Development and experimental verification of a revised granularity equation in electrophotography

Kenneth Riehl
DEVELOPMENT AND EXPERIMENTAL VERIFICATION OF A REVISED
GRANULARITY EQUATION FOR ELECTROPHOTOGRAFY

KENNETH RIEHL JR.

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DEVELOPMENT AND EXPERIMENTAL VERIFICATION OF A REVISED
GRANULARITY EQUATION IN ELECTROPHOTOGRAPHY

by

Kenneth Riehl Jr.

A Thesis submitted in partial fulfillment of the requirements for the degree of Bachelor of Science in the School of Photographic Arts and Sciences in the College of Graphics Arts and Photography of the Rochester Institute of Technology.

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Signature of Author

Imaging and Photographic Sciences

Certified by

Thesis Advisor

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DEVELOPMENT AND EXPERIMENTAL VERIFICATION OF A REVISED
GRANULARITY EQUATION FOR ELECTROPHOTOGRAPHY

by

Kenneth Riehl Jr.

Submitted to the
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at the Rochester Institute of Technology

ABSTRACT

An equation to describe the granularity of an electrophotographic imaging system has been developed to include the density variance attributed to the paper and toner surfaces. Granularity, \( G = \sigma_D^2 \), measurements from two conventional electrophotographic copiers were used to validate the relationship. The results show good agreement with the overall shape of the \( G \) versus density curve, however the values are not consistent with estimated toner particle size. Clustering of the toner particles is suggested as the cause of the high \( G \) values.
Acknowledgements

The author would like to express his deep appreciation to Mr. Peter Engeldrum, a part-time teacher at the Rochester Institute of Technology, and consultant: for his excellent advisement and guidance throughout the project.
Dedication

The author would like to dedicate this project to his parents for all of their love and support, and to Tracy whose loving encouragement and understanding made it bearable.
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I) Introduction

The most fundamental method used to characterize image noise is the Wiener spectra of the density fluctuations. When the Wiener Spectrum is constant at low spatial frequencies and the scanning aperture area, A, is large, the product of the variance of the density fluctuations and the aperture area is a constant, G. This relationship is known as Selwyn's Law, named after E.W.H. Selwyn who first developed it in 1935. It can be shown under these assumptions, that G is equal to the scale, or zero spatial frequency value of the Wiener Spectrum; ie $G = WS(0,0)$.

The relationship of granularity, G, or Wiener Spectrum, versus density, D, is of interest to workers in the imaging field. One of the earliest attempts at developing a G vs. D curve was by Siedentopf. According to Siedentopf's relationship:

$$G = KDa[1 + (\sigma^2/\bar{a}^2)],$$

(1)

where; $K = \log e$, $\bar{a}$

$a = \text{average area of toner particles}$,

$\sigma^2_a = \text{the variance of } a$,

the granularity increases linearly with density. This constant increase in G with density, is not observed with electrophotographic images. The data of Goren and Szczepanik, in particular, show a decrease in the measured G values as the density approaches $D_{max}$.

[see figure 1] Experimentation also suggests that there are density
fluctuations associated with the paper, or support, of the toned image, at minimum density, and at maximum density when there is enough toner to completely cover the paper. The fluctuations observed at maximum density in electrophotographic images, are probably due to the morphology of the toner pile, which is most heavily influenced by the method of fusing. These considerations have prompted a theoretical and experimental investigation into the measured granularity of electrophotographic images.

A) Theory

The total granularity can be formulated as the sum of three components: the fluctuations in density due to toner particles, the density fluctuations due to the substrate surface, and the density fluctuations due to the toner surface. Formally, this can be be represented by:

\[ G(\text{total}) = G(\text{particles}) + G(\text{toner}) + G(\text{paper}) \] (2)

where it is implied that the particle, toner, and paper fluctuations are independent.

The starting point for the derivation for the granularity vs density relationship is the reflectance equation of Castro and Lu;

\[ \bar{R} = R \exp(-\bar{n}a/A) + R [1 - \exp(-\bar{n}a/A)] \] (3)

where:

\( \bar{R} \) = the average reflectance,
\( R \) = the reflectance of the paper,
\( \bar{n} \) = the average number of toner particles,
\( a \) = the average area of the toner particles,
A = the measurement aperture area.

This equation is well suited since it contains all of the parameters of interest; eg. the toner particles and the reflectance of the paper and toner. The approximate reflectance variance for this model can be obtained by taking the partials of the reflectance in relation to the equation's four parameters; \( \bar{a}, \bar{n}, R_P, \) and \( R_T \). This is shown below:

\[
\sigma^2 = \left( \frac{\partial R}{\partial \bar{a}} \right)^2 \sigma^2_{\bar{a}} + \left( \frac{\partial R}{\partial \bar{n}} \right)^2 \sigma^2_{\bar{n}} + \left( \frac{\partial R}{\partial R_P} \right)^2 \sigma^2_{R_P} + \left( \frac{\partial R}{\partial R_T} \right)^2 \sigma^2_{R_T}
\]  

(4)

where:

\( \left( \frac{\partial R}{\partial \bar{a}} \right) \) = the first partial of \( R \) with respect to the average particle number,

\( \left( \frac{\partial R}{\partial \bar{n}} \right) \) = the first partial of \( R \) with respect to the average particle area,

\( \left( \frac{\partial R}{\partial R_T} \right) \) = the first partial of \( R \) with respect to the toner reflection at \( D_{\text{max}} \),

\( \left( \frac{\partial R}{\partial R_P} \right) \) = the first partial of \( R \) with respect to the paper reflection at \( D_{\text{min}} \),

\( \left( \sigma^2 \right) \) = the variance of the paper surface, \( D_{\text{min}} \),

\( \left( \sigma^2 \right) \) = the variance of the toner surface, \( D_{\text{max}} \).

