Visual modulation transfer function as a predictor of acuity

Thomas A. Sebring
VISUAL MODULATION TRANSFER FUNCTION
AS A PREDICTOR OF ACUITY

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Science in the School of Photographic Arts and Sciences in the College of Graphic Arts and Photography of the Rochester Institute of Technology

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ABSTRACT

The use of fourier and modulation transfer techniques implies the ability to characterize an optical system and predict the output resulting from various inputs. In this study, the optical system is defined as a projection system, rear projection screen, and the human eye. A determination of modulation thresholds of visibility for a series of sinusoidal (in transmittance) test objects results after some manipulation in the modulation transfer function of the complete system. This result is then used to predict the subject's ability to resolve conventional acuity challenges. These conventional test objects are then
manufactured for projection in the system. Correlation between predicted acuity and observed acuity as tested may now be determined. The result indicates the degree to which optical designers may rely on modulation transfer and fourier techniques to predict the visual performance of a system.

Presentation of targets was made using a modified slide projector. Changes in modulation were achieved by addition of a veiling glare projector. Subject response consisted of a forced choice, yes or no, as to whether modulation was perceived in the field of view. Thresholds of perception were defined as that modulation correctly observed 50% of the time. Two-bar targets were produced, and characterized by microdensitometer traces. Using the discrete fourier transform, these targets were convolved with the spread function defined by the M.T.F. Resulting absorption profiles permitted an estimate of image degradation. When essential details of the object were lost it was predicted that visual resolution would fail. The accuracy of this prediction was tested by presentation of the two bar targets to the subject.

Results indicate that while modulation transfer and fourier techniques permit rapid assessment of an optical
system, that many factors affect subjective visual response. In testing it was found that while a high degree of correlation exists between predicted resolution and actual resolution, variability inherent in threshold measurement precludes precision in prediction.
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INTRODUCTION

Research into visual acuities has enjoyed dramatic and rapid advancement due to the recent application of fourier techniques to describe the subjective response of the eye to given stimuli as modulation transfer characteristics. Correlation of varied published results is difficult as variations in systems and techniques result in great disparity between them. In addition, many studies attempt to separate the optical and psychophysical elements contributing to response in an effort to better understand these systems. It is the performance of the eye, together with an instrument, however, which is of interest to the designer of optical equipment intended for visual use. This paper outlines a method for determining the confidence which may be assigned to predictions of visual acuity made using the modulation transfer function of the system, mathematical modeling of the input and discrete fourier techniques. The ability to rely on these methods of simulation would provide the optical engineer with additional tools for use in system design.
BACKGROUND

Pre-fourier research into visual acuities correctly defines characteristics of target composition and presentation which affect subjective visibility. Target size, contrast (modulation), surround luminance, target shape, adaptation time, temporal variation of response, and pupil diameter are some elements which affect the visual process. Additionally, psychophysical characteristics such as variability between viewing sessions, subject motivation, order of target presentation, and suggestibility of subjects have been shown to interfere with objective quantification of results.\(^1\) It becomes apparent that most of these parameters must be closely controlled that the significance of those remaining may be demonstrated.

Problems inherent in the characterization of vision become additionally complicated when the spatial frequency response is the observed parameter. Geometries of target presentation, methods of observer evaluation, and apparatus used in presentation all contribute to a lack of concordance among published results. (See fig. 1) Published accounts
of visual M.T.F. determination may be loosely grouped into two types. In the first, the observer matches the contrast which exists to a visual reference, also present in his field of view. The second method involves the presentation of sinusoidal targets at varying frequencies and modulations in an attempt to determine the modulation threshold of perception at different spatial frequencies. The inverse of the relationship between the modulation threshold of perception and spatial frequency is the modulation transfer function. Several reasons exist for the choice of the contrast threshold method here. Apparatus required for target presentation in contrast matching experiments are more complex due to requirements of simultaneous presentation of reference patches. Further, it may be argued that variability introduced by a second object within the field of view may preclude precise determinations.

