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Gianfranco Bertone
NASA/Fermilab Theoretical Astrophysics Group

David Merritt
Rochester Institute of Technology

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Time-Dependent Models for Dark Matter at the Galactic Center

Gianfranco Bertone\textsuperscript{1} and David Merritt\textsuperscript{2}

\textsuperscript{1}NASA/Fermilab Theoretical Astrophysics Group, Batavia IL 60510, USA
\textsuperscript{2}Department of Physics, Rochester Institute of Technology, Rochester, NY 14623, USA

The prospects for indirect detection of dark matter at the Galactic center with gamma-ray experiments like the space telescope GLAST, and Air Cherenkov Telescopes like HESS, CANGAROO, MAGIC and VERITAS, depend sensitively on the mass profile within the inner parsec. We calculate the distribution of dark matter on sub-parsec scales by integrating the time-dependent Fokker-Planck equation, including the effects of self-annihilations, scattering of dark matter particles by stars, and capture in the supermassive black hole. We consider a variety of initial dark matter distributions, including models with very high densities ("spikes") near the black hole, and models with "adiabatic compression" of the baryons. The annihilation signal after $10^{10}$ yr is found to be substantially reduced from its initial value, but in dark matter models with an initial spike, order-of-magnitude enhancements can persist compared with the rate in spike-free models.

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There is compelling evidence that the matter density of the Universe is dominated by some sort of non-baryonic, "dark", matter, the best candidates being weakly interacting massive particles (WIMP). Numerical $N$-body simulations suggest dark matter (DM) density profiles following broken power laws, $\rho \propto r^{-\gamma}$, with $\gamma \approx 3$ in the outer parts of halos and $1 \lesssim \gamma \lesssim 1.5$ ("cusps") inside the Solar circle. Although these profiles reproduce with sufficiently good accuracy the observed properties of galactic halos on large scales, as inferred by rotation curves, little is known about the DM distribution on smaller scales, where the gravitational potential is dominated by baryons. The situation at the Galactic center (GC) is further complicated by the presence of a supermassive black hole (SBH), with mass $\sim 10^{6.5} M_\odot$, whose sphere of gravitational influence extends out to $\sim 1$ pc.

The prospects for indirect detection depend crucially on the distribution of DM within this small region. The flux of gamma-rays from the GC, from the annihilation of DM particles of mass $m$ and annihilation cross section $\sigma v$ at angle $\psi$ with respect to the GC, $\Delta \Omega$ is the solid angle of the detector, and $K$ is a normalizing factor, $K^{-1} = (8.5\text{ kpc})(0.3\text{ GeV}/\text{cm}^3)^2$. We denote by $\mathcal{J}_5$ and $\mathcal{J}_3$ the values of $\mathcal{J}$ when $\Delta \Omega = 10^{-5}\text{ sr}$ and $10^{-3}\text{ sr}$ respectively; the former is the approximate field of view of the detectors in GLAST, and in atmospheric Cherenkov telescopes like VERITAS and HESS, while the larger angle corresponds approximately to EGRET.

DM densities that rise more steeply than $\rho \propto r^{-3/2}$ near the GC imply formally divergent values of $\mathcal{J}$, hence the predicted flux of annihilation products can depend sensitively on any physical processes that modify the DM density on subparsec scales. Although the analysis of DM indirect detection is usually performed under simplifying assumptions on the DM profile – extrapolating the results of numerical simulations with power-laws down to subparsec scales – several dynamical processes may influence the distribution of DM at the GC, including the gravitational force from the SBH, adiabatic compression of baryons, and heating of the DM by stars.

Here, we focus on the evolution of the annihilation signal due to two physical processes that are almost certain to strongly influence the form of the DM density profile near the GC: DM self-annihilations; and gravitational interactions between DM particles and stars. Both processes act on a similar time scale ($\sim 10^9$ yr) to modify $\rho(r)$ on the sub-parsec scales that are most relevant to the indirect detection problem. While these two mechanisms both tend to lower the DM density, we find that interestingly high densities can persist over a particular range of $(m, \sigma v)$ values. The time-dependent profiles discussed here may also have important consequences for the prospects of observing an extra-galactic gamma-ray background.