Using the approximation:

\[
\sigma^2_D = k \sigma^2_R / R
\]  

(5)

where \( k = \log \) to the base 10 of \( e = 0.43429... \), the granularity due to the particles can be shown to be (see appendix A for complete details):
\[ G(\text{particles}) = K (1 - \frac{R}{\bar{R}})^2 \ln\left[ \frac{(R - R_0)}{(\bar{R} - R_0)} \right] \bar{a}\left(1 + \frac{\sigma}{\bar{a}}\right) \]  

Note that as \( R \) goes to zero and \( R_p \) goes to one, the equation simplifies to:

\[ G(\text{particles}) = K\bar{a}D \left(1 + \frac{\sigma}{\bar{a}}\right) \]

which is Siedentopf's granularity formula (Equation (1)).

The granularity in the toner fluctuations, using the same approach, is found to be (see appendix A):

\[ G(\text{toner}) = \frac{(AK/R)^2 - \sigma^2}{(1 - R/R_p)^2 / (1 - R_p/R)} \]  

where \( (\sigma)^2 \) is the reflectance variance of the toner surface, and \( A \) is the area measurement of the aperture. Note that this term goes to zero as the average reflectance, \( \bar{R} \), approaches the paper reflectance, \( R_p \). This term is dependent on \( A \), and does not become significant until the maximum density of the image is approached. The \( (\sigma)^2 \) parameter is largely dependent on the type of image fusing, since the toner variance is primarily dependent upon the surface properties of the sample.

The paper granularity is expressed as:

\[ G(\text{paper}) = \frac{(AK/R)^2 - \sigma^2}{(1 - R/R_p)^2 / (1 - R_p/R)} \]  

where \( (\sigma)^2 \) is the reflectance variance of the substrate. Now observe that, in relation to the \( G(\text{toner}) \) term, the paper granularity is insignificant at \( D_{\text{max}} \), but becomes important as the reflectance of the paper is approached.
The final granularity equation is written as the sum of the G’s of the parts (Equations (6), (7), and (8)):

\[
G(\text{total}) = K \left( \frac{1-R}{R} \right)^2 \ln \left( \frac{R-R}{\bar{R}-R} \right) \left( 1 + \frac{\sigma_a}{\bar{a}} \right) G(\text{particles}) \\
+ \left( \frac{AK}{\bar{R}} \right) \left( \frac{1-R}{R} \right) \left( \frac{1-R}{R} \right)^2 \left( \frac{1-R}{R} \right)^2 \sigma_p \sigma_T | \sigma_T \sigma_R |
\]

The granularity equation (5), due to only G(\text{particles}) yields G vs D curves as shown in figure 2, plotted in normalized form. Note that the granularity goes to zero at both \(D_{\text{min}}\) and \(D_{\text{max}}\). The introduction of the two new parameters of paper and toner variance are expected to change the shape of these curves to better fit the actual data.

Figures 3 and 4 are plots of normalized G(toner) and G(paper), equations (7) and (8), respectively. Note that the plot of G(toner), equation (7), becomes significant at only the high densities. This is the opposite of the G(paper) term, equation (9), which is significant at the low densities.
FIGURE 1

Data from Goren and Szczepanik
Normalized $G$ vs. $D$ curves for $G(\text{particles})$ of Reflectance Dot Model and Various Imaging Element Densities

FIGURE 2
FIGURE 3

Normalized G vs. D curves for G(toner) of Reflectance Dot Model and Various Imaging Element Densities

FIGURE 4

Normalized G vs. D Curves for G(paper) of Reflectance Dot Model and Various Imaging Element Densities
II) Experimental

A) Targets

The targets used to obtain the electrophotographic images were Munsell Neutral Value Patches. These are spectrally flat, matte patches available in 32 different density values from the Munsell Corp. They were utilized for two important reasons: 1) they are spatially, extremely uniform in density, and 2) they have small density increments between patches. Typically, electrophotographic copiers have a high contrast Dout vs Din curve, thus leaving a small margin of input density latitude to achieve uniform samples. The small density increment between patches is necessary to insure that samples of all output densities can be obtained. [see figure 5]

B) Samples

The electrophotographic copiers that were chosen to image the Munsell targets were the Xerox Models 9200 and 1075. There were two criteria that were examined when testing different systems. One was the need for the copier to produce a maximum density that completely covered the paper, and the second was a spatially uniform density output over the full density scale. The second, good toner coverage at maximum density, is needed to be able to measure the toner reflectance variance as required by the model. Many readily available copiers were surveyed, but none, other than the two Xerox copiers, were able to meet the criterion needed to experimentally assess the model.
C) Microdensitometer

The densitometer that was used to scan the samples was a reflection micro densitometer, built by the Xerox Corp. Some of the main characteristics of the instrument are listed below:

- **Efflux optical system**: 5x
- **Illumination source**: 3000 degrees K; tungsten halogen
- **Illumination angle**: 45 degrees from six positions around an annulus
- **Objective numerical aperture**: 0.116
- **Density/reflectance output**: 0.000 to 2.000
- **Movement**: X,Y motion in stepped intervals of 12.7um, controlled by a Wang Computer

The scanning aperture selection was based on three criteria; 1) the minimum densitometer electronic noise, 2) the quantization error, and 3) the estimated minimum value of the measured granularity.

