Thermal Characterization and Parametric Optimization of a Thermal Bimorph for use in Mirco-Robotics Applications

Owen E. Brown
Thermal Characterization and Parametric Optimization of a Thermal Bimorph for use in Micro-Robotics Applications

By

Owen E. Brown

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Approved By:

Dr. Wayne Walter
Department of Mechanical Engineering

Dr. Ferat Sahin
Department of Electrical Engineering

Dr. Satish Kandlikar
Department of Mechanical Engineering

Dr. Edward C. Hensel
Department Head of Mechanical Engineering

Department of Mechanical Engineering
Rochester Institute of Technology

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ABSTRACT

The thermal bimorph actuator is a multi-layer Micro-Electro-Mechanical (MEMS) device used to achieve out-of-plane mechanical displacements in response to a thermal input. This device is one of the simplest MEMS devices to manufacture. Previous investigations of thermal bimorph actuators have studied the best materials to use based on ease of deposition, and the overall effect on the devices. The current work presents an optimization of the thermal bimorph (Aluminum and Polysilicon) actuatorgeometry, for a target application of a micro robotics actuator. The application of bimorph actuators to micro robotics demands high efficiency in the conversion from thermal energy to mechanical displacement, since low efficiencies require larger power supplies and reduce the payload capacity of the micro-robot. Two subsystems with significant impact on bimorph actuator efficiency include the thermal mass of the substrate (modeled with three parameters) and the relationship between various geometrical dimensions (modeled with four parameters) of the actuator leg. Each subsystem is optimized using a transient finite element analysis of the coupled thermal and mechanical response. Parametric studies were used to investigate the response curve of the target functionals and then optimized using a local steepest descent algorithm. The optimized system results in a nominally 200% higher payload capacity with 350% stiffer mechanical characteristics. Results of the investigation demonstrate the need for more accurate material properties at the micro-scale. The mass of the power supply required to achieve sustained micro robot motion using thermal bimorph actuators currently exceeds the corresponding payload capacity of the device.
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## Table of Contents

Chapter 1 Introduction ........................................................................................................... 1  
Chapter 2 Background Information ....................................................................................... 3  
  2.1 Thermal Leg Devices ........................................................................................................... 3  
  2.2 Geometry ............................................................................................................................. 3  
  2.3 Commonly Used Terminology ............................................................................................... 4  
Chapter 3 Literature Review ...................................................................................................... 8  
  3.1 Analytic Methods Applied to the Bimorph problem .............................................................. 8  
  3.2 Experimental Methods Applied to the Bimorph Problem ...................................................... 11  
Chapter 4 Objective ................................................................................................................. 16  
Chapter 5 Model Setup ............................................................................................................ 18  
  5.1 Device Geometry and Symmetry ............................................................................................ 18  
  5.2 Meshing and Mesh Convergence ......................................................................................... 20  
  5.3 Boundary Conditions .......................................................................................................... 22  
  5.4 Material Properties ............................................................................................................. 23  
  5.5 Lumped Capacitance and the Timoshenko Beam Models ..................................................... 27  
Chapter 6 Thermal Boundary Condition Development ............................................................. 31  
  6.1 Modeling the Heater ............................................................................................................ 31  
  6.2 Radiation Cooling ............................................................................................................... 34  
  6.3 Convection Cooling ............................................................................................................. 35  
  6.4 Fluid Deflections ................................................................................................................. 41  
Chapter 7 Model Uncertainties and Sources of Error ............................................................... 44  
  7.1 Continuum Mechanics Assumption .................................................................................... 44  
  7.2 Finite Element Model Assumptions ..................................................................................... 46  
  7.3 Loading Uncertainty ............................................................................................................ 47  
  7.4 Uncertainty in Material Properties ..................................................................................... 48  
Chapter 8 Model Results and Comparison ............................................................................ 51  
  8.1 Bulk Substrate ................................................................................................................... 51  
  8.2 Radiation Effects ................................................................................................................. 57  
  8.3 Convection Cooling With Radiation .................................................................................... 62  
  8.4 Comparison of Models to Experimental Results ................................................................. 65  
Chapter 9 Optimization Method and Recommendations ........................................................ 68  
  9.1 Substrate Optimization Results ............................................................................................. 79  
  9.2 Beam Optimization Results ................................................................................................ 81  
Chapter 10 Conclusions ........................................................................................................... 87  
Chapter 11 Recommendations for Future Work ..................................................................... 90  
References ................................................................................................................................ 91  
Appendix A: Material Properties ............................................................................................... 94  
Appendix B: Comparison of Beam Correlations ....................................................................... 96  
Appendix C: ANSYS Optimized Beam Code .......................................................................... 98  
Appendix D: Optimization Results ............................................................................................. 102
Table of Figures

Figure 2.1 One device labeled with dimensions .......................................................... 4
Figure 2.2 Deflection response to input forcing, with different types of displacements labeled .......................................................... 5
Figure 2.3 Definition of deflection angle geometries .................................................... 6
Figure 2.4 Symmetry of the devices in the patterned array .......................................... 7
Figure 3.1 Sample thermal bimorph geometries ......................................................... 12
Figure 5.1 Deflection as a function of device thickness ............................................... 19
Figure 5.2 Mesh convergence for structural deflections in the thermal bimorph .......... 21
Figure 5.3 Symmetry conditions applied to the device ............................................... 22
Figure 5.4 Device deflection with linear and nonlinear material properties ................ 24
Figure 5.5 Thermal response of the device ............................................................... 25
Figure 5.6 Transient deflection of the device modeled with different Young’s Modulus values ........................................................................................................... 26
Figure 5.7 Lumped capacitance thermal circuit ............................................................ 28
Figure 6.1 Effect of various cooling mechanisms ....................................................... 35
Figure 6.2 Buoyant flow developing around the device ............................................. 39
Figure 6.3 Air temperatures around the micro robotics array ..................................... 40
Figure 6.4 Comparison of the convection coefficients ............................................... 41
Figure 6.5 Fluid induced deflections in the device .................................................... 42
Figure 8.1 Heat flow into the substrate during device heating ..................................... 53
Figure 8.2 Heat flow from the substrate during device cooling .................................. 53
Figure 8.3 Transient thermal response of the device ............................................... 54
Figure 8.4 Maximum device temperature varying with the change in dimension of the device ............................................................................................................. 55
Figure 8.5 Heating and cooling time for the devices when modeled with varying amounts of substrate ........................................................................................................... 56
Figure 8.6 Thermal response of the device when cooled with radiation only ............. 58
Figure 8.7 Dynamic temperature of the devices ....................................................... 59
Figure 8.8 Mechanical deflection of the tip of the beam .......................................... 60
Figure 8.9 Rise and relaxation times for various power inputs to the system cooled with radiation ........................................................................................................... 61
Figure 8.10 Free convection thermal response ......................................................... 63
Figure 8.11 Steady state temperatures of the device ............................................... 64
Figure 8.12 Beam deflection of the free convection models ....................................... 64
Figure 8.13 Rise and relaxation time of the devices when cooled with radiation and free convection ........................................................................................................... 65
Figure 8.14 Comparison of the Ataka results to the ANSYS reproduction ............... 67
Figure 9.1 Micro robotics application of the micro leg array ..................................... 68
Figure 9.2 Dimensions of the etched substrate ....................................................... 71
Figure 9.3 Optimization process complexity ............................................................ 74
Figure 9.4 Beam parameters used in the device optimization (drawing not to scale) ...... 75
Figure 9.5 Beam stiffness varied by thickness of each layer of material in the beam .... 76
Figure 9.6 Beam deflection for varying layer thickness, fixed length and width T=300°C... 77
Figure 9.7 Effect of etching on the bulk substrate .................................................... 79
Figure 9.8 Device results before and after optimization .............................................. 80
Figure 9.9 Functional value at varying thickness of the beam layers, with beam lengths (100,
550, and 1000 microns) and width fixed at (50, 200 and 350 microns) ............................. 81
Figure 9.10 Beam functional value as a function of the length and width of the device....... 82
Figure 9.11 Beam functional values for at varying layer thickness with physical constraints
shown ..................................................................................................................................... 83
Figure 9.12 Beam functional values for at varying layer thickness with physical and design
constraints .............................................................................................................................. 84
Figure 9.13 Dimensions of the deposited heater ............................................................... 85
Figure B.1 Device deflection for 300 degree temperature change ................................. 97
Table of Tables

Table 3.1 Reported thermal bimorph geometry ................................................. 13
Table 3.2 Thermal bimorph static deflection .................................................. 14
Table 3.3 Reported thermal bimorph dynamic deflection ................................ 15
Table 6.1 Heater test setup using +5V input .................................................. 33
Table 6.2 Resistive heating results using +5V input ........................................ 33
Table 7.1 Effect of varying Young’s Modulus on the optimization functional .... 49
Table 8.1 Setup of the test of bulk substrate effects on the device response ..... 52
Table 8.2 Parameters used to characterize the radiation model ...................... 57
Table 8.3 Thermal input into the free convection model .................................. 62
Table 9.1 Fixed state variables for device optimization ................................... 69
Table 9.2 Model constraints for the device optimization .................................. 70
Table 9.3 Etch size parameters ...................................................................... 73
Table 9.4 Beam optimization parameters ....................................................... 78
Table 9.5 Important optimization results ......................................................... 85
Table 9.6 Final dimensions of the device after optimization ......................... 86
Table A.1 Properties of air at standard pressure ............................................ 94
Table D.1 Bulk substrate optimization results .............................................. 102
Table D.2 Beam optimization results .............................................................. 104
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Units</th>
</tr>
</thead>
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<tr>
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**Subscripts**

- **bot**: Bottom beam layer
- **conv**: Convecting surfaces
- **cs**: Cross section
- **etch**: Etch pit in bulk substrate
- **gen**: Heat generated within the device
- **in**: Energy transferred into the system
- **out**: Heat energy lost from the system
- **top**: Top beam layer
- **rad**: Radiating surfaces
- **s**: surface
- **stored**: Heat stored in the device
- **sub**: Device substrate
Chapter 1 Introduction

Micro Electro Mechanical Systems, MEMS for short, are a classification of devices, which have been manufactured using micro fabrication techniques. While these devices might seem like a work of science fiction, they are quite real. Every advance in micro manufacturing technology allows for the development of new device applications. Even though MEMS devices currently have only a small number of commercial uses, projected markets for MEMS technology are in the billions of dollars. People have daily contact with MEMS devices and do not even realize it. For example, the sensors used to deploy airbags are MEMS devices. The growing field of devices and markets for these devices has led to a necessity to predict the behavior from a system standpoint. This necessity has led to the use of finite element programs to model the devices.

In the field of MEMS devices, there are few devices as hard to develop as the thermal bimorph actuator. The principle is simple: add heat to a beam made from two different materials laminated together to form a beam and deflection is the end result. While this process seems fairly simple, the complexity due to thermal mechanical interactions is actually quite high. This complexity has made modeling the devices a difficult task. The advancement of numerical modeling techniques has allowed made it possible to model these complex systems with more accuracy than ever before.

Unlike other types of MEMS devices, thermal MEMS are significantly more difficult to model. The difficulty modeling thermal devices is caused by the application of boundary conditions. Thermal devices have boundary conditions with some similarities to those applied to electromechanical devices. While electromechanical devices, like the comb drive actuator, may seem similar to the thermal bimorph because they both convert electrical energy into mechanical displacements, they are very different in many respects. In the comb drive actuator, the electrical charge, which is the driving force, stays where it is directed. This happens because isolating the electrical charge with electrical insulators is easily accomplished. When modeling thermal actuators, a common mistake is assuming thermal potential is as easily isolated as electric potential. If the comparison of the thermal to electrical resistance is made, the thermal resistance in the devices is orders of magnitude
lower than the electrical resistance. This means the devices will conduct heat much better than they will conduct electricity.

Beyond the problem of heat being dissipated around the device, other problems can occur in the modeling of these devices, which are significant to the performance of the model and are easy to overlook at first glance. For instance, plate-bending effects have been shown to effect the tip deflection of the devices. The properties of the device materials also vary as a function of temperature. There are environmental effects, like static electricity, can prevent motion of devices or cause unwanted motion in the devices. In the case of thermal actuators, other devices on the chip can cause substrate heating which in turn causes thermal devices to deflect. All of these effects need to be accounted for when developing a model of the thermal device.

The benefits from overcoming the modeling problems are just as great as the challenges in developing a successful model. The devices have applications in almost every field of study from sensing devices, from measurement of boundary layer velocities, to developing control surfaces for micro air vehicles. IBM has developed a method for using MEMS cantilever devices to imprint on a polymer chip and store up to one terabyte on a surface no bigger than a postage stamp.

The focus of this work is two fold. The first portion of the work involves developing a model, which shows good correlation to previously published works. This will be accomplished through a thorough study of the conditions affecting the devices. The second part of the work is to optimize this model to provide the most efficient device structure for micro motion devices. These applications include micro robot and ciliary motion arrays. By performing this analysis and documenting the method used to create a high quality device model, the methods put forth in this thesis can be applied to the modeling of similar thermal MEMS devices.
Chapter 2 Background Information

Before diving into the problem of optimizing the thermal bimorph actuator, the terms used to describe the device and its behavior need to be defined. This background information section outlines the terminology used in describing the geometry, actuation, and behavior of the devices, and provides a reference for the rest of this work.

2.1 Thermal Leg Devices

Cantilever legs are one of the most commonly used MEMS devices. Within the category of cantilever leg actuators, thermal leg actuators are a common type of device. While there are several types of cantilevered thermal actuators for both in plane, and out-of-plane motion, the focus of this study is on a common type of out-of-plane actuator known as the thermal bimorph actuator.

Thermal bimorph actuators are nothing new. For years they have been used in thermal switching operations. The most common use of the thermal bimorph actuator is to switch off clothes dryers when a predetermined temperature is reached. Thermal bimorphs present a simple method to convert a temperature change to a mechanical displacement. The basic theory behind the thermal bimorph is very simple. Different materials expand at different rates when subjected to a temperature change. If these materials are joined together, stress is produced. This stress leads to a subsequent displacement in the device. This displacement causes the motion that is desired from the thermal bimorph for applications from thermal switching to the micro robotics application studied in this work.

2.2 Geometry

In order to understand the terminology used in describing the devices used in this study, the abbreviations used in describing the device need to be presented. The formal definition of each part of the device will be explained later in this chapter. The geometry of the device is very simple. The red layer, typically aluminum, is deposited on the blue layer, representing the silicon substrate. The yellow layer in the case of this study will be patterned on top; however, it can be added before the other layers depending on the desired device geometry. The thermal bimorph contains all three of these layers and extends from the large bulk substrate portion of the device. The length, width and thickness of the substrate, are denoted
by the subscript, sub. Similarly the beam length and width are distinguished by the subscript, beam. The thicknesses of the two mechanical layers in the thermal bimorph are denoted with subscripts as well. The top and bottom layers of the device are labeled with, top and bot respectively.

