An Active FET Receiver Front End Mixer

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AN ACTIVE FET RECEIVER FRONT END MIXER

by

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ABSTRACT

In many recent receiver designs, the front-end mixer determines the receiver's sensitivity and its susceptibility to distortion from large input signals. Two balanced FET mixers are developed for such an application; a common source configuration and a common gate configuration. Each exhibits a range of 110 dB between the available input power necessary for $10 \, \text{dB}_{\text{S+N}}^{\text{N}}$ sensitivity and the available input power of a two-tone signal which causes intermodulation products down 40 dB from the desired signals.

An expression for the conversion transconductance of a junction FET used as a mixer is first derived from the device transfer characteristic equation. The advantages of a balanced mixer configuration are then discussed.

A noise equivalent circuit which describes the sources of noise in each of the proposed mixers is developed. This leads to the calculation of noise factor and hence, sensitivity of the common source mixer as a function of $R_s$, the generator resistance.

The dependence of distortion on $R_s$ for the common source mixer is then found and plotted. A comparison of this plot and the sensitivity vs. $R_s$ plot shows that the maximum dynamic range results from a choice of $R_s$ equal to 200 ohms.

The dynamic range of the common gate mixer is investigated and found to be equal to that of the common source mixer. The sensitivity, however, is found to be 6 dB worse.
The effects of frequency dependent terminations on the mixer ports are examined and found to be quite important to mixer performance. The bias problem is solved with provision made for the variation in parameters from device to device.

The methods of testing the mixer are then discussed and test results are shown to agree excellently with the results predicted by the theoretical analysis.

A brief description of FET operation and an observation on the calculation of noise factor are presented in the appendices.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>viii</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>2</td>
</tr>
<tr>
<td>II. THE FET AS A MIXER</td>
<td>5</td>
</tr>
<tr>
<td>Development of the Junction FET</td>
<td>5</td>
</tr>
<tr>
<td>Calculation of Conversion Transconductance</td>
<td>5</td>
</tr>
<tr>
<td>Balanced Mixers</td>
<td>10</td>
</tr>
<tr>
<td>III. NOISE ANALYSIS</td>
<td>12</td>
</tr>
<tr>
<td>Sensitivity and Noise Factor</td>
<td>12</td>
</tr>
<tr>
<td>Sources of Noise in FET's</td>
<td>15</td>
</tr>
<tr>
<td>Noise Equivalent Circuit</td>
<td>17</td>
</tr>
<tr>
<td>Noise Factor Calculation</td>
<td>19</td>
</tr>
<tr>
<td>Sensitivity vs. $R_s$ for Common Source Mixer</td>
<td>23</td>
</tr>
<tr>
<td>Sensitivity of Common Gate Mixer</td>
<td>24</td>
</tr>
<tr>
<td>Summary of Noise Analysis</td>
<td>28</td>
</tr>
<tr>
<td>IV. LARGE SIGNAL HANDLING CAPABILITY</td>
<td>30</td>
</tr>
<tr>
<td>Intermodulation and Crossmodulation Distortion</td>
<td>30</td>
</tr>
<tr>
<td>Balanced Mixer Distortion</td>
<td>36</td>
</tr>
<tr>
<td>V. DYNAMIC RANGE COMPARISON</td>
<td>39</td>
</tr>
<tr>
<td>iv</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>Comparison</td>
<td>39</td>
</tr>
<tr>
<td>Bandwidth Check</td>
<td>40</td>
</tr>
<tr>
<td>VI. FREQUENCY DEPENDENT TERMINATIONS</td>
<td>41</td>
</tr>
<tr>
<td>VII. BIASING AND CIRCUIT COMPONENTS</td>
<td>43</td>
</tr>
<tr>
<td>Component Sensitivity</td>
<td>50</td>
</tr>
<tr>
<td>VIII. EXPERIMENTAL PROCEDURE AND RESULTS</td>
<td>53</td>
</tr>
<tr>
<td>Results</td>
<td>53</td>
</tr>
<tr>
<td>Discussion</td>
<td>58</td>
</tr>
<tr>
<td>IX. CONCLUSIONS</td>
<td>64</td>
</tr>
<tr>
<td>APPENDIX I</td>
<td>66</td>
</tr>
<tr>
<td>Brief Description of FET Operation</td>
<td>66</td>
</tr>
<tr>
<td>APPENDIX II</td>
<td>70</td>
</tr>
<tr>
<td>An Observation on the Calculation of Noise Factor</td>
<td>70</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>75</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Common Source FET Balanced Mixer</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Common Gate FET Balanced Mixer</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Simple FET Mixer</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>Noise Equivalent Circuit of FET</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>Balanced Mixer Noise Equivalent Circuit</td>
<td>19</td>
</tr>
<tr>
<td>6</td>
<td>Section of Noise Equivalent Circuit</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>Input Conductance, $g_s$</td>
<td>23</td>
</tr>
<tr>
<td>8</td>
<td>Dynamic Range of Common Source Balanced Mixer</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>FET Equivalent Circuit</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>FET Equivalent Circuit with Shorted Output</td>
<td>26</td>
</tr>
<tr>
<td>11</td>
<td>I.M. Distortion</td>
<td>31</td>
</tr>
<tr>
<td>12</td>
<td>Distortion Model of Mixer</td>
<td>33</td>
</tr>
<tr>
<td>13</td>
<td>U222 Characteristic</td>
<td>44</td>
</tr>
<tr>
<td>14</td>
<td>FET Biasing Example</td>
<td>46</td>
</tr>
<tr>
<td>15</td>
<td>Bias Circuit</td>
<td>47</td>
</tr>
<tr>
<td>16</td>
<td>U222 Characteristic</td>
<td>48</td>
</tr>
<tr>
<td>17</td>
<td>Improved Bias Circuit</td>
<td>49</td>
</tr>
<tr>
<td>18</td>
<td>Input Transformer</td>
<td>52</td>
</tr>
<tr>
<td>19</td>
<td>Output Transformer</td>
<td>52</td>
</tr>
<tr>
<td>20</td>
<td>Sensitivity and Noise Factor Test</td>
<td>54</td>
</tr>
<tr>
<td>21</td>
<td>I.M. Test</td>
<td>55</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>22</td>
<td>Crossmodulation Test</td>
<td>56</td>
</tr>
<tr>
<td>23</td>
<td>Spurious Response Test</td>
<td>59</td>
</tr>
<tr>
<td>24</td>
<td>Spurious Response Data</td>
<td>61</td>
</tr>
<tr>
<td>25</td>
<td>Spurious Response Data</td>
<td>62</td>
</tr>
<tr>
<td>26</td>
<td>Spurious Response Data</td>
<td>63</td>
</tr>
<tr>
<td>I</td>
<td>N Channel FET</td>
<td>67</td>
</tr>
<tr>
<td>II</td>
<td>FET Characteristics</td>
<td>68</td>
</tr>
<tr>
<td>III</td>
<td>FET Transfer Characteristic</td>
<td>69</td>
</tr>
<tr>
<td>IV</td>
<td>Input Noise Equivalent Circuit</td>
<td>70</td>
</tr>
<tr>
<td>V</td>
<td>Two-Port Noise Equivalent Circuit</td>
<td>71</td>
</tr>
<tr>
<td>VI</td>
<td>Improved Input Noise Equivalent Circuit</td>
<td>73</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Noise Factor Parameters</td>
<td>24</td>
</tr>
<tr>
<td>II</td>
<td>Mixer Data</td>
<td>57</td>
</tr>
</tbody>
</table>
# LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DS}$</td>
<td>FET drain to source voltage</td>
</tr>
<tr>
<td>$V_{GS}$</td>
<td>FET gate to source voltage</td>
</tr>
<tr>
<td>$V_P$</td>
<td>FET pinchoff voltage</td>
</tr>
<tr>
<td>$I_D$</td>
<td>FET drain current</td>
</tr>
<tr>
<td>$I_{DSS}$</td>
<td>FET drain current with $V_{GS} = 0$</td>
</tr>
<tr>
<td>$\ell$</td>
<td>the FET bias voltage $V_{GS}/V_P$</td>
</tr>
<tr>
<td>$g_{mc}$</td>
<td>conversion transconductance</td>
</tr>
<tr>
<td>$G_c$</td>
<td>conversion power gain</td>
</tr>
<tr>
<td>$R_s$</td>
<td>generator or source resistance</td>
</tr>
<tr>
<td>$F$</td>
<td>noise factor</td>
</tr>
<tr>
<td>$S_i/N_i$</td>
<td>input signal to noise ratio</td>
</tr>
<tr>
<td>$S_o/N_o$</td>
<td>output signal to noise ratio</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>a specific $(S_o + N_o)/N_o$</td>
</tr>
<tr>
<td>$I_{dn}$</td>
<td>thermal noise output current</td>
</tr>
<tr>
<td>$G_{11}^s$</td>
<td>input conductance of common source FET</td>
</tr>
<tr>
<td>$g_{11}$</td>
<td>input conductance of FET in either configuration</td>
</tr>
<tr>
<td>$I_{gs}$</td>
<td>shot noise of the gate leakage current</td>
</tr>
<tr>
<td>$I_{gn}$</td>
<td>capacitively coupled gate noise</td>
</tr>
</tbody>
</table>
\( F_0 \) optimum noise factor