These variances will set a minimum value on the G that can be measured. This can be stated as:

$$ G = A \left( \sigma_e^2 + \sigma_q^2 \right) $$

The electronic noise variance is a function of aperture size, so it was measured with an aperture that was approximated to have the same area as the final aperture, giving a variance, of 2.0E-7. The quantize error was assumed to follow a uniform probability density function with a variance $\sigma_q^2$ equal to $(.001)^2 / 12$. A minimum G of 2.0um^2 D^2 was assumed. Solving equation (10) for A, and subtracting
the appropriate values yields:

$$A = \frac{2}{\left[(2.0E^{-7}) + (0.01)/12\right]} = 7.07E+6\text{um.}$$

This area, divided by five (the system magnification), yields the effective scanning aperture area. The final effective scanning aperture diameter, of approximately 2\text{mm}, requires an actual diameter of 10\text{mm}. [For further details see appendix C] The aperture was constructed by punching a 10\text{mm} diameter hole in a piece of aluminum shim stock and then fastening this to the aperture wheel in the optical head of the microdensitometer. [This is described in detail in appendix D]

D) Measurement Procedure

Immediately after the targets were imaged on the copiers, it was evident that the middle density samples did not have the desired uniformity in density. This makes the granularity constant unrealistically high; i.e. there is an added density fluctuation that is due to gross non-uniformities, not to particles. It was also noticed, however, that each sample usually had a small area of uniform density. This uniform density area was the major premise of a procedure created to establish density limits on each target prior to scanning. The major feature of the data collection program is to scan the uniform density area of each sample, establish upper and lower cutoff density values, and then sequentially test all data points as the rest of the target is scanned.
The test operates under the assumption that there is a minimum area of 150 square mm where the mean density is constant and the fluctuations are due to particles, toner, and paper, only. Forty density data points are collected and sorted in ascending order. The highest data point is tested against the next highest. If they are the same, the highest data point, density value, is taken as the upper density limit. If they are different, the next highest density value is taken as the limit. Applying the same approach to the lowest valued data point yields the lower density acceptance limit. This procedure is equivalent to histogramming the data and taking the central 95% \((2*1/40 = 0.05)\) of the data. This method was selected to minimize the assumption regarding the probability distribution of the density data.

After the uniform area was scanned, and the upper and lower cutoff boundaries established, the rest of the target was scanned, and the mean and variance of the density fluctuations were determined. [see appendix E for further details]

Before the granularity was calculated using \(G = \sigma_D^2\), the electronic noise variance was subtracted from the final density variance of the target. A regression analysis was made for the noise by taking 1000 data points at 10 calibration plaque densities and computing the density variance. Since the microdensitometer table was off, the input signal (density) was constant, so the density variance was only due to the electronic noise in the system, and quantization. The least squares fit between electronic noise variance and density was computed, and is shown in figure 5. The goodness of fit \(R^2\) was calculated to be .9993. This equation was
incorporated into the program so that for any sample density the electronic noise is readily computed. The granularity of each sample is then found through the multiplication of the density variance with the scanning aperture area, after subtraction of the noise.
FIGURE 5

Typical Contrast Curves for electrophotographic copiers
Regression for Electronic Noise
III) RESULTS

The measured granularity, G, values for the Xerox 1075 and the 8200 are shown in figures 7 and 8. Comparison of the data with the model, equation (9), requires the parameters of average toner particle area (a), and toner area variance ($\sigma_a^2$). These could not be independently estimated so the effective area factor:

$$\tilde{a}(1 + \sigma_a^2/a)$$

was back calculated from the model, assuming the model to be correct. This area factor was calculated from:

$$[G_{\text{measured}} - G_{\text{toner}} - G_{\text{paper}}]/[K (1-R/T \bar{R})^2 \ln[(R-T)/(T \bar{R} - R_T)]]$$

for each sample. An average area was computed from the samples of each copier.

This area factor was computed to have an equivalent average diameter of 177.5μm for the 1075 data, and 178.5μm for the 8200 data. Using these average area factors and the measured paper and toner reflectance variances, the G vs D curves were calculated using equation 9. Figure 7 shows the measured data points and the model predictions for the Xerox 8200 copier. Figure 8 shows the model and measured data results for the Xerox 1075 copier.
Model Predictions and Measured Data for Xerox 8200
FIGURE 8

Model Predictions and Measured Data for Xerox 1075
IV) DISCUSSION

The G vs D curves in figures 7 and 8 show some interesting results. Figure 7 is the G vs D curve for the Xerox 8200 samples. The curve for G(total), equation (9), represented by the solid black line, fits the data reasonably well. Over the middle densities measured G values have a high variance, as evidenced by the large scattering of points. This is believed to be from the non-uniformity of the samples. If the limit selection procedure was not applied over exactly the same area for each sample, the upper and lower bounds would shift according to the mean value of the area chosen. This often changed the density variance of the sample. Even with this large density variance, the G vs D curve generated from the G(total) equation fits the data reasonably well. This is especially evident at the minimum and maximum densities. At these two regions, the G(particles) equation goes to zero, while it is clear that the data does not. The G(total) equation is a much better fit to the data, particularly in these limiting density regions.

