5-1-1979

Determining the modulation transfer function of a lens-film combination using the diffraction effects of a coherent imaging system

Mark Washburn

DETERMINING THE MODULATION TRANSFER FUNCTION OF A LENS-FILM COMBINATION USING THE DIFFRACTION EFFECTS OF A COHERENT IMAGING SYSTEM

by

Mark E. Washburn

Rochester Institute of Technology

A thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Science in the School of Photographic Arts and Sciences in the College of Graphic Arts and Photography of the Rochester Institute of Technology

May, 1979

Signature of the Author Mark Washburn

Photographic Science and Instrumentation

Certified by Name Illegible

Thesis Advisor

ABSTRACT

This study will present a simple, quick, and inexpensive way to producing sinusoidal test targets, which can then be used in measuring the image quality of a photographic system by way of a coherent system. The sinusoidal test targets will be produced from a series of bar targets using Fourier Optics, and a comparison study will be made between measuring MTF with a laser and a microdensitometer. A defocused target series will also be produced to note the change in MTF of the photographic system.

The results indicate that sinusoidal test targets can be made quickly and inexpensively, and then can be used to provide meaningful data in evaluating a system. It was found that the laser method used in measuring the MTF of a photographic system was found to be superior to the microdensitometer in convenience, in speed, and in noise reduction. However, the microdensitometer can perform tests which the laser cannot, which still makes the microdensitometer an essential and invaluable tool in measuring the image quality of a system.

TABLE OF CONTENTS

LIST OF TABLES	iv
LIST OF FIGURES	v
INTRODUCTION	vi
BACKGROUND	vii
THEORY	ix
EXPERIMENTAL PROCEDURE	xv
1) Producing the gratings	xv
2) Checking the purity of the targets to be produced	xvi
3) Producing the sinusoidal test targets	xvii
4) Measuring MTF with the laser	xxi
RESULTS/ANALYSIS	xxiii
CONCLUSION	xxxvii
APPENDIX	xxxviii
1) Fourier Analysis Applications	xxxix
2) Sine target Calculations (Forward Doolittle Method)	xxxxi
3) Blur Circle Calculation	xxxxv
4) Sine target calculations	xxxxvi
ACKNOWLEDGEMENTS	xxxxvii
REFERENCES	xxxxviii

LIST OF TABLES

1.	$\mathcal{F}$ [rectangular window function]	x
2.	$\mathcal{F}$ [grating function]	xi
3.	U(f) x G(f)	xii
A1.	Sum of squares	xxxxi
A2.	Randomly chosen values	xxxxi
A3.	Sum of squares	xxxxii
A4.	Use of sum of squares	xxxxiii
A5.	Complete Doolittle Table	xxxxiii

LIST OF FIGURES

1.	Plot of U(f) vs. f	x
2.	Plot of G(f)	xi
3.	Plot of G(f) x U(f)	xii
5.	Geometrical layout of sine targets	xvii
6.	Laser set-up	xxi
6b.	Modulation Purity check	xxiv
7.	f = 7.8 c/mm	xxv
8.	f = 2.0 c/mm (defocused)	xxvi
9.	f = 32.6 c/mm	xxvii
10.	f = 14.4 c/mm	xxviii
11.	MTF curves measured from the microdensitometer	xxx
12.	MTF curves measured from the laser	xxxi
13.	Defocused MTF curves from microdensitometer	xxxii
14.	Defocused MTF curves from laser	xxxiii
16.	Image Modulation vs. β^2	xxxv
A10.	Physical and Geometrical layout of blur circle	xxxxv

INTRODUCTION

The purpose of this experiment is two fold: the first objective is to show that a simple method can be used to produce sinusoidal test targets to evaluate photographic systems, and the second objective, is to use the targets that have been produced to measure the modulation transfer function (MTF) of a photographic system without the aid of a microdensitometer.

One of the most widely used methods of measuring the MTF of a system involves the use of a microdensitometer. But this can involve much time and money in scanning a number of sine targets and edges, then proceeding to determine their modulation, both the object and imaged sine targets. An alternative method virtually eliminates the use of a microdensitometer which can quickly and accurately measure the MTF of a photographic system - by way of a laser.

The study to follow presents a method which produces sinusoidal targets quickly and inexpensively, in order to evaluate the quality of a photographic system, by a very simple method.

BACKGROUND

There are many ways in which to evaluate a photographic system which produces an image, such as; resolution, information capacity, graininess, acutance, tone reproduction, and modulation transfer function (MTF).

In this study, modulation transfer function will be used, which measures the reduction in contrast of an imaging system[1]. In terms of an expression for the modulation transfer function of a photographic system, the modulation transfer function is the ratio of the modulation of an image of an object to the modulation of that object[2]. Or,

MTF = image modulation / object modulation

If an ideal system was available, it would be obvious that the MTF (which is measured as a function of frequency) would be equal to 1.0 at every frequency. Unfortunately, a system has yet to be designed where the MTF remains at 1.0 for a short frequency range, never mind for all frequencies.

Yet, knowing the MTF of an imaging system is a valuable tool. It can be used in comparison to other systems in terms of frequency response in order to select the proper system. The modulation transfer can be measured for each element of a system as well as an entire system. Examples of elements of a system might be a lens or film. Therefore, MTF is a valuable tool in that it measures the quality of a system or its elements.

There are several methods that are used in measuring the MTF of an imaging system, two of which are edge analysis and the use of sinusoidal targets[3].

The modulation of a sine target can be determined by scanning it on a microdensitometer and then plotting transmittance versus position. Then, by imaging that target of known frequency on a recording material and scanning that material, plotting an effective exposure versus position trace by going back through the characteristic curve of the recording material, a modulation of the image can be determined. Thus, by having the ratio of the modulation of image to object, the MTF has been determined for that frequency. By determining the MTF for a number of frequencies, an MTF curve can be produced. The reason that only transmittance and effective exposure can be used in determining modulations and not density, is because density is non-linear, whereas, transmittance and effective exposure have linear responses.

In determining the modulation of an image of an object or the object itself, the mathematical approach would be,

$$m_o = t_{max} - t_{min} / t_{max} + t_{min}$$ [1]

where t is the transmittance of the object. The modulation of the image can be expressed by,

$$m_i = H_{max} - H_{min} / H_{max} + H_{min}$$ [1]

where H is the effective exposure for the image of the object. Thus, MTF = m_i / m_o.

If the system is linear (whereby a sinusoidal input into the system produces a sinusoidal output), the MTF of the microdensitometer can be eliminated by dividing it out.[1] Another method, although not the best, is edge analysis. This is done by scanning an edge and converting the trace in terms of transmittance versus position. Then, if the trace is differentiated and Fourier transformed, the modulation transfer function of the microdensitometer can be determined. The differential of the scan is in actuality the line spread function of the microdensitometer. By definition, the MTF is equal to the Fourier transform of the line spread function of a system or an element in the system.[1] One of the drawbacks of using edge analysis is that noise is introduced into the evaluation of the system in differentiating the scan of the edge.

The MTF of the system is finally determined when the MTF of the microdensitometer is removed.

THEORY

The background discussion relates directly to the study at hand, in that the two methods that can be used in measuring MTF, edge analysis and use of the sinusoidal targets, are used in this study.

After looking over the method used in making the sinusoidal targets, a question that might arise could be, "why does this method work"? One must turn his attention to the spatial frequency domain for the analysis. From the experimental procedure, sinusoidal intensity distributions have been produced from square wave targets or gratings. To understand why this works, Fourier Optics must be introduced into the analysis. Distances are chosen so that, by illuminating a square wave target with a fluorescent light source, a shadow image is produced on a screen whose spread function has a width one third the period of the original target. In actuality, the Fourier transform of the rectangular window function (whose width is one third that of the original grating's period) has been multiplied by the Fourier transform of the grating function. This is expressed by,

$$\mathcal{F}[u(x) * g(x)] = U(f) \cdot G(f)$$

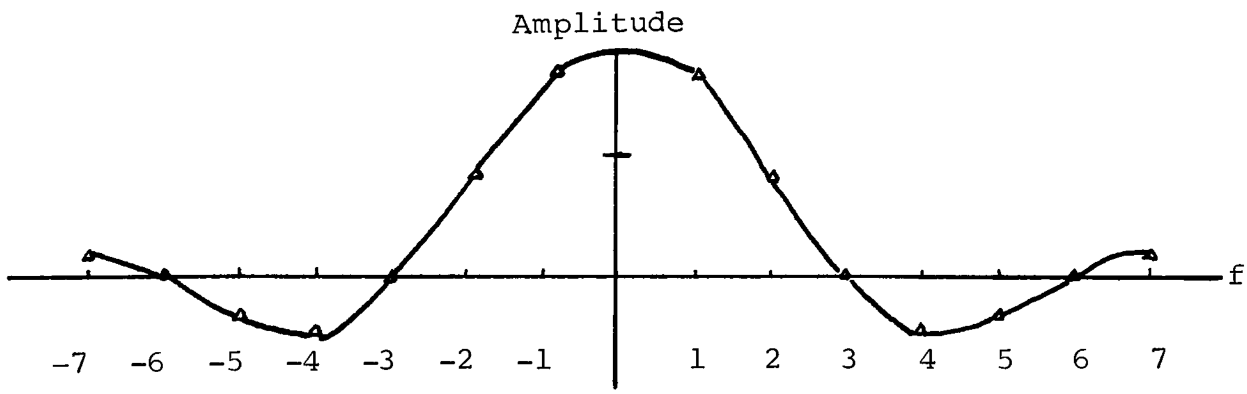
where $u(x) * g(x)$ is the convolution of the two functions, $U(f)$ is the Fourier transform of $u(x)$ and $G(f)$ is the Fourier transform of $g(x)$[3,6]. The rectangular window function has been represented by $u(x)$ and the grating function has been represented by $g(x)$. It is this convolution which produces a sinusoidal intensity distribution on the diffusion screen. For the actual calculations of multiplying the Fourier transforms of the two functions (the grating function and the rectangular window function), see the Appendix under FOURIER ANALYSIS APPLICATIONS.

The Fourier transform of a rectangular window function of width p/3 is a sinc function. The actual function U(f) is equal to $\sin(\pi f/3)/\pi f/3$ for p = 1. The values of the function for different frequencies are as illustrated in Table 1, and the graphical representation of U(f) is shown in Figure 1.