\( g_0 \) source conductance which yields \( F_0 \)

\( R_{So} \) bias resistor

\( V_{GS0} \) dc bias voltage \( V_{GS} \)

\( R_{I.M.} \) ratio of amplitude I.M. products to amplitudes of desired signals

\( V_{I.M.} \) voltage for each tone of a two-tone signal which causes \( R_{I.M.} = 40 \) dB

\( V_{xmod} \) voltage of AM signal which produces 1% crossmodulation

\( P_{I.M.} \) available power of a two-tone signal of amplitude \( V_{I.M.} \).
I. INTRODUCTION

The radio frequency spectrum has become quite crowded with signals ranging from those which are barely detectable to those which originate in nearby high power transmitters. The need for receivers which will detect a very weak signal without being distorted by large undesired signals is, therefore, apparent. This problem is especially prevalent in military communications because several systems often must operate from within a small area.

This characteristic of a receiver, called its dynamic range, has received much attention in recent receiver designs. One significant improvement has resulted from the elimination of R.F. amplifiers with their poor selectivity, and hence, susceptibility to distortion from large undesired signals. This, however, has left the burden of sensitivity and signal handling capability to the receiver's front end mixer.

Wide dynamic range mixers have lately been developed utilizing low noise, excellent square law devices, such as Schottky diodes and field effect transistors (FET). Presented in this paper is such a mixer which was developed for use in the 2 MHz - 12 MHz band with an i.f. frequency of 26.5 MHz.

The performance of two balanced mixer configurations will be analyzed and compared. For convenience, full schematics of the two proposed configurations are presented in Figures 1 and 2.
For a description of the components see Section VII.
For a description of the components see Section VII.
Development of the Junction FET

The junction-gate FET was first described in a paper by W. Shockley in 1952. Earlier work had been done on field effect devices in the 1930's by J. E. Lillienfeld and in the latter 1940's by Shockley, but these devices did not exhibit good enough performance to be practical. During the war the field effect devices did not receive much attention, because the emphasis at that time was on the development of the point contact transistor and other solid state devices which apparently were an outgrowth of the early work by Shockley.

An excellent approximation to the junction FET transfer characteristic, equation 1, was derived by R. D. Middlebrook, and it is this expression which will be used in the following calculations.

\[ I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \] (1)

For a brief review of FET operation, see Appendix I.

Calculation of Conversion Transconductance

An expression for the conversion transconductance of an FET used as a mixer can now be derived using equation 1. One possible circuit is shown in Figure 3. The bias must first be set at \( V_{GS}/V_P = \rho \), where \( \rho \) is such that the input signals will not drive the device into saturation or cutoff. If two signals, \( S_1(t) \) and \( S_2(t) \), are now injected on the
Simple FET Mixer

Figure 3
FET gate with r.m.s. amplitudes $A_1V_P$ and $A_2V_P$,

\[ S_1(t) = A_1V_P \cos \omega_1 t \]  
\[ S_2(t) = A_2V_P \cos \omega_2 t \]  

the gate to source voltage can be written

\[ V_{GS} = V_P + A_1V_P \cos \omega_1 t + A_2V_P \cos \omega_2 t \]  

Substituting into equation (1) yields

\[
\frac{I_D}{I_{DSS}} = (1-\rho + A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2
\]

\[ = (1-\rho)^2 + 2(1-\rho) (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)
+ (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 \]  

Examining the third term only

\[ I_D \propto (A_1 \cos \omega_1 t)^2 + 2A_1A_2 \cos \omega_1 t \cos \omega_2 t
+ (A_2 \cos \omega_2 t)^2 \]  

Remembering an identity from trigonometry we can express the second term in equation 6 as

\[ I_{DSS} \propto A_1A_2 \left[ \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t \right] \]  

Equation 7 shows that a frequency conversion has taken place. Two
new frequencies \((\omega_1 + \omega_2)\) and \((\omega_1 - \omega_2)\) have been created with amplitude \(A_1A_2\). The complete expression for the drain current is now

\[
\frac{I_D}{I_{DSS}} = (1-\rho)^2 + 2(1-\rho) (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)
+ (A_1 \cos \omega_1 t)^2 + (A_2 \cos \omega_2 t)^2
+ A_1A_2 \left[ \cos(\omega_1 + \omega_2) t + \cos(\omega_1 - \omega_2) t \right]
\]

(8)

If \(S_1(t)\) were a local oscillator signal, L.O., of constant amplitude and if \(S_2(t)\) were a desired R.F. signal, the output at a desired intermediate frequency \((\omega_1 - \omega_2)\) would be

\[
I_{\text{Di.f.}} = I_{DSS} A_1A_2 \cos(\omega_1 - \omega_2) t
\]

(9)

As expected the i.f. current is proportional to the amplitudes of both input signals.

The conversion transconductance, \(g_{mc}\), is defined as the ratio of the output i.f. current to the desired input voltage.

\[
g_{mc} = \left| \frac{I_{\text{Di.f.}}}{A_2 V_p} \right|
\]

(10)

Substituting for \(I_{\text{Di.f.}}\) from equation 9

\[
g_{mc} = \frac{I_{DSS} A_1}{V_p}
\]

(11)

The conversion gain of such a mixer would be
\[ G_c = \frac{\text{i.f. power out}}{\text{available r.f. power in}} \]  \hspace{1cm} (12)

\[ \left| \frac{I_{\text{Di.f.}}}{2} \right|^2 \frac{R_L}{(\frac{A_2V_p}{2})^2} \frac{1}{R_s} \]  \hspace{1cm} (13)

\[ G_c = 4 \ g_{mc}^2 \ R_L \ R_s \]  \hspace{1cm} (14)

Where \( R_s \) is the source resistance.

More specifically, a Siliconix U222 FET with the following parameters:

* \( V_p = 8V \)

* \( I_D = 130 \text{ ma} \)

\( R_s = 100 \text{ ohms} \)

\( R_L = 2500 \text{ ohms} \)

when used as a mixer with a one volt L.O., \( A_1 = 1/8 \), would from equations 11 and 14, have a conversion transconductance of

\[ g_{mc} = 2000 \text{ micromhos} \]  \hspace{1cm} (16)

and a conversion gain of

\[ G_c = 4 \text{ or } 6 \text{ dB} \]  \hspace{1cm} (17)

*From manufacturers specification sheet U221 - U222.*
Balanced Mixers

Several improvements can be made on the above mixer by utilizing a balanced configuration. Two possible circuits of this type are shown in Figures 1 and 2.

Perhaps the most significant improvement is the cancellation of noise from the L.O. port which includes voltages at the r.f., i.f., and image frequencies.

Referring to Figures 1 and 2 we see that the i.f. signal is coupled out of the circuit through a push-pull transformer. Therefore, the i.f. currents resulting from the two halves of the balanced circuit must be in the proper phase or cancellation will take place within the transformer. This is exactly what happens to the three noise signal components from the L.O. port. Since the r.f. noise signal and the L.O. signal are both single ended inputs, they result in drain currents at the i.f. frequency which cancel. The same is true of the image frequency noise component. The i.f. frequency component results in i.f. drain currents without being mixed. It also cancels, though, because the input is single-ended, while the output is push-pull. This cancellation makes it unnecessary to add high and low pass filters at the L.O. port, which is indeed an attractive result, for in some cases practical filters can not be realized that will pass the L.O. signal and reject the r.f., i.f. and image frequencies.