The results of the Xerox 1075 data look virtually identical to that of the 8200. The data curves start and stop at the minimum and maximum densities, respectively, at some finite granularity value. This is again a reasonably good fit by the model. The model predictions are better over the middle densities for this copier, compared to the 8200, probably because the samples were more uniform.
Although the model fits the data well, the area factor is much too large. It was calculated that the equivalent diameter of the 8200 samples was 178.6\,\mu m, and the 1075 samples was 177.5\,\mu m. The actual diameters of the toner particles were measured under a microscope to range from about 20 to 40\,\mu m. The factor difference between the equivalent diameters and the measured diameters has been attributed to two phenomena, one is clustering, and the other is the paper spread function.

Clustering is when the toner particles are not randomly distributed, but tend to agglomerate together in groups and cause, in effect, a larger area particle. The simplest model is one due to Shaw where he assumes an average number of particles/clump of $\bar{m}$. The $G$ vs $D$ equation for completely opaque elements and perfectly transmitting base, is given as:

$$G = KD\bar{m}[1 + 1/\bar{m} + (1/\bar{m})\left(\frac{J/\bar{a}}{a}\right)^2]$$

To the first order, the toner area is increased by a factor equal to the average number of particles/clump. For the data, $a\times m = 2499$ square \mu m, but a particle diameter of 20\,\mu m has an area of 314 square \mu m. The ratio of these two yields an $m$ of 79 particles per clump. This, however does not include the paper spread function factor which can increase the area by a factor of about one to three. Assuming a paper spread factor of about 3.0, the average number of particles/cluster is calculated to be about 26 (79/3), which is not unreasonable as judged from photomicrographs of the images.
V) CONCLUSIONS

A simple model was developed to determine the granularity vs density curve for electrophotographic imaging systems. The characteristic feature of this model is an increase and then decrease in granularity, G, as the density increases. This result is contrary to Siedentopf's relationship which predicts a continuous increase in G with density. A peaking in the G vs D curve is found to be a function of the maximum density of the image (toner) elements.

Included in the model, were the contributions of the reflectance variances of the paper and toner surfaces. The data suggests that these two terms are essential to account for the variance at the minimum and maximum densities.

The model fits the data curves well when the average area is deduced from the measured data. However, the particle areas are high when compared to the basic toner and element area. Clustering models suggest that the high area can be attributed as an agglomeration of the toner particles.
VI) REFERENCES


5.) P.G.E., personnel communication.


VII) APPENDIX

Appendix A

Derivation of Granularity Equation

Starting with the basic reflectance equation:

\[ \overline{R} = \frac{R \exp(-\overline{\alpha}/A) + R (1 - \exp(-\overline{\alpha}/A))}{P} \tag{A1} \]

which was derived by Castro and Lu, equation (Al), in their paper "Reflection of Light from Toned Paper". The equation for the reflectance density variance is derived as a function of the variances of the average toner element area and number, and the variance of the paper and toner surfaces. This is shown below:

\[ \sigma^2 = (\frac{\partial R}{\partial \overline{a}})^2 (\overline{a}/A) + (\frac{\partial R}{\partial \overline{\alpha}})^2 (\overline{\alpha}/A) + (\frac{\partial R}{\partial T})^2 T + (\frac{\partial R}{\partial P})^2 P \tag{A2} \]

where the first partial are expressed as;

\[ (\frac{\partial R}{\partial \overline{a}})^2 = (R - R ) \frac{2}{T} (\overline{a}/A) \exp(-2\overline{\alpha}/A), \tag{A3} \]

\[ (\frac{\partial R}{\partial \overline{\alpha}})^2 = (R - R ) (\overline{\alpha}/A) \exp(-2\overline{\alpha}/A), \tag{A4} \]

\[ (\frac{\partial R}{\partial T})^2 = (1 - \exp(-\overline{\alpha}/A)), \tag{A5} \]

\[ (\frac{\partial R}{\partial P})^2 = \exp(-2\overline{\alpha}/A). \tag{A6} \]
Substituting $A_3-A_6$ into $A_2$, we obtain the approximate expression for the reflectance variance:

$$
\sigma^2 = (R - R_T) \exp(-2\bar{a}/\Lambda) \left[ (\bar{\sigma}/A) \sigma + (\bar{a}/\Lambda) \sigma \right]_T^P
+ \exp(-2\bar{a}/\Lambda) \sigma + (1 - \exp(-\bar{a}/A)) \sigma
$$

(A7)

The number of elements within the aperture is a Poisson random variable, where $\sigma^2 / \bar{a}$. The variance of the average area is the population variance divided by the sample size: $\sigma^2_a = \sigma^2 / \bar{a}$. Therefore:

$$
\sigma^2 \approx (\bar{R} - R_T) \exp(-2\bar{a}/\Lambda) \left[ \bar{\sigma}/\bar{a} + \bar{a} \right] + (1 - \exp(-\bar{a}/\Lambda) \sigma + \exp(-2\bar{a}/\Lambda) \sigma
$$

