/5X,'R')
930 FORMAT(/5X,'R0,R1,R')
940 FORMAT(5X,1P6E12.3)
END IF
IF (KODLIN.EQ.0) CALL GUAS(NN, NB, NE, A, R, Y)
IF (KODLIN.EQ.1) CALL SEIDEL(NN, NB, NE, A, R, Y, Y0)
IF (IFRQ.EQ.NFRQ.OR.NFRQ.EQ.1) THEN
   IFRQ=0
   CALL OUTPUT(NN, NE, T, Y, SIG)
ELSE
   DO 70 I=1,NN
   Y0(I)=Y1(I)
   Y1(I)=Y(I)
   END DO
   DO 80 J=1,NBOUND
   EN(1,J)=EN(2,J)
   EN(2,J)=EN(3,J)
   END DO
   WRITE(9,*), STOP,(ZER(I),I=1,NN)
   WRITE(10,*), STOP,(ZER(I),I=1,NN)
   STOP
   END
SUBROUTINE ELEMENT(N,H,M,K1,K2,K3,EF)

THIS SUBROUTINE IS USED TO CALCULATE THE ELEMENT STIFFNESS MATRIX, THE ELEMENT DAMPING MATRIX, AND THE ELEMENT MASS MATRIX.

REAL*4 M(N,N),C(N,N),K1(N,N),K2(N,N),K3(N,N),EF(N)

H2=H*H
H3=H*H2

*....MASS MATRIX [M]

RA=RO*AREA
M(1,1)=156.0
M(1,2)=22.0*H
M(1,3)=54.0
M(1,4)=-13.0*H
M(2,2)=4.0*H2
M(2,3)=-M(1,4)
M(2,4)=-3.0*H2
M(3,3)=M(1,1)
M(3,4)=-M(1,2)
M(4,4)=M(2,2)

DUM=H/420.0
DO 10 I=1,N
DO 10 J=1,I
10 M(J,I)=DUM*M(J,I)
M(I,J)=M(J,I)

*....DAMPING MATRIX [C]

C(1,1)=-0.5
C(1,2)=0.1*H
C(1,3)=0.5
C(1,4)=-C(1,2)
C(2,2)=0.0
C(2,3)=C(1,2)
C(2,4)=-H2/60.0
C(3,3)=0.5
C(3,4)=C(1,2)
C(4,4)=0.0

DO 20 I=2,N
DO 20 J=1,I-1
20 C(I,J)=-C(J,I)

*....STIFFNESS MATRICES [Ki] i=1,2,3.

K1(1,1)=12.0
K1(1,2)=6.0*H
K1(1,3)=-12.0
K1(1,4)=K1(1,2)
K1(2,2)=4.0*H2
K1(2,3)=K1(1,2)
K1(2,4)=2.0*H2
K1(3,3)=K1(1,1)
K1(3,4)=K1(2,3)
K1(4,4) = K1(2,2)

K3(1,1) = -6.0/(5.0*H)
K3(1,2) = -1.1
K3(1,3) = -K3(1,1)
K3(1,4) = -0.1
K3(2,2) = -2.0*H/15.0
K3(2,3) = 0.1
K3(2,4) = H/30.0
K3(3,3) = K3(1,1)
K3(3,4) = 1.1
K3(4,4) = K3(2,2)

DO 30 I=1,N
DO 30 J=1,I
K1(J,I) = K1(J,I)/H3
K1(I,J) = K1(J,I)
K2(J,I) = C(J,I)
K2(I,J) = C(I,J)
K3(I,J) = K3(J,I)
30 END DO

K3(2,1) = -0.1
K3(4,3) = 0.1

ELEMENT FORCE MATRIX
Fa = Ra*G*H/2
EF(1) = Fa
EF(2) = Fa*H/6
EF(3) = EF(1)
EF(4) = -EF(2)

READ [M], [C], [Ki]

IF (IWRT.EQ.1) THEN
WRITE (6, 610)
WRITE (6, 640) ((M(I,J), J=1,4), I=1,4)
WRITE (6, 620)
WRITE (6, 640) ((C(I,J), J=1,4), I=1,4)
WRITE (6, 630)
WRITE (6, 640) ((K1(I,J), J=1,4), I=1,4)
WRITE (6, 640) ((K2(I,J), J=1,4), I=1,4)
WRITE (6, 640) ((K3(I,J), J=1,4), I=1,4)

