On Numerical Simulation of The Dynamics of Bottles In Conveyor Systems

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ABSTRACT

The present paper concerns the dynamic behavior of bottles transported by conveyor systems. Such systems are widely applied in numerous industries. However, if systems are not properly designed or controlled, multiple undesired phenomena, like clogging or severe bottle scuffing, may occur in the bottle bulk. These effects are eventually strongly linked to the dynamic behavior of and interaction between bottles. In order to propose a simulation model to support the design process needed for bottle conveyors, the plane mechanical behavior of bottles in a parametrically defined plant is considered. Generation of geometry, contact simulations by penalization and Coulomb friction is included in the model, which is solved by numerical integration of the Newton-Euler equations. The model is developed applying basic concepts from mechanics and numerical analysis and is therefore well-suited for implementation in any software for scientific programming. Results obtained with the model represent the expected physical effects for soft bottles well. A strategy for improvement of the computational efficiency of the method for modeling of high stiffness bottles is outlined.

KEY WORDS

unconstrained dynamics, numerical simulation, bottle conveyors

INTRODUCTION

Conveyor systems for handling of bottles are of major importance in numerous industries such as the glass container production and beverage industries. The scale of the required bottle handling lines ranges from very small in supermarket bottle sorting systems to enormous plants capable of handling many thousands of bottles per hour in modern breweries. Owing to high speeds and output rates, the conveyor equipment in such bottling plants is failure-sensitive with availabilities of 92-98 %.
However, multiple undesired strength-related phenomena may occur in the bottle bulk. Among the most noticeable are:

- **Bottle scuffing**, by which tangential contact forces leave wear traces, that might reduce the bottle lifetime significantly
- **Bottle crushing**, where radial forces may lead to vertical movements and cause bottles to be shot out of the conveyor system. Furthermore, in extreme cases, crushing may cause structural collapse of bottles
- **Impact forces on bottles** due to sudden contact, which may smash bottles of brittle materials like glass
- **Plant clogging**

These effects may to some extent be investigated experimentally on basis of measurements recorded for actual plants. For engineering analysis it is desirable to apply analytical and numerical principles from strength of materials for failure prediction, where finite element analysis in many cases would be a logical choice. While investigating how to develop design calculation models, it occurred to the authors that the force input for any strength assessment of any type of bottle is quite difficult to obtain by measurements.

Therefore, investigations of how to develop a simulation model for assessment of the dynamics causing the undesired phenomena listed above are presented in this paper. The main focus will be on geometry generation, since calculation of forces acting on and between the bottles can be defined surprisingly easy using penalization methods once the geometry is specified. Forces may either be impulsive/dynamic or quasi-static caused by ‘slow contact’. However, both types of forces are required as input for engineering design and are extremely dependant on the dynamic behaviour and operating conditions of the considered bottle conveyor system.

A number of commercial programs can be applied for analysis of bottle conveyor systems on basis of either multibody dynamics or discrete element methods. However, since these often are not available to an industrial engineer, the analytic equations for geometry generation in the present paper are believed to be of general interest for engineers in the packaging industry.

Conveyor belts and guiding rails are treated separately allowing for modelling of accumulation tables and guiding rails on belts.

The objective is to obtain a model which is simple to implement and contains the most common features in bottle conveyors. These are:

- **Bottles**
- **Straight belts**
- **Guiding rails/barriers**
- **Curved belts surrounded by barriers**
  (also enabling modelling of turn-tables)

Fig. 1: Large scale bottle conveyor plant in a brewery

**DEVELOPMENT OF A SIMULATION MODEL**

Only the bottle motion in the plane of the conveyor belt and the bottle rotation around an axis perpendicular to this plane will be considered. A similar approach was followed by Livramento [1]. This is sufficient for calculation of the contact forces. On the other hand, motions related to bottle...
tipping cannot be modelled. The tipping forces and corresponding overturning moments can be estimated, and simple analytical methods will then be sufficient to check for bottle tipping. The dynamics of the bottles is modelled using concepts from multibody dynamics which are summarized by Nikravesh [2] and Suresh et al. [3].