Parameters directly affecting visual modulation transfer as established in the literature include; Viewing distance and length of presentation, Average brightness of targets and Sine vs. Square-Wave targets. No record is found of attempts to use the derived modulation transfer characteristics to predict optical performance of the eye at a different task. This may be due in part to the fact that
variability in subjective measurement of optical thresholds is notoriously poor. Indeed as shown in Bryngdahls work\textsuperscript{3} of 1964, variability in the data collected lacked enough concordance to even result in one well-defined average response characteristic. This may be due in part to the fact that Bryngdahl, as others,\textsuperscript{2} attempts to subtract the characteristics of the presentation system from those of the viewer, rather than treating the entire system as is done here.

As described above, much work has been done in describing the response of the human visual system in terms of modulation transfer characteristics, yet it is significant that little has been published about the use of the relationship to predict or describe the eye's response to a given stimulus. Such a stimulus in its simplest sense might be a resolution challenge.

In situations where the target consists of distinct, separate parts between which the background subsists, the ability to perceive this separation is termed a resolution acuity challenge. Additional visual acuities exist in shape discrimination acuity and vernier or alignment acuity.\textsuperscript{7} Fourier analysis techniques imply the ability to model targets presented in tests of these parameters. Providing that requirements of linearity, homogeneity, and isotropy are met,
then Fourier and modulation transfer mathematics allow the prediction of subjective visual response to given targets. The visual systems' response with respect to brightness, location on the retina, and image orientation is anything but linear. Obviously, some constraints are required to permit the application of these techniques.

The near logarithmic response of the eye to variations in brightness must be obviated by the use of small perturbations in the target field that linearity over some small range of brightness may be postulated. Nonhomogeneity of the retina and thus the visual system presents another difficulty and render results valid only for the small region near the optic axis. The anisotropic nature of the human eye due either to optical astigmatism or nervous anomalies may be reduced by corrective lenses to the point where sensitivity to modulation is altered no more than 15% by target orientation. Alternatively, results may be considered valid for only one target orientation. Within these constraints, the use of the visual sine-wave modulation transfer function to predict subjective perception is postulated to be valid.

Were it possible to predict visual performance in this manner, it would provide the optical engineer with
the tools to evaluate the quality of an imaging system including an observer. This hopefully would represent a quantum improvement over best case/worst case analysis of visual resolution. The following sections of this document illustrate the techniques used and the results obtained in evaluation of this procedure.
METHODS AND PROCEDURES

Work necessary to determine the correlation between predicted visual acuity and actual visual acuity is generally divided into three parts. The first part consists of a determination of the combined modulation transfer function of an observer and a projection system. The second part involves the use of the results derived in the first part to predict visual response to conventional resolution test objects. The third phase is a presentation of those conventional test objects to the same observer in the same system and a test of the correlation between predicted acuity and the observed actual acuity. Methods and procedures for the performance of these sequential operations are also divided into three general classifications. These are: 1) Physical requirements which relate to the ability to present the required images to the observer in a measurable, repeatable fashion, 2) Psychophysical requirements embrace the elements which must be controlled to assure viable quantification of subjective interpretation, 3) Mathematical considerations include the calculation of required quantities and functional relationships and statistical determination of significance.
Apparatus

Projection System

It was decided to use the system illustrated in figure (2a). The projector was in one room while observation was made from another. This minimizes stray light problems from the projector and minimizes clues which might alter subject response. A rear projection screen (30cm x 25cm) was placed in a plywood door between the rooms, centered at eye level for a seated observer of average height. In order that conditions of homogeneity and isotropy be fulfilled a viewing jig (figure 2b) was constructed which assures repeatability of subject position and orientation. It was decided to use an artificial pupil in an effort to minimize the introduction of variability due to adaptation and spherical aberration of the eye. 3.75m.m. was chosen for the diameter as this was an "average" value in the literature. During early phases of experimentation, it became apparent that the aperture was far enough from the eye that it was functioning as a field stop. The viewing jig was then modified to place the aperture at the eyelash of the observer, i.e., as close to the eye as possible. The eye-piece of this jig also limited the cone angle of vision to a region within the boundaries of the screen.
**Figure 2a - Projection System (Top View)**

- **a** = Projector (Object)
- **b** = Projector (Veiling Glare)
- **c** = Photometer
- **d** = Rear Projection Screen
- **e** = Viewing Fixture
- **f** = Observer