![Figure 2.1 One device labeled with dimensions](image)

This diagram also shows the coordinate system referenced throughout the documentation. Generally the length of the devices are in the x direction, the width of the devices are in the z direction and the thickness and the displacements are in the y direction.

### 2.3 Commonly Used Terminology

This section describes the terminology commonly used to describe the thermal bimorph actuator. Each term described here explains a portion of the device behavior. An effort has been made to use common terms from literature to describe the thermal bimorph actuator.

Device deflection is a key focus of this research. There are several types of deflection explained in this documentation. Each deflection describes something different, however
nature of the terminology can be somewhat confusing. Because these terms can be confusing, the terms commonly used in this study for the different types of device deflection are explained in detail below. Figure 2.2 shows the displacements of the tip of the device relative to the surface of the wafer plane. This system output is compared against the square wave input into the system.

Figure 2.2 Deflection response to input forcing, with different types of displacements labeled

**Static Displacement**: is the distance the tip of the device has lifted above the substrate. The static displacement is caused by the built in stresses caused by the material deposition process of the deposited layer on bottom layer of the beam. This is the deflection the device will see when there is no power flowing to the device and it is at thermal equilibrium with the surroundings. In Figure 2.2, the dark blue diamonds represent the static deflection portion of the curve. These blue diamonds are not present in the when the device is being powered and while cooling to a nominal temperature.

**Dynamic Deflection**: is the distance the tip of the device has traveled from the initial static deflection. Conditions for dynamic deflection of the device are the resistance heater is powered, the device has completely heated and the device is at thermal equilibrium with its surroundings. This portion of the curve shown in Figure 2.2 is represented with the light blue x characters.
**Transient Deflection**: is the distance the tip of the device is from the static deflection point at any instant during the heating and cooling phase of the device. During the heating phase of the cycle, the resistance heater powers the device, and in the cooling phase, the device is dissipating the latent energy stored in the device. The transient deflection of the device is represented in Figure 2.2 as green triangles.

Another portion of the device behavior that is sometimes difficult to understand is the reporting of the deflection of the devices. Based on the measurement tools available and the type of analysis that is being performed by the author, the reporting method could include any convenient method of describing the geometry. Often times this convenient method is given as a deflection angle. There are several types of deflection angles authors have used. These angles are described in this section to enable ease of conversion from the form of data in literature, to the form used in this work. Figure 2.3 shows the angles and the reference lines that they are measured from.

![Diagram](image)

**Figure 2.3 Definition of deflection angle geometries**

**Planer Deflection Angle**: is defined as the angle formed by the wafer plane and a line connecting the two ends of the curled beam.

**Radial Deflection Angle**: is defined as the angle formed by projecting lines to both ends of the cantilever from the center of the arc formed by the curled beam.
Tangential Deflection Angle: is the angle formed by the wafer plane and a line which intersects the wafer plane and is tangent to the tip of the device.

Bulk substrate: refers to the portion of the wafer that is not etched away during processing. The beams protrude from this structure, and it represents the majority of the mass in the system.

Beam: is the extended cantilever consisting of a deposited top layer and a poly-silicon bottom later.

Resistance Heater: is the element deposited on the device that converts electrical current into heat, shown as the yellow layer in Figure 2.1.

Device: refers to the one symmetric division of the larger device array, this is best illustrated in Figure 2.4.

**Figure 2.4 Symmetry of the devices in the patterned array**

Array: all of the devices with the same symmetry conditions repeated along symmetry planes to make up the mechanism.
Chapter 3 Literature Review

The thermal bimorph actuator is not a new concept, nor is it new to the field of MEMS. Researchers have looked at the problem of the thermal bimorph in several different ways. Work has been done to develop models for the deflection of the devices. These models were developed from beam equations. Experimental studies of the thermal bimorph are also common because these devices are easily manufactured. In many cases, the devices are simply fabricated and tested to avoid some of the problems associated with modeling the devices. This section will look at analytical and experimental methods for extracting data from the thermal bimorphs.

3.1 Analytic Methods Applied to the Bimorph problem

Thermal bimorph actuators have been used long before the invention of finite element modeling software, MEMS devices, or even the computer as we know it today. Because these devices were useful for thermal switching applications, methods needed to be developed to calculate the deflection in these devices. Several methods of modeling bimorphs are presented in the literature. The early models of the devices were derived from beam equations for the devices. More recently, finite element models of the devices have become the method of choice for modeling the thermal bimorph actuator.

Timoshenko was one of the first to present a model for the deflection of the thermal bimorph [1]. His work presents a model that relates the layer thickness of the device to the radius of curvature. Riethmüller and Benecke were the first to apply the Timoshenko bi-layer beam model to MEMS devices [2]. Since then, other authors have used this model for comparison of finite element models and the experimental results of the bimorph beams[3-5]. These models are the most commonly used models for the thermal bimorph for the MEMS application. It is the most widely used thermal bimorph beam model in MEMS devices, particularly in some of the early MEMS thermal bimorphs literature.

Other beam models are used in the literature to predict the behavior of the thermal bimorph actuator, and these methods are briefly discussed here. Multi-layer models exist, like the one presented by, Chan and Li, for modeling devices with a stiffness layer between the main layers [6]. This model was used to predict the behavior of the devices presented in
Zhau et al [7]. The thermal bimorph model presented in Chu was developed from experimental results and compared with the results from finite element analysis [8]. Schweitzer et al. used this model with some success to model their thermal bimorphs [9]. Similar equations for the thermal bimorph cantilever were developed by Burgreen [10]. An additional method found in the literature for modeling the deflection is not presented in Appendix B due to the number of assumptions used. This method, presented by Suh et al., uses a thin slice of a bi-layer round plate with a differential temperature to find the deflection [11].

Models more sophisticated than the beam model have appeared in literature, which implement finite element models to solve the thermal bimorph problem. These methods include both thermal and structural models for the devices. Some of the methods presented use modeling techniques which are fairly antiquated. In the time since these papers were written, the computational power of computers has grown significantly. Because of these advances, more research has gone into these fields, and better modeling techniques now exist to perform many of the same types of simulations, faster and more accurately.

One of the first papers modeling the thermal bimorph MEMS device, presented by Funk et al., provided a comprehensive outline of the modeling process [12]. This paper suggests a method to deal with highly coupled electro-thermal actuators as well as the nonlinearities that are seen in the micro-device. Funk et al. outlines the steps that are taken in defining the model parameters, with particular emphasis on the meshing and material properties. Despite using an extremely coarse mesh in the model and noted problems with the method, the model was able to predict the behavior of the device to a high degree of precision.

Funk et al. also deals with the thermal aspect of the devices, and was the earliest author to do this [12]. The thermal domain is simulated to include free air as a body capable of dissipating heat from the model. There is no actual fluid simulation done on the device. In their models, Funk et al., modeled the fluid as a solid element with very negligible mechanical properties. To paraphrase the process, the surrounding air is modeled as a solid that can conduct heat, but does not interfere with the mechanical stresses and displacements in the system. Fluid structural interfacing was done in this way because fluid elements were not part of the finite element methods used. Since Funk et al. were looking at a single beam
mechanism, and flow does not develop for just one device. Modeling the fluid, however, will work quite well for the case of free convection over the device [12].

Other finite element models of thermal bimorphs have been developed for applications with less similarity to the micro robotics application presented in this study. These models differ primarily from the application of the boundary conditions to the models of the devices. Liu and Huang developed a model with radiation and convection cooling of the device [13]. They report convection is responsible for approximately 1% of the overall cooling, and radiation accounts for 0.1% of the heat lost from the system. They report conduction is the major driver for cooling, though they do not report how this condition was applied to their model. However, since they report a significant temperature gradient in the beam, it seems likely that they have used a constant temperature boundary at the wall where the beam interfaces with the substrate, as was done in other simulations performed by other authors. Chen, et al. simulated the bulk substrate however, it was held at a constant temperature, so they observed much the same effect as holding the wall at a constant temperature (293 K) [14].

Popa, et al. also chooses to neglect radiation in their model, which considers the reaction to shaped input forcing of their blade actuator [15]. They did, however, simulate the displacements of the devices at very high frequencies, and reported that this kept the temperatures in the devices very low, so neglecting radiation is a reasonable assumption in this case. They also report that they did not simulate any fluid around the device. Instead, they chose to use approximate values, which seemed appropriate to test the response characteristics, but not to simulate the actual model. They were able to simulate the resistance heaters with some success, and used a nonlinear model for electrical resistance. They showed a reduced input power into the system. They were also able to fit their experimental model to the FEA model with some success.

Schweitzer et al. looked into optimum beam deflection [9]. Using a bimorph beam equation model, the deflections of the device were plotted against one another to find the combination of layer thicknesses for the materials presented which would give the best displacement. By plotting displacement as a function of top layer thickness for several bottom layer thicknesses, the relationship between the layer thickness and the tip deflection can be determined. Based on this relationship the curves can be varied until a maximum
deflection for the device can be found. This optimization is used only to increase the deflection of the devices.

Two other models found in the literature were optimized using ANSYS. Liu and Huang presented an ANSYS optimization of the heater placement in their devices [13]. This optimization was done to increase the linearity in their digital voltage to deflection conversion actuator. The other method involves the optimization of the V-groove actuator. These actuators, initially presented by Ebefors are a widely used alternative to the thermal bimorph actuator [16, 17]. Khire presents optimized results of the V-groove actuator, with various applied conditions around the V-groove, to improve the heating and cooling of the device [18, 19].

The analytical models presented for the thermal bimorph actuators show device modeling techniques are lacking. Most authors did little modeling in favor of building and testing the thermal bimorph. The beam models used did little to account for the heating effects in the system, and the models reported reflect this. The only model not making sweeping assumptions about the thermal behavior of the devices, was simulated using antiquated methods for finite element analysis. The optimization methods used do not account for the device powering, and left much to be desired. Therefore there is a real need to derive a feasible method for finite element analysis of the thermal bimorph, and optimize the device results.

3.2 Experimental Methods Applied to the Bimorph Problem

Almost all of the authors who studied the thermal bimorph actuator built test devices to experiment on. Depending on the intended application of the actuator, the relevance of the results may be limited. Other times the experimental values are not reported in a manner suitable for reproducing the experimental results. The results of the data, produced by previous experiments and found in literature, are presented in a tabular form in order to simplify the reporting process.

The first important step in discussing experimental results is discussing the experimental setups. In this case, the experimental setups described are mostly beam geometries. Along with providing length and width, authors have provided measured layer thickness for the device layers present. Thermal bimorphs can have a wide variety of geometries; from very simple two layer devices, to very complex beams with numerous
layers deposited on top of one another to insulate the layers from one another. Examples of the types of geometries presented in the literature are shown in Figure 3.1.

The geometry in Figure 3.1 shows some of the geometries which approximate the ones used in the literature, and shows how the layers are arranged in the different devices. For specific details of each device geometry please refer to the specific documentation referenced in Table 3.1.

The geometry presented in each paper is best represented in tabular form in Table 3.1. The two primary layers in the device are the bottom and top layer. These are the layers which have the differential coefficients of thermal expansion, and drive the motion in the thermal bimorph. For each of these layers the material and material thickness is listed. For the heaters in the devices, the material and the thickness are reported. The location and specific geometry of the heater layer was not always listed, however many papers present diagrams similar to Figure 3.1 and can be referenced if necessary. The category in the table for other layers refers to any other material deposited on the devices. These layers refer to insulation layers deposited to keep heater layers from shorting through other deposited layers, and stiffness layers used to increase the stiffness of the deflected beam.
Table 3.1 Reported thermal bimorph geometry

<table>
<thead>
<tr>
<th>Author</th>
<th>Bottom Layer Thickness (µm)</th>
<th>Top Layer Thickness (µm)</th>
<th>Heater Layer and Thickness (µm)</th>
<th>Other Layers and Thickness (µm)</th>
<th>Device Length (µm)</th>
<th>Device Width (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riethmüller [2]</td>
<td>Si 4</td>
<td>Au 1.8</td>
<td>Poly-Si 0.5</td>
<td>Si₃N₄</td>
<td>500</td>
<td>100/80*</td>
</tr>
<tr>
<td>Riethmüller [2]</td>
<td>Si 4</td>
<td>Au 2.5</td>
<td>Poly-Si 0.5</td>
<td>Si₃N₄</td>
<td>500</td>
<td>100/80*</td>
</tr>
<tr>
<td>Schweitzer [9]</td>
<td>SiO₂ 1</td>
<td>Metal 0.43</td>
<td>Deposited Layer</td>
<td>No Protective Layer</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Suh [11]</td>
<td>PIQ-L200 4.5</td>
<td>PIQ-3200 4.5</td>
<td>TiW 0.09</td>
<td>Al Si₃N₄ 0.78 0.13</td>
<td>430</td>
<td></td>
</tr>
<tr>
<td>Bühler [5]</td>
<td>SiO₂ 1.35</td>
<td>Al 1.6</td>
<td>Poly-Si 0.3</td>
<td>-</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>Zhou [7]</td>
<td>Par 0.3</td>
<td>Pt 0.2</td>
<td>Ti -</td>
<td>Par 0.1</td>
<td>2000</td>
<td>100</td>
</tr>
<tr>
<td>Jain [20]</td>
<td>SiO₂ 1</td>
<td>Al 0.5</td>
<td>Poly-Si 0.5</td>
<td>-</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>Ataka [21]</td>
<td>PolyB 2.2</td>
<td>PolyA 3.6</td>
<td>Au Ni 0.2/0.1</td>
<td>Cr 0.05</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>Liu [13]</td>
<td>Si 1</td>
<td>Al/Au 1</td>
<td>Poly-Si 0.5</td>
<td>Si₃N₄</td>
<td>240</td>
<td>84</td>
</tr>
</tbody>
</table>

*Bottom layer width/Top layer width

Table 3.1 shows several initial problems with the device reporting, as are seen in the geometric data. Authors reported the inclusion of one or more layers, but did not report the specifics of these layers. Because of this reporting error it is impossible to follow up on the work presented by two of these authors. Another glaring reality is no two authors have presented devices which are comparable in size or construction. Several devices are comparable in length and width but the materials used are entirely different. This makes it impossible to match the results from one study to the next.