The analysis will in the present case be limited to include bottles of circular cross-sections. The chosen approach therefore to some extend resembles the basic principles applied for modelling using DEM as described by Kozicki and Donzé [4].

**Plane equations for bottle motion**

The bottles are modelled as a system of rigid bodies interacting through contact. All bottle motions are in a modelling sense unconstrained. The accelerations and the forces acting on the bottles are linked together by the Newton-Euler equations. In global Cartesian coordinates, these are for the \( j \)’th bottle in the \( i \)’th simulation time step given by

\[
\begin{bmatrix}
m^{(j)} & 0 & 0 \\
0 & m^{(j)} & 0 \\
0 & 0 & j^{(j)}
\end{bmatrix}
\begin{bmatrix}
a_x^{(j,(l))} \\
a_y^{(j,(l))} \\
a_z^{(j,(l))}
\end{bmatrix}
= 
\begin{bmatrix}
F_x^{(j,(l))} \\
F_y^{(j,(l))} \\
M_z^{(j,(l))}
\end{bmatrix}
\]

(1)

in which \( m \) denotes the bottle mass, \( a_x \) and \( a_y \) the linear acceleration in the x- and y-directions and \( a_z \) the angular acceleration of the bottles. Furthermore, \( F_x \) and \( F_y \) denote the resulting forces acting on the bottle respectively in the x- and y-directions and \( M_z \) the force moment around the bottle centroid. When the forces acting on the bottle are known, the acceleration can be calculated from equation 1 on the form

\[
a^{(j,(l))} = (a_x^{(j,(l))} \ a_y^{(j,(l))} \ a_z^{(j,(l))})^T
\]

(2)

It is now required is to calculate the velocities on the form

\[
v^{(j,(l))} = (v_x^{(j,(l))} \ v_y^{(j,(l))} \ \omega_z^{(j,(l))})^T
\]

(3)

and the positions on the form

\[
r = (r_x^{(j,(l))} \ r_y^{(j,(l))} \ \Theta_z^{(j,(l))})^T
\]

(4)

In equation 3 and 4, \( \omega_z \) and \( \Theta_z \) denote respectively the angular velocity and the angular acceleration of the bottle. Since equation 1 will not have an analytical solution for the considered case, the \( v \) and \( a \) vectors must be determined by numerical integration. Multiple algorithms are available for this and are commonly available in software for scientific programming. The simplest algorithm is the Euler integrator, in which \( v \) is updated linearly in small time steps of length \( \Delta t \)

\[
v^{(j,(l+1))} = v^{(j,(l))} + \Delta t a^{(j,(l))}
\]

(5)

The positions are updated in a similar fashion by

\[
r^{(j,(l+1))} = r^{(j,(l))} + \Delta t v^{(j,(l+1))}
\]

(6)

in which the updated velocity is applied for the position update. This small modification from the original explicit forward Euler integrator increases the accuracy significantly.

The contact forces will be added by the penalization method. This implies, that rigid bodies are allowed to penetrate each other. However, when this happens, contact forces are simulated as a spring force term dependant on the penetration depth and a damping force term dependent on the velocity of the considered bodies. The main disadvantage of this method is that contact simulation requires very small time steps. On the other hand, it enables easy simulation of non-impulsive actions, which are more difficult to represent in a physically realistic fashion if methods based on impulse and momentum are applied.

In order to calculate the contact force, the geometries of belts and barriers must be defined in a fashion enabling easy contact detection.

**Regularized Coulomb friction**

The friction forces in the system will be accounted for using the Coulomb model, in which the friction is velocity independent. The forces
applied due to friction will then solely be a function of the normal force $N$ and the dynamic friction coefficient $\mu$. see Figure 2. However, the model has as defect that friction at very low velocities will shift direction. The model is therefore on original form not well-posed for implementation in numerical models accounting for stick-slip effect.