**Figure 2b - Viewing Jig (Side View)**

- **a** = Chin Rest
- **b** = Aperture
- **c** = Forehead Rest
- **d** = Detector
- **e** = Base

**Figure 2c - Veiling Glare Projector**

- **a** = Tube (12")
- **b** = Lamp (12V, 15W)
- **c** = Transformer
- **d** = Dimmer
The projector required modification for two reasons:

1) The illumination needed for experimentation in about the mesopic range of vision was much less than that afforded by the 300 watt bulb. 2) Field illuminance was determined to be extremely uneven, introducing modulation unrelated to the test object. Pursuant to these requirements the projector lamp was replaced with a 40 watt, spherical, outside frosted lamp. Further, a piece of .125" opal glass was placed in the optical path before the slide to diffuse the source. These modifications corrected the faults found in the projector.

Variation of the modulation of the test images was achieved by the use of a veiling glare projector (figure 2c). The 12 volt 15 watt bulb is powered by a triac controlled transformer. This permitted the open gate level of veiling glare illuminance to be matched to that of the image projector, an adjustment discussed later. Modulation changes as calculated and tabulated in the mathematical section were achieved by the introduction and removal of neutral density filters from the paths of the projectors. The intent was that a filter removed from one beam and placed in the other would result in the same average illuminance at the screen.

Measurement of open gate projector illuminances and
monitoring of the intensity levels of the target presentations was done using a United Detector Technologies radiometer, and a table of these intensities for each treatment is found in the appendix. Photopic filtration was used to simulate visual perception and to include a rough test of spectral consistency.

Variation of projector/viewer geometries permitted a large variation of image frequencies on the retina. This fact was used in presentation of the bar target image to eliminate variability caused by differences between the targets.

Targets

Test objects sinusoidal in transmittance were provided by the Eastman Kodak Company. The production and characterization of these objects is described in an article by R. L. Lamberts of Kodak.\(^8\) Patches of various frequencies were excised from the supplied target. Kodalith masks were fabricated to prevent light spillage around the patches. These components were assembled in standard 2 inch slide mounts. Microdensitometer traces of these test objects were made and used in determination of the objects' modulations and frequencies. These criterion are tabulated in the appendicies.
Double bar targets were chosen as conventional resolution targets. Three strips of .125" Geotype graphic tape were applied to a large white sheet of paper with their edges just touching. The center piece was then removed creating an equal space between two dark bars. This target was photographed with Kodak film type Technical Pan, processed, and then duplicated on Ortho Pan. Microdensitometer traces of the target used begin the prediction sequence in the appendix.

Psychophysical Considerations

Experimental Design

The repeatability of viewing conditions was considered elemental in achieving acceptable confidence of results. Subject viewing geometry was maintained constant by use of the aforementioned viewing fixture. Further, the artificial pupil arrangement limited the width of field of view. This effect is shown in figure 2b. The effect is that of maintaining a viewing field of constant cone angle to the observer. This permitted modification of the magnification of the eye by changing viewing distance. Though viewing distance was determined to be a significant factor affecting spatial resolution in some studies$^9,5$ later work demonstrates$^6$ that if the cone angle of vision is held constant then within reason-
able limits, viewing distance fails to be significant. This fact was used in the presentation of bar targets where viewing distance was altered rather than target size.

The division of the two rooms for projection and viewing minimized several spurious inputs which could have affected viewer response. Stray light from the projection screen was virtually below the capabilities of the United Detector Technologies photometer to detect, (less than $10^{-4}$ microwatts) at the viewing position. The subject could not observe any details of target manipulation which might supply cues for correct response. Viewing sessions began with a dark adaptation time of twenty minutes and a rest of five minutes between experimental runs was allowed.\footnote{1} Initially the subject was advised of the presentation of a target vocally, but as the observer could hear the automatic changing mechanism of the projector, this was deemed sufficient notification. The subject was informed that somewhere in the experimental run that a blank slide would be presented. Functionally, a slide of much higher frequency than was ever detected was used for this "blind". The order of slide presentation was varied for each run, that the subject not make determinations of visibility based on foreknowledge. The modulation was progressively lowered stepwise until no modulation in the field of view was perceived. The subjects
response was a forced choice, "yes" the target was observed, "no", no modulation was observed. The right eye of the subject having "normal" uncorrected vision was used exclusively. No advisement of correct responses was made as it was unknown which targets at what modulations should be observed.