610 FORMAT (/5X,'ELEMENT MASS MATRIX [M]:')

620 FORMAT (/5X,'ELEMENT DAMP MATRIX [C]:')

630 FORMAT (/5X,'ELEMENT STIF MATRIX [K]:')

2X,1PE13.4)
END IF
RETURN
END
SUBROUTINE GLOBFX(N, NE, NN, NB, M, K1, EF, GM, GK1, GF)

* THIS SUBROUTINE IS USED TO ASSEMBLE THE GLOBAL MATRICES BASED
* ON THE ELEMENT MATRICES FROM THE PREVIOUS SUBROUTINE.

COMMON/CONTR/NFRQ, IWRT
REAL*4 M(N,N), K1(N,N), IX, EF(N)
REAL*4 GM(NN, NB), GK1(NN, NB), GF(NN)
DIMENSION B1(2,6), B2(2,6)
COMMON/MATDIM/RO, EE, AREA, IX, HT, P, G, P0, Vin

* INITIALIZE
DO 5 I=1,2
   DO 5 J=1,6
      B1(I,J)=0.0
      B2(I,J)=0.0
  5  END DO

* PRELIMINARIES
   RA=RO*AREA
   FX=EE*IX
   DO 10 I=1,N
      DO 10 J=1,N
         M(I,J)=RA*M(I,J)
         K1(I,J)=FX*K1(I,J)
  10  END DO

* FIRST AND LAST PAIRS
   DO 20 I=1,2
      IP2=I+2
      K=NN-2+I
      DO 20 J=1,N
         JP2=J+2
         GM(I,J)=M(I,J)
         GK1(I,J)=K1(I,J)
         GM(K,JP2)=M(IP2,J)
         GK1(K,JP2)=K1(IP2,J)
  20  END DO

* BASIC UNIT
   DO 30 I=1,2
      IP2=I+2
      DO 30 J=1,NB
         JM2=J-2
         IF(J.LE.4) THEN
            B1(I,J)=M(IP2,J)
            B2(I,J)=K1(IP2,J)
         END IF
         IF(J.GT.2) THEN
         END IF
  30  END DO

* REPEAT BASIC UNIT
   NEM1=NE-1
   DO 40 I=1,NEM1
  40  END
DO 40 K=1,2
L=2*I+K
DO 40 J=1,NB
GM(L,J)=B1(K,J)
GK1(L,J)=B2(K,J)
END DO

* GLOBAL FOR MATRIX
GF(1)=EF(1)
GF(2)=EF(2)
GF(NN-1)=EF(3)
GF(NN) = EF(4)
DO 50 I=3,NN-3,2
GF(I) = EF(3)+EF(1)
GF(I+1) = EF(4)+EF(2)
ENDDO

* PRINT OUT
IF(IWRT.EQ.1) THEN
WRITE(6,610)
WRITE(6,630) ((GM(I,J),J=1,NB),I=1,NN)
WRITE(6,620)
WRITE(6,630) ((GK1(I,J),J=1,NB),I=1,NN)
END IF