In this same manner the L.O. signal is isolated from the r.f. port, greatly reducing the amount of local oscillator power which
reaches the antenna. With suitable filters at the r.f. port, radiation at the L.O. frequency ceases to be a problem.

The r.f. signal is, of course, also isolated from the L.O. port. If it were not, filters would have to be added which would prevent dissipation of r.f. power. Similarly the i.f. signal is isolated from the L.O. port again preventing power loss.

The advantages of a balanced configuration, therefore, make its use in front end mixer designs very desirable. Indeed, the cancellation of L.O. noise is most significant, but in order to optimize the noise performance of the complete balanced circuit, the analysis in Section III is necessary.
III. NOISE ANALYSIS

Sensitivity and Noise Factor

Before the noise performance of the two proposed circuits can be investigated, an understanding of noise factor, sensitivity, and the relationship between the two is necessary.

The noise factor of a two-port network is defined as the input signal to noise ratio divided by the output signal to noise ratio.

\[ \text{Noise factor} = \frac{S_1/N_1}{S_o/N_o} = F \]  \hspace{1cm} (18)

It is, therefore, a measure of the degree of degradation of signal to noise ratio by the two-port network. For example, a transistor amplifier raises the power level of the signal and noise at its input, and in addition degrades the signal to noise ratio by introducing noise which is created in or caused by the circuit itself. In order to compare the noise performance of individual circuits, a measure of this degradation is required, hence the noise factor parameter.

The noise factor of a network can also be thought of as the ratio of (1) the total noise power at the output to (2) the noise power at the output which is due solely to noise at the input.

\[ F = \frac{\text{total noise power output}}{\text{noise power output due to input noise}} \]  \hspace{1cm} (19)

\[ F = \frac{N_o}{GN_1} = \frac{N_o}{(S_o/S_1)N_1} \]  \hspace{1cm} (20)
This form is more convenient for noise factor calculations.

The noise factor of two cascaded stages can be found using the following equation: \(^3\)

\[
F_T = F_1 + \frac{F_2 - 1}{G_1}
\]  (21)

where \(F_T\) is the total noise factor, \(F_1\) is the noise factor of the first stage, \(F_2\) is the noise factor of the second stage, and \(G_1\) is the power gain of the first stage.

The sensitivity of a receiver is a measure of its ability to detect small desired signals. The inherent noise of the receiver limits this ability because a small signal will not be discernable if its power is not comparable to the noise power in the signal path.

Because the sensitivity and total noise factor of a receiver are both measures of its noise performance, they must be related. A one to one relationship exists, however, only after the bandwidth and operating temperature of the receiver have been specified.

Sensitivity is usually specified as the minimum available power from a source which is necessary to obtain a given signal plus noise to noise ratio at the receiver output. With a ratio of \(\alpha\) specified

\[
\frac{S_0 + N_0}{N_0} = \alpha
\]  (22)

\[
\frac{S_0}{N_0} = \alpha - 1
\]  (23)
substituting into equation 18,

\[
\frac{S_1}{N_1} = F (\alpha - 1) \tag{24}
\]

Since \(S_1/N_1\) is a ratio of powers at the same impedance level, \(R_s\), it can be written as a ratio of the mean square voltages.

\[
\frac{e_s^{-2}}{4kTR_s \Delta f} = F (\alpha - 1) \tag{25}
\]

where \(e_s^{-2}\) is the mean square open circuit signal voltage and \(4kTR_s \Delta f\) is the mean square thermal noise voltage due to the resistance, \(R_s\). The available input power is now

\[
\frac{e_s^{-2}}{4R_s} = kT \Delta f F (\alpha - 1) \tag{26}
\]

which expresses the relationship between noise factor, \(F\), and sensitivity.

In the analysis that follows it is desirable to find the sensitivity of a receiver which incorporates either of two proposed mixers. To accomplish this a noise equivalent circuit for the Siliconix U222 FET will first be developed. This will allow us to calculate the noise

\(k\) is the Boltzmann constant \(1.38 \times 10^{-23}\) joules/degree Kelvin.

\(T\) is the temperature in degrees Kelvin.

\(\Delta F\) is the receiver bandwidth.
factors of the two mixer configurations and then the sensitivity of a receiver incorporating either of the mixers can be found from equation 26. This is only valid if we assume that the mixer determines the total noise factor of the receiver. Equation 21 shows that this assumption is legitimate if the mixer is followed by a low-noise, high-gain circuit.

Sources of Noise in FET's

Before an equivalent circuit, which describes FET noise performance, can be drawn, the sources of noise in FET's must be investigated to determine the extent to which each will contribute to the total noise power.

The primary source of noise in FET's is thermal noise of the channel. Van der Ziel⁴ has shown that the current flowing in the shortcircuited output of an FET which is due to this thermal noise of the channel can be expressed as

\[ I_{dn} = 4kT_{g_{\text{max}}} \Delta f \cdot Q(V_G, V_D) \]  

(27)

That it should take this form is not surprising because the thermal noise current³ due to a conductance \( g \) is \( i = 4kTg \Delta f \). The additional factor in the FET thermal noise expression, \( Q(V_G, V_D) \) is an arbitrary constant of value one or less. Its value is determined by the choice of \( V_G \) and \( V_D \) or bias, and is approximately 0.7 for operation in the saturated (current source) region. \( g_{\text{max}} \) is the maximum transconductance at the particular bias voltage \( V_G \). The term \( g_{\text{max}} Q(V_G, V_D) \) can be considered the equivalent conductance of the FET which produces the
thermal noise current $I_{dn}$.

Equation 27 was derived through consideration of noise voltages produced along the channel which modulate the channel width and hence produce an amplified noise voltage at the drain. The expression is theoretically valid only for bias conditions below saturation (pinchoff) but Van der Ziel’s experiments showed that the expression is "nearly correct in the saturated part of the characteristic as long as the field strength in the cutoff part of the channel is not too large".\textsuperscript{4} This expression will, therefore, be used to evaluate the amplitude of channel thermal noise for the U222 FET. The agreement between theory and experiment will be shown to be excellent in Section VIII.

Another important source of noise in FET’s is that due to capacitively coupled gate noise.\textsuperscript{5,6} At moderately high frequencies thermal noise voltages along the channel will be capacitively coupled to the gate causing a current to flow in the short-circuited input. Considering the source of this noise current, one would suspect that it would be partially correlated with the thermal noise current $I_{dn}$. Brunke and Van der Ziel\textsuperscript{6} showed this to be true, however, they also concluded that the correlation was so slight that it could be ignored in a noise analysis of an FET circuit. The gate noise current was shown to be simply thermal noise due to the device input conductance $G_{11}^\text{s}$ or

$$\frac{f}{gn} - \frac{\sqrt{2}}{4kT} G_{11}^s \Delta f$$

(28)
Here the notation $G_{11}^s$ represents the input conductance of the device connected in a common source configuration. This must be emphasized because the capacitively coupled gate noise does not change when the device is connected in a common gate configuration. Hence, the term $G_{11}^s$ which describes this noise, will appear in the calculations for the noise factor of both mixers. The term $g_{11}$, however, will be used to represent the input conductance of either configuration.

Other noise sources in FET's are excess or $1/f$ noise and shot noise of the gate current. The $1/f$ noise is negligible for operation at frequencies above audio$^7$ and the shot noise of the gate current which can be expressed as $I_{gs} = 2qI_g \Delta f^7$ is insignificant when compared to the thermal noise due to $g_{11}$.

The thermal noise of the channel is essentially constant in amplitude over a wide frequency range, and therefore, the limiting factor for noise performance at high frequencies is the gain cutoff point of the transistor.$^6$ Since our frequency range of interest is below this cutoff point and above the range where $1/f$ noise must be considered, the only two sources of noise which must be described in the analysis are thermal noise of the channel and capacitively coupled gate noise.