The results found in literature are reported in a tabular manner. This will allow for convenient comparison of the device results. The method column in the table references the measurement technique used to define the device deflections. These techniques were described earlier in Section 2.3. The measurement device section refers to the tool used to measure the results.
Table 3.2 Thermal bimorph static deflection

<table>
<thead>
<tr>
<th>Author</th>
<th>Method</th>
<th>Measurement Device</th>
<th>Deflection Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schweizer [9]</td>
<td>Planer</td>
<td>SEM Photographs</td>
<td>45°</td>
</tr>
<tr>
<td>Suh [11]</td>
<td>Deflection</td>
<td>SEM Photographs</td>
<td>95-125μm *</td>
</tr>
<tr>
<td>Bühler [5]</td>
<td>Deflection</td>
<td>UBM Optical Microscope</td>
<td>14 μm</td>
</tr>
<tr>
<td>Jain [20]</td>
<td>Radial</td>
<td>-</td>
<td>17°</td>
</tr>
<tr>
<td>Ataka [21]</td>
<td>Deflection</td>
<td>SEM Photographs</td>
<td>250 μm</td>
</tr>
</tbody>
</table>

* Results for winged actuator at the centerline and wing tips respectively

The results presented in literature show a wide spectrum of device results for the devices. Some layer combinations have high deflections, while others have low ones. Despite presenting one of the smallest devices, Bühler had a 14 μm deflection, which is more than a third of the device length. Ataka showed an even more astounding 250 μm displacement from a 500 μm device. The winged actuator, presented by Suh, also had high deflections relative to the length. These deflections were increased by the use of the wings at the actuator tip. One problem with the reporting of results in the literature, is none of the authors commented on the error in their measurements. Another problem is there is no mention if the authors studied one device or several and then took the average deflection. If the deflections were averaged there is no mention of the standard deviation in the results.

Dynamic results require several things to be reported to be replicated. Static beam deflection required only information about the beam geometry to accurately predict the deflection. Dynamic deflection requires much more information to reproduce. Dynamic deflection requires the heating and cooling scheme for the device, and the data reported for the static deflection, to accurately predict the device results. In many cases the power used to heat the devices is provided, but not the cooling. Because the cooling condition cannot be approximated for the devices without knowing the specifics of the setup at hand, hope of matching these results is slim. Table 3.3 shows the dynamic behavior of the thermal bimorphs presented in literature.
Table 3.3 Reported thermal bimorph dynamic deflection

<table>
<thead>
<tr>
<th>Author</th>
<th>Power (mW)</th>
<th>Length (microns)</th>
<th>Deflection (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riethmuller [2]</td>
<td>200</td>
<td>500</td>
<td>-50</td>
</tr>
<tr>
<td>Jain [20]</td>
<td>50</td>
<td>700</td>
<td>-198.8</td>
</tr>
<tr>
<td>Ataka [21]</td>
<td>3.125</td>
<td>500</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 3.3 shows fewer authors have presented dynamic results for the devices than did for the static deflection. The major problem with the results presented in this section is that no mention is made of whether or not the results presented are referenced from the wafer surface, or from the location of the statically deflected device tip. Because the method used in determining the dynamic deflection is not known, it is hard to determine the true deflection above the reference plane without making an assumption about the measurement technique used to find the deflection.

As a whole the method for reporting the behavior of these devices is often ambiguous. Key information about each of the devices presented is left out of the published literature. Whether it is the device spacing in the array, the power input to the devices, or the environment in which the device is tested, many of these conditions needed to repeat the experiments are not reported. This makes the process of comparing to experimental results nearly impossible. Ataka was the only author who presented enough data about the geometry of the experimental devices to reproduce any results [21]. Therefore the Ataka results will be used for comparison to the models developed in this study.
Chapter 4 Objective

The overall goal of this work is to develop a quality model for a thermal bimorph actuator, which accurately predicts the device behavior, and be used to optimize the device for the micro robotics application. There are several factors to be taken into account during the development of this model. Based on previously proposed models of thermal leg MEMS devices several there are several factors, which have been excluded during the application of finite element model. This has led to error in these models. To develop a quality model of the actuator, these sources of error must be included. These factors will make the model more complex, however there are also steps that can be taken to simplify the model, using boundary conditions.

- Demonstrate the ability to reduce the finite element model complexity by using boundary conditions to reproduce heating effects caused by a resistance-heating element.
- Show the effect of non-linear material properties on the modeling of the device.
- Follow up previous works suggesting a need to model the beam in three dimensions to capture three dimensional thermal and plate bending effects.
- Show a need to include radiation, which is commonly neglected, in the thermal bimorph actuator models.
- Understand how the flow conditions of the fluid surrounding the device affect the device behavior both thermally and mechanically.
- Develop a lumped capacitance model, which uses classical heat transfer and beam equations, to determine the time steps which should be used in the finite element analysis and verify the results of finite element models.
- Understand the effect of the bulk substrate on the thermal behavior of the system, and justify the inclusion of the substrate in the device model.
- Apply the methods used to study the effects of various boundary conditions on the devices to model and compare with published devices as well as devices manufactured within our research group in order to validate the model.
- Optimize the model for the micro robotics application where the minimized functional is a function of power consumption, cycle time, dynamic deflection and beam stiffness.
The objectives outlined in this section can be applied to other models of similar MEMS devices. It is hoped the procedures in this document lay out a guideline of how to develop the models of thermal devices. Taking these basic concepts and developing them for each actuator allows them to be applied across a wider scope than just the thermal bimorph actuator.
Chapter 5 Model Setup

In this section the theory behind the model presented in this document is discussed. There are several facets to the model generation important to extracting quality results. One is accurately representing the device geometry with the finite element model. Another key factor is getting a quality mesh on the device geometry. The device is dependant on appropriately applying the boundary conditions to the model. The last thing needed for success in modeling is a basic model can be used to check the results of the finite element model analysis. Developing and checking each of these parts of the model allow for successful device modeling.

5.1 Device Geometry and Symmetry

Determining how to model the device is more complicated than one might think. Many factors need to be considered in the modeling of the device. While there are many ways geometry can be simplified into a finite element model, few produce quality results. Unfortunately each method of modeling makes assumptions about the behavior of the device. In order to develop a finite element model capable of predicting the behavior of the devices, these assumptions need to be understood. By identifying the assumptions, which can and cannot be made about the devices, the correct modeling method can be chosen.

The first major decision about how to model the devices is whether to use a two or three-dimensional model of the device. The two-dimensional model of the device is much simpler to model, but it also has the most assumptions involved with it. Two-dimensional models assume the conditions are all the same through the thickness of the device. Three-dimensional models are more accurate at modeling the thermal behavior of the devices.

The major assumptions needed, to make the two-dimensional model work, are based around the beam not being affected by the thickness of the device. The heat flow in the device flows from the beam into the bulk substrate, which has a different thickness. The way heat radiates from surface to surface in the device is also three-dimensional. There are also three-dimensional beam-bending effects have been show to vary as a function of width. Plate-bending effects, in thermal bimorph actuators, have been published by Hou and Chen [22]. Their study showed a linear an increase in deflection of the devices with increase in
beam width. Their data was for material dissimilar to the materials used in this study, and the range of widths studied was much higher. The effect described by Hou and Chen was investigated using the beams in this study and a three-dimensional plate-bending model. The results of this checking the plate bending effects in the device are shown in Figure 5.1.

![Graph showing tip deflection as a function of device thickness](image)

**Figure 5.1 Deflection as a function of device thickness**

Figure 5.1 shows several things about the width effect on the device. The first is the trend of increased tip deflection seen in Hou and Chen's results is repeatable for smaller width variations. This also shows using a two-dimensional model is infeasible for accurately determining the deflections of the device.

Since it has been determined two-dimensional models will not account for heat flux in the z-direction, and the plate bending effects of the model a three dimensional model must be used. The assumptions used in this three-dimensional model are reasonable. One assumption used is the minor imperfections caused from the manufacture of the devices are negligible. These imperfections would include things such as raggedness along the edges where the beam has been released from the substrate. The other assumption about the three dimensional model is symmetry can be applied to the device model.

Section 2.3 discusses the difference between a device and an array of devices. The reason the entire array is not simulated for the cases discussed in this work is simple. The
overall array of devices consists of many devices, and to apply a finite element model to all of these devices would increase the element count by several orders of magnitude. While modeling the array of devices provides a good idea of what is going on in the entire array, it does not show what is happening locally with each device any better than modeling a single device from the array with symmetry boundary conditions.

Symmetry does several things to the device model. The foremost reason to use a symmetry condition in the devices is reducing the element count and as a result the simulation time decreases. Another reason to use symmetry is to reduce the time it takes to apply boundary conditions to the model. More occurrences of the same device will lead to applying the same conditions over and over to many surfaces in the array. Because both of these problems can be eliminated by the use of symmetry, without any degradation of the model, it is a wise idea to take advantage of symmetry. The device has a plane of symmetry down the center of the beam, which is not used. The device cannot be further split down the center because of the method used to solve for radiation. Radiation is applied as a radiation boundary condition to account for heat from the beam radiating between the surfaces of the device. The radiosity solver cannot account for radiation crossing symmetry conditions, so symmetry is limited to breaking the array up by the individual device.

5.2 Meshing and Mesh Convergence

Device meshing is absolutely critical to proper device modeling. No matter how well the rest of the device in set up, a poor mesh of the device will ruin the results. It is critical the elements used to mesh the device are of the correct type, and the element size is small enough to capture the device geometry accurately.

There are several types of elements used in this study and each is used for a very specific purpose. The elements used were chosen based on recommendation in the software documentation [23]. The most commonly used element is the coupled field brick element, ANSYS element type Brick 5. This three-dimensional element allows for the coupling of thermal, mechanical, and electrical loads. This element would be used exclusively for the device modeling if three-dimensional elements were not stiff in bending. Because a goal of this study is to extract mechanical deflections out of the system, an overly rigid stiffness matrix will skew the results, making them impossible to compare to experimental data. In these cases the thermal loads can be transferred onto a multi-layered nonlinear shell element,
ANSYS element type, Shell 91. This element type is specifically for multi layered plates in bending. The last of the elements used in the device modeling are the fluid elements used to simulate the cooling and the fluid induced deflections. The ANSYS element used for these calculations is the Fluid 141 element.

A factor which is as important as choosing the correct elements is proper meshing. The mesh is checked to make sure there is no difference between the results produced by a model with the element size chosen and smaller elements. Performing this check verifies the results of the study are not effected by the mesh applied to the model. For each new model setup a mesh convergence test is run. A sample of how the mesh density effects the results of the device are shown in Figure 5.2.

![Figure 5.2 Mesh convergence for structural deflections in the thermal bimorph](image)

Figure 5.2 shows a change in the device deflection as the number of elements in the model changes. The graph shows the expected trend, there is an initially a high amount of error in the poorly meshed model, with a decrease in the error as the number of elements increases.
5.3 Boundary Conditions

The boundary conditions applied to the model lie at the heart of the device modeling, and are as important as having accurate models, meshing, and material properties. Boundary conditions are crucial to simplifying the device model. This section deals with the implied boundary conditions, used in developing the physical model. The derived boundary conditions, such as convection coefficients, will be discussed in the next chapter. The conditions presented here allow for the building of the finite element model.

One of the steps in modeling the device makes some assumptions about the thermal structural interactions in the device, and therefore for consistency, must be included in the discussion of the modeling process. The glue operation used in ANSYS modeling combines the nodes from the two objects being joined together. These combined nodes allow for the flow of heat, stress, and current across the boundaries of the glued layers. When there is an interface between two different materials, there is some loss of heat across the boundary between those layers. Compared with the amount of power flowing into the devices, these losses are very small. Because these loses are small they are neglected in the model of the device.

The other implied condition is the symmetry applied to simulate the larger array of devices. Symmetry conditions, shown in Figure 5.3, for thermal and structural models, are commonly used in device modeling, therefore, they will only be briefly explained here.

Figure 5.3 Symmetry conditions applied to the device
Figure 5.3 shows the symmetry boundary conditions used to model the device. Thermal symmetry conditions involve adiabatic walls. A symmetric device should have the same thermal conditions on both sides of the wall. Because the temperature on either side of the boundary is the same, no heat will flow across the wall, and the wall is said to be adiabatic. Mechanical symmetry boundary conditions restrict the deflection of the device normal to the surface, and the rotation about the other principle directions.

5.4 Material Properties

Material properties for the device are complicated. For each material used, there are eight material properties affecting the overall device model. The values for these properties are found in Appendix A, however some discussion of these properties is given in this section of the documentation because of the importance of these properties. The focus is placed on the effect of nonlinear material properties. Extra attention is paid to the Young’s Modulus of the deposited layer in the device since it has been the subject of recent research.

Most material properties are known to vary as a function of the temperature in the device. Some of these properties vary by as much as 70% of the value at room temperature. In order to find the effect of nonlinear material properties on the device model a simple test is defined. The model is run with both single point and nonlinear material properties, and the results are plotted against one another. By using the same geometry of the devices used in the rest of the study, the results of using nonlinear material properties on the actual system can be shown.

The model with the nonlinear material properties shows slightly reduced temperatures, deflections, and response time from the model without the material nonlinearity. There are three material properties that differ between the linear model and the nonlinear one, and have an effect on the device response. Two of these properties, Young’s Modulus and the Coefficient of Thermal Expansion effect the beam deflection. The specific heat of the device influences the thermal behavior of the devices. The thermal conductivity of the materials in the device also change significantly as a function of temperature. The size effects of these devices still limit the significance of these properties even with this change.

First varying property is the Young’s Modulus of the deposited layer. For small temperature changes about the temperature where the bulk value was measured, the Young’s
Modulus can be considered to be the bulk value. Larger changes in temperature can significantly reduce the Young’s Modulus. This reduction in Young’s Modulus causes reduced device deflections at higher temperatures, because the top layer can no longer sustain the same amount of force as it did at lower temperatures. This decrease in material stiffness is not as significant in the substrate material because it has a crystalline structure that makes its material properties more stable at higher temperatures. The reduction of the value of the Young’s Modulus will cause the device to deflect back towards the zero stress point of the bottom layer material. At these temperatures the top layer will no longer be able to sustain the forces required to bend the bottom layer, which has not been affected by the higher temperatures. The coefficient of thermal expansion increases as a function of temperature in the aluminum top layer, but this increase is not large enough to counter the effect of the decreased Young’s Modulus. The results of this phenomenon are shown in Figure 5.4.

