Geometry and vision to Galen

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GEOMETRY AND VISION TO GALEN

BY

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Thanks to Nathan Lyons. It is mostly by working with him that my interests in the interpretation of images developed.

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per Arnolfo
# Table of Content

- **Chapter 1 - Introduction**
  1 - Theory, practice, communities, and the meaning of photographic imagery. 1
  2 - Instruments and exemplars. 5
  3 - Structure and method. 16

Notes. 19

- **Chapter 2 - Observation and Proof before the Greeks**
  1 - Introduction. 21
  2 - Egypt. 23
  3 - Babylon. 37

Notes. 48

- **Chapter 3 - Geometry and Remote Sensing**
  1 - Observation and the measurement of distance. 52

Notes. 64

- **Chapter 4 - Imagery and Visual Evidence in Early Greek Thought**
  1 - Introduction. 64
  2 - Vision as exposure. 67
  3 - From witness to proof. 73

Notes. 84

- **Chapter 5 - Greek Theories of Vision**
  1 - Alcmaeon. 86
  2 - Empedocles. 89
  3 - Democritus. 96
  4 - Plato. 108
  5 - Aristotle. 119

Notes. 137
Chapter 6 - HELLENISTIC THEORIES OF VISION

1 - Institutionalization of science:
   systematicity and quantification .................. 143
2 - Deduction, demonstration, and geometry .......... 156
3 - Plato, Aristotle, and Euclid ..................... 161
4 - Euclid's Optics .................................. 176
5 - Ptolemy: optics and astronomical observations 188
6 - Galen: anatomy and geometry ..................... 206

Notes ................................................. 232

- BIBLIOGRAPHY ..................................... 240
Chapter 1 - INTRODUCTION

1 - Theory, practice, communities, and the meaning of photographic imagery

One of the common notions about the photographic medium is that it produces images which often have a proof-like status. However, a series of problems are raised as soon as we consider what are the processes which legitimize the "truthfulness" of photography.

It is true that photography is usually accepted as legal or scientific "proof" because of the optical and chemical processes peculiar to the medium (the point-to-point "correspondence" between the image and the depicted object), but this is not always the case for, in other situations, the acceptance of the proof-like status of a photograph is instead largely context-dependent. This is particularly evident in the media environment where photographic and electronic imagery in general derives its status of proof largely from the authority of the media themselves, even though--paradoxically--media develop their own authority (at least initially) also from the shared belief about the inherent "scientific" truthfulness of the medium.
This suggest that—at least in a broader social context—the proof-like status of photographic imagery does not derive from theory only but rather from the interaction between the theory and the practice(s) of the communities which use (and/or are used by) the medium. More precisely, certain practices are initially justified by the theory (which is imported from another community), but then develop their own contexts of justification in which references to the initial theory (if present at all) are difficultly explicable.

This complex interaction can be better (if not only) grasped in the context of specific communities which use the medium for specific purposes. This because, in general, the transmission of justificatory (and often myth-like) arguments for the proof-like status of photographic images reaches a community from another wider one to which the former is both culturally and economically related. As suggested by Sekula's analysis of the meaning-making processes which operate around and within photographic archives\(^1\), the meaning of such imagery derives from an almost entirely bureaucratic ("normal") process, whose justification is only indirectly theory-related. In such a context, the "theory" of the medium is not important as such, but rather as an instance of the overall "rationality" entailed by the structure and
organization of the communities which make use of that medium.

My concern here is to approach this interaction between the theory and the practice of a medium from an historical point of view, extending the field of the inquiry to the development of theories of vision and geometrical optics, and to their relation with the design and use of optical instruments as well as with the evaluation of their reports.

I will both follow the development of the notion of demonstration as produced within an axiomatic deductive system and the development of pre-geometrical theories of vision, showing their final merging into geometrical optics which is the backbone of both modern theories of visual perception and physical optics, which--ultimately--is an important component of the theory through which modern media are designed and "proved".

By doing so, I will show that historically we do not find a theory which stands aside the practice of the use of optical instruments, which "explains" them and give the necessary tools for the evaluation of the instrumental reports. What we find, instead, is that the use of an instrument mediates the development of a theory which then mediates both the further development of the instrument and the evaluations of its reports. Following Feyerabend's
thesis, we can say that theory does not only move **out** of a practice, but it actually **remains such**. This because the difference between theory and practice is of structure and not of kind.

From here we get to the notion that the instrument is not just a simple extension of the senses, but rather an active tool, like a machine which does not observe the world for us but rather **produces** observational reports which we are taught how to read once we belong to that community. In other terms, the world we perceive through an instrument or a medium is not an "extended" world but an artificial one, and the range of practices which develop around the use (and the development) of the instrument play an important role in separating **artificiality** from **arbitrariness**. Borrowing Kuhn's terminology, I would say that the instruments play sometime the role of the **exemplar**.

I believe that the camera obscura worked as the basic exemplar for the theories of geometrical optics. In fact the understanding of the behavior of the light rays passing through the pinhole and forming the image on the opposite plane, was crucial to the development of the awareness of the processes of image formation within the eye which would then bring to both Kepler's development of the concept of retinal image and
of the lenses' optical behavior. The non-arbitrariness of the exemplar was guaranteed by a wide range of uses within astronomy and land survey of optical instruments—primarily the camera obscura itself\(^5\) and the dioptra—which both rely on the use of pinholes.

2 - Instruments and exemplars

Evidently the problem is the analysis of the dynamics of the interaction of theory and practice.

Feyerabend states that "reason and practice are not two different kinds of entities but parts of a single dialectical process."\(^6\) He exposes quite clearly the limitations of traditional approaches to the philosophy of science which rely on a mythified (and mystified) notion of rationality, and shows the relevance of non-intellectual factors as well as political issues to such dialectical process. However the participants to that process are better described than the structure of their interactions.

Kuhn's notion of exemplar could be a better tool for describing some of the dynamics which develop between theory and practice. He introduced such a notion to precise one of the meanings of the term paradigm he had widely used in his The Structure of Scientific Revolutions
trying to show how actively the professional training within a specific scientific community (tacitly) supplies content to a series of operations which are generally supposed to be purely syntactical. One of his examples is the development of analogies with successfully solved problems in a "comparable" field of research which justify the adaptation of a more general law to a new case. For instance Galileo:

"...found that a ball rolling down an incline acquires just enough velocity to return it to the same vertical height on a second incline of any slope, and he learned to see that experimental situations as like the pendulum with a point-mass for a bob. Huygens then solved the problem of the center of the center of oscillation of a physical pendulum by imagining that the extended body of the latter was composed of Galilean point-pendula, the bonds between which could be instantaneously released at any point in the swing."

Each of these "translation" are not just syntactical transformation, but they entail the addition of a specific empirical content. Evidence of this problem can be found in the fact that students often cannot "see" the application from the formalism they can read, but they need to be supplied of that kind of knowledge through training in problem-solving. The solution of problems is what "points" to the proper analogies that allow for the application of the formalism. Kuhn's hypothesis is that such a training produces something like a
gestalt-modification so that the processing of stimuli into data is "programmed" in a way that the criterion of similarity suggested by the exemplar is tacitly introduced in the student gestalt.

Kuhn's emphasis on the fact that the empirical content of each generalization of a formalism is different an not "derived" syntactically from the first case of such generalization ("Empirical content must enter formalized theories from the top as well as the bottom") also suggests an interesting approach to the relationship between theory and practice during the evaluation of instrumental reports. These considerations seem to confirm that instruments are not just "extensions" of our perception even though we usually say that we "see" through an electronic microscope or that we "see" an electron in a cloud chamber.

Hanson has exposed a series of differences between "to see" and "to see as" in connection with different problems, instruments, and contexts of discovery, while Kuhn has pointed to the fact that:

We do not see electrons, but rather their tracks or else bubbles of vapor in a cloud chamber. We do not see electric currents at all, but rather the needle of an ammeter or galvanometer.

The process which allows us to bridge the gap
between our "natural" perception, and the "extended" one is not just the precise knowledge of the "syntax" of the instrument which produces the report, but also a specific kind of training of the same kind considered before.

With these considerations, Kuhn describes the role of the tacit knowledge of the community as entailed by the professional practice and how and where it interacts with the "theory". However his primary concern in these arguments is to clarify the concept of paradigm as the structure of the cognitive identity of the scientific community, and therefore the practice and the theory which are shown to interact are internal to the community. External influences are certainly not excluded, nevertheless they need to be mediated by the paradigm. This attitude evidently refers to Kuhn's necessity of offering a satisfactory articulation of the relativism introduced by the admission of the role of practice within the scientific enterprise. In fact he treats the scientific community as a body and--by studying its anatomy, physiology, and psychology--tries to account for its behavior, which is the production of science. The "evolutionary" connotation of the relationship between theory and practice is particularly evident in the "Postcript 1969" where he says that:

In many environments a group that could not
tell wolves from dogs could not endure. Nor would a group of nuclear physicists today survive as scientists if unable to recognize the tracks of alpha particles and electrons.13

In a sense the notion of salience of a similarity and the capability of "seeing" it is teleologically related to the professional survival of the community, in fact in "Metaphor in Science" he says that:

Only through the multiplicity of such exposures can the student acquire what other authors...refer to as the feature space and the knowledge of salience required to link language to the world 14

Evidently the "multiplicity of those exposures" and the related notion of salience derives from a professional training of the kind considered earlier.

However, the study of the exemplars adopted by a community—especially in a context of full institutionalization of science—may reveal something about the presence of interests which are also related to a practice. A.Pickering has proposed a model of interaction between interests and exemplars, where he suggests that:

An "interest", then is a particular constructive cognitive orientation towards the field of discourse. As a shorthand description one can refer to an exemplar being so constructed that it "intersects with the interests" of some particular group or groups.15
But Kuhn's model can be useful also for a structured analysis of the results of external influences on the community, primarily by studying the "importation" of exemplars which have been originated outside the community, either in other scientific groups, or from the practices of other professionals who are not engaged in scientific activity.

This is my concern here, also because, by dealing with a period in which the scientific activity is largely pre-paradigmatic, the notion of "community" is so loose that practically everything is (by default) external to it. However, this set of problems is of particular relevance also for the historigrahical practice, because, to be able to evaluate the potential relevance of the study of exemplars for the understanding of the interactions between theory and practice, or between external influences and internal orientations and commitments of a scientific community, it is necessary to develop some kind of historically-grounded hypothesis about how such a relationship may have developed in time.

We can detect a pattern of "exemplars-exchange" among communities also within contemporary science, where those exemplars do not stand just as successful solutions of typical problem of a scientific community, but their relevance is extended also to a much wider set of social
communities. I believe that a significant example is offered by the adoption of the "computational metaphor" as an exemplar for the neo-formed community of Cognitive Sciences. Evidently the computer has a double (and doubly successful) status: as a structure of computational procedures (like a Turing machine) it is an exemplar for the theory of computability; and, as an actual computer, it is an exemplar of a specific kind of solution of problems of information management which has an evident social and political connotation. The two aspects are probably inseparable especially once we consider the teleological dynamics of the community's fund-raising.

Beside the economical implication of the choice of an exemplar over another--which can develop only in a social context characterized by the presence of a scientifically-informed technology--we see that the "world" of the scientist has always been historical and not natural, for it is conceived as the domain of the theories and the practices of that period.

For instance we find that Aristotle thought that speed varied in proportion to the ratio of force to resistance (and that this created difficulties if the motion took place in the vacuum\textsuperscript{16}), but he was also living in an environment in which (with the exception of arrows)\textsuperscript{17} most of the movements were quite slow and
with quite a friction. His "experiments" are often "performed" by fishermen hauling boats on the shore (which shows quite clearly why he was talking of force and resistance and their ratio). His unit of force is "man" and his notion of speed was evidently thought in terms of the duration of the hauling task. To think of the absence of resistance in a world constituted by practices like this, it was not an abstraction but a dream.

I am not particularly convinced by the theses about the craft-origin of many of the theories developed in the early phases of the scientific revolution, but what seems to be out of question is that a new set of practices presented the scientist with a different world in which certain theories could be thinkable and their examples could be seen. For instance we find Galileo saying that:

SIMP. I would like to hear your reason for putting the projectiles of fire arms, i.e., those using powder, in a different class from the projectiles employed in bows, slings, and crossbows, on the ground of their not being equally subject to change and resistance to the air.

SALV. I am led to this view by the excessive and, so to speak, supernatural violence with which such projectiles are launched; for, indeed, it appears to me that without exaggeration one might say that the speed of a ball fired either from a musket or from a piece of ordnance is supernatural. 18

The term "supernatural" in Galileo does not have any magical connotation. He simply means that such a speed
cannot be obtained with "natural" means, for he believes such a speed to be superior to the terminal velocity of free-falling bodies in air.

This seems to be a literal example of the "supernaturality" (i.e. historicity) of the scientist's physical world.

It is also a common notion that Galileo's mechanics was less "intuitive" than Aristotle's, but what was less "intuitive" were also the phenomena he had to face and which were produced by a technological (and socio-political) practice.

The introduction of fire-arms did not extend the range of observable phenomena; it created them (and not others), with a process analogous to the "historicization of nature" which Marx indicated by saying that:

The cherry-tree, like almost all fruit-trees, was, as is well known, only a few centuries ago transplanted by commerce into our zone, and therefore only by this action of a definite society in a definite age it has become "sensuous certainty"...

This pattern of examples suggests that phenomena are to an important extent a product of the practice of a community which (in the beginning) is society itself. The same cannot be easily said about scientific theories. In fact, it is quite difficult (especially in an historical period in which the scientific community is not fully
structured) to trace the relationship between the development of scientific theories and other practices, as evidentiated by the controversial results of Zilsel's work\textsuperscript{20}. This because the relationship between scientific theories and other practices does not occur according to a fixed pattern, for also such a pattern develops (unlinearly) in time.

We cannot think of the role of the "computational metaphor" for the Cognitive Sciences as commensurable with the role of the "hauling metaphor" for Aristotle's physics, or with the influence of the development of fire-arms on Galileo's dynamics. These contributions of external practices on the development of scientific theories are much different once we evaluate them in their proper context, which is, by considering what was the structure of the scientific community and its position in society at that time. In fact it is such a structure that mediates external contributions and internal components around the concept of relevance and salience. Consequently a science is "mature" not only because somebody can lay down its specific axioms, but also because its community is fully structured. Geometrical optics is a good example of this process. Euclid developed it on an axiomatic base, but it was then articulated by scientists belonging to different communities (astronomy,
astrology, medicine) which held different paradigms and were trained with different problems and exemplars (though still about vision). This produced often incommensurable readings and articulations of those axioms.

However, by "community" I do not mean a citadel-like object, something which isolates the scientists from the outer context. What I mean is that the interactions between the inside and the outside are mediated not in terms of individual interests, orientations, but precisely in terms of relevance, salience, and ultimately of professional survival (or expansion) of the whole community.

As a sketchy summary of the relationship between instruments and exemplars, we can consider instruments to be among that range of results of practices (not necessarily scientific) which contributes to the production of new range of phenomena (and not the extension of old ones). An instrument can become an exemplar (being "translated" in the theory and integrated into the practice of the community) if it offers a "relevant" analogy with the problems contemplated by the program of the community. The notion of "relevance" is not a fixed one but changes together with the structure and the social role of the community.
3 - Structure and method

Most of these considerations have their translation in terms of historiographical problems. Paradigms are useful tools for the historical analysis of the activity of a community (from an internal point of view), even though they can easily introduce the misleading notion of "normal" history, by propagating among historical evidences what the historian supposes the paradigm to have been. However, in dealing with the historical period up 200 AD, the possibility to identify anything more than paradigm-sketches is probably unrealistic, primarily because of the scarce articulation of scientific communities. In Greece the only science which was probably practiced within a community-like structure was medicine. Its social relevance made its practitioners to become professionals who often organized themselves in guild-like structures. Greek Astronomy was also professionally practiced, but the "circular motion paradigm" that is sometimes attributed to it, covers several important discontinuities, the most important being the shift from a quasi-qualitative attitude before Hipparchus to a systematic and quantitative one in the Hellenistic period.

The identification of a paradigm for the theories
of vision is an hopeless task because those theories were
developed within (and in connection with) a range of
different professional practices and schools of thought.
As Lindberg puts it:

"We do not know all the motivations that
underlie these earliest Greek discussions, but it
seems clear that blindness and eye disease
stimulated medical thought on the subject of
vision and gave birth to the science of
ophthalmology; that an interest in epistemology and
psychology led philosophers to examine mankind's
most noble and dependable sense, the sense of
vision, and to analyze its functioning in physical
terms; and that the artist's concern for
scenography and the astronomer's concern for
accurate celestial observations induced
mathematicians to formulate a mathematical theory
of perspective."

The materials presented here cover Lindberg's
synopsis even though they are articulated according to a
different methodological attitude.

The development of geometrical optics has been
approached here from two quite different but converging
points of view. The development of pre-geometrical
theories of vision has been treated with an "internalist"
attitude; looking for certain recurrent problems with the
different solution offered by the different authors and
trying to identify paradigm sketches.

Instead--with an "externalist" approach--I have tried to trace some of the aspects of the interaction
between scientific theories and professional practices in
this early period. In fact, the history of geometrical optics offers the possibility to trace the development of the notion of demonstration as entailed by euclidean geometry in one of its first applications to the explanation of a physical, physiological, and psychological process. The point is of particular interest because it refers to the process of development of formal reasoning out of the interaction of a number of practices and concerns within a changing landscape of social conditions.

Among the various components of the social context, I have tried to point at certain issues which pertain to the development of institutionalized scientific communities and to the subsequent structure of programs of research.

Unfortunately, the limited period of time (weighted in terms of "density" of scientific activity) considered here made possible only to indicate certain pattern which were going to be more visible only later.
NOTES TO CHAPTER 1


2 P. Feyerabend, Science in a Free Society, London, 1978, p. 26, "...the difference between 'reason' and something 'unreasonable' that must be formed by it or can be used to put it in its place arose from turning structural differences of practices into differences of kind.... What is called 'reason' and 'practice' are therefore two different types of practice".


5 The same camera obscura was used for astronomical observations, especially for eclipses. Alhazen has left a detailed description of such a practice.

6 P. Feyerabend, op. cit., p. 25.


9 'Tacit' here does not mean 'secretive' or anything like that because the practices Kuhn is talking about are perfectly explicit. Instead it means that such a knowledge cannot be "seen" by looking at the formalisms.


12 T. S. Kuhn, The Structure of Scientific Revolutions, p. 196.

13 T. S. Kuhn, ibid., p. 195-6.


17 The problems of the observation of fast moving body was a crucial problem primarily because of the lack of a precise instrument of time reckoning. A. Koyre' in Metaphysics and Measurement, London, 1968, pp. 89-117, has shown that also Galileo and his immediate successors relied on preconceived ideas when evaluating the results of their experiments, because of their large margin of indetermination.


Chapter 2 - OBSERVATION AND PROOF

BEFORE THE GREEKS

1 - Introduction

Before considering Euclid's development of geometrical optics in the hellenistic period, we need some background in the earlier theories and practical applications of geometry and optics in astronomy, land surveying, and chartography. This because the application of a geometrical model to vision was most probably initiated not from the philosophical speculations about the process of cognition through vision, but rather from a concern in the formal justification of observational reports by treating them accordingly to the method and propositions of Euclid Elements. By tracing the uses and the contexts in which optical observations were initially performed and utilized without much formal concern about their theoretical status, we may develop some understanding of the intellectual and social change which stimulated the development of Euclid's Optics and his concerns about "proving" vision.
The pattern of transmission of early mathematical, geometrical, and astronomical knowledge from the Middle East to Greece is still unclear, and not all the reports we have about the trips of Thales, Democritus, Plato, and Eudoxus to Egypt to learn astronomy and geometry from the astronomers/priests at Heliopolis are believed to be accurate. Nevertheless, it is clear that the early development of geometry took place in Egypt and Babylon where an agrarian economy essentially based on a very extensive and state-controlled system of irrigation developed a complex system of land registration. The geometry of the land surveyor and the arithmetics of the scribe probably constituted the early tools of the astronomer.

We do not have detailed information about the instruments used for those observations\(^1\), but it is reasonable to assume that they did not go under relevant developments before the hellenistic period. One of the first surveying handbooks of the Greek world is the *Metrica* by Hero of Alexandria (250? A.D.) whose dioptra (an improvement of an already existing device) was one of the most sophisticated but scarcely used surveying devices. The major source of information about astronomical instruments is Ptolemy (85?-165? A.D.), who describes some of them in his *Almagest* \(^2\) and
Planishaerium, where he also refers to Hipparchus' (180-125 BC) "horoscopic instrument", suggesting that he already knew the astrolabe. We do not have information about previous instruments specifically designed for astronomical purposes, and it is probable that they were basically similar to the ones of the surveyors. Moreover the determination of the orientation of a specific plot of land requires some astronomical knowledge which suggests that in the beginning the astronomer and the surveyor shared not only the same instruments but also similar notions. In any case the later development of optical instruments for astronomy did not introduce radical modifications in the sighting components, but it rather tended to develop the computational aspects of those instruments, so that a single observation of a celestial body could produce automatically a wider range of information. There is no evidence of a theory of geometrical optics before the Hellenistic period, even though all these instruments were evidently constructed and employed under the tacit assumption that something was putting the viewer in "touch" with the viewed object along a straight line.

2 - Egypt
Egyptian astronomy made scarce use of geometry, and—beside the development of an accurate calendar—did not achieve particularly significant results.

The positions of the stars were plotted on a grid so to work as a night clock: the "meridians" indicated the hours, while the parallels gave the stars' elevation. It is quite probable that their "latitude" and "longitude" were not derived from angular measurements, but rather from subjective evaluations of direct observations. R.A. Parker has reconstructed the procedure of astronomical observations followed by the early Egyptian astronomers as:

On a suitable viewing platform, probably a temple roof, two men would sit facing one another on a north south line. The northernmost would hold a sighting instrument like a plumb bob before him and would call out the hour when a star had reached either a meridian or one of the lines before or after as sighted against the target figure.

After 1500 B.C. the water-clock largely replaced astronomical observations which continued to be performed almost for calendarial purposes only.

The emphasis on the accuracy of the calendar and the much lesser concern with a full and precise stars' cataloguing (like the ones later developed by Hipparchus and Ptolemy) can be explained by considering the context in which the Egyptian astronomers were operating.
Egyptian cosmology offered a picture of the heavens and the gods which strongly confirmed the actual structure of the Empire. The two sides of the picture were connected through the god-like figure of the king. An accurate calendar was therefore necessary to synchronize heavenly and earthly life at recurrent dates of particular importance for the social life, such as the annual flood of the Nile. Egyptian religious and political behavior was developed around the concept of *maat* which entailed the concepts of both natural and legal order. The orderly configuration of the heavens was matched both by the highly articulated politico-bureaucratic structure of the Egyptian state and by the seasonal regularities of nature. The king-god is the keeper of *maat*. His role is to maintain the teleological order of nature, society, and the heavens.

It is not accidental that the new pharaoh was enthroned on the first day of the year and that this event was considered (and depicted) as a new creation, as a new useful re-ordering of the whole world according to *maat*. By encoding this order as displayed by the course of heavens and of seasonal phenomena, the calendar becomes also an expression of *maat* and it is consequently associated to the figure and role of the king. The relationship between the pharaohs, the natural phenomena,
the celestial order and the "good life" is evident in this fragment of a hymn for the new king's accession to the throne:

Be gay of heart, the entire land, for the goodly times have come! A lord has been given to all lands!.... The waters stand and are not dried up, and the Nile carries a high [flood]. [Now] the days are long, nights have hours, and the moons come regularly. The gods are at rest and happy of heart, and people live [in] laughter and wonder!.

This passage emphasizes the importance of order, both in society and in the sky. Order is meant to be a periodic recurrence of events, and the periodic repetition of phenomena suggests the presence of a constant cause. When phenomena recur eternally—such as the movement of the heavens—the cause needs also to be eternal, which is, ontologically justified. As a result kingship is natural in the same way the sun rises naturally each day.

Analogously, the notion of the heavens as an organism is mirrored by the structure of the state. Everybody has its specific role which is also the basis of both his individual and social identity as well as of the form of his cognitive approach to natural phenomena. As Max Weber puts it:

"every individual is bound to the function assigned to him within the social system; and therefore every individual is in principle unfree." or:
"In general, the individual in Egypt was from early times a tool of pharaonic power; he and his property were no more that entries in a cadaster."

Then, if the heavenly organism is alive, it means that it has an individual will, which implies that it cannot be analyzed but rather is understood (or sympathetically comprehended) through a dialogue with it. If the Nile does not rise, it is because it does not want to, which means that it needs to be interrogated about the reasons of its behavior through the mediation of oracles. This attitude was probably transferred into astronomical investigations, because—if a planet has its own will—the study of its movements trying to understand the pattern of its orbit would result in a limitation of that planet's (or that god's) will.

However, to limit our attention to the consideration that mythopoeic thought did not allow planetary motion to be observed with scientific attitude but only in terms of dialogue, tends to underestimate both the loaded connotation of this dialogue and the identity of the dialoguants. A socially relevant activity such as astronomy cannot be considered per se, but rather in its relationship with the social context through which the astronomers derive their "identity". Going back to Weber's remarks, we can argue that if individuals assume an
identity in terms of their social role, it follows that their work--also the intellectual one--assumes the status of a procedure. Both individuals and their work share the same characteristic of being typical. This is also partially confirmed by the absence in Egypt (and Babylon) of the notion of authorship in scientific and technological achievements, which instead--in a much different social context--was going to be recognized in the Greek world.

We can expect that the "form" of the questions posed by Egyptian astronomers to be probably incommensurable with the one of later astronomers, and that this incommensurability cannot be attributed only to the fact they were "ancient" and mythopoeic, but it rather needs to be referred to the different role of astronomy and astronomers within those societies.

The intellectual behavior of Egyptian astronomers cannot simply explained in terms of mythical and religious reverence toward celestial bodies, for we cannot easily understand the real content of the religiosity of technical astronomers. These kind of explanations are usually introduced by whiggish historians of science who first de-contextualize religious, mythological, or astrological concerns to re-introduce them later as blinding eyeglasses to explain the slow-downs and detours
from the linear progress of western science. We can rather think of a professional mentality which incorporates, mediates and supports religious components which otherways could not probably survive the demistificatory impact of routine-like activities. To maintain an explanatory value, religious concerns needs to become part of a professional ritual. In a social context in which religion and cosmology are not fundamental for the justification of the political structure of the state, the role of the astronomers' community is much different as well as the content of its paradigm. In fact we find Plato who--referring to Anaxagoras--says that:

...those who study these objects in astronomy and the other necessary allied arts become atheists through observing—as they suppose—that all things come into being by necessary forces and not by the mental energy of the will aiming at the fulfillment of good.  

A more reasonable explanation of the attitude of Egyptian astronomers can be found in their training. The intricate bureaucratic structure of the Egyptian Empire required the systematic use of written records in most of its activities. This brought into existence a numerous class of scribes who were trained on textbook-papyri which presented and solved "typical" problems. R.Gillings tells us that most of these papyri end as:"The manner of reckoning it", or "Manner of working
out", or "These are the correct and proper proceedings", and "That is how you do it". In Babylon, which is characterized by a social and political context which shows several similarities with the Egyptian empire, we find the same concerns for bureaucracy and we also find specific evidences of the scribes being trained on clay-tablets-textbooks presenting typical solutions for typical problems. Interestingly enough the Babylonian textbooks were ending with what Neugebauer translate as: "this is the procedure", which is strikingly close to Gillings' findings. These endings are functionally equivalent of the "Q.E.D" or "Q.E.F" as found at the end of the propositions of Euclid's Elements.

The difference is that the function of the deductive demonstration is here performed by the official authority which issued the textbook defining both the socially-teleologically relevant problems and their form of solution. Therefore, the reason for the absence of the notion of proof was a social one. Consequently the difference between problem-solving procedure and formal explanation which has been often used as a criticism to the unscientific intellectual attitude of ancient civilizations is the product of an a-historical translation of the problem.
However, the fact that these scribes were hired by the same authority which issued the standard, cannot be trivially taken to imply that they followed certain procedures under an economic pressure only. I believe the situation was quite more complex. The textbook is a sort of "social contract" and the scribes by "subscribing" it were also receiving a role-related identity. The same cosmological myths were not ingredients of an oral culture, but they were instead taught and actually compiled at scribal schools. They were an important component of the ideological formation of the scribe and not of the peasant-serf whose political consensus was of no relevance.

To understand the importance of the opportunity of a career as a scribe we need to remember that the notions of citizenship and legal identity in antiquity were drastically different from our contemporary ones. For instance a usual from of "identity" was entailed by the concept of *idia* which:

> was based on the principle that every individual must have an official domicile; that is, he must be officially entered on the rolls of a community and hence be responsible for helping that community meet its assigne labour duties. He who did not have a domicile therefore had his property confiscated, and he and his family became directly subject to pharaonic authorities.

In other terms, *idia* is identity as a servant.
Scribes were reaching a much better social and legal status, special privileges, and their job was often inheritable by other members of the family, as shown by this fragment:

"Put writing in thy heart, so that thou mayest protect thine own person from any [kind of] labour and be a respected official." and "...the scribe, however, he is the one who directs the work of everybody [else]. He pays out taxes by writing, so that he has no [real] obligations."

It is very probable that land surveyors were scribes who went through a further training. Some astronomers were priests, which means that they too had a specific training and a strong community awareness. The development noted by M.Rostovtzeff of professional organizations, which begun under the pharaohs and reached their full importance in the hellenistic period probably contributed to the consolidation of this group identity. Because of a much different social and economical context that we will briefly consider later, both the structure and contents of Greek education was radically different. For instance, the "scribe culture" was fully developed only in the hellenistic period.

Going back to consider what it meant to be a surveyor in ancient Egypt, we need to remember that the same notion of maat which legitimizied the king's authority and his role as keeper of natural and social order was
also applied to any social object related to the king. For instance, land was virtually under the king's authority and protection (at least in the New Kingdom) and (with the exception of the estates owned by the temples) it was cultivated for the king by serf-like peasants. The social context casted a religious significance on the property of land (by far the most important kind of property) for we find fragments of prayer like:

Cursed is he, that removeth his neighbour's land mark. Amen. 17

We also know that votive stones called "kudurus" were placed as land-marks in Babylon 18 where we also find records of disputes about land boundaries and inheritance divisions since Hammurabi 19. It is in fact from a letter of Hammurabi (1792-1750 BC.) that we get to know of the early legal role of the surveyor in this kind of law suits. This was later developed by Roman jurisprudence to the point that the surveyor assumed sometime the double role of technician and judge 20.