In order to compensate for this defect, regularization is applied for low velocities, so the friction varies continuously between 0 and a chosen low velocity slip limit $v_{lim}$. The applied friction force vector is then given by

$$\mathbf{F}_\mu = \begin{cases} -p(v)\mu\frac{v}{||v||}N & v \leq v_{lim} \\ -\mu\frac{v}{||v||}N & v > v_{lim} \end{cases} \quad (7)$$

The negative sign and the normed velocity vector ensure, that the friction force is directed opposite the direction of motion. The regularization function $p(v)$ is chosen as a polynomial of second order on the form

$$p(v) = av^2 + bv + c \quad (8)$$

This has been demonstrated to work well, see [5]. However, other transition curves are commonly applied and can be perfectly justifiable. Another common choice is for example a transition curve based on the atan-function.

The constants in equation 8 can in order to ensure a smooth transition be determined by the conditions

$$p(0) = 0 \quad p(v_{lim}) = 1 \quad \frac{dp(v_{lim})}{dv} = 0 \quad (9)$$

The constants can with these conditions substituted into equation 8 be determined algebraically and are given by

$$a = -\frac{1}{v_{lim}^2} \quad b = \frac{2}{v_{lim}} \quad c = 0 \quad (10)$$

More complex velocity dependant friction models have been developed, see [6] and [7], in order to obtain an improved fit between measurements and theory. These however, rely on measurements and with little information about an actual plant available, it is usually pointless to apply these models.

For friction on the lower face of the bottles, the normal force is obviously simply mass times gravity. For friction acting on the sides of the bottles due to barrier or bottle contact, it will be demonstrated in section 2.4.3 how the normal force is calculated by the penalization method.

Plant geometry generation

Before a method for contact detection and force calculation is selected, the plant geometry must be specified. Guides and barriers are easily defined as straight lines between two points.

Straight belts will be defined as box shaped geometry with the length direction along the x-axis. The belt will afterwards be rotated around an axis perpendicular to the plane of the plant in a predefined angle. The principle is illustrated in Figure 3 (left sketch). Eventually, a belt with four
corner points is obtained. These are denoted

\[ p^{(h)}_1, p^{(h)}_2, p^{(h)}_3, p^{(h)}_4 \]

Curved belts surrounded by barriers are defined as an arc-shaped area, see Figure 3 (right sketch). This type of belt is defined in terms of the end points on the outer and inner radius

\[ p^{(w)}_{1,i}, p^{(w)}_{2,i}, p^{(w)}_{3,0}, p^{(w)}_{4,0} \]

The mathematical description of the plant geometry will be derived in section 2.4. A summary of the key input parameters for generation of the complete plant model is presented in Table 1.

**Contact detection**

**Contact detection for straight belts**

A plant with \( n \) bottles contained in a space with \( m \) conveyor belts surrounded by \( w \) barriers in a plane Cartesian coordinate system will be considered. This corresponds to a top view of the bottles. For each of the \( m \) belts, a local coordinate frame with a longitudinal \( \xi \)-axis and a transverse \( \eta \)-axis is attached as belt coordinate system

Belt-to-bottle contact detection is easily conducted in belt coordinates, while this is more difficult in global coordinates. Therefore, the bottle positions are transformed to belt coordinates. Considering the position vectors defined in Figure 3, the position of the \( j \)'th bottle in the \( i \)'th time step is given by

\[ r^{(j,i)}_{xy} = r^{(h)}_{rP,j} + r^{(j,i)}_{P} \]  \hspace{1cm} (11)

The relative position of the bottle with respect to the belt corner, which in belt coordinates is the local left point, is given by

\[ r^{(j,i)}_{P} = A^{(h)}_{\xi\eta} r^{(j,i)}_{\xi\eta} \]  \hspace{1cm} (12)

In equation 12, \( A \) is the plane transformation matrix. This is in terms of the belt angle with respect to the global x-axis given by

\[ A^{(h)} = \begin{bmatrix} \cos \alpha^{(h)} & -\sin \alpha^{(h)} \\ \sin \alpha^{(h)} & \cos \alpha^{(h)} \end{bmatrix} \]  \hspace{1cm} (13)

On basis of equations 11 and 12 the bottle position in belt coordinates is given by

\[ r^{(j,i)}_{\xi\eta} = (A^{(h)})^{-1}(r^{(j,i)}_{xy} - r^{(h)}_{P}) \]  \hspace{1cm} (14)

A bottle is in contact with a belt if the x-coordinate in belt coordinates is between 0 and the belt length and the y-coordinate is in the range between 0 and the belt width. Contact forces are then added by the principle described in section 2.2. The velocity required for the direction and the regularization function is the velocity of the bottle relative to the belt

\[ v^{(j,i)}_{xy} = v^{(j,i)}_{x} - v^{(h)}_{band} \]  \hspace{1cm} (15)

It follows that when belt and bottle move with equal velocities, there will be no frictional effects acting.