Table 1

$\mathcal{F}$[rectangular window function]

f	U(f)	f	U(f)
0	1.00	± 5	-0.17
± 1	0.83	± 6	0.00
± 2	0.41	± 7	0.12
± 3	0.00		
± 4	-0.21		



Plot of U(f) vs. f

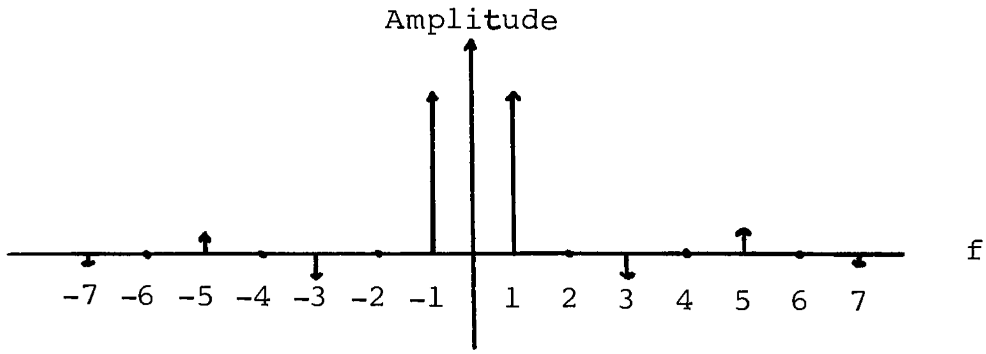
Figure 1

With the original grating function of period p = 1, summing all the harmonic components to get the Fourier series coefficients, the Fourier transform of the grating function has been calculated at discrete, corresponding frequencies.[3] The amplitude of the Fourier transform of the grating function G(f) at the nth harmonic frequency is equal to $\sin(n\pi/2)/n\pi/2$. Table 2 shows the values for the function at a DC level of 0.50 and Figure 2 shows the graphical representation of the function. The product of the two transforms is what produces a sinusoidal intensity distribution. This can be demonstrated by actually multiplying the two functions and plotting the results. Table 3 and Figure 3 show the tabulated values of the product of the two functions and the graphical representation.

Table 2

$\mathcal{F}$[grating function]

n	G(f)	n	G(f)
0	0.50	± 4	0.00
± 1	0.32	± 5	0.06
± 2	0.00	± 6	0.00
± 3	-0.11	± 7	-0.05



Plot of G(f)

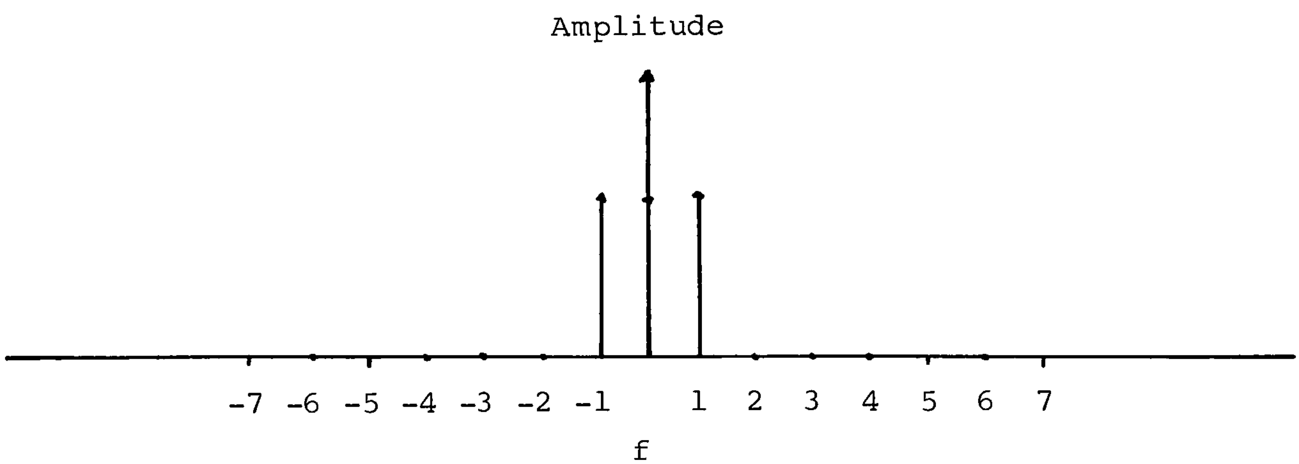
Figure 2

The amplitude spectrum in Figure 3 needs now to be transformed from the spatial frequency domain back to the spatial domain. To do this, it must be realized that the three major "spikes" in Figure 3 represent a cosine function with the fundamental frequency of the original square wave target[6]. By multiplying the Fourier transforms of these two functions together, the higher harmonics are, in essence, filtered

Table 3

U(f) x G(f)

f	U(f)	G(f)	U(f) x G(f)
0	1.00	0.50	0.50
±1	0.83	0.32	0.26
±2	0.41	0.00	0.00
±3	0.00	-0.11	0.00
±4	-0.21	0.00	0.00
±5	-0.17	0.06	-0.01
±6	0.00	0.00	0.00
±7	0.12	-0.05	-0.01



Plot of U(f) x G(f)

Figure 3

out and all that remains is the fundamental frequency and the level. This is the beauty of this method. The two delta functions, at $f = +1$ and at $f = -1$, can be transformed to $H(x'') = Ae^{i2\pi fx''} + Ae^{-i2\pi fx''} = 2A\cos(2\pi fx'')$, where A is the amplitude of the fundamental component. The delta function at $f = 0$ is the DC level of the cosine wave. The amplitude of the fundamental component was calculated to be equal to 0.26, rather than 0.25, because of the very small contributions of the higher harmonics that had not been filtered out completely. Therefore, the final expression for the exposure onto the film is,

$$H(x) = 0.50 + 0.52 \cos(2\pi fx)$$

After the sine targets have been produced, the photographic system can be evaluated by using MTF. A different method was used to measure MTF rather than the customary microdensitometer approach. A laser was used, instead, to measure the MTF of the photographic system. There are several reasons for this selection of methods.

One of the advantages of using the laser is convenience. Rather than scanning targets and tracing them back through the characteristic curve of the film, MTF can be directly measured from the point spectrum produced when the sine targets are illuminated by the laser's beam. Another advantage is the reduction in the influence in noise. This is due in part because the image noise is fairly "flat", whereas, the image contains a point or line spectrum. Consequently, it is easy to separate the two (image noise and the point spectrum). Something which goes along with using the laser is that it allows the experimenter to check the purity of the sine targets being used. This is done by looking at the point spectrum.[7] Any even harmonics present indicate harmonic distortion. Another advantage, other than a check for purity and reduction in noise, is that the laser covers a much larger area in the target than the microdensitometer. Thus, because it averages a much larger area, it reduces the amount of noise still further.

In using the laser to determine the MTF of the photographic system, an expression for the image and object intensity distributions can be developed. The object intensity function can be expressed by,

$$\sigma(x,y) = a + b\cos(2\pi f x)$$ [4]

where x and y are the coordinates in object space. The ideal image intensity function, free of diffraction-effects and aberrations, can be expressed by,

$$\sigma(x',y') = a + b\cos(2\pi f' x') \qquad m_2 = b/a$$ [4]

where x' and y' are the coordinates in image space and f' is the frequency in image space. The realistic image intensity function, which takes into account aberrations and diffraction-effects, is expressed by,

$$\sigma(x',y') = a + bM\cos(2\pi f' x' + \phi) \qquad m_1 = bM/a$$ [4]

where ϕ is a phase change. The ratio of m_1/m_2 is the MTF of the photographic system, because if $m_1 = bM/a$ and $m_2 = b/a$, then it follows that m_1/m_2 equals the MTF of the system, where MTF = M.[4] It should be noted that both M and ϕ are functions of frequency.

EXPERIMENTAL PROCEDURE

The experimental procedure can be broken down into three sections which consist of: 1) producing gratings of various frequencies, 2) producing sinusoidal targets from the gratings, and 3) measuring the MTF of the photographic system by use of a laser and also the microdensitometer as a comparison.

Producing the gratings:

The basic materials needed in producing gratings is a large viewbox, black scotch tape (5 mm wide), an 8" x 10" camera and some 8" x 10" lithographic film.

On the large viewbox was taped long strips of black scotch tape, spaced apart the same thickness as the tape used. By using a ruler incremented in millimeters, the tapings can be spaced apart with reasonable accuracy.

An 8" x 10" camera, with a focal length of 10 inches, was used to produce gratings. Ilford lithographic film was used, because all that of interest is black bars on clear background. In order to produce gratings of certain frequencies, the proper magnification had to be computed. This was done through use of the focusing equation, which is $1 / s' = 1 / s + 1 / f$.

By decreasing the spacing between the bars of the gratings wish to be made, the spatial frequency will be increased because the spatial frequency is inversely related to the width of a bar and a space. Knowing this, the gratings can be produced.

Thus, each grating of a certain spatial frequency was reduced by a certain factor to produce gratings which logarithmically increased in frequency, because the MTF changes as a function of frequency logarithmically. Each grating was photographed twice and the two images were taped together (which resulted in out-of-register problems) to avoid reducing the area of the gratings in reducing the spacing of the bars on the gratings. Processing was done in Kodalith developer for 3 minutes at room temperature. All exposures were made at f/11 for three seconds. In order to reduce flare as much as possible, the unused areas on the viewbox were blacked out by the use of black construction paper and the emulsion side of the film was faced into the viewbox.

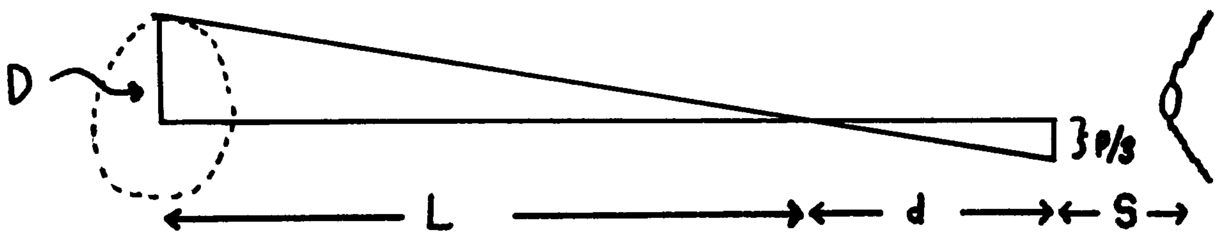
Checking the purity of the targets to be produced:

Rather than taking the time to Fast Fourier transforming the sinusoidal targets that are to be produced to get their Amplitude spectrum and determining the purity of each target, another method less time-consuming and nearly effective was used.