610 FORMAT(/5X,'GLOBAL MASS MATRIX [GM]')
620 FORMAT(/5X,'GLOBAL STIF MATRIX [GK1]')
630 FORMAT(2X,1P6E12.3)
END
SUBROUTINE GLOBVR(N,NE,NN,NB,X,C,K2,K3,GK1,GC,GK)
*
* THIS PROGRAM IS USED TO ASSEMBLE THE VARIABLE GLOBAL
* MATRICES BASED ON THE ELEMENT MATRICES FROM SUBROUTINE
* 'ELEMENT'.
*
REAL*4 C(N,N),K2(N,N),K3(N,N)
REAL*4 GK1(NN,NB),GC(NN,NB),GK(NN,NB)
COMMON/MATDIM/RO,EE,AREA,IX,HT,P,G,P0,Vin
COMMON/DELTAS/DT,H
COMMON/CONTR/NFRQ,IWRT,CONTROL
*
INITIALIZATION
DO 5 I=1,NN
DO 5 J=1,NB
GC(I,J)=0.0
GK(I,J)=0.0
5 END
*
PREREQUISITES
IF (CONTROL .EQ. 4) THEN
READ(7,*) V1,V2,V1D,V2D,P,PD
SLOPV=(V2-V1)/X
SLOPD=(V2D-V1D)/X
DV=H*SLOPV
DD=H*SLOPD
V(I)=V1+DV/2.0
VD(I)=V1D+DD/2.0
DO 10 I=2,NE
10 V(I)=V(I-1)+DV
VD(I)=VD(I-1)+DD
ELSE
DO 10 I=2,NE
10 V(I)=V1
ENDDO
P = P0
EPSD = 0.0
END IF
*
COMPUTE TAU,Q,LMD PER ELEMENT
RA=RO*AREA
FX=EE*IX
EPSD=PD/(EE*AREA)
DO 20 I=1,NE
Q(I)=RA*V(I)
QDI=RA*VD(I)
TAU(I)=Q(I)*V(I)-P
LMD(I)=QDI+Q(I)*EPSD
20 END
*
FIRST PAIRS OF ROWS
DO 30 I=1,2
DO 30 J=1,N
GC(I,J)=2.0*Q(1)*C(I,J)
GK(I,J)=LMD(1)*K2(I,J)+TAU(1)*K3(I,J)
30 END DO

*.....LAST PAIRS OF RAWS
DO 40 I=1,2
K=NN-2+I
IP2=I+2
DO 40 J=1,N
JP2=J+2
GC(K,JP2)=2.0*Q(N)*C(IP2,J)
GK(K,JP2)=LMD(N)*K2(IP2,J)+TAU(N)*K3(IP2,J)
40 END DO

*.....IN BETWEEN
NEM1=NE-1
DO 50 I=1,NEM1
IP1=I+1
DO 50 K=1,2
L=2*I+K
KP2=K+2
DO 50 J=1,NB
JM2=J-2
IF(J.LE.4) THEN
GC(L,J)=2.0*Q(I)*C(KP2,J)
GK(L,J)=LMD(I)*K2(KP2,J)+TAU(I)*K3(KP2,J)
END IF
IF(J.GT.2) THEN
GC(L,J)=GC(L,J)+2.0*Q(IP1)*C(K,JM2)
GK(L,J)=GK(L,J)+LMD(IP1)*K2(K,JM2)+TAU(IP1)*K3(K,JM2)
END IF
50 END DO
DO 60 I=1,NN
DO 60 J=1,NB
60 GK(I,J)=GK(I,J)+GK1(I,J)

*.....WRITE
IF(IWRT.EQ.1) THEN
WRITE(6,610)
WRITE(6,630) ((GC(I,J),J=1,NB),I=1,NN)
WRITE(6,620)
WRITE(6,630) ((GK(I,J),J=1,NB),I=1,NN)
610 FORMAT(/5X,'GLOBAL DAMPING MATRIX [GC]')
620 FORMAT(/5X,'GLOBAL STIFNESS MATRIX [GK]')
630 FORMAT(2X,1P6E12.3)
END IF
RETURN
SUBROUTINE NUMARK(N, NB, NN, GM, GC, GK, A, B, D)

THIS SUBROUTINE IS USED TO TRANSFORM A SYSTEM OF DIFFERENTIAL EQUATIONS INTO A SYSTEM OF LINEAR ALGEBRA EQUATIONS.

REAL*4 A(NN, NB), B(NN, NB), D(NN, NB)
REAL*4 GM(NN, NB), GC(NN, NB), GK(NN, NB)
COMMON/NEWMARK/BETA, GAMMA
COMMON/Deltas/DT, H
COMMON/CONTR/NFRQ, IWRT, CONTROL

PREREQUISITES

D2 = DT*DT
D11 = 1.0
D12 = DT*GAMMA
D13 = D2*BETA
D21 = -2.0
D22 = DT*(1.0 - 2.0*BETA + GAMMA)
D23 = D2*(0.5 - 2.0*BETA + GAMMA)
D31 = 1.0
D32 = DT*(-1.0 + GAMMA)
D33 = D2*(0.5 + BETA - GAMMA)

....COMPUTE [A], [B], [C]