PR

(A8)

where $((R - R_T)/\Delta R = \exp(-\bar{a}/\Lambda)$, using $\sigma = k \bar{\sigma}/\bar{a}$, we get:

$$
\sigma^2 = k \left[ 1 - (R / \bar{R}) \right] \ln[(\Delta R / (R - R_T)) \bar{a}/A] + \bar{\sigma}/\bar{a}
$$

(D)

$$
+ k / \bar{R} \left[ (1 - (\bar{R} - R_T) / \Delta R) \right] \sigma + ((R - R_T) / \Delta R) \sigma
$$

TR

(A9)

Finally, by multiplying equation (A9) by the aperture area, $\Lambda$, the granularity, equation (9), follows:

$$
G = [1 - (R_T / \bar{R})] \ln[(R - R_T) / (\bar{R} - R_T)] k \bar{a}[1 + \bar{\sigma}/\bar{a}]
$$

(T)

$$
+ AK / \bar{R} \left[ (1 - (\bar{R} - R_T) / \Delta R) \right] \sigma + (R - R_T / \Delta R) \sigma
$$

PR

(A10)
Appendix B

Targets and Sample Generation

The input targets that were needed had to have uniform density and a small density increment between each target. The uniform density was needed to better insure uniform output density (for the calculation of \( G \)), and the small changes in density were needed to assure that some input density would fall on the straight line portion of the \( \text{Din} \) vs. \( \text{Dout} \) curve of the copier. Most copiers have high contrast \( \text{Dout} \) vs \( \text{Din} \) curves, as shown by figure 4. There must be input density on the straight line portion of this curve to produce output density that is not at a minimum or maximum for the system. This will enable the determination of granularity over the middle densities. After the input targets were obtained, they were then imaged (copied) on two electrophotographic systems, the Xerox Models 1075 and 8200.

It was originally planned to make these input targets by exposing photographic paper to various exposure levels of light. However, it was soon learned that there was already patches of varying uniform density on the market. These are the 32 matte finish Munsell Neutral Density Patches. They fit the necessary characteristics in that they were not only extremely uniform, but also had small increments in density between patches. They range in density from .0458 to 1.6021 and have a density increment ranging from .0289 to .0935. Figure 11 shows five Munsell patches and the samples that were generated from them on the Xerox 8200 copier.
Note the uneven density distribution on the middle density samples.

After the input targets were obtained, the next step was to image them on an electrophotographic copier. This was not as easy as it sounds, in that over ten different copier systems were tried before the Xerox 8200 and 1075 were decided upon. The method of testing different systems was to take a low, medium, and high density target and subjectively compare the output of different systems. Most of the copiers could not produce a high uniform Dmax. This was a critical test for the choice of systems because the project deals specifically with the Dmax area of the output. Most of the copiers were eliminated on this characteristic alone. Physically it is their inability to put enough toner on the paper to get a uniform layer at Dmax. The second major criteria for the choice of systems was the uniformity of the middle densities. This is the hardest area of electrophotographic reproduction and yielded the worst results. The Xerox 8200 was also bad in this respect, as the results show, but it was chosen for its high uniform Dmax. The best output was obtained from the Xerox 1075 which had high uniform Dmax and the best middle density reproduction range of any copier system tested.
Appendix C

Aperture Calculation

From the basic granularity equation, \( G = \frac{2}{D} \), it is apparent that the granularity constant is a function of the density variance and the scanning aperture area. Solving for \( A \), we get \( A = \frac{G}{\sigma_D^2} \).

Knowing this, and using two basic assumptions, the scanning aperture can be computed. The first assumption is that for electrophotographic imaging systems, the minimum granularity was assumed to be \( 2 \mu m^2 D^2 \). The second assumption is made in the calculation of the smallest measurable variance to yield the largest usable aperture area. It is known that variances add, so that the final variance term will be composed of two smaller variances; the variance due to the quantizer and the electronic noise. The quantizer variance is calculated by taking the difference between the quantized levels \((.001)\), squaring this and then dividing by twelve. This relationship is true providing a uniform distribution is assumed for the quantizer error. Solving for \( A \) will yield the final effective aperture area, but to find the actual physical size, the effective area must be multiplied by the magnification of the system (5x). Using these values, the calculation of the scanning aperture area and diameter is as follows:

\[
\begin{align*}
\text{Quantizer Variance} &= (.001)^2/12 \\
\text{Assumed Electronic Noise} &= (.005) \\
\text{Minimum Granularity} &= 2\mu m^2 D^2 \\
A &= \frac{G}{\sigma_D^2}
\end{align*}
\]
\[ A = 2/\{0.001\}^2/12 + (0.005)^2 \times 1 = 7.973E+4 \text{ um}^2 \]

\[ \pi r^2 = 7.973E+4 \text{ um}^2 \quad r = 0.16\text{mm} \]

Effective Diameter = 0.32 mm
Actual Diameter = 0.32 * 5 = 1.6 mm

The calculated aperture diameter of 1.6 mm was used as a starting point for which to get an accurate measurement of the minimum electronic noise in the system. The noise is dependent on aperture area so a pre-existing aperture that was close in area to that of the aperture calculated above was used to measure the electronic variance. This was done by measuring the variance of the densities on the ten calibration plaque densities, over time. The minimum electronic noise was measured to be 2.0E-7. This was then input back into the calculations above, and yielded a new maximum aperture diameter of approximately 15 mm, 3 mm effective. The final aperture selected had a 2 mm effective diameter.
Appendix D