A target was scanned at a very low frequency (2.9 cycles per millimeter) with a photometer. A plot was then made to see what kind of modulation was produced using this method of making sine targets. The modulation was determined by dividing the peak level by the average level of the sinusoid, or the AC level by the DC level. The values used in producing a sine wave of a certain purity were in units of foot-candles per meter squared, but this is really irrelevant. This information will tell us the amount of purity in the targets made, in general. It should be noted because of the problem of out-of-register of the higher frequency targets, that as the targets get higher in frequency, the purity will also diminish. The higher frequency targets cannot be tested for purity in this manner though, because the periods are too small to use a photometer to test purity. In the final analysis in this section for a purity check, the modulation of the image of that sinusoidal intensity distribution (which has a frequency of 2.9 cycles per millimeter) will be superimposed onto the modulation of the sinusoidal intensity distribution on the diffusion screen to compare modulations between object and image of the object.

<u>Producing the sinusoidal test targets</u>:

The producing of the sine targets requires only basic geometry. From the geometrical layout below in Figure 5, it can be readily be seen that the distance L = dD3/p, where L is the distance from the flourescent tube, d is the distance from the grating to the diffusion screen, S is the distance from the sinusoidal intensity distribution produced on the diffusion screen to the lens of the camera which will image this distribution, and p is the desired period which equals 1 / f . The reduction onto the 35 mm film was twenty times and the focusing equation was once again used in determining and proper subject distance. The value of D is of course fixed because the same flourescent tube was always used, i.e. 38 mm.



Geometrical layout of sine targets

Figure 5

By referring to the Appendix under SINE TARGET CALCULATIONS, a table of values shows the calculated values for d and L at each of the ten frequencies that were produced. The exposure can be determined by placing a photometer behind the lens in the camera, measuring the illuminance, and changing the aperture size to get the illuminance level needed to get on the straight-line portion of the characteristic curve produced when the film was processed. The aim was to have the average density of each sine target to fall at step 11 on the characteristic curve. The film used to image these sinusoidal intensity distributions was Kodak Type Plus-X Pan film. Once the exposures were made at the computed distances, another section of the film was exposed to a step tablet and processed in D-76 diluted 1:1 at 66^{o}F for 7 and one-half minutes. Once the film was thoroughly dried, it was scanned on a Joyce-Loebbel microdensitometer to determine the frequencies of the targets (as mentioned in the Appendix) and their modulations.

Two wedges were used, with gradients of 0.43 density units per centimeter and 0.131 density units per centimeter, in the scanning of the targets on the microdensitometer. These gradients were used to determine the density ranges of each target. The density readings on the Joyce-Loebbel microdensitometer were then converted to diffuse density values. To do this, a sensitometric strip was scanned on the microdensitometer and also measured for density on the MacBeth TD 504 macrodensitometer. A linear regression was performed using the Doolittle Method (see Appendix under SINE TARGET CALCULATIONS) to determine an equation of best fit. The final equation was,

$$Y = 0.87X + 0.30$$

where Y is the diffuse density (macrodensitometer density) and X is the machine density (microdensitometer density)

The modulation was computed by tracing back through the characteristic curve of the film used, and the modulations were computed through the relation,

$$m = H_{max} - H_{min} / H_{max} + H_{min}$$

where H is the effective exposure.

The results obtained with the microdensitometer are still not complete. The MTF of the microdensitometer needs to be removed from the modulations already computed. This can be done by scanning an edge. An NBS (National Bureau of Standards) edge was scanned where the MTF was evaluated to a cutoff frequency of 200 cycles/mm, which is far beyond the actual cutoff frequency needed. A computer was used to determine the MTF. In order to avoid aliasing, the effective slit width used in scanning was less than five microns. The sampling interval selected was based on $x = 1 / (2f_{max})$, and resulted in providing one data point every 2.6 microns. To determine the proper aperture setting to give an effective aperture of five microns, the magnification needs to be taken into account. Once the trace of the edge has been made on the graph paper, the arm magnification must also be considered to determine the proper sampling interval on the graph paper.

To get a plot of transmittance versus position for the edge, a density versus position plot is needed. But this density produced from the scan is machine density, not diffuse density. Therefore, the stepped density edge was scanned on the microdensitometer and compared to the density values measured from the MacBeth TD 504 macrodensitometer. After linear regression, the equation of line of best fit was found to be,

$$Y = 0.69X + 0.04$$

where, once again, Y is diffuse density and X is the machine density.

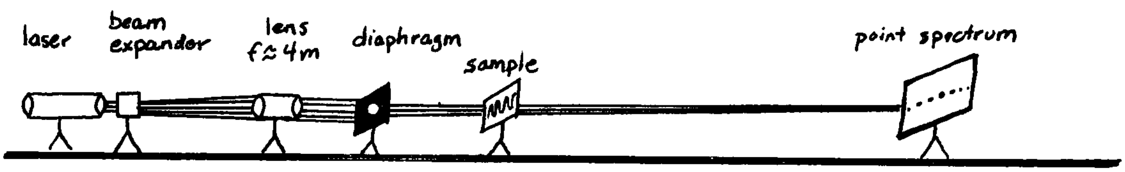
After the density values were picked off the edge scan at a selected interval as mentioned before, the density values were converted to diffuse density, which were then converted to transmittance by the relation of $T = 10^{-D}$, where T is transmittance and D is density.

Besides taking into consideration the MTF of the microdensitometer to determine the actual modulation of the photographic system being studied, but also the slit width used in scanning all the targets. There needs to be a correction factor made for each target because not all the targets were scanned at less than five microns as was the case when the MTF of the microdensitometer was determined. As a result, the cutoff frequency has been reduced lower than 200 cycles/mm if the slit width was increased for certain frequencies (which it was). By utilizing the relation $T(f) = \text{sinc}(\pi wf)$, the modulations can be corrected by taking the reciprocal of T(f) and multiplying this by the modulations obtained after having divided out the MTF of the microdensitometer. Finally, the MTF of the photographic system as a function of frequency was plotted out and compared to the results obtained through the laser method.

Another section involved producing a blur circle of a certain diameter on each of the sine targets in the series to be produced. The objective was to observe the change in modulation (which means a change in MTF) as the frequency of the sine target was increased. The blur circle's diameter was set at a "middle" frequency of the sine target target series produced earlier. The "middle" frequency is about 10 cycles/mm, therefore, the blur circle needs to be 0.10 mm in diameter. Once again, the proper distances to produce a certain condition (namely, a blur circle of 0.10 mm) were based on geometrical calculations. See Appendix under BLUR CIRCLE CALCULATIONS. Knowing the new image distance from calculations based on a 0.10 mm diameter blur circle and the focal length, the new subject distance can be computed from the focusing equation. The new subject distance is 1000 mm, or 1 meter. After these blur circles were produced on film, scanned on the Joyce-Loebbel microdensitometer, and analyzed, it was observed that the moculation at each frequency declined rapidly as the frequency increased. The modulations were measured to determine the fall-off of the MTF curve compared to the in-focus MTF curve.

<u>Measuring MTF with the laser</u>:

The laser was set up as illustrated below in Figure 6.



Laser set-up
Figure 6

Because the beam of the laser used is only about one millimeter in diameter, a beam expander was used to expand the beam to about a two millimeter beam diameter. The lens used to focus the beam was a long focal length lens of about four meters. A point spectrum was observed. The actual modulation is proportional to the measured intensity ratio, which is the ratio of the amplitude of the fundamental frequency to the amplitude of the level. See the Appendix under CALCULATING THE MTF OF SINE TARGETS USING A LASER for additional information. The photometer used measured the amplitudes in cd/m^2 , but the type of measurement is really irrelevant, whether it be in foot-candles or microwatts. What is important is that the same type of measurement be used throughout the measurements, in measuring the modulations.

In calculating the MTF, or the modulation of the image in our case (because the modulation of the object is equal to 1.0), the relationship m equals $2\sqrt{\beta^2} / 1 + \beta^2$, will be used. The detailed analysis can be found in Swing's determination of MTF using a coherent source. In the relationship above, β^2 is the ratio of measured amplitudes of the fundamental frequency to the level. Thus, the MTF can be directly measured from a point spectrum produced from a series of sine targets through the use of a laser.

RESULTS / ANALYSIS

The first results that need to be discussed are the purity of the targets which were produced. A comparison study, between the sinusoidal intensity distribution produced on the diffusion screen and the image of that distribution, was performed to test for purity. One target was tested, which was a representation of the rest of the targets, although it should be noted that alignment problems increased as the frequency decreased in the actual making of the targets. The graphical comparison of the two targets (object and image) is illustrated in Figure 6b. The dashed line indicates the image modulation and the solid line indicates the sinusoidal intensity distribution which was scanned with a photometer that was finally imaged onto 35mm film. Both are modulations are from the same grating frequency of 0.15 cycles/mm. The computed modulations for the object and its image were calculated to be 0.63 and 0.67, respectfully. The numbers themselves have really no significance other than the fact that the object modulation should be normalized to 1.0. What is important is that the shape of the two plots are very similar. This says much for the purity of the target. Why the modulation of the image is greater than the modulation of the object on the plots is a problem that needs to be further investigated. Allowance must be considered for the exact alignment of the two plots because of the fact that the two targets were not plotted at the same position along the targets and that the targets are not perfect.