DO 10 I = 1, NN
  DO 10 J = 1, NB
    D(I, J) = D31*GM(I, J) + D32*GC(I, J) + D33*GK(I, J)
  10  END DO

....WRITE [A], [B], [D]

IF(IWRT.EQ.1) THEN
  WRITE(6, 610)
  WRITE(6, 640) ((A(I, J), J = 1, NB), I = 1, NN)
  WRITE(6, 620)
  WRITE(6, 640) ((B(I, J), J = 1, NB), I = 1, NN)
  WRITE(6, 630)
  WRITE(6, 640) ((D(I, J), J = 1, NB), I = 1, NN)
ENDIF

610  FORMAT(/5X, ' [A]')
620  FORMAT(/5X, ' [B]')
630  FORMAT(/5X, ' [D]')
640  FORMAT(5X, 1P6E12.3)

END
SUBROUTINE MULT(N,M,NE,A,X,Y)

REAL*4 A(N,M),X(N),Y(N)

*.....FIRST PAIR
DO 20 K=1,2
SUM=0.0
DO 10 J=1,M
10 SUM=SUM+A(K,J)*X(J)
Y(K)=SUM
20 END DO

*.....LAST PAIR
NM2=N-2
NMM=N-M
DO 40 K=1,2
SUM=0.0
I=NM2+K
DO 30 J=1,M
30 SUM=SUM+A(I,J)*X(NMM+J)
Y(I)=SUM
40 END DO

*.....IN-BETWEEN PAIRS
DO 100 II=2,NE
I0=2*(II-1)
L0=2*(II-2)
DO 50 K=1,2
SUM=0.0
I=I0+K
DO 50 J=1,M
L=L0+J
SUM=SUM+A(I,J)*X(L)
50 END DO
Y(I)=SUM
60 END DO
100 END DO
RETURN
END
SUBROUTINE SEIDEL(N,M,NE,A,R,X,X0)

THIS SUBROUTINE IS USED TO SOLVE A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS USING GAUSS-SEIDEL METHOD.

DIMENSION A(N,M),R(N),X(N),X0(N)
COMMON/SEIDEL/ERRCRT,MAXITR
COMMON/CONTR/NFRQ,IWRT,CONTROL
KOUNT=1

DO 10 I=1,N
   X0(I)=X(I)

DUM=A(1,1)
SUM=0.0
DO 20 J=2,M
   SUM=SUM+A(1,J)*X(J)
X(1)=(R(1)-SUM)/DUM
DUM=A(2,2)
SUM=A(2,1)*X(1)
DO 30 J=3,M
   SUM=SUM+A(2,J)*X(J)
X(2)=(R(2)-SUM)/DUM

DO 60 I=2,NE
   I0=2*(I-1)
   L0=2*(I-2)
   DO 50 K=1,2
      SUM=0.0
      I=I0+K
      IF(K.EQ.1) IPIV=3
      IF(K.EQ.2) IPIV=4
      DUM=A(I,IPIV)
      SUM=0.0
      DO 40 J=1,M
         IF(J.EQ.IPIV) GO TO 40
         L=L0+J
         SUM=SUM+A(I,J)*X(L)
   40 END DO
   X(I)=(R(I)-SUM)/DUM