As it can be expected, Egypt developed a central cadaster and system of land and men registration early during the Old Kingdom which was the core of a complex system of taxation 21 and distribution of public charges. The importance of the cadaster and consequently of the surveyors' work was particularly stressed by the
crucial role of irrigation and of the mapping of the responsibilities of its management.

But Egyptian civil law was poorly articulated for it was evidently comprehended by maat. Consequently the notion of legal proof in land-related law suits was evidently an arbitrary. In general:

..the manner in which justice was administered made it inevitable that eventually the prevailing rule would be "no men without a master." 22

Analogous to what we have seen in the case of astronomical investigations, in this case there is also no need for proof because its function is replaced by confirmation through authority. Together with proof, also the notion of responsibility tends to dissolve. Once the surveyor performed the measurement accordingly to the procedure he has been trained on, he had "done his job" respecting the "ritual" of the profession. He was not responsible for the production of a proof. As in the case of the astronomer also the surveyor performs typical observations not of objects but rather of socially connoted items.

The lack of interest for the notion of proof is evidently related to a particular social attitude about causality. In fact, the scarce articulation of civil law implies the scarce awareness of the concept of legal
causality, which in turn does not stimulate the
development of the notion of proof as related to the
output of socially relevant technical activities such as
land measurement. On the other hand, the role of the
astronomer and his training as a member of a community
prevents him to pose untypical questions. This would have
meant to extend the language of his community so to be
able to articulate criteria of causality and proof which
could be accepted by a wider (or rather different)
community which instead did not exist in that society.
They were short of audience which could have actually made
use of those developments. This because, as Weber puts it:

Even in the oldest extant documents we can see
that while there were privileged classes in Egypt,
there were no juridically free communities
equivalent to the Greek polis or kome.  

Here I am not suggesting the equivalence between
the concept of proof in law and in the physical sciences,
but rather that they both are thinkable (and
actually thought ) only in specific historical stages of
social emancipation. If the emancipation from the
mythopoeic thought implies the progressive development of
the awareness of the mediative character of the cognitive
process, viz. of cognitive identity, this is also
paralleled by the establishment of a social and legal
identity. To get to the point where proofs are needed in a
choice-making context, it is necessary to have already
developed the notion of choice, and to be able to
recognize it as such. In other terms, the notion of proof
can exist only together with the one of choice, and with a
community which can hold that choice. This in turn
requires the social corpus to be articulated in
communities with a legal status that allows them the
possibility of holding different choices. Evidently this
was not the case with ancient Egypt.

As a result of this scarce development of the
notion of proof and legal responsibility, it is not
surprising that we do not find any interest about the
assumptions and the processes implied by the act of taking
a measurement with and optical instrument. In a sense the
instrument too does not have an identity (a "syntax") yet.
The concept of light does not go beyond its usual
metaphysical connotations such as "good", "keeper of
life", etc., and we do not find any trace of inquiries
about the process of vision among the few medical
papyruses. Geometrical notions of angle and line are still
conceived in terms of dimensions of actual objects.

The concept of proof as a result of a deductive
inference such as the one entailed by Euclidean geometry,
was not developed only because of the scarce abstractness
of geometry, but also because of the multiple approach to
causality which was peculiar of ancient Egyptian thought. In such a form of thought, the causes of a process are referred to a number of different agents. Each of them "contributes" to the overall causation, but these contributes are very often at odds with each other. This lack of coherence in causal explanations is found not only in the approach to natural phenomena but it is also reflected in the scarce awareness of causal ordering in the structure of the state. The bureaucratic chaos typical of the administration of the Egyptian empire shows that responsibilities were incoherently distributed among ever-conflicting roles without much awareness of the problem. Frequent conflicts were not "explained" by tracing responsibilities, but they were rather adaptively and paternalistically solved or supressed. In turn, the administrative chaos also reflects the scarce articulation of civil law, suggesting again that causality and proof have their roots in both intellectual and social emancipation.

3 - Babylon

In Babylon too the cosmological myths played an important role in the justification of the structure of the state. As T. Jacobsen puts it:
The only truly sovereign state, independent of all external control, is the state which the universe itself constitutes, the state governed by the assembly of the gods. This state, moreover, is the state which dominates the territory of Mesopotamia; the gods own the land, the big estates, in the country. Lastly, since man was created especially for the benefit of the gods, his purpose is to serve the gods. 24

In this context the secular rulers are nominally only "governors" and need to receive the orders from the gods. From here the systematic practice of divination and astrology. Here we do not find any concept equivalent to the Egyptian maat, on the contrary dramatic contrasts take place in the heavens and are tragically projected on the hearth in the form of wars, invasions, and storms. Therefore the messages that the gods send to men are not expected to be of univocal interpretation for they represent a state of continuous tension and danger. This introduced the need for a sort of statistical inference based on the omen previously observed so to be able to evaluate the present message. This concern is reflected in the systematic keeping of astronomical and calendrical records. Neugebauer 25 has found that in early tablets these records are often kept on more columns inclusive of calendrical dates, astronomical records, relevant events and omen. But Wiseman 26 notes that the concern for
records to be used for astrological prediction is much wider, for we also find that dates of all public events, accessions, deaths, mutinies, famines and plagues, height of the rivers' level, wars, battles, religious ceremonies, royal decrees, prices, and other pertinent facts were recorded by Babylonian scribes.

The need for predictions was a political issue because cosmological patterns were also related to meteorological conditions and therefore to the height of the rivers which was the most important "entry" of Mesopotamian economy. We need now to consider how this highly bureaucratized and record-oriented social context influenced the development of geometry and of the investigation of natural phenomena.

Babylonian arithmetical geometry shows the same "empirical" approach to the analysis of space found in ancient Egypt\(^{27}\), however the notion of "empirical" needs some further qualification.

It is true that we find clay tablets with mathematical texts for the training of scribe where an area is arithmetically added to a distance, but we also find that most of the sample problems are developed around the same initial numerical values, as to point to the fact that the procedure and not the result is the issue. With this peculiar choice of data, they try to show that these
are not "real" problems but "abstract" procedures. To understand this contradictory features we need to put them in context. For instance the apparent "concreteness" and shown by the addition of an area to a length becomes "abstractness" once we consider it within its proper system of classification. For instance J. Goody while studying the relationship between Babylonian and Egyptian commercial records and the related form of thought, has noted that:

Quality tends to suffer a further reduction to quantity when items with very different material properties are equated as contributions or as taxes and then totalled up by means of a set of common units. In Egypt, writes Wolley, "All taxes were paid in kind and stored in the royal magazines; it is illuminating to find that all the goods thus brought in, grain, cattle, wine, linen, are invoiced indiscriminately as "labour"; in other words, they are put on precisely the same basis as the corvee whereby Pharaoh's serfs, the people of Egypt, were called to build a pyramid or to clean out a canal."  

Therefore "area" and "length" did not have an "Euclidean identity" but they where rather two items gathered under the heading "property", and therefore they were "coherently" added together. The scribe did not need to "prove" anything, but only to respect the form of classification and the procedures of his profession.

These considerations suggest that the notion of abstractness may take different forms by picking up
different "exemplars" offered by the specific historical and social context in which the emancipation from mythopoeic thought takes place; in a similar fashion to what E. Durkheim and M. Mauss noticed about primitive classification in language.\textsuperscript{30}

But in the results of Babylonian astronomers we can also find other connections between the social context and the "form" of explanation of astronomical phenomena. One of these can be found in the use of the sexagesimal notation which—being developed for the bureaucratic administration of the Babylonian Empire—reflected its "ideology". It is in fact probable that the Babylonians developed few general arithmetic procedures and then applied them practically to all the problems of social relevance by changing the parameters of the "program". For instance Neugebauer says that lists of coefficients have been identified where we can find:

\ldots coefficients needed for "bricks" of which there existed many types of specific dimensions, then coefficients for "walls", for "asphalt", for a "triangle", for a "segment of a circle", for "copper", "silver", "gold", and other metals, for a "cargo boat", for "barley", etc. Then we find coefficients for the "diagonal", for "inheritance", for "cut reed" etc.

This coefficients correspond to a form of classification which defines what are the socially relevant items to be identified and eventually processed.
In this list geometry is treated as a set of parameters for a standardized arithmetical procedure. To divide a field or to divide an inheritance was "syntactically" the same process (also because fields were generally divided up during the distribution of inheritances).

Two of the components of the "Babylonian paradigm" seem to be delineated: on one hand we have all sorts of records, and on the other hand we find arithmetic as the standard procedure for the management of those records. It is quite probable that arithmetic--together with a powerful positional notation--was a quite more efficient tool than geometry for the solution of the problems faced within that context.

The quantitative and "non-demonstrative" character of Babylonian astronomy is not only related to the need of precise predictions about the positions of heavenly bodies, but it can also be partially seen as the result of a "record-gestalt". In fact J. Goody suggests that the very notion of systematic prediction may have developed out of the use of lists:

The lists in the Ugarit tables.....record tributes and other types of receipt, as well as outgoing payments. Other lists indicate not past income or outgoing payments, but future ones e.g. quotas and rations, what people should pay and should receive; such prospective lists constitute plans or programmes, rather than simple records, though the two are not to be kept altogether distinct.
He then also suggest that the representation of the management of a property in list form:

not only provides a record of a transaction (and hence the possibility of a debt) or of and estate at a particular time, it also institutes a more formalized way of conceiving that transaction or that property.

Once we realize the social importance of astronomical predictions in Babylon, it is not unreasonable to think that the planets' positions were considered as resources to be recorded and arithmetically processed in lists. In a sense, Babylonian astronomers were "astronomical bookeepers" and the ephemerides were part of the national "budget".

Moreover the list-gestalt could fit quite naturally the astronomers' mental framework because the phenomena they were dealing with could not be grasped with few random observations, but only by looking at a long list of systematically arranged astronomical records. The ephemerides are not just the result of the astronomers work, but also his major tool, and the notion of explanation takes the form of a relationship among records in the language currently available, which is arithmetic.

We do not have specific information about the training of the Babylonian astronomers, but from the
analysis of some scribal textbooks we can understand that
the list-gestalt was deeply rooted into its education.
Goody for instance suggests that scribes were trained to
develop any sort of arrangements of items under different
kinds of headings by working on lists and arrays. The
total uselessness of many of those headings suggest that
again the emphasis was on the process and not on the
specific problem.

There is another consideration which points to the
influence of the social context on the form assumed by the
notion of "explanation".

Kuhn says that to Christian thought the Copernican
Revolution meant "to break the continuous chain of created
being."34, similarly, a cosmological theory with
qualitative components in Babylon would have probably
meant a drastic revision of the cosmological myths which
were the grounds on which society was politically
structured. In this context the astronomers' community
shares most of the political paradigm; therefore a
paradigm shift would probably mean a really revolutionary
change. A quantitative approach based only on computation
of former records was instead acceptable precisely because
it was not proposing--at least officially--qualitative
hypotheses that may have contradicted the cosmological
myths.
With this premises, the so-called System B zig-zag function for the prediction of planetary positions does not seem to be a lesser form of "explanation" than the a-priori assumption about the circular motion of the heavens which Ptolemaic astronomy employed primarily to perform computations and compile ephemerides.

Beside the more inquisitive attitude and the more accurate results, we see that the work of Babylonian astronomers is still typified by their social function, in fact, even though that society allowed and actually needed the development of astronomy, it strongly influenced the form of the approach to the problem. Astronomy was still expected to produce role-related results; it still was an "official" procedure and it is not casual that the available ephemerides are all signed by the scribe-compiler/computer, and have been excavated in places which are thought to be official archives.

At this point it may be interesting to develop some comparative analysis between cosmological myths, socio-political contexts, and the notions of proof and causality in Egypt and Babylon.

The different role and results of astronomy in Babylon may be also related to the different content of the cosmological myths and in particular to the fact that it did not contemplate any concept equivalent to the
Egyptian *maat*. The reason why no cosmological order is contemplated there may be referred to the history of the civilizations which populated at different times that area. If the concept of *maat* was developed in Egypt it is also because that country could be actually presented as isolated from the rest of the world and therefore endowed of a perennial order. Egyptian cosmological myths stress the geographical closure of the country limited on one side by the sea and on the other three sides by desert. Non-Egyptian people are non-human and the cosmos is exactly that portion of sky above Egypt. Myths generally work till when they are not inescapably challenged, and the concept of *maat* was made safe also by the scarce and controlled contacts with the outside.

The geographical area around Babylon instead did not offer any natural defense, and invasions of foreign peoples happened quite frequently. The cosmological myth could not talk about perennial order but rather about perennial conflict and uncertainty. The myth needed to be more "elastic" if it did not want to break under the challenge of other cultures.

Evidently the exposure to foreign world views is not a sufficient condition for the development of a more conscious investigation of natural phenomena. In fact the implications of these "messages" need to be read, which
means that there needs to be an enough emancipated language to allow this translation. Nevertheless a continuous interaction with different peoples because of commerce and the absence of centralized political structure, replaced instead by a pattern of small local powers, contributed to the much different world view of the Greeks.

In a context where the political, economical, and geographical situation does not allow for an authoritative isolation, decisions need to be taken out of agreement rather than ontologically entailed by the procedure.
NOTES TO CHAPTER 2


3. Two different instruments are usually denoted by the term "astrolabe": the armillary sphere which was known to Ptolemy (85-165 ca.AD), and the plane astrolabe which was supposed to have been developed much later. The plane astrolabe is designed around the stereographic projection whose development was generally attributed to the Greek astronomer Hipparchus (180-125 BC). O. Neugebauer has shown in "The Early History of the Astrolabe", *Isis*, XI, 1949, pp. 240-56, that Hipparchus should be credited also of the invention of the plane astrolabe.

4. O. Neugebauer, *ibid*.


Frankfort, op.cit., p.87.
16 H.I.Marrou, A History of Education in Antiquity, London, 1956, p.42. The difference in the structure and content of education in classical Greece and the one of the hellenistic period--when the study of science becomes more institution-ized and systematic--is particularly evident, and it has something to do also with the development of a much more bureocratic administration following the passage from the polis to the empire. Scribe culture would have been useless in the polis, but of vital importance in the empire. The relationship between education/training to work in a bureocratic structure shapes also the mental framework of the scholar/scientist as we may detect from the much different attitude in natural philosophy as practiced in Classical Athens and later in Alexandria. We need to keep in mind the impact of an all-comprehensive bureocracy in Egypt and Babyl on on the form of thought of their astronomers.
18 O.A.W.Dilke, ibid., p.19.
20 O.A.W.Dilke, ibid., p.78.
21 Taxes were generally computed as a function of the size of the land and the quality and quantity of the yearly flood of the Nile. Therefore the size of the field was generally measured each year. In the Hellenistic period the complexity of the cadaster grew to the point that not only the land was registered, but the kind of crops cultivated in it as well as how much seed was used--i.e. supplied by the state--for that field. The paesant needed to register even his animals. Taxes were at that point calculated in Alexandria from all the data received from the provinces. The amount of scribes and surveyors required is evidently impressive, which means that their training was systematic and not approximate. For more informations see: M.Rostovzeff, op.cit., or W.W.Tarn, Hellenistic Civilization, New York, 1952, Chapt.5.
27 Geometry was actually a branch of arithmetics. As noted by O. Neugebauer, to divide an inheritance or a field was a formally equivalent problem. M. Clagett, in his Greek Science in Antiquity, New York, 1955, p. 30, notes that Babylonian arithmetic did not make use of the equivalent of our "0" until its latest period. They probably left the space blank when they needed to signify the absence of quantity (something like "1 1" instead of "101"). This suggests that they were thinking of numbers as marks for a set of things and could not abstract the concept of number from the concept of quantity--no quantity, no mark. This tells something about their "concrete" attitude about geometry. In the same way a number is a quantity, a geometrical figure is a set dimensions, i.e. the set of the measures of the edges of the figure. To think of an abstract geometrical figure would probably mean to think of nothing. However, even though the lack of "generalization" in Babylonian arithmetical geometry is a matter of fact, the relationship of this evidence with the mythopeic form of thought needs some further qualification. Otherwise we risk to introduce a context-free notion of abstraction which then generally ends up being our kind of abstraction. Most of the historiographical work about Babylonian science (for instance, S. Toulmin and J. Goodfield, The Fabric of the Heavens, New York, 1961, pp. 23-51.) displays this sort of bias and offer apparently coherent results only by trimming out critical evidences.

28 O. Neugebauer, op. cit., p. 43. A similar case is found in: R. J. Gillings, op. cit., p. 159, where he points to the fact that several Egyptian textbooks propose numerical values which are totally unrealistic. Such a choice of values seems to tend to the same result of the recurrent values noted by Neugebauer, which is, to de-emphasize the particular problem and indicating the "general" procedure. As noted suggested by Neugebauer, the consideration of a mathematical approach as abstract or empirical is a mere problem of translation: Babylonian algebra does not "look" like ours and therefore it becomes "primitive".


31 O. Neugebauer, op. cit., p. 45.

32 J. Goody, op. cit., p. 87.

33 J. Goody, ibid., p. 88.


35 O. Neugebauer, ibid., pp. 92-138.

36 To talk about quantitative forecast instead of scientific explanation is to miss the mutual relationship between the
two. It is like to suggest that it is possible to make predictions through a generic "grinding" of astronomical observations, as if numbers and observations spoke by themselves. But Neugebauer is clear about the fact that we do not have any information about the considerations that allowed for the development of the System B and that anyway Babylonian astronomers were aware of certain causal relationships between the motions of different planets. In fact, if a forecast shows a reasonable agreement with the phenomena, it means that the process which produced it represents a reasonably accurate causal nexus between the items represented by the terms of the computation. Moreover the accuracy of the ephemerides show a pattern which means that Babylonian astronomers were refining their grasp of causal relationship between the observational reports.

37 O. Neugebauer, *ibid.*, pp. 54-70.

38 Foreign trade was generally managed by the Pharaoh and concentrated in few coastal cities. Merchants were not generally allowed into the country. In earlier periods the Pharaoh was sending the army in foreign countries to gather the necessary raw materials which were then distributed through a network of state-owned warehouse. The distribution of seeds and any other kind of items from these warehouse stressed the paternalistic role of the king and confirmed the belief in maat.
Chapter 3 - GEOMETRY AND REMOTE SENSING

1 - observation and the measurement of distance

Greek mythology was no less articulated that the Egyptian or the Babylonian, but its role was not as important as it was in those countries. Even in the period of the Achaean kings the political organization was too fragmented to need legitimation through cosmological myths. We can get a sense of the deep differences between these two kinds of political context by comparing the content of the Enuma Elish$^1$ with Hesiod's Theogony$^2$.

Greek cosmological myths still attributed religious importance on stars and planets, but the early introduction of the four elements (fire, air, water, earth) as original and orderly components of the cosmos allowed the development of a non strictly religious approach to astronomy. Moreover Greek economy was largely based on exchange$^3$ and the many contacts with other
civilizations made them aware of the cultural relativity of myths as it was clearly stated by Xenophanes as:

Mortals consider that the gods are begotten as they are, and have clothes and voices and figures like theirs. The Ethiopians make their gods black and snub-nosed; the Thracians say theirs have blue eyes and red hair. Yes, and if oxen and horses or lions had hands, and could paint with their hands, and produce works of art as men do, horses would paint the gods with shapes like horses, and oxen like oxen....

Much later, Aristotle in the Politics made a similar point:

All men say that the gods have a king because they themselves either are or were formerly under the rule of a king: for they imagine not only the forms of the gods, but also their ways of life to be like their own.

Xenophanes and Aristotle's attitudes were not anti-religious, but they rather acknowledged certain processes of projection of human experience into the domain of religion and myth, and ultimately suggested the existence of a real, non-anthropomorphic god. This criticism of anthropomorphism helped for the transition from person-like cosmological principles to element-like ones, allowing for a wider methodological perspective in the study of natural phenomena.

However, in this new context natural philosophy was not a topic of political relevance and in fact—
the exception of medicine—it held a marginal position until the development of the schools of Plato and Aristotle.

Greek philosophy developed around the dialectic of the changing and the unchanging, the one and the many, in which the apparent chaotic, orderless and everchanging earthly phenomena were contrasted by the stillness and incorruptibility of the heavens. This pattern was also incorporated in Plato's very influential theory of ideas as well as in his cosmology. Aristotle adopted the same basic model but attempted a more fruitful mediation between the two polarities by introducing the notion of causality in his multi-sphere universe hierarchically ordered in terms of degree of movement and corruptibility. As Koyre' puts it:

Heavens and earth are two different things. This is why mathematical astronomy is possible, but mathematical physics aren't.

In this context the all process of vision was forced in an ambiguous situation. Questions like "what is light about?", or "what is the nature of the visual ray?" or "how are we able—in the sublunar sphere—to perceive the heavenly one?" could not be easily answered for they all refered to a theoretically problematical mediation between those two spheres. This explains why Plato
developed a metaphysics of light as we can find it in the Timaeus.

The difficulties became sharper with the development of geometrical optics in which the duality between the abstract geometrical model of vision and its physical aspects needed to be mediated with a space that was far from homogeneous and isotrophic as the Euclidean one.9

Since the Presocratics, observations through optical instruments were employed for a number of different disciplines, but—differently from Egyptian and Babylonian practices—they were often analyzed with geometrical methods. Among Diels' Fragmente der Vorsokratiker 8 we find that

"[Thales (624ca-546ca BC)] described and determined the position of constellation of the Auriga, used by the Phoenicians to steer their ships." and that:

"Eudemus, in his History of Geometry [now lost] credited Thales for this theorem because the method by which he is supposed to have measured the distance of ships at sea requires the use of this theorem."10(fig3).

The fragments suggest that the first application of remote sensing by means of geometrical interpretation of observational reports was developed out of the interest in navigational problem. At this point to observe does not mean just to point to an object, but rather to relate
geometrically that observation with others, so to derive information about size, distance, and position which would not have been directly available.

It is the early recognition of what Kepler would later call the "triangulum distantiae mensorium" (the triangle which measures the distance) which he incorporated in his theory of the retinal image. The content of Thales theorem was later introduced in the 4th axiom of Euclid's Optics as "Things seen under a larger angle appear larger, under a smaller angle appear smaller, and under equal angles appear equal."

Other fragments suggest that Thales himself applied this theorem to a range of problems involving measurement at distance:

"Thales of Miletus succeeded in measuring the height of the pyramids by measuring the shadow in the moment when it has the same length of the body that projects it.", and

"Once you placed a pole at the limit of the shadow projected by the pyramid and, by the fact that the sun rays reach them [the pyramid and the pole] and form two triangles, you [Nilossenus to Thales] demonstrated that the pyramid and the pole bear each other the same proportion as their shadows do."

Proclus claims that there was no proper demonstration for this theorem before Euclid developed one and included it in his Elements, which suggests that—even though the geometrical articulation of visual
measurements was already available—its acceptance was based on common sense rather than on a formal demonstration.

As noted by W.F. Theisen\textsuperscript{12}, Thales method is not only articulated into a theorem in Euclid's \textit{Elements}, but it is practically re-presented in his \textit{Optics} in a series of propositions about remote sensing. In fact, Prop.19 is about "How to determine a given height when the sun is shining", and Prop.21 deals with "How to measure a given length" by optical means.

Other fragments confirm that Thales was also concerned with the study of astronomy as well as of its applications to the solution of practical problems of sailing techniques and for the improvement of the calendar. In fact:

"Following the tradition, Thales was the first among the Greeks to engage in the study of physical problems..... it is reported that he did not leave any written text except the so-called Nautical Astrology.", and that "with few [geometrical?] lines he discovered great things such as the length of the seasons....., the oblique pattern of constellations.... the annual return of the sun.."\textsuperscript{13}.

Thales was credited of the first prediction of an eclipse, but other Presocratics also shared Thales' interest in astronomical studies. We find that his immediate follower and perhaps student Anaximander:
"Discovered the gnomon and placed it at Sparta, in an opportune site for the study of the shadows... to indicate solstices and equinoxes. He also built [water] clocks."; and we also find that "The first [geographers] after Homer, says Herathostenes, were two: Anaximander, friend ans fellow-citizen of Thales, and Eucateus of Miletus; the first made a world-map..", and that "Anaximander was first to find the ratio between size and distance [of planets] as referred by Eudemus."14

In fact it was Anaximander who first proposed a "mechanical" astronomical system constituted of rings of fire surrounded by some kind of haze which made them invisible. Fire flows out only in few spots, which we perceive as stars or planets. He also gave figures for the mutual distance of these 3 rings of fire. Their diameters were respectively 9, 18, and 25 times the earth's diameter. The number "3" shows up too often to make it believable that Anaximander calculated those distances from observations.

This is probably the first instance of the problem of the determination of astronomical distances. Because of the difficulties in finding the third point of reference (which instead was easily found for measurements on the earth as shown by Thales' example) it became one of the crucial problems of technical astronomy. This explains why many authors who dealt with geometrical theories of vision (Ptolemy, Alhazen, Kepler) were also professional
astronomers interested in the "triangulum distantiae mensormium".

It has been suggested that Babylonian astronomers were trimming down observations to fit the predictions suggested by their arithmetical interpolations, and that they tended to compute rather than observe. These attitude is easily found also in the work of the Greek astronomers up to Ptolemy (and even later\textsuperscript{15}). In the same way Anaximander was pythagorically "seeing" distances in terms of multiples of 3, since Plato astronomers were seeing orbits in terms of combinations of circles and evaluating observations accordingly. Recently, after an analysis of Ptolemy's treatment of errors R.R.Newton has drastically claimed that:

All of his observations that Ptolemy uses in the Syntaxis are fraudulent, so far as we can test them. Many of the observations that he attributes to other astronomers are also frauds...Thus Ptolemy is not the greatest astronomer of antiquity, but he is something still more unusual: he is the most successful fraud in the history of science.\textsuperscript{16}

Here we begin to encounter the problematic distinction between "to see" and "to see as", and--as we will consider soon--this applies not only to the "what" but also to the "how" of vision. This because the various theories of vision developed by Greek philosophers were directly connected with their theories of knowledge. They
tried to explain how we get to see what we should see.

The list of fragments considered before suggests the use of observations with instruments for technical astronomy, calendrical studies, time reckoning, astrological predictions, geography, and the development of sailing techniques, however, it does not say much about the social concerns beyond these investigations as well as about their actual articulation.

It is quite probable that the development of sailing techniques was relevant for Greek economy and that this interest may have supported the application of geometry to the determination of the ship's position at sea, however, many other activities which involved optical measurements were of much less importance both to the economy and the political life of the Greek world. For instance geography had mostly a speculative interest to the early Greeks for we do not have traces of the systematic development of official archives with geographical and anagraphical data about territory and population. Such structures which were typical of the eastern empires were totally useless in a fragmented political configuration like the Greek. It is only much later that we find Strabo (63?BC-21?AD) praising the relevance of geography for military purposes, but his audience (and sponsors) were the Romans.
Land survey was not a major activity because the Greek political structure did not need the mapping of manpower, land, and animals as in Egypt or Babylon, and the form of taxation did not require the development of a precise cadaster.

Astrology was not practiced to justify a precise political structure as in Egypt or Babylon, but rather for personal horoscopes or meteorological forecasts for agriculture. The Greeks did not seem to have much concern with the precision of the calendar either. In fact Thucydides\(^\text{17}\) criticized and Aristophanes\(^\text{18}\) made fun of the way the calendar was managed by Athens' authorities. The revival of the interest in astronomy and the important achievements of Eudoxus and Callippus took place in the 4th century probably also in connection with a stronger and generalized interest in astrology.

This limited interest in astronomy is also reflected by the scarce development of optical instruments. Moreover Herodotus claims\(^\text{18}\) that even the few ones available—the gnomom and the sundial—had been imported from the East. With these premises it is even problematic to understand on what basis Eudoxus was able to achieve his results later in the 4th century. In fact astronomy can develop only out of systematic observations which implies the presence of a program of research which
can hardly develop without some kind of institutionalization of science.

Greek astronomy instead did not have either a reliable set of records or a precise program because the first "mechanical" astronomical system early proposed by Anaximander did not allow for a workable articulation. It has been suggested that possibly some astronomical records from the East became available to the Greek astronomers around 350 BC and that Plato's belief in the geometrical structure of the cosmos allowed for the beginning of a program for astronomy. In the Republic he claims that:

...in astronomy, as in geometry, we should employ problems, and let the heavens alone if we would approach the subject in the right way and so make the natural gift of reason to be of any real use. That...is a work infinitely beyond our present astronomers.

This program was taken over by Eudoxus and Callippus who both were Plato's students or associates at the Academy.

To Plato geometry is an innate component of human soul, a reflection of the geometrical structure of the cosmos. On the contrary geometry and in particular deductive thought have a earthly history. For instance, Euclid's deductive method is a direct product of Aristotle's method of reasoning whose roots can be traced
back to the historical development of the methods of correct (or convincing) reasoning within philosophy and politics.

It is now important to follow this historical process and its influences on the development of theories of vision.
NOTES TO CHAPTER 3

1 In: The Babylonian Genesis, ed. A. Heidel, Chicago, 1963
2 Hesiod, Theogony, Indianapolis, 1953.
3 On the contrary, one of the reasons that contributed to maintain the cultural closure in Egypt and Babylon was also the agricultural nature of their economy. This suggests that cultural closure can be obtained only through a parallel economic isolation.
5 Aristotle, Politics, 1252b24ff.
7 If we can observe the stars in the superlunary sphere made up of living and divine aether while being "down here" in the sublunary sphere constituted of the four non-living elements, this means that there is some kind of movement across the different spheres which seems to be problematic for Aristotle's notion of causality.
9 Max Jammer in his Concepts of Space, Cambridge, Mass., 1954, pp. 23-24, says that: "The idea of coordinates in the plane seems to go back to pre-Greek sources... it would therefore be only natural to expect some reference to spatial coordinates in Greek mathematics. But in the whole history of Greek mathematics no such reference is found. Longitude (mekos) and latitude (platos) as spherical coordinates on the celestial sphere or on the earth's surface were obviously used by Eratosthenes, Hipparchus, Marinus of Tyre, and Ptolemy, being the ideal two-dimensional system for concentric spheres in Aristotle's world of spherical symmetry... The use of three-dimensional coordinate system, was not thought as reasonable until the seventeenth century... when the concept of space had undergone a radical change. Undoubtedly, Greek mathematics dealt with three-dimensional objects; Euclid himself... saw perhaps in the construction and investigation of the Platonic bodies the final aim of his Elements. Yet space, as adopted in mechanics or in astronomy, had never been geometrized in Greek science. For how could Euclidean
space, with its homogeneous and infinite lines and planes, possibly fit into the finite and anisotropic Aristotelean universe?".