Samples of the microdensitometer scans of the targets can be found in Figures 7 through 10, which show the modulations of in-focus and out-of-focus targets from low to high frequencies. Special attention needs to be drawn to

Figure 6b

Modulation Purity Check

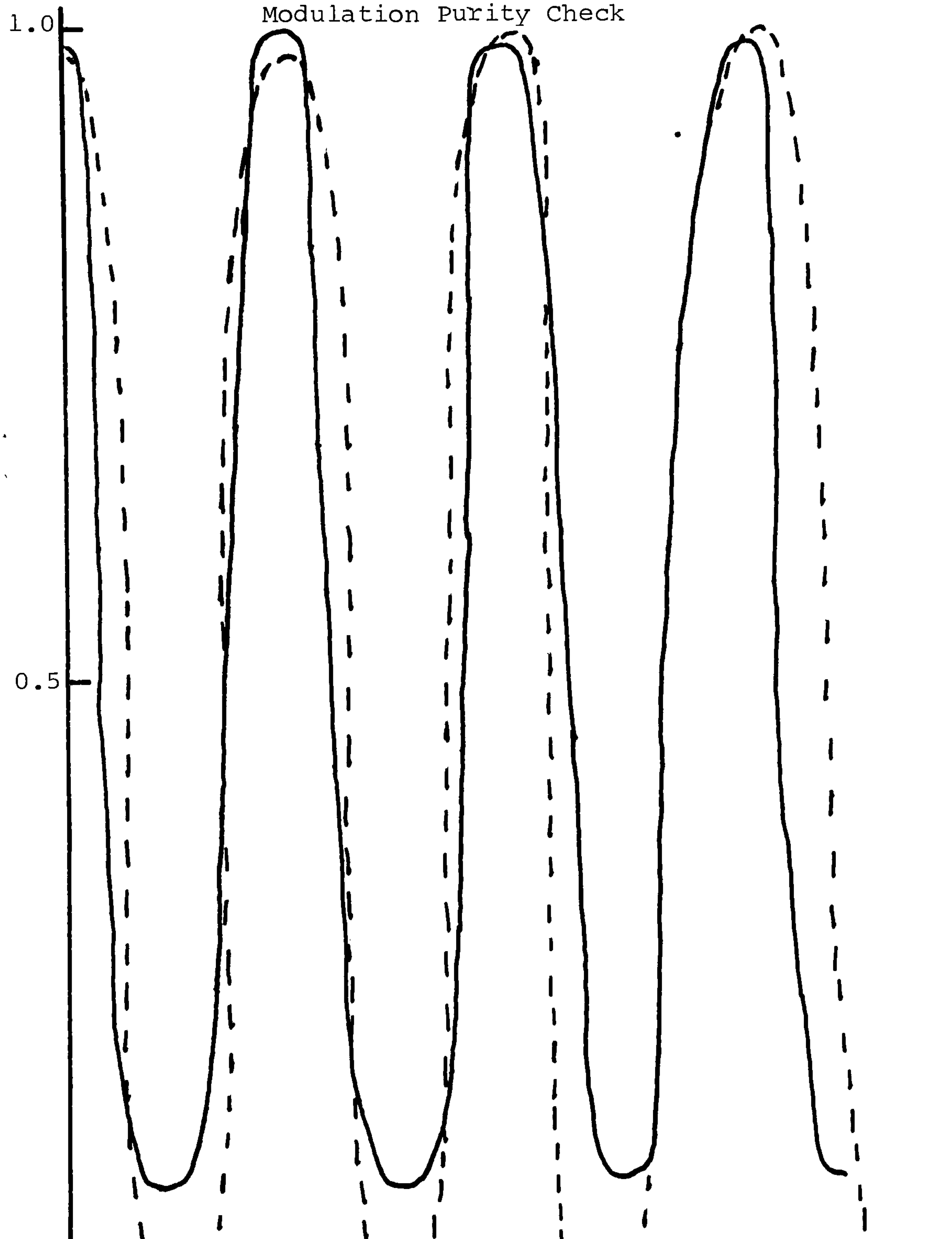


Figure 7
f = 7.8 c/mm

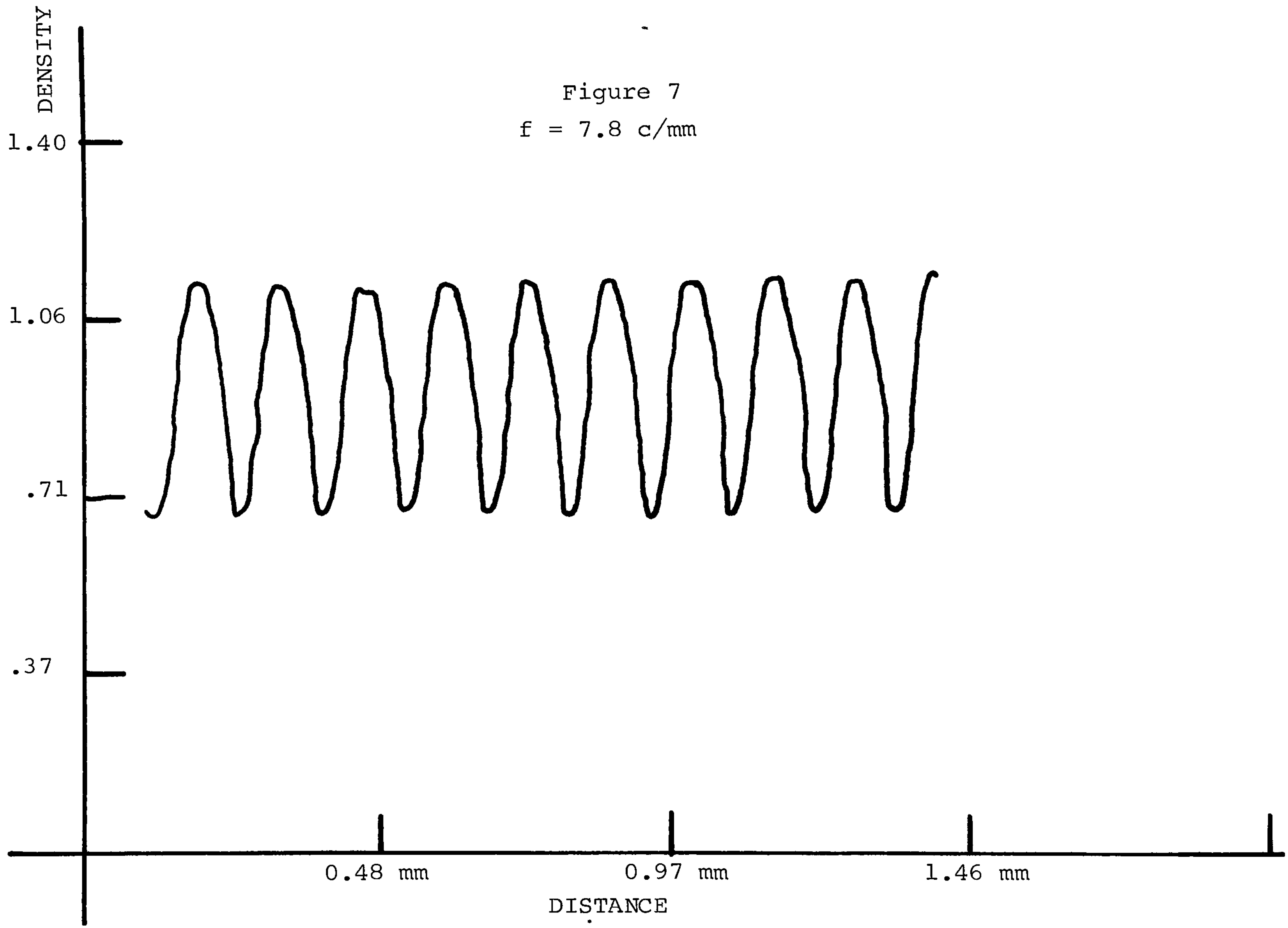
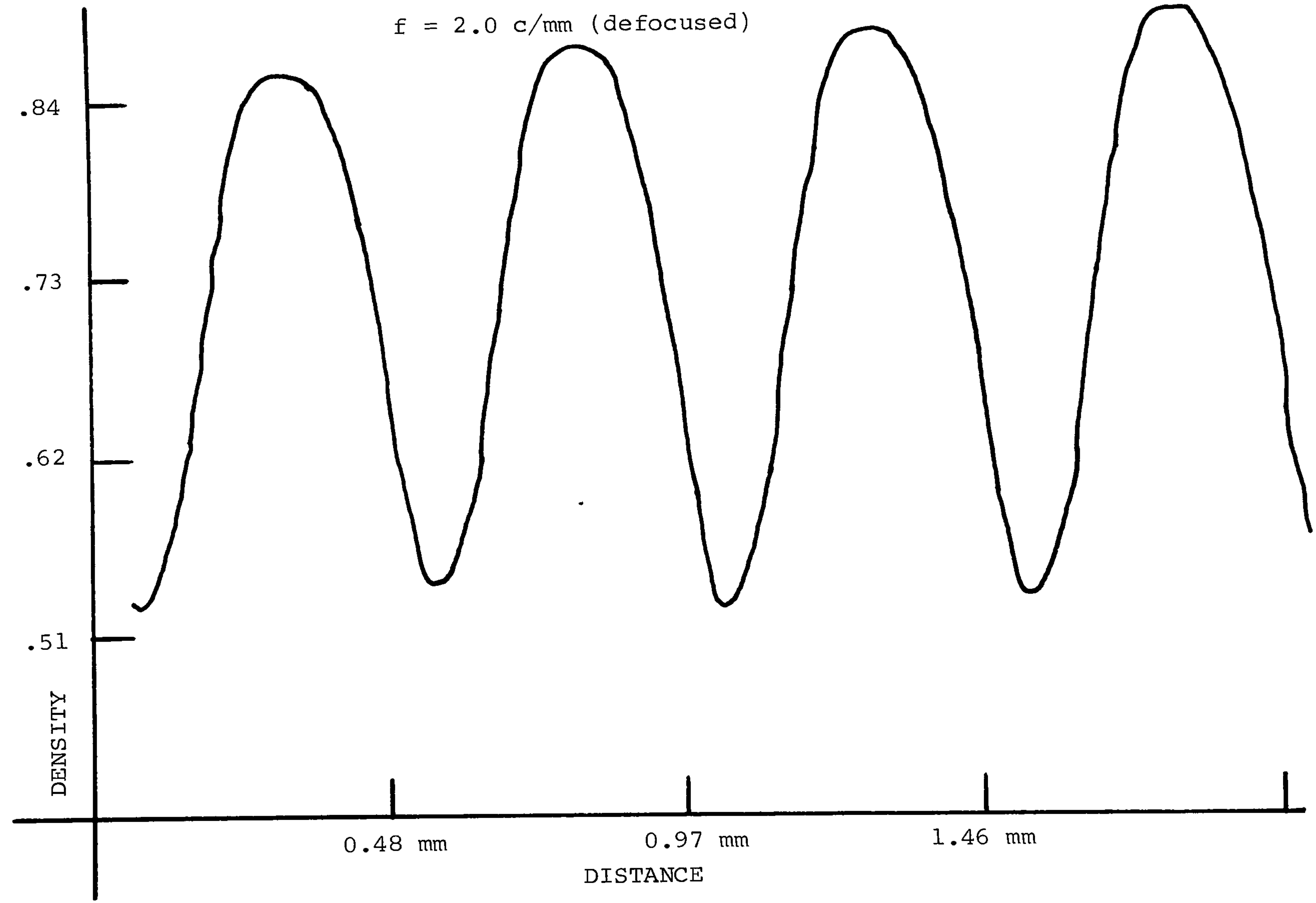


Figure 8

f = 2.0 c/mm (defocused)