Mathematical

Target Frequencies

Calculations of target frequencies to be presented were based on: 1) Previous frequencies used in visual M.T.F. determination.\textsuperscript{10} 2) Projector Magnification, and 3) The magnification of the average human eye.\textsuperscript{11} Projector magnification was determined empirically by the manufacture, characterization on measuring microscope, projection, and image measurement of a slide made of a machinists rule. The projector magnification throughout at a distance from the screen of 3.05 meters was found to be 27.4X. A series of visual magnifications based on the formula: \( \frac{1}{m} = \frac{s-f}{f} \) and the estimate of visual focal length, \( f = 17 \text{mm} \)\textsuperscript{11} were determined. It was found during initial phases of experimentation that frequencies as high as those predicted by Gubisch\textsuperscript{10} would not be observed in this system. Target frequencies may be observed in table 2 in the appendix.
**Target Modulation**

Though the variable transmittance sinusoidal test objects manufactured by Lamberts are characterized in his article, the origin of the specimens available was unclear. Thus it was determined to characterize the frequency and modulations of each sample. A microdensitometer trace of each sample was made and referenced through a callibration curve of machine vs. actual density. The resulting differences between maximum and minimum density allowed verification of modulation using the table 17.3 from the S.P.S.E. Handbook. Conversion to transmittance of these maxima and minima followed by averaging yielded the average transmittance of each sample. This information is also in table 2 in the appendix.

**Presentation Modulation**

Variations in the modulation at which each target would be presented were made by adjusting stepwise the amount of contribution to the resulting image made by the object projector with its sinusoidal slide and the veiling glare projector. Modulation at the screen was determined from a mathematical model of the presentation. The form of this model is as follows:
Modulation at Screen = \frac{IgNg + IoTmaxNo - IgNg + IoTminNo}{IgNg + IoTmaxNo + IgNg + IoTminNo}

where: 

Io = open gate illuminance of object projector  
Ig = open gate illuminance of veiling glare projector  
Tmax = maximum transmittance of slide  
Tmin = minimum transmittance of slide  
Ng = total transmittance of neutral density placed in the path of the veiling glare  
No = total transmittance of neutral density placed in the path of the object projector

If we further stipulate that Io = Ig, that is that the open gate illuminance of the two projectors are set equal before the experiment begins we will obtain the least variation in total illumination when neutral density filters are moved from one path to the other. Additionally, using mathematics of modest rigor the previous expression becomes:

\begin{align*}
\text{Modulation at Screen} & = \frac{Tmax - Tmin}{Tmax + Tmin} \\
& \quad \times \frac{2Ng}{No (Tmax + Tmin) + 1}
\end{align*}

where \( \frac{Tmax - Tmin}{Tmax + Tmin} \) is clearly the modulation of the test object and \( \frac{Tmax + Tmin}{2} \) is the average transmittance.
A minor problem exists in that the accumulation of neutral density stepwise results in exponentiation of transmittance, thus precluding exact constance of the sum of illumination as filters are moved back and forth. A constant check during testing however revealed that the integral illuminance at the viewing position had an average of \(0.159 \times 10^{-3}\) lux and a standard deviation of only \(0.018 \times 10^{-3}\) lux. Table 2 in the appendix illustrates the modulation of each target and table 3 shows the modulation at the screen during each treatment.

**Discrete Fourier Transforms and Predictions**

Discrete fourier transforms of bar targets were necessary in order that convolution with the experimentally obtained spread function of the system could be performed. Additionally back transforms of this product were required to obtain predicted subjective absorption profiles of the degraded bar images. An Apple computer and a fast fourier transform program supplied by Dr. E. Granger of Eastman Kodak Company and utilizing decimation in time by twos were used for this work. The capability of this program to determine both forward and backward transforms was beneficial. A microdensitometer trace of the bar target was converted to actual percent absorption. This profile
was then sampled and transformed using 32 points. The resulting function was multiplied frequency by frequency with the previously obtained M.T.F. This product was then backtransformed, resulting in the degraded image. It is the amount of degradation of this image which provides the basis for a prediction regarding resolvability. (See prediction sequence in appendix.)
EXPERIMENTAL

The aim of this experiment was to determine how well modulation transfer characteristics of a system consisting of a projection scheme and an observer would predict the resolution of a two bar target. Methods of presentation of targets has been previously discussed. One judge was used throughout as this permitted increased confidence in the correlation between prediction and performance, given a fixed number of samples.