**Noise Equivalent Circuit**

The noise equivalent circuit of a field effect transistor is one which includes the two noise generators $I_{dn}$ and $I_{gn}$ in addition to the $^7$ $I_g$ is the drain to gate leakage current.
generators and admittances which describe the devices two-port parameters. If the $y$-parameter equivalent circuit for the device is chosen, the generators $I_{dn}$ and $I_{gn}$ are included as shown in Figure 4, because they represent the short-circuit output noise current and the short-circuit input noise current respectively.\footnote{\textsuperscript{6}}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Figure 4}
\end{figure}

This noise equivalent circuit can now be used to calculate the noise performance of any circuit which utilizes an FET.

It is now possible to obtain a noise equivalent circuit for the balanced mixer configuration by replacing the FET's of Figures 1 and 2 by the equivalent circuit of Figure 4. The result, which is shown in Figure 5, is valid for either mixer configuration.
Noise Factor Calculation

Figure 5 can now be used to derive an expression for the noise factor of the balanced mixer circuit. It has been shown previously in this section that noise factor can be calculated by finding the total mean square current in the shorted output and dividing that by the portion in the output which is due to the source. Note that in the following calculation it is assumed the capacitance of the transformer and that of the devices has no effect on the noise factor. This assumption is valid if the capacitive reactance is large with respect to $1/g_s$, or if the capacitive effect is neutralized as by series peaking, for example. This will be examined further in Section V.

If the input and output transformers each have turns ratios of one to one to one, the total mean square current in the shorted output is
\[ I_{nT}^2 = 2 (g_{mc}^2 E_1^2 + I_{dn}^2) \]  

(29)

The dependence of \( E_1 \) on the three input current generators must now be found. The most straightforward approach is to apply the superposition principle.

The part of \( E_1 \) which depends on one of the \( I_{gn} \) generators, \( E_{ig} \), can be calculated from the circuit in Figure 6.

\[ g_{11} \]

\[ 4g_s \]

\[ 1:1:1 \]

\[ I_{gn} \]

\[ g_{11} \]

\[ E_1 \]

\[ 4g_s \]

\[ g_{11} \]

\[ I_{gn} \]

\[ g_{11} \]

\[ E_1 \]

Figure 6
\[ E_{ig}^2 = \frac{I_{gn}^2}{(4g_s + 2g_{11})^2} \] (30)

There are two such generators and the voltage due to both is therefore

\[ 2E_{ig}^2 = \frac{2I_{gn}^2}{(4g_s + 2g_{11})^2} \] (31)

The contribution of the source generator is found in the same manner to be

\[ E_{ig}^2 = \frac{4I_s^2}{(4g_s + 2g_{11})^2} \] (32)

Substituting equations 31 and 32 into equation 29 yields

\[ I_{nt}^2 = 2g_{mc}^2 \left[ \frac{2I_{gn}^2}{(4g_s + 2g_{11})^2} + \frac{4I_s^2}{(4g_s + 2g_{11})^2} \right] + 2 \overline{I_{nd}^2} \] (33)

The mean square current due to the source generator is now the second term in equation 33 or

\[ \left( 2g_{mc}^2 \right)^2 \frac{4I_s^2}{(4g_s + 2g_{11})^2} \] (34)
The noise factor is now found by dividing equation 34 into equation 33

\[ F = 1 + \frac{I_{gn}^2}{2I_s^2} + \frac{4I_{nd}^2(g_s + g_{11/2})^2}{g_{mc}^2 I_s^2} \]  

(35)

Substituting equations 27, 28 and \( I_s^2 = 4kTg_s \Delta f \) yields

\[ F = 1 + \frac{G_{11}^s}{2g_s} + \frac{4g_{max} Q(V_G,V_D)(g_s + g_{11/2})^2}{g_{mc} g_s} \]  

(36)

and setting

\[ R_{nc} = \frac{g_{max} Q(V_G,V_D)}{2 g_{mc}^2} \]  

(37)

The noise factor of the balanced mixer is now:

\[ F = 1 + \frac{G_{11}^s}{2g_s} + \frac{4R_{nc} g_s (g_s + g_{11/2})^2}{g_s} \]  

(38)

The sensitivity as a function of noise factor was previously found to be

\[ \frac{e^{-2}}{4R_s} = kT \Delta f F (\alpha - 1) \]  

(39)

Substituting from equation 38
\[
\frac{e_s^2}{4R_s} = kT \Delta f (\epsilon - 1) \left[ 1 + \frac{g_{11}^s}{2g_s} + \frac{4R_{nc}(g_s + g_{11/2})^2}{g_s} \right] \tag{40}
\]

which expresses sensitivity as a function of the source admittance, \(g_s\), where \(g_s\) is as shown in Figure 7.

![Figure 7](image)

**Sensitivity Vs. \(R_s\) for Common Source Mixer**

When the parameters of equation (40) are assigned the values of Table I, the sensitivity of the common source mixer can be plotted vs. \(R_s\) as in Figure 8.
Table I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\text{max}}$</td>
<td>10,000 micromhos</td>
</tr>
<tr>
<td>$g_{\text{mc}}$</td>
<td>2,000 micromhos</td>
</tr>
<tr>
<td>$Q(V_G, V_D)$</td>
<td>0.7</td>
</tr>
<tr>
<td>$G_{11}^S$</td>
<td>21.7 micromhos</td>
</tr>
<tr>
<td>$g_{11}$</td>
<td>21.7 micromhos</td>
</tr>
<tr>
<td>$k$</td>
<td>$1.38 \times 10^{-23}$</td>
</tr>
<tr>
<td>$T$</td>
<td>290° K</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>3,000 Hz</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>10</td>
</tr>
</tbody>
</table>

Note that the sensitivity is -114 dBm at an $R_s$ of 200 ohms which corresponds to a noise factor of 34.7.

**Sensitivity of Common Gate Mixer**

Sensitivity vs. $R_s$ of the common gate mixer can also be found from equation 40. However, it is first necessary to derive an expression for $g_{11}$, the input conductance of a common gate FET, and $G_c$ the conversion gain of the common gate mixer.

The equivalent circuit for a common gate FET is shown in Figure 9. For our purpose the output is assumed a short circuit for the R.F. signals, as shown in Figure 10. $Z_{in}$ is now derived by first writing an expression for $V_{sg}$

*The output is tuned at the intermediate frequency.*
Dynamic Range of Common Source
Balanced Mixer

Figure 8
\[ V_{sg} = \frac{I_s - g_m V_{sg}}{g_{DS} + j\omega c_{SG}} \]  
\[ (41) \]

\[ V_{sg} (g_m + g_{DS} + j\omega c_{SG}) = I_s \]  
\[ (42) \]

and now,

\[ z_{in} = \frac{1}{g_m + g_{DS} + j\omega c_{SG}} \]  
\[ (43) \]

\[ g_{11} \approx g_m \]  
\[ (44) \]

As shown in Section VII the devices are biased at \( g_m = g_{11} = 10,000 \) micromhos or \( R_{11} = 100 \) ohms.

The conversion gain of the common gate single-ended mixer is found in the same manner as \( G_c \) was found for the common source mixer in Section II.

The single-ended common source mixer was found to have a conversion gain of 6 dB, equation 17. If the device were now switched to the common gate configuration, \( g_{mc} \) would not change, but the input voltage would drop because of the higher input conductance of the common gate configuration (equation 44). The available input power would not change though, while the output power, which depends on the input voltage, would drop.

At \( R_s = 1/g_s = 100 \) ohms, for example, the source is matched to the 100 ohm input impedance of the common gate FET. The input voltage is, therefore, one-half what the voltage would be if it were driving the
high impedance of a common source device. The conversion gain is, therefore, 6 dB less than that of the common source mixer or 0 dB.

The conversion gain of a balanced common gate mixer is the same as that for the single-ended mixer if the source and load impedances are adjusted properly. The gain is, therefore, 0 dB with $R_s$ equal 200 ohms. This is now the maximum possible conversion gain for this mixer because the source impedance is matched. Increasing or decreasing $R_s$ will result in a lower conversion gain.