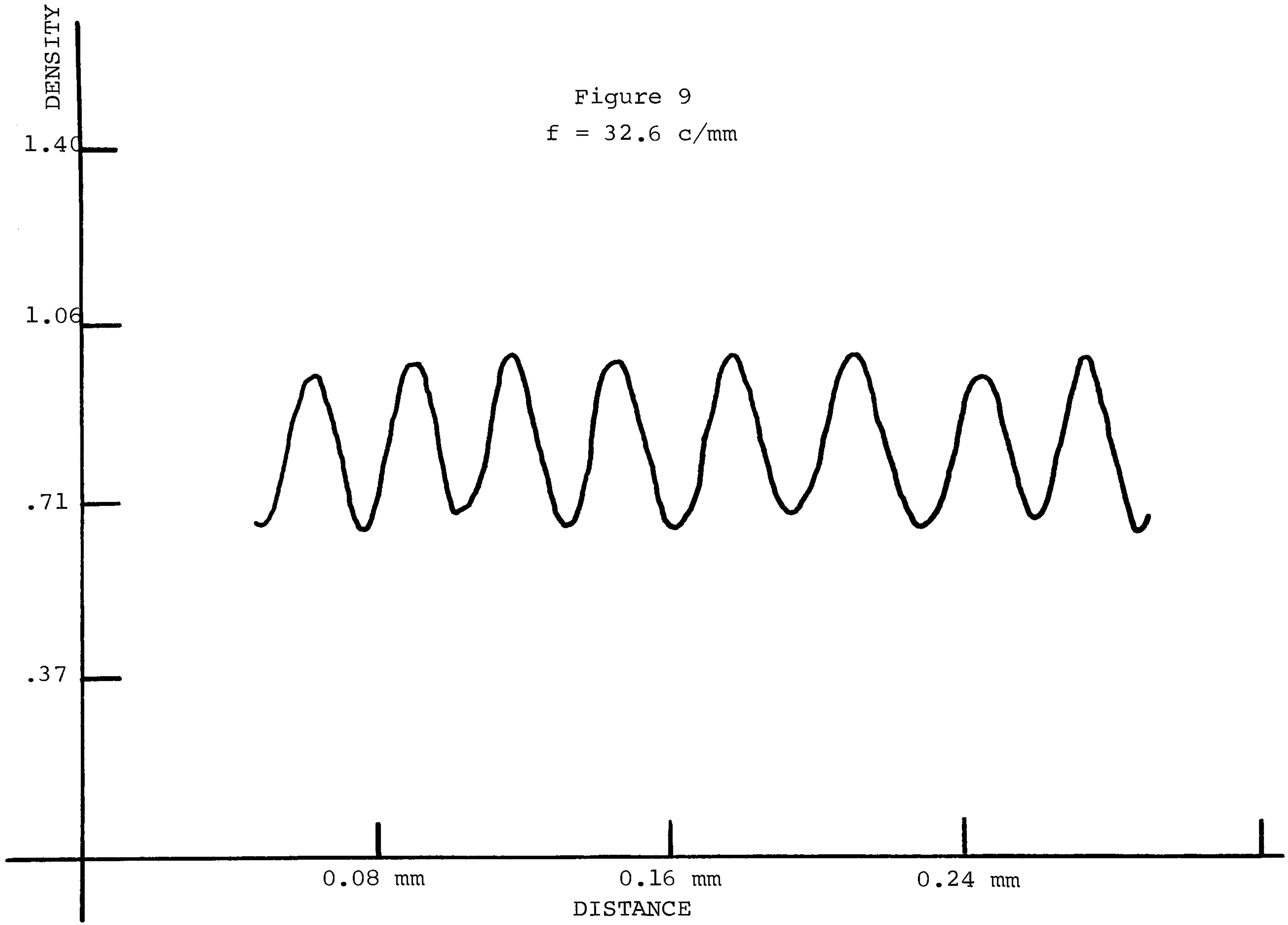


Figure 9

f = 32.6 c/mm

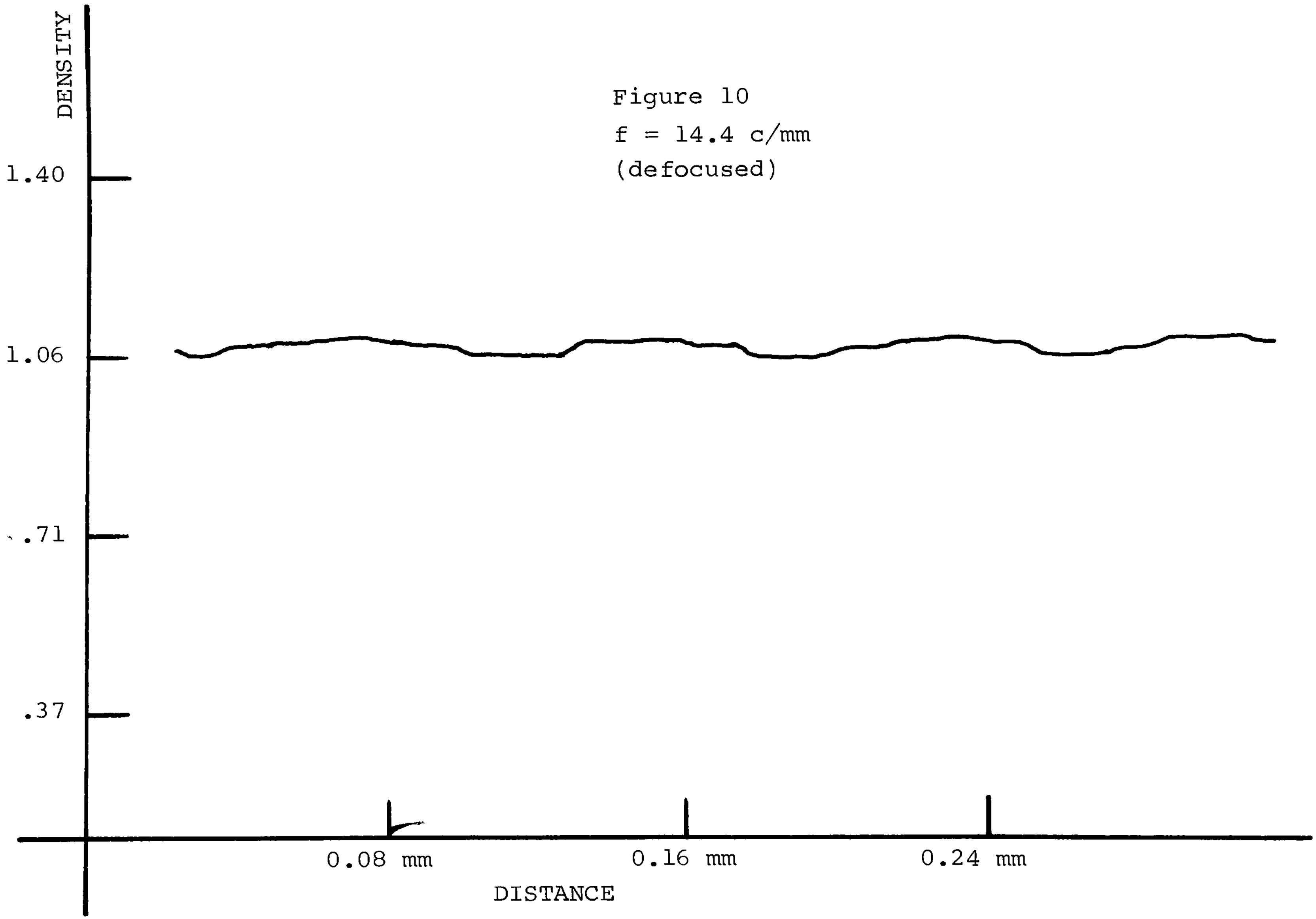


Figure 10
f = 14.4 c/mm
(defocused)

Figure 10, which shows the rapid decay in modulation due to the lens being defocused. Notice also the high modulation, in contrast to Figure 10, of the defocused target in Figure 8. The lower frequencies are essentially unaffected by the defocused lens. The remainder of the scans of the targets and the targets themselves can be found in the notebook for this experiment.

The second area that needs to be discussed is the quality of the photographic system, which was measured through the use of MTF. The MTF curves can be found in Figures 11 through 14. Each curve was normalized a second time (the first time because the object modulation had to be normalized from 0.63 to 1.0) because of high MTF values, at the higher frequencies used in the experiment. The resultant MTF curves produced by the microdensitometer show unusual dips and high points. This could very easily be due to the fact that each target produced did not receive the same effective exposure, because of problems with the photometer used. Special note must be made of the cut-off frequency of the defocused MTF curve. Rather than cutting off at the expected 10 cycles/mm, it cut off at about 28 cycles/mm. Speculation suggests that the noise from the microdensitometer, the problem with exposure, and the possiblity that the blur circle produced was not exactly equal to 0.10 mm in diameter, might produce this cut-off frequency from the microdensitometer.

The MTF curves from the measurements made with the laser in Figure 12, are comparable to the microdensitometer, except in the low end of the frequency range. This problem can be explained in the actual MTF calculations with the laser. The relation, $m = 2\sqrt{\beta^2}/ 1 + \beta^2$, has a linear relation with image modulation at values exceeding $\beta^2 = 0.15$.