![Figure 5.4 Device deflection with linear and nonlinear material properties](image)

It is clear from Figure 5.4 Young’s Modulus is the dominant material property in beam portion of the model. The coefficient of thermal expansion of the device continues to increase as the device is heated, but the small increase in the coefficient of thermal expansion of the top layer is far outweighed by the decrease in Young’s Modulus.
The other temperature dependent property is the specific heat of the device. Due to the small amount of mass, the aluminum layer will not store much heat. The substrate makes up most of the mass in the system, so the thermal storage properties of this portion of the device are critical to the device response. The thermal response of the device is shown in Figure 5.5

![Temperature vs Time Graph](image)

**Figure 5.5** Thermal response of the device

As Figure 5.5 shows, the model with nonlinear material properties takes longer to reach steady-state, than the model with linear material properties. The cause of this is the specific heat increase in the device. As the specific heat increases, the amount of energy stored in the device also increases. Because the rate of energy being transferred into the device has remained constant, the devices will heat slower in the nonlinear case. The decreased rise time is not very significant in the system, since the change in material properties over the temperature range is still small.

Since Young’s Modulus is so dominate in the deflection of the device, and there is some recently presented research suggesting the bulk value of the Young’s Modulus is not applicable for deposited thin aluminum films, these results are presented in this work for completeness. There are two schools of thought on the value of the thin film properties of
aluminum. Most authors have reported using, with success, a thin film Young’s Modulus very close in value to the bulk value, to model the aluminum layer in the thermal bimorph actuator. Other authors have presented experimental results of tensile testing showing a reduced value of the Young’s Modulus in thin film. The value of this reduced value of the Young’s Modulus is consistently around 30 GPa [24-28]. A plot of the tip deflection when the device is modeled with each value of Young’s Modulus is shown in Figure 5.6.

![Figure 5.6 Transient deflection of the device modeled with different Young's Modulus values](image)

Figure 5.6 shows significant changes between the two device models based solely on changing one material property in the device model. As expected the response times are the same, but the deflection of the tip of the device is different in each case. The 30 GPa case, which represents the thin film Young’s Modulus results, shows reduced deflections from the 70 GPa or bulk parameter model.

Based on the data presented in this section choosing the material properties for modeling the device is more complicated than one would hope. The decision to use bulk parameters is based on the success of previous studies using bulk parameters for device modeling. Nonlinear material properties are used in the modeling of the devices. The optimization of the devices is limited to 320°C in hopes of limiting the extreme effects of the nonlinear material properties in the devices.
5.5 Lumped Capacitance and the Timoshenko Beam Models

In order to check the finite element model for consistency, a method for confirming the model of the device has been derived of the parameters used in the modeling of the device. The method used is a lumped capacitance method. Lumped capacitance is a widely used method for modeling the heating and cooling of thermal devices. Because of this acceptance and the common use of this modeling technique for thermal devices this method will be used to validate the thermal model of the device. Several beam models have been derived for modeling the mechanical deflections of the beam. These models are presented in Appendix B of this study and compared to one another. Of the models presented, the Timoshenko bi-layer metallic beam equation is the one chosen as the best to model the devices.

Selecting a beam equation to compare the device results to proved to be difficult because several methods of modeling thermal bimorphs exist. The two most commonly used equations are the Timoshenko model and the Chu model. The Timoshenko beam model is a derived model for the deflection of the bimorph actuator, and the Chu model is based on empirical data. The Chu correlation is used only within one research group and does not seem to have gained wide acceptance [8, 9]. The Timoshenko model has been used, successfully, to model thermal bimorphs in several works. A complete comparison of these beam models is given in Appendix B.

There are several reasons for presenting a non-finite element solution to the problem of the thermal bimorph actuator. The main reason for using the lumped capacitance model is it simplifies the problem to a level where the exact principles being used can be easily explained and analyzed. All too often, the complexity of finite element method and the loads applied to the model make an exact solution to the problem nearly impossible, and the output is considered to be correct regardless of whether it actually is or not. Thus the lumped capacitance method is provided as a safeguard against error in the finite element method. The other reason for using the lumped capacitance method is transient data can be gathered rapidly, to pick out the points of study for the resource intensive finite element models.

The lumped capacitance model is a theoretical method for simplifying transient thermal models [29]. The basis of this theory comes from a direct solution of the energy balance equation, Equation (5.1).
\[ Q_{in} + Q_{gen} = Q_{out} + Q_{stored} \] (5.1)

The energy balance equation is conservation of energy applied to the device being studied. The energy that goes into the system or is generated in the system either needs to be stored or lost from the system. This basic principle can manipulated by knowing how much energy is input, generated, stored, and lost from the system. The end result of this manipulation incorporates all of the conditions on the device. With some assumptions about the geometry of the device, a lumped capacitance model of the device temperature can be easily applied.

For a three-dimensional model lumped capacitance will allow for a simplified solution to the transient thermal problem. In order to do this, certain assumptions about thermal gradients in the device must be made. A diagram of a simplified thermal circuit for the device is shown in Figure 5.7.

![Lumped capacitance thermal circuit](image)

Figure 5.7 Lumped capacitance thermal circuit

Figure 5.7 shows the thermal circuit of the device. In the lumped capacitance method the temperature, or thermal potential, is analogous to voltage. Thermal resistance and capacitance correspond to electrical resistance and capacitance.

The basis of the lumped capacitance method is a simple assumption about how the heat distributes in the device. In order for the method to work the temperature difference across the device must be small. The Biot number, \( Bi \), is used to determine if this condition
is met. The Biot number is defined as the ratio of conduction resistance to convection resistance and is defined in Equation (5.2). In order to for the conditions to be considered appropriate for the application of the lumped capacitance model, the Biot Number must be less than 0.1.

\[ Bi = \frac{hL_{\text{char}}}{k} \leq 0.1 \]  

(5.2)

In Equation (5.2), \( h \) represents the convection coefficient, \( L_c \) represents the characteristic length of the device, and \( k \) is the conduction coefficient of the material in the device. Since several materials are used in the thermal bimorph actuator and the geometry of the device is irregular, one might expect problems in the application of Equation (5.2). However, in MEMS, the characteristic length of the devices dominates this equation. Therefore even in when there is a high convection coefficient and the thermal conduction is low, the Biot number will still meet the condition for lumped capacitance.

The boundary conditions used in the finite element model of the device are substituted into Equation (5.1) to form the equation used as the lumped capacitance model, Equation (5.3).

\[ q_{s,in}A_{s,\text{flux}} = hA_{s,\text{conv}}(T - T_{\infty}) + \varepsilon\sigma A_{s,\text{rad}}(T^4 - T_{\infty}^4) + \rho C_p V \frac{dT}{dt} \]  

(5.3)

In Equation (5.3), the heat input term is defined by \( q_{s,in} \), the heat flux term, and, \( A_{s,\text{flux}} \) the surface through which the heat flux flows. The heat loss term consists of the convection term and the radiation term, both of which are involve the device temperature, \( T \), and the temperature of the surroundings, \( T_{\infty} \). The rest of the convection term consists of the convection coefficient, \( h \), and the convecting area, \( A_{s,\text{conv}} \). The radiation term involves the surface emissivity of the material, \( \varepsilon \), the Steffan-Boltzman constant, \( \sigma \), and the radiating surface area, \( A_{s,\text{rad}} \). The energy storage term is defined by the density of the device, \( \rho \), the specific heat of the material, \( C_p \), and the volume of the device, \( V \).

In order to be of use Equation (5.3) must be solved for the device temperature, \( T \). The non-linearity of the model prevents Equation (5.3) from being solved by simple integration. The model is then put into MATLAB so a numerical solution can be derived using a 4th order Runge-Kutta solver with a fixed 0.01 second time step (one-one thousandth of the time that the simulation is to be run). The output of the MATLAB integration process will provide the
base thermal analysis for comparison during the study. The results of the lumped capacitance model will be labeled as “Theory” in the graphs in this documentation.

The thermal model of the device is then coupled with the beam bending equation presented by Timoshenko [1]. By coupling the thermal response with this mechanical model, a method is developed capable of checking the dynamic and transient results of the finite element model.
Chapter 6 Thermal Boundary Condition Development

Some of the boundary conditions used in this study are slightly more complicated than those presented in the previous chapter, which outlined the conditions applied to simulated the device geometry. This chapter covers the derivation of boundary conditions requiring defined geometry and material properties from the previous chapter. The model presented in Chapter 5 is used in the derivation of these conditions. Three of the boundary conditions studied in this chapter are thermal boundary conditions. The fourth condition is the direct result of the fluid flowing over the device. These conditions simplify the model significantly by removing electrical and fluid degrees of freedom from the overall model of the device. This will save on processing time and significantly reduce modeling time in the devices.

6.1 Modeling the Heater

The most complicated assumption to simplifying the model by boundary conditions is using a heat flux boundary condition to model the effect of a resistance heater. One of the reasons for using this assumption is that it removes a significant amount of geometric complexity form the device model. This leads to lower element counts and faster simulation time. The only major assumption required is the heat from the deposited heater goes exclusively into the underlying layer. This assumption will be explained in detail and tested using modeling techniques.

The ability to generate heat by running current through a resistive element lies at the heart of some technology we take advantage of every day. Thermally actuated MEMS devices rely on this effect to generate the heat used to generate deflections. These heaters are typically a deposited metal or other electrically conductive substance with a very small cross sectional area. The small cross section restricts the flow of current and increases the resistance of the heater.

The amount of heat generated in the heater can vary based on several parameters. The first parameter influencing the resistance of heater is the path length, $L_{path}$. Longer path length in the heating element leads to greater distance for the electrons to travel to reach zero potential. The farther the electron travels the higher the device resistance. Increasing the device's resistance reduces the amount of power needed to heat the devices. The key
parameter needed for determining the resistance is the cross sectional area, $A_{CS}$, of the resistive element. The smaller the conductive element’s cross-section, the more difficult it is for electrons to flow, thus increasing the resistance of the element. The last parameter directly affecting the resistance of the heater is the electrical resistivity, $\rho$, of the material. Resistivity is the actual material property governing how much resistance per unit length there is in a particular material. Resistivity is highly susceptible to thermal changes. From these parameters, the following relationship for the resistance, $R$, of the heating element is given in Equation (6.1):

$$ R = \frac{\rho L_{path}}{A_{cs}} $$  \hspace{1cm} (6.1)

The resistance that is defined by Equation (6.1) can take advantage of Ohm’s Law to find the power generated in the heater if either the voltage or the current input is known. Since the power supply commonly used in testing the devices is a voltage source, the voltage, $v$, is used as an independent variable in the simulations. Therefore the power, $P$, for each heater can be derived using Equation (6.2):

$$ P = \frac{v^2}{R} $$  \hspace{1cm} (6.2)

Equation (6.2) is generally used as the power lost across a resistor. Since this resistor is heating the device it is assumed most of the heat from the resistor is stored in the device, and not lost to the surroundings. In order check this assumption, a test is devised to quantify the loss. The resistance of the heating element is found from Equation (6.1). Knowing the resistance of the patterned heating element, Equation (6.2) is used to find the power generated. The power generated from Equation (6.2) is referred to as the predicted power.

A model is then generated to calculate how much power is actually transferred into the device. The resistance heater is modeled on the beam. The same voltage used to find the predicted power using Equation (6.2), is applied to the ends of the resistance heater model. A convection coefficient is applied to the surfaces of the device and the heat flux through the heater/beam interface is measured. Since the interface area is known the heat flux can be converted into power. The power flowing through this interface is called the generated power. The model is simulated and the power calculated using both methods for all six cases presented in Table 6.1.
Table 6.1 Heater test setup using +5V input

<table>
<thead>
<tr>
<th>Resistor Material</th>
<th>Number of Heater Legs</th>
<th>Thickness of heater layer (microns)</th>
<th>Width of heater legs (microns)</th>
<th>Approximate Path Length (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>2</td>
<td>0.3</td>
<td>20</td>
<td>640</td>
</tr>
<tr>
<td>Al</td>
<td>4</td>
<td>0.3</td>
<td>8.5</td>
<td>1250</td>
</tr>
<tr>
<td>Al</td>
<td>6</td>
<td>0.3</td>
<td>5.4</td>
<td>1855</td>
</tr>
<tr>
<td>TiW</td>
<td>2</td>
<td>0.3</td>
<td>20</td>
<td>640</td>
</tr>
<tr>
<td>TiW</td>
<td>4</td>
<td>0.3</td>
<td>8.5</td>
<td>1250</td>
</tr>
<tr>
<td>TiW</td>
<td>6</td>
<td>0.3</td>
<td>5.4</td>
<td>1855</td>
</tr>
</tbody>
</table>

The resistive-heating model produced results, which show the resistance heater can be modeled as a boundary condition. Table 6.2 shows how well these results compare to one another. The minor losses of heat in the deposited heater model are small enough to be considered negligible. Because these results match so well, the simplification of these boundary conditions can be made with confidence. This simple check of the boundary conditions saved days worth of processing time by eliminating the need to model heater geometry. This heater geometry adds no real benefit to the model, demonstrating how important performing checks on boundary conditions can be to simplify complex models.

Table 6.2 Resistive heating results using +5V input

<table>
<thead>
<tr>
<th>Heater</th>
<th>Predicted Power (mW)</th>
<th>Generated Power (mW)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 leg Al</td>
<td>8680.6</td>
<td>8658.5</td>
<td>0.2541 %</td>
</tr>
<tr>
<td>4 leg Al</td>
<td>1888.9</td>
<td>1885.1</td>
<td>0.2059 %</td>
</tr>
<tr>
<td>6 leg Al</td>
<td>808.6</td>
<td>806.6</td>
<td>0.1804 %</td>
</tr>
<tr>
<td>2 leg TiW</td>
<td>532.6</td>
<td>531.3</td>
<td>0.2573 %</td>
</tr>
<tr>
<td>4 leg TiW</td>
<td>115.9</td>
<td>115.7</td>
<td>0.2505 %</td>
</tr>
<tr>
<td>6 leg TiW</td>
<td>49.62</td>
<td>49.49</td>
<td>0.2624 %</td>
</tr>
</tbody>
</table>

During the modeling of these heaters a phenomenon was observed in the corners where the heater turns 90°. At these locations, there are points where the heat generated is high on the inside corner and low on the outside corner. Other authors have reported seeing
this effect in their devices. This temperature difference occurs because the current in the device wants to flow closer to the inside of the corner of the heater loop because of the shortened path to ground. Therefore the inside corner generates more heat than the outside corner. This area of non-uniform heating is only a tiny fraction of the heating element, and the results are valid for the model.

Based on the results presented in this section there is no need to model the resistance heater on the device. The predicted power in the system is can be found from the solution of Equation (6.2), and corresponding heat flux will accurately model the heater from a thermal standpoint.

### 6.2 Radiation Cooling

Heat loss due to radiation is normally neglected in most thermal analysis because temperatures are too low cause significant radiation losses. Thermal bimorph MEMS devices see very high changes in temperature. Because of these higher temperatures radiation effects can come into play so it is important to include them in the model. This section demonstrates the need to include these effects and how radiation is applied to the model with the radiation boundary condition in ANSYS.