The noise factor at values of $R_s$ other than 200 ohms is therefore of little importance, because equation 21 shows that the total noise factor of two cascaded stages increases rapidly as the loss in the first stage increases. Another reason for considering the noise factor for $R_s = 200$ ohms is that the maximum dynamic range occurs for this value. This will be shown in Section V.

The sensitivity of the common gate balanced mixer at $R_s = 1/g_s = 200$ ohms is now found from equation 40 by substituting from Table I except with $g_{11} = 10,000$ micromhos. The sensitivity is -108 dBm corresponding to a noise factor of 139.

**Summary of Noise Analysis.**

The sources of noise in FET's were described and an equivalent circuit for the device which quantitatively represented these sources was developed. This led to the noise equivalent circuit of the balanced mixer and to the calculation of the noise factor as a function of $R_s$. It was then possible to obtain sensitivity as a function of
$R_s$ because the relationship of sensitivity and noise factor had been previously derived. The noise factor, and hence, sensitivity of the common gate mixer were found to be 6 dB worse than for the common source mixer at a particular value of $R_s$ (200 ohms).

The lower limit of the dynamic range has now been established. The upper limit or the large signal handling capability will be examined in Section IV.
IV. LARGE SIGNAL HANDLING CAPABILITY

Intermodulation and Crossmodulation Distortion Relationship

When large signals are present at the mixer input, distortion is produced in the circuit. Another measure of the quality of a receiver front end mixer is, therefore, the amount of distortion produced for a given amplitude of input signal. This aspect of mixer design will now be examined and a quantitative result will be obtained for the FET mixer.

The two types of distortion which must be considered are intermodulation distortion, I.M., and crossmodulation distortion.

I.M. is the result of two input signals at frequencies $f_1$ and $f_2$ combining in a non-linear device to give products at $(kf_1 \pm 1f_2)$, where $k$ and $l$ are integers. The third order products, $(2f_2 - f_1)$ and $(2f_1 - f_2)$, are of the most concern because they can be very close to the desired signals, $f_1$ and $f_2$ making it impossible to remove them with selectivity.

An example of I.M. in a tuned amplifier is shown in Figure 11. The amplifier is driven with a large two-tone signal causing I.M. products at $(2f_2 - f_1)$ and $(2f_1 - f_2)$.

Crossmodulation is defined as the transfer of the sidebands of a large A.M. undesired signal to a desired signal. This also is caused by the third order curvature of the device transfer characteristic.

The reason for the choice of square law devices should now be evident. The second order curvature is needed for frequency conversion,
Tuned Amplifier with Center Frequency $f_0$.  

Two-tone Input  

Amplifier Output with I.M. Distortion  

Figure 11
but third order curvature must be kept at a minimum so that large signals can be accepted with little I.M. and crossmodulation distortion occurring.

In a mixer the I.M. and crossmodulation distortion products actually are not produced until the desired signal has passed through the mixer twice. The i.f. frequency is generated on the first pass, and fed back to the input, through the drain to gate capacitance for example. The I.M. products and crossmodulation at the i.f. frequency are then generated on the second pass. This suggests that the mixer be modeled as a perfect mixer which exhibits the proper second order curvature, followed by a non-linear two-port having the proper third order curvature as in Figure 12. An expression for the relationship between I.M. and crossmodulation distortion will now be derived by closely examining the non-linear two-port network.

The transfer characteristic of the two-port can be written as a Taylor series

\[ I_2 = c_1 v_1 + c_2 v_1^2 + c_3 v_1^3 + \cdots \]  

(45)

If a two-tone signal is now substituted for \( v_1 \), I.M. distortion will result.

\[ v_1 = V \cos \omega_a t + V \cos \omega_b t \]  

(46)

Substituting and retaining only those terms with frequencies in the vicinity of \( \omega_a \) and \( \omega_b \)
\[ I_2 \propto c_1(V \cos \omega_a t + V \cos \omega_b t) + \]
\[ c_3 \left[ \frac{9}{4} V^3 \cos \omega_a t + \frac{9}{4} V^3 \cos \omega_b t + \frac{3}{2} V^3 \cos (2\omega_b - \omega_a) t \right] \quad (47) \]

\[ \text{Pure Second Order Curvature} \quad \text{All Orders of Curvature} \]

\[ \text{Non-linear Two-Port} \]

\[ \text{Non-linear Two-Port} \]

Figure 12
The amplitude of the desired signal, $\omega_a$, at the second port is now $c_1 V$, neglecting $c_3 9/4 V^3$ relative to $c_1 V$. This is justified if the two-port network has much less third order curvature than it does first order curvature, a property of most practical devices. The amplitude of the I.M. product at $(2\omega_a - \omega_b)$ is $3c_3 V^3/2$ and the ratio of the desired signal amplitude to the distortion signal amplitude is

$$R_{I.M.} = \frac{2c_1}{3c_3 V^2}$$ (48)

$R_{I.M.}$ now expresses the degree of I.M. distortion as a function of the input signal amplitude and the coefficients of the Taylor series representing the device transfer characteristic.

If the distortion products are down 40 dB* from the desired signal,

$$R_{I.M.} = 100$$ (49)

and the amplitude of the desired signals which produced the distortion are now,

$$V_{I.M.} = \sqrt{\frac{150 c_1}{c_3}}$$ (50)

Finding an expression for crossmodulation distortion\(^9\) can be done by returning to equation 45 and substituting

\(^*\)An arbitrary choice which has been found satisfactory for S.S.B. voice communications.
\[ v = V_1 \cos \omega_1 t + V_2 (1 + m \cos \omega_n t) \cos \omega_2 t \]  

(51)

which is the sum of a desired signal at \( \omega_1 \) and an undesired A.M. signal at \( \omega_2 \). Only those terms in the vicinity of \( \omega_1 \) will be retained.

\[ I_2 \propto c_1 V_1 \cos \omega_1 t + \]

\[ (V_1 \cos \omega_1 t) \left[ \frac{3c_3}{2} \left( \frac{V_1^2}{2} + V_2^2 \left( 1 + \frac{m^2}{2} + 2m \cos \omega_n t + \frac{m^2}{2} \cos 2 \omega_n t \right) \right) \right] \]

(52)

Neglecting \( \frac{3c_3}{2} \left( \frac{V_1^2}{2} + V_2^2 \left( 1 + \frac{m^2}{2} \right) \right) \) relative to \( c_1 \), as before,

\[ I_2 \propto (3c_3 V_2^2 m \cos \omega_n t + c_1) V_1 \cos \omega_1 t \]

(53)

and

\[ I_2 \propto c_1 \left[ \left( \frac{3mc_3}{c_1} \frac{V_2^2}{2} \right) \cos \omega_n t + 1 \right] V_1 \cos \omega_1 t \]

(54)

Now, for \(*\) crossmodulation

\[ \frac{3(m)c_3}{c_1} V_2^2 = 0.01 \]

(55)

* An arbitrary choice which has become somewhat a standard.
and the amplitude of the undesired signal which is necessary to produce the crossmodulation is

$$V_{x\text{mod}} = \sqrt{\frac{c_1}{3000 (m) c_3}}$$  \hspace{1cm} (56)

If the undesired signal were 30% modulated, \( m = 0.3 \) and

$$V_{x\text{mod}} = \sqrt{\frac{c_1}{90 c_3}}$$  \hspace{1cm} (57)

from 50 and 57 the ratio \( V_{x\text{mod}} \) to \( V_{I.M.} \) is

$$\frac{V_{x\text{mod}}}{V_{I.M.}} = \frac{\sqrt{\frac{c_1}{90 c_3}}}{\sqrt{\frac{c_1}{150 c_3}}} = 1.29$$  \hspace{1cm} (58)

In short the amplitude of a 30% A.M. modulated signal necessary to produce 1% crossmodulation is 1.29 times the amplitude of each tone of a two-tone signal which produces I.M. products down 40 dB from the desired signals.

**Balanced Mixer Distortion**

\( V_{I.M.} \) for the FET balanced mixer was experimentally found to be 400 mV per tone when measured across the complete secondary of the input transformer. The available input power necessary for I.M. distortion products down 40 dB from the desired signals can now be found
as a function of $R_s$, where $R_s$ is the real part of the impedance seen looking into the secondary of the input transformer. See Figure 7.