Figure 11

MTF curves measured from the microdensitometer

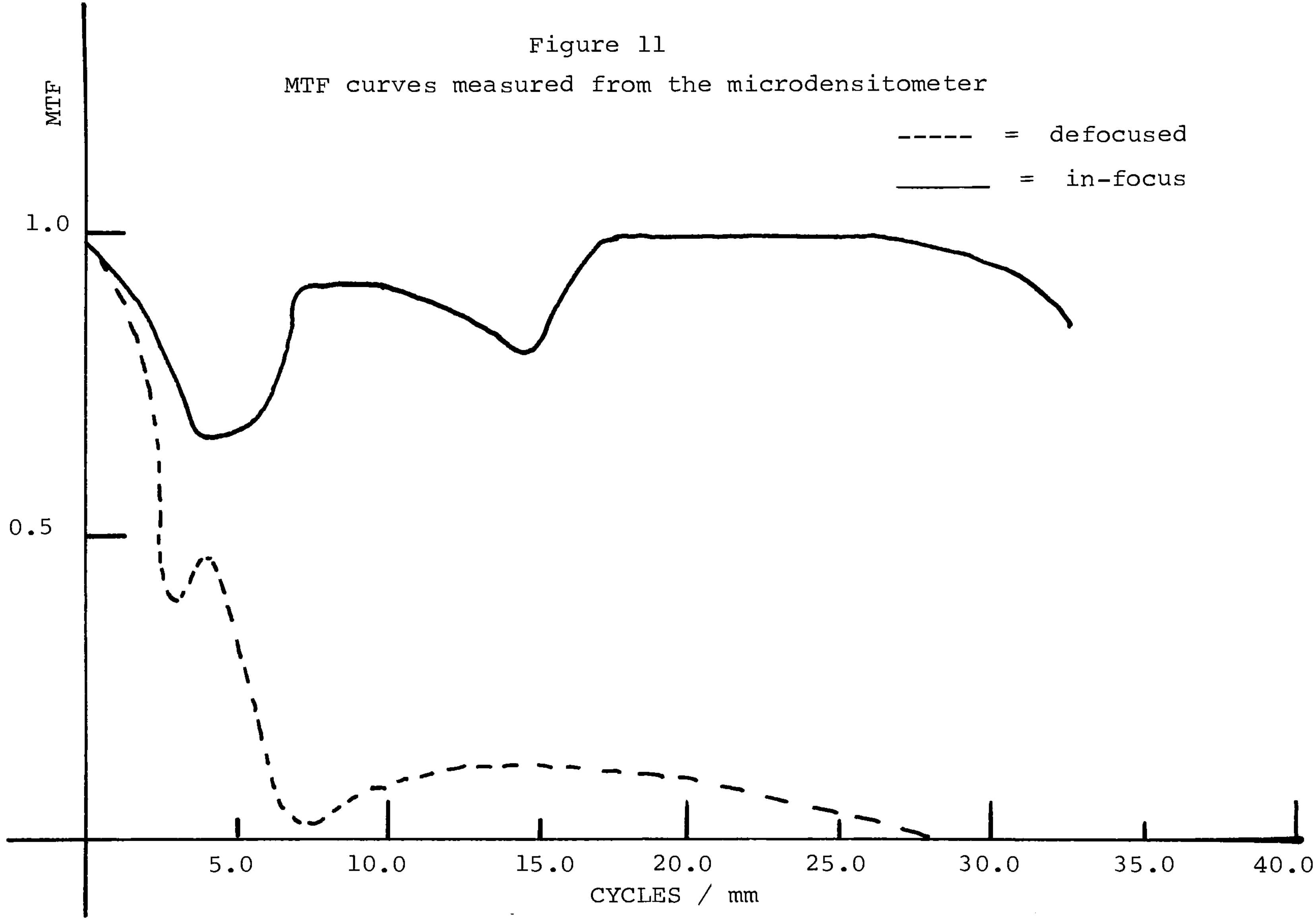
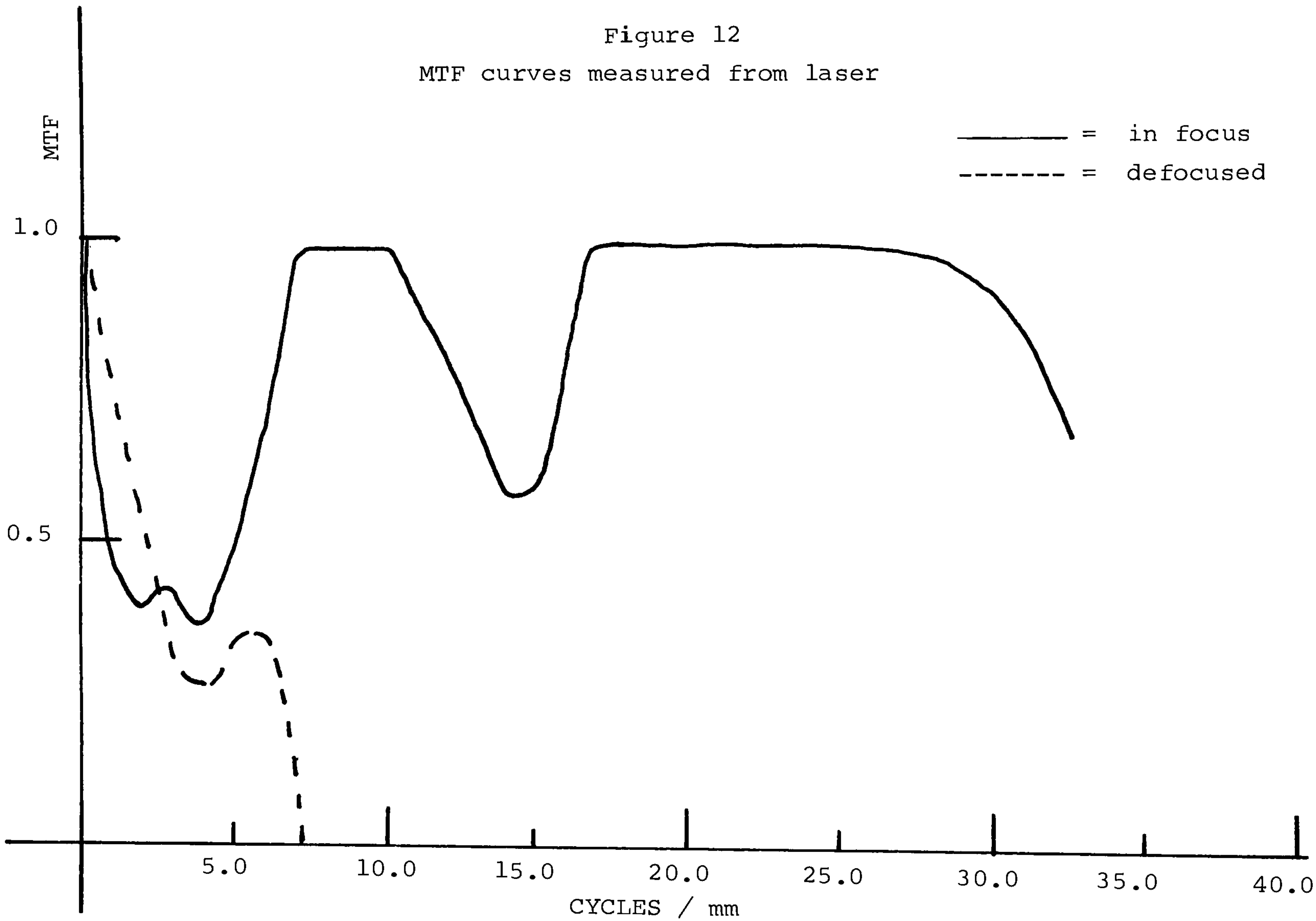