**Contact detection for curved belts**

The principle applied for belt-to-bottle contact detection in curved belts is fairly simpler than the ones applied for straight belts. The belt angle \( \phi \) is defined as increasing in the counter clockwise direction, see Figure 3. This can be calculated in multiple ways. In the present context, it is easily obtained utilizing the inverse signed tangent function (often referred to as atan2).

A bottle is in contact with the belt, if the angle of the bottles position vector with respect to the global x-axis is the range between \( \beta_{ini} \) and \( \beta_{ini} + \beta \) and the distance to the center of the circle, which the belt races form, is in the range between \( R \) and \( R+b \).
In order to determine the belt center, 
\[ C(x_c^{(w)}, y_c^{(w)}) \]
the belt inner radius \( R^{(w)} \) and the end points must be applied for calculation of the equation of the circle spanned by the inner race of the belt. The equation governing the geometry is given by
\[(x - x_c^{(w)})^2 + (y - y_c^{(w)})^2 = R^{(w)} \tag{16} \]

The endpoints of the belt must fulfill this equation. We therefore have
\[
\begin{align*}
(x_1^{(w)} - x_c^{(w)})^2 + (y_1^{(w)} - y_c^{(w)})^2 &= (R^{(w)})^2 \tag{17} \\
(x_2^{(w)} - x_c^{(w)})^2 + (y_2^{(w)} - y_c^{(w)})^2 &= (R^{(w)})^2 \tag{18}
\end{align*}
\]

Isolating the \( y \)-coordinate, we obtain the expression
\[ y_c^{(w)} = C_1^{(w)} + x_c^{(w)} C_2^{(w)} \tag{19} \]

In this equation, the two constants \( C_1 \) and \( C_2 \) are given by
\[
\begin{align*}
C_1^{(w)} &= \frac{(y_2^{(w)})^2 + (y_1^{(w)})^2 - (x_1^{(w)})^2 - (y_1^{(w)})^2}{2(y_2^{(w)} - y_1^{(w)})} \\
C_2^{(w)} &= \frac{x_1^{(w)} - x_2^{(w)}}{y_2^{(w)} - y_1^{(w)}}
\end{align*}
\]

The following expression is derived for the \( x \)-coordinate
\[
\begin{align*}
\begin{align*}
a^{(w)} (x_c^{(w)})^2 + b^{(w)} x_c^{(w)} + c^{(w)} &= 0 \tag{20} \\
\end{align*}
\end{align*}
\]
in which the constants are given by
\[
\begin{align*}
a^{(w)} &= 1 + (C_2^{(w)})^2 \\
b^{(w)} &= -2x_1^{(w)} + 2C_1^{(w)}C_2^{(w)} - y_1^{(w)}C_2^{(w)} \\
c^{(w)} &= (x_1^{(w)})^2 + (y_1^{(w)})^2 + (C_1^{(w)})^2 - 2y_1^{(w)}C_1^{(w)} - (R^{(w)})^2
\end{align*}
\]
This equation can be solved analytically. Obviously, the belt is not uniquely defined, since the equation has two solutions (corresponding to different signs of the curvature). Choosing one, the geometry of the inner race can be generated by the parametric equation

$$\begin{align*}
    x &= x^{(w)}_c + R^{(w)} \cos(\gamma) \\
    y &= y^{(w)}_c + R^{(w)} \sin(\gamma)
\end{align*}$$

for $\beta_{ini} \leq \gamma \leq \beta_{ini} + \beta$. The belt normal and tangent required for contact and friction force calculation can be obtained directly from this expression.

**Bottle-to-bottle contact detection**

Bottle-to-bottle contact is detected by calculation of the distance between the centroids of two bottles using an Euclidian norm. If this distance is smaller than two times the bottles radius, the bottles are in contact, see Figure 4-A. The penetration depth $\delta$ normal to the tangent plane of the impact is recorded, since this will be required for calculation of contact forces.