The common source mixer has a very high input impedance and, hence, the available power at the input which will lead to a voltage of $V_{I.M.}$ per tone at the transformer output is

$$P_{I.M.} = 2 \left( \frac{V_{I.M.}}{2} \right)^2 \left( \frac{1}{R_s} \right)$$

(59)

with,

$$V_{I.M.} = 0.4 \, \text{V}$$

(60)

$$P_{I.M.} = \frac{0.08}{R_s}$$

A plot of equation 60 is shown in Figure 8.

A plot of crossmodulation is also shown in Figure 8. The available power for 1% crossmodulation should be 2 dB (1.29 in dB) higher than the power in one tone of the two-tone signal. However, since there are two-tones the power of that signal is doubled (3 dB) and the crossmodulation plot ends up 1 dB below the I.M. plot.

The common gate mixer which we have agreed in Section III to investigate only at $R_s = 200$ ohms has an input impedance which matches $R_s$. $V_{I.M.}$ therefore, will appear from source to source when the available input power is
\[ P_{I.M.} = \frac{2(V_{I.M.})^2}{200} \]  

(61)

With \( V_{I.M.} \) equal to 400 mv

\[ P_{I.M.} = 1.6 \text{ mv} = + 2 \text{ dBm} \]  

(62)

Which is four times the power which caused the same distortion in the common source mixer, at an \( R_s \) of 200 ohms.

The large signal handling capability of each configuration has now been established and we can compare the dynamic ranges as in Section V.
V. **DYNAMIC RANGE COMPARISON**

**Comparison**

The limits of sensitivity and signal handling capability for the common source mixer have now been established and are shown in Figure 8. The maximum dynamic range occurs where the slopes of the two plots are the same. If the range between the sensitivity and the I.M. plots is considered, we find that the maximum is 110 dB at an $R_s$ of around 200 ohms.

This is a very desirable result because a wide band input transformer must be built, and it has been shown by Ruthroff\(^10\) that such a transformer with a two to one step up from 50 ohms ($R_s = 200$ ohms) is quite easily realizable. The resulting sensitivity is $-114$ dBm or 0.45 microvolts into fifty ohms which is excellent. $P_{I.M.}$ is $-4$ dBm, which corresponds to a two-tone signal of 100 mv per tone into fifty ohms.

The dynamic range of the common gate mixer has also been established, but only at one particular value of $R_s$, 200 ohms. However, it should not be too difficult to see that this value of $R_s$ also leads to very nearly the maximum possible dynamic range, for this mixer as it did for the common source mixer.

In Section III the sensitivity of the common gate mixer at $R_s$ equal to 200 ohms was found to be $-108$ dBm and in Section IV $P_{I.M.}$ was found to be $+2$ dBm. This is a range of 110 dB, exactly that of the common source mixer.
In summary, the dynamic ranges of the two mixer configurations are equal. The sensitivity of the common source mixer is 6 dB better than that of the common gate mixer, but the large signal handling capability of the common-gate mixer is 6 dB better than that of the common source mixer.

**Bandwidth Check**

A check should now be made to determine if the bandwidth specification can be met with \( R_s = 200 \) ohms. The specification for this application is for a bandwidth of 10 MHz. The receiver range is to be 2 MHz to 12 MHz.

The maximum input capacitance, \( C_{GS} \), for the Siliconix U222 is given as 20 pf.* The total capacitance from gate to gate is, then, 10 pf. The input frequency response will be down three dB at a frequency where the reactance of the total \( C_{in} \) is equal to \( R_s \). With \( R_s = 200 \) ohms this occurs at 80 MHz. In other words it will be no problem to meet the bandwidth requirement. This also justifies the assumption that the capacitance would not effect the noise factor.

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*Manufacturer's specification sheet Siliconix U221 - U222.
VI. FREQUENCY DEPENDENT TERMINATIONS

Frequency dependent terminations are needed at the mixer ports to maximize sensitivity, maximize the conversion gain, protect the circuit from large out-of-band signals, and minimize I.M. distortion.

In the analysis of Section III it was assumed that the input noise at the R.F. port contained no thermal noise at the image frequency. This assumption is valid if a termination is used which will short the image noise to ground. For our application the image band is 55 MHz to 65 MHz, and therefore, a low pass filter which will pass the desired 2 MHz to 12 MHz and provide a short for this band is used.

It is also important to minimize the amount of I.F. power which is dissipated in the R.F. and L.O. ports. It was shown in Section II that the L.O. port is balanced to the I.F. port and hence, power dissipation there is no problem. This is not true, however, of the I.F. power at the R.F. port, and therefore, a short must be provided at the I.F. frequency. This can be accomplished with the same low pass filter that shorts the image frequencies. The result will be the best possible conversion gain.

The low-pass filter at the R.F. port is also needed to protect the circuit from large signals above 12 MHz, and an additional filter will be needed to protect the circuit from those large signals which are below 2 MHz.

The I.F. port also needs a frequency dependent termination. As was discussed in Section IV the I.M. distortion is created only after
feedback of the I.F. signal. Obviously no selectivity can remove the I.M. products generated in this manner. However, it is also possible to generate I.M. distortion by feeding back the second harmonic of the input signal. For example, a two-tone signal, $f_1 + f_2$ at the R.F. port would create signals at its second harmonics as it passed through the mixer. If these harmonics are fed back to the input, they can be combined with the fundamentals on the second pass. On still another pass the I.M. products are converted to the I.F. frequency. Therefore, if a short is provided at the i.f. port for the R.F. signal second harmonics, the I.M. generated in this manner will be insignificant when compared to that generated in the usual manner. For our application this is done with a tuned circuit at a resonant frequency of 26.5 MHz.
VII. **BIASING AND CIRCUIT COMPONENTS**

As in any active network, the devices must be biased at a point on their characteristic curves which gives them the desired a.c. parameters.

In our choice of a suitable bias point, the dependence on bias of the following must be considered: input impedance of the common gate devices, amplitude of local oscillator drive, and balance of the circuit.

The input impedance of the devices in the common gate configuration can be set to the desired 100 ohms by proper choice of operating point. Figure 13 is a plot of the U222 transfer characteristic.* At the bias point, $V_{GS0}$, $I_D$, the input impedance is found by drawing a line tangent to the curve. The slope of this line $\frac{V_{GS}}{I_D}$ or $g_m$ can now be used to calculate $Z_{in}$, because $Z_{in} = 1/g_m$ as is shown in Section III. It must be remembered that the large L.O. voltage swing tends to shift the d.c. bias point and an average $g_m$ must be used.

Also dependent on bias is the allowable local oscillator power. The L.O. must not drive the devices too far into cut-off or the full conversion gain will not be realized. Clipping or cutting off the local oscillator waveform reduces the power at its fundamental frequency and, therefore, reduces the conversion gain. In addition, clipping shifts the bias point of the device causing a decrease in input impedance.

*From manufacturer's specification sheet Siliconix U221 - U222.
Figure 13

U222 Characteristic

$I_D$ (ma)

$V_{GS}$ (volts)

$R_{SO} = 500 \Omega$
This will reduce the gain of the common gate mixer even farther because the input will become mismatched. Some clipping can be tolerated, however, for a good deal is necessary to cause a significant decrease in impedance.

One might suspect the clipping of the L.O. waveform would cause distortion. That it doesn't is evidenced by the fact that mixers with excellent crossmodulation characteristics have been built utilizing an FET biased near pinch-off. 11

As shown in Section II, the circuit must be balanced for proper operation. The variation in parameters from device to device makes it imperative that a balance control be a part of the bias circuit, so that the bias, and, hence, g_m can be adjusted over a small range to bring the circuit into balance.

The transfer characteristics of the U222 also vary with temperature. The two solid lines in Figure 13 indicate the extremes of the parameter variations from device to device at room temperature. That is to say, any U222 at room temperature would have a characteristic curve which would fall on or between the two solid lines. The dotted lines indicate the temperature dependence of the curves.

The solution to the bias problem is most easily arrived at through a graphical analysis. A single FET can be self-biased with a resistor, R_so, connected from the source terminal to ground. See Figure 14. The resulting operating point can be determined by plotting a line on the device characteristic which passes through the origin and has slope
Figure 14

$R_{so}$. For example, a U222 with $V_p$ of -10V and an $I_{DSS}$ of 260 ma at room temperature would be biased at 12 ma by an $R_{so}$ of 500 ohms as shown in Figure 13.