NM1=N-1
NM2=N-2
NM7=N-M
MM1=M-1
MM2=M-2
DUM=A(NM1,MM1)
SUM=0.0
DO 70 J=1,M

/*...LAST PAIRS*/
IF(J.EQ.MM1) GO TO 70
00060  L=NM7+J
00061  SUM=SUM+A(NM1,J)*X(L)
00062  X(NM1)=(R(NM1)-SUM)/DUM
00063  DUM=A(N,M)
00064  SUM=0.0
00065  DO 80 J=1,MM1
00066   L=NM7+J
00067   SUM=SUM+A(N,J)*X(L)
00068  80 END DO
00069  X(N)=(R(N)-SUM)/DUM
00070  *....CHECK CONVERGENCE
00071    ERRMAX=0.0
00072    DO 90 I=1,N
00073      ERRI=X(I)-X0(I)
00074      ABSERR=ABS(ERRI)
00075    ERRMAX=AMAX1(ABSERR,ERRMAX)
00076  90 END DO
00077    IF(ERRMAX.LE.ERRCRT) KOUNT=0
00078    IF(ERRMAX.GT.ERRCRT) THEN
00079       KOUNT=KOUNT+1
00080    IF(KOUNT.GT.MAXITR) THEN
00081       KOUNT=0
00082    WRITE(6,610)
00083  610 FORMAT(//5X,'**** NO CONVERGENCE ****')
00084    END IF
00085    END IF
00086    IF(IWRT.EQ.1) THEN
00087     WRITE(6,620) KOUNT
00088  620 FORMAT(/2X,'ITERATION NO.:',14)
00089     WRITE(6,630) (X(I),I=1,N)
00090  630 FORMAT(2X,1P10E11.3)
00091     WRITE(6,640) ERRMAX
00092  640 FORMAT(2X,'ERROR=',1PE12.3)
00093    END IF
00094    IF(KOUNT.NE.0) GO TO 100
00095    RETURN
00096    END
SUBROUTINE GUAS(N,M,NE,A,R,Y)

THIS SUBROUTINE IS USED TO SOLVE A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS USING GAUSS-ELEMINATION METHOD WITH BACK SUBSTITUTION.

DIMENSION A(N,M),R(N),Y(N)

PRELIMINARIES
NM1=N-1
MM1=M-1
MM2=M-2

ELEMINATION

FIRST PAIR
DO 10 K=1,2
KP1=K+1
DO 10 I=KP1,MM2
C=A(I,K)/A(K,K)
DO 5 J=KP1,M
A(I,J)=A(I,J)-C*A(K,J)
R(I)=R(I)-C*R(K)
10 END DO

IN-BETWEEN
DO 50 II=2,NE-1
I0=2*(II-1)
DO 50 K=1,2
KP1=K+1
IPV=I0+K
JPP=2+K
IF(K.EQ.1) THEN
IPVP1=IPV+1
JPVP1=JPP+1
C=A(IPVP1,JPV)/A(IPV,JPV)
DO 20 J=JPVP1,M
A(IPVP1,J)=A(IPVP1,J)-C*A(IPV,J)
R(IPVP1)=R(IPVP1)-C*R(IPV)
20 END IF
DO 40 IP=1,2
I=(I0+2)+IP
C=A(I,K)/A(IPV,JPV)
DO 30 J=KP1,MM2
JJ=JPV+J-K
A(I,J)=A(I,J)-C*A(IPV,JJ)
R(I)=R(I)-C*R(IPV)
30 END DO
40 END DO
50 END DO

LAST PAIR
I0=2*(NE-1)
DO 70 K=1,3
IPv=I0+K
JPP=2+K
IPVP1=IPv+1
JPVP1=JPP+1
DO 70 I=IPVP1,N
C = \frac{A(I, JPV)}{A(IPV, JPV)}

DO 60 J = JPV1, M

A(I, J) = A(I, J) - C * A(IPV, J)
R(I) = R(I) - C * R(IPV)
END DO

*.... BACK SUBSTITUTION

*.... FIRST PAIR
R(N) = \frac{R(N)}{A(N, M)}
R(NM1) = \frac{(R(NM1) - A(NM1, M) * R(N))}{A(NM1, MM1)}

*.... IN-BETWEEN
DO 90 II = 1, NE - 1
IB = 2 * (NE - II)
DO 90 K = 1, 2
KB = 3 - K
I = IB + KB
IF(K .EQ. 1) IPV = 4
IF(K .EQ. 2) IPV = 3
SUM = 0.0
IPVP1 = IPV + 1
DO 80 J = IPVP1, M
L = I + J - IPV
SUM = SUM + A(I, J) * R(L)
R(I) = (R(I) - SUM) / A(I, IPV)
END DO

*.... FIRST PAIR
DO 100 K = 1, 2
KB = 3 - K
SUM = 0.0
DO 95 J = KB + 1, 4
SUM = SUM + A(KB, J) * R(J)
R(KB) = (R(KB) - SUM) / A(KB, KB)
END DO

*.... OK NOW TO Y
DO 110 I = 1, N
Y(I) = R(I)
RETURN
END
SUBROUTINE OUTPUT(N,NE,T,Y,SIG)

**

THIS SUBROUTINE IS USED TO WRITE THE OUTPUT RESULTS FOR PLOTTING.