**Barrier-to-bottle contact detection**

Contact detection between belts and bottles can be conducted simply by calculation of the

<table>
<thead>
<tr>
<th>Table 1: Input parameters for plant model</th>
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<tr>
<td><strong>Number</strong></td>
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<td><strong>Bottles</strong></td>
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<td><strong>Linear belts</strong></td>
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<td><strong>Barriers</strong></td>
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<td><strong>Curved belts</strong></td>
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<tr>
<td>(angle increasing in counter clockwise direction)</td>
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distance between the centroid and each barrier, see Figure 4-B. In the present context, the analytical formula for the distance between a line and a point was applied. Contact is detected if the distance is smaller than bottle radius.

For each time step, contact detection is performed by checking if each bottle is in contact with any other bottles, bars and barriers. After contact detection has been performed, the forces are on vectorial form added by the expression

$$F = \sum F = \sum \frac{N}{||n||} - \sum c \frac{v^*}{||v^*||} + \sum F_\mu$$  \hspace{1cm} (22)

The first term in equation 22 represents contact forces occurring due to barrier-to-bottle and bottle-to-bottle contacts. These are directed along the normals $n$ to the surfaces in contact. Applying the penalization method, the magnitudes of those are for linear contacts given by

$$N = k\delta \hspace{1cm} (23)$$

However, a non-linear expression obtained from experiments or for example from FEA could also be applied.

The second term in equation 22 constitutes the damping forces accounting for energy dissipated in the impacts and is denoted by a star to indicate, that angular velocities must be converted to linear velocities. In the present context this has been done by using the bottle radius, which strictly speaking in a theoretical sense is inaccurate, but has produced reasonable results.

The third term in equation 22 represents the friction forces. These may arise from either belt contact accelerating the bottles in the xy-plane, or from tangential friction due to barrier-to-bottle or bottle-to-bottle contact. Each of the tangential frictional forces will, since these are acting on the bottle outer surface, produce a moment around the bottle centroid. These are added adequately to account for bottle rotation.

RESULTS

In this section, it will be elaborated how the proposed method was implemented as numerical algorithm. Furthermore, an example of the results obtained with the method is presented.

Numerical implementation

The numerical model described in section 2 was implemented in a Matlab script allowing for generation of arbitrary plane geometries on basis of the developed plant elements. Matlab was in particular chosen
due to the ‘ready-to-use’ graphic tools enabling easy visualization. The model could might as well be implemented in GNU Octave, Python or any other software suitable for scientific programming. Both the implicit-Euler integrator described in section 2.1 and Matlabs build-in ODE solvers were tested. The best results were obtained using a solver custom built for stiff problems due to the contact stiffness parameters. The reduction in computational time was however remarkably small when applying a commercial solver based on higher order Runge-Kutta integrators. The reason for this is surely, that contact detection by far is the most time consuming sequence of the program, and especially detection of bottle-to-bottle contact has proven computationally costly. This is by no means surprising, since contact detection involves checking contact for all bottles against all bottles for each time step. For many simulations, it was estimated that the computational time required for contact detection exceeded 90% of the total computation time.

**Example of obtained numerical results**

In order to demonstrate the capability of the developed method, a test plant geometry was defined, see Figure 5. The test plant consists of a fast running single liner bottle feeder, a slow running accumulation table with a slow running belt, and a fast running single liner belt constituted by both straight and curved belt sections. The fast running single liner section feeds the bottles back into the accumulation table to form a closed-loop system.

The simulation was conducted with fictive parameters chosen for test purposes. A mass of $m=0.2$ kg was chosen for all bottles with the corresponding moment of inertia approximated by the expression for a thin walled hollow cylinder $I=mr^2$. The coefficient of dynamic friction for belts was set to $\mu^{(b)}=\mu^{(w)}=0.4$, and friction for bottle-to-bottle and barrier-to-bottle contact was specified to $\mu^{(c)}=0.1$. A slip limit velocity $v_{\text{lim}}=10$ mm/s was chosen.