A U222 having a characteristic like the one labeled typical in Figure 16 would be biased at 10 ma. The tangent to the curve at the bias point has a slope of just under 10,000 micromhos. This would be the desirable bias point for each FET in the mixer if each had such a characteristic. The local oscillator would shift the bias point up to where the tangent line has a slope of 10,000 micromhos thereby yielding an input impedance of 100 ohms. The figure also shows that the one volt local oscillator would not drive the device into cutoff. For the bias voltage, -5.6 volts, is 2.4 volts away from the cutoff voltage, -8.0 volts. This, of course, is near the ideal case. Let us now examine the worst case.

The worst case occurs with one FET in the mixer having a pinchoff voltage of -10 volts, and the other a pinchoff voltage of -6 volts. For this situation different source resistors, $R_{so}$, must be used to bring
the circuit into balance, which will occur when the devices have the same input impedance. As shown in Figure 16 this can be done with one $R_{so}$ at approximately 300$\Omega$ and the other approximately 700$\Omega$. Note that the tangent lines will have approximately the same slope. A balance potentiometer is, therefore, necessary so that this case and any other combination can result in a balanced circuit.

A bias circuit which could result in equal $R_{so}$'s of 500$\Omega$ or could be adjusted to 250 ohms and 750 ohms is shown in Figure 15.

![Figure 15](image)

The circuit of Figure 17, however, gives the same result with one resistor eliminated.
Figure 16
Another bad case which must be considered is the one for which the FET's have equal pinchoff voltages of 6V. At -40°C the bias voltage, $V_{GS_{0}}$, on each device will be -4.4 volts as shown in Figure 16. Remember $R_{so}$ is 500 ohms for each when the characteristics are equal. The difference between the pinchoff voltage, $V_p$, and $V_{GS_{0}}$ for this case is 1.6 volts. Because $V_{GS_{0}}$ will always be at least this far away from $V_p$, the one volt local oscillator can never drive the FET's into pinchoff.

The r.f. chokes in Figure 17 prevent power loss of the desired signal in the bias circuitry, and also make it unnecessary to consider the thermal noise of the bias resistors in the noise analysis. Of course, they are needed only for the common gate mixer as shown in Figures 1 and 2.

It was found that a typical mixer biased in this fashion with a
B+ of 20 volts required a 23 ma current from the supply. This indicates that the graphical procedure provides an adequate solution to the bias problem.

**Component Sensitivity**

To have a good yield of working circuits from the manufacturing division, the designer must insure that the parameters of his circuit remain relatively constant for small variations in the component values. This problem has become quite simple for the FET mixer with the addition of the balance potentiometer. It has already been shown how this adjustment compensates for the wide variation in the FET characteristics.

The variation in the resistor values can easily be as much as 10% without dropping the worst case \((V_p - V_{GS0})\) to 1.4 volts, the absolute minimum. This variation would cause a change in \(g_m\) of less than 1% as can be seen in Figure 16.

The variation in output capacitance from device to device is of little concern because the trimmer capacitor (5-25 pf) has a 5 to 1 range.

The balance potentiometer and the trimmer capacitor, therefore, remove any concern about component sensitivity.

The only components of the circuit which have not yet been described are the two transformers and the coupling capacitors.

The input transformer\(^{10}\) is composed of a pyroferric toroid of CF-121-06 material and three strands of # 30 wire. The wires are first twisted together and then six turns are placed on the toroid evenly
spaced. The wires are phased as shown in Figure 18.

The output transformer is composed of a pyroferric toroid of carbonyl SF material (PT 310-156-125) and again three strands of \# 30 wire. However, only two of the wires are twisted together, for this transformer. Ten turns of the pair are then placed on the toroid evenly spaced to form the primary. The secondary is two turns of \# 30 wire. Figure 19 shows the phasing of the three wires.

The value of the coupling capacitors is not critical. For our application 0.1 microfarads will suffice.
Input Transformer

Figure 18

Output Transformer

Figure 19
VIII. EXPERIMENTAL PROCEDURE AND RESULTS

Results

Both a common source mixer and a common gate mixer were constructed as in Figures 1 and 2 with $R_s = 200$ ohms for each. The sensitivity and signal handling capability of each was experimentally determined so that a comparison could be made to the calculated values.

The $10 \text{ dB } \frac{S+N}{N}$ sensitivity of each configuration was measured using the equipment shown in Figure 20. With the HP606 signal generator replaced by a 50 ohm termination, a reference was set on the RMS voltmeter. The generator was then connected to the circuit and its output was increased until the RMS meter reading had increased by 10 dB. The available power in dBm was then recorded from the HP606 meter.

The measured sensitivity of the common gate mixer was found to be -113 dBm or 0.5 microvolts into 50 ohms. That of the common source mixer was -117 dBm or 0.3 µV into 50 ohms.

A comparison of the signal handling capabilities of the two configurations was made with the equipment shown in Figure 21. The amplitude of the input two-tone signal was increased until the spectrum analyzer indicated that the I.M. distortion products were 40 dB below the desired signals.

The input signal was then removed and measured across a 50 ohm load. The common source mixer was able to accept a signal at -4 dBm while the common gate mixer accepted a signal at +1 dBm.

The equipment in Figure 22 was used to determine the crossmodulation
Figure 21

Intermodulation Test

FET Mixer

20 dB Pad

Low Pass Filter

Two-tone Generator

HP 608E Signal Generator

Boonton 91C RF Voltmeter
Cross-Nutation Test

Figure 22

56

HP5060 Signal Generator 50% A.M.

Hybrid Coupler

Adjustable Pad

HP 5100A Synthesizer

Local Oscillator HP 6088

26.5 MHz Crystal Filter

511 Collins Receiver

HP Wave Analyzer

FET Mixer
point of the common gate mixer. A reference was first set on the wave analyzer meter, which was tuned to the audio frequency of the 30\% AM modulated input signal generator. The input level was set at approximately 40 dB above the sensitivity level. Notice equation 54 indicates that crossmodulation is independent of desired signal level. The HP606 was then tuned to another frequency within the 2 MHz to 12 MHz band.* The synthesizer was now set to the desired frequency with an output equal to that for which the reference was set. The amplitude of the 30\% A.M. signal was then increased until the wave analyzer gave an indication which was 30 dB below the reference (1\% crossmodulation). The available power of the A.M. signal was then measured at the output of the hybrid coupler and recorded.

In the above manner the crossmodulation point was determined to be +1 dBm or 250 mv.

Other pertinent data which was taken is shown in Table II.

Table II

<table>
<thead>
<tr>
<th></th>
<th>Common Gate Mixer</th>
<th>Common Source Mixer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversion gain</td>
<td>2 dB</td>
<td>6 dB</td>
</tr>
<tr>
<td>L.O. to R.F. isolation</td>
<td>28 dB</td>
<td>22 dB</td>
</tr>
<tr>
<td>frequency response</td>
<td>flat 2-12 MHz</td>
<td>flat 2-12 MHz</td>
</tr>
<tr>
<td>B+</td>
<td>20V</td>
<td>20V</td>
</tr>
<tr>
<td>DC Current</td>
<td>23 ma</td>
<td>23 ma</td>
</tr>
</tbody>
</table>

*Not at a frequency where a spurious response would occur.
The spurious response rejection of the mixer was determined with the equipment shown in Figure 23. A reference was first set on the audio voltmeter with the input generator at the desired frequency. The generator was then swept across the 2 MHz to 12 MHz band with its amplitude as much as 90 dB above the desired input level. When a spurious response was found, the generator amplitude was set so that the audio voltmeter would return to the reference. The signal generator amplitude was then recorded. Special precaution was taken to prevent harmonics of the signal generator from getting into the mixer. This was accomplished by substituting any of several low pass filters with cutoff frequencies in the 2 MHz to 12 MHz band.

The data for these separate desired frequencies are shown in Figures 24, 25 and 26. The height of each line indicates the relative output that would result from an input signal of that particular frequency. This is assuming that all the inputs are equal in amplitude to the desired signal.