Heat losses due to radiation are usually easy to neglect in heating models. Convection and conduction are usually dominant in the cooling of devices. Larger devices develop significant buoyant flow because of their size, and can conduct heat away from hot areas. Thermal bimorphs have none of these characteristics. The power input to the devices is very high. Because the array of devices is only touching the surface it is walking on, the contact is very limited the devices have little hope of losing heat through conduction. Convention will remove a significant amount of heat from the devices. Because the devices have temperatures so much higher than their surroundings some heat will be lost through radiation to the surroundings. When convection and radiation losses are compared as part of the total cooling for the device it becomes apparent radiation is a significant factor in the cooling of the thermal bimorph actuator. The cooling effect of both radiation and convection is shown in Figure 6.1.
The radiation boundary condition in ANSYS can be applied using the Radiosity solver in ANSYS. This method uses the device geometry to calculate view factors to determine how much energy is radiated to the surroundings, and to other surfaces in the device. The process calculates the heat transferred into and out of each element where the radiation boundary condition is applied.

The theory behind the Radiosity solver in ANSYS is fairly simple. The surface emitting the radiation is divided up into sections. Then view factors are determined based on the orientation of the sections. The view factor takes into account the hemispherical emsivity of the surfaces and calculates the amount of radiation energy which can be transferred between surfaces. Some of the energy will be transferred between the surfaces, some is reflected off the surface, but most is lost directly to the surroundings.

6.3 **Convection Cooling**

Buoyancy driven flow is a significant contributor to the behavior of the thermally actuated MEMS device. This section will take a look at how the thermal behavior of the devices is affected by free convection, and how those convection boundary conditions are checked and
applied to the finite element model. Because of the complexity of free convection several different methods are used to check these conditions and the results are compared to one another for consistency before they are applied to the model.

The goal of developing the free convection conditions for the device is to accurately model the cooling of the micro robot at the array level. If the devices are modeled as a single entity no flow will develop because the amount of heat flowing out from a single device is not enough to drive buoyant flow. In these cases the cooling in the air around the device is conduction to the fluid only. However, since there is a larger array of devices in the case of the micro robot, the size of the device heated, and the amount of heat flowing from the device is much larger than it would be from a single beam. If the micro robot is considered to be a 2cm by 2cm array of devices, and modeled as a flat plate, the convection coefficients and subsequent cooling can be determined. In order to model the actuator effectively, several basic assumptions about the environment that the model is being run in must be made to simplify the model.

- Bulk fluid is infinite and quiescent, which means that the plume or warmed fluid caused by the free convection will be able to fully develop without interference from other fluid effects or other objects. Also, it assumes that the fluid outside the plume is at a steady constant temperature.

- Variation in temperature of fluid in the plume does not cause the viscosity of the fluid to change. This allows the viscosity to be input into the equations as a constant, interpolated from the referenced tables in Appendix A.

- Fluid behaves as an ideal gas. This simplifies the equation for the density of the fluid in the plume.

- Convection coefficient is independent of the thermal gradients caused by Joule heating and the various cooling methods applied to the device. The temperature on the entire surface of the actuator can be considered to be the average surface temperature.

- The beam in the flow is assumed to be flat, and the transient displacement of the beam is negligible.

- Interactions between the plumes from each surface are neglected in the calculation of the other plumes. Therefore, the flow for each surface must start at the same conditions as the bulk fluid in the analysis.
The average convection coefficient $\bar{h}$ of the surface can be shown to be a function of the dimensionless average Nusselt number ($\overline{Nu}$) [29]. The equation that relates these parameters is shown below:

$$\bar{h} = \frac{\overline{Nu} \cdot k}{L_c}$$  \hspace{1cm} (6.3)

In order to determine the free convection boundary conditions, several parameters must be determined. The non-dimensional parameter, the Grashof number, is a ratio of the buoyancy force caused by the changing gas temperature to the viscous forces in the fluid medium. The characteristic dimension, $L_{\text{char}}$, is determined for the micro robot by taking the ratio of surface area to perimeter. This parameter accounts for the device size effects in development. The equation that defines the Grashof number, $Gr$, is shown in (6.4):

$$Gr = \frac{g \beta (T_s - T_\infty) L_{\text{char}}^3}{\nu^2}$$  \hspace{1cm} (6.4)

The term, $g$, is the gravitational acceleration, which is the driving force behind the buoyancy effects of the fluid. The volumetric thermal expansion coefficient $\beta$ defines how the gas expands and contracts. (For an ideal gas, $\beta$ can be represented as the reciprocal of absolute temperature; for non-ideal gasses and liquids, $\beta$ values must be looked up in a table). The temperature of the surface of the micro robot and the bulk fluid are represented as $T_s$ and $T_\infty$, respectively. Dynamic viscosity of the fluid is denoted as $\nu$.

Once the Grashof number has been determined, the Prandtl number is defined for the fluid. The Prandtl number is the non-dimensional ratio of the momentum and thermal diffusivities. This term relates the amount of velocity and thermal energy which can be transported through the boundary layer. The Prandtl number term is defined by the following equation:

$$Pr = \frac{\nu}{\alpha}$$  \hspace{1cm} (6.5)

where $\alpha$ is the thermal diffusivity of the convective medium.

The Rayleigh number, $Ra$, is defined to characterize free convection in terms of the Grashof and Prandtl numbers. The Rayleigh number is used as the dimensionless, independent parameter for free convection.
Free convection equations have been validated for a given range of Rayleigh numbers. While the Rayleigh numbers for the micro robot array are below the range specified, an assumption is made that these equations will still predict within some range the convection coefficients. Making this assumption will allow the use of Equations (6.7) and (6.8) which are typically used only with Rayleigh numbers on the order of \((10^4 < Ra < 10^7)\) and \((10^5 < Ra < 10^{10})\) respectively. Equation (6.7) is used for the top of a hot plate and Equation (6.8) is used for the bottom of a hot plate [29].

\[
\bar{Nu} = 0.54 Ra^{1/4} \quad (6.7)
\]

\[
\bar{Nu} = 0.27 Ra^{1/4} \quad (6.8)
\]

For the CFD comparison of the cooling in the micro robot a 10 cm by 10 cm cube of air is used as the convective medium. The wall temperature of the air is kept at a constant 20 °C and the pressure at the walls is kept at standard pressure, so that the flow can develop as it would in free air. Figure 6.2 shows the flow development over the device.
Figure 6.2 Buoyant flow developing around the device

Figure 6.2 shows the movement of air around the micro robot array due to density changes around the micro robot. As expected cool air flows towards the center of the device, heats, then flows upward. Flow is the fastest as it leaves the top surface of the beam, which signifies heat from the top micro robot has been transferred to the fluid making air less dense. Figure 6.5 shows the temperature in the surrounding fluid as the hot air flows upward, away from the device.
Again the temperature in the fluid just above the device is warm, but the air cools as it moves away from the hot surfaces. The air is then cooled by the surroundings.

The CFD analysis was run for several temperatures and the average convection coefficient for the micro robot surfaces was calculated. In each of the simulated cases the results from these simulations were shown to be within a few percent of the values calculated from Equations (6.7) and (6.8). This proves the assumptions made about the ranges of the Rayleigh number for the system were suitable and the simple equations used to calculate the free convection over the larger array of devices are valid. Figure 6.4 shows the correlation between the calculated device heat transfer coefficient and the simulated model convection coefficients.
6.4 Fluid Deflections

When buoyant flow develops over these devices it is important to know what this flow does to the device deflection. If the forces placed on the device by the interaction with the flowing fluid are high the fluid might deflect the devices. This section looks at the effect of fluid flow over the surface of the devices at various speeds.

The method for determining the amount of deflection caused by the fluid involves coupling two solution methods; a fluid solution and a structural solution. The fluid side is a two dimensional fluid flow around the beam. The structural side is a three-dimensional beam problem similar to the beam used to calculate the effects of the changing beam width. It was hoped a two dimensional solution could be applied to both the fluid and structural problems using the FSI (Fluid Structural Interface) boundary condition to couple the two problems [23]. However, problems with the meshing prevented the FSI method from being used. When this method was applied to the model, element consistency problems prevented the model from meshing dynamically. Mesh errors developed in the second iteration, and the solutions failed.

This data is very important to the study of these devices, another method was devised to couple the fluid and structural problems together. Each part of the problem (structural and fluid) was modeled separate from the other. The structural side was solved first for the
deflection out of the wafer plane. This deflection was then applied to a fluid model, which was used to calculate the pressure on the deflected beam surface. Then this pressure could be applied, as a load, to the beam problem, which was solved for the fluid deflection. This process of applying the beam geometry to the flow and the forces from the fluid solution to the beam was repeated until the deflection of the beam converged to a constant value from one iteration to the next. This method performs the same process that the fluid-structural solver would, except it requires extra post processing to apply the pressures derived in the fluid model to the structural model.

The results of this portion of the study showed several interesting things. The deflection caused by the fluid was hypothesized to be much higher than it was found to be. This is due to several things. With fluid deflection, the size of the beams once again is important. The small size of the devices makes the overall deflection of the beams very small despite the high pressures associated with the flow conditions that were observed over the devices. Since the devices have such a small surface area under pressure, the force from the fluid is reduced to the micro-Newton scale, and is easily handled by the stiff devices.

![Graph](image)

**Figure 6.5 Fluid induced deflections in the device**

The forced convection flow condition is considered to be the worst case, in terms of causing fluid deflection in the beam. It is unlikely that speeds this high would ever be used.
to cool the devices. Knowing these conditions are the worst the beam will see, and the
deflection caused by the fluid flow is quite low, it can be neglected. In the case of forced
convection it is important to note that there may be a small deflection caused by the fluid
surrounding the device, but not a significant one.
Chapter 7 Model Uncertainties and Sources of Error

There are several sources of error in the model of the thermal bimorph actuator, including modeling, finite element discrimination, models used for convective cooling, error in the reporting of material properties and with the basic assumptions regarding the applicability of the continuum mechanics governing assumption. This section will quantify the errors associated with each of these sources, and their impacts on the models used in the optimization of the devices.

7.1 Continuum Mechanics Assumption

MEMS scale devices teeter dangerously on the boundary where continuum and quantum mechanics meet. As MEMS devices become smaller the effect of crystal growth and atomic structure become increasingly important to model. This section will examine whether the continuum mechanics model used so far is appropriate, and how significant quantum effects are to the current problem. Literature citations will be used to identify where the underlying assumptions of the continuum mechanics finite element model break down. As MEMS devices move closer to the nano scale, finite element modeling should yield to a molecular dynamics approach for understanding the behavior of the devices. Molecular dynamics have the advantage of better describing the internal workings of the device, when the scale of the devices approaches the scale of the molecules making up the body. A molecular dynamics method would be the most accurate way to model a thermal bimorph actuator. The primary disadvantage of molecular modeling is that current computer technology cannot feasibly model billions of atoms. Continuum mechanics model is preferable from a computational standpoint if the model will produce similar results.

The main issue with the application of a finite element model is the validity of the continuum mechanics assumptions. These assumptions are all based on the fairly simple concept that boils down to neglecting the interaction of matter at the molecular level. Homogenous materials are the cornerstone of the finite element models presented in this work, however with device sizes as small as the ones presented, these assumptions should be validated.
The main homogeneity issue on the MEMS scale is determining at what point the operation of the device becomes affected by the discrete atoms making up the device itself. Lai suggests the size at which the continuum mechanics assumption breaks down is not easily quantified by the number of molecules in a volume [30]. Instead he suggests that a case by case application based on experimental results is the best way to determine what size the assumption for continuum mechanics breaks down. While it is always preferable to do experimental tests, the point of modeling these devices is to predict the behavior before building models.

Several authors who have studied micro devices have stated that for most applications on the MEMS scale continuum mechanics assumptions hold unless there is unforeseen effect [31, 32]. For instance in micro gears, the gears themselves are on a level where the internal structure behaves according to the rules for continuum mechanics. The contact between the surfaces of the gears violates material continuum assumptions, and they can not be used to model these boundaries. According to both authors, high frequency applications (e.g. vibrational frequencies above approximately 20kHz) are another case when these assumptions tend to break down.

A better approach for the modeling application presented here may be the one presented by Fung, who quantifies the length scale at which continuum mechanics assumptions fail [33]. He suggests using a dimension at least two orders of magnitude larger than the molecular dimension to insure the validity of the continuum mechanics assumption. Since experimental data has been previously presented stating the continuum assumption begins to fail at 10 nm for a silicon device, the two order of magnitude rule can be tested [31]. The size of a silicon atom is 118 pm, or 0.118nm [34]. Increase this value by two orders of magnitude and yields 11.8 nm. For the thermal bimorph actuator herein, the smallest linear dimension of the silicon layer is 1,000 nm so it is well above. Applying the two order of magnitude rule to the aluminum layer, will produce a minimum dimension for the continuum assumption of 14.8 nm. Since the minimum thickness of the aluminum layer of the thermal bimorph actuator is 100 nm the continuum assumptions should hold in the case of the thermal bimorphs studied.
7.2 Finite Element Model Assumptions

The process of reducing actual geometry to a finite element model always has several assumptions associated with it. One of the fortunate things about MEMS devices in this respect is the geometry is patterned on the devices, so the geometry is easily approximated by simple geometric objects. The major issue in the modeling of the devices is the connection of the deposited layers to one another.

The meshing of the model was discussed in section 5.2. The error associated with discretizing the model is presented in this section. The h-convergence test of the model showed the effect of the element count on the model. If the element size in the mesh of the beam is too small the devices will be too stiff and the deflections will be low. The meshes used in the devices are all well above the converged mesh size to be conservative in the device modeling. The element type is chosen based on the recommendation of the ANSYS manual. If solid model elements had been used, the devices would be overly stiff, so the shell elements are used to mesh the beam. The thermal elements used are the only type available to model the effects. The thermal mesh size is also conservative to avoid errors associated with the meshing.

It was previously mentioned the layers in the device are “glued” to one another using an ANSYS operation of the same name. However, it is known heat does not transfer perfectly across this interface, due to the thermal boundary resistance between the layers [35]. Fortunately this effect is not critical to the device behavior of the thermal bimorph. If this effect is more prevalent than expected in the actual multilayer device, this effect will improve the device results. The temperature differential shown between layers would be beneficial to the device deflection. The top layer would have a higher temperature than the bottom layer of the beam. For higher deflections, the top layer needs to expand more than the bottom layer. Since this temperature differential exists, it will help the differential expansion of the layers, increasing the force in the beam. This increased force should drive higher displacements in the devices.