Determination of M.T.F.

Seven targets ranging in frequency from 17.53 to .287 cycles/mm (on the retina) were presented at a total of eleven different modulations. The highest frequency target was never observed as modulated, and thus provided a, "blank" presentation. This made each run last approximately fifteen minutes. Objects were presented as long as the subject felt was required to judge the presence of modulation in the field. The thirty recorded trials occurred over a two week period and varied between two a day to six. It was felt that concentration fell off after more than two runs resulting in
casual and shoddy observation. Phenomenal reports of the subject were recorded on a tally sheet and then tabulated. Variation of modulation was achieved as mentioned, by the removal of neutral density filters from one projector path and installed in the other. These changes were made in increments of .1 neutral density. As detailed in the following section on data analysis, the relationship between modulation and frequency at the threshold of sensation resulted in the modulation transfer function of the system.

Testing of Conventional Resolution

Manufactured bar targets were presented to the observer in the same apparatus. No veiling glare was added in these presentations, yet the overall illumination from the object projector was reduced by the addition of neutral density to produce the same illumination from the screen as in the case of the M.T.F. determinations. Five different size bar targets were projected and viewed at six increasing distances. Microdensitometer traces of the bar targets showed that adjacency and line growth effects in processing had made them not symmetrical. It was decided then to use one correct target at varying distances for the criterion of conventional resolution. It must be remembered that the cone angle of vision at
each distance was limited by the viewing apparatus. The
determined limit of visual resolution with this observer
and apparatus is discussed in the data analysis section next.

Prediction of Visual Resolution

The prediction process has already been stated as one
of modeling, convolution and evaluation. While no problem
was encountered in producing the degraded image profiles,
evaluation of these profiles in an attempt to predict success
or failure of visual resolvability was more complex. The
form of the degraded image as expected showed an increasing-
ly demodulated center region (between where the bars had been)
as viewing distance increased. The point at which this de-
gradation inhibited resolution is a major conclusion of this
work and as such is discussed in that section. Generally,
however, it was postulated that an analogy existed between
the classic double star resolution problem and the resolva-
bility of two bars. This opened the way for an application
of Rayleigh's and later Sparrows\textsuperscript{12} criterion to the problem.
The correlation between predictions based on these two axioms
and actual resolvability is discussed in the Analysis of Data.
ANALYSIS OF DATA

The analysis of data for this project proceeds step-wise in three major increments. Recall that the first phase or experimentation involved the determination of the modulation transfer function of a projector-screen-observer system. This was to be derived from forced choice, yes or no, responses to the visual presentation of sinusoidal test objects demodulated stepwise by the variation of veiling glare levels. Having obtained this characteristic of the system, it was then used to determine the form of the subjective degraded image of two bar targets of varied size. Some characteristic deformation of the original image was then selected as an indicator of how much degradation signified the limit of visual resolution. Finally, the bar targets themselves were presented to the subject, resulting in an actual test of resolution limits. The correlation between hypothetical predicted and actual resolution thresholds must then be evaluated as an indicator of prediction process quality.

MTF Determination

The form of response which the observer could make was
a forced yes or no upon notification of a presentation. The form of this data, as expected varied from 100% yes for high modulation, low or middle frequency to 0% yes with low modulation, high frequency, with a few non-uniform responses about the threshold modulation for each frequency. Some frequencies were never observed and, as such, were eliminated or used as "blank" presentations. Tabulated percentages of "yes" responses appear in table 4 in the appendix.