Let $f(r, v, t)$ be the mass density of DM particles in phase space and $\rho(r, t)$ their configuration-space density, with $r$ the distance from the GC, i.e. the distance from the SBH. We assume an isotropic velocity distribution,
\( f(r, v, t) = f(E, t) \), where \( E \equiv -\ddot{r}^2/2 + \phi(r) \) is the binding energy per unit mass and \(-\phi(r)\) is the gravitational potential, which includes contributions from the stars in the Galactic bulge and from the SBH. We assume that \( \phi \) is fixed in time, i.e. that the mass of the SBH has not changed since the epoch of cusp formation, and that the stellar distribution has also not evolved. The first assumption is commonly made based on the observed, very early formation of massive black holes (e.g. \[12\]). The second assumption is motivated by the expectation that the Galactic center \[11\] and both the first and third terms on the right hand side of eq. 3a depend in what follows. Eq. 3 was advanced in time via a backward differential scheme coupled with the method of lines to reduce the partial differential equation to a system of ODEs \[22\] (these models are labelled “N” in Table I). The most recent simulations \[24, 27\] (see also Refs. \[24, 27\]) suggest a power-law index that varies slowly with radius, but the normalization and slope of these models at \( r \approx r_h \) are essentially identical to those of models with an unbroken, \( \rho \propto r^{-1} \) power law inward of the Sun. We took \( R_\odot = 8.0 \) kpc for the radius of the Solar circle \[28\].

Since the total mass budget of the inner Galaxy is dominated by baryons, the DM distribution is likely to
TABLE I: Properties of the halo models. “N” and “A” stand for NFW and adiabatically contracted profiles, respectively. The subscripts “c”, “sp” are for profiles with core and spike respectively. Core radius $r_c$ is in units of $r_h$. Density at $R_0$ is in units of GeV cm$^{-3}$. $\mathcal{J}$ and $\mathcal{J}_s$ are values of $\mathcal{J}$ averaged over windows of solid angle $10^{-3}$ sr and $10^{-5}$ sr respectively and normalized as described in the text. The final two columns give $\mathcal{J}_s$ in evolved models for $\sigma_v = 0$ (no annihilations), and for $(\sigma_v, m) = (3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}, 50 \text{GeV})$ (maximal annihilation rate), respectively.

<table>
<thead>
<tr>
<th>$\gamma_c$</th>
<th>$\gamma_{sp}$</th>
<th>$r_c$</th>
<th>$\rho(R_0)$</th>
<th>$\log_{10} \mathcal{J}_s(\mathcal{J}_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1.0</td>
<td>-</td>
<td>0.3</td>
<td>2.56 (3.51) 2.56 (3.50) 2.56 (3.50)</td>
</tr>
<tr>
<td>$N_c$</td>
<td>1.0</td>
<td>10</td>
<td>0.3</td>
<td>2.54 (3.33) 2.54 (3.33) 2.54 (3.33)</td>
</tr>
<tr>
<td>$N_{sp}$</td>
<td>1.0</td>
<td>2.33</td>
<td>-</td>
<td>9.21 (11.2) 3.86 (5.84) 2.56 (3.52)</td>
</tr>
<tr>
<td>$N_{c,sp}$</td>
<td>1.0</td>
<td>2.29</td>
<td>0.3</td>
<td>6.98 (8.98) 2.61 (3.88) 2.54 (3.33)</td>
</tr>
<tr>
<td>$A$</td>
<td>1.5</td>
<td>-</td>
<td>0.5</td>
<td>5.80 (7.75) 5.26 (7.03) 5.23 (6.98)</td>
</tr>
<tr>
<td>$A_c$</td>
<td>1.5</td>
<td>10</td>
<td>0.5</td>
<td>4.96 (6.27) 4.96 (6.27) 4.96 (6.27)</td>
</tr>
<tr>
<td>$A_{sp}$</td>
<td>1.5</td>
<td>2.40</td>
<td>0.5</td>
<td>14.8 (16.8) 9.25 (11.3) 5.25 (7.02)</td>
</tr>
<tr>
<td>$A_{c,sp}$</td>
<td>1.5</td>
<td>2.29</td>
<td>0.5</td>
<td>9.99 (12.0) 5.21 (6.96) 4.96 (6.27)</td>
</tr>
</tbody>
</table>