An inflow of 100 tightly packed bottles with

*Fig. 5: Test plant geometry for numerical simulations*
diameters \( D_{\text{bottle}} = 130 \text{ mm} \) was simulated with a distance of \( 1 \text{ mm} \) between the bottles on the in-line. The linear bottle stiffness was set to \( k_{\text{bottle}} = 5 \text{ kN/m} \) and the linear stiffness for barrier-to-bottle contact was set to \( k_{\text{barrier}} = 10 \text{ kN/m} \). A tiny bit of damping was added for numerical stabilization for each impact with \( c = 0.1 \text{ kg/s} \). The selected case does from a practical perspective correspond to a rather large, but quite soft bottle with linear radial stiffness properties moving on a rough belt.

Various time steps were tested for a simulation time \( \Delta t = 20 \text{ s} \). The obtained results are shown in Figure 6 and Figure 7 for a total number of time steps of \( n = 500000 \) where convergence had been obtained.

The bottles can be observed to propagate onto the slow running belt on the accumulation table and slowly move forward occasionally hitting each other triggering the contact conditions and leading to impulsive force responses. The velocities in global \( x \)- and \( y \)-directions of the \( 1^\text{st} \) and \( 50^\text{th} \) bottle fed into the system is shown in Figure 8 and Figure 9. The observed peaks are indications of the impulsive interactions between the bottles. Once reaching the end of the accumulation table, the bottles stock and form patterns similar to those observed in actual bottle sorting plants.

The bottles typically enter the single liner loop belt in groups of two, by which the bottle entering from the upper side slips onto the belt with high velocity causing it to be shot back and forth between the walls of the narrow section. The bottle entering from the lower side enter the loop belt with a lower speed in a smoother fashion. This behaviour seems dependant of the chosen angles of the guides (green lines) on the right side of the accumulation table.

All forces from the obtained analyses can be saved and applied as input for strength assessment of the bottles.

**REQUIRED MODEL IMPROVEMENTS AND SCOPE OF FUTURE RESEARCH**

The present model has in section 3 been demonstrated to produce results which qualitatively correspond well to the expected physical behaviour of soft bottles on a conveyor belt. However, in order to compare the obtained results to the actual behaviour of similar systems, key input parameters for modelling of stiffness, friction and damping must be identified experimentally. Furthermore, a method, probably based on visual motion capture, for detection of the actual behaviour of a bottle conveyor must be developed.

Glass bottles are likely to have a significantly higher stiffness than the ones applied for generation of the presented results. Actual bottle stiffness parameters may range in the scale of \( 10^7 - 10^8 \text{ N/m} \). Simulations with stiff parameters are in principle possible with the present implementation, but are estimated to run for days on computers with a single processor core.

Two different approaches may be followed in order to analyse systems with higher stiffness:

The established model implementation can be optimized to increase the computational speed. In particular, the contact detection holds a large optimization potential. First of all, contact detection might not have to be performed for each time step as long as forces are updated based on contact detection from a previous step. Approaches for ‘search for closest neighbour’ from modern DEM techniques could furthermore be considered a possibility for speeding up the algorithm.

As alternative approach, the system could be analysed with equivalent stiffness parameters lower than the actual, if such can be obtained for example by using dimensional analysis. This approach would however require experimental verification.
Fig. 6: Simulated results, positions of bottles on the accumulation table for different time steps
Fig. 7: View of bottle positions in the entire plant for t=20 s (end of simulation)

Fig. 8: Velocity response in the x-direction (left) and y-direction (right) of the first bottle entering the plant

Fig. 9: Velocity response in the x-direction (left) and y-direction (right) of the 50th bottle entering the plant
CONCLUSIONS

A model for the plane behaviour of bottles on a conveyor belt has been presented. The model is based on principles from basic dynamics and numerical analysis and is easy to implement in software for scientific programming.

On basis of the obtained results, it has been concluded that the results correspond qualitatively well to the physical behaviour of actual systems. A qualitative comparison cannot be conducted at the time of writing due to lack of experimentally obtained measurements.

The possibilities for improving the model for faster analysis of stiffer systems have been identified, and will form the basis for the future research of the authors.

REFERENCES