It was considered acceptable to plot the percent yes response vs. modulation for each frequency as an "s" shaped curve (figures 6 thru 11) following Flook,\textsuperscript{13} though it is suspected that the "normality" of response distribution is due to the similarity of the binomial distribution to the normal at large sample sizes.\textsuperscript{14} Further, the approximation is best when the probability of correct response approaches .5 as it does if we define the visibility threshold to be that level of modulation for each frequency at which it is observed 50 percent of the time. Finally, these points represent averages and as such must be normally distributed. The standard deviation of the 50% correct response point is estimated by:

\[ S_x = \frac{p(1-p)}{n} \]

where \( p \) = probability of correct response

\( n = \) number of samples
This quantity is represented on the modulation transfer graph (fig. 3). We encounter here one of the limiting factors of the prediction process. The result of this estimate of variability is that we must sacrifice either confidence or precision to maintain a modulation transfer function which we may use for the prediction process. Note that this alarming width of confidence limits exist despite a sample size of 30 trials. This is, however, the disadvantage of threshold determinations as opposed to contrast matching.

From the series of graphs just discussed, modulation at threshold levels for each frequency target may be interpolated. A plot of spatial frequency against the reciprocal of threshold modulation is the modulation transfer function. Note that the modulation transfer function of this system falls off considerably more quickly than that also illustrated (by Gubisch10), for approximately the same artificial pupil diameter. Two possible reasons for this exist in the fact that Gubisch used strong white light as opposed to low light levels used here and that the transfer characteristics of the rear projection screen are included in this determination. The proportional relationship between resolution and illumination2 and the poor modulation transfer of diffuse materials support these conjectures.
Prediction Process

The prediction sequence in the appendix (figures 4 thru 10), begins with the microdensitometer trace of the bar target used for mathematical modeling. This trace was sampled and converted to percent absorption vs. position, as shown in the next graph. These values were used as input to a 32 point fast fourier transform. The spacing of each data increment was determined as the following. The width of the target used was .46mm on the slide. Project magnification increased the image size on the screen. Viewing magnification, however, as a function of viewing distance decreased the image size on the retina. For successful convolution operation it was required that the target fill only half of the 32 point window. Therefore the total width of the bar target on the retina covered 16 pts and $\Delta X$ was found to be:

$$X = \frac{\text{width at retina}}{16}$$

This quantity determined the frequency increment as

$$f = \frac{1}{N\Delta X} \text{ where } N = 32 \text{ points.}$$

Clearly, these quantities depend on viewing distance as given a fixed object size, magnification is inversely proportional to viewing distance. The result was that the
transforms hold more higher frequency components as viewing distances increased. The relatively low frequency cut off of the previously determined M.T.F. dictates however that high frequency components will not be transferred. The result is increased image degradation as retinal image size decreases.

The use of these predicted subjective absorption profiles presents a difficult task. Observing similarities, however, between these profiles and those of two overlapping point images at varying separation it can be proposed that some variation of Rayleigh's or Sparrows criterion might be applied. (see figure 4). While Raleigh's criterion is uncomplicated, it is generally considered a pessimistic approximation of visual resolution. Sparrow's more realistic criterion states that the two points will be just resolved when the central declavity between the two maximums just fails to appear.12 Previous to this point, the profile has displayed considerable similarity to the individual sinc\(^2\) type shapes of the individual sources. It is felt this similarity exists because the mach phenomenon causes the subjective impression of increased density after the first minimum.4 As the retinal image becomes smaller, however, similarity ceases. As the viewing distance increases further, the profile merely spreads instead of reaching a coincidental peak as the two-source model does.
Graphing Sparrow's criterion as a predictor of acuity then we would place the size of bar target overall on the retina on the ordinate and the proportion of trials correctly resolved on the abcissa. If we place the 50% resolved proportion at the image size dictated by Sparrow's rule, (.18 mm in this case) we have one point of a prediction curve.

If it is then stated that Rayleighs' criterion will be resolved all of the time (i.e. 1.0), a line which it is proposed will represent a predictor of acuity results. A higher order model would certainly be more likely to fit data which again will be distributed in an approximately normal manner, but correlation between predicted and actual acuity should be sufficient to demonstrate the relative merit of this technique.