have been influenced by the baryonic potential and its changes over time. The “adiabatic-growth” model [10] posits that the baryons contracted quasi-statically and symmetrically within the pre-existing DM halo, pulling in the DM and increasing its density. When applied to a DM halo with initial $\gamma_c \approx 1$, the result is a halo profile with $\gamma_c \approx 1.5$ inward of $R_0$ and an increased density at $R_0$ [24, 30, 31]. Adiabatically contracted halo models are labelled “A” in Table I. Alternatively, strong departures from spherical symmetry during galaxy formation might result in strong enhancements in $\mathcal{J}$-values, giving $\mathcal{J}$-values depend appreciably on the particle physics model only when the initial DM density has a “spike” around the SBH; in other cases the central density is too low for annihilations to affect $\mathcal{J}$. Particularly in the case of maximal $\sigma_v$, the final $\mathcal{J}$ values are found to be modest, $\log_{10} \mathcal{J}_s \lesssim 5.3$ and $\log_{10} \mathcal{J}_s \lesssim 7.0$, compared with the much larger values at $\tau = 0$ in the presence of spikes.

We also carried out integrations for the set of benchmark models derived in [32] in the framework of minimal supergravity (mSUGRA). Although other scenarios (supersymmetric or not) predict different parameters for the DM candidate, the values of $\sigma_v/m$ are often approximately the same. Light DM candidates [37], for example, have masses smaller than 20 MeV if they are to be responsible for the 511 keV emission from the Galactic bulge [33], but they also typically have cross sections much smaller than the thermal cross section in the early universe, which implies that $\sigma_v/m$ falls again in the same range discussed above. Heavy candidates, like those proposed to explain the HESS data [32, 10], have masses in the 10–20 TeV range, and thermal cross sections, so that they fall again in the same range of $\sigma_v/m$. Fig. 1 shows the final DM density profile for each of the benchmark models, starting from DM models $N_{sp}$ and $A_{sp}$; the latter model is the “adiabatically contracted” version of the former. While adopting the maximal annihilation rate effectively destroys the spike and produces $\mathcal{J}$ values as low as those of spike-free models, other benchmark models with smaller $\sigma_v/m$ result in strong enhancements in $\mathcal{J}$.

Fig. 2 shows the evolution of the dark matter density at a radius of $10^{-5} r_h \approx 2 \times 10^{-5}$ pc, starting from $\mathcal{J} \sim r^{-2.33}$ spike ($\rho \sim r^{-1}$ cusp). Two values were taken for the initial density normalization at $r = r_h$, $\rho(r_h) = (10, 100) M_\odot\text{pc}^{-3}$. The self-annihilation term in Eq. (16a) was computed assuming $m = 200$ GeV, $\sigma_v = 10^{-27} \text{cm}^3\text{s}^{-1}$. The early evolution is dominated
by self-annihilations but for \( t \gtrsim 10^9 \text{yr} \) \( \approx T_{\text{heat}} \), heating of dark matter by stars dominates. The change in \( \mathcal{J}_5 \) (Fig. 2b.) is dramatic, with final values in the range \( 10^3 \lesssim \mathcal{J} \lesssim 10^5 \).

We define the boost factor \( b \) as \( \mathcal{J}/\mathcal{J}_N \), with \( \mathcal{J} \) the value in the evolved model and \( \mathcal{J}_N \) the value in a \( \rho \propto r^{-1} \) (spike-free) halo with the same density normalization at \( r = R_{\odot} \). Figure 2 shows boost factors at \( \tau = 10 \) for each of the models in Table I. We found that the dependence of \( B \equiv \log_{10} b \) on \( \sigma v/m \) could be very well approximated by the function

\[
B(X) = B_{\text{max}} - (1/2)(B_{\text{max}}-B_{\text{min}})\{1+\tanh[C_1(X-C_2)]\}
\]

with \( X \equiv \log_{10}(\sigma v/10^{-30}\text{cm}^3\text{s}^{-1})/(m/100\text{GeV}) \). Table II gives values of the fitting parameters at \( \tau = 10 \) in each of the models with a spike. The boost factor is independent of \( \sigma v \) for low \( \sigma v \), since annihilations are unimportant in this limit, and also for high \( \sigma v \), since annihilations effectively destroy the spike.