10 Callimachus,[fr.191 Pfeiffer].; Proclus, in Euclid , 352, 14.

11 Pliny Nat.His. , XXXVI 82.; Plutarch, Conv. VII, sap.2, p.147 A.


13 Symplicius, Phys. 23,29. Other commentators attribute the Nautical Astrology to Focus of Samos that lived approximately a century earlier.; Apuleius, Flor. 18.

14 Diogenes Laertius, II 1-2.; Strabo, I 7.; Simplicius, De Caelo 471,1.


18 Aristophanes, Nu. 615 ff.

19 Herodotus, II 109.


21 Plato, Republic , VII, 530.
Chapter 4 - IMAGERY AND VISUAL EVIDENCE IN EARLY GREEK THOUGHT

1 - Introduction

After this outline of the development of methods of measurement at distance, we need to consider what "to see" meant to the Greeks and how this concept developed along with their philosophy. We will then trace how geometry, after having been accepted as a model for measurement and description of physical entities, was also adopted for the analysis of the process of vision. However, the whole process cannot be understood only as a result of the successful development of geometry. It is also the previous articulation of theories of knowledge that allowed Euclid to focus on the geometrical description of the behavior of light, without putting much effort in the study of cognitive and psychological aspects of vision.

The early Greek philosophers, instead, had to face a wide range of different problems at the same time. Their theories of vision were directly connected with their hypotheses about knowledge and cosmology and,
consequently, they also reflect the historical development of the differentiation between the individual and the environment, between reality and representation.

The understanding of the articulation of the concepts of analogy and proof is fundamental for tracing the development of geometrical optics. In fact--by preceding Archimedes' work on statics by few decades--geometrical optics is the first instance of analogy between a precise geometrical model and a physical process. The adoption of a geometrical analogy shows that the model is no more thought of as a "thing" but as a modular system whose properties are abstract enough to be encoded in a set of rules which define the range of coherent transformations allowed for that model. The development of the notion of proof is also related to this process.

2 - Vision as exposure

The Iliad and Odyssey show that the Greeks in Homer's time--like the Egyptians and the Babylonians before them--went through a stage of mythopoeic thought. However, the emancipation from it took a form quite different from the ones considered earlier, and that form can be regarded as the origin of western thought.
As we can expect, the cognitive relationship between man and environment is not initially thought in terms of an internal reflection on the objects produced by perception, but rather as a dialogue based on affinity. This affinity is not like the fitness between the nature of an external stimulus and its receptor, but it is closer to the notion of sympathy as we find in magic. As noticed by Snell\(^1\), in this early period which corresponds to the epic literature, "reality" is still seen through the myth. Then the myth progressively became a source of similes, and these similes then develop into analogies which constituted the basis for logical thought.

Snell has shown that Homer uses several verbs to describe what in English language is denoted by "to see"\(^2\). Perception is thought of as a "dialogue", and in fact the language of this period contains verbs which denote the attitude, the mood, the intention of this dialogue. Verbs which describe the expression of a look: authoritative, attentive, distracted, scared; or others whose meaning could be paraphrased as: "his glance falls on an object", or "it turns toward something", and "he casts his glance on someone". The verbs which denote a mode are not used in the first person, they are used as nouns to describe a process. Instead others refer to the experience of seeing something, and are used in the first person.
Being vision a dialogue, these verbs have a meaning which often refer to the object seen or rather talked to. The viewer does not have a specific identity, he is a component of the scene. Consequently the notion of vision as representation is unconceivable, and in fact we cannot find in this period a verb which denotes the viewing function of the eye. Eyes are the seat of expression and not of sight.

In post-Homeric literature most of the verbs of mode are dropped and other verbs begin to denote more specifically the act of vision. The verb teorein which develops out of the noun teoros, which meant "to be a spectator", assumes the meaning of "to observe", showing that the presence and the identity of the seeing subject begins to be acknowledged more independently both from the context and from the object of sight.\(^3\)

The notion of perception could not develop in Homer's time also because of the lack of a unified concept of soul or mind. Psyche does not mean soul but rather life: it comes from psuchein (to breathe). Somehow related to psyche, we find noos, the capacity of perceiving and thinking images. Evidently there is no clear linguistic distinction between mental images, perceptual images, and objects\(^4\), and consequently no precise awareness of the process of representation itself.
Similarly, the body is perceived as an agglomerate of limbs but not yet as an organism. Each organ has its own function, but the terms of the interaction are still unclear. The eyes are both the organ of sight and sight itself. Traces of this line of thought can be found later in the Hippocratic and Galenic doctrine of the "faculties".

This parcelization of consciousness is also found in the absence of "internal conflicts" in Homeric heroes. If the hero is undecided, it does not mean that he is experiencing a conflict within his mind, but rather that his psyche wants something but the hands or the feet want something different. What we call "thought" was not considered to be articulated within the psyche but rather provoked by some external agents (gods) who could easily "send" different impulses to different limbs.

The lack of the awareness of thought as an internal process explains why Greek culture and language in homeric period did not have any unifying notion such as the logos. As we may expect, Homer's notion of "knowledge" was a result of a sympathetic dialogue between the "individual" and the environment. From noos -- the faculty of perceiving and producing images -- we find the verb noiein, which means to comprehend, to get into. Cognition is therefore intended to be a process of
immedesimation. Like "comprehends" the like. The
Presocratics applied this notion both to perception—"like
perceives the like"—and to cognition in general. As
Empedocles puts it: "Knowledge pertains to the similar,
ignorance to the dissimilar". ¹

The notion of thought as produced by external
factors implies a strong reduction of the concept of
individual will and responsibility. As Snell puts it:

When the Homeric hero, after duly weighing his
alternatives comes to a final conclusion, he feels
that his course is shaped by the gods.

Hesiod begins to develop the notion of causality
but not the one of decision. His Theogony is actually a
cosmogony in which—in the form of a genealogical tree of
the family of gods—he is trying to describe the order of
nature. His notion of genealogical cause is then
articulated by the Presocratics in terms of affinity and
attraction, even though with a less evident sexual
connotation.

This form of causation gives a picture of the
world as ruled by a patriarchal authority based on power
rather than law. Consequently, the notion of decision and
the correspondent inquiry about the causes of the matter
to be decided is virtually excluded. It is interesting to
note that this form of thought presented by Homer and
Hesiod reflects quite closely the social structure of the period in which it was developed. Such a structure was in fact centered around the *oikos*, which was a kind of extended household of the family (genos) of the local lord. There was no written law and all authority was concentrated in the patriarchal figure of the lord. Only those who belonged to the oikos had some kind of legal status. Even though in a less structured and authoritative way, Homer's myths and Hesiod's *Theogony* play a role analogous to the cosmological myths in ancient Egypt and Babylon. One of the reasons of their lesser authority is not to be found in the content of their text, but rather in the fact that there was no central political authority to endorse and raise them to a law-like status.

To Snell the notion of individuality develops in the period between Homer and the Presocratics, and precisely with the *arcaic lyric* which flourished during the early development of the *polis*.

The lyric introduces the notion of personal point of view. Here the poet is both observer and judge of what he sees. The individual is now a citizen and his being responsible implies the awareness of the concept of decision. In turn, this assumes the capability of making judgements about causes (which need to be "represented" somewhere).
The lyrics do not refer to conflicts between psyche and hands or feet like the ones described by Homer: the conflicts are now internalized. With the development of individual identity came also the unity of the mind, which is the first stage toward the conceivability of an entity like the logos.

Anacreon writes: "Again I love and I love not; I rave, nor do I rave."


Consciousness begins to be less "a thing" and more and equilibrium between opposites. The contents of consciousness (like Sappho's Eros) are still of divine origin, but, at the same time, they are progressively becoming the psychological experience of love. The concept of identity as equilibrium between opposites is then extended from microcosm to macrocosm (and vice versa) as in Empedocles' notion of cosmic equilibrium between love and strife. Similarly, in the Polis, "Zeus" begins to be used as a name for law.

2 - From witness to proof

We need now to consider the role both of analogy and of the social context in the emancipation of individual consciousness from mythopoeic thought.

Once the several psychic and cognitive activities
are attributed to the workings of a unified and independent mind, one can begin to think that also the "outside" is controlled by a similar form of unified rationality, which can also be understood. It is only when people begin to question the divine cause of their decisions and begin to develop a different form of law that they also begin to think of a rationality immanent in nature.

Being cosmology and theory of knowledge closely related, we find that the Presocratics applied the notion of logos (or related concepts) to both domains and connected them through microcosm-macrocosm analogy. G.E.R. Lloyd\textsuperscript{11} has analyzed the imagery used by Presocratics in their cosmological theories, and has arranged it under three categories: social and political images, vitalistic notions, technological images. It is difficult (or probably even irrelevant) to understand the precise "logical" status of these analogies, particularly in this linguistic context in which working analogy and metaphor are not yet separated. It seems that the notion of law as order in the organism of the polis is applied to the order of the organism of the cosmos. Anaximander, in a fragment which probably deals with the law-like form of motion of celestial bodies, says that:

For they pay the penalty and recompense to one
another for their injustice, according to the assessment of time.

It seems that Parmenides thought of the equilibrium between light and dark, day and night as a contract between equals, and that Empedocles described the relationship between love and strife (the two agents which maintain the equilibrium between the four elements) as an oath. Heraclitus says that:

the sun will not overstep his limits; otherwise the Erynes, the servants of Justice, will find him out.

These analogies between the polis and the cosmos evidently work both ways. These cosmological patterns are now developed by citizens who do not think of order just as an authoritative and patriarchal notion like the Egyptian maat. However the concept of a more "natural" and less divine law is difficult to be articulated, and some analogies with natural phenomena would certainly be useful. The structure of the heavens can become (if opportunely read) a model for a rationally justifiable notion of law or logos. Similarly, contracts between natural elements could replace the innate patriarchal and genealogical causation like the one presented by Hesiod. Interestingly enough the Greek term aitia meant both cause and legal responsibility.
The overall process reminds of the justification of earthly states through cosmological myths we have seen in ancient Egypt and Babylon, but we find instead a drastic difference in the referent for the notion of order and law. Once the "patriarchal paradigm" is dropped, another more "artificial" referent for a different concept of order and law needs to be found (or at least assumed accordingly to a radically new form of common sense). What needs to be developed is not a code which articulates an already given form of authority, but a constitution. In fact:

The period from the seventh to the fourth century is one of unprecedented activity throughout the Greek world, in the formulation, discussion, revision and, at times, overthrow of legal and constitutional codes.

This "program of research" is not only limited to the ethical or political context, on the contrary—through the analogy between microcosm and macrocosm, between man, state, and cosmos—it is "naturally" extended to natural philosophy.

These arguments suggest that the identity of the mind, the possibility of representing opposites in a form of thought is not just a linguistic or intellectual achievement. Instead, it is the result of an interaction between Greek arcaic mentality and its social environment.
It is evident that the polis and its related concepts of law and decision based on contrasting arguments (order and necessity) could not develop without the mental and linguistic capability of representing such concepts. However—looking back at the many instances of social conditioning of mentality— it is unprobable that the development of individual consciousness and of the related notion of law/logos is simply the result of the "natural" growth of the seed of Greek language—as Snell (almost mystically) seems to think. Consciousness is both a tool and a result. Otherwise, we probably need to agree with Popper's germane belief that the development of western rationality and science—"and there seems to be no other"—did start "with bold theories about the world"20.

The shift from the immedesimation of a faculty with a thing (like eyes with sight), to the dialectical mediation of two opposites within the same entity, introduces the concept of things as having (containing) different qualities rather than the constitutive faculty only. As Snell puts it:

The new thing was that properties, instead than being parcelled out among various subjects, were now concentrated upon one unique figure.21

This attitude is evidently reflected in Empedocles'
elements which—each with different qualities—are distributed in different bodies in different proportions, determining the specific characteristics of that body.

At this point the concept of analogy is no longer between two objects in their fixed and irriducible qualities, but rather between certain specific characteristics of theirs. However, this does not mean that abstract geometry could be conceivable yet. In fact, it is possible to compare the "straightness" of two bodies without dealing with their constitution, and it is also possible to say that there is "something straight" about a light ray comparing it to a stick, but there are no linguistic instruments already available to represent an abstraction such as "geometrical line".

The possibility of abstractness is closely related not only to linguistic instruments but also to the possibility of producing a very specific exemplar. Actually, the Presocratics did not "produce" these exemplars, but rather found them in other practices (political, technological, etc.) and "translated" them into a different theory and practice. Moreover, the use of technological analogies (and artificial examples in general) offered a more specific, reproducible and sharable term of reference. In other terms, abstraction is construction.
However, the passage from mythical similes—which contained aspects of arcaic Greek society and consciousness—to more specific analogies with technological or juridico-political content, is not a symptom of an abstract notion of intellectual development, but rather of a profound change in the socio-intellectual landscape from where these "pictures" were taken.

Some fragments of Empedocles allow to trace some of the aspects of this intellectual and linguistic development within the study of the process of vision. The comparative analysis of his work with the one of the other Presocratics suggests that this transition was ununiform both in pace and direction. For instance Empedocles adopts the form of the mythical homeric simile, but replaces its content with a technological one. In a fragment he presents the process of vision like:

As when a man who intends to go out on a stormy night makes ready a lantern, a flame of blazing fire, fitting to it panes to screen it from every wind, and these scatter the breaths of the winds that blow, while the light leaping out [throught the thin walls of polished horn or a thin tissue of linen]--as much of it as it is finer--shines across the treshold with unfailing beams: so then [when the eye was created] did the primal fire, enclosed in membranes, trap the round pupil in delicate tissues, which are pierced through with marvellous passages and which keep back the deep surrounding water while they let through the fire--as much of it as is finer.

Beside the Homeric style of the introduction, we
understand that the subject matter is not the man who is holding the lantern in the storm, but rather the analogy between two technical processes which are "narrated" to introduce his "sieve" theory of vision.

The absence of anthropomorfic characters in these analogies is the symptom of a precise linguistic concern. Only when he gets to describe philia --the force of attraction between elements that is also responsible of vision--he cannot find an abstract concept already available in Greek language, nor a mechanical example which could convey his concept, and therefore he forcibly goes back to an anthropomorphomorphic image. He says that she is.

acknowledged as being inborn in the limbs of mortals, and by her they have a gentle disposition and achieve works of peace, calling her by the names of Joy and Aphrodite.

But he is tries this statement be understood as pointing to an abstraction, in fact he continues by saying that:

contemplate her with your mind, and do not sit gazing with your eyes.

The two similes of the lantern and of Aphrodite as philia show that--beside the evident linguistic struggle of conveying abstract notions--the notion of "truth" is no
more thought of in terms of evidence of an image, but rather as a result of an inquiry around an analogy.

Empedocles' notion of explanation is basically a reduction of a process to the "natural" behavior of the elements involved in that process. In the case of vision we find fire and water which are moved by philia through the eye's pores.

The technical nature of the model is particularly important because it directs the reduction of the process to the behavior of its elements along familiar lines. More importantly the fact that the model is man-made rather than natural implies that it already entails a language. Looking ahead at Kepler's theory of the retinal image modeled around the behavior of the camera obscura, we may detect a pattern of interaction between the development of a theory and its exemplar or instrument which is both the keeper (or the "image") of former results and the mediator of new ones.

Evidently this process of reduction is not coincident with the picture of the model. It is suggested by it, but it needs to be "read" (rather than "seen") on the model accordingly to a certain language.

The distinction between the representation of a process and its reading became progressively clear and—one century later—Plato pointed explicitly at the
difference between image (i.e. verbal narrative with the use of visual models) and demostrative account of a process. The same critical attitude about visual models or reports is shown few years later by Aristotle who both proposed a method for witness' criticism and attacked directly Empedocles' imagery on the grounds that it was either unclear or arbitrary.

We can refer this new attitude also to the development of rethoric and dialectic as instruments of the political life of the polis.

Approximately at the time Plato drew the distinction between image and demonstration we find that Athenian jurisprudence was changing its notion of evidence from personal witness (as it was in Homer) to written documents.

To Homer, if somebody was "exposed" it meant that he was also "impressed" and therefore that he kept a "picture" of the event. The practice of torture was based on this conviction. The introduction of written documents as legal evidence marked also the abolition of torture for juridical purposes.

Few decades later, Euclid proposed geometry as a precisely defined model which could be composed to fit a wide range of objects or processes which could be described in their static aspects. With Aristotle we also
find enough awareness of both the internal nature of thought and of perception as a mediative process (and not just a "faculty") between soul and environment. This made possible to apply the geometrical method of demonstrative inquiry to the process of vision itself. This reduction of vision to geometry was proposed by Euclid's Optics.
NOTES TO CHAPTER 4

2 B. Snell, ibid., p.1.
3 This stage of linguistic development sees the wider use of adverbs. We do not find the verb of mode anymore, but rather the standard verb which denotes the act of seeing is joined by an adverb which introduces the attitude or the psychological status of the viewer.
4 This can also be noted by the importance and also the reality generally attributed to dreams by the ancients and by contemporary primitive tribes. Behind this attitude we can detect again the scarce development of the awareness of individual consciousness, to the point that it is impossible to draw a precise line between not only between the object "out there" and its perceptual, but also between the experience of reality and the experience of a dream.
5 Teophrastus, 10,[B107].
8 B. Snell, Op.cit., p.237, says that in the earlier Homeric period there was no specific linguistic form to express a causal relationship but "At first the logical element is merely understood from the context: as a second step, certain words which had at first had a different function came to represent the latent logic; and finally this logic, now overtly expressed, becomes an object of reflexion."
9 B. Snell, ibid., p.60.
10 The presence of Law as subject matter of the Greek tragedy shows the public concern with the investigation of this new concept which before the polis was not articulated, but rather comprehended by the arbitrary authority of the patriarch. Particularly evident is the contrast between the old divine law and the research of a more rational, "natural" one.
16 It is worth noting the difference for instance between the violent, absolute power of Hesiod's Zeus and Plato's Demiurge. The Demiurge in fact organizes the world taking as example the eternal, orderly ideas. It is also well-known that Plato thought of the cosmos to be the model of a rationally ordered state.
17 If the Law of the polis was in a sense "artificial" this is because "natural" meant "mythical". Snell op.cit., p.111-112, finds this strong concern for the criticism of the paradoxical aspect of divine law in the Tragedy, and Feyerabend draws a comparison between Zeno's and Sophocles paradoxes. Evidence is taking the shape of the result of an inquiry and not just "what we see".
19 The macrocosm-microcosm analogy was very early developed within medicine as well as the notion of equilibrium between opposites. Because of lack of space, the range of vitalist notions which were included by the Presocratics in their cosmologies and natural philosophies, will be scarcely considered. For further information see G.E.R. Lloyd, op.cit., "Vitalist Notions: the Cosmos as a Living Organism", pp.232-272.
22 Snell shows that linguistic items are not invented ex-novo but they are rather developed by changing meaning and use of already existent terms or syntactical connectors. Probably it is this consideration what takes him to see the development of consciousness as a matter internal to language.
23 Fr.84.I-II, quoted in G.E.R. Lloyd, op.cit., p.325. The common source of this fragment is Aristotle, De Sensu, 437b26ff, but the various translations emphasize different aspects of the simile. Here are some of them: B. Snell, op.cit., p.214; Aristotle, De Sensu and De Memoria, trans. G.R.T. Ross, Cambridge, 1906, p.49; J.I. Beare, Greek Theories of Elementary Cognition, Oxford, 1906, pp.15-16. Both Lloyd and Beare give a brief discussion of certain problems entailed by the translation.
26 Aristotle, Rhetoric, III 18 1418b 39ff.
27 Aristotle, Sens. 437b 9ff; Ga. 747a 34ff; Top. 127a 17ff.
Chapter 5 - GREEK THEORIES OF VISION

1 - Alcmaeon

The first account of the process of vision is of the physician Alcmaeon of Croton (500? BC). Hippocrates reports he maintained that:

Many are these membranes around the front part of the viewing eye and they all are transparent. By means of this transparency, light and everything luminous is reflected, and through this reflection we get to see.

By introducing a sympathetic similarity, Alcmaeon tries to explain the occurrence of a visual sensation as an effect of the formation of the image of an object on the corneal lens. The process of vision is presented as an unmediated "exposure", and the eye is compared to a mirror whose surface needs to be humid to allow for the formation of the image. In agreement with the doctrines of the Coos' school of physicians, he considers the brain to be the site of the soul and, by identifying the liquid around the meninges with the liquid found around the eye, he connects the eye and the soul through the optical nerves which he takes to be hollow. The meninges' liquid is the sympathetic carrier both of the faculty of sight from the soul to the
eye, and of the visual sensation back to the soul.

His training as a physician shown by his knowledge of the anatomy of the eye and the brain\textsuperscript{2} is probably one of the reasons which led him to approach the process of vision from the "inside", focusing on the connection between the soul and the eye rather than on the nature of light, on the characteristics of the transparent medium, or on the process of reflection of the object on the corneal lens\textsuperscript{3}. The presence of a "medical paradigm" behind his direction of research becomes more evident once we consider that also Galen (seven centuries later) adopted a similar form of explanation of the by replacing Alcmeon's meningeal liquid with the Stoics' pneuma.

Alcmaeon's emphasis on the role of the brain as the starting point for his explanation of vision can be better understood by placing it in the context of his theory of knowledge. For instance he says that:

\begin{quote}
Concerning things unseen the gods have certainty, whereas to us as men conjecture [only is possible].
\end{quote}

which suggests that even though vision is still directly associated with knowledge—like in Homer's notion of witness—there is a less skeptical attitude about the possibility of getting to know what cannot be seen. Alcmeon's attitude offers an early evidence of the
transition from the notion of evidence as image to evidence as inquiry.

He believes that human cognition is not just a sympathetic exposure to the environment, which he instead attributes to the animals. He thinks that the soul can keep memories which—if left quiet—tend to solidify becoming knowledge. This attitude derives also from Alcmaeon's training as a physician, for he is used to deal with internal diseases (which are processes he cannot see) by drawing inductive inferences from experience. 5

Consequently he does not focus on the process of image formation because to him the brain and the inferences he could draw from the memories there "solidified" are a more important cognitive tool: they allow him even to conjecture about the "unseen".

However, Alcmeon does not develop a more articulated notion of "accordance" between the sensitive organ and the outside world. He brings the sensitive fluid down from the soul to the eye, but then he relies on a very literal form of sympathetic analogy which is the mirror-like image. He probably assumes that the sensitive liquid which "produces" the image can "feel" it too. It is only with Empedocles (490-430 BC) that we find the first attempt to explain the behavior of the sensitive organs as a process of mediation rather than reflection.
2 - Empedocles

In trying to mediate Parmenides and Heraclitus' opposite positions about the changing and/or unchanging nature of the physical world, Empedocles (490-430 BC) starts a program which will be later developed by the Atomists. He is concerned to find (or to define) some kind of unchanging entities whose combination would account for the changing aspects of nature. He finds them in the four elements (fire, air, water, earth) which he assumes to be pulled together by love and separated by strife.

He still explains the process of vision by linking its parts by means of sympathetic affinities, but—as we have seen before—these are articulated around the model of the lantern. Cognition and perception—which at this point are still undifferentiated—are both referred to sympathy. In fact: "Knowledge pertains to the similar, ignorance to the dissimilar"⁶, and "Like perceives like"⁷. To articulate this sympathetic link he first takes blood (and not the brain) to be the seat of the soul, and then assumes that blood contained all the same four elements which constitute the world.
With this sympathetic "exchange" of elements and with the related notion of balance Empedocles is able to develop a quite more complex explanation of perception.

He maintains that we do not hear simply because the external sound makes the internal ear to vibrate, but rather because our ear is already vibrating autonomously. Perception ceases to be an "exposure" and becomes a sympathetic accordance.  

Similarly he explains some of the aspects of vision in terms of an accordance between the "visual power" (still a faculty) of the eye and the external light. The connection is again a sympathetic one: light is similar to fire, which is akin to the visual fire contained in the eyes (in fact we see sparks if we hit them). The affinity is then reinforced by a further correspondence between the water (as an element) in the world and water (the corpus vitreum) inside the eye.

Like the lantern, the corneal lens has a series of very small pores through which the visual fire goes out and reaches the objects creating vision, and, as the wind which could not get into the lantern and blow off the fire, water cannot leak out being too thick for those too small pores.

However, the description of Empedocle's theory of vision is still uncertain. The quantity of fragments is
limited and the later reports often reflect the personal attitude of the commentators (Plato, Aristotle, Theophrastus) rather than the original content of the theory. As noted by J.I. Beare, it is not clear whether the internal visual fire goes out and reaches the objects or it is the emanation of external objects that gets into the eye through the pores. Other authors mediated the two positions too-literally by suggesting that the outgoing visual fire and the incoming luminous emanations meet somewhere between the eye and the object.

Empedocles' belief that every object had pores, and that it emitted some kind of emanation or influence, suggests that perception takes place when the pores of the receiving subject fit the emanations of the object. If so, the role of the eye's internal fire would be to preset the pores of the corneal lens and to establish a sympathetic connection with the external fire, in a similar fashion to the assonance between an internal and external vibration which he takes to be the cause of hearing. This explanation credits Empedocles of having introduced the concept of sense specificity.

Different organs have different pores and they "perceive" different emanations, i.e. aspects, of the same object, which means that Empedocles begins to be aware that objects are not just a name or an image, but a point
of aggregation of different qualities.

This reading of Empedocles' theory of vision is also shared by Theophrastus who interprets his position as:

The pores of the eye are arranged alternatively as those of fire and water. By passage through the fiery pores we perceive the white objects, whereas through the watery we perceive the black objects. Each sense perception has to fit its end organ.

Even though the remark that:"each sense perception has to fit its end organ" sounds too-teleological not to be the result of Teophrastus' own interpretation, the report is considered to be basically accurate. Empedocles seems to take color as the object of perception, and that the two different elements of the eye and the related pores can operate some selection among them. It is interesting to note that Empedocles considers only those features of the objects that he can reduce to some relationship with the two elements (fire and water) which he takes to be the "carriers" of vision. Other aspects of vision such as the evaluation of the distance of an object or the recognition of its shape are not "seen". The analysis of these problems was to be developed only later by Democritus, who tried to explain different perceptions by means of different geometrical configurations of the atoms reaching the eye.
The reading of Empedocles' theory of vision in terms of a process of filtering of luminous emanations coming from the external objects, contradicts quite evidently the simile of the lantern where the visual fire is described as going out of eye/lantern to illuminate the street with "unfailing beams". Perhaps Empedocles thinks that the visual fire goes out, reaches the object and is then reflected back on the pupil\textsuperscript{15} which is humid because of the internal water. The external fire-light may have sympathetically joined the visual fire and "helped" it to reach far away objects.

A possible mediation of the two apparently contrasting readings\textsuperscript{16} can be developed by considering once again the notion of balance.

Theophrastus says that Empedocles thought that:

\begin{quote}
The construction of the eyes, which are composed of these elements, is not always equal. In some eyes the fiery factor is more centrally, in others more peripherally located. Therefore some animals see better during the day, others during the night.\textsuperscript{17}
\end{quote}

The balance between internal fire and water determines the quality of vision, and Theophrastus confirms this by saying that:

\begin{quote}
Even the animals which have fire in excess are dim sighted during the day since fire within the eye has been further increased by the daylight; this covers and occupies the passages of water. The same thing happens by night to those animals
\end{quote}
which have water in excess, because fire is now overtaken by water.

This shows—if we can trust Theophrastus—that Empedocles thought of visual fire and sunlight as one fire. When sunlight is strong it gets into the eye—after being reflected on the objects—by passing through its pores, but at night—as in the case of the lantern—when sunlight is absent there is less "pressure" and the visual fire goes out trying—unsuccessfully—to reach the objects. This reading may reconcile the two apparently opposite accounts of vision presented by Empedocles. However, we need to consider that some of his accounts that we take to be paradoxical may be a residual result of the "multiple approach" typical of mythopoeic thought.

This may be one of the reasons why Empedocles does not seem to draw a clear distinction between sunlight and other kinds of emanations from objects. The presence of some mythopoeic attitudes in Empedocles' thought is also suggested by his extension of perceptual features to the whole body and in his tendency to read physical causality as the effects of a living will. As Alexander of Aphrodisia puts it:

Hempedocles thinks that iron is attracted toward the magnet due to the effluvia that are radiated by both of them also due to the fact that the poruses of the magnet are symmetrical to the effluvia radiated by the iron.
Beside the striking similarity between the treatment of the behavior of the magnet and of the process of vision, it is also evident that Empedocles was not sensitive to the existence of different kind of emanations. He maintains that it is the eye's water that allows us to see dark objects and that the eye's fire is responsible of the perception of light-coloured objects, but, then, he explains the effects other kinds of emanations which do not convey either "darkness" or "clearness" with the same model he utilizes for vision.

This contradiction suggests the opposite to be true. He does not explain the effect of emanations as a particular case of his theory of vision, but, vice-versa, he explains vision by "personalizing" the treatment of emanations with the addition for color sensitivity. This seems to be supported by a fragment in which Aetius maintains that Empedocles said that:

> Often women fall in love with statues or images and give birth to babies who look like those images.²⁰

Image-related effects can be propagated by means other than light. Light, love, and magnetism are all emanations²¹ which can be (problematically) selected by opportune affinities and pores of adequate sizes.
Between the individual and the environment there is now a filter (literally a sieve\textsuperscript{22}), but what is mediated of the external emanation is its entrance and not its effect (as shown by the incongruencies between different kinds of emanations, sizes of pores, and related effects). The individual is no more just comprehended in the environment as it was in Homer, but his consciousness is still conceived as a filter and not as an active interface between him and the environment.

3 - Democritus

Several aspects of Empedocles' philosophy and theory of vision are then developed by Leucippus(440? BC) and Democritus(460-360? BC) who became the founders of philosophical school of the Atomists. Probably also influenced by Anaxagoras'(500-432?BC) notion of the divisibility of the being, they approached the problem of change by dividing Empedocles' elements down to a furtherly irreducible size. The One was replaced by an infinite number of ones which are the atoms, and the changing aspects of reality are explained in terms of different and changing configurations of unchanging atoms.
The atoms do not have any quality except size and shape. All the other qualities do not exist per se but they are subjectively produced by the interaction of the senses with the various configurations of atoms.