The other major assumption in these devices also has to do with the deposition of one layer on top of another. Because these layers are so thin, any surface defect on the target layer will cause a discontinuity in the layer above. This discontinuity would cause the modeled geometry to be invalidated, and the actual device could have stresses due to this
discontinuity which cannot be predicted. Manufacturing process defects are impossible to predict in MEMS devices, as they are on the macro scale. Since these defects are impossible to predict, commenting on the effects of these manufacturing defects is moot.

7.3 **Loading Uncertainty**

The convection coefficient for the devices is derived assuming a constant array temperature and uniform heating of the array. This data is then compiled into a table of the convection coefficients as a function of array temperature which is linearly interpolated between the calculated points. This method for modeling the cooling of the devices is wrought with assumptions about what happens in the devices. It is therefore important to demonstrate the effect of under and over predicted cooling of the micro cantilever devices which over and under estimate the convection coefficient.

A device which is under-cooled during the optimization process will cause problems with the device models becoming too hot at steady state. If the cooling is underestimated the devices will take longer to cool, and less time to heat. The heating cycle for the devices is very short in the devices, even with different convection rates on the surfaces. The cooling portion transient cycle fills the majority of the cycle time for even the best cooled devices shown in the study. Therefore an underestimate of the device cooling will cause the models to take longer to cool, and the cycle times will increase.

A convection model with over estimated surface coefficients would predict slower rise times and faster cooling times, and the devices would have lower temperatures than in actuality. Again if the rise time is said to only be a small portion of the cycle time, and the cooling portion is considered the dominant factor in cooling, these devices will show better results of the optimization functional than the actual model.

The overall effect of incorrectly predicting the convection coefficients for the device will not change the outcome of the substrate optimization. The curves generated in the device simulations over the range of optimization parameters will show similar trends to the device optimization. The optimization procedure could assume any surface convection coefficient and still arrive at the same dimensional value, because the small changes in geometry at the device level do not effect the derivation of the convection at the array level.
7.4 Uncertainty in Material Properties

A potentially large source of uncertainty in the model is the material properties. The properties of materials discussed in this study are constantly disputed between experts in the field of MEMS devices [24, 25, 28, 36]. These properties can vary based on any number of factors in the processing of the materials on to the wafer. These models all assume there is no oxidation in the materials in the device. Some of the material properties have a significant effect on the device behavior, while others are less influential. The beam is the most significantly effected by inaccuracies in the material properties, while the optimization trends for the beam are not heavily dependant on the material properties.

The thermal characteristics of the model are controlled by two properties the thermal conductivity and the specific heat. The thermal conductivity of the material effects how the heat transfers within the device. Because the size of the devices has such a significant effect on how quickly heat transfers within the devices, the thermal conductivity of the solids in the device models would need to change by several orders of magnitude, to have a significant effect on the cooling of the devices. In the event of a severe decrease in the thermal conductivity the assumptions used to derive the cooling model will fail. This extreme amount of change in a material property is not likely in the actual devices. Thus, errors in the thermal conductivity will not have a negative impact on the optimization results of the thermal bimorph actuator.

The specific heat has an effect on the thermal behavior of the devices. As this material property changes, so does the amount of energy stored in the devices. A change in the specific heat in the substrate will cause error in the thermal model and the substrate optimization functional value. An increase in the specific heat will cause an increase in the amount of energy stored before the device reaches steady state temperature. Since power is being applied at a known rate, and this rate is not varied throughout the optimization an increase will cause higher device rise times. The same is true for the relaxation time; the more energy stored in the devices, the more energy will need to be dissipated in order to cool the devices. Errors in the specific heat will cause a proportional error in the optimization results, although the resulting trends remain accurate.

Young’s Modulus is the most critical material property to the device optimization. As was seen before in the nonlinear material property section, there is a significant
relationship between Young's Modulus and device behavior. This parameter is influential to
driving the stiffness of the device, and the forcing in the devices. For simplicity in
visualizing the effect of Young's Modulus this section will refer to a beam with 1 micron
thick layers for the situations described in this section. Also the coefficients of thermal
expansion will be set fixed at 2 μm/m-K and 1 μm/m-K for the top and bottom layer
respectively. With these parameters set at fixed values the effect of uncertainty Young's
Modulus is analyzed. The Young's Modulus of the top layer, $E_{top}$, will be compared to the
Young's Modulus of the bottom layer, $E_{bot}$. The results of the analysis of the effect of the
relationship of the Young's Modulus are shown in Table 7.1.

<table>
<thead>
<tr>
<th>Relationship of Young's Modulus</th>
<th>Effect of Relationship on Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{top} &gt;&gt; E_{bot}$</td>
<td>Bottom layer fails due to high internal stress</td>
</tr>
<tr>
<td>$E_{top} &gt; E_{bot}$</td>
<td>Bottom layer has higher stresses than the top layer, and the deflection is reduced</td>
</tr>
<tr>
<td>$E_{top} = E_{bot}$</td>
<td>Best distribution of internal stresses and best optimization functional value</td>
</tr>
<tr>
<td>$E_{top} &lt; E_{bot}$</td>
<td>Top layer has higher stresses than the bottom layer, and the deflection is reduced</td>
</tr>
<tr>
<td>$E_{top} &lt;&lt; E_{bot}$</td>
<td>Top layer fails due to high internal stress</td>
</tr>
</tbody>
</table>

Table 7.1 shows the basic effect of various pairings of Young's Modulus for the
dissimilar materials. For the geometry used in the optimization of the devices the devices see
only the second case presented in this table, $E_{top} > E_{bot}$. Uncertainty in the Young's Modulus,
was previously reported in Section 5.4 would cause more beam designs to be ruled as
infeasible from the optimized sets because of high stress.

The last material property which affects the behavior of the devices is the coefficient
of thermal expansion. The differential thermal expansion causes deflection in the device.
Any error in the coefficients of thermal expansion would cause the differential between the
two values to change; it will effect the internal forcing of the device. If the differential
between the values for the top and bottom layer increases, so will the forcing, and if it
decreases the forcing in the device will reduce. An increase in the differential between
coefficients of thermal expansion will cause stresses to increase and the functional for the
beam to reduce, and a decrease in differential will have the opposite effect.
To sum up the effects of the uncertainty in the material properties used during the optimization process, the effects of concern have been shown to minimally influence the outcome of the optimization. For small changes in material properties, the results of both the beam and the bulk substrate optimization will remain the same. In order for uncertainty in reported material property values to significantly affect the optimization outcome, each bulk parameter would have to vary significantly. A better understanding of the thin films behavior of the materials used in the devices would help quantify any errors in the model.

The assumptions made in the modeling of the devices, will lead to a conservative model of the device performance. The results of the optimization are likely to improve based on the ability to eliminate these assumptions from the models of the devices.
Chapter 8 Model Results and Comparison

This chapter sums up what happens when the conditions derived in the previous chapters are used in the modeling of the devices. These methods will be used to characterize the devices used in this study. These methods are also applied to the models of similar devices fabricated in previous studies, and the results of the model are compared to these experimental results. The effects of the boundary conditions applied to the model are presented in this section. Each of the conditions is simulated over a wide range of parameters to study not only the values, but trends in the modeling. The results show the modeling techniques presented provide results that match trends in the experimental results.

8.1 Bulk Substrate

The bulk substrate is very important in the modeling of the beam; it controls how much heat is stored in the system, since it provides the majority of the mass in the system. The thermal response is heavily dominated by the bulk substrate. This section will show how the substrate acts as a heat sink when the beam is being heated. Also discussed is the beams effect on the cooling rate of the device substrate when the device is not powered. The results presented in this section show a distinct need to report the device geometry in a different manner than is currently the standard.

The modeling procedure to analyze the effect of the bulk substrate is simple, hold all of the parameters constant and change the amount of substrate. In all cases, the power input into the system is 1.123 mW, which corresponds to a input heat flux of 50,000 W/m². This power value was chosen based on predicted heater geometry capable of producing approximately 1.12 mW of power. This power level will be shown later in the study to be approximately the power needed to heat the device prior to optimization. This power level is capable of heating the device to a steady-state temperature, which the device can withstand without thermal failure. Since this is a good approximation of the device power needed to actuate the device, it is chosen as the baseline power for the bulk substrate analysis.

There were two other models tested in the study of the bulk substrate. These were the conditions where just the beam portion of the device was modeled. Two different sets of boundary conditions were used to attempt to model the device accurately, without the bulk
substrate. A constant temperature boundary condition was used at the cantilever wall. The result was extreme temperatures in the device, and the entire cycle time of the device was several nanoseconds. Similar results were seen when the device was modeled with an adiabatic wall instead of the bulk substrate at the fixed end of the beam. Based on what is known about the experimental device responses, modeling the beam without the substrate was ruled out entirely. The parameters of the different substrate models are shown in Table 8.1.

Table 8.1 Setup of the test of bulk substrate effects on the device response.

<table>
<thead>
<tr>
<th>Substrate Length (μm)</th>
<th>Substrate Width (μm)</th>
<th>Volume (μm³)</th>
<th>Cooling Surface Area (μm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>1540435</td>
<td>54928</td>
</tr>
<tr>
<td>150</td>
<td>150</td>
<td>3415435</td>
<td>67428</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>6040435</td>
<td>84928</td>
</tr>
<tr>
<td>250</td>
<td>250</td>
<td>9415435</td>
<td>107428</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>13540435</td>
<td>134928</td>
</tr>
<tr>
<td>350</td>
<td>350</td>
<td>18415435</td>
<td>167428</td>
</tr>
</tbody>
</table>

Table 8.1 shows the model setups for the devices simulated. For each test the substrate length and width are increased by 50 microns. The size of the devices is varied in this way to keep the device in proportion to the shape of the actual devices. The volume and surface area of the device are also calculated and added into this table. These parameters are included, since they have a direct influence on the device model.

The results of the device modeling show the trends that were expected. During the heating phase, heat flows from the hot beam to the cold substrate. During the cooling phase the process is reversed. Since the thin beam will lose heat very quickly, the stored thermal energy in the substrate is partially dissipated from the beam. The bulk substrate is acting as a heat sink during the heating phase, and a heat source during the beam cooling. This effect shows the bulk substrate regulates the speed the device heats and cools. The heat flow during the heating and cooling phases of the device are shown in Figure 8.1 and Figure 8.2 respectively.
Figure 8.1 Heat flow into the substrate during device heating

Figure 8.2 Heat flow from the substrate during device cooling
The rise times for devices modeled with varying bulk substrate change dramatically based on the amount of bulk substrate that is used in the modeling of the system. With no bulk substrate, the device had a very small rise and cooling time (about 0.1 seconds for each). The maximum amount of bulk substrate tested had rise and cooling times two orders of magnitude higher than the devices without the bulk substrate. A wall with a constant ambient temperature was also tested and the response times were very low; on the order of nanoseconds, and these results are not included on the graph.

Figure 8.3 shows the surface generated from the transient results of the beam devices when modeled with varying amounts of bulk substrate. There is a direct relationship between the time to heat to steady state and the volume of the device.

The steady-state temperatures for devices also vary significantly based on the amount of bulk substrate included in the models. The lowest temperature is for the device with the largest amount of substrate tested at 326 °C and the highest temperature, 697 °C, is
developed in the beam without the bulk substrate. When the constant temperature wall was used in the model, the temperatures developed were much lower than the beams with the adiabatic wall, only having a small temperature increase when actuated of around 7.5 °C. This is because the constant temperature wall has the ability to act as a heat sink. This heat sink can remove an infinite amount of heat from the device.

![Figure 8.4](image)

Figure 8.4 Maximum device temperature varying with the change in dimension of the device

Figure 8.4 shows the maximum temperature in the device decreases with the amount of substrate modeled. In energy conservative systems at steady state, only heat in and out of the system are important. So the increased mass in the device can have no effect on the steady state temperature. The surface area of the device, which increases as the result of increasing the size of the bulk substrate, can affect the steady state temperature. The more cooling surface area the device has, the more energy will be lost. The more energy lost, the more heat is required to reach a constant temperature.

While the steady state temperature is only a function of the surface area of the device, the cycle time is influenced by both the surface area and the volume. The heat loss from the system is still a function of the surface area. The energy storage in the system also becomes
important to the cycle time of the device. The heating of the device is less affected by this because the heat flows from the hot beam into the cool substrate. The cooling phase is significantly affected because heat slowly flows back out of the beam. The time taken in heating and cooling of the devices as a function of substrate size is shown in Figure 8.5.

![Figure 8.5](image)

**Figure 8.5** Heating and cooling time for the devices when modeled with varying amounts of substrate

The blue curve represents the heating phase of the cycle, and shows a small increase in the cycle time of the devices. The pink curve represents the cooling time, which shows a very large increase in the cooling time of the devices, as the amount of substrate increases.

The results presented in this section show just how critical it is to accurately model the substrate of the device. The modeling of the substrate affects both the temperature of the device, and the time it takes to heat the device to those temperatures. From a mechanical deflection standpoint the bulk substrate is of little consequence, however in order to match the thermal response of the device, there are few things as important to the model.
8.2 **Radiation Effects**

The device is modeled with radiation cooling only for several reasons. The main reason for doing this is the devices are commonly measured in an evacuated chamber, so the cooling is dominated by radiation. By simulating the devices with radiation only cooling the measurable device results are easily obtained and compared to the measured results of the device. The results are compared at various power inputs and frequencies to show the device behavior when the devices are used in a vacuum.

While the micro robotics application does not need to study only radiation cooling on the devices, some test cases for the devices are conducted in a vacuum. Because there is no fluid around the device to lose heat to, through conduction, a model with just radiation effects is used to simulate this condition. The device is modeled with a varying power input into the system, so the results can be characterized over a range of heating conditions. The power inputs to the device are listed in Table 8.2.

**Table 8.2 Parameters used to characterize the radiation model**

<table>
<thead>
<tr>
<th>Heat Flux (W/m²)</th>
<th>Power input (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>0.225</td>
</tr>
<tr>
<td>50000</td>
<td>1.123</td>
</tr>
<tr>
<td>100000</td>
<td>2.246</td>
</tr>
<tr>
<td>250000</td>
<td>5.616</td>
</tr>
<tr>
<td>500000</td>
<td>11.232</td>
</tr>
<tr>
<td>1000000</td>
<td>22.464</td>
</tr>
</tbody>
</table>

The models are run at these power levels and characterized for a 150 second pulse with a 33% pulse width. This pulse width allows for 50 seconds to heat, and 100 seconds to cool. The emissivities of the specific materials are included in the model to account for the difference in the radiation from the materials. The average temperature in the beam is reported in the graphs of the beam response. The bulk substrate is left out of the average, because it reduces the average temperature below what is actually seen in the beam. This difference, when including the temperate of the bulk material, is caused by the size of the substrate skewing nodal and mass averages of the device temperature.
The thermal device results show some interesting trends in the transient temperature of the device. The predicted transient temperatures are shown in Figure 8.6.