Figure 12

MTF curves measured from laser



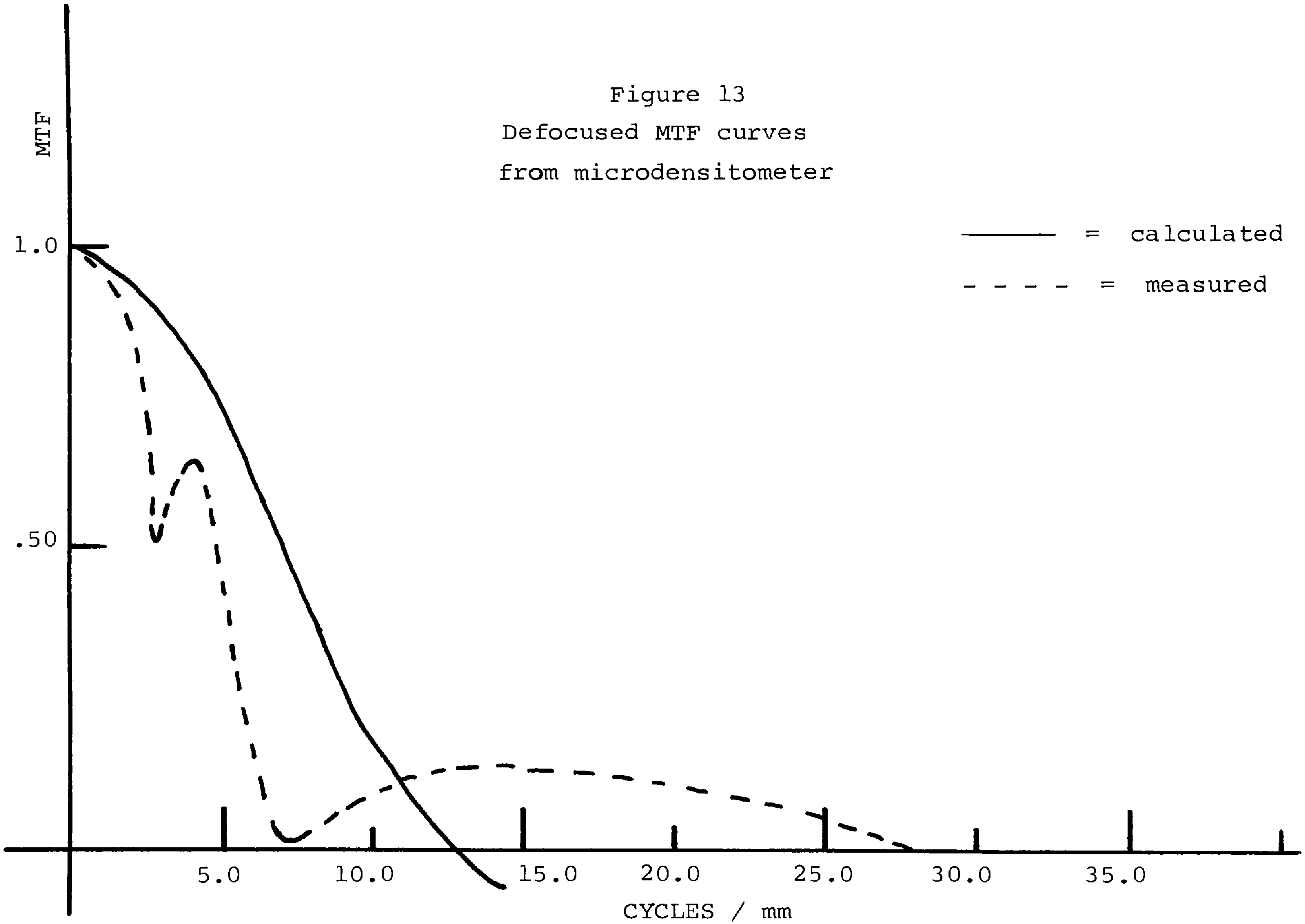


Figure 13
Defocused MTF curves
from microdensitometer

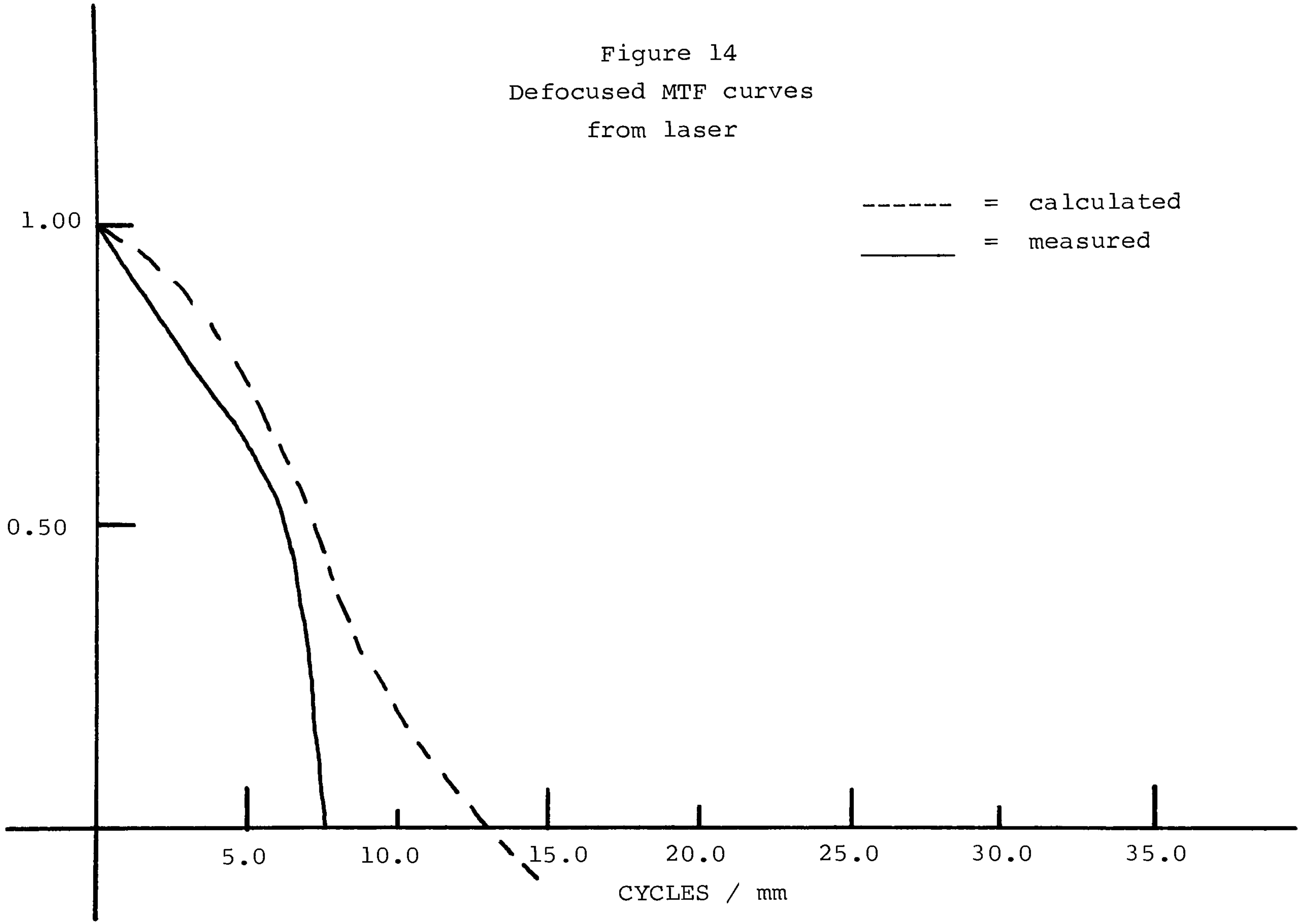
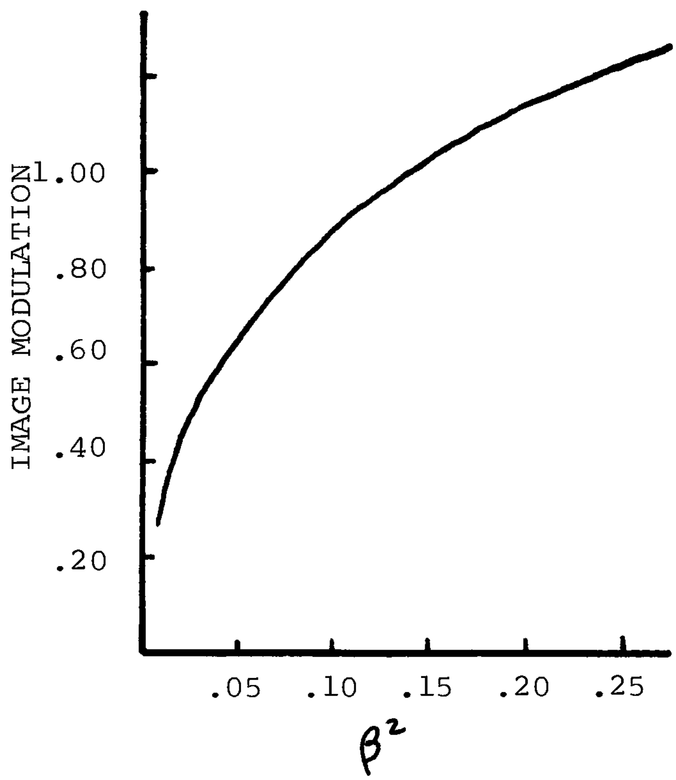


Figure 14
Defocused MTF curves
from laser

Less than this and a factor (which changes as β^2 changes) needs to be introduced to give the actual image modulation. This is illustrated in Figure 16, relating image modulation to β^2. Besides the low end of the frequency range, the same dipping and rising occurs as in the MTF curves generated from the microdensitometer. The consistency in the shapes of the MTF curves from the two methods used shows that the methods' measurements were agreeable, but that there was a problem in the producing of the targets. Once again, exposure from bad photometric readouts is the prime suspect. Also, the defocused MTF curves produced from the laser supports the findings that less noise is introduced into the system when a laser is used for MTF measurement, in comparison to the microdensitometer usage.

Finally, the defocused MTF curves need to be discussed. The curves generated from the microdensitometer and laser marked "measured" on the graphs in Figures 13 and 14 are simply the ratio of out-of-focus to in-focus MTFs, whereas, the calculated curve was computed from a Bessel function. The Bessel function was $2J_1(w)/w$, where w equals πdf, d being the diameter of the blur circle, f being the frequency, and $J_1(\ )$ being the first order Bessel function. According to Rayleigh's criterion, unless the OPD (optical path difference) is less than one-quarter of a wavelength, the quality of the image (thus, the MTF) will be degraded.[2] In our case, the OPD is greater than one-quarter wavelength but less than one wavelength, because the curves would show more agreement if the OPD were greater than one wavelength. The negative values of the Bessel function (which cannot all fit on the graph) indicate spurious resolution, where lines are discernible but clear images are not. This characteristic of an optical system is frequently observed in defocused, well-corrected lenses whose image of a point produces a

Figure 16
Image Modulation vs. β^2



uniform, illuminate circular blur .

The results obtained with the laser in measuring the defocused MTF are in more agreement with the calculated MTF curve than with the microdensitometer results. This could be due to the reduction in noise with the laser method, in comparison to the noise in the microdensitometer.

CONCLUSION

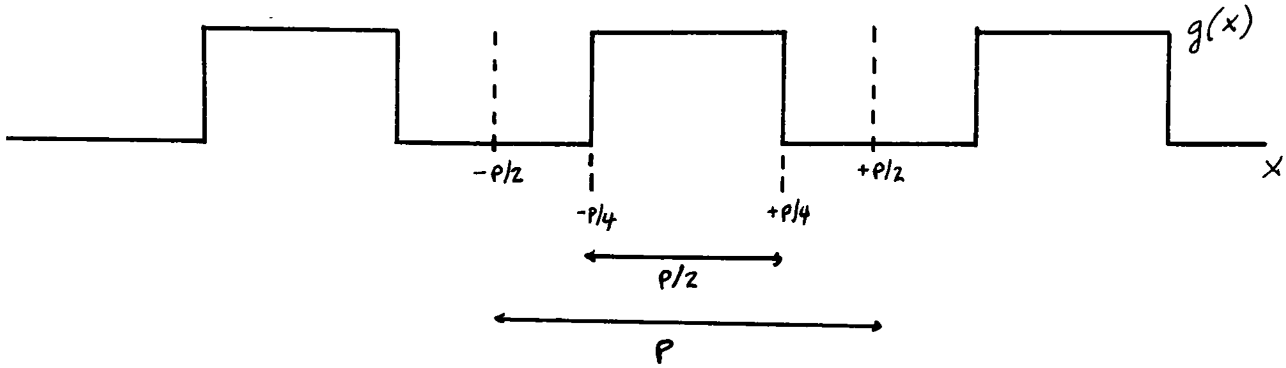
The sinusoidal test targets that were produced shows that sinusoidal test targets can be produced, both inexpensively and with speed. Although the targets were not of the finest quality, they still allowed a person to determine the quality of a system without sacrificing reliable data.

The sinusoidal targets could not have been produced using this method without Fourier Optics laying the groundwork for the theory. The method used in making the targets clearly illustrates the power of Fourier Optics. The fact that any periodic or aperiodic function can be manipulated to meet the needs at hand is a powerful tool. And this is only one simple application to Fourier Optics. Many other applications can be made to Fourier Optics which can solve the most complex problems.

The actual MTF results that were obtained (outside of the defocused series) were not very good. Yet, the out-of-focus MTF curves clearly illustrated the degradation of MTF by having a defocused lens. At very low frequencies, the MTF is unaffected, but at higher frequencies, the MTF suddenly goes to zero. The fact that the defocused MTF curve measured from the laser was comparably close to the Bessel function (compared to the MTF curve from the microdensitometer) indicated there was elimination of noise.

All in all, in measuring MTF, the laser was far more impressive than the microdensitometer in that it offered convenience and rapid, direct measurement of MTF from the coherent system. Still, it needs to be kept in mind that there is no question but that the microdensitometer is an essential element in measuring the image quality of a photographic system.