Differently from Alcmaeon who was a physician primarily concerned with the cognitive functions of the sense organs, Democritus seems to focus on the problem of the articulation of the being without compromising its integrity. After having achieved this within his atomistic theory, he moves to explain other processes—such as psychological ones—in terms of atoms’ behavior. Unfortunately his works related to the theory of knowledge like About the Mind, Perception, Colors, About the Different Arrangements and the Concept of Form are now lost.

In his work on vision, he improves significantly on Empedocles’ description of how the qualities of the perceived object are modulated on the luminous emanations, but he does not go beyond his understanding of the process by which emanations are transformed into sensations.

Atoms move in all directions not out of will, but rather driven by the collisions against each other. This had been a crucial concept for Democritus and his followers when they tried to account for the origin of the cosmos moving from the assumption of the total lack of
will in the atoms. He faces this problem again when he tries to explain the behavior of living matter and the process of human cognition. In fact he makes an exception and introduces a special kind of atoms—the soul-atoms—which have some degree of voluntary behavior. Following the teachings of Coos' medical school, he takes the brain to be the seat of the soul, but he also allows for a certain number of soul-atoms to be dispersed in the whole body. Even though the soul-atoms are discrete elements, they have—within the process of perception—the equivalent role of Empedocles' blood. They "intercept" the emanations which pass through the pores. Plutarch reports that Democritus:

> explained dreams as the result of the influences of *eidola* [images] which penetrate through pores deeply into the body and then come up [to the brain] and produce vision during the sleep.

This suggests that, like Empedocles, Democritus takes perception to happen both through the body and the sense organs. The important difference between the two is that Democritus' emanations are quite more structured. The *eidola* are precisely these structured emanations by which he explains both visual perception and the imagery of dreams. They are configurations of atoms which "maintain" the shape of the object which casts them. Accordingly to
alexander of aphrodisia (225? ad) leucippus and democritus...

attributed sight to certain images of the same size of the object, [eidola] which were continually streaming off the objects of sight and impinging on the eye.24

other commentators--such as theophrastus--say that the eidola is a "front" of air pushed forward by the outgoing stream of atoms. he calls this pattern of compressed air deikela 25. the concept of deikela has attracted the attention of contemporary authors like van hoorn26 who has read in it the first instance of the use of an intermediary medium instead of a material emanation from the object. however, the difference is illusory for the deikela need to be sustained, which means that there needs to be a continuous flow of atoms from the object to maintain it. the deikela seems to be a side-effect of the eidola which--being a stream of material atoms--pushes forward the air it encounters between the object and the eye.

the eidola, being a pattern of emanation which maintains the coherent shape of the emanating object, offers a first solution to the problem of how vision could bring more precise information about the object of sight, beside its darkness, brightness, or constitutive elements. empedocles' problematic negotiation between visual fire
and emanations from the object is here— at least nominally— solved. Democritus' theory shows also a more evident awareness of individual consciousness. The introduction of soul-atoms as receptors shows that perception is loosing its connotation of dialogue with the environment.

However, there are problems. For instance, the fact that soul-atoms have an ontological status and are virtually unknowable, implies that we cannot get to know how sensation is produced in their interaction with the eidola. Moreover, the introduction of the eidola is a quite elegant solution of the problem of the interaction between light and on other kind of emanations, but Democritus does not—and probably cannot—suggest how this mediation may take place. In the available fragments we read of eidola streaming from objects but there is no clear indication of the role of sunlight in the process, or how the materiality of light interacts, joins, or reacts with the other material emanations and, finally, ends up producing a coherent perception.

It seems that this series of problems encountered and left unsolved by Democritus points to the limits of his paradigm which— when dealing with perception— can save coherence only by dropping content and heuristic value.

For instance, Democritus takes the eidola to touch
the pupil and that the image there created—i.e. the image reflected on the humid surface of the eye—is then perceived by the soul-atoms, which is exactly Alcmaeon's early position developed one hundred years earlier and clearly reflecting a mythopoetic attitude.

The solution was dated in Democritus' own time for Aristotle (even though almost a contemporary of his) attacks him also on this point:

Democritus... is right in his opinion that the eye is of water; not however when he goes on explaining seeing as mere mirroring. The mirroring that takes place in an eye is due to the fact that the eye is smooth, and it really has its seat not in the eye which is seen, but in that which sees. For the case is merely one of reflection.... It is strange too, that it never occurred to him to ask why, if his theory be true, the eye alone sees, while none of the other things in which are reflected do so.

He develops his criticism of the corneal image in other passages, by saying that sight is neither a kind of "contact", nor the image on the eye, because, if we bring the object in touch with the eye we do not see anything.

Other criticism pointed at the fact that if every objects emanate their own material eidola in every direction, how can it happen that they do not collide with each other, but instead we perceive all the objects distinctly? Others questioned the possibility that an eidola, which is supposed to be of the same size of the
body, could shrink to the point of being contained over the pupil?

Even though some of this criticism develops from an awareness of the geometrical aspects of vision which became available only later, what is still surprising is that Democritus—who is credited by Archimedes of some significant work on the conics—did not develop such an awareness himself.

The point becomes more obscure once we consider that Democritus did not have only a good understanding of geometry, but he actually used optics in his astronomical and astrological work. In fact Vitruvius and Geminus wrote about his work in calendrical studies and astrology and Ptolemy used some of Democritus' astrological work.

We may understand the limits of Democritus paradigm and propose a solution of these apparent incongruities by considering the implications of another passage by Aristotle:

Democritus is not correct in his view that, if the space between the object and eye were pure void, an ant could be seen clearly in the sky.

This suggests that Democritus—even though well equipped in geometry—cannot not frame the process of vision in geometrical terms because of his atomistic
paradigm. To him, the behavior of light and the process of formation of a coherent perceptual image needs to be faced in terms of atoms' movement and not as a geometrical transformation.

The example of the ant shows that he does not think in terms of angle of vision, which is by considering how the angle under which and object is seen tends to diminish accordingly to the object's distance from the viewer. Instead he faces the problem of the reduction of the resolution of sight with the increase of distance in terms of motion of atoms. We do not see distant objects clearly because of the atoms of air between us and the ant which are colliding with the eidola/deikela of the ant making it to blurr. He does not talk about distance and sizes but about collisions.

If an atom cannot be destroyed, it means that it can be seen from everywhere. The only reason it may not be seen is that it has been deviated from its path toward the eye. The adoption of the attitude later developed by Euclid would have led him to consider that the "visibility" of an atom tends to zero if the distance becomes very large. But this is exactly the kind of problem he was trying to avoid by postulating that matter is not infinitely divisible. 

To Democritus, the approach to vision along a
geometrical model would have been suicidal. He had to save the atom.

This shows that at that time the materialistic and the geometrical approach to vision collided incommensurably on the nature of light. We will this problem in several other occasions within the work of those authors who adopted Euclid's framework but continued to question the relationship between the material nature and the geometrical behavior of light. What Democritus avoided was an early instance of the problem of correspondence rules.

However, the problems of Democritus' theory of vision do not emerge in every context. In fact under the term "vision" we find different ranges of experience for different purposes, and ultimately different kinds of professional common-sense. For instance, Democritus' remark that if space was pure void we were to see an ant in the sky, could have made perfect sense to an astronomer accustomed to observe stars whose relative size he could be approximated to a point. To observe a point trying to track its movement is not like to study the qualities of an object trying to understand how we get to perceive those qualities. Again, to an astronomer who is practically aware of the troubles produced by the refraction of light through the atmosphere, the example of
the ant was probably not useful but it surely would have
not contradicted his common-sense, while a painter working
at realistic theatrical scenarios would have probably
thought of it as laughable. This suggests that the choice
of a theory of vision is not only a matter of how
precisely it accounts for the qualities of the physical
world which are looked for by that community, but its
choice entails some assumption regarding what the concept
of quality is about.

The application of a geometrical model to vision
probably developed out of a notion of quality which is
somehow related to spatial coherence, and it is not casual
that was Euclid to introduced it. The progran which moves
from this assumption looks for a process which allows for
the transmission of a coherent image, which is for the
perception of an object and its own size and with the
proportions between its parts coherently reproduced.

Democritus holds a different notion of quality,
and to him coherence is refered to the structure in which
atoms are organized. There are few fragments which cast
some light on what Democritus may have had in mind.

Sextus Empiricus says that Democritus considered
the principle of like-attracts-like to be active both in
the domain of living beings and among the inanimate "..as
we can see with seeds in a sieve and pebbles on the
sea-shore\textsuperscript{35}. To Democritus it is "natural" for pebbles to be sorted by the waves on the beach in layers each of a certain size, and it is also "natural" to seeds to be sorted by a sieve in sets of similar size. The interesting point is that this "naturality" is not considered as a passive disposition. The sieve and the waves are not presented as sorters. On the contrary, he suggests that the sieve and the waves are just amplifying the tendency to coherent organization inherent to pebbles, seeds, and--evidently--to atoms.

It is a matter of "system of coordinates". To him, the primary "cause" of the sorting process is the atoms' own structure, and not the size of the sieve's holes. We can now understand why he approaches the coherence of visual perception not as a result of a "geometrical sieve" but rather of the coherent cohesion of the atoms constituting the eidola.

Even though Democritus claims that normal atoms have a totally mechanistic behavior, when he gets to explain perception, he attributes them several sympathetic qualities which--paradoxically--make them "alive". Probably this paradox is a result of his atomism which--not being rich enough to articulate explanations of complex phenomena--needs to re-introduce previously dismissed sympathetic affinities to fill up the gaps. In a
sense, the difference between atoms and soul-atoms seems to be less sharp than what he probably wanted to, and perceptual experience is presented more as the result of a coherent (almost voluntary) structure of the eidola rather than of the active and "thoughtful" behavior of soul atoms.

There is a pattern of continuity among Empedocles' simile of the eye as a lantern, or between his notion of perception as the filtering effect of the eye's pores, and Democritus' concept of sieve. The common attitude is to compare perception to a static tool and not to an adaptive process. This is obtained by emphasizing the role of the organization of the "outside" rather than the process of selection controlled by the "inside". If we take a step ahead in time, we find Alhazen who thinks of perception as a point-to-point relationship between the object of sight and its internal image, but his notion of image-formation (even though a much more sophisticated one) is still sieve-like one. The refraction of the light-rays through the several lenses of the eye is the crucial factor for the formation of the perceptual image, instead he takes it only as a mean to weaken out the "spurious" ones. The filter is still a passive one.

It is only with Kepler that refraction is thought to make all the possibly useful rays to converge into a
coherent retinal image. Perception is no more a fixed and sympathetic filter but an adaptive, active and self-controlled process. It ceases to be a magic tool and becomes a machinery, following the changing concept of individual consciousness.

In the Hellenistic period, the influence of Democritean optics was limited to the Epicureans only. The interest in atomistic philosophy developed again only in the Renaissance and gained interest through the crucial developments in astronomical theory and the crisis of Aristotelean physics. The new concerns for the concept of force and the rejection of Aristotelean space made Atomism and its "corpuscolar" theory of light to be reconsidered. In fact Descartes in his Dioptrique tried to integrate Euclid's "geometry" of light with Democritus' atoms.

4 - Plato

Plato refuses the Atomists' articulation of the being in terms of a parcelization of matter, and tries instead to work around Parmenides' affirmation of the unity of being by substituting it to a multiplicity of ideas. The notion of being as ideas and not as matter introduces the well-known dicotomy between real knowledge
and sensorial experience.

Plato refuses Democritus' philosophy as a whole and consequently looks back to Empedocles for a model for theory of vision. Actually, he does not improve significantly on Empedocles, and his relevant contribution to the understanding of visual perception is rather to be found in the way he connects it to a more complex theory of knowledge. Nevertheless—despite of his scarce interest for the cognitive content of sense reports—Plato dealt with vision as a tool to link the everchanging a corruptible earthly world to the realm of eternal ideas as represented in the regularity of the cosmos.

The extent of Plato's borrowings from Empedocles is particularly evident in the account of the process of vision he gives in the *Timaeus*:

They arranged that all fire which had not the property of burning, but gave out a gentle light, should form the body of each day's light. The pure fire within us that is akin to this they caused to flow through the eyes, making the all eye-ball, and particularly its central part, smooth and close-textured so that it would keep in anything of a coarser nature, and filter through only this pure fire. So when there is daylight round the visual stream, it falls on its like and coalesces with it, forming a single uniform body in the line of sight, along which the stream from within strikes the external object. Because the stream and daylight are similar, the whole so formed is homogeneous, and the motions caused by the stream coming into contact with an object or an object coning into contact with the stream penetrate right through the body and produce in the soul the sensation we call sight.
The crucial issue in the interpretation of this passage is the meaning of "stream" and "contact", which is actually quite more complex than what the modern translation seems to suggest. For this purpose we need to consider Plato's theory of vision in the context of his theory of knowledge.

One of the reasons that probably made Plato to assume the extromissionist position is that by introducing the visual fire as an agent of cognition, he implicitly presents perception as an active process. The relevance of this becomes more clear once we understand that Plato considers knowledge to be largely a-priori. Therefore the only (weak) possibility to bridge the gap between the chaotic earthly world and the domain of the eternal ideas is by assuming perception to be somehow informed by the same ideas it is trying to "see" in the objects. Evidently this can done more easily if vision was an active process which "goes out and find things" accordingly to an a-priori program. It is not surprising that we find confused reports of Plato's theory of vision in terms of "tentacles reaching out of the eyes".

Sensation has a controversial status: it is a faculty of the soul, but it takes place through a bodily (and therefore fallacious) organ, thus reflecting the
duality between ideas and corruptible world typical of Plato's theory of knowledge.

Imagination (phantasia) is the faculty which creates images (as forms) taking as a model the innate ideas preserved in the soul. This a-priori images are then "matched" with the environment through the sense organ, generally with approximate and/or erroneous results.

This implies that what can be seen is only what can be recognized as fitting one of the ideas-forms contained in the soul. The ideas in the soul are the link of intelligibility between the individual and the environment being the copies of the ideas on which the Demiurge shaped the world. Their role within Plato's theory of vision is comparable to the one of the sympathetic affinity between the visual fire and the external light in Empedocles'. In this sense we may say that Empedocles' pores which "recognize" one element from another are functionally similar\textsuperscript{38} to the Platonic ideas-related forms which--through the sense organ--"recognize" the objects which reflected that form. These considerations introduce also the crucial but subtle difference between image and form in Plato.

The comparison becomes more informative once we consider that Plato's forms are seed-like items, and that, similarly, Empedocles' elements were meant to be orderly
principles and were often compared to seeds. This emphasizes once more the essential link between theory of knowledge and theory of vision, between the way both "reality" and its representation are explained.

This interaction is particularly evident in Plato because of his notion of ideas as forms which introduces the analogy between images, forms, and ideas, which is—ultimately—between the body's eye and the mind's eye. The relationship between light as sunlight and light and a sympathic link between the corruptible and the eternal world is well articulated in this passage of the Republic:

[Socrates to Glaucon] ......then reflect; has the ear or voice need of any third or additional nature in order that the one may be able to ear and the other to be heard?
Nothing of the sort.

............
But you see that without the addition of some other nature there is no seeing or being seen?

............
Of what nature are you speaking?
Of what which you term ligth, I replied.
True, he said".

After having introduced the difference between sight and the other senses because of the special status he attributes to light, they both agree that it is appropriate to the sun, the most noble among planets, to be the one to emit light. Then they consider the relationship between sun and sight:
[Socrates] Neither sight nor the eye in which resides is the sun?
[Glaucon] No.
Yet of all the organs of sense the eye is the most like the sun? 
By far the most like.
And the power which the eye possesses is a sort of effluence which is dispensed from the sun? 
Exactly.

.............
Why, you know, I said that the eyes when a person directs them towards objects on which the light of day is no longer shining, but the moon and stars only, see dimly, and are nearly blind; they seem to have no clearness of vision in them?
Very true.
But when they are directed towards objects on which the sun shines, they see clearly and there is sight in them?
Certainly.

And the soul is like the eye: when resting upon that on which truth and being shine, the soul perceives and understands and is radiant with intelligence; but when turned towards the twilight of becoming and perishing then she has opinion only and goes blinking about, and is first of one opinion and then of another and seems to have no intelligence?

Just so.

The dialogue goes on, and after few paragraphs Plato introduces the well-known myth of the cave.

These passages show that there is an intricate network of cross-references among the concepts he is presenting. The sun makes vision possible, not only by emanating light, but also by sending an effluence to the eye and endows it with the visual power. Like Empedocles, Plato first relates light to fire as an element, then he sets a sympathetic analogy between the "power" of vision
and fire by assuming that some kind fire is contained in the eye. Therefore when the sun is shining, it emanates light which illuminates the object of sight, and at the same time--by means of sympathy--it "calls" the visual fire out of the eye, and they then go together to "see" the object. This coalescence is not material entity because vision is not a sort of touch but rather a matching process between the form created by the phantasia and the form of the object. In a sense the light of the sun and the visual fire are two converging beams with concur to "enlighten" the object.

Vision has a priviledged status among the senses exactly because it is the less mechanical and therefore the most a-priori of the senses. Moreover, its medium is light, which has a special status in Plato's theory of knowledge. It is exactly this lack of mechanicity and its inherent a-priori character what makes vision impossible to be explained.

Here we have a coalescence of analogies. In the same way light as an emanation of fire calls the visual fire out of the body's eye, the light of truth emanates evidence which calls up the mind's eye. As most of Plato's metaphors, light is neither sunlight nor the light of truth, but both. The other analogy between microcosm and macrocosm which is encapsulated in this dialogue is the
notion of visual fire as the "soul" (faculty) of the eye because of its affinity to light, truth, and ideas.

The Timaeus offers further material to clarify the relationship between vision and knowledge. Plato says that regular orbits are the image of rational thought and that the Demiurge...

gave each divine [heavenly body] two motions, one uniform in the same place, as each always thinks the same thoughts about the same things, the other forward, as each is subject to the same and uniform 41.

The regular motion of the stars is the image of eternity. Regularity is also the image of rational thought, hence eternity is also the image of rational, perfect, thought. His cosmology too is an image of his theory of ideas, and vision—being mediated by light—is the sympathetic connection between the cosmos, ideas, and men 42. Again, vision is not just either visual sensation or a metaphor for knowledge but both. It confirms its privileged position among the other senses because knowledge is represented in the heavenly sphere: we can see it but we cannot reach it (mechanically).

We can now understand why Plato is the first to point at the difference between image and proof. Proof to him is the true image which is coincident with the form and with the idea. The ideas preserved in the soul (as
well as the related forms and images) are true, while the
images we perceive with the corruptible body about the
corruptible world are not proofs. Plato does not think of
images as copies: images are either true or false, and if
an image is true (i.e. coincident with an idea) the
question of "originality" is pointless.

In the Timaeus we also find that:

And in the second of the orbits from the
earth, god lit a light, which we call sun, to
provide a clear measure of the relative motions of
the eighth revolutions, ...... and to enable the
appropriate living creatures
to gain knowledge of number from the
uniform movements of the same ... for I reckon
that the supreme benefit of which sight is
responsible is that not a word of all we have said
about the universe could have been said if we had
not seen the stars and sun and heavens. As it is
the sight of day and night, the months and the
returning year, the equinoxes and solstices
caused the invention of numbers, given us the
notion of time .

Plato expresses these regularities of the motion
of the heavenly bodies in terms of uniform circular
motions, and it is from the observation of these
regularities that human beings develop their sense of
numbers, time, and geometry.

Therefore Plato considers geometry to be entailed
in the plans of the Demiurge who organized the cosmos out
of chaotically dispersed matter. He does not apply
geometry to the study of astronomy, but he rather offers
an ontological justification of geometry through his cosmology. Geometry is a form, not a tool.

It is interesting at this point to ask why he does not think of geometry as the "form" of vision. Probably it is a problem of "world view", and its origins can be found once again in Plato's theory of knowledge.

If geometry is one of the forms the Demiurge took as example while creating the world, it means that geometry participates with the soul of the cosmos which is permanent, immutable, and therefore static (as ideas are). Geometrical forms are; they do not become.

Now, to apply geometry to vision, it is not just to apply geometry to the behavior of light rays, but rather to apply it to visual cognition, and in the context of Plato's theory of knowledge human cognition is transitory and/or erroneous. It is a process of becoming, not of being. Instead, geometry is a form of the being and cannot be applied to the becoming.

This suggests that the application of a geometrical model to natural phenomena is possible only when the items that are going to be translated in that model are reasonably "unloaded". Plato's notion of light is far from being possibly treated within Optics. It has too much content; from lighting to eternal truth and justice. In a sense Plato's theory of knowledge makes even
more acute those problems generated by the incorporation of sunlight and "cognitive" emanations from objects we already noticed with Empedocles and Democritus.

The notion of "reasonable load" of a model needs to be considered in the context the model is developed and/or applied. However, it is evident that the dismissal of most of the cognitive aspects of vision was particularly important in making Euclid's *Optics* thinkable.

The concerns about the cognitive aspects of vision was re-introduced after Euclid and "modulated" on the geometrical model, but at that point cognition through vision had already assumed a much different meaning than the one attributed by Empedocles, Democritus, and Plato. Probably one of the reasons of this change was the geometricization of matter (reflected also in Plato's five regular solids-elements) which introduced a notion of space not as the measure of things but rather as their structure. As Max Jammer puts it:

Although "Platonic matter" was sometimes held to be a kind of body lacking all qualities (Stoics, Plutarch, Hegel) or to be the mere possibility of corporeality (Chalcidius, Neoplatonists), critical analysis seems to show that Plato intended to identify the world of physical bodies with the world of geometrical forms. A physical body is merely a part of space limited by geometric surfaces containing nothing but empty space. Stereometric similarity becomes the ordering principle in the formation of
macroscopic bodies. Euclid's *Optics* seems to develop this program. The cognitive aspects of vision are not concerned with the "essence" of things or with the elements which constitute them, but rather with their stereometric structure, with their positions in space and with their spatial relationships with the surrounding objects. Empedocles' elemental sieve is replaced by a stereometric one.

Plato's theory of vision became particularly influential for the early medieval inquiry on vision and light, especially for those authors such as Grosseteste who developed a metaphysics of light. Plotino's Hellenistic reinterpretation of Plato with an emphasis on light as an actual emanation of divine knowledge, as well as the later integration of Platonism within Christianism operated by Augustine contributed to the interest of medieval philosophers for Plato's theory of vision.49

5 - Aristotle

With Aristotle we find the beginning of the divorce between theories of visual perception and geometrical analysis of the behavior of light.

Aristotle tries to mediate the duality between being and not-being, between unchanging and changing, by
introducing the notion of potential being. With this assumption, movement and change can be seen as a passage from potentiality to actuality. Change is no more thought of as chaos, but on the contrary it is seen as a form of teleological transformation.

Consequently—quite differently from Plato—his theory of knowledge attributes a relevant role to the sensorial experience of the everchanging physical world. The drastic distinction between the realm of ideas and sensorial experience is dismissed. Light looses the peculiar status it had in Plato and the study of its behavior can be approached in empirical terms. For similar reasons, sight does not maintain its metaphysical connotation and can be treated as a kind of movement of light rays which enter the eye producing the visual sensation.

This change of attitude is evident in the De Sensu where he attacks directly the earlier theories of vision, particularly on the notion of visual fire and of light as a form of fire:

The explanation in the Timaeus, that the sight issuing from the eye is extinguished in the darkness, is quite without point, for what can the extinction of light mean? Heat and dryness are annulled by damp or cold, as we see in the case of the fire and flame in burning coals; but neither of these is a characteristic of light. If they are and we do not detect their presence owing to the smallness of their amount, light would of
necessity be extinguished in broad light too, when it was wet, and darkness would increase in frosty weather.

This criticism shows that Aristotle refuses to explain vision in terms of elements' behavior, and in fact he articulates his theory of perception with a clear concern with both the notion of causality as entailed in his potentiality-actuality duality, and with his concept of space as a plenum.

He considers space to be a medium that has the potentiality of being transparent; light is the actual state of its transparency, and it entails visibility. The object of sight—once illuminated—is the cause of the shift of the state of the medium from potentiality to actuality. The corpus vitreum inside the eye—being transparent—is taken to be the receptor. Its transparency is supposed to be connected with the one of the outer medium. When the air shifts to transparency, the same happens to the corpus vitreum inside the eye, and it is through this chain of transparencies that vision takes place.

From the eye, the visual sensation is then brought to the soul which Aristotle—like Empedocles—thinks to be around the heart. He takes visual sensation to reach the brain through the optical nerves, and then to continue toward the heart through the blood vessels which connect
it to the brain. He thinks the brain to be bloodless and therefore senseless. It is:

a refrigerator of the natural hotness of the body. The eye is an offspring of the brain because it is cool.\(^{51}\)

The description of the sequence of transitions from potentiality to actuality which finally produces vision is particularly problematic because perception is not just the movement or transformation of an object but rather of the representation of the surrounding environment. It is not easy to consider this process as continuous, and difficulties emerge in the identification of the causes, the form, and the matter of all the various stages of the process.

Aristotle is perfectly aware of these difficulties and, by pointing to the different aspects of the process, he probably tries to frame it. For instance he claims that:

perception consists in being moved and acted upon, for it is held to be a species of qualitative change.\(^{52}\)

but also that:

manifestly the sensible object simply brings the faculty of sense into active exercise; in this transition, in fact, the sense is not acted upon or qualitatively changed.\(^{53}\)
Willem van Hoorn suggests that a mediation between these two apparently paradoxical positions can be obtained by considering that:

In general the term "actuality" has two meanings in Aristotle: in one sense it refers to the possession of something e.g. knowledge—a disposition—and in the second sense it refers to the exercise of something, e.g. knowledge—a manifestation.

Which means that the eye sees (is in the actual state of seeing) both because it is brought to see by the presence of an object in front of it, and because it has a disposition to see. We can rephrase it by saying that the eye has the potentiality of seeing not in the sense that an apple has the potentiality to fall from the tree.

In fact when the eye sees something, it does not change or move as an apple which falls. When the object of sight is gone, the eye is still the same while the apple is in a different position (state).

This is why Aristotle says that the sense is acted upon and it is not. He tries to explain this unusual form of causation by de-materializing the cause itself, because a material cause cannot produce an immaterial change. In fact, the cause of vision is not the object qua object, but rather its form. This also implies the adoption of an unusual kind of potentiality: a potentiality that can switch to actuality when "touched" by a form, in a fashion
that brings up again the well-known problem of the substantiality of the aristotelean forms. Under the shape of this hyper-sensitive potentiality he is trying to account for a kind of disposition which in many senses is an actual activity. In other terms, he introduces the notion of individuality under the shape of an a-priori disposition to perception.

Differently from Empedocles and Democritus, he rejects the notion of cognition in terms of material emanations which stay (almost magically) for the object which emanates them. To Aristotle the object (and the cause) of perception is not matter but form and qualities. Sympathy between elements, which was at the base of earlier theories of vision, is replaced by Aristotle with a correspondence between forms which also introduces the notion of individual as a cognitive agent. This because the faculty of the senses is a potentiality, which means that senses can perceive what triggers them, but they cannot sense themselves. Differently, the soul can become aware of its own individuality not by thinking of itself (because would be again a potentiality which is aware of itself), but rather by being aware of a teleologically lesser potentiality such as the senses. In a proto-Cartesian fashion, we find Aristotle saying that:

and if to perceive that we perceive is to
perceive that we exist.  

This awareness is possible only because of Aristotle's concept of cognitive being which is far away from the homeric individual who was "exposed" to or "impressed" by the environment. The consciousness of the individual develops from him being able to compare and criticize the sense reports. As he puts it clearly:

We possess a faculty or power accompanying all the individual senses, in virtue of which power one sees that he sees, or hears that he hears, or in general perceives that he perceives. It is in virtue of this common power that one does so; for assuredly it is not by the special sense of seeing that one sees that he sees.

This hierarchy of cognitive "powers" begins with perception but can be found also within perception itself. The a-priori power is placed on a higher cognitive level than the materials it is supposed to process, in fact:

the sensitive subject... once generated possesses perception exactly in the same sense we possess knowledge. And to have actual perception corresponds to the exercise of knowledge.

Evidently the initial problem is still the crucial one. How can the transition from potentiality to actuality be explained? What does it mean to say that the eye is acted upon?

Aristotle explains that "to be acted
upon" (paschein) has different meanings:

Sometimes it means a sort of destruction by the contrary, sometimes it is rather a preservation of what is potentially existent by what is actually existent and like it, so far as the likeness holds of potentiality when compared with actuality. For it is by exercise of knowledge that the possessor of knowledge becomes such in actuality: and this either is not qualitative change .... Hence it is not right to say that which thinks undergoes change when it thinks, any more than that the builder undergoes change when he builds.

Knowledge--as well as vision--is both caused from the outside and developed from an internal seed. More than a transformation we should probably talk about "dynamic matching". In fact:

in one sense what is acted upon is acted upon by what is like it, in another sense by what is unlike it...that is to say, while being acted upon it is unlike, after it has been acted upon it is like the agent.

Therefore the percipendum is unlike the sense, while the perceptum is alike it. The problem left open is how is it possible that only the form of the object could be the cause of vision. The other examples he proposes--like the wax that receives the seal and is then indistinguishable from the seal itself--fail to convey a significant analogy because they refer to a material transformation which is instead dismissed by his account of perception. He carefully avoids to present perception as a physical imprinting (like Democritus'), but he
describes it as an activity which is only triggered from the outside and then developed internally and with internal energy. This problem is probably also the origin of the troublesome status of light in the process of vision.

Aristotle then negates the process of thought to be a form of change, because the presence of change implies that something imperfect (not fully actualized) is moving toward its final actualization. Thought is rather a form of energeia which is the preservation of something already perfect. It is a movement that (not being material) does not require time. The faculty of vision is an energeia too, which means that it is the preservation of what already perfect. Evidently what is perfect is not the form of the object of vision but rather the power of the sense; in fact, the contrary would contradict Aristotle's teleological hierarchy. This seems to confirm that it is the perceptum that shapes the percipiendum and not the opposite, as Aristotle suggested before.

All these apparently paradoxical statements of the kind "it is, but it is not" suggest that the mediation between the strongly a-prioristic disposition of the power of the senses and the actualization of a particular visual sensation cannot be explained, but rather implied by the teleological fitness of the senses with the physical
world. Ultimately this is what allows Aristotle to escape Plato's deadlock between the chaotic world of senses and the realm of eternal ideas.