![Figure 8.6 Thermal response of the device when cooled with radiation only](image)

Figure 8.6 shows the devices heat very quickly and cool very slowly. The models showed high temperatures for the higher power inputs. These higher temperatures demonstrate that the devices can overheat. These devices should not heated by more than short bursts of high power. With the pulse width presented, inputs of 5.6mW or more can cause thermal failure. In these devices, thermal failure occurs at around 600 °C.

The steady-state temperatures produced by ANSYS are higher than the temperatures that are predicted by the theoretical model. This temperature difference is due in part to the beam curling up on itself, and radiating between the bulk substrate and the bottom of the beam. The lower power models have lower temperatures, and see less of this effect, since they have lower deflections. These lower deflections lead to less energy being trapped in the system. These differences are best seen in Figure 8.7.
The difference in the temperature between the theoretical model (pink line) and the finite element model (blue diamonds) can be seen in Figure 8.7, which clearly shows these small deviations become noticeable after 5.616 mW.

What cannot be seen from this diagram is the temperatures corresponding to these power inputs cause very high deflections in the device. When the deflection of beam is high, the beam has bent beyond the wafer plane and is radiating energy directly to the bulk substrate. The bulk substrate is also radiating energy back to the beam. The theoretical model cannot account for this exchange because. The model can only account for heat lost from the surface due to radiation, and not the transfer between surfaces via radiation. This small amount of error does not invalidate this theoretical model, for the purposes of this study, because the error is both small and predictable. Also heating the device to temperatures high enough to cause this deflection would cause the device to fail. Under normal operating conditions, the devices will never see temperatures this high.

The deflection of the devices is high when cooled with radiation, as a result of the higher temperatures developed in the devices. As was previously mentioned, these
deflections are often high enough to curl the beam back beyond the wafer plane. Deflections this high are not needed in the micro robotics application, since any deflection further than the wafer plane is wasted. The plot of the deflections relating to the power inputs presented are shown in Figure 8.8.

![Graph showing mechanical deflection of the tip of the beam](image)

**Figure 8.8 Mechanical deflection of the tip of the beam**

Figure 8.8 represents the distance that the beam has traveled in the negative y direction when it reaches dynamic displacement for the applied conditions. As expected, the curve has the same shape as the temperature curve. The tip deflection shown is the magnitude of the beam displacement after it has been fabricated. This does not show the initial curvature of the beam, only the absolute dynamic displacement. The initial static displacement at the tip of the beam, 67.5 microns, is factored into the absolute displacement magnitude shown in Figure 8.8.

The last figure included in this section, Figure 8.9, shows the rise and relaxation times for the device. One of the most obvious results of the device analysis is it takes longer to cool the device than to heat it. During the heating phase, energy input into the system is high; so much so the cooling term almost appears insignificant at the lower temperatures. Because the beams need high temperatures for the radiation effects to be significant, the
beam temperature is high at steady state. It important to understand the difference in rise and relaxation times when attempting to force the beams at high frequencies. If the desired result is to bring the beam back to the initial temperature, the power will need to be turned off for longer than it is turned on for. By powering the device for a small fraction of the device cycle, the device can return to its initial temperature before heating again.

![Graph](image)

**Figure 8.9 Rise and relaxation times for various power inputs to the system cooled with radiation**

Figure 8.9 shows the devices cool very slowly and the increase in time required to cool down increases with the steady state temperature of the device. The heating speed is entirely a function of power input. The devices are heated to higher temperatures, and heat faster when the power input increases. Both the theoretical and modeled rise and relaxation times are correlate well to one another.

As a final thought on radiation effects in the system, the beams cooled with radiation only, reach much higher steady-state temperatures than the beams cooled with both radiation and convection. The cooling times are very slow for the radiation cooled systems as well. If the beams are to be forced at varying frequencies, then the effects of doing this will need to be accounted for. This can be done by varying the pulse width of the input or accounting for the change in deflection caused by the adjusted cooling. Power input is important to note, since in all cases studied here, the devices are susceptible to thermal overload. Also, it is important to suggest that if these devices are to be used in an enclosed environment, the
enclosure should be able to absorb the radiated energy. For higher responses out of radiation only devices, active cooling via conduction should be utilized in order to protect the devices from the high temperatures they are capable of producing in large arrays. As in all of the other devices, if these beams are made part of an array with a large distance between the beams, the radiation effect can be enough to cool the devices.

8.3 Convection Cooling With Radiation

This section looks at the results device models with boundary conditions similar to those seen in the device when it is used in the micro robotics application. These results will show the effect of adding free convection into the device model along with the radiation effects described in the previous section. By modeling the devices in this way, the device behavior can be understood. This will provide a baseline for the results of the optimization to be compared with.

Table 8.3 Thermal input into the free convection model

<table>
<thead>
<tr>
<th>Heat Flux (W/m²)</th>
<th>Power input (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>0.225</td>
</tr>
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<td>100000</td>
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<tr>
<td>250000</td>
<td>5.616</td>
</tr>
<tr>
<td>500000</td>
<td>11.232</td>
</tr>
<tr>
<td>1000000</td>
<td>22.464</td>
</tr>
</tbody>
</table>

The power input conditions used in the convection cooling model are the same conditions used in the radiation only models of the device. The modeling techniques are identical, with the exception that convection boundary conditions are added to the model simulations. The input forcing of the device is a 100 second cycle time with a 20% pulse width. The results of the modeling with free convection as well as radiation are shown in Figure 8.10.
Figure 8.10 Free convection thermal response

Figure 8.10 shows similar trends to the models simulated with only radiation. The temperatures reached at steady state are lower than with just radiation. The cooling times are also significantly lower than the relaxation times with radiation only.

For the most part, the results of the device with free convection are similar to the results of the device model with radiation at steady-state. The trends found in the free convection model show similar high temperatures to the ones seen in the radiation only models. These temperatures are only slightly reduced from the values in the devices modeled with radiation only. These high temperatures are most likely the result of higher temperature models being dominated by the radiation effects of the device. The steady state temperatures of the device are shown in Figure 8.11.
Figure 8.11 Steady state temperatures of the device

Figure 8.11 Shows the dynamic temperatures of the device based on both ANSYS temperatures and on the theoretical model of the device. The corresponding deflections of the device are shown in Figure 8.12
The behavior of the devices during the transient portion of the heating and cooling tells a different story. Radiation alone is effective at cooling the devices at high temperatures. At lower temperatures radiation cooling is less effective, and the devices take a long time to cool. In the case of the devices modeled with free convection, radiation still cools the devices at high temperatures. Free convection heat loss from the devices continues to cool the devices at lower temperatures. Because of free convection there is a significant increase in the amount of heat lost at low temperatures. Figure 8.13 shows that there is a dramatic drop in the cooling time of the devices (around 30% less than the devices with just radiation). The time to heat the devices is virtually unchanged. This shows that the free convection around the devices helps cool the devices without being detrimental to the heating of the devices. This result is desirable for the micro robotics application.

Error! Objects cannot be created from editing field codes.

**Figure 8.13 Rise and relaxation time of the devices when cooled with radiation and free convection**

While Figure 8.13 shows the same trends as the radiation models did, the relaxation time is affected significantly by the addition of the convection cooling. While a small increase in the rise time was seen, it is not significant in the model. This behavior is adventagious for the micro robotics application. The micro leg devices will be able to dissipate energy faster than if only radiation was a factor in the devices. This will lead to faster cycle times and higher walking speeds in the micro robot arrays.

The results of the devices modeled with free convection show flow developing over the surfaces of the arrayed devices. This flow is significant enough to reduce the relaxation time of the devices. The rise time for the devices is not significantly effected by convection cooling. Radiation still plays a part in the cooling of the devices when convection cooling is added to the model, because the device temperatures are still high. While the devices can still develop temperatures high enough to cause device failure, these high temperatures can be predicted, and reduced by applying the appropriate power input to the devices.

### 8.4 Comparison of Models to Experimental Results

This section outlines the behavior of the device model when compared with previously published experimental results. None of the models presented in literature have the same geometry or materials as the devices presented in previous sections. Because the
conditions presented in other works are dissimilar the ones in this study, this section will show the basic method presented in the previous sections to model a specific device geometry can be modified to accommodate small changes in geometry and materials properties of the device.

The model used for comparison is the Ataka bimorph, a polyimide-polyimide bimorph. As was previously stated the reason for choosing this model is not similarity to the device geometry used in this work. The reason this model is used is the completeness in reporting of the experimental parameters. Since a differential beam temperature was given, the device deflections can be compared against a beam model with this temperature differential applied to it.

The model representing the Ataka devices has temperatures corresponding to the power inputs from the device. This allows for direct loading of the thermal portion of the device model. By loading the model directly the unknowns from the experiments carried out can be removed from the model. Without the model being reported in this way there would be no way to compare the results. The results of the modeling compare very well to the actual beams presented by Ataka. The results of the modeling are plotted against the reported experimental results in Figure 8.14.
The beam deflections shown in Figure 8.14 have excellent correlation to one another. The lower power model predicted the device behavior better than the higher temperature model. The cause of this discrepancy is most likely due to nonlinear material property errors at high temperatures. For the comparison of these models the nonlinear effects are not taken into account, because the material models available consist of single point data for the polyamides used.
Chapter 9 Optimization Method and Recommendations

A model for the thermal actuation of a bimorph cantilever has been defined and verified, the design of these devices can be optimized for a common application. One design application of thermal bimorph actuators is for micro robotics locomotion. The micro robot utilizing thermal bimorphs is difficult to optimize because of the large number of design parameters required of the system. Several authors have built and tested micro robots, however none have reported any optimization process on the thermal bimorphs before applying them to the micro robot[2, 3, 6, 11, 37]. Many of the optimization parameters are contradictory in some aspects and as such, compromises will be required from each portion of the system during the optimization process. Figure 9.1 best shows the actuation process for the micro robotics application of the bimorph cantilever.

Figure 9.1 Micro robotics application of the micro leg array

Figure 9.1 shows a multi-step cycle to induce motion from the proper voltage input into each leg of the micro robot. In step one the device is not powered, the legs are deflected statically. In step two the rear set robot legs are heated leaving the front legs to hold up the entire weight of the robot and any load it might be carrying. In step three all legs are powered causing the robot to move forward as the legs are heated. In step four the rear legs are turned off allowing them to cool back to the temperature of the environment; step four
also results in the forward motion of the device. During step five the remaining powered legs are turned off and the robot remains at rest, in a new location, ready to begin another cycle.

In order to understand why each process is being implemented in the optimization of the micro robot array a brief explanation of why each of the goals of these processes is important to the device optimization. One of the most important things for micro motion devices is to have a very large static deflection in the cantilever beam. The large static deflection is important because it allows for the most movement in a single actuation cycle. The next major factor in the device motion is the dynamic displacement of the device. The higher the dynamic deflection of each leg, the more the micro robot will move with each deflection cycle. Cycle time is also critical to the optimized motion, because it dictates the speed the micro robot can travel. Stiffness is also important to the motion of the micro robot. While increasing beam stiffness ($K$) will make deflecting the thermal bimorph more difficult, increased stiffness will result in increased load carrying capability. Load capacity is especially important in the micro robot case when much of the load is made up of a power supply for the devices. The last component to the optimization of the devices is the efficiency of these devices. It is very important to increase the power output of the devices, and decrease the system input power, since there is a limited supply of power to the devices.

The desired system characteristics are not the only factors governing the behavior of the device. Limiting factors also control the device behavior. There are two types of factors applied to the model. The fixed state variables are limits imposed on the optimization based on the material properties of the device and its surroundings. Unlike the fixed state variables, which limit the system to the physically possible realm, system constraints are limitations imposed on the system. Some constraints are limits imposed by the manufacturability of these devices, and include such things as how thin layers can be made and how close geometry can be patterned to one another. For the sake of brevity, the specific fixed state variables are listed in Table 9.1.

Table 9.1 Fixed state variables for device optimization

<table>
<thead>
<tr>
<th>State Variable</th>
<th>Value</th>
<th>Applies to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Strength Al</td>
<td>450 MPa</td>
<td>Aluminum layer</td>
</tr>
<tr>
<td>Yield Strength Poly-Si</td>
<td>1100 MPa</td>
<td>All poly-silicon in the system</td>
</tr>
<tr>
<td>Max Temperature</td>
<td>660°C</td>
<td>Aluminum</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>20°C</td>
<td>Entire model</td>
</tr>
</tbody>
</table>
The fixed state variables used here are chosen for very specific reasons. For instance the yield strength of the materials is used, because above this stress level there is permanent deformation of the devices and they will not return to the original shape. The maximum temperature of 320 °C is chosen as the highest temperature the device is allowed to reach. By choosing this temperature, the plasticity effects of the material at higher temperatures are avoided. The ambient temperature assumes the micro robot is being used in a room temperature lab at 20 °C.

The model constraints are chosen based on previously fabricated device arrays. The length, width and thickness of the substrate are fixed at values used in the manufacture of device arrays. Other constraints are based on the ability to etch out and pattern the materials used in the device. The constraints are listed in Table 9.2.

**Table 9.2 Model constraints for the device optimization**

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Value</th>
<th>Applies To</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etch angle</td>
<td>57°</td>
<td>All anisotropic etching</td>
</tr>
<tr>
<td>Heater to beam edge</td>
<td>5 μm</td>
<td>Heater layer</td>
</tr>
<tr>
<td>Distance between heater legs</td>
<td>2 μm</td>
<td>Heater layer</td>
</tr>
<tr>
<td>Minimum heater thickness</td>
<td>0.1 μm</td>
<td>Heater layer</td>
</tr>
</tbody>
</table>

The process used in the optimization of the devices is fairly simple. The optimization uses a decoupled system. Based on previous success using a lumped capacitance method, combined with the Timoshenko beam equation, the bulk substrate portion of the optimization has been decoupled from the structural beam portion of the equation. Assuming the thermal gradients across the device are minimal allows the model to be decoupled. The beam will add cooling to the entire system but it will not change the optimized results of the bulk substrate. Subsequently the beam deflections can be made independent of the substrate by the application of a constant temperature in the beam.