We now apply these results to the study of high-energy \( \gamma \)-rays from dark matter annihilations at the GC. An early detection by the EGRET collaboration, of a \( \gamma \)-ray source coincident with the position of the SBH [41], has not been confirmed by a subsequent analysis [42]. However, Air Cherenkov Telescopes like HESS, CANGAROO and VERITAS have all detected a source coincident within their angular resolution with the GC SBH. In particular the HESS data suggest a spectrum extending up to 10 TeV, with no apparent cut-off [43]. It is difficult to interpret the observed emission as due to DM annihilation, since usual DM candidates are lighter than the required 10 TeV, and since the spectrum is quite flat. The latter problem can be solved by considering processes like \( \chi \chi \rightarrow \ell \ell \gamma \) [39], where \( \ell \) is a charged lepton, a channel heavily suppressed for neutralinos, but open for Kaluza–Klein particles. Although the contribution of the total flux is small (the channel is suppressed by a factor \( \alpha/\pi \) with respect to the annihilation to charged leptons), the corresponding photon spectrum is very flat, with a sharp cut-off at an energy corresponding to the particle mass. The other problem, i.e. the high dark matter particle mass required to reproduce the HESS data, can be solved in the framework of some specific theoretical scenarios, such as those proposed in Refs. [39, 40].

![FIG. 1: Evolved DM density profiles at \( \tau = 10 \) (roughly \( 10^{10} \) yr) starting from two initial DM profiles (see Table I and text). Colored curves: benchmark models; dashed lines: \( \sigma v = 0 \); dotted curves: \( \sigma v = 3 \times 10^{-26}\text{cm}^3\text{s}^{-1} \), \( m = 50 \text{ GeV} \); thick line: initial DM density.](image1.png)

![FIG. 2: (a) Evolution of the dark matter density at a radius of \( 10^{-5} \rho_h \approx 2 \times 10^{-5}\text{pc} \) in a \( \rho \sim r^{-2.33} \) spike, for \( m = 200 \text{ GeV} \), and \( \sigma v = 10^{-27}\text{cm}^3\text{s}^{-1} \). The upper(lower) set of curves correspond to an initial density normalization at \( r_h \) of 10(100) \( M_\odot \text{ pc}^{-3} \). In order of increasing thickness, the curves show the evolution of \( \rho \) in response to heating by stars; to self-annihilations; and to both processes acting together. Time is in units of \( T_{\text{heat}} \) defined in the text; \( \tau = 10 \) corresponds roughly to \( 10^{10} \) yr. (b) Evolution of \( \mathcal{J} \) averaged over an angular window of \( 10^{-5} \) sr.](image2.png)

**TABLE II: Parameters in the fitting function for the boost.**

<table>
<thead>
<tr>
<th>( \Delta \Omega )</th>
<th>( 10^{-3} )</th>
<th>( 10^{-5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{\text{min}} )</td>
<td>( B_{\text{max}} )</td>
<td>( C_1 )</td>
</tr>
<tr>
<td>( N_{\text{sp}} )</td>
<td>-0.02</td>
<td>1.31</td>
</tr>
<tr>
<td>( N_{e,\text{sp}} )</td>
<td>-0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>( A_{\text{sp}} )</td>
<td>2.16</td>
<td>6.29</td>
</tr>
<tr>
<td>( A_{e,\text{sp}} )</td>
<td>1.96</td>
<td>2.22</td>
</tr>
</tbody>
</table>
case, a boost factor of order $10^3 \lesssim b \lesssim 10^4$ is required to match the observed normalization, for particle masses of order 10 TeV and cross sections of order $3 \times 10^{-26}$ cm$^2$ s$^{-1}$. Figure 2 shows that such boost factors are achievable in the adiabatically compressed DM models, $\rho \sim r^{-1.5}$, especially if a spike is initially present, although the spike is not required. We note that the particle models discussed above could easily evade the synchrotron constraints discussed in [14, 15, 16, 17]. Looking for example at Fig. 6 of Ref. [17], we note that the synchrotron constraint is weaker for heavier masses, and the annihilation rate, in the case of the evolved $A_{sp}$ profile discussed above, is suppressed by many orders of magnitude with respect to the case discussed in [15], corresponding to a non-evolved $N_{sp}$ profile (Table I). A detailed analysis of indirect detection of supersymmetric and Kaluza-Klein DM in light of this work will be presented elsewhere.

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FIG. 3: Boost factors $b_\tau (\Delta\Omega = 10^{-5})$ as a function of $\sigma v/m$ at $\tau = 10$ for the DM models of Table I. Hatched region is the approximate boost factor required to explain the HESS $\gamma$-ray detection if the particle mass is $\sim 10$ TeV and $\sigma v = 3 \times 10^{-26}$ cm$^2$ s$^{-1}$ (vertical line).