The actualization of vision is an *awakening* more than an *impression* because the faculty of sight—though bound to the organ of sight—has the capability of perceiving *all* the possible forms of objects because it is on a higher level of teleological hierarchy. It is a *metaform*, so to speak.

The articulation of individual consciousness is now evident, and it is mirrored by the hierarchical organization of the items of knowledge which entails the privileged status of the human being. Such a hierarchy was scarcely developed in Empedocles, and was instead thought of in terms of *consciousness of matter* by Democritus. What is outside the individual is no more *brought* inside through a sympathetic process, but it is rather *perceived* by a teleologically-informed faculty of perception. It is exactly by assuming that perception is teleologically informed that Aristotle solves the problem of the *projection* of a priori forms on the perceived object. As a matter of fact, those forms are a-priori, but they *fit* the objects without any need to "trim" them down because there is a teleological affinity (almost a "brotherhood") among them.
The relationship between what is seen and what is known, or between perception and knowledge, is widely articulated by Aristotle. His notion of "universal" is not transcendental as in the case of Plato's ideas, but it is developed with induction from sense reports. In turn, cognition is often necessary to achieve a full reading of perception because most of the senses can perceive directly only the so-called common sensibles which are motion, rest, number, figure, and size. Most of the other qualities need to be know by inference from these basic data as well as from experience. This is made possible both by Aristotle's notion of perception as perception of forms which de-emphasizes the materiality of the object while stressing its different qualities, and by the primary role attributed to common sense as the structure of experience. For instance, we see gall as yellow (directly) and we feel it as bitter (directly); but the next time we see gall (directly), we do not need to taste it again for we know it is bitter through an indirect inference.

With Aristotle we begin to find the awareness of the effects of common sense on the judgements we normally make out of visual percepts. As noted by Feyerabend:

...the process by which universals are "established" in the soul depends on particulars and "low level universals" already imprinted in
it. An idiosyncratic history of perception will therefore lead to idiosyncratic perceptions later on. Also the senses, being acquainted with our everyday surroundings, are liable to give misleading reports of objects outside this domain. This is proved by the appearance of the sun and the moon; on earth large but distant objects in familiar surroundings such as mountains are seen as being large, but far away. The moon and the sun however, "appear to measure one foot across even to men who are in health, and know [their] real measurements". The discrepancy is due to the imagination which is "some kind of movement....caused by actual sensations" but "it may be false....especially when the sensible object" appears in unusual conditions, such as large distance and removed from the supervision by the "controlling sense". A combination of unusual conditions and absence of control thus leads to illusions; for example, patterns on the wall are sometimes seen as animals.

Light has a crucial and problematic role in Aristotle's theory of vision. As we considered earlier, Aristotle criticises Democritus' notion of eidola by saying that is not by putting the object directly on the eye that we get to see it. We need instead a medium to behave like a "cognitive distance", so that only the form of the object (and not the object itself) is brought in "contact" with the sense of vision. Light needs to be selective (of the form) but transparent. It needs to transport something without being the cause of anything. We are again faced with a paradox of the kind "it is, but it is not", and it is solved accordingly. Light cannot be a body (because that would translate Aristotle's theory in
Democritus'), nevertheless its role is indispensable, which means that it cannot be "nothing". Aristotle answers in the usual way, by saying that light is just **actuality**: the actuality of the transparent. Now, one may ask what is the transparent about, because Aristotle's space is a plenum........

Such a notion of light could be hardly applied to a geometrical model, and in fact Aristotle did not. Later Aristotelians, like Alexander of Aphrodisia, Themistus, and Simplicius who became more sensitive to the articulation of the process of vision offered by Euclid's *Optics*, needed to replace Aristotle's notion of **kinesis** (which is the transition from potentiality to actuality) with the **locomotion** of the luminous form of the body to the eye.

However, Aristotle seems to hold a different notion of light when he deals with physical rather than perceptual processes. Once light is considered outside this critical chain of potentialities and actualities which lead to vision, it can be approached **qua** geometrical line. What remains unclear is whether or not these approaches to the study of light were mutually commensurable.

In his *Physics* Aristotle introduces the relationship between different scientific disciplines and
their subject matters and methods. Among them he touches upon Optics and its use of geometry for the study of light:

Physicists, astronomers, and mathematicians, then, all have to deal with lines, figures and the rest. But the mathematician is not concerned with these concepts qua boundaries of natural bodies, nor with their properties as manifested in such bodies. Therefore he abstracts them from physical conditions.

What is abstracted is not the global form of the object, but only certain qualities of its. It is a process of modelization. Those "selected" qualities are processed in a formalized context, and the results (even though abstract) are applicable back to the objects. In fact, he goes on with a criticism of Plato's notion of total and therefore unworkable abstraction:

Now the exponents of the philosophy of 'Ideas' also make abstractions, but in doing so they fall unawares into error; for they abstract physical entities, which are not really susceptible to the process as mathematical entities are. And this would become obvious if one should undertake to define respectively the mathematical and the 'ideal' entities, together with their properties; for the concepts 'odd', 'even', 'straight', 'curved', will be found to be independent of movement; and so too with 'number', 'line', and 'figure'.

The opposition between Plato's total and Aristotle's selective abstraction becomes more evident when Aristotle introduces the application of geometry to
physical matters:

The point is further illustrated by those sciences which are rather physical than mathematical, though combining both disciplines, such as optics, harmonics, and astronomy: for the relations between them and geometry are, so to speak, reciprocal; since the geometer deals with physical lines, but not _qua_ physical, whereas optics deals with mathematical lines, but _qua_ physical not _qua_ mathematical.

Aristotle justifies the possibility of geometrical abstraction as a formalized articulation of common sense, and not because (as in Plato) geometry was ontologically entailed in the order of the cosmos. It is again common sense that indicates the workable analogies, which is, whether or not a geometrical model "fits" a physical process. This implies that a natural phenomenon is not equivalent to its geometrical model, but it can be treated as such for certain purposes and if within a certain discipline. Aristotle's concept of mathematical and geometrical models shows the beginning of the notion of correspondence rule.

To Aristotle, light could be dealt with _qua_ geometrical line, but probably visual perception could not be studied _qua_ a form of geometrical projection, therefore he is interested in geometrical optics only within the study of natural phenomena.

C.B. Boyer\(^6\) has considered Aristotle's
specific concern with the geometrical treatment of optical problems such as the behavior of mirrors, or the explanation of the rainbow. He maintains that Aristotle was well aware of the law of reflection—already outlined by Plato in his *Timaeus*—even though there is no evidence of a quantitative formulation of such a law in his works. In an interesting passage of his *Problemata* he also put forward an analogy between mechanics and optics that was going to be used again by Alhazen and later by Descartes in his *Dioprice*:

Now every object rebounds at similar angles, because it is travelling to the point to which it is carried by the impetus which was imparted by the person who threw it; and at that point it must be travelling at an acute angle or at a right angle. Since then the repelling object stops the movement in a straight line, it stops alike the moving object and its impetus. As then in a mirror the image appears at the end of the line along which the sight travels, so the opposite occurs in moving objects, for they are repelled at an angle of the same magnitude as the angle at the apex (for it must be observed that both the angle and the impetus are changed), and in these circumstances it is clear that moving objects must rebound at similar angles.

It is interesting to note that here Aristotle talks about "the line along which sight travels". This unexpected platonism confirms what said earlier, which is that once the process under scrutiny is not directly related to visual cognition but rather to the behavior of light, it does not matter in what direction light is
travelling. The split between theories of vision and the study of the behavior of light was later developed by the Hellenistic "geometers" of vision.

The last consideration about Aristotle's notion of light is in relation to his concept of multi-sphere space. Behind the notion of light qua pure actuality, or light qua geometrical line, there is a careful avoidance of the materiality of light which would have been problematic with his cosmology. If light had a material nature, how can we see the stars if a material light was to go through several spheres each constituted of different kinds of matter?

A concern for the same problem can be detected in his notion of light as energeia. As we have seen before, energeia pertains to something that switches from potentiality to actuality being already perfect. Energeia does not imply any movement during this transition because movement pertains only to what is perfecting itself through actualization. The transition of light from potentiality to actuality does not require any time and does not imply any movement. It is this absence of movement which save Aristotle a problematic explanation of the cause of the movement of light through several heterogeneous spheres.

However, this ad-hoc definition of light may have
had a positive influence on the development of the awareness of the refraction of light through different media and/or cosmological spheres.
NOTES TO CHAPTER 5

1 Hippocrates, De loc. in hom. 2[VI 278], and De carn. 17 [VIII 606].

2 His background accounts also for his description of hollow optical nerves which could have been not a forceful arrangement of evidences to fit the model of explanation, but rather a confusion between nerves ans blood vessels during dissection. Alcmaeon's mistake was widely transmitted to later philosophers also because of problems of interpretation of his text. In fact he talks about "poroi" which have been read as "pores", nerves, vessels. Also Aristotle in his account of the connection between the sense organ and the soul seems to fall into these problems.

3 Alcmaeon is sometimes considered to be a Pythagorean, and his reference to the corneal image seems to confirm this. The few fragments available report that the Pythagorean were the first to introduce the theory of the visual fire contained in the eye which reached out to perceive the object. The visual fire was then reflected back to the eye to form the mirror image on the cornea. Nevertheless this reading of Alcmaeon's theory seems to contradict other of his medical works. For instance it seems that he thought of the fire in the eye not as a sensitive element but rather as an irritation. For more information about Alcmeon's theory of vision see: J.I. Beare, Greek Theories of Elementary Cognition, Oxford, 1906, pp.11-13.


5 The beginning of induction is generally attributed to Hippocrates and his followers. The inductive method is clearly presented in the Hippocratic treatise Tradition in Medicine and the meticulous listings of meteorological conditions, symptoms and patterns of developments of illnesses as found in the various books of Epidemics, show an evident concern in orderly record to be used for inductive prognosis. But from the reading of Epidemics is also difficult to evaluate the specific achievements of the Hippocratic inductive method because it does not seem that the physician was doing much more than observing and recording. The frequency of deaths is impressive, and the more fortunate endings of the illness seem quite casual.

6 Teophrastus, 10, [B 107].

8 The influence of Pythagorean doctrines of harmony is evident here. The soul is the harmony of the body and it also shares some of the harmony of the cosmos which is then reflected in the senses which are effective because "in tune" with the world.

However, behind the introduction of the concept of balance and accordance we can see the development of the "unity of the mind". In fact, it is only through a development of consciousness which allows for the representation of a dialectical process, that those concepts could be made thinkable.

9 Teophrastus, *De Sens.*, 26 sg.[Dox.345].

10 The sympathetic chain between the outside and the inside is already set by means of this correspondence between internal and external fire and water. Moreover he describes the creation of the eyes by saying that "Divine Aphrodite [the personification of love] solidified the eyes from certain elements and then fastened them [to the head] with bolts of love."(Fr.86, quoted in G.E.R.Lloyd, *Polarity and Analogy*, Cambridge, 1966, p.275) His verb "to solidify" refers to the salt which is dried by the sun. This is a further support to the chain of sympathy because fire itself took part in the creation of the organ of sight.


12 This shows that vision was considered to be a particular case of "influence". The doctrine of the influences is particularly related to astrology, and in fact it is interesting to note that we can find astrological overtones in most of the theories of vision up to Kepler. Even though astrology is generally criticized as a sort of evil for science, it actually played an important role in the development of post-aristotelean science because it maintained alive the notion of action at distance that--except very particular cases--was negated by Aristotelean philosophy. In particular, the development of the notion of force of attraction that was so important for Kepler's astronomical model based on non-circular motion was taken over from Gilbert's studies on magnetism and applied to astronomy because of Kepler's own interest in astrological sympathy. We need to remember that the study of magnetism was kept alive exactly because of non-scientific purposes, such as magic, astrology and the speculations about perpetual motion. Mary Hesse dealt with the relationship between the development of the concept of field of force and the theory of emanations in her *Forces & Fields*, Totowa, NJ, 1965, particularly pp. 74-79.

The specific references to "influences" within the studies of optics and vision can be easily found in Plato, Al-Kindi, Grosseteste, R.Bacon, and Biagio Pelacani. Very often beside
treatises of optics and vision we also find works such as Al-Kindi's *De Radiis Stellatis*, or Grosseteste and Bacon's *Multiplicatio Specierum*, which specifically deal with aspects of the theory of influences.


15 We do not have evidences of this but we speculate that this may be the case for the theory of the pupillary image was adopted before him by Alcmæon and after him by Democritus. It is not just a chronological relation when we consider that there is a pattern of continuity between the theories of Alcmæon, Empedocles, and Democritus.

16 Fragments that support the "intromissionist" reading of Empedocles' theory are: Aristotle, *De Sensu*, 437b 9; Teophrastus, 18[Dox.500 sgg.]; Aetius, IV 13, 4 [Dox.403].


18 Teophrastus, *ibid.*, p.8. Siegel, *op.cit*, p.151, thinks that behind this belief there is a precise empirical observation. He claims that Empedocles--being a physician--"must have observed that dark pigmented eyes have better vision during the bright daylight, but eyes with a blue iris have better performance at night." From blue-eyed to "which have fire in excess" is explained by considering that in Greek the color of blue eyes is denoted by"glaucos" which is generally translated as "gleaming".


20 Aetius, V 12,2 [Dox.423].

21 Both this fragment and the earlier one by Alexander, show a clear pattern of relationship between the study of the process of vision and the one of "influences".

22 It is interesting to note that several philosophers who adopted some kind of atomism, or who at least held the corpuscular nature of matter, use the "exemplar" of the sieve to account for processes involving some kind of transformation which could be seen as a selection. An example that comes to mind here is Descartes' criticism of Galen's physiology by replacing his "natural faculties" with sieves.

23 Plutarch, *Quaest.* , conv.VIII 10,2 p 734F and 735A.


25 Teophrastus, *On the Senses*, Stratton's ed., p.109-11, says that "For the reflection does not arise immediately on the pupil. On the contrary the air between the eye and the object of sight is compressed by the object and the visual organ, and thus becomes imprinted since there is always an effluence of some kind arising from everything. Thereupon this impressed
air, because it is solid and is of a hue contrasting with the pupil, is reflected in the eyes, which are moist."

26 Willem van Hoorn, As Images Unwind, Amsterdam, 1972, p. 27.

28 In fact we find that Aristotle—while criticizing him—(see note 27) claims that "he seems to have attained to no clear general theory of the mirroring and reflection of objects", De sensu, 438a 9-10, trans. G.R.T. Ross, p. 51. Other translations put more emphasis on the fact that the lack of a theory of mirroring was not a fault of Democritus, but there was no one available. The same description of the mirror's behavior given by Plato in his Timaeus is only qualitative and it is not clear at all whether or not he knew of the quantitative form of the law of reflection.

29 Archimedes, de mechan. theor. ad Eratosth. meth., [ed. Heiberg, (Herm.), XLII, p. 245, 23].
30 Vitruvius, IX 5, 4.; and Geminus, isag., p. 218, 14.
31 Ptolemy, astron. apparit. epileg., [there in the appendix] p. 275, 1.
32 Aristotle, On the soul, 419a 15, quoted in J.I. Beare, op. cit, p. 27. 32.
33 Actually we find some perplexity about this problem also in Euclid even though probably from a different perspective. Euclid in fact holds that the visual rays moving from the eye to be discrete. Even though he may introduce this because it helps him to explain why distant object are seen less clearly than near-by ones, by saying that they are reached by a lesser number of rays, it also show that he still had problems to deal with the notion of "density". To him it was probably odd to conceive that an angle which at its vertex has no "area", it then may cover an infinite space. In a sense it sounds like that each "line" or ray not to be immmaterial but expanding as a cone while getting far away from the vertex. In this sense I think that Euclid's assumption of discrete visual rays reflects a concern with the problem of the immateriality/materiality of the geometrical line taken to be the model for the visual ray. Even though Ptolemy emendated this point of Euclid's by introducing the notion of pyramid of vision as a continuous "solid" body, the problem emerges again with Kepler. In his Ad Vitellionem Paralipomena, Frankfurt, 1604, p. 9 he says that "Lucis motus non est in tempore, sed in momento", which is that light's speed is infinite, but he also says that it slows down while moving away from the origin. He clearly does not talk about the intensity of light, but he splits light as-a-ray from light-as-a-surface. The ray of light is immaterial, it is pure movement and it maintains infinite speed anytime and anywhere, while light as "illumination" is a surface and
therefore its speed decreases with the distance. He is clear
about this:"Lucis radius nihil est de luce ipsa ingrediente",
which is that the ray of light is not made up of light. In
a sense he explains the decreasing intensity of light with
the decrease of its speed as a surface. If we translate
"illumination" with "visual rays per square inch" we are
again at Euclid's early problem and quite close to Demo-
critus' ant in the sky.

34 Ptolemy and Alhazen, even though basically assuming the
framework of Euclid's Optics they were also sensitive to
the problems of the diffusion of wealening of the resolution
of sight because of the medium in which lights propagates.
For instance Ptolemy in Optics Book II, sec.49 says that:
"Et cum aspexerimus eam sine interpositione, magis uidebitur
quam cum inter nos et illam fuerint aliquae res subtiles
quas usus penetrat et aliquantum ei resistunt.",(p.36 of
L'Optique de Claude Ptoleme , ed A.Lejeune, Louvain, 1956),
Alhazen comments on the influence of the medium in Opticae
thesaurus , Liber 1, Prop.41, but he seems to refer spec-
ifically to refraction.

35 Sextus Empiricus, M. VII 117 f., DK 68 B 164., quoted in
G.E.R.Lloyd, op.cit, p.270.

36 An important contribution to this revival was the re-dis-
covery by the humanist Poggio Bracciolini of a manuscript
of De rerum natura of the Epicurean Latin poet Lucretius.
Epicurus theory of vision was quite plainly borrowed from
Democritus, and Lucretius gives an extensive exposition of
it. The eidola is here presented as a thin skin emanating
from the bodies without being the body itself, like the skin
snakes drop once a year.


38 By using "functionally similar" I am trying to compare
which are incommensurable among them like Plato's form and
Empedocles' elements, by pointing to the analogies between
their roles in their programs. I am not suggesting that by
doing this sort of comparative analysis we may detect the
problem which is dealt in different ways through different
programs, but rather that we can trace the development of
this incommensurability by tracing it from the common his-
torical starting point.

39 Plato, Republic , VI, 507-508, trans.B.Jowett, Modern

40 Plato, ibid. , p.248-249

41 Plato, Timaeus ,40, trans. D.Lee, Hardmonsworth, 1965,
p.55.

42 A further connotation of light is in terms of "Justice"
Plato in fact considers the cosmos as a perfect model of
political and juridical rationality; therefore light does
not only mediate individual but also social and political
knowledge.
142

Plato, Timaeus, 39 ff, p.54.
Plato, Timaeus, 47 ff, p.65.
This is also confirmed by the fact that Plato took geometrical forms to be innate in the soul.
In fact even though Plato's description of reflection in the imaeus is not quantitative, it is reasonable to suppose that the overall knowledge of geometry within the academia was not scarce, at least from what we can grasp from Aristotle. Moreover Euclid's Optics is by far simpler than his Elements, and anyway he worked only few decades later, which means that Plato's disregard for geometrical optics cannot be simply attributed to his scarce proficiency in geometry, but rather to a different philosophical attitude.

Another typical influence of Plato's theory of vision on the development of medieval optics was the distinction between lux and lumen; between light as illumination and light as emanation. This duality developed from the medieval interpretation of the difference between light as illumination and fire as radiation or emanation that Plato inherited from Empedocles. The notion of visual fire was also taken over by esoterism and popular magic and was transformed in the spell-like emanations from the eyes of witches, snakes, and lovers.

Aristotle, ibid., 416b 33-35.
Aristotle, ibid., 431a 4-5.
Willem van Hoorn, op.cit., p.76.
Aristotle, quoted in W. van Hoorn, ibid., p.96.
Aristotle, ibid., 417b 17-19.
Aristotle, ibid., 417b 2-15.
Aristotle, ibid., 417a 18-20.
Aristotle, De Somn., 458b28; cf. de an., 428b 4 ff.
Aristotle, De an., 429a 5.
Aristotle, ibid., 428b 30f.
Aristotle, de somn., 460b 17.
Aristotle, Physics, II,II,193b 30-35.
Aristotle, ibid., II,II,194a 1-5.
CHAPTER 6 - HELLENISTIC THEORIES OF VISION

1 - Institutionalization of science: systematicity and quantification

The proto-encyclopaedic program of Aristotle's Lyceum, with an emphasis on method, on the ordering of disciplines and subject matters, and on the historical development of those disciplines, is then adopted and institutionalized by Alexandria's Museum. Before then, in Classical Greece, schools of philosophy were not official institutions, and even the well-known medical school of Coos seems to have been a guilt-like network of master physicians and their pupils.

Patronage developed in Greece with the tyrants of the Archaic period and survived in different forms till the Roman conquest. Taxation was not carried out with a precise and well articulated procedure even in the Classical Period, but there was instead the implicit obligation of the wealthier class to "take care" of certain public expenses. It is probable that patronage was one of the forms of this implicit taxation.
Institutionalized patronage of systematic research rather than of performing arts as represented by Alexandria's Museum, shows a profound cultural and political change between the Classical and the Hellenistic Period.

At the death of Alexander the Great in 323 BC., his empire was partitioned among three of his generals, and Egypt went to Ptolemy Soter. He did not introduce major modifications in the political and administrative structure of the country, but he rather fit himself in the already existing role of the king. Probably advised by Demetrius of Phaleron—who was a former tyrant in Athens (317-310 BC), a Peripatetic philosopher, and who helped Theophrastus in obtaining the legal status for the Lyceum—Soter established the Museum as a center of scholarship with an aristotelean orientation.

However, there are differences in the interests pursued at the Museum and at the Lyceum, and they can be partially referred to the different form of patronage which endowed the two institutions.

The basic features of Egypt's economy during the reign of the first of the Ptolemies were characterized by the same extreme centralization we already found in the pharaonic period, with the difference that now Alexandria was the administrative and political center of the state.
Commerce, industry, prices, banking, the distribution of raw materials and finished products was controlled by the state, and the king—like the pharaoh before him—considered himself to be the actual owner of Egypt's land. These policies were quite extraneous to the political thought and economic attitudes of Classical Greece.

The economic policy of the polis was generally concerned only in securing the supply of products—mainly corn—indispensable for the safe survival of the community. The remaining range of trading and manufacturing activities was basically left uncontrolled. It has been noted¹ that the Greeks did not have a precise awareness of economical processes and some of the few resolutions taken at Athens in matter of foreign trade often proved to be counter-productive. The very notion of profit in terms of a systematic accumulation of capital was developed in Athens just before the Hellenistic period. Such a development was sharply criticized by both Plato and Aristotle who saw in it the transition from the homo politicus to the homo oeconomicus, or, in general, to the "professional" who was to become one of the typical figures of Hellenistic civilization.

However, this attitude about economic matters had probably something to do with the notion of civic identity
developed in the polis which—at least in Athens—rested on the implicit assumption of the necessary role of metics and slaves in trade and production. The analysis of banking procedures of Classical Athens shows that there was nothing capitalistic about trade. Harbours were busy, but a large amount of the goods were in transit. What was remarkable was the number rather than the value of transactions. The Greek of the polis were more consumers than producers, and in fact we cannot find evidences of any accumulation of capital comparable to the ones of ancient middle-eastern empires.

Hellenistic Egypt offers a totally different landscape. The figure of the homo politicus never existed there, and the Ptolemies had no interest in introducing it. They instead concerned themselves in improving the efficiency the old bureaucratic structure. The citizen did not participate to the political life of state, whose administration was instead totally managed by professionals: from the strategoi (warfare specialists), to the numerous engineers in charge of construction and irrigation works, to the administrators of the royal estates. The division of labour was articulated to the point that certain jobs (like the peasant) became void of any decisionality. He sown the quality and the quantity of seed he was given but he was not even in charge of the
field's irrigation, which was instead centrally managed by an engineer².

Philadelphus—the second of the Ptolemies—reformed the bureaucratic structure of the state and furtherly articulated the centralized control of the economy, and introduced written administrative procedures (entolai). The complexity of the cadaster and of the land's management became impressive. Not only the land is registered, but also how much seed was sown, how much irrigation it got, how regular was the growth of the crops at various periods, who was in charge of that parcel of land in that season, etc. From the cadaster's data, the annual amount of taxes (the so-called "role of perception") was then computed.³

From the records of a large estate we find that also the planning of crops was centrally controlled. The dioiketes Apollonius (ministry of finance of Ptolemy Philadelphus) writes the oikomonos Zenon (manager of the estate) that:

> the king has ordered us to sow the land twice. As soon as you gather the crops, irrigate the soil immediately by hand, or if that is impossible, allow as many tollenos (shadoofs) as possible to be operated and irrigate the land, but don't keep the water on the fields longer than five days. After irrigation sow the three-months wheat. Write me when you have succeeded in gathering the first crops.
The Ptolemies maintained alive, even though in a more secular form, the ancient notion of maat which legitimizes the absolute power of the king and of whom received it from him. In fact, we find a dioiketes saying to an oikonomos that: "No one has the right to do what he wants, but all is regulated for the best." which reminds of Weber's remarks about the notion of identity in ancient Egypt. However, it is particularly important to note the paradigmatic role of authority in the ideology of the "managerial class". For instance it is highly improbable that "the king has ordered" Apollonius to "sow the land twice", but it is rather Apollonius that takes the king into the picture as a "postulate" to justify his decision, knowing that it will be an effective move.

This and other analogous evidences suggest that the mental framework peculiar to the social context who supported the institutionalization of science in the Hellenistic period was closer to the one of ancient Egypt or Mesopotamia rather than to the one of the Greek polis.

This organization of economy together with the Ptolemaic financial and monetary policy primarily concerned with the accumulation of precious metals in the Royal Bank, allowed for a concentration of funds and for the possibility of patronage on a scale previously unknown.
to the Greeks.

However, it would probably be a mistake to take the Museum to be a "research center" effectively engaged with problems of relevance to the sponsor. On the contrary, we can hardly think of it as a productive investment. Nevertheless, the Museum's "encyclopaedic" program reflected some of the features of the widespread specialization and division of labour typical of Hellenistic Egypt, or that the methodicity and systematicity of Aristotle's program was perfectly fitting that socio-political context.

Strabo says that:

The Mouseion is part of the royal quarter and it has a cloister and an arcade and a large house in which is provided the common meal of the men of learning who share the Mouseion. And this community has common funds, and a priest in charge of the Mouseion, who was appointed previously by the king but now by Caesar.

Therefore—like its Greek models—the Museum had a legal status similar to a temple run by a brotherhood of the muses and was endowed by royal funds which were probably independently managed and free from taxation. The Museum was a peculiar institution because of its exceptional fundings, however, according to Rostovtzeff, the tendency to form and support groups and collegiate institutions was a typical feature of the Hellenistic
culture. Professionalism, and the consequent corporativism, was the common denominator among associations of people working in the administration of the state, in the army, in the navy, the other associations of "technitai" (physicians, lawyers, musicians, poets, etc.) and collegiate institutions like the Museum. Even though Radet and Ramsay suggested that the roots of these associations can be traced to the pre-hellenistic guilds which developed in certain Greek cities, it is evident that this tendency was particularly enhanced by the presence of different ethnical communities (politeumata) in Alexandria (Egyptians, Jews, Greeks) which naturally tended to maintain their cultural identities (like the Greeks with the Gymnasia) and often even their own laws and magistrates (as with the Jews). This configuration was also particularly useful to the Ptolemies because it led to a fragmentation of the social body along professional and corporative lines, which helped for its own political control by de-emphasizing broader political concerns. This process both contributed and reflected what Rostovtzeff calls the transition between the Greek homo politicus to the Hellenistic homo domesticus, oeconomicus, and technicus. The orientation of religion, literature, and philosophy reflected quite accurately this cultural and political
change.

With the development of this pattern of professional communities, which extends also within the scientific environment, the notion of "paradigm" becomes more visible. At this point a community of scholars-scientists is not simply defined by its intellectual orientation but also by it being supported as a community by an institutionalized form of patronage. However, the available historical evidences are too poor to allow for an articulated analysis of the process of institutionalization of scientific activity. For instance, the information about the actual activity of the Museum are so scarce that, except in few cases, we cannot reconstruct what was the form of association between numerous scholars and scientists like Euclid, Archimedes, Erathostenes, Galen, Ptolemy, Hero, and Marinus, with the Museum.

However—even though on a smaller scale—also the other Hellenistic monarchies showed a similar attitude to patronage and institutionalization of culture as confirmed by abundant evidences of migrations of scholars. This suggests that it is reasonable to talk about a wide-spread hellenistic mentality, even in the intellectual domain.

The impact of the hellenistic emphasis on systematicity can be easily found in Hipparchus' new
quantitative astronomy, which focuses on different problems and adopts different methods than Eudoxus' pre-Hellenistic astronomy. Pliny points to the novelty of Hipparchus' encyclopaedic catalogation of stars by stressing that:

he dared to do something that would be rash even for a god, namely to number the stars for his successors and to check off the constellations by name.\[11\]

Hipparchus' work incorporates also another concern typical of hellenistic science, which is quantification. Beside Neugebauer's thesis about Hipparchus' development of the astrolabe, there are reliable evidences that he also introduced the dioptra and begun to take observations which Ptolemy regarded as the first reliable ones in Greek astronomy. His concern for the accuracy of the instruments is not just the result of his interest in "keeping up with technology" because, once we consider that the astronomers before him were using the gnomon and the sundial, it becomes evident that his work was establishing a totally different approach to the study of astronomy.

The interest for quantification and prediction was a minor one in Greek astronomy, as we have noticed by the scarce precision of Athens' calendar. This is also confirmed by considering the form of explanation Eudoxus' was seeking for with his sophisticated geometrical system
for astronomy. A. Aaboe has noticed that his "hippopede" introduced to account for the retrograde motion of the planets was evidently only a nominal solution because the motions of Venus and Mars were blatantly left unsolved. To him, the evidence of this problem suggests something about Eudoxus' real concerns. In fact:

I wish to emphasize this point strongly because I believe it reveals that the purpose of these models was to serve as a qualitative description of planetary motion.

which is quite different from Hipparchus' concerns because: "It appears that what Hipparchus was engaged in was to adapt geometrical astronomical models of a certain kind to new purposes."\(^{13}\), and prediction was one of these new purposes.