* *

COMMON/MATDIM/RO,EE,AREA,IX,HT,P,G,P0,Vn
COMMON/Deltas/DT,H
REAL*4 Y(N),SIG(NE)
REAL LEN
WRITE(6,610) T
610 FORMAT(/5X,'TIME=',1PE12.3,/5X,'---------------',/2X)
WRITE(6,620) (Y(I),I=1,N-1,2)
620 FORMAT(2X,'DISP',1P10E11.3)
WRITE(6,640) (Y(I),I=2,N,2)
640 FORMAT(2X,'SLOP',1P10E11.3)
*......FOR POST PROCESSOR
IF (T .GT. 0.301 .AND. T .LT. 0.302) THEN
WRITE(6,*)
DO 77 1=1,N,2
LEN = H*((I-1)/2)
WRITE(97,*) LEN, Y(I)
77 CONTINUE
ENDIF
IF (T .GT. 1.201 .AND. T .LT. 1.202) THEN
WRITE(6,*)
DO 78 1=1,N,2
LEN = H*((I-1)/2)
WRITE(98,*) LEN, Y(I)
78 CONTINUE
ENDIF
IF (T .GT. 1.801 .AND. T .LT. 1.802) THEN
WRITE(6,*)
DO 79 1=1,N,2
LEN = H*((I-1)/2)
WRITE(99,*) LEN, Y(I)
79 CONTINUE
ENDIF
WRITE(12,* T,Y(1)
WRITE(33,* T,Y(33)
WRITE(41,* T,Y(41)
RETURN
END
PROGRAM AXIAL
This program is used to provide the program WEB1 with those variables in the axial motion and the boundary conditions at the two ends for the vibration analyses of phase shift and transient effects.

Variable description:

- J1, J2: polar moment of inertia of rollers
- B1, B2: viscous damping of rollers 1 & 2
- HD1, HD2: radii of rollers 1 & 2
- TF: frictional torque of driven roller
- K: axial stiffness of web
- UNBL1, UNBL2: unbalanced forces of roller 1 & 2
- TRQ0: initial applied torque
- TRQ1: final torque applied (for transient analysis only)
- OMG01, OMG02: initial angular velocities of rollers 1 & 2
- P0: initial tension in the web
- TEND: end time
- DT: time increment
- T0, T1: start & end time of changing torque
- TSLP: time rate of changing torque
- V1, V2: web velocities at the contact point with roller 1 & 2
- V1D, V2D: time rate of web velocities at the contact point with rollers 1 & 2
- P: web tension
- Q1, Q2: shear forces at the left and right ends
- M1, M2: moments at the left and right ends
- PRAT: time rate of web tension