Discussion

The experimental results are in excellent agreement with the results predicted by the theoretical analysis.

The only measured value which requires discussion is the conversion gain of the common gate mixer. 0 dB was predicted, but 2 dB was measured. It is believed that the input impedance of the mixer was higher than desired and the conversion gain, therefore, increased. This also explains why the sensitivity is not 6 dB worse than the common
Figure 23
source mixer as was predicted.

It is pleasing to note that, the measured dynamic ranges of the two mixers are essentially the same at 113 dB.
Spurious Response Rejection

Desired: 2 MHz
L.O.: 28.5 MHz

Figure 24
Spurious Response Rejection

Desired: 5 MHz

L.O.: 31.5 MHz

Figure 25
Spurious Response Rejection

Desired: 12 MHz
L.O.: 38.5 MHz

Figure 26
IX. CONCLUSIONS

Two FET balanced mixer configurations have been analyzed, and each has been found to be quite adequate for use in a receiver front end.

An expression for the conversion transconductance of a junction FET used as an active mixer was first derived from the device transfer characteristics. This led to the calculation of the conversion gain of a simple single-ended FET mixer with 6 dB as the result.

The two configurations were then proposed; a common source mixer, and a common gate mixer, and the advantages of a balanced circuit were discussed.

The sources of noise in FET's were described and an equivalent circuit for the device which quantitatively represented these sources was developed. This led to the noise equivalent circuit of a balanced mixer which was valid for either configuration. Using this circuit it was then possible to obtain noise factor, and hence, sensitivity as a function of $R_s$ for the common source mixer. The common gate mixer sensitivity was analyzed at only one value of $R_s$ because this was the only value which resulted in a practical circuit.

The large signal handling capability of the two configurations was then analyzed and the distortion as a function of $R_s$ was found for the common source mixer. The distortion of the common gate mixer at $R_s = 200$ ohms was then discussed.

A comparison of the sensitivity vs. $R_s$ plot and the I.M. distortion vs. $R_s$ plot for the common source mixer showed that the maximum dynamic
range occurred for $R_g$ equal to 200 ohms. This range between sensitivity and I.M. distortion for each mixer was then found to be 110 dB. The common source mixer sensitivity at this value of $R_g$ was -114 dBm and common gate mixer sensitivity was -108 dBm.

The effects of frequency dependent terminations on the mixer ports were then examined and found to be quite important to the circuit's performance.

A graphical solution to the bias problem was presented which provided for the variation in parameters from device to device and insured that the devices would operate at a desirable point on their characteristic curve.

The methods of testing the mixer were discussed and test results were shown to agree excellently with the results predicted by the theoretical analysis.
APPENDIX I

Brief Description of FET Operation

Junction FET's work on the principal of conductance modulation. With the gate-source junction reverse biased, a depletion region is created on each side of the channel as shown in Figure I. The conductance of the channel is proportional to the width of this region, which is in turn proportional to $V_{GS}$, the gate to source voltage. The conductance can, therefore, be modulated by varying $V_{GS}$.

With a positive drain to source voltage, $V_{DS}$, and with $V_{GS} = 0$ volts, a depletion region will still extend into the channel because of the IR drops in the channel material. When $V_{DS}$ is increased under these conditions until the depletion regions on either side of the channel just touch, $V_{DS}$ at that point is defined as the pinchoff voltage, $V_p$. Increasing $V_{DS}$ further does not increase the drain current for it reaches a maximum value, $I_{DSS}$, and remains there for $V_{DS} > V_p$ as shown in Figure II.

The value of $I_D$ for $V_{DS} > V_p$ can now only be changed by varying $V_{GS}$. For example, changing $V_{GS}$ from 0 V to $-V_p$ would change the drain current from $I_{DSS}$ to a near zero value.

Shockley's expression for the FET transfer characteristic is:

$$I_D = I_{DSS} \left[ 1 - 3 \left( \frac{V_{GS}}{V_p} \right) + 2 \left( \frac{V_{GS}}{V_p} \right)^{3/2} \right]$$ (1)
N Channel FET

Figure I
An excellent approximation to expression (i) which was derived by R. D. Middlebrook is:

\[ I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2 \]  

(ii)

Figure III is a plot of the two curves showing that the approximation is indeed a good one.
\[ \frac{I_D}{I_{DSS}} \]

\[ (1 - \frac{V_{GS}}{V_P})^2 \]

\[ 1 - 3 \frac{V_{GS}}{V_P} + 2 \left( \frac{V_{GS}}{V_P} \right)^{3/2} \]

Figure III
APPENDIX II

An Observation on the Calculation of Noise Factor

The noise equivalent circuit of Figure IV is often used for noise factor calculations.\textsuperscript{12,13} It will be shown that this leads to a correct result only if it is assumed that the input admittance of the circuit is very small.

![Figure IV](image)

Shortcircuiting the output of the noise circuit yields a total mean-square current of

\[ \overline{I_n^2} = \overline{I_s^2} + \overline{\overline{I_n^2}} + \overline{E_n^2} |Y_s|^2 \]  \hspace{1cm} (iii)

assuming no correlation of the sources. The noise factor is now found by dividing equation (iii) by the source mean square current.
\[ F = 1 + \frac{I_n^{-2}}{I_s^{-2}} + \frac{E_n^{-2}}{I_s^{-2}} \left| Y_s \right|^2 \] \hspace{1cm} (iv)

An expression for noise factor will now be derived using the circuit of Figure V which also represents a linear noisy two-port.

![Figure V](image)

Again shortcircuiting the output

\[ \overline{I_{nT}^{-2}} = \left| Y_{21} \right|^2 \overline{V_1^{-2}} + \overline{I_{2n}^{-2}} \] \hspace{1cm} (v)

but

\[ \overline{V_1^{-2}} = \frac{\overline{I_s^{-2}} + \overline{I_n^{-2}}}{\left| Y_s + Y_{11} \right|^2} \] \hspace{1cm} (vi)
and now

\[ I_{nT}^{-2} = \left| \frac{Y_{21}}{Y_s + Y_{11}} \right|^2 \left( \frac{I_s^{-2} + I_n^{-2}}{I_s^{-2} + I_{2n}^{-2}} \right) + I_{2n}^{-2} \]  

(vii)

Dividing by the mean square current in the short which is due to the source yields,

\[ F = 1 + \frac{I_{n}^{-2}}{I_s^{-2}} + \frac{I_{2n}^{-2}}{I_s^{-2}} \left| \frac{Y_s + Y_{11}}{Y_{21}} \right|^2 \]  

(viii)

Note that \( F \) is now proportional to the input admittance \( Y_{11} \) which is certainly a more pleasing result.

The correspondence between equations (iv) and (viii) is more easily seen when one considers that \( E_n \) of equation (iv) was used to represent the noise current which appears in the short-circuited output when the input is also short-circuited. In other words

\[ E_n = \frac{I_{2n}}{Y_{21}} \]  

(ix)

or

\[ I_{2n} = E_n Y_{21} \]  

(x)

Substituting this expression into equation (viii) yields
Equations (iv) and (xi) will now be equal if it is assumed that

\[ Y_{11} \ll Y_s. \]

The real purpose of this discussion is to show that the circuit of Figure VI which has all the advantages of the simple noise equivalent circuit, makes it unnecessary to assume \( Y_s \ll Y_{11} \) in order to yield the correct noise factor.

\[ F = 1 + \frac{I_n^2}{I_s^2} + \frac{E_n^2}{I_s^2} \left| Y_s + Y_{11} \right|^2 \quad (xi) \]

Calculating noise factor as before

\[ \overline{I_n T}^2 = \overline{I_s}^2 + \overline{I_n}^2 + \overline{E_n}^2 \left| Y_s + Y_{11} \right|^2 \quad (xii) \]
Comparing to equation xi we see that the equivalent circuit of Figure VI does indeed yield the correct noise factor with no assumptions.

\[ F = \frac{I_{nT}^2}{I_s^2} = 1 + \frac{I_n^2}{I_s^2} + \frac{E_n^2}{I_s^2} \left| Y_s + Y_{11} \right|^2 \]  

(xiili)

and
REFERENCES