The origin of this new concern may be also referred to the development of astrology in the Hellenistic period, and in fact both Hipparchus and Ptolemy wrote on the subject. However, this different approach to the study of astronomy can be also referred to other contextual influences.

Hellenistic astronomy is generally considered to be the evolution of the circular-motion paradigm introduced by Plato's school, but it also entails a strong, even though less visible, feature of Babylonian astronomy. In fact, the interest in systematic stars'
cataloging and of astronomical forecast was also developed out of the importation of data (and attitudes) of Babylonian astronomy during the Hellenistic period. This incorporation is not causal, but it reflects a certain congruence between the Babylonian and Hellenistic approaches to the systematic study of natural phenomena, which in turn refers to the similarly institutionalized status of the scientific communities in those two contexts.

Another example of the impact of the "Hellenistic paradigm" can be found in the study of geography.

Hipparchus and Ptolemy did not share only the interest in astronomy and astrology, but also in mathematical geography. The introduction of mathematical geography and its clear differentiation from chorography (historical-ethnographical geography) as emphasized by Ptolemy in his Geography points directly to the fact that also the description of space is becoming systematic. It is not only in the use and in the concerns behind geographical studies (commerce, warfare, state administration), but also in their "form" that we can recognize the Hellenistic paradigm. Geography became a syntax of space, a form of geometrical projection of the earth on a plane, an accurate and reliable (bureacratic) procedure. In fact—as ptolemy himself indicated—
by following the method, you do not need to be an artist to draw a map anymore:

Chorography needs an artist and no one represents it rightly unless he is an artist. Geography does not call for the same requirements, as any one, by means of lines and plain notations can fix positions and draw general outlines.

As a summary: the Hellenistic context in which the notion of demonstration was articulated and included in the method of physical science, was characterized by the widespread development of professional communities and by the institutionalization of science. Even though the administrative structure of Hellenistic monarchies was not very different from the ancient middle-eastern empires, many of the communities within the social body could hold different identities and interests. This because from the clash of cultures which took place in the Hellenistic period, the existence of an omni-comprehensive and homogeneizing system of beliefs came to an end. The weakening of a "state paradigm" in a social structure which—unlike the one of Classical Greece—was highly bureaucratized, may have reinforced the role of professional paradigms whose development could have been facilitated by the presence of standardized and systematic operational procedures typical of a bureaucratic environment. The concept of demonstration and its need for
a commonly shared set of initial assumptions, found the
proper context for its full articulation in the structure
of the professional communities. It is this same context
which supported the encyclopaedic attitude found in
Hipparchus' work, in Euclid's Elements and Optics, or in
Ptolemy's Almagest, Optics, and Geographia.

2 - Deduction, demonstration, and geometry

The earliest form of deductive reasoning in Greek
philosophy is generally attributed to Parmenides. His
arguments move from drastic assumptions about the being or
not being of certain items and are then deductively
propagated. The "demonstration" is generally obtained in a
negative from, which is, by showing that a certain
reasoning brings to a contradiction of the fundamental
assumptions and consequently its complementary is taken to
be true. Zeno and Melissus developed this form of reasoning
into the well-known paradoxes whose avoidance played a
relevant role in the development of later Greek
philosophy. Those paradoxes indicated that the definitions
of the premisses of the "language-game" were essential for
the construction of a formally correct reasoning and--at
the same time--they also showed that a correct reasoning
could also be content-free (or content-less).
The impact of these experiences is also found in the work of Aristotle who (even though concerned in empirical research) defined certain concepts of his (like space) precisely not to be caught in Eleatic-style paradoxes. This concern for formal coherence sometimes took him to conceive "explanations" that were nominal and with scarce empirical content.

The difficulties about the distinction between correct, effective, and convincing reasoning were well known to the Greeks, and became more so with the full development of rhetoric in the mid-fourth century BC. However, this awareness took different shapes and focused on different aspects of the chain of reasoning. Archimedes felt the need for a precise legitimization of his results by a deductive demonstration, but he also realized the limits of formal reasoning as an heuristic tool. In his famous letter to Erathostenes he claims that he often developed his conclusions by using mechanical analogies (he actually "constructed" his theories) and that only later he proved them geometrically.

The application of a geometrical model to physical problems, as in Archimedes' work on statics, Euclid's *Optics*, and Ptolemaic astronomy, tries to transfer the accuracy of an axiomatic deductive reasoning to the analysis of natural phenomena, but it also adds further
problems. Beside the difficulties for the identification of the initial assumptions, it is also presented the problem of the introduction of the correspondence rules between the abstract geometrical model and the actual but geometrized process. However, it is exactly the introduction of a precise form of modelization what developed the awareness of the problem of the correspondence rules. This was clearly recognized by Aristotle who claimed that not all the initial assumptions can be referred to the "genus" of the subject-matter, but some of them, like the notion of geometrical point and line are "external" to that genus. In other terms the model is not the thing represented by that model.

Evidently the two problems of the production of axioms (both objects and rules) and the indication of the correspondence rules are closely related in several ways. From the consideration of the historical development of geometrical optics, we see that at first there is a tendency of the axiomatic model to suggest strongly a certain modelization of the physical phenomena (in Euclid's *Optics*, what is "saved" is the model rather than the phenomena), then, with the further articulation of the explanation of the phenomenon, a modification (either explicit or implicit) of the initial assumptions takes place. The empirical content of the explanation is
sometimes found in terms of "pressure" on the correspondence rules, which may then trigger a revision of the premisses of the model. The history of axiomatic geometry before and after Euclid suggests that there are quite relevant variations in the level of accepted implicitness in the workings of the axiomatic structure. However, such implicitness does not necessarily means "richness of content" for, instead, it could be caused by lack of articulation of the process. The determination of the border between the two is extremely complex, and a formal approach needs to be integrated with a historical analysis of the content and structure of the common sense of that community.

The evidences of the historical evolution of the notion of demonstration within Greek thought indicate that there was quite a wide spread concern for the determination of the foundation of demonstrative reasoning, but that the the solutions proposed at different times were quite different and probably even incommensurable. Even the reliability of geometrical demonstrations was not a given. In fact Plato in the Phaedo has Simmias to say that:

I am well aware that accounts that base their proofs on what is probable are impostors: unless one is on one's guard against them, they deceive one very badly, in geometry and in everything else.
instance the one of Prop.1) cannot be understood by referring to the initial assumptions, but it rather needs to be inserted into a "narrative" which includes explicit but "unofficial" psychological, physical, and physiological elements.\footnote{50}

Another important case of ambiguous interaction between geometrical and psychological notions can be found in Definition 4 where he claims that "Things seen under a larger angle appear larger, under a smaller angle appear smaller, and under equal angles appear equal". This definition became the subject-matter of frequent arguments up to Kepler; Al-Kindi tried to prove it while other authors thought of it as either unclear or incompleate. From a geometrical point of view, its status is unclear because it takes in considerations angles but not distances. It does not tell us how to understand whether we are looking at a large but distant object or a small but near one. Evidently it implies the presence of some kind of process of recognition of psychological nature. In fact if we can recognize what we are looking at, we can also figure out its distance because we already know its size. Instead, if we are not familiar with that object we need at least to recognize something around it, so that we can compute its proper size by considering its spatial relationships with the other object we already know.
Moreover, the term of reference for the computation of distance out of angles, does not need to be recognized as an item which is contemplated among the initials assumptions. On the contrary it could be any kind of "accident" which happens to be out there, suggetsing that the application of the axiomatic model depends upon some "splinter of content" which cannot be formalized within it.

The problem of the angle-to-distance relationship entailed by the fourth definition has been crucial to all theories and practices in which the definition of the scale was a relevant but difficult matter. We already pointed at Thales' need for the third point to be able to use his method of remote sensing, but the more difficult problems in computing distances out of angle were met in astronomy where the scarce familiarity with the observed objects was emphasizing the problems entailed by the fourth axiom. In fact it is Copernicus who suggests how the beliefs about what is observed may influence considerably how distances are computed through Euclid's theory.

No one doubts that the sphere of the fixed stars is the most distant of the visible things. As for the order of the planets, the early Philosophers wished to determine it from the magnitude of their revolutions. They adduce the fact that of objects objects moving with equal speed, those farther distant seem to move more
slowly (as is proved in Euclid's Optics). They think that the moon describes her path in the shortest time because, being nearest to the Earth, she revolves in the smallest circle. Farthest they place Saturn, who in the longest time describes the greatest circuit.

Here Copernicus is referring to Proposition 57 which is proved through Definition 4, and suggests that both the application of such a proposition and the acceptance of its results is based on the assumption that those objects are "moving with equal speed". In other terms, the scale is defined by "recognizing" the uniform motion of the planets.

Another difficulty in the application of Definition 4 can be found in the writing of the Renaissance's architects who were trying to map the space over a plane with a system of perspective which could entail the computation of scale.

However, the limitations of Euclid's theory need to be checked against its aims. If we consider the content of the 61 propositions, it emerges that his primary concern is in matching the geometrical relations of objects as perceived with the same relations among the same objects in the physical world, and in doing so he touches on perceptual illusions or idiosyncrasies (an interest shared by many Hellenistic authors) like for instance the case of parallel lines which are instead
perceived as converging at the infinite. Nevertheless, he is not concerned in what these objects are about or how they behave, but rather with their stereometric properties, in how these objects occupy and are placed in space (as we can see from his interest in binocular or stereoscopic vision).

Most of the propositions included in the *Optics* are problemata rather than theoremata, but with different degrees of generality. Some of them—generally the initial ones—are almost theorems of vision, while others are the evident formalization of actual problems of the surveying practice, like Propositions 19-20-21-22, in which he articulates Thales' methods of measurement at distance. It is precisely the heterogeneous nature of the material included in the propositions what suggests that the *Optics* does for the application of geometry to problems of measurement at distance what the *Elements* does for geometry and arithmetic, which is, that it reorganizes older and newer results of much different origin by trying to integrate them into a model which tries to be axiomatic.

In the same way the axiomatic lacunae of the *Elements* were neutralized once in the context of the "straightedge and compass paradigm", the various ambiguities found in the *Optics* need to be
checked not against the older Greek theories of vision, but rather against the professional paradigm of the technical astronomers and land surveyors. The practice, the problems, and the procedures developed around the use of the gnomon, the sundial, the mekhet and the dioptra was the "peg and rope" of the Optics.

Therefore, the application of a deductive model to physical and psychological process makes the identification of the axioms more complex. The increased difficulties are not just a function of the number of items which are described and included in the initial assumptions, but rather of their content and qualities.

These problems result in an emphasis on the role of a kind of tacit knowledge which is developed by the members of the community not just by reading the axioms and "understanding" them, but rather by going through a long process of training in which the use of instruments and the belief in the "truthfulness" of their output play a relevant role.

5 - Ptolemy: optics and astronomical observations

Ptolemy's work in optics offers a case study for tracing the historical modification of a theory once it is transferred between two different professional communities.
We will try to follow this process by considering certain differences between Euclid and Ptolemy's theory of vision, and by referring them back to the differences between the geometer and the astronomer's paradigms.

Ptolemy's Optics has reached us only in an incomplete and often unsatisfactory late twelfth century Latin translation from the Arab by Eugene of Sicily. The first book which introduced the axioms and dealt with the difference between light and visual rays is now lost, but it has been partially reconstructed by A. Lejeune from the remaining text and from later sources. The second book deals with monocular and binocular vision, the third and fourth with reflection, and the fifth with refraction. Lejeune excludes any doubt about the authorship of the work.

Because of his professional concern in astronomy, Ptolemy could not deal with the process of vision just on theoretical terms, as Plato and Aristotle had done before him, nor he could just "geometrize" it leaving most of the content out, as Euclid had done. Instead, he needed to rely on a theory of visual perception which could account for the content of a wide range of experiences, supplying him with a tool to evaluate astronomical observations. In fact—with the exception of Archimedes' brief remarks about the problems in using the dioptra—Ptolemy's
work in optics represents the first example of a scientist's systematic concern with the evaluation and criticism of the limits of his method of observation.

He accepts Euclid's basic framework, but tries to articulated it into a more complex theory of vision which is, at least nominally, placed in the broader context of the whole system of perception. He looks back at the Aristotelean notion of sense-specificity and, after having considered that different senses responds to different qualities of the objects, he recognizes that a global perception can take place only through the mediation of different sense reports mediated by common sense. This re-evaluation of common sense is symptomatic of an empirical paradigm as the one of the astronomer, and it also shows an attempt to find a coherent explanations to a series of perceptual experiences which cannot be accounted for by a geometrical model.

He considers the problem of sense-specificity trying also to identify the object of the process of vision. He is not satisfied with Euclid's assumption that the eye is directly sensitive to angles and lines and he tries to identify what is that which behaves as a geometrical line and can be perceived through vision.

He assumes color--and not spatial relations--to be the object of vision. Space is not a primary quality of
the object, but it is rather perceived as the "form" of color. He is professionally committed to the stereometric investigation of space, but his space is not populated by empty solids like Plato's. To him, color is not created by light or by our sense of vision, but it is an objective quality of the objects. He tries to demonstrate it with an argument ad absurdum: if colour was created by light it means that an object placed in a fixed position in relation to a light source would always maintain the same color, but the change of color of the camaleon or of somebody's face proves the contrary.

This concept of color is also what allows him to establish a relation between the visual ray and light (again along aristotelean lines). He probably already knows that light and visual rays have the same geometrical behavior out of his professional use of the dioptra. He probably noticed that both light and visual rays pass through its pinnholes with a similar pattern of behavior, however, he does not take this to mean that they are the same "thing". This because, being aware of the uneveness of the visual field, he cannot identify light with the visual rays. He separates the two as:

quo [visus et lumen] differunt in virtutibus et motibus eorum

but if we read (with Lejeune) "virtutibus" as
referring to potentiality and "motibus" as referring to a passage to actuality, we see that Ptolemy does not relate them by means of sympathy (as in Empedocles or Plato), but rather by an Aristotelean matter-form relationship. Lejeune introduces Ptolemy's notion of the relationship between color and light as:

Ce son deux especes d'un meme genre: le lumineux. Au contact la lumiere se coleur et le coleur s'illumine. L'une passe a l'espece de l'autre et reciproquement.

And Ptolemy presents the differences and affinities between light and visual rays in a very similar way:

"ut [visus ey lumen] sibi communicent", or "communicat sibi ipsi"

which Lejeune paraphrases as:

La lumiere se communique en quelque sorte a elle-meme, pusque le coleur n'est qu'une autre sort de lumineux.

Therefore the potentiality-actuality relationship formerly applied to color and light is now extended (somehow more sympathetically) to light and visual ray. The three fundamental objects of vision: light, visual ray, and color are mutually and homogeneously related. They are all aspects of the luminous which reminds
of Aristotle's relation between light and the actuality of the transparent, and of the stoics' pneuma. The relationship Ptolemy sets up among these objects may be correctly criticized because of its nominality, however it allows him to recognize the difference between light and visual ray and—at the same time—to apply the same geometrical model to both of them because of their common mode of propagation.

The emphasis on light and color brings him to assume that different degrees of illumination correspond to different degrees of visibility. From this consideration, he develops a notion of visual ray quite different from Euclid's. If the latter thought that clarity of vision was proportional to the number of discrete rays which reached the object, and that there could be no visual sensation if the object was small enough to fall between two visual rays, Ptolemy instead introduces a concept of a continuous flux of visual rays which could not be separated by each other. With this hypothesis he avoids one of the geometrical incongruences of Euclid's, and also introduces perceptual content in the geometrical description of vision.

The limits of perception are not determined by a geometrical "gap", but by a threshold of perceptual sensitivity which—even though reached when the angle of
vision becomes extremely small—it is also dependent upon other non-geometrical factors. Here we may detect the influence of the astronomical paradigm.

Ptolemy was well aware of the problems created by atmospheric refraction, and in fact we do not find in the Almagest instances of crucial astronomical observations taken when the celestial bodies were close to the horizon\(^59\) refraction. He was also well aware of the apparent "pulsing" of the luminosity of stars, and therefore he could not believe that visibility was just a matter of minimum angle. In the Almagest --probably after having considered the psychological and physiological effect of stress on the astronomer's concentration after a long program of observations--he also expresses doubts about the constancy of the performance of eye-sight\(^60\).

It is from the awareness of this range of problems that he tried to investigate the mutual relation of the medium and the physiology and psychology of vision around a geometrical model.

We know from Damianus\(^61\) that Ptolemy did not accept the rectilinear propagation of light or visual ray as a postulate, but proved it with an instrument whose description reminds quite precisely of the dioptra. With this physicist-like approach, Ptolemy assumed that it is not sufficient for the object to be connected graphically
to the eye with geometrical a line to produce visual sensation, but the visual ray need to touch upon and to be stopped by the object.

The awareness of the effects of refraction together with his notion of different degrees of visibility, makes him to consider the role of the medium placed between the eye and the object. He talks quite precisely about "accumulatio and congregatio [radiorum]"\(^2\), which does not mean—as in Democritus—that visual rays tend to maintain their structure because of some natural tendency (like the pebbles on the shore), but rather that they are initially denser\(^3\) and that later they get confused and dispersed by the interaction with the medium. The process is mechanical rather than vitalistic. Ptolemy's visual rays are quite close to force-vector or projectiles and do not resemble at all Plato or Empedocles' "tentacular" visual fire.

The interesting difference of his position from the older ones is the much increased number of describable causes which visibility can be referred to. In order we find: illumination, the refraction in the medium, the threshold of perceptual sensitivity, and, finally, the position of the object within the visual field.

It is from the analysis of the uneveness of the
clarity of vision over its field that Ptolemy introduces another important improvement upon Euclid. In fact he assumes that because of the diffusion of visual rays through the medium and because of their pattern of propagation, the visual cone has not an uniform behavior, but a priviledged direction, which is the one of its axis. The rays far away from the axis are "debilitated".

This conclusion allows him to improve also on geometrical grounds. Euclid did not give any precise referent for judgements such as "high", "low", "left", or "right" rays under which the object was perceived, because the cone was not considered as having any specific orientation. Differently, with Ptolemy, the axis of the cone is the referent for all the other angular measurements. His description of the process can be paraphrased as:

The visual cone perceives the position of the object in terms of the position of its vertex and axis, and through the angular distances (relatives to both the vertex and the axis) between all the rays which impinge upon the object.

Which is quite more precise than Euclid's "Things seen under higher rays appear higher." It is interesting to note that it is precisely the consideration of physiological issues like the uneveness
of the visual field what takes him to improve on the geometrical aspects of the theory; and that it is the analysis of physical problems such as the difference between light and visual ray, or the study of the propagation of light and its interaction with the medium, what allows him to improve on Euclid's axiomatic structure by showing that certain postulates were not such, and that other definitions were far from clear.

The other significant difference between Euclid and Ptolemy is about the perception of distance. The psychological content implicitly entailed by Euclid's fourth definition ("things which are seen under a larger angle appear larger...") is explicitly admitted by Ptolemy (probably out of his awareness of the problematic evaluation of astronomical distances).

The problem is originated by the fact that the eye can tell the angle under which an object is perceived but not the length of the visual ray which touches it. Ptolemy admits that the determination of the distance is a matter of judgement rather than proof, but he tries to show that the subjective component inherent to such a process is mediated by common sense. In fact, an object can be perceived also by the other senses, and the different sensations are then organized (in an Aristotelean fashion) by common sense, which finally determines the spatial
position of the object. He is clear about the fact that there is some active process of adjustment which develops out of experience ("ex consuetudine fit") like the one which allows for the perception of moving objects. The role of this process of adjustment becomes particularly important in mediating the apparent displacement of objects in space as produced by binocular vision.

Ptolemy is professionally concerned with the problem of the perception of distance, and his theory of the axis of the visual cone makes him even more aware of it. In Euclid's case, the determination of the object in space was done through the perception of its contour, however, being no privileged direction of visual rays, there was no system of reference upon which to take angular distances which could yield information about the surface relief of such an object. Instead Ptolemy realizes that by means of the binocular parallax he could find the "triangulum distantiae mensurium", but he does not articulate the point satisfactorily probably also because the amount of that parallax is insignificant for the solution of his problems, which is, the determination of astronomical distances. In fact he treats the problem of the insufficient amount of binocular parallax as another instance of scarce perceptual sensitivity which falls below the threshold of the minima sensibilia.
His examples are evidently drawn from the problems of his profession: equal but very small angles can be taken to be different, and different angles can be taken to be equivalent; or that—under very small angles—the perception of relief disappears and we see the moon and the sun as flat disks even though they have a convex surface. These considerations of observations under minimal angles suggests that he could not have believed the sphere of the fixed stars to have an uniform diameter, but that it was perceived as such through an illusion of visual perception.

Other considerations about the relationship between observed angles and distances are found in the Almagest where he stresses that an intrinsically difficult problem like the evaluation of distance from angle can be made even more critical by the effect of refraction:

..the same angular distances appear greater to the eye near the horizon, and less near the zenith, and so for this reason it is clear that they can be measured sometimes as greater and sometimes as less than the real angular distance.

Moreover planets cannot be perceived by means of other senses but vision, therefore the evaluation of distance relies on a form of common sense which is not of "sensorial" but rather theory-related. The earlier quote from Copernicus is a good example of the problems which
If geometry does not move from "sound" assumptions, it may easily become a sophisticated form of deceptive reasoning under the appearance of "scientific" reliability.

The point here is not the consideration of "deception through geometry", but rather to suggest that the reliability of deductive reasoning needs to be considered in its historical development and not through modernizing eyeglasses. In fact the same role of Euclid's *Elements* has often been de-contextualized while looking for a historical watershed between pre-rational and rational thought. More recent works have instead tried to approach Euclid's notion of deductive reasoning within its own historical development.

For instance G.E.R. Lloyd suggests that:

..his *Elements* are not merely an axiomatic, but also an explicitly hypothetical system, in this sense at least that it was one was one based on postulates and common opinions which include propositions that he must have known to have been questioned or denied by other Greek thinkers.  

A. Seidenberg while analysing the structure of the *Elements* claims that:

If Book I was supposed to have been founded axiomatically, we can only conclude that this was mostly ineptly done. On the other hand, it is
clear that the Book itself is masterfully conceived. The author, beginning with Proposition 1, wastes no words, goes directly to his goal, keeps to essentials, and does not allow himself to get sidetracked.  

I. Mueller compares the modern axiomatic method as proposed in Hilbert's *Grundlagen* with the structure of Euclid's *Elements* disseminated with implicit assumptions and, after having introduced the structural interpretation of modern geometry, says:

I will be contrasting the Greek use of the axiomatic method with the modern one and arguing that Greek mathematics should not be interpreted in terms of structure.

It is now important to consider more in detail the historical development of axiomatic method before Euclid.

3 - Plato, Aristotle, and Euclid

Even though we find the first instance of deductive reasoning in Parmenides, the concern for the identification of the reliable premises for that kind of reasoning can be considered to be both a direct result and a reaction to the development of the "art of speaking" which developed particularly in fifth-century Athens.
Aristotle\textsuperscript{20} says that rhetoric was primarily focusing on the production of correct and convincing speeches in law-courts, public assemblies and ceremonies. However, it is probable that debates occurred also in many other contexts, and that this resulted in a generalized criticism of traditional values\textsuperscript{21}.

Even though Plato's criticism of the rhetoreticians, and of the Sophists in particular, seems to be misrepresentative of their thought and concerns, it is evident that there was quite a production of convincing rather than correct arguments within the political and social activities of the polis. From here the (problematic) concern to draw a line between the probable and the true, and for the transformation of the art of speaking into a method of reasoning.

In the Republic Plato opposes dialectic to rhetoric, the first looks for truth the second for victory. The dialectician needs to manage the dialogue properly, he needs to answer the questions clearly and to the point, and cannot ask questions which implicitly suggest the answer. By being "synoptic", which is by being able to relate similarities and differences between the arguments developed in the dialogue, the dialectician may get to the recognize "idea" which entails the explanation of the matter dealt with. Plato does not think of the
axiomatic method to be the best remedy against the unrigorous reasoning of the rhetoreticians. However, he admits its consistency:

You are aware that students of geometry, arithmetic, and the kindred sciences assume the odd and the even and the figures and three kinds of angles and the like in their several branches of science; these are their hypotheses, which they and everybody are supposed to know, and therefore they do not deign to give any account of them either to themselves or others; but they begin with them, and go on until they arrive at last, and in a consistent manner, at their conclusions.

He compares the axiomatic method with the one of the dialectician which leads to:

that other sort of knowledge which reason herself attains by the power of dialectic; using the hypotheses not as first principles, but only as hypotheses—that is to say, as steps and points of departure into a world which is above hypotheses, in order that she may soar beyond them to the first principle of the whole...

To him, the axioms are only the beginning and not the justification of the rest of the reasoning. If the dialectician is able to grasp the idea, he will also find the explanation of his own initial hypotheses, which instead Aristotle takes to be undemonstrable. From this difference between the axiomatic and dialectic method Plato draws the conclusion that:

..the habit which is concerned with geometry
and the cognate sciences I suppose that you would term understanding and not reason, as being intermediate between opinion and reason. 24

Aristotle instead approaches the problem with a quite different attitude. He is primarily concerned in identifying a set of hypotheses which can be assumed without demonstration and from which a deductive chain of reasoning can be later developed.

The different attitudes of Plato and Aristotle about the status of these hypotheses can evidently be traced back to their theories of knowledge. Plato does not take common sense to have any serious cognitive content, therefore he cannot admit the reliability of hypotheses which are inductively developed out of common sense. As indicated from the passages of the Republic, his notion of demonstration is not conceived as the construction of a deductive argument, but rather as a leap into the realm of ideas from a set of considerations "sinoptically" organized. To him there cannot be any fixed notion except the ideas.

Aristotle instead has a different consideration of common sense and therefore tries to build on it.

He draws a clear distinction between demonstration and dialectics in his Topics 25 where he says that the demonstrative syllogism is based on true premises, while the dialectical syllogism moves from generally accepted
notions. The crucial and much criticized aspect of Aristotle's method of deductive reasoning as articulated in the Prior and Posterior Analytics is precisely the identification of the true premises of deduction. In a more or less explicit way he proposes two methods. The first one moves from perception. As we read in the last chapter of the Posterior Analytics:

Now we have said earlier that it is not possible to understand through demonstration if we are not aware of the primitive, immediate, principles. But as to knowledge of the immediates, one might puzzle both whether it is the same or not the same—whether there is or it is not understanding of each, or rather understanding of the one and some other kind of thing of the other—and also whether the states are not present in us but come about with us, or whether they are present in us but they escape notice. Well, if we have them, it is absurd; for it results that we have pieces of knowledge more certain than demonstration and yet this escapes notice. But if we get them without having them earlier, how might we become familiar with them and learn them from no pre-existing knowledge? For that is impossible, as we have said in the case of demonstration too. It is evidently impossible, then, both for us to have them to come about in us when we are ignorant and have no such a state at all. Necessarily, therefore, we have some capacity, but do not have one of a type which will be more valuable than these in respect of certainty.

Interestingly enough, we find again the notion of capacity which we already encountered at the basis of Aristotle's theory of vision in the form of a faculty or energeia: as an already perfect potentiality which could account for perception without requiring any
movement or physical modification of the organ of sense.

In fact he continues as:

And this evidently belongs to all animals; for they have a connate judgemental capacity, which is called perception. And if perception is present in them, in some animals retention of the precept comes about, in others it does not come about. Now for those in which it does not come about, either wholly or with regard to that for which [retention] does not come about in them, there is no knowledge outside perceiving; but for some perceivers, it is possible to grasp it in their minds. And when many such things have come about, then a difference comes about, so that some come to have an account from the retention of such things, and others do not.

So from perception there comes memory, as we call it, and from memory experience; for memories that are many in number form a single experience. And from experience, or from the whole universal that has come to rest in the mind [there comes] a principle of skill and understanding.

It is that notion of common sense we already found as the agent of coordination of the inputs of the different senses what gathers the several perceptions of the same category in a cluster from which (in time) the universal emerges

not as something different but as something of the same kind but stronger. In fact:

..as in a battle when a rout occurs, if one man makes a stand another does and then another, until a position of strenght is reached. And the mind is such as to be capable of undergoing this.

He then extends this process of clustering and
reinforcement to induction. In this way he is able to offer an explanation of the inductive process which produces universals out of particulars without admitting any leap or discontinuity in kind between the input and the output. This also rests on his initial assumption about the faculty of the senses as an already perfected potentiality. In fact, this faculty is innately able to "see" universals and to produce perceptions which are weak (but not approximated) copies of them. In fact:

..there is a primitive universal in the mind (for though one perceives the particular, perception is of the universal--e.g. of man but not of Callias the man); again a stand is made in these, until what has no parts and is universal stands--e.g.such and such animal [stands], until animal does, and this [a stand is made] in the same way. Thus it is clear that it is necessary to us to become familiar with the primitives by induction; for perception too instils the universals in this way.

Plato's leap in the quality of knowledge between opinion and truth is replaced by Aristotle with an addition in time of "quantities" of the same kind of knowledge. It is through this form of additive induction that we develop some of the correct premises for deductive reasoning.

However, not all the sciences move from the same initial hypotheses, even though they share a basic core of them. The identification of the additive hypotheses of the
particular sciences develops from a critical analysis of the generally accepted opinions about that science. As he says in the *Topics* 31, it is in this process that the importance of the form of dialectical reasoning become evident. It is this second range of hypotheses to introduce the paradigmatic components of the process of demonstration.

Aristotle identifies the three kinds of initial assumptions as *definitions*, *axioms*, and *postulates* 32, which are then adopted with some modification by Euclid in his *Elements* as *definitions*, *postulates*, and *common opinions* 33. The interpretation of these differences is of crucial importance, but unfortunately it is made problematic by the lack of evidence about Euclid's direct opinions about them.

Aristotle's axioms and Euclid's common notions include self-evident truths like "if equals are subtracted from equals the remainder is equal", which apply to most sciences. There are then other principles which have the status of an axiom which are peculiar to the subject-matter ("genus") of a particular science:

By first principles in each genus I mean those those the truth of which is not possible to prove. What is denoted by the first [terms] and those derived from them is assumed; but as regard to the existence, this must be assumed for the principles but proved for the rest. Thus what a unit is, what the stright [line] is, or what a
triangle is [must be assumed]; and the existence of the unit and of magnitude must also be assumed, but the rest must be proved.