C
V1, V2, V1D, V2D, P, PRAT ....... FILE 7
Q1, M1, Q2, M2 ............ FILE 8

INTEGER TRANS
PARAMETER NMX=2000
PARAMETER NEQ=3
DIMENSION XX(NEQ), F(NEQ)
DIMENSION R1(NEQ), R2(NEQ), R3(NEQ), R4(NEQ)
REAL*4 J1, J2, B1, B2, HD1, HD2
COMMON/PARAM/J1, J2, B1, B2, HD1, HD2
COMMON/OTHER/TF, K, UNBL1, UNBL2
COMMON/INPUT/TRQ0, TRQ1, TSLP, T0, T1
C..... READ/WRITE DATA
READ (5,*) TRANS ! This is a control number for transient analysis.
READ (5,*) J1, J2, B1, B2, HD1, HD2
READ (5,*) TF, K, UNBL1, UNBL2
READ (5,*) OMG01, OMG02, P0
READ (5,*) TEND, DT
READ (5,*) TRQ0, TRQ1, T0, T1
WRITE(6,600)
600 FORMAT (/2X, 'INPUT DATA: ')
WRITE (6, 610) J1, J2, B1, B2, HD1, HD2
610 FORMAT (/2X, 'SYSTEM PARAMETERS: ')
1 /2X, 'J1=', 1PE12.3, ' (KG-M**2)',
2 /2X, 'J2=', 1PE12.3, ' (KG-M**2)',
3 /2X, 'B1=', 1PE12.3, ' (N-S-M)'
00058   4   /2X,'B2=',1PE12.3, ' (N-S-M)',
00059   5   /2X,'R1=',1PE12.3, ' (M)',
00060   6   /2X,'R2=',1PE12.3, ' (M)'
00061 WRITE(6,620) TF,K,UNBL1,UNBL2
00062   620 FORMAT(/2X,'OTHER INFORMATION:',
00063           1   /2X,'FRICTIONAL TORQUE=',1PE12.3, ' (NM)',
00064           2   /2X,'WEB AXIAL STIFF K=',1PE12.3, ' (NM)',
00065           3   /2X,'UNBALANCE IN ROL1=',1PE12.3, ' (KG-M)',
00066           4   /2X,'UNBALANCE IN ROL2=',1PE12.3, ' (KG-M)'
00067 WRITE(6,630) OMG01,OMG02,P0
00068   630 FORMAT(/2X,'INITIAL CONDITIONS:',
00069           1   /2X,'OMEGA1=',1PE12.3, ' (RAD/S)',
00070           2   /2X,'OMEGA2=',1PE12.3, ' (RAD/S)',
00071           3   /2X,'P(TEN)=',1PE12.3, ' (N)',
00072 WRITE(6,640) TEND,DT
00073   640 FORMAT(/2X,'TEND,DT....',1P2E12.3)
00074 TSLP=(TRQ1-TRQ0)/(T1-T0)
00075 WRITE(6,650) TRQ0,TRQ1,T0,T1,TSLP
00076   650 FORMAT(/2X,'TRQ0=',1PE12.3, ' (NM)',
00077           1   /2X,'TRQ1=',1PE12.3, ' (NM)',
00078           2   /2X,'T0=',1PE12.3, ' (SEC)',
00079           3   /2X,'T1=',1PE12.3, ' (SEC)',
00080           4   /2X,'TSLP=',1PE12.3, ' (NM/S)'
00081 C.....INITIALIZATION
00082 WRITE(6,660)
00083   660 FORMAT(/10X,'TIME',5X,'OMEGA1',6X,'OMEGA2',10X,'P',
00084       *10X,'Q1',10X,'Q2')
00085 XX(1)=OMG01
00086 XX(2)=OMG02
00087 XX(3)=P0
00088 T=0.0
00089 CALL FUN(T,NEQ,XX,F)
00090 CALL OUTPUT(T,NEQ,XX,F)
00091 C.....NUMERICAL INTEGRATION
00092 DO 20 WHILE (T.LE.TEND)
00093 CALL RK4(T,DT,NEQ,XX,R1,R2,R3,R4,F)
00094 CALL OUTPUT(T,NEQ,XX,F)
00095   20 END DO
00096 STOP
00097 END
The fourth-order Runge-Kutta method