**Actual Visual Resolution**

The results of presentation of bar target images to the observer are seen in table 5 in the appendix. The prediction of response is graphed against retina image size in figure 5 on the next page. Also appearing is the actual graph of retinal image size vs. proportion resolved, obtained in 15 tests of detectability at each image size (viewing distance). Clearly a higher order model is required, particularly to include regions of retinal size where resolution is nearly certain or nearly impossible. A plot was now
Figure 4

(Clearly resolved)

(Rayleigh)

(Sparrow)

(Not resolved)

(from Hecht + Zajac)
Target width on retina vs % of presentations resolved (mm)
Predicted Proportion Predicted vs Proportion Actually Resolved

line of best fit = .684 + .3X

Correlation Coefficient = .983
made of % predicted response vs. % actual response for each retinal size (figure 6). Linear regression performed on this data yielded a line of best fit with a correlation coefficient of .983. It appears that a strong correlation then does exist between prediction of visual resolution by this technique and actual visual resolution. Observation of the difference between predicted and observed resolution leads to speculation on the nature of, and reasons for, the underestimation. The nature of this discrepancy might constitute a cause for further investigation.
Derivation of Resolving Power from M.T.F.

Another technique of predicting the actual resolving power of a system given that systems modulation transfer function is outlined in the S.P.S.E. Handbook of Photographic Science and Engineering. A graph of modulation transfer vs. spatial frequency is plotted representing the M.T.F. of the system. The same axes are now used to plot the threshold curve of the system. That is the modulation required for resolution of each frequency of sinusoidal target. Recall that the M.T.F. in this experiment was derived from the threshold curve. This threshold curve is shown in figure 3. The intersection of these two curves represents the resolving power of the system, in this case about 3.25 cycles/m.m. (at the retina). It is obvious that this method represents considerable underestimation of the limit of visual resolution in this system. It is hypothesized that error inherent in the threshold determination is compounded as the M.T.F. used was derived from the same threshold data. This technique is better suited and intended for use when the M.T.F. of a lens may be objectively determined and used with threshold data related to emulsion characteristics of a film. The resolving power resulting in this case represents that of the combined system.
CONCLUSION

It has been demonstrated that this prediction technique for visual acuity is a feasible tool to be developed further. Restrictions exist on the accuracy with which modulation transfer and indeed any subjective data may be collected using a binomial mode-sampling scheme. Decreases in error of M.T.F. determination will yield more repeatable results and a higher order model designed around Rayleigh's and Sparrow's criteria would hopefully more closely predict visual resolution. Modeling and production for resolution challenges more closely resembling functional visual tasks is a real world application of fourier and modulation transfer techniques which should dictate the course of continued work in this field.

2) M. Davidson, Perturbation Approach to Spatial Brightness Interactions in Human Vision, J.O.S.A., 58; 1301, (Sept. 1968)

3) O. Bryngdahl, Characteristics of the Visual System Psychophysical Measurements of the Response to Spatial Sine-Wave Stimuli in the Mesopic Region. J.O.S.A., 54; 1152, (Sept. 1964)


5) A. Wantanabe, et. al., Spatial Sine-Wave Responses of the Human Visual System, Vision Research, 8; 1245 (1968)


12) E. Hecht and A. Zajac, Optics, Addison and Wesley Publishing Co., Reading, Mass.; (p. 360)


APPENDIX
(figures)
Figure 7

50% Pt = 27% Modulation

Modulation %

%
Figure 10

50% P% vs observed (%)

50% Pt = 36% modulation
figure 12

Bar Target

% Absorption

figure 13

Predicted subjective response profile

Viewing distance = 15"

% absorption vs spatial displacement
figure 14

Predicted Subjective Response Profile

Viewing distance = 2.5 feet
% absorption vs. spatial displacement

figure 15

Predicted subjective response profile

Viewing distance = 4'
Rayleigh's Criteria fulfilled
figure 16

Predicted Subjective Response Profile
Viewing distance = 5'
Sparrow's criteria fulfilled

figure 17

Predicted subjective response profile
Viewing distance = 6'
APPENDIX

(tables)
Table 1

**Illuminance at Viewer**

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Table 2
Characteristics of targets

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<th>Slide frequency (cycles/mm)</th>
<th>Frequency on retina (cycles/mm)</th>
<th>Slide modulation (%)</th>
<th>Slide average transmittance (%)</th>
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Table 4

% "yes" response for each treatment

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Table 5

Resolution of Bar Targets

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