Here Aristotle draws a difficult distinction between certain characteristics of the genus which are included among the definitions. Some of them describe a thing while others imply that the thing described is actually existing. To "be such and such" and to "exist" mean something different, and in fact Euclid treats the first as a definition and the second as a postulate, but not as a common notion. This because the actual existence of a geometrical line is not so as self-evident as the fact that if equal are subtracted from the remainder is again equal. Postulates are in a sense more artificial that common notions but they are nevertheless necessary. The nature of this necessity will be clearer later.

Not all the initial assumptions fit commonsense with the same easiness, and it is not clear whether or not they can all be derived inductively or dialectically from it. The problem becomes more evident when Aristotle introduces the difference between hypothesis and postulate, because in this case he distinguishes between assumptions grounded in common sense and others accepted through the respect for the teacher's authority. In fact:

Now anything that the teacher assumes, though it is a matter of proof, without proving it
himself, is a hypothesis if the thing assumed is believed by the learner, and it is moreover a hypothesis, not absolutely but relatively to the particular pupil; but if the same thing is assumed when the learner either has no opinion on the subject or is of contrary opinion, it is a postulate.

Aristotle by saying that the assumption the pupil accepts under the authority of the teacher is "matter of demonstration" tries to avoid the charge of arbitrariness in the choice of the initial assumptions, but he does not really show how all these items external to common sense can actually be "matter of demonstration". This point can be better understood by considering again the difference between "being something" and "existing" as introduced by Aristotle and adopted by Euclid in dealing with definitions.

Aristotle says that definitions are self-evident characteristics of the subject matter of a particular science, like for instance line and point in the case of geometry. By saying that the definition of a thing does not imply its existence, he tries to draw a line between mental and external items, a distinction confirmed by his assumption that not all speeches are hypotheses about something real. However, demonstration needs a "medium" because something needs to be written or drawn to be proved. This probably does not apply to modern mathematics but it surely does to Euclid's Elements and Optics where,
as recently noted by Mueller\textsuperscript{36} and Theisen\textsuperscript{37}, the "recognition" of the validity of the proof is often closely related to the contextualized reading of the figures. Aristotle tries to minimize this "realism" of the demonstration by de-emphasizing the actual importance of the drawing and saying that:

\begin{quote}
\dots the geometer does not conclude anything from there being this line which he himself has described, but \textit{from} what is made clear through them.\textsuperscript{38}
\end{quote}

But even though the demonstration lies not in the pattern of lines on the paper but in the correspondent mental process, such a figure remains necessary. From here he is obliged to assume that the definitions of certain items such as line, points etc. are different from the other nominal ones, because they do not just say what a certain item is about but that it also \textit{exists}. In other terms, by the fact that the notion of demonstration is developed accordingly to Aristotle's teleological concept relating items to their real causes, it takes the shape of a genealogical tree, or rather a particular branch of the genus of the subject matter.

The demonstration is a shift from the potentiality to the actuality of a geometrical figure and of the concept it refers to. It is the actual construction (synthesis) of a branch of geometry, therefore he needs
not only to describe the "bricks" for that construction, but also to postulate their actual existence. This same concern is found in Euclid who first introduces the line and the point among the definitions, and then he also postulates the possibility of actually drawing those figures, so to be able to build up the demonstration.

The problem is to understand where the nominal definitions come from, or how somebody may develop them. In Aristotle they have a limbo-like status, for they gain actual existence only once inserted into the synthetic genealogical tree of demonstration. As Trendelenburg says:

The nominal definitions of geometry have only a provisional significance and are superseded as soon as they are made genetic by means of construction.

This situation seems to point back at the implications of Archimedes' method, which maintained that things are first understood in reality by means of some kind of analogy, and are then formally proved with a geometrical demonstration. This suggests that Aristotle's definitions are real items which have been found, embalmed, and then shelved, waiting to be revived by the process of demonstration. They are nominalized rather than nominal.

This ambiguous distinction between nominal and real definition probably refers to the more general
problem of Aristotelean philosophy about the real or nominal status of the universals, and points once again to the difficulties pertaining the formation of concepts out of perception and experience.

The structure of Euclid's *Elements* has been found to be quite different from modern axiomatic geometry, and one of the main difference is the relevant "structural" role played in the first by the notion of construction.

A. Seidenberg suggested\(^{40}\) that the process which led to the identification of some of Euclid's postulates was not an abstract induction from a number of perceptions, but rather the use of peg and cord for the solution of geometrical problems. Seidenberg grounds his hypothesis on the consideration that several ancient sources refer to the early use of geometry in the measurements necessary for the construction of temples and altars. Such a measurement was a ritual itself and the peg and ropes used for that were "official" instruments. But whether the first task of the geometer was the outlining of a temple's area or the delimitation of a field, it is quite probable (and I think confirmed by the very structure of the *Elements*), that it was the use of peg and cord that defined the range of possible constructions and therefore of demonstrations. Peg and cord were part of the "exemplar".

It is also clear that it was the set of implicit assumptions developed from the paradigmatic use of these instruments that filled the many lacunae of the axiomatic structure of the *Elements*. In fact, once you know that certain transformations can be done only in the way your (and the all community's) instruments allow you to, there is no need to describe how these transformation should be carried on. In fact Mueller has noted that:

> The emphasis on construction in Greek geometry is connected with the absence of absolute existence assertions like the second part of Hilbert's axiom I,3 which asserts the existence of three non collinear points. In the geometry of the *Elements* the existence of one object is always inferred from the existence of another by means of a construction.\(^41\)

In other terms Euclid's notion of existence of geometrical objects is analogous to the possibility of their construction with straightedge and compass.

Going back to the problem of the initial assumptions, these last consideration point to the fact that the use of an instrument for the solution of a problem is what mediates the process of abstraction which leads to the generalization of that problem. In other terms the notion of abstraction seems to be also a specialized form of experience mediated by the instrument used in that experience. The instrument--being closely connected with the problem faced with it--becomes part of
the exemplar, similarly to the use of lists in Babylonian astronomy, or to Kepler's study of the camera obscura in relation to his theory of vision.

The presence of a subdivision in Euclid's *Elements* between *theorēmata* and *probēmata* (theorems and problems), together with a scarce concern about the notion of generalization could be related to the same reasons. In fact the propositions end either with QED ("which was required to be proved") or with QEF ("which was required to be done") and it is probable—going back to what we have already considered about Babylonian and Egyptian mathematics—that the *probēmata* were a form of proof still closely related to the solution of a problem through actual construction or drawing of the figure. What links the *probēmata* to the more abstract construction of the *theorēmata*, is still the emphasis on the procedure. In the *theorēmata*, the procedure follows the same rules but now it "invents" figures rather than reproducing the schema of a practical problem (it "produces" rather than "re-produces"). It is the paradigmatic belief that the procedure can be repeated, that the instrument can be applied on and on to the same solution of the same problem that makes the consideration of the problem of generalization to be superfluous. The problem of generalization becomes a problem of "computability".
Therefore the notion of demonstration is paradigmatic but not arbitrary. It is a notion that cannot be developed by an individual qua individual but qua member of a professional community. This because the process of demonstration does not need to be passively "accepted" but it rather needs to be understood and then performed through a precise professional training. The institutionalization of science and the formation of professional communities offered the proper context for this development.

4 - Euclid's Optics

Euclid worked in Alexandria during the reign of Ptolemy Soter (305-285 BC.), and, even though he is primarily known for his Elements, he also wrote on astronomy, music, conics, optics and catoptrics. Unfortunately, most of these works are lost or available only in recensions from other authors.

T.L. Heath suggests that he studied geometry in Athens with Plato's pupils, and it is probably there that he became familiar with the earlier works in the elements of geometry. Earlier Elements are generally attributed to Hippocrates of Chios (450?-400? BC) and, in a much more articulated form, to Theodius of Miletus, who wrote the
geometrical textbook for Plato's Academy.

This may justify Proclus' claims that he was a Platonist\(^42\). Nevertheless his concern with method refers directly to Aristotle's work and in a strong sense Euclid can be taken to be the developer of the geometrical approach to optics which we briefly considered in Aristotle's *Physics*.

The *Optics* is generally considered to be a genuine work of Euclid even though doubts have been raised in the past about the lesser rigor of its demonstrations once compared with the ones of the *Elements*. The existence of Theon's recension of both Euclid's *Optics* and *Catoptrics* --which often happened to be attributed to the same Euclid--supported the doubts about the authorship of the work.\(^43\)

The importance of the *Optics* is not to be found only in the fact that it constitutes the first axiomatic and demonstrative study of the process of vision, but also in the deep influence it played in the later development of the theory of vision. It represents both a methodological discontinuity with the former approaches and it constitutes the basic framework on which Ptolemy, Alhazen, Bacon, Witelo, Peckam, Pelacani and finally Kepler developed their theories. The study of Renaissance texts on perspective shows that Euclid's *Optics* was
carefully studied and that the solution of the problem of the accurate computation of scale of objects at different distance (the law of "rimpicciolimento") was often referred to Proposition 9 ("Equal and parallel magnitudes located at different distances from the eye do not appear proportional to their distances from the eye").

Moving from a short passage of Vitruvius which reports Democritus' interest in problems of perspective in relation to theatrical backdrops and from Geminus' (first century BC) consideration of scenography as a branch of optics, it has been suggested that Euclid's work is not just the source of Renaissance perspective, but its very development grew out of analogous interests in the Greek world.

Following the requirements of Aristotle's method, Euclid presents only those aspects of the "genus" of optics which differ from the "genus" of geometry. In fact the definitions included in the Optics are not all-inclusive because Euclid takes for granted that the reader is already familiar with all the apparatus of definitions, axioms, and common notions of the Elements. The Optics begins with nine (sometimes seven) oroi or definitions:

1. Straight lines drawn from the eye proceed in space over great distances.
2. The figure contained under the visual rays is
a cone having its vertex in the eye but its base at the limits of the object viewed.

3. Those objects are seen upon which sight falls and those are not seen upon which sight does not fall.

4. Things which are seen under a larger angle appear larger, under a smaller angle appear smaller, and under equal angle appear equal.

5. Things seen under higher rays appear higher, and things seen under lower rays appear lower.

6. Similarly, things seen under rays more on the right appear more to the right, and things more on the left appear more to the left.

7. Things seen under many angles are seen more clearly.

8. All visual rays have the same speed.

9. An object is not visible under every angle whatsoever.

The first two definitions entails the basic structure of Euclid's model. Visual rays are originated (either irradiated or just "drawn") from the eye. The pattern of these rays is a cone in which the object of sight is "framed". Vision is primary referred to the pattern of angles among the various rays which "touch" the various part of the object. For the first time vision is not the "understanding" of the object, but the determination of its placement in space through a form of remote sensing based on the computation of angular distances.

The Optics lacks of the apparatus of postulates and common notions which are instead present in the Elements. This is not only because Euclid refers implicitly to the one of the Elements, but primarily
because some of the definitions of the *Optics* are actually common notions or postulates.

It is probable that the problematic distinctions among initial assumptions proposed by Aristotle could not be maintained once the subjects matter of those assumptions are of very different nature and their relationship to the "genus" is extremely complex. In fact, together with physical hypotheses like (1) we find other assumptions like (4) which has a definite psychological content.

Once the geometrical model is applied to a physical process, it is not only the problem of correspondence rules to emerge, because the very assumptions of the deductive reasoning need to include some items which will make possible the management of those rules of correspondence. Probably Euclid was faced by the necessity of extending the number of "headings" under which to categorize the various assumptions, and instead he solves the problem by gathering all of them under the same category. For instance (1) would probably be both a definition and a common notion because it implies the existence of what it describes, (2) is certainly also a postulate because the thing which is described and whose existence is assumed is a counter-intuitive notion. Some of the other definitions
cannot fit any of Aristotle's categories of assumptions because of their content. For instance, how can the verb "to appear" be part of and axiomatic system? This difficulties have been more or less explicitly recognized by the translators. For instance Cohen and Drabkin point to the fact that in the context of the Optics the oroi are both definitions and assumptions; Theisen calls them directly "assumptions"; and P.Ver Eecke says that "Ces definitions (oroi) sont plutot des hypotheses ou des postulats".

Evidently Euclid tries to deal, as far as he can, with geometrical matters only, but he cannot explain the whole process of vision geometrically, and he is obliged to insert several items of ambiguous content among his assumptions. However, the difficulties are not only in what he considers, but also in the problems he avoids to limit the inconsistencies of his model.

For instance, he does not consider the "lines" which proceed from the eye to be material, and the Greek verb used to describe their behavior is prospiptosi which means "to draw to meet", which is exactly the verb he uses in the first postulate of the Elements when he assumes the physical existence of the geometrical line. Similarly, we do not find any consideration of the medium in which this propagation takes place, for it was probably
compared to a blank papyrus on which those lines could be drawn. He also introduces the verb "to appear" within the definitions, but he does not give a geometrical interpretation of the workings of the eye, which is simply taken to be the "point" from which the lines which create "appearances" are radiated.

However, we cannot just say that Euclid approached the study of vision geometrically and operated the necessary reductions. In fact several of the problems of his model do not derive from the drasticity but rather from the contradictoriness of those reductions.

For instance, his primary concern with the geometrical aspects of vision takes him to consider the rectilinear propagation of light as a definition. By treating it as a purely geometrical object, he avoids the experimental confirmation of this hypothesis, which instead was carefully sought by Ptolemy, Alkindi, and Alhazen through the study of the behavior of the shadows and of the camera obscura. Things get more complex when he applies his concept of matter-less visual ray to the actual process of vision, because at this point he needs to negotiate the evidence that vision is not evenly sharp on the whole visual field and that the intensity of light decreases with the distance from the source. The matter-less visual ray fits his notion of continuous and
infinitely bisectible angle given in the *Elements*, but such a notion of angle would not easily account for both the experience of vision and the reduction of the intensity of light through diffusion. Consequently, in Proposition 1 he claims that "Nothing that is seen is seen at once in its entirety" and goes on explaining it saying that visual rays are discrete and that they scan the object like the beam of a tv monitor. In a sense he has "atomized" his matter-less visual ray against its own assumptions in the *Optics* and *Elements* to be able to produce a "geometrical" explanation of a physical and physiological process. Moreover by inserting the notion of scansion which is not exactly a geometrical one, he brings in also the concept of time. To make sense of this kinematic component he cannot freeze, he paradoxically gets to explain it with a psychological argument like: "...since the rays are carried rapidly [over the object] it seems to be seen all at once". 49

This example—which can be easily replicated by considering other propositions—shows that the limits of Euclid's *Optics* are not to be found also in the fact that he cannot maintain the internal consistency of his model if he wants to save its content. Not only he "contextualizes" the status of the geometrical objects in an adaptive manner, but his demonstrations (like for
emerge in those contexts, and the analysis of Ptolemy's work in astronomy offers further material about how the process of the evaluation of instrumental reports is influenced by those beliefs rooted in specific form of professional common sense.\(^{72}\)

It has been noted that his commitment to the epicyclic paradigm influenced his treatment of astronomical observations, but also his *Optics* contains materials for similar considerations. For instance, once he realizes the uneveness of the visual field, he tries to determine the pattern of his distribution experimentally, but he reads those results through his and Euclid's assumption that visual rays propagate in a conical pattern. He ends up concluding that the visual field extends 45' in each direction around the axis of the cone.\(^{73}\) This assumption makes him blind not only to the variation of latitudinal and longitudinal width of the field of vision, but also to the fact that the "base" of the cone of vision is far from flat because of the effect of distance on the resolutive power of the eye. Lejeune seems to have identified the problem when he says that:

Ptoleme s'imaginait donc mesurer une grandeur constante et ne songeait nullement à varier les conditions d'expérience.\(^{74}\)

The same adaptive evaluation of experimental
evidences can be found in his study of the law of refraction. He attempted a systematic series of tests, but it has been suggested that he could not have observed what he claims he did. As noted by Lloyd:

"...when he presents what he claims as the results of detailed experiments to determine the amounts of refraction from air to water, from air to glass and from water to glass, these results have clearly already been adjusted to tally with the underlying theory. Here, then the observations have been interpreted before they are recorded."

Beside the elegant but quite doubtful quantitative treatment of refraction (which Ptolemy's himself does not seem to have trusted if he did not make any systematic correction in his *Almagest*), he seems to have had some qualitative awareness of the implications of refraction for the internal workings of the eye. Probably moving from Archimedes' doubts about the fact that the apex of the cone of vision could be identified on the pupil, and from some sort of approximate physiological knowledge, Ptolemy realized that the different density of the parts of the eye had something to do with the process of vision. However, he did not try to negotiate the anatomy of the eye with the process of vision, but he rather tried to reduce the problems which could be generated by refraction.

The notion of central and normal ray is a
recurrent one in Ptolemy and this has probably something to do with (or at least it fits) his concern with refraction. In fact, by assuming the apex of the visual cone to be coincident with the center of curvature of the cornea, he assumed that all the visual rays were normal to its surface, and therefore refraction could not take place in them.

It is also quite evident that he treats both refraction and other aspects of vision in mechanical terms. For instance not only the visual ray needs to be "stopped" by the object to be able to produce vision, but the central ray of the visual cone is the "strongest" because it "hits" the object along a normal direction. Similarly, refraction weakens off the rays that are not normal to the line of contact of the two different media, in the same way that the lateral rays of the visual cone (the ones not normal to the base of the cone) are weaker and further weakened by the interaction with the medium. Evidently to Ptolemy refraction is a special kind of wear which is particularly effective on the weaker, non-central, or non-normal rays.

He also adopts mechanical analogies to explain the concept of illumination which he compares to the notion of concentration of forces. Visual rays are treated as force-vectors as in his explanation of the "stronger"
sharpness of binocular vision which he attributes to the double concentration of visual rays on the object of sight. However he does not make use of these mechanical analogies when he tries to explain other aspects of binocular vision. In fact, after dismissing Euclid's hypothesis that the visual reports of the two eyes are just "added" in an unspecified manner, he clearly admits the existence of the apparent displacement of the object's position in space and claims that the unification of the two visual fields takes place also through psychological processes. Even though his explanation of what this "addition" is about is quite unsatisfactory, it implies the recognition that what we perceive to be the place of the object in space is actually a psychological construction. Psychology is no more either a bumper-like and unstructured process which mediates what cannot be accounted geometrically, or, worse, the cause of errors and deceptive illusions. Illusions are slowly becoming accepted as the standard.

Ptolemy is probably the first to introduce a systematic and experimental attitude into the study of optics and vision, even though as we have seen, his interpretation of those experimental reports is quite problematic. We have noticed that his interest for optics moved out of his professional commitment to astronomy, and
that in the *Almagest* he was often acknowledging observational problems related either to the instruments' or to eye-sight's inaccuracy. In the same way Plato and Aristotle treated vision as part of their theories of knowledge, Ptolemy considered it as the observations' evaluation section of his astronomy.

The continuous interaction between the problems he faced in astronomy and optics shows that despite the fact optics had its own axioms, it was not an independent science. The form and the results of his experiments in optics confirms this interaction. For instance Lejeune has noted\(^79\) that the results of his experimental investigation on the shape and extension of the field of vision are perfectly matched in the *Almagest* by his considerations of astronomical visibility. However, the most interesting example comes from the consideration of how the instruments he was using as an astronomer guided the investigation about important aspects of his theory of optics.

Archimedes offers an earlier example of how the use and the reflections about the behavior of the instrument, may trigger a criticism of the very bases on which such an instrument could be used. In fact by using the dioptra—which was going to become a classical instrument for Hipparchus, Ptolemy, and Heron—he
questioned the fact the eye could be considered as a point by assuming the vertex of the cone of vision to be on the surface of the cornea. It is by studying the behavior of the instrument that he got to the conclusion that if we are able to "see" the output of that instrument, it means that the workings of the eye are more complex than what supposed.

The use of the diotra seems to have left some traces within Ptolemy's work too. Lejeune, after having tried to identify the instruments he used in his experiments, concludes that:

Il est vraisemblable qu'il faisait également constater qu'un objet n'est visible au travers des deux pinnules d'une dioptrre qu'a^ la condition de se trouver sur la droite passant par celles-ci. C'est le principe même de son procédé de mesure des angles d'incidence et de réflexion, procédé en tous points parallèle à celui utilisé pour la réfraction.

The fact that Ptolemy took his diotra as the exemplar which could mediate the investigation of the fundamental aspects of optics is confirmed by a pseudo-euclidean work on catoptrics generally attributed to Theon, whose first definition says that:

Soit suppose que le rayon visual est une droite dont toutes les parties medianes se butent a des limites.

which Lejeune translates slightly differently as:
Le rayon visuel est une droite dont tous les points intermédiaires masquent les extrémités.

which shows more clearly that the two pinholes placed on the same viewing device of the dioptra were already used as the basic "paradigmatic" instruments for the experimental demonstration of the rectilinear propagation of light.

This last example confirms the paradigmatic nature of axiomatic systems, for we see that both the choice and the content of the initial assumptions is also mediated by the professional practice developed around the use of the instrument.

In other terms the dioptra was to Ptolemy what the straightedge and compass was to Euclid, what the camera obscura was to Kepler, or--going back to the Presocratics--what the lantern was to Empedocles.

6 - Galen: anatomy and geometry

Galen (129?-199 AD) was almost a contemporary to Ptolemy but assumed a totally different approach to the explanation of the process of vision, for he moved from the physiological and anatomical analysis of the organs of vision and tried to integrated it with a geometrical model.
His system of physiology incorporates several concept of Stoic philosophy, and one of them, the **pneuma**, is of crucial importance for the understanding of his analysis of the process of vision.

The philosophy of the Stoics cannot be considered as a homogenous system, nevertheless they maintained quite a broad agreement about the notion of pneuma. The Stoics introduced a hierarchy among the elements of the cosmos: air and fire were considered to be the active, earth and water the passive ones. The pneuma is precisely a mixture of air and fire which *gives form* and keeps together the other two elements. The Aristotelean continuum is made active. The innovation is important for it allows a new range of possible solution of the problem of movement which was crucial in Aristotle's theory of vision. The pneuma participates of both in animate and inanimate bodies; it is a quite sophisticated development of the sympathetic relations among elements proposed by Empedocles. The important novelty is the kind of relationships among bodies which are made possible by their common "pneumatic" nature.

To the Stoics, everything in the cosmos is kept together in a dynamic equilibrium produced by the tension produced by the pneuma. In a sense, they articulated and extended the well-known homeric notion of psiche as breath
to the whole cosmos through a microcosm-macrocosm analogy. S. Sambursky gets to the point of comparing the pneuma—probably by isolating its "modern" aspects—to the modern notion of field of forces. This form of dynamic equilibrium, of cohesive tension, introduces a notion of space in which movement can be propagated as a wave on a tense string. This characteristic of pneumatic space is potentially interesting in relation to the the duality between perception as produced by the "touch" of material emanations or as change of state in the luminous medium and in the visual "faculty".

Chrysippus introduces the first important feature of the pneuma for the process of vision when he says that:

The structure of matter is simply air, for bodies are bound together by air. Likewise all that is bound together in a material structure derives its quality from the binding air which in iron is called hardness, in stone thickness, and in silver whiteness.

There is a shift from the Atomistic notion of configuration to the Stoic concept of synthesis: here it is the pneuma to give forms to substances through synthesis, and for the same reason—once used as the medium of vision— it can also "perceive" them. The pneuma is everywhere and especially in the air which, for this reason, becomes a tense medium. As Aetius puts it:
The Stoics say that the air is not composed of particles, but that it is a continuum which contains no empty space. If struck by a puff of breath it sets up circular waves which advance in a straight line to infinity, until all the surrounding air is affected just as a pool is affected by a stone which strikes it. But whereas in this case the movement is circular, the air moves spherically.

It is precisely this concept of wave-motion that characterizes most of the Stoic theory of perception which will be later adopted and modified by Galen. Through this notion they try to solve both to the duality between material movement and change of state, and to the two-way perceptual transmission that we have found in Empedocles, Plato, Euclid, and Ptolemy. This because a wave, once struck an obstacle, moves back to the origin. The disturbing problem is that this model works much better in a pool than in an unlimited space. However, the observation of a "standing wave" in a limited environment, could offer a model for that combination of stilness and activity, for that tensional motion (tonike' kinesis), which was an appealing potential solution of a series of problems. The use of the pneuma for the explanation of psychological and physiological processes was particularly interesting because the domain of those processes (the human body) was actually limited.

Most of the Stoics (but not Galen) took the heart to be the seat of the hegemonikon, the "ruling part of
the soul", and maintained that there was a two-way communication between the sense organs and the hegemonikon based on this tensional motion. In this way perception—as Aristotle was trying to prove with his notion of the "actualization of the already perfect"—could be considered not as a material modification, but as a change of state. Chrysippus has been credited of saying that:

In the same way as a spider in the centre of the web holds in its feet all the beginnings of the threads, in order to feel by close contact if an insect strikes the web, and where, so does the ruling part of the soul, situated in the middle of the heart, check on the beginnings of the senses, in order to perceive their messages from close proximity.

This notion of tense matrix was then quite interestingly applied by Chrysippus to the problem of the nature of perceptual "impressions". Cleanthes—another Stoic—held that an impression was a material imprint like the one of a seal on the wax, but Chrysippus opposed him claiming that:

It is by no means absurd that the same body be submitted at one and at the same time to a very large number of modifications. In the same way as air, when many sounds are uttered, is submitted to innumerable and different strokes and holds at once many modifications, so the hegemonikon undergoes an equivalent experience when presentations are formed by it in various ways.

Chrysippus' fragment reminds also of the pool in
which different perturbations of the water surface maintain a pattern of interference from which—in each point of the pool—the pattern of those perturbations can be computed. This introduces the quite innovative hypothesis that different impressions can be differently modulated at the same time on the same portion of medium. Impressions are not things which occupy a space, but a state of the psychic pneuma. Implicitely the temporal factor is introduced into the picture. The Stoics applied the "pool-model" also to inanimate bodies, in fact Philo says that pneuma:

...begins in the center of the body and extends outwards to its boundaries, and after touching the outermost surface it turns back till it arrives at the same place from which it started.

Therefore the pneuma is not just a cohesive tension, but a wave-like one.

With these premises, the explanation of the process of perception by means of this tensional motion is quite automatic. Both the explanation of hearing and sight is evidently approached in terms of waves diffused in the air, but—in the case of vision—the determination of what this waves are about is by no means easy. It is at this point that this interesting model shows its limits.

However the wave-model does not necessarily
exclude the geometrical treatment of the process. We have seen that Chrysippus thought of the propagation of waves in the spider-web to be able to contain the information about the direction of the "perturbation", and something similar could probably have been applied to the propagation of a "visual wave" which hits an obstacle and comes back as in a radar. Moreover the pattern of wave propagation is perfectly coherent with Euclid's results because the direction of wave-motion is perpendicular to its front. However, probably also because of a series of philosophical difficulties that the Stoics raised about geometry, the integration of the two approaches was never satisfactorily achieved, and their pneumatic theory of vision did not go beyond a nominal and sympathetic explanation of the process. We are told that:

According to Chrysippus... sight is due to the light between the observer and the object observed conically. The cone forms in the air, with its apex in the eye of the observer and its base in the object observed. In this way the signal is transmitted to the observer by means of stressed air, just as [by feeling] with a stick.

This adaptation of Euclid's model to a pneumatic medium did not satisfy all the Stoics either, and in fact Galen dismissed the comparison with the stick by claiming that a stick (even though it offered a good example of two-way communication) is not sensitive to color. What
Galen will maintain is the basic concept of sensitivization of air by means of the pneuma.

In fact the Stoics thought that visual perception took place after the pneuma was sent from the hegemonikon to the eyes, where the surrounding air was put in a state of tension by the contact with the pneuma-filled eyes. Vision is not just a matter of light, because air needs to be sensitized and made ready to transmit by means pneumatic tensional motion. However, the problem of the relationship sunlight-pneuma is not of difficult solution (at least nominally), because sunlight is a kind of fire which, in turn, is a component of the same pneuma. Alexander of Aphrodisia wrote the Stoics maintained that:

The illuminated air becomes more powerful because of the mixture [with fire] and can propagate the sensation through pressure whereas as dark air is slack and cannot be stressed by vision.

Galen philosophy of physiology contains several features of the Stoicism. He first encountered it in his early training in philosophy before becoming professionally involved in medicine, and--once he moved to Rome--he probably continued to operate in an intellectual environment which was strongly sympathetic with Stoic philosophy.

But, more importantly, Stoicism fit quite
coherently his physiological paradigm. For instance his theory of the four humors—which he took over from Hippocratic medicine—contained a deterministic connotation which found many point of contact with the Stoics' own determinism, which was a consequence of the conparticipation of the pneuma with the whole cosmos. However the major reason of Galen interest with Stoic philosophy is to be found in the range of physiological explanations made possible by the adoption of the notion pneuma. For instance, beside the interesting explanation of nervous transmission, the dynamic equilibrium peculiar to pneuma allowed Galen to offer a coherent analysis of the crucial physiological problem of muscular motion.

Galen's theory of vision develops primarily from the careful consideration of the anatomy of the organ of vision, whose interpretation is strongly influenced both by his pneumatic philosophy and by his commitment to a teleological interpretation of the structure of human body. However his notion of teleology is much less philosophical than Aristotle's. It seems that—probably out of his experience as gladiators' physician in Pergamum, and as military physician with Marcus Aurelius—he considered usefulness primarily in terms of safety and protection of vital organs from external
injuries. These concerns can be traced also in the interpretation of the eye's anatomy, but they do not play a primary role here as they do in other parts of his physiological system.

Galen dealt with vision in his On Vision (now lost), in De placitis Hippocratis et Platonis, and in On the Usefulness of the Parts of the Body. At the end of Tenth Book [The Eyes] of the On the Usefulness, he tries an unsuccessful negotiation of his pneumatic theory with Euclid and Ptolemy's theories of geometrical optics.

Galen held different opinions than the Stoics' about the nature, kinds and distribution of the pneuma in the human body. He thought that the vital pneuma (zotikon pneuma) is absorbed from the outer air primarily through respiration and is delivered through blood vessels to the brain. Once there, it becomes cerebral pneuma (psychikon pneuma), which is the essential active component of all the perceptual processes. With this description of the transformation of the pneuma, he explicitly dismisses the Stoics' belief that the heart was the seat of the soul, which he instead—like Alcmaeon—places in the brain. He also opposes the Stoics by maintaining that the cerebral pneuma was not subdivided in other more specialized pneumas (visual, auditory, etc.) as instead held by
Chrysippus. Galen explains sense-specificity—in an almost Aristotelean fashion—by maintaining that the sensitivity to different qualities of the objects was not referred to different qualities of sensitive pneuma, but rather to the interaction between a single kind of pneuma with different structures of sense organs. However, he equally modifies Aristotle's hypothesis that conscious perception took place in the sense-organs: to him in fact, sense-organs are just specialized "terminals". The production of conscious sensation takes place in the brain as effect of the "returning wave" of the pneuma from the sense-organs.