APPENDIX



$$a_n = \frac{1}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} f(x)\cos\left(\frac{\pi n x}{L}\right)dx$$

where $P = 2L$

$\therefore \quad L = P/2$

and $L/2 = P/4$

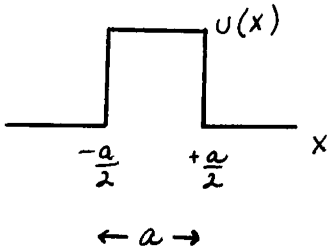
$$G(f) = \frac{1}{P/2}\int_{-\frac{L}{2}}^{+\frac{L}{2}} (1)\cos\frac{\pi n x}{P/2}\,dx$$

$$= \frac{2}{P}\int_{-P/4}^{+P/4} (1)\cos\left(\frac{2\pi n x}{P}\right)dx$$

$$= \frac{2}{P}\left(\frac{P}{2n\pi}\right)\sin\frac{2\pi n x}{P}\Bigg|_{-P/4}^{+P/4}$$

$$= \frac{2}{n\pi}\left[\sin\left(\frac{\pi n}{2}\right)\right]$$

$$G(f) = \operatorname{sinc}(\pi n/2)$$



Assume $a = p/3$

$$U(f') = \int_{-a/2}^{a/2} (1)\, e^{-i2\pi f' x}\, dx$$

$$= \frac{1}{i2\pi f'} e^{-i2\pi f' x} \Big|_{+a/2}^{-a/2}$$

$$= \frac{1}{i2\pi f'} \left[e^{i2\pi f' a/2} - e^{-i2\pi f' a/2} \right]$$

$$= \frac{1}{\pi f'} \sin(2\pi f' a/2)$$

$$= \frac{1}{\pi f'} \sin(\pi f' a)$$

$$\therefore U(f') = a \,\mathrm{sinc}(\pi f' a)$$

If $a = p/3$

then,

$$U(f') = \frac{p}{3} \,\mathrm{sinc}(\pi f' p/3)$$

let $U(f) = \mathrm{sinc}(\pi f p/3)$

SINE TARGET CALCULATIONS
(FORWARD DOOLITTLE METHOD)

To determine the line of best fit for a linear regression, sets of data are needed which will make dependent and independent variables Y and X, respectfully. This is because the equation is,

$$Y = b_0X_0 + b_1X_1 \qquad \text{where } X_0 = 1$$

Begin by forming a sum of squares table, as in Table A1 below.

Table A1
Sum of Squares

	X_0	X_1	Y
X_0	X_0^2	X_0X_1	X_0Y
X_1		X_1^2	X_1Y
Y			Y^2

X_0 is the number of sets of data. For the example in the experiment, there were 13 sets of density readings from the macrodensitometer and the microdensitometer. Therefore, $X_0 = 13$. Table illustrates a set of randomly chosen values to demonstrate the Doolittle Method.

Table A2
Randomly chosen values

(continued on next page)

Table A2

Randomly chosen values

X_0	X_1	Y
1	1	5
2	2	2
3	5	3
4	7	4
5	15	10
6	6	12
7	2	13
8	1	14
9	10	20
10	1	12
11	2	9
12	3	11
13	5	7

Complete the sum of squares table by determining: X_0X_1, X_0Y, X_1^2, X_1Y, Y^2. When these values have determined, reassemble the sum of squares table with the values inserted, but leave out the Y^2. It should appear like Table A3 below.

Table A3

Sum of squares

	X_0	X_1	Y
X_0	13	11	12
X_1		10	9

Draw a line under the bottom set of values like illustrated on the previous page, recopy bottom line and divide the entire top line by the value furthermost to the left (namely, 13), as illustrated in Table A4.

Table A4
Use of sum of squares

13	11	12
	10	9
13	11	12
1	0.85	0.92

Draw a line under the bottom set of values. The middle value (0.85) is a pivot point multiplier. Multiply 0.85 by 11 in the X_1 column, third row and subtract 10 (in X_1 column) from this value. Put below 0.85. Multiply pivot point multiplier by 12 in third row in the Y column and subtract 9 from this value. Put below 0.92. The complete table should appear as below in Table A5 when the last line is divided by the number below the pivot point multiplier.

Table A5
Complete Doolittle Table

X_0	X_1	Y
13	11	12
	10	9
13	11	12
1	0,85	0.92

Table A5 (continued)

Complete Doolittle Table

X_0	X_1	Y
	0.65	-1.20
	1.00	-1.90

To determine the coefficients for $Y = b_1X_1 + b_0X_0$, refer to table. The values in the table are the values of the variable X and Y. Therefore, by referring to the bottom line, $X_1 = 1$ and $Y = -1.90$. If $b_0 = 0$ in this line, then

$$b_1(1) = -1.9, \text{ or, } b_1 = -1.9.$$

To determine b_0, go to the third line from the bottom, where $X_0 = 1$, $X_1 = 0.85$, and $Y = 0.92$. Therefore,

$$b_0X_0 + b_1X_1 = Y, \text{ or,}$$

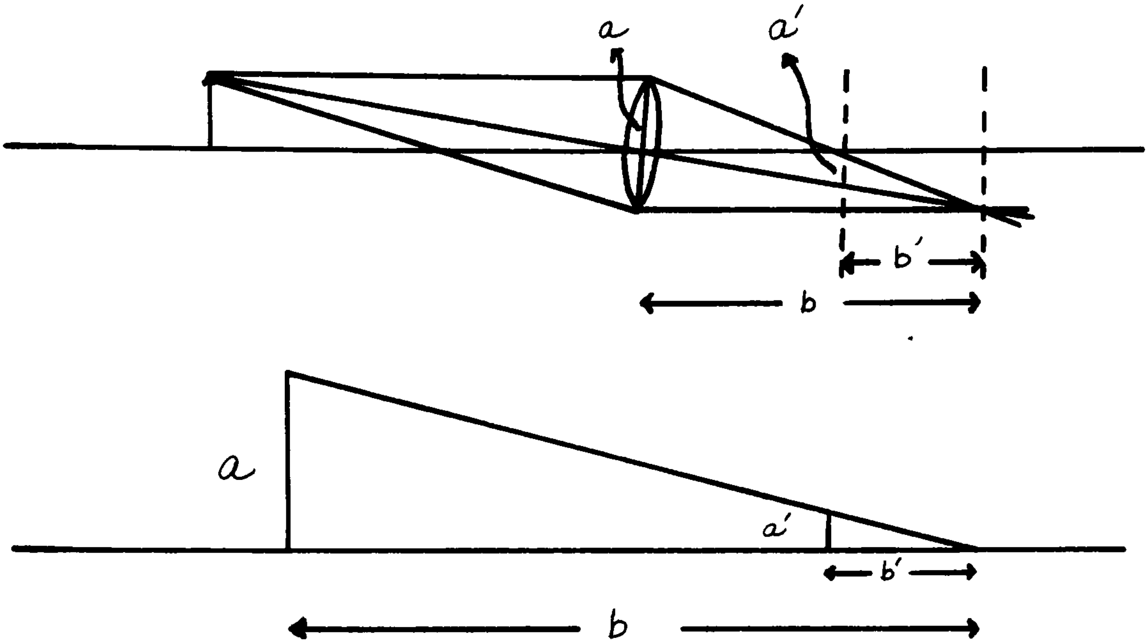
$$b_0(1) + (-1.9)(0.85) = 0.92$$

By simple arithmetic, $b_0 = -0.69$. Therefore, with this data, the line of best fit would be,

$$Y = -1.9X - 0.69$$

The slope of the linear regression is -1.9 and a y-intercept of -0.69.

BLUR CIRCLE CALCULATION



PHYSICAL AND GEOMETRICAL LAYOUT
OF BLUR CIRCLE

Figure A10

From the illustration above, it follows that,

a'/ a = b'/ b, therefore, b' = (a'/ a) b
Then the new image distance must be s' = b - b'. The new subject distance can be computed from the focusing equation because s' and f are now both defined.
The variable a is the opening of the aperture, a' is the blur circle diameter and b is the old image distance.

SINE TARGET CALCULATIONS

f	d	L	exposure
2 c/mm	21 mm	278 mm	f/11 for 1 sec.
2.9 c/mm	21 mm	379 mm	f/11 for 1 sec.
4 c/mm	21 mm	507 mm	f/11 to 16 for 2 sec.
5.7 c/mm	21 mm	731 mm	f/16 for 4 sec.
7.4 c/mm	13 mm	585 mm	f/11 for 4 sec.
9.8 c/mm	13 mm	765 mm	f/5.6 for 1 sec.
14.4 c/mm	5 mm	447 mm	f/8 for 1 sec.
17.2 c/mm	5 mm	637 mm	f/5.6 for 1 sec.
28.1 c/mm	5 mm	782 mm	f/8 for 1 sec.
32.6 c/mm	3 mm	675 mm	f/8 for 1 sec.

diameter of flourescent tube D = 38 mm

CALCULATING THE MTF OF SINE TARGETS USING A LASER

f	β^2	$2\beta/1+\beta^2$	β
2.0	0.125	0.246	0.015
2.0*	0.230	0.437	0.053
2.9	0.136	0.267	0.018
2.9*	0.104	0.206	0.011
4.0	0.117	0.231	0.014
4.0*	0.083	0.165	0.006
5.6	0.200	0.385	0.040
5.6*	0.112	0.221	0.012
7.4	0.345	0.617	0.119
7.4*	0.000	0.000	0.000
9.8	0.345	0.617	0.119
9.8*	0.000	0.000	0.000
14.4	0.188	0.363	0.035
14.4*	0.000	0.000	0.000
17.2	0.351	0.625	0.123
17.2*	0.000	0.000	0.000
28.1	0.343	0.614	0.118
28.1*	0.000	0.000	0.000
32.6	0.225	0.428	0.051
32.6*	0.000	0.000	0.000

* denotes out-of-focus targets

ACKNOWLEDGEMENTS

A very special thanks needs to be extended to Dr. Edward Granger, for his patience and direction in supervising this study.

Also, many thanks to my fellow students and professors in the Photographic Science and Instrumentation department who supplied little bits of information which helped in better understanding of the project at hand.

Finally, a special thank you to Professor John Carson, for his help in setting up the laser and other additional advice provided.

REFERENCES

1) M. Abouelata, "The Microstructure of the Photographic Image", Neblette's Handbook of Photography and Reprography, Van Nostrand Reinhold, 1977, pp. 197 - 233.

2) W. Smith, Modern Optical Engineering, McGraw - Hill, 1966, pp. 311, 322 - 323.

3) M. Spiegel, Schaum's Outline Series Mathematical Handbook, McGraw - Hill, 1968, p. 131, 174 - 175.

4) E. H. Linfoot, Fourier Methods in Optical Image Evaluation, Focal Press, 1964, pp. 12 - 20.

5) J. Goodman, Introduction to Fourier Optics, McGraw - Hill, 1968.

6) E. Hecht and A. Zajac, Optics, Addison - Wesley, 1974, pp. 401 - 411.

7) J. C. Dainty and R. Shaw, Image Science, Academic Press, 1974, pp. 247 - 249.

8) S. Davis, Diffraction Grating Spectrographs, Holt, Rhinehart and Winston, 1970.

9) R. E. Swing and M. C. H. Shin, "The Determination of Modulation Transfer Characteristics of Photographic Emulsions in a Coherent Optical System", Selected Readings in Image Evaluation, 1976.

10) R. N. Bracewell, The Fourier Transform and its Applications, McGraw - Hill, 1978.