Aperture Construction

From appendix C, it is known that an aperture diameter of 10mm, yielding an effective aperture diameter of 2mm, is needed for the final scanning aperture of the system. The was constructed out of a five inch by five inch piece of aluminum shim stock, .002 inches thick. The microdensitometer's optical head was then taken apart to provide access to the aperture plate. This plate has eight apertures on it, the two largest being apertures eight and seven, having a one inch diameter hole and a 1.27 by 1.27 cm square hole, respectively. The attenuation for each aperture on the wheel was different, so aperture seven was chosen as the opening over which to place the new aperture because it was closer in area to the new aperture. The finished aperture is shown below in figure 9.

FIGURE 9

Photograph of Finished Aperature
The new metal shim stock was cut to fit over aperture seven, and a 10mm diameter hole was punched through the center to become the new aperture. The finished aperture is shown below. This was then taped on the aperture wheel over aperture seven. The final installation is shown in figure 10, with the new aperture on the top center, aperture eight to the right, and a smaller aperture slit in the lower corner.

FIGURE 10
Photograph of Aperture in Densitometer
Appendix E

Data Collection

1.0 Objective

The objectives of the computer program were: a) to set up a density acceptance bounds, b) to automatically scan a target without any outside interaction, c) and to compute the necessary statistics for the computation of G. Also, remembering that the electronic noise is additive with the target density variance, a noise vs density relationship had to be experimentally determined to calculate the noise at any given target density. This enabled for the correction of the measured sample variance.

2.0 Program Description

The first task was to make the microdensitometer scan a target automatically. A characteristic of this particular microdensitometer is that it operates by taking discrete data points over a given number of steps, where one step is 12.7um. Knowing this, and the scanning aperture diameter, the number of steps needed to separate the data points, insuring no overlap, can easily be calculated. Then, the target dimensions, length and width, were converted into aperture steps, yielding the number of available data positions on the horizontal and vertical.
With the automation solved, the data selection technique had to be constructed. The need for this arises from the practical reality that many of the electrophotographic targets were far from uniform. [see figure 11] This problem was solved by implementing a density selection test.

An assumption was made that if the samples had a uniform area where 40 or more data points could be collected, then this area would serve as the standard of accepting or rejecting all other densities over the sample. The methodology was that 40 densities would be recorded over this uniform area. They were then put into a frequency histogram array, and arranged from Dmin to Dmax. An alpha risk of .05 (.025 for each tail) was used to determine the cutoff frequency of one, for both ends. Thus if the high density frequency was only one, then the density occupying the frequency bin below that bin would be the cutoff Dmax. If there was a density frequency of two or more in the upper bin then that bin becomes the cutoff. The same happens for the Dmin level cutoff. Note that $1/40 = .025$, which was the basis for selection of this number.

This test is probably the most critical factor in determining the samples' measured granularity. The range of accepted densities over the sample is determined by the data selection technique, which establishes density boundaries on an observed uniform area on the sample. The uniform area has to be chosen with extreme care so that: 1) it can be found again (for subsequent scans of the sample, if required) and 2) it represents the lowest density range on the sample. If the boundary is enlarged, the range of accepted densities is increased, hence increasing the density variance. This change in
variance is directly related to the final targets' granularities, from the computation of $G$, $(G = \sigma_D^2)$.

Early attempts to understand the high $G$ values prompted a detailed investigation into the method of establishing the density limits. An error was found in the code and the limit setting procedure was subsequently revised. Due to the strong dependence of the measured granularities on this procedure, the density limits were improved to another significant figure (from .01 to .001). A bubble sort method was implemented, replacing the frequency histogram. This sorts the data from low to high densities, and then performs the same statistical logic that was used in the frequency histogram. The Wang Computer only allowed for a frequency array dimensioned at 200 values, thus the density could only range from 1 to 2 with a delta $D$ of .01. The bubble sort method needs only an array with 40 positions thus enabling the data bounds to be of the exact same precision (.001) as the incoming densities.

After the density acceptance bounds are established, the rest of the target is scanned. Again, due to the Wang's memory limitations, the large quantities of data could not be stored as individual data points. The problem was solved by immediately summing the densities after each horizontal scan. First they are stored in an array, then checked to be within the acceptance boundaries, and if they are, they are summed and sum squared. This process is repeated for each horizontal scan of the target. At the end of the scanning process the mean and variance of the target density is computed. Before the granularity is computed the electronic noise at that target density is calculated via an
empirical noise-density function, and subtracted from the sample's density variance. Multiplication of this converted variance by the aperture area, A, yields the sample granularity, G.

3.0 Electronic Noise Calculation

The electronic noise is calculated given the target mean density. The variance was calculated from 1000 data points taken over time for ten different densities. The least squares fit between the electronic noise and the average density is shown in figure 6, and written as:

$$\sigma^2 = 3.028E-7 \exp(1.4986D + 0.5126D^2)$$

where: $$\sigma^2$$ = the electronic noise variance

$$D$$ = the average density

The goodness of fit ($$R^2$$) for this regression is .9993.