Nevertheless, by explaining sense-specificity, he re-introduces strong sympathetical analogies which remind of Empedocles. In fact, talking about the sensitivity of different senses he claims that:

They are not however all altered by every perceptible thing; rather the bright luminous sense instrument is altered by colors, the airlike instrument by sounds, the vaporous instrument by odors, and in a word, like is perceptible by like.

It is not the pneuma but the sense-organ to be specialized. However, this emphasis on the material structure of the organ (which is clearly a result of Galen's primary concern with physiology), re-introduces the problem of the "modification" of such an organ during the process of perception. Chrysippus had found an elegant
but nominal solution and Galen, re-introducing anatomical reality in the picture, brings the same old problem up again. Probably because of his "anatomical paradigm", Galen considered perception and thought to be somehow more imprint-like than what Chrisippus had previously suggested:

In substance the encephalon is very like the nerves, of which it was meant to be the source, except that it was softer, and this was proper for a part that was to receive all sensations, form all images, and apprehend all ideas. For a substance easily altered is most suitable for such actions and affections, and a soft substance is always more easily altered than one that is harder.

The notion of perceptual impression is now presented as an ambiguous mediation between a change in the status of the medium of perception and an imprint in the cerebral medium. Galen tries to solve Aristotle's ambiguous explanation of perception in terms of the non-material modification of an already perfect faculty of perception, by substituting Aristotle's perfect and extremely sensitive innate faculty of perception with an extremely soft and sensitive nervous substance which is closely but unclearly related to the cerebral pneuma. But--like Aristotle--he falls in a similar difficulty. In fact in the same way Aristotle could not easily dismiss the materiality of the cause of perception, Galen cannot
avoid to treat pneuma as a material entity of which, however, he does not offer any further description. His physiology, despite the relevant empirical content, is still largely metaphysical.

Galen takes the bright and transparent to be the sensitive component of the organ of vision and identifies it with the crystalline lens (also in accordance to his professional experience of cataracts). The object of vision is not just the position of the bodies and their shape, but, primarily, their color. It is color which can effect the transparent, in a fashion quite similar to what already proposed by Aristotle.

From here Galen's criticism of Chrysippus' concept of vision in terms of the touch of a stick-like, extremely thin, cilinder of air made tense by the presence of visual pneuma. In Galen's theory, instead, it is the cerebral pneuma what makes the lens to be sensitive and the air to be tense. Such pneuma is brought from the brain to the eyes through the optical nerve which...

is a part of the brain, like a branch or offshot of a tree, and the member to which the part is attached receives the power from the part into the whole of itself and thus becomes capable of discerning the things that touch it. Something similar happens also in the case of the air that surrounds it. When it has been illuminated by the sun, it is already an instrument of vision of the same description as the pneuma arriving from the brain; but until it is illuminated it does not turn into a sympathetic instrument in accordance
with the change effected by the outflow of the pneuma into it. The Stoics, then, must not say that we see by means of the surrounding air as with a walking-stick.

The role of sunlight in the process of the sensitization of the air in the presence of pneuma, reminds both of Aristotle notion of the diaphanus and of Plato's coalescence of visual fire and sunlight. The aristotelean connotation seems however to be stronger: sunlight is considered as a *dynamis*, a "sensory power", something similar to Aristotle's *energeia*. It makes another *energeia* (which is pneumatised air) to switch to actuality. When air is illuminated it becomes an instrument of vision like pneuma itself.

The other problem which Galen tries to minimize is the relationship between pneuma and air. They are evidently related, but a better articulation of such a relation would oblige him to define better the nature of pneuma. In fact he often uses different terms than "air", like when he outlines the process by which pneuma is able to sensitise the space between the eye and the object:

It appears likely that the pneuma arriving in the eye [from the brain] unites itself instantly with the surrounding [periechonti] and alters it, accordingly to its own nature, but it does not extend itself over a great distance.  

In other passages he seems to solve the
problem through another sympathetic analogy, by saying that air is a continuous element constituted by contiguous particles of different size. What is responsible for the transmission of visual perception is the most subtle parts of air, which seems to match sympathetically the subtlety of the cerebral pneuma which he takes to be the most air-like. However he does not offer a more precise explanation of the actual relation between these two "subtleties".

However, the origin of most of these problems cannot be referred to Galen's articulation only. It is Chrysippus' notion of tensional motion that looses a good part of its actual interest once it is moved from nominal to factual explanations of natural phenomena. Nevertheless it is true that Galen's major contributions to the theory of vision are to be found in the emphasis on the anatomy of the eye, rather than in his coherent arrangement of the philosophical aspects of those problems.

The evaluation of the actual contribution of Galen's accurate study of the anatomy of the eye to the development of more comprehensive theories of vision is not an easy task. It is evident that his anatomy was very functional for medical and surgical problems, but it is also clear that his a-priori interpretation of the anatomical evidences (not unsimilar to Ptolemy's treatment
of experimental reports) did not help him to develop a consistent theory of vision. His phisiology of the eye needed in fact to be revolutionized by F. Platter in 1583 to make the idea of retinal image possible.

Galen sets up a close anatomical analogy between the eye and the brain (probably justifying it with the excellence of sight among the other senses) and develops it by assuming a one-to-one relationship between the membranes which surround the brain and the eye. The eyes are two round offshots of the brain, and therefore the retina, being the most internal membrane of the eye is taken to be an actual part of the brain. Its being in direct contact with the lens guarantees an immediate transmission of visual sensation to the brain; in fact:

In appearance the retina is indeed like a net, but not like a membrane at all, neither in color nor in substance. One would surely believe, if one detached it and rolled it together, that one had before oneself a detached portion of the brain. Its chief function, that for which it was sent down by the brain, is to perceive the alterations which occur in the crystalline body [lens] and to communicate these [to the brain].

Galen assumes that the eye needed more pneuma than the other sense organs and therefore "sees" a channel in the optical nerves—probably inferring it from the small depression visible in the eye at the point the optical nerve "opens up" and flattens into the retina. However he
is not the first to "see" them, because the traditional belief that optical nerves were "poroi" goes back even to Alcmaeon, who needed to bring down "water" from the brain to keep the eyes humid and sensitive.

His lack of understanding of the role of refraction in having the external image to converge in an internal one (in the case of the retinal image), led him to explain the actual process of visual sensation with a more sophisticated version of the notion of mirror image. Actually he does not take the pupil to be the mirror as Alcmaeon or Democritus did, in fact the image is perceived by the crystalline lens, but we are not told how that process may take place beside through the usual sympathetic analogies. The same hypothesis about the role of the lens is primarily developed and maintained through a precise interpretation of anatomical evidences which he also base on a related preparation of the specimen to be dissected. R. Siegel has followed the instruction given by Galen for the dissection of the eye and has noticed that:

The flat ciliary body is a disk-like vascular structure which occupies in the intact eye the space between the sclera and the posterior part of the iris.... The ciliary body is attached with its outer margin to the retina... to which it is connected by innumerable fibers. During preparation of the specimen the connections between ciliary body and lens remained intact, but those to the retina were torn, since the retina remained inside the eyeball covering the choroid membrane.... The radially arranged black
spokes of the ciliary body form a very beautiful pattern and seemed to Galen as a convincing demonstration of the functional connections between retina and lens. It was Galen's mistake to infer a functional relation from these anatomical relations.

However it was quite natural to Galen—because of his commitment to a teleological view—to see those functional relations. Moreover, the lens was bright and clear and therefore its choice as the sensitive part of the eye was also fitting the pneumatic components of his paradigm.

The impression is that Galen arrived at the analysis of the process of vision with a precise system of physiology already well articulated, and that he applied it in the least contradictory way to his anatomical knowledge of the eye, referring the explanation of the crucial points to the undeterminate nature of the pneuma. In fact it is difficult not to relate his findings about the pores of the optical nerves with the more famous pores in the intraventricular septum of the heart, and suggests that his concern in saving his phisiological paradigm played a continuous and important role in his interpretation of anatomical evidences.

However we also need to remember that most of the eye diseases known to Galen were external ones with the exception of the cataract, and that his explanation of vision does not contradict the experience of his
professional practice. On the contrary, the crucial role
of the lens in his theory of vision reflects the awareness
of the dangerous consequences of its diseases.

In the last part of the Tenth Book of
On the Usefulness, Galen approaches quite
reluctantly--apparently only because ordered by a god--the
geometrical aspects of vision:

I have explained nearly everything pertaining
to the eyes with the exception of one point which
I intended to omit lest many of my readers be
annoyed with the obscurity of the explanations and
the length of the treatment. For since it
necessarily involves the theory of geometry and
most people pretending to some education not only
are ignorant of this but also avoid those who do
understand and are annoyed by them, I thought it
better to omit the matter altogether. But
afterward I dreamed that I was being censured
because I was unjust to the most godlike of the
instruments and was behaving impiously toward the
Creator in leaving unexplained a great work of his
providence for animals, and so I felt impelled to
take up again what I had omitted and add it to the
end of this book.

He stresses even more clearly the adversity of the
community of the physicians to mathematical and
geometrical demonstrations by saying that:

..if it depended on me, I would omit
demonstrations requiring astronomy, geometry,
music, or any other logical discipline, lest my
books should be held in utter detestation by
physicians. For truly on countless occasions
throughout my life I have had this experience:
persons for a time talk pleasantly with me because
of my work among the sick...but when they learn
later on that I am also trained in mathematics,
they avoid me for the most part and are at all glad to be with me. 108

These premises are significant to understand Galen's attitude in dealing with the geometrical aspect of vision. Even though he displays a full understanding of geometrical matters, it appears evident that it is not in geometrical terms that he had thought of vision. However, probably out of the awareness of the popularity of geometrical optics at the time, he tries to offer a geometrical interpretation of his pneumatic theory of vision.

In trying to negotiate the two approaches he introduces major modification in his former theory—primarily in regard with the transmission of visual impression to the optical nerve—but the final results seem to have lost part of the (nominal) coherence of the pneumatic approach without having added much to the already available geometrical theories of vision. The main reason of this lack of synthesis can be found in the scarce understanding of the role of refraction in the process of image formation, for this was the only way to integrate the physiological and geometrical aspects of vision.

Ptolemy's theory of vision has in fact shown that, without having a full understanding of the anatomy of the
eye, he probably failed to see the application of the law of refraction (which he had somehow grasped) to the physiological behavior of the eye. This brought him to look at psychological explanations—following Aristotle's notion of common sense—to fill the gap between the experience of vision and its possible description in geometrical terms. On the contrary Galen was professionally uninterested in geometrical and psychological matters; instead he looked for explanations in terms of physiological interpretation of anatomical evidences. The physiological and geometrical components needed to be matched, but evidently the paradigms of the various scientific communities dealing with vision did not allow for the interaction among different approaches and concerns. In fact Kepler studied and understood the process of image formation in the camera obscura, but it was Felix Platter's discovery of the photosensitivity of the retina what allowed him to see the application of the behavior of the camera obscura to the process of vision.

At first, Galen faces the problem of the integration of the pneuma with the geometrical line; a problem which reminds of Aristotle's unresolved troubles in applying geometry to the diaphanous, and eventually to the study vision. As noted earlier, the problem was not of direction of propagation, because the direction of
propagation of the wave-like tensional motion of the pneuma was rectilinear. The problem was instead about the nature of the ray, about how the pneuma could be reduced to such a geometrical object. The problem was made more complex by the empirical/realistic attitude of Stoic philosophy which tended to question the abstractness of geometrical forms. We have seen that Ptolemy—probably out of his professional necessity of applying geometry to a variety of physical objects—made a clear distinction between the model and the process: what was geometrical about the sunlight and the visual rays was their mode of propagation and few other qualities. This is evident when he treats both light and visual flow with the same formalism making clear at the same time that they are quite different objects.

But the pneuma is a more complex object, especially because of its wave-like propagation, and even though its direction is rectilinear, Galen probably felt that the reduction of a "wave" to a geometrical line to be a too adventurous one. In the same way he substituted "air" with "surrounding" to avoid the problem of defining the relationship between that object and pneuma, he replaced "visual ray" with "visual impression" ("gramma" with "opsis"), and assumed that it was the opsis what travelled along a geometrical line.
However, the problem is only temporarily avoided for it comes up again when he tries to describe how visual perception can be referred to the geometrical behavior of the opsis.

Even though he assumes with Ptolemy the nature of the visual flow to be continuous (the only solution which could be possibly coherent with the continuous nature of the pneuma) he cannot maintain his former explanation of visual perception based on the pneuma. Once he accepts the assumptions of the geometrical tradition of Euclid and Ptolemy, he also accepts the fact that the eye is considered to be the vertex of a visual cone or pyramid. Therefore, the sensitive element cannot be an extended and (then) geometrically uncharacterized surface like the crystalline lens. For the same reasons, the object of perception cannot be anymore a perturbation of the air which is pneumatically reflected in the eye in terms of a modification of the transparent lens. The two systems seem to be incommensurable, and in fact—without the understanding of the process of refraction inside the eye—they are actually so.

Galen is well aware of these difficulties, and in fact dismisses most of his explanations of the pneumatic behavior of the eye. He cannot mediate the two approaches and therefore assumes a drastic geometrization of vision,
which he then literally extends to anatomy and physiology. Through a newly functional reading of the anatomy of the eye, he assumes that the central opsis (probably a Ptolemaic influence) could pass straight through the lens and get into the optical nerve which he takes to be on the axis of the pupil. Actually the beginning of the optical nerve is quite out of axis and it is scarcely believable that such an anatomical evidence would have escaped Galen. However, he then continues the drastic geometrization of anatomy even in the path of the optical nerves, which he takes to be practically rectilinear. Evidently he dismisses his former hypothesis about the sensitive lens: now the point is to have the visual impression to travel straight up to the brain, where--pneuma or not--it could be somehow "felt". However he does not say how a single "ray" could entail the information of the whole field of vision.  

The presence of the chiasma, the crossing point between the two optical nerves immediately after they come out of the brain, offered him a chance for a further geometrical interpretation of the propagation of the visual sensation through the system eyes/optical nerves/brain. In fact he reads this "X"-like pattern as two geometrical lines which intersect each other, which (according to the Book XI of Euclid's Elements) implies
that they lay on the same plane. From this he is able to recognize that:

The pupil, the root of the eye, from which the nerves begin to spread [into the retina] and, thirdly, the junction of the two optic nerves entering the anterior brain lie in a straight line. They proceed on the same plane to form the eyes in the correct position that neither pupil is higher.

Strangely enough, this personalized reading of anatomy helps him to explain binocular vision. In fact he assumes double-vision to take place when the pupils of the two eyes diverge vertically rather than laterally. With this assumption, the axes of the eyes, the two optical nerves, and the chiasma laying all on the same plane looks really teleological (or rather provvidential).

However, at this point, his functional reading of anatomy does not seem to be simply the result of a professionally-conditioned mental framework, because from the rethoric (and sometimes arrogance) of Galen's text it becomes evident that he was aware of hiding the weakness of his interpretation behind the facade of geometrical "complexities". In fact:

...Now if there is anyone who does not understand what I have said, he clearly does not know even the elements of geometry. It would be a long task if I were to write demonstrations of such things, and indeed a person would not understand these either unless he has studied a great deal beforehand.....You must, then, learn
the demonstration from Euclid and when you have learned it, come back to me and I will show you in an animal these two straight lines, the channels from the encephalon.

The attitude reminds of Aristotle curse-like criticism of those who would doubt about the indemonstrability of axioms:

The axiom is the most firmly established of all principles. It is ignorance alone that could lead any one to try to prove the axioms....If any one thought he could prove them, he could at once be refuted; if he did not attempt to say anything, it would be ridiculous to argue with him: he would be no better than a vegetable.

It's a matter of professionalism.
NOTES TO CHAPTER 6

2 M. Rostovtzeff, A Large Estate in Egypt in the Third Century B.C., Madison, 1922, p. 67. Rostovtzeff reports a letter of Zenon, the manager of the estate, to Kleon, the chief engineer of the nome, where he says that the water is high and that he is therefore obliged to open the sluices without the special permission of the engineer. The fact that the management of the irrigation system was a task of the engineers is also often stressed by M. Weber in his Agrarian Sociology of Ancient Civilization.
4 M. Rostovtzeff, A Large Estate in Egypt in the Third Century B.C., Madison, 1922, p. 49.
5 Later in the Hellenistic period and before the Roman conquest, Egyptian kings tended to reassert some of the pharaonic aura by having their figure to be associated with religious roles. They again tried to revive the concept of maat for political purposes, at least for the section of population which still followed Egyptian religion.
7 In fact the property of the land was attributed to the king only nominally. The estate Apollonius is talking about he actually his own, for the king gave it to him even though probably only for the duration of his life.
8 Strab. 793-4. Quoted in P. M. Frazer, Ptolemaic Alexandria, Oxford, 1972, p. 315. Frazer suggests that Strabo's description was based on personal knowledge.
10 M. Rostovtzeff, ibid., p. 1076.
12 A. Aaboe, "Scientific Astronomy in Antiquity", in

13 A. Aaboe, ibid., p. 37.


20 Aristotle, Rhetoric, 1355 b 26 f.

21 A more detailed analysis of the relationship between rhetoric and the criticism of traditional values and magical beliefs, see G. E. R. Lloyd, op. cit., Chapt. 1.


23 Plato, ibid., p. 252.
24 Plato, ibid., p. 253.
25 Aristotle, Topics, 100 a 27 ff.

27 Here in Chapter 5, Section 5 - Aristotle.
29 Aristotle, ibid., 100 a 13-14, p. 81.
30 Aristotle, ibid., 100 a 16-20/100 b 1-5, p. 81.


32 For the evaluation of a number of problems and different views about the notion of initial assumption see: T. L. Heath, The Thirteen Books of Euclid's Elements, 3 Vols, Cambridge, 1925, pp. 117-124.


34 Aristotle, Posterior Analytics I, 10, 76 a 31-36, quoted in T. L. Heath, op. cit., p. 117.
35 Aristotle, ibid., I, 10, 76 b 28-32.
37 W. R. Theisen, The Medieval Tradition of Euclid Optics, unpublished dissertation, University of Wisconsin, Dept. of
History, 1972, p.51, notes that: "Often the proofs are incomplete, in many unjustified statements. The reader must rely heavily on a careful interpretation of the diagrams".


41 I. Mueller, op.cit., p.15.

42 Euclid's platonism is also suggested by his work on astronomy Phaenomena of a strict eudoxian derivation, and a work on harmonics in the Pythagorean tradition. It has also been often suggested that Euclid's notion of visual ray reminds of Plato's estromissionist theory of vision, but probably such a similarity is quite nominal for it seems that Euclid adopted that position because was the less problematic within his own premises.

43 For more philological information, as well as for the transmission and the translation of Euclid's Optics in the west, see W.R. Theisen, op.cit., pp.2-51.

44 The influence of the study of scenography on the development of geometica optics has been suggested by Lindberg and Van Hooten with minor emphasis, but it has been emphasized by Lejeune and Thiessen.

45 This set of definitions is taken from W.R. Theisen, op.cit., pag.185, but others--generally including only the first seven--can be found in M.R. Cohen, I.E. Drabkin, A Source in Greek Science, Cambridge, 1966, p.257, Euclide L'Optique et la Catoprique, translation introduction and notes by P. Ver Eecke, p.1-2 and can be compared with the definitions of Theon's recension at p.57. Other translations can be also found in Ovio, L'ottica di Euclide, Milano, 1918.


47 W.R. Theisen, op.cit., p.185.

48 P. Ver Eecke, op.cit., p.1, footnote 1.

49 W.E. Theisen, op.cit., p.187. From this position he then develop Prop.3 "Every object has a certain limit [of visibility] and when this is exceeded [the object] is no longer seen." In other terms the assumption of the discreteness of the propagation of the visual ray implies that things which fall among two adjacent rays are not seen. This in turns the assumption of the discreteness of the objects because he introduced such a assumption to explain
a phenomenon of physiological and psychological nature, which does not have anything to do with objects. As a last analysis, we can say that it is the first unsatisfactory choice of the discrete pattern of emission which is the cause of several of his apparent Platonism, and of the several actual problems.


56 Ptolemy, op. cit., 12,18, quoted in A. Lejeune, op. cit., p.30 and p.64.

57 A. Lejeune, op. cit., p.64.

58 This analogy is not directly with Galen, who worked after him, but it rather suggests that Ptolemy adopted some of the features of older theories of vision developed by physicians. For instance also his notion of color as produced from the mediation of black and white reminds of Empedocles' theory of vision which to R. Siegel incorporates few information typical of the medical profession. Moreover is Empedocles the first one to think of the central visual rays as stronger, a position which is fundamental in Ptolemy and which is also found soon after in Galen. Therefore Ptolemy seems to be informed about what was the physicians' observations about vision. His notion of "the luminous" seems to incorporate elements of the stoics' pneuma, but it is not clear whether such an influence came from his study of vision or it is something he absorbed in Alexandria's eclectic philosophical context. Nevertheless it is interesting to note that such a notion seems to be half way between Aristotle and Galen, so it may be that Galen picked up from him the idea of how to use the pneuma for the explanation of vision.

59 G.E.R. Lloyd, "Observational error in later Greek science", in J. Barnes, J. Brunswig, M. Burnyeat, M. Schofield, eds. Science and Speculation, Cambridge, 1982, p.134-4. Lloyd says that Ptolemy makes some confused remarks about the possibility of distortions when objects are viewed close to the horizon. "In I 3, I i II.20ff., he says that heavenly bodies appear larger when they are near the horizon, and at I3.3ff. he explains this as due not to their being closer to the earth (which they are not), but to the evaporation of the moisture which surrounds the earth (an effect which he claims is similar to the increased apparent size of objects seen in water)."
60 G.E.R. Lloyd, op. cit., p.135, in footnote 12 he says that
Ptolemy in VIII 6, I ii 203.15ff. "draws attention..to the
discrepancies between different observers attempting to
determine, for instance, the heliacal risings and settings of
heavenly bodies."

61 Damianus says that:"Ptolemeee l'a demontre au moyen d'ap-
pareils dans son traite d'optique et l'on peut en outre
s'en rendre compte par le raisonnement." in A.Lejeune, op-
cit., p.38. The study of the rectilinear propagation was
primarily attempted through the study of shadows which was
both an astronomical problem like for instance in connect-
ion with the study of eclipses.
What is important to note is that nobody paied much atten-
tion to the penumbra that was produced during those experi-
ments and which could have been considered as a disturbing
evidence. That is also what astronomers were also doing
when dealing with eclipses. It was a paradigmatic blind-
ness.

62 Ptolemy, op.cit., 23, 29:...non videtur aliquid maius aut
minus propter multitudinem radiorum visus et paucitatem
eorum tantum, cum non fuerit diversitas quantitas radiorum
ex quantitate anguli, sed ex accumulatione et congregacione
eorum.

63 The notion of a variable density is a problematic one
once applied to the process of vision, and Ptolemy probably
tried to avoid the problems which troubled Euclid by taking
"denser" to mean "stronger" in a mechanical sense.

64 It is not clear whether or not the substituition of the
cone with the pyramid of vision was actually done by Ptole-
my or it was produced by an error of translation.

65 Ptolemy, op.cit., 80, 7-9:...cuius causa est debilitas
quae accidit ex reverberatione, et iam diximus, quod hac
de causa debiliori aspectu res apparent....

66 Ptolemy, ibid., 17, 7: Visus quoque discernit situm cor-
porum, et cognoscit eum per situm principiorum suorum, quae
iam diximus, et per ordines radiorum a visu cadentium super
illa, videlicet quae fiunt in longitudine secundum quantita-
ten radiorum, qui procedunt a capite visibilis pyramidis,
et [quae] fiunt in latitudine et profunditate secundum dis-
tantias eorum ab axe consimiles ordine.
I have primarily based the english paraphrase on Lejeune's
"le cone visuel distingue la position des corps par la po-
ositions relatives [a ce origines] des rayons qui tombent sur
ces corps." A.Lejeune, op.cit., p.87.

67 It is the first part of Definition 5 of Euclid's Optics .
See the section on Euclid in this chapter.

68 It can also be said that Ptolemy's error was in a sense
only nominal. He did not have any notion of retinal sensi-
tivity and therefore he called "visual ray" the cause of
the phenomenon he was unable to better grasp. The choice
was evidently wrong, however it took him to out of an im-
passe and allowed him for some further productive step.
Nevertheless it is also true that it was not just a nomi-
nal mistake because we cannot simply replace "retinal sensi-
tivity" for "visual ray", because such a translation would
collide with his alchemical adjustments about the relation-
ship between light and visual ray. It is also interesting
to note that the contradiction would develop around items
which were (and probably still are) untestable. The experi-
imental mapping of retinal sensitivity instead would prob-
ably show a reasonable agreement to what Ptolemy thought
the pattern of the propagation of visual rays to be.

69 The concept is probably derived from Aristotle. For a
more detailed analysis see A. Lejeune, op.cit., pp. 116-
121.

70 Other examples of related problems in A. Lejeune, ibid.
pp.119-120.

71 Ptolemy Almagest , 210. 3ff., quoted in G.E.R.Lloyd,
op.cit., p.135.

72 Other informations about Ptolemy treatment of astro-
nomical observation in comparison to other scientists see
G.E.R.Lloyd, op.cit., and Magic Reason and Experience ,
Treatment of Astronomical Observations", in Archive for His-

73 See A. Lejeune, op.cit., pp.41-51.

74 A. Lejeune, ibid., p.51.

75 See both A. Lejeune, ibid., and G.E.R.Lloyd, op.cit. .

76 G.E.R.Lloyd, ibid., p.151.


79 A. Lejeune, ibid., p.45.

80 A. Lejeune, ibid., p.40.

81 Euclide, L'Optique et la Catoprique , trans. P. Ver Eecke,

82 A. Lejeune, op.cit., p.41.

83 Stoic philosophy is not usually considered for it contrib-
utions to the development of science. However S. Sambursky,
in his The Physical World of the Greeks , New York, 1956,
and Physics of the Stoics , London, 1959, has attempted a
revaluation of the importance of the contributions to the
Stoics to the later developments of science. In particular
he emphasizes the conceptual novelties inherent to the con-
cept of pneuma.

84 The strong concern with astrology developed out of the
Stoics' determinism can be traced back also to the sympathet-
ic nature of the concept of pneuma. The pneuma is in fact
dispersed all through the cosmos like a drop of wine in the
sea: which means that everything shares something with every-
thing else. This in turn suggests that everything is influenc-
ed by everything. The Stoics found several empirical evidences of their determinism especially in the relationship between terrestrial and celestial bodies, like for instance between the moon’s motion and the tides. This sympathetic connotation of the pneuma is then found in the Stoics’ theories of vision. In fact in the same way that Empedocles was explaining vision in terms of emanations of the elements of the object of sight which could be “felt” by the correspondent elements (or element-related pores) in the organ of sense, the Stoics used the pneuma—which is the common nature of all objects—as a sympathetic link between consciousness and the environment. The relation between optics and the theory of emanations which was developed by the Presocratics, continues in a more articulated form, with the Stoics.

85 S. Sambursky, The Physical World of the Greeks, p. 135. He claims that “This tension (tonos)… is the most significant distinguishing quality of the pneuma, by force of which it becomes an entity not altogether unlike the concept of a physical field in contemporary science.” However M. Hesse in her Forces and fields, Totowa, NJ, 1965, pp. 75-77, puts much less emphasis on the analogy.


91 Philo, Quod deus sit immut., 35 (Arnim, II, 458), quoted in S. Sambursky, ibid., p. 31.

92 R. Siegel in his Galen on Sense Perception, New York, 1970, p. 39 says that “There is an indication in the Ancient Literature that the Stoics considered the Theorem of infinite thinness in order to find a compromise between the concepts of pneuma and light rays”. In fact Stoicism was basically an empiricist philosophy which had problems to accept the abstract status of geometrical objects such as lines and angles.


94 This model is re-presented much later in Descartes’ Dioptrique and Treatise of Man.

95 Alex. Aphr., De anima, 131, 32., quoted in S. Sambursky, Physics of the Stoics, p. 28.

96 Galen in fact, probably applying the pneumatic notion of tensional motion to more practical cases, proposes the difference between static and dynamic equilibrium. For instance
a man swimming against the current at the same speed of the flow of the water, looks still to somebody standing on the shore, but such a stillness, such an equilibrium is unlike the one of a object left undisturbed and therefore still. From this he says that the stillness of a limb like an arm needs to be considered like the one of the swimmer; the arm is still because it is kept still through an equilibrium of opposite forces.

97 De placitis Hippocratis et Platonis, in Opera Omnia, Kuhn ed., Vol.5.
99 "Like perceives like" is precisely found in a fragment by Empedocles. See Chapt.3,4 of this thesis. The correspondence is not casual because Galen's analogies between the structure of the sense-organs and the qualities of the object of sensation are actually sympathetic. The notion of pneuma--as it has been considered earlier--contributes to the adoption of those analogies.
100 Galen, On the Usefulness..., p. 402.
101 Galen, ibid., p. 398.
104 R.Siegel, ibid., p.78.
106 R.Siegel, ibid., pp.52-53.
108 Galen, ibid., p.502.
110 R.Siegel/ ibid., p.99.


- Before Philosophy, Chicago, 1949.
Hanson, N.R., Patterns of Discovery, Cambridge, 1958.
- Euclide et Ptoleeme, Louvain, 1948.
Lindberg, D.C., Theories of Vision from Al-Kindi to Kepler, Chicago, 1976


- The Exact Sciences in Antiquity, Providence, RI, 1957.


- A Large Estate in Egypt in the Third Century BC, Madison, 1922.


Schul, F., "Perche' l'antichita' classica non ha conosciuto il 'macchinismo'?", in De Homine, No.2-3, 1962.

Seidemberg, A., "Peg and cord in ancient Greek geometry", in


Siegel, R.E., "Theories of vision and color perception of Empedocles and Democritus: some similarities to the modern approach", in Bulletin of History of Medicine, XXXIII, 1959, pp.145-160.


Van Hoorn, Willem, As Images Unwind, Amsterdam, 1972.


- "The genesis of the concept of physical law", in Philosophical Review, LI, 1942, pp.245-279.
- "The genesis of the concept of scientific progress", in Journal of the History of Ideas, VI, 325-349.