SUBROUTINE RK4(T, DT, N, XX, R1, R2, R3, R4, F)
DIMENSION XX(N), F(N), R1(N), R2(N), R3(N), R4(N)
DIMENSION DUM(10)
DO 10 I=1, N
DUM(I) = XX(I)
10 END DO
CALL FUN(T, N, XX, F)
DO 20 I = 1, N
R1(I) = DT*F(I)
XX(I) = DUM(I) + R1(I)/2.0
20 END DO
T = T + DT/2.0
CALL FUN(T, N, XX, F)
DO 30 I = 1, N
R2(I) = DT*F(I)
XX(I) = DUM(I) + R2(I)/2.0
30 END DO
CALL FUN(T, N, XX, F)
DO 40 I = 1, N
R3(I) = DT*F(I)
XX(I) = DUM(I) + R3(I)
40 END DO
T = T + DT/2.0
CALL FUN(T, N, XX, F)
DO 50 I = 1, N
R4(I) = DT*F(I)
XX(I) = DUM(I) + (R1(I) + 2.0*R2(I) + 2.0*R3(I) + R4(I))/6.0
50 END DO
RETURN
END
```
C SUBROUTINE FUN(T,N,X,F)
DIMENSION X(N),F(N)
REAL*4 J1,J2,K
COMMON/PARAM/J1,J2,B1,B2,HD1,HD2
COMMON/OTHER/TF,K,UNBL1,UNBL2
COMMON/INPUT/TRQ0,TRQ1,TSLP,T0,T1
CALL INPUT(T,TRQ)
F(1) = (TRQ - B1*X(1) - HD1*X(3))/J1
F(2) = (HD2*X(3) - B2*X(2) - TF)/J2
F(3) = K*(HD1*X(1) - HD2*X(2))
RETURN
END

C SUBROUTINE INPUT(T,Y)
COMMON/INPUT/TRQ0,TRQ1,TSLP,T0,T1
IF(T.LE.T0) THEN
  Y=TRQ0
ELSE IF(T.GT.T0.AND.T.LT.T1) THEN
  Y=TRQ0+TSLP*(T-T0)
ELSE IF(T.GE.T1) THEN
  Y=TRQ1
END IF
RETURN
END
```
SUBROUTINE OUTPUT(T,N,X,F)
DIMENSION X(N),F(N)
REAL*4 J1,J2,K
COMMON/PARAM/J1,J2,B1,B2,HD1,HD2
COMMON/OTHER/TF,K,UNBL1,UNBL2
C..HISTORIES OF OMG1,OMG2,P TO FILES 56,55,24
OMG1=X(1)
OMG2=X(2)
P =X(3)
ALF1=F(1)
ALF2=F(2)
PRAT=F(3)
IF (TRANS .EQ. 1) THEN
WRITE(56,*),T,OMG1
WRITE(55,*),T,OMG2
WRITE(24,*),T,P
ENDIF
C..OTHERS FOR WEB1
VI =HD1*OMG1
V2 =HD2*OMG2
V1D=HD1*ALF1
V2D=HD2*ALF2
WRITE(7,610) V1,V2,V1D,V2D,P,PRAT
Q1,M1,Q2,M2 TO FILE 8
OT1=OMG1*T
OT2=OMG2*T
S1 =SIN(OT1)
S2 =SIN(OT2)
C1 =COS(OT1)
C2 =COS(OT2)
Q1 =+UNBL1*(OMG1*OMG1*S1-ALF1*C1)
Q2 =+UNBL2*(OMG2*OMG2*S2-ALF2*C2)
ZER=0.0E0
WRITE(8,*),Q1,ZER,Q2,ZER
610 FORMAT(4X,1P6E12.3)
C..LAST INFORMATION
IF (TRANS .EQ. 1) THEN
CALL INPUT(T,TRQ)
WRITE(27,*),T,TRQ
ENDIF
RETURN
END
PROGRAM CONVER
This program is used to convert the data of the responses of the web-roller system from the time domain into the frequency domain.

Variables Description:

* N number of data point
* IN file control number
* DT time increment
* T(I) time array
* SEQ(I) displacement array
* FREQ(I) frequency array
* COEF(I) amplitude in the frequency domain

```
!INTEGER N, I, IN!
!REAL PI, DFREQ, DT, TN!
!PARAMETER (N=1930, PI=3.14159)
!REAL COEF(N), SEQ(N), FREQ(N), T(N), seq1(N)
!EXTERNAL FFTRF

READ(5,*) IN !Get control number
IF (IN .EQ. 0) THEN
  C reading input file by unit number
  C file unit 12 is the first node data (X/1 = 0.0)
  C file unit 33 is the node 33 data (X/1 = 0.8)
  C file unit 41 is the node 41 data (X/1 = 1.0)
DO 10 I=1,N
  READ(12,*) T(I), SEQ(I)
10 CONTINUE
ELSE IF (IN .EQ. 1) THEN
  DO 11 I=1,N
    READ(33,*) T(I), SEQ(I)
11 CONTINUE
ELSE IF (IN .EQ. 2) THEN
  DO 12 I=1,N
    READ(41,*) T(I), SEQ(I)
12 CONTINUE
ENDIF
DFREQ = 1/(2*T(N))
CALL FFTRF(N, SEQ, COEF)
DO 30 I=1,N
  COEF(I) = ABS(COEF(I))
30 CONTINUE
WRITE (98,*) FREQ(I), COEF(I)
DO 40 I=2,N
  FREQ(I) = FREQ(I-1) + DFREQ
  IF (FREQ(I) .LT. 220) THEN
    WRITE (98,*) FREQ(I), COEF(I)
  ENDIF
40 CONTINUE
STOP
END
```