4.0 Program Output

Figure 12 is typical of the data outputs that were computed by the data collection program. A brief explanation follows. The first numbers of the I.D. are the day and month of the data acquisition and the next number is the target patch Munsell Value number. If the I.D. has "dup" labeled before the patch number, this means simply that the same patch is being scanned again. The discrepancy between which copier's data (8200 or 1075) was being used was solved by only scanning one of the two sets of data on any one day. The copier that was being utilized each day is recorded in the
laboratory notebook. Some of the other data on the print out are: probability of upper and lower bounds, number of data points used, mean density, variance, electronic noise, and granularity.

5.0 Data Collection Procedure

An outline on the standard procedure used for the data collection on the Xerox Macro/Micro Densitometer is as follows.

A. Warm-up 1/2 hour before using

B. Calibration of the microdensitometer
   1. place calibration standard on table
   2. focus on standard
   3. move aperture to center of zero adjust patch
      a. switch trigger selector to internal
      b. switch function selector to density
      c. filter select should be on green
      d. aperture select should be #7 (new aperture)
   4. adjust high voltage knob to read .07 +/- .005 on meter
   5. switch function select to dark current
   6. switch filter knob to D
   7. adjust dark current knob to obtain a meter reading of .001 +/- .002
   8. switch function select to density
   9. switch filter to green
  10. adjust master knob to a meter reading of .070 +/- .001
  11. switch trigger selector to external

C. Load collection program "Den 3" into Wang from tape

D. Run program
1. place sample on platen under cross hairs on viewing screen

2. focus on sample

3. input length and width of the uniform density area to be scanned when prompted by program
   a. move the sample to align the bottom right of the uniform area in the bottom left of the viewing screen
   b. hit return to automatically scan the uniform area

4. input sample I.D.

D. Scan Sample

1. move the bottom right of the sample to the bottom left of the viewing screen

2. input the length and width of the target

3. hit return to automatically scan target

4. if another scan desired input "yes"; goes to section of "input target I.D."

5. if done, input "no"; program ends
FIGURE 11
Photograph of Munsel Patches and 8200 Samples

ID: 3/18-1075 9.0
LOWER LIMIT = .132
UPPER LIMIT = .167

NUMBER OF DATA POINTS = 260
AVERAGE DENSITY = .1504576923077
THE ELECTRONIC NOISE = 3.83850674E-07
AVERAGE REFLECTANCE = .70720009068
VARIANCE = 9.06915725E-05
STANDARD DEVIATION = 9.52321230E-03
GRANULARITY = 261.4855732951

FIGURE 12
Example of Program Output
Appendix F

Program Listing

2 REM THIS PROGRAM WRITTEN TO CALCULATE GRANULARITIES FOR ELECTROPHOTOGRAPHIC IMAGING SYSTEMS.
4 REM WRITTEN BY KEN RIEHL, WITH PETER ENGELDRUM AS ADVISOR
5 REM CHANGED LINE 1240 TO HOLD 12 PATCHES ON 1 TAPE 2-10-81

6 REM THIS EDITION FINISHED 2/10/84
10 REM M/M SUBROUTINES PACKAGE 1 PART PROGRAM BY J.P. 8-29-78
20 COM Z7:SELECT PRINT 005:PRINT HEX(030AOA):IF Z7 >0 THEN 1
30 COM M8$6,W8$6,R8$5,S8$10:REM TABLE (MOVE, WAIT) AND METER READ
40 W8$=HEX(00000000000099):S8$=HEX(0D):REM SET WAIT, STATUS
50 COM B8,M8,N8,X8,Y8:R8,B8,M8,N8=1:REM BACKGROUND, MAG, LOCATION
60 COM R8(200),F8(1),R8:R8=1:REM ()'S TO STORE READINGS
70 REM TO USE DEFFN'1 SET R8()>=30 TO USE DEFFN'4 SET F8(150)
80 COM D8$30,K8$,M9$,S9$,F9$,CO:REM DATE, KEY, MESS, CALIB
90 COM I8$(12,2)11,L8$(12,2)2,I8$:REM ID. & FORMAT TEST
100 COM T8$(4)64, T8, U8: INIT(AA) T8$(()): T8, U8=1:REM TAPE SETUP
110 REM
120 REM PLACE MAIN PROGRAM BETWEEN LINES 130 & 4000
130 DIM L(2)
140 INPUT "CALIBRATE THE MICRODENSITOMETER, PRESS ANY KEY TO CONTINUE", Y
150 REM D1=DIAMETER OF APERTURE IN STEPS
160 D=1916
170 D1=D/12.7
180 N1=3000: S3=INT(D1+5)
190 REM S4=ACTUAL DIST. BETWEEN DATA POINTS
200 S4=S3*.0127
210 INPUT "PATCH ID", I1$
220 INPUT "PLACE TARGET ON TABLE; PRESS ANY KEY TO CONTINUE", W
230 INPUT "LENGTH AND WIDTH OF UNIFORM AREA (MM)", X, Y
240 INPUT "MOVE THE UNIFORM AREA TO THE BOTTOM LEFT OF THE VIEWING SCREEN; PRESS ANY KEY TO CONTINUE", W
250 X=INT(X/S4)
260 Y=INT(Y/S4)
270 REM X*Y>40 IF NOT GET LARGER AREA
280 IF X*Y>=40 THEN 310
290 PRINT "SELECT ANOTHER AREA"
300 GOTO 230
310 C=0
320 R8=1
GOSUB '1(1,0,1)
R8=1
FOR J=1 TO Y
GOSUB '1(S3,0,X)
C=C+X
IF C>40 THEN 420
GOSUB '0(0,-S3)
GOSUB '0(-(X-1)*S3,0)
NEXT J
I1=0
FOR J=1 TO 39
IF R8(J)>=R8(J+1) THEN 500
T1=R8(J)
T2=R8(J+1)
R8(J)=T2
R8(J+1)=T1
I1=1
NEXT J
IF I1=1 THEN 420
IF R8(1)=R8(2) THEN 550
L(2)=R8(2)*.001
GOTO 560
L(2)=R8(1)*.001
IF R8(39)=R8(40) THEN 590
L(1)=R8(39)*.001
GOTO 600
L(1)=R8(40)*.001
SELECT PRINT 215(80)
PRINT "---------------*
ID: ",I1$
R8=1
PRINT "LOWER LIMIT =",L(1)
PRINT "UPPER LIMIT =",L(2)
PRINT "*
SELECT PRINT 005(64)
L(1)=L(1)*1000
L(2)=L(2)*1000
INPUT "LENGTH AND WIDTH (MM) OF THE TARGET TO BE SCANNED ; PRESS ANY KEY TO CONTINUE",F,T
INPUT "MOVE THE TARGET TO THE BOTTOM LEFT HAND CORNER AS SEEN THROUGH THE SCREEN; PRESS RETURN",Y
F=INT(F/S4)
T=INT(T/S4)
S1=0
S2=0
C=0
R8=1
FOR S=1 TO T
GOSUB '1(S3,0,F)
FOR N=1 TO F
IF R8(N)<L(1) THEN 870
IF R8(N)>L(2) THEN 870
S1=S1+R8(N)
S2=S2+R8(N)*R8(N)
C=C+1

IF C=N1 THEN 930
NEXT N
GOSUB '0(0,-S3)
GOSUB '0(-(F-1)*S3,0)
R8=1
NEXT S
SELECT PRINT 215('0)
PRINT "NUMBER OF DATA POINTS =",C
M1=S1/C*.001
REM ELECTRONIC VARIANCE CORRECTION
V=3.02B3E-7*EXP((1.4986+0.512694*M1)*M1)
B1=((C*S2-S1*S1)/(C*(C-1)))*.000001-V
IF B1<0 THEN 1120
C=.7B5398*D*D*B1
B2=SQR(B1)
PRINT "AVERAGE DENSITY =",M1
PRINT "THE ELECTRONIC NOISE = ",V
PRINT "AVERAGE REFLECTANCE = ",10**(M1)
PRINT "VARIANCE = ",B1
PRINT "STANDARD DEVIATION = ",B2
G=.7S398*D*D*B1
PRINT "GRANULARITY = ",G
SELECT PRINT 005(64)
IF A*="Y" THEN 210
IF A$<="N" THEN 1080
END
PRINT "THE VARIANCE IS NEG."
SELECT PRINT 005(64)
GOTO 1080
DEFFN'0(X9,Y9):REM TABLE MOVE
X7=INT(ABS(X9*M8)+.5):X9=SGN(X9)
IF X7 >0 THEN 5030:X9=1
Y7=INT(ABS(Y9*N8)+.5):Y9=SGN(Y9)
IF Y7 >0 THEN 5050:Y9=1
STR(M8*,6)=HEX(99)
BIN(STR(M8*,5))=Y9+(X9+3)/2
BIN(STR(M8*,4))=Y7-256*INT(Y7/256)
BIN(STR(M8*,3))=INT(Y7/256)
BIN(STR(M8*,2))=X7-256*INT(X7/256)
BIN(M8*)=INT(X7/256)
SELECT PRINT 4EE:PRINT M8*:SELECT PRINT 005
X7=X7*SGN(X9)/M8:X8=X8+X7
Y7=Y7*SGN(Y9)/N8:Y8=Y8+Y7
RETURN
DEFFN'1(A9,B9,C9):REM PUT C9 READINGS IN R8()
DATA SAVE /4EE,W8$:A6=EXP(7)
$G10/25A(C610,S8$)R8$:CONVERT R8$TO A6
R8(R8)=A6/B8
IF C9< 2 THEN 5290
GOSUB '0(A9,B9):SELECT PRINT 4EE
B6=R8+1
C6=R8+C9-1:GOTO 5240
PRINT M8$:X8=X8+X7:Y8=Y8+Y7:B6=B6+1
PRINT W8$:A6=EXP(7)
$G10/25A(C610,S8$)R8$:CONVERT R8$TO A6
R8(B6)=A6/B8
IF B6<=C6 THEN 5230
SELECT PRINT 005
R8=R8+C9:RETURN
VIII) VITA

Kenneth Riehl Jr. grew up in the Hudson River Valley, in Pleasantville, N.Y. He attended Briarcliff High School from 1976 to 1980. He graduated in the top half of his class, with special honors in photography. In September of 1980 he entered the Rochester Institute of Technology in the major of Photographic Science and Instrumentation. As of the time of this writing he is expected to graduate on May 19, 1984 from the program of Imaging and Photographic Science, and pursue a career in the imaging sciences.