The model of the bulk substrate used for the device is fairly simple. The physical limits on the bulk substrate are set as constraints for this subsystem. The bulk substrate is constrained in this way because when the devices are fabricated the device spacing characterized by the bulk substrate may be beyond the control of the designer. Reduced
substrate mass will reduce cycle time and have higher steady state temperatures, as shown with the simulation results presented in Section 8.1. Since the results of changing the bulk substrate geometry parametrically have already been shown in this document there is no need to include it in the parametric optimization discussion. The optimization of the bulk substrate focuses on the etch pit on the top surface of the device. The etch pit will allow the device to heat and cool faster, but will cause the device to lose more heat during the heating phase due to the increased surface area. A diagram of the etch pit can be seen in Figure 9.2.

![Diagram of etched substrate](image)

**Figure 9.2 Dimensions of the etched substrate**

In Figure 9.2 the substrate is shown without the leg portion of the device. The abbreviations for the etch pit geometry are the etch pit length $L_{etch}$, width of the etch pit, $W_{etch}$, and the depth of the etch pit, $D_{etch}$.

After the bulk substrate has been optimized the beam subsystem can be optimized. Unlike the bulk substrate, the beam subsystem is more complex, with the beam geometry influencing both the forcing of the device and the stiffness of the device. The beam subsystem model consists of the mechanical layers of the beam, and neglects the heater layer. Once the bulk substrate and the beam have been optimized a heater is devised capable of supplying the required power to the device, which will produce the temperatures seen in the optimization.
The basis of the bulk substrate optimization focuses on speed of actuation. For a given power input into the device, it will take a certain amount of time for the device to reach the desired temperature, $T_{\text{max}}$. In this case this time will be referred to as the rise time, $t_{\text{rise}}$. The other portion of the cycle involves the amount of time for this device to cool back to the ambient temperature, $T_{\infty}$. Since the time to cool back to room temperature will be very long depending on the amount of energy stored, another temperature, $T_{\text{relax}}$, is defined as 110% of the ambient temperature in degrees Celsius. Based on this relaxation temperature the other portion of the cycle is defined, $t_{\text{relax}}$, and the time for a complete cycle, $t_{\text{cycle}}$, is shown in Equation (9.1).

$$t_{\text{cycle}} = t_{\text{rise}} + t_{\text{relax}}$$

(9.1)

Again the goal of the optimization is to significantly reduce the cycle time of the device for a fixed input power. A function has already been defined which can be minimized. The functional for the optimization of the bulk substrate, $F_{\text{sub}}$, is simply a function of the cycle time.

$$F_{\text{sub}} = t_{\text{cycle}}$$

(9.2)

The parameters used in the optimization of the bulk substrate are fairly simple to understand. The ranges of the substrate used are specifically set up to prevent the etching from getting within 25 microns of the edges of the bulk substrate. Patterning the etch pits is done this way for several reasons. The first reason is to prevent the device from etching through the substrate. A second reason for limiting the etch pit is to allow room to pattern connecting leads to each leg in the larger device array. If this space was not left for this purpose reduction of the cross section area of the heater could occur which would lead to a loss of heater efficiency. A third reason to limit the geometries to this range is to prevent self-intersecting geometries. A self-intersecting geometry could occur if the depth of the etch pit was very deep and either of the length or width was very small.

By limiting the etch length and width to the large end of the range of possible values two goals can be accomplished. The first is it eliminates the self-intersecting geometry, which is a problem when building the model parametrically. Self-intersecting geometry can occur when a combination of variables is introduced that generates an etch pit that could not be etched. For instance if the depth is very large and the length or width is small the geometry will not be realistic and the model will fail to build. The maximum etch pit depth
is well within the acceptable range for the minimum length and width values, given in Table 9.3. By starting with a large etch pit the speeds of the optimization is increased.

Table 9.3 Etch size parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum value in Range</th>
<th>Maximum value in Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etch Length</td>
<td>100 µm</td>
<td>150 µm</td>
</tr>
<tr>
<td>Etch Width</td>
<td>100 µm</td>
<td>150 µm</td>
</tr>
<tr>
<td>Etch Depth</td>
<td>0 µm</td>
<td>140 µm</td>
</tr>
</tbody>
</table>

The device model is to be parametrically optimized in ANSYS, the results should be checked against a simpler method. A simpler method has already been used to check the results of previous ANSYS models. By applying the lumped capacitance model to the optimization process a series of isometric surfaces can be generated to show the effect on the device of changing the three parameters used to define the etch pit geometry. This assumes the lumped capacitance model will continue to accurately predict the device behavior with the addition of an etch pit.

There are two main differences between this ANSYS optimization, and the model characterization performed in the previous chapters. First the models must be built parametrically for all ANSYS optimization. The second major difference the post processing should be an automated process, so that it can be completed within the optimization run. The post processing is of particular importance because it is where the processing of the variables influencing the functional takes place. The critical data is found by comparing the temperature of the beam to the simulation time, and the appropriate time is extracted from the system. Once the data has been obtained from the post-processed results, it is applied to the functional being minimized.

The process ANSYS uses to find the minimum value of the functional is fairly simple, and does not require copious amounts of user input. This is different from the way modeling is normally performed, where the user is specifying inputs at all steps in the modeling process. All of the processes which normally would require some user input need to be coded into the batch file that controls the optimization of the device. Figure 9.3 shows the extra steps in running an ANSYS optimization.
Select Random Optimization Process

Chose Random Variables for Parameters Where $s^*$ are random numbers between 0 and 1

$\begin{bmatrix}
    P_1 \\
    \vdots \\
    P_n
\end{bmatrix}
= 
\begin{bmatrix}
    \frac{P_{lax}}{P_{max}} \\
    \vdots \\
    \frac{P_{lax}}{P_{max}}
\end{bmatrix} + 
\begin{bmatrix}
    s^*_1(P_{lax} - P_{max}) \\
    \vdots \\
    s^*_n(P_{lax} - P_{max})
\end{bmatrix}$

Apply random parameters to model and simulate

ANSYS Simulation And Data Extraction

Parametric Model Input with initial parameter values

Loop For 50 Iterations

Select best set and determine new best set using by determining the gradient of the objective function

Select Gradient Optimization Process

Record Parameters State Variables and Functional Values

$\nabla F = \left[ \frac{\partial F}{\partial P_1}, \frac{\partial F}{\partial P_2}, \ldots, \frac{\partial F}{\partial P_n} \right]$

$\frac{\partial F}{\partial P_i} = \frac{F(P) + \Delta P_i - F(P)}{\Delta P_i}$

Converged?

Figure 9.3 Optimization process complexity [23]

Several things can be said about the optimization process used. The process is more complex for the optimization process, due to the limited user input required after the parametric model is initially built. After the parametric model is applied to the system the optimization runs without user input until the model converges. Because no user input is needed the batch file must be written so the process can extract the numeric values of the terms used to quantify the functional. This output is then used to build the functional, and the optimization proceeds based on the value of the functional built from the results of the previous model.

The optimization of the beam subsystem uses two optimization tools to find the minimum of the functional for the device. The random variable tool is used to map the design space by selecting random starting points for the optimal searches of each parameter, $(L_{etch}, W_{etch},$ and $D_{etch})$. After promising minima have been identified using the random variable tool, the gradient search is used to find the minimum of the functional on the range

74
based on the results observed in the random search process. The process of using these two tools is used for both the optimization of the bulk substrate and the beam sub system portion of the device. As more random starting points are considered, we become increasingly confident in the location of the global extremum can be located.

The parameters involved in the beam optimization are shown in Figure 9.4. The length of the beam, $L_{beam}$, runs in the x-direction, the width of the beam, $W_{beam}$, runs in the z-direction and the thickness of the layers are in the y-direction. The top layer is defined as $D_{top}$, and the bottom layer is defined as $D_{bot}$.

![Figure 9.4 Beam parameters used in the device optimization (drawing not to scale)](image)

Recall that several factors are important during optimization of the beam subsystem for the micro robotics application. The first is the stiffness of the beam. The stiffness is one of the more complex parts of the beam behavior. The complexity comes is due to stiffness being needed to hold up the device, but being detrimental to the overall deflection of the device. A stiff beam will have reduced static and dynamic deflections because the internal forcing does not increase with thickness as fast as the stiffness. Figure 9.5 quantifies the beam subsystem stiffness as a function of the layer thickness.
Figure 9.5 Beam stiffness varied by thickness of each layer of material in the beam

Figure 9.5 illustrates that the maximum stiffness will occur when the beam has both layers at the maximum thickness, as expected. The stiffness of the device increases rapidly as a function of layer thickness. If high stiffness were the only goal of beam optimization, the optimization would have this simple solution. However, other factors must be considered in optimizing the beam subsystem for the micro robotics application.

The beam deflection is another key factor in the micro robotics. Both the static and dynamic deflections are crucial to this application of the bi-morph. Unlike the stiffness, where the increasing the layer thickness increases the stiffness, it is more difficult to find the appropriate layer thickness to drive high deflection, it is more difficult to find the appropriate layer thickness to drive high deflection. Because deflection is a function of both the stiffness and the forcing, the beam optimization needs to focus on finding a balance between the stiffness from the layers and the force generated between the layers. Figure 9.6 shows the surface created when mapping the beam deflection as a function of the thickness of each layer.
Figure 9.6 shows the device has the greatest deflection when the stiffness is the lowest. Conversely, the higher stiffness lowers the device deflection. This also contradicts the need to have high stiffness beams, since it will reduce the ability to produce high deflections in the micro robotics application with a significant payload. The functional chosen for the beam optimization must account for this contradiction.

Since it is desired to have a large force from the leg and a large deflection in the leg, the logical progression is to use energy storage as a way to combine these two parameters. And the method chosen to do this is by using a strain energy method. The strain energy, expressed as $U$ in Equation (9.3), takes into account the amount of energy stored in the device by deflecting it. By choosing this as part of the functional it is easy to maximize the amount of work done by the beam. This is similar to the amount of work stored in a leaf spring; with the difference being the beam itself is driving the motion of the device. Equation (9.3) defines the strain energy in terms of the stress and strain stored in the device.

$$U = \frac{1}{2} \int \left( \sigma_{xx} \varepsilon_{xx} + \sigma_{yy} \varepsilon_{yy} + \sigma_{zz} \varepsilon_{zz} + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz} \right) \, dV$$  \hspace{1cm} (9.3)

In Equation (9.3) involves the axial stresses, $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$, and axial strains, $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$, as well as the shear stresses $\tau_{xy}, \tau_{xz}, \tau_{yz}$, and shear strains $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$. In this case the
integration is performed over the volume of the device, \( V \), and then divided by two to arrive at the strain energy.

Reducing the power input into the system is also a goal of the beam optimization. Since larger devices will require more power it is important to only make the devices as large as necessary to achieve the goals of the optimization. Since the larger devices will dissipate more heat, they will need more power to heat them. Limiting the beam surface can reduce this problem. Including the upper and lower surface area of the beam in the functional will limit the beam to a reasonable length. Since there is so little surface area on the sides of the beam, this area is ignored in the optimization functional.

\[
A_{\text{beam}} = 2(L_{\text{beam}} + W_{\text{beam}}) \tag{9.4}
\]

The beam functional is finally taken to be the ratio of surface area to strain energy as shown in Equation (9.5). Minimizing the functional, \( F_{\text{beam}} \), is achieved by maximizing the strain energy, while minimizing the surface area, as desired.

\[
F_{\text{beam}} = \frac{4(L_{\text{beam}} + W_{\text{beam}})}{\int y (\sigma_{xx} \varepsilon_{xx} + \sigma_{yy} \varepsilon_{yy} + \sigma_{zz} \varepsilon_{zz} + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz}) \, dV_{\text{beam}}} \tag{9.5}
\]

The parameters for the beam optimization are constrained to establish the design space for the beam subsystem. The thickness of each layer included in the optimization model, is limited by its manufacturing constraints ability. Since the top layer will need to be vapor deposited, the thickness of the Aluminum top layer is limited to the range of 0.1 to 1.0 microns. The thickness of the bottom layer of the beam subsystem is limited at the low end to avoid problems with etching through the device, and limited at the upper end to 4 microns so that stiffness does not overwhelm the functional. The length and width of the beam are also constrained to keep the beam within the typical range of the micro robotics applications. The constraints on the design parameters are summarized in Table 9.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum value in Range</th>
<th>Maximum value in Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length</td>
<td>100 ( \mu )m</td>
<td>1000 ( \mu )m</td>
</tr>
<tr>
<td>Beam width</td>
<td>50 ( \mu )m</td>
<td>350 ( \mu )m</td>
</tr>
<tr>
<td>Top layer thickness</td>
<td>0.1 ( \mu )m</td>
<td>1 ( \mu )m</td>
</tr>
<tr>
<td>Bottom layer thickness</td>
<td>1 ( \mu )m</td>
<td>4 ( \mu )m</td>
</tr>
</tbody>
</table>
The ANSYS optimization process for the beam is similar to the process used in the optimization of the bulk substrate. The model is built parametrically and the optimization functional is programmed into a batch file, so the optimization process may proceed with little need for user input.

**Substrate Optimization Results**

The results of the bulk substrate optimization show reducing the amount of substrate in the system can improve the cycle time of the device. While this decrease in cycle time is small compared to the overall cycle time of the device, it will still help improve a slow actuation cycle. The improvement occurs when the etch pit is as large as possible. The effect of the size of the etch pit is illustrated by Figure 9.7.

![Figure 9.7 Effect of etching on the bulk substrate](image)

A few important observations can be drawn from Figure 9.7. The first is the etch pit will become as large as possible to minimize the functional for the devices. This was
expected, since a large etch pit will reduce the device volume and increase the surface area of the device, both of which contribute to faster response time. The increased area does not appear to have a significant negative effect on the rise time of the devices. From this result it is safe to conclude that any increase which may have been seen for the increased surface area is offset by the loss of system mass.

The thermal response of the device has been optimized and based on the trends shown previously it appears the device has been successfully optimized. Plotting the thermal cycle for the device in its entirety best shows the amount of improvement in the device behavior based on the etching. Figure 9.8 shows a single cycle of the device plotted against the response of the original device before optimization.

![Figure 9.8 Device results before and after optimization](image)

The comparison of the two curves plotted in Figure 9.8 shows the type of improvement desired from the optimization process. During the heating phase of the curve the optimized beam heats at a higher rate than the original beam did. This process shows that the optimized device can move through the same distance in a quicker time when heated than the initial device. This decrease in time was a major goal of the optimization process, and from this graph it shows that it has been completed. The other goal was to see a marked
improvement in the cooling time. The cooling portion of the curve for the device with the optimal substrate etch also reaches the cooled temperature well before the curve for the device with no substrate etching beam. While the curves do not show any extreme changes in the device performance the optimized substrate model clearly shows the type of improvement desired from the device.

**Beam Optimization Results**

The optimization of the beam is slightly more complex than the substrate portion of the device. There are four parameters varied in the beam optimization, and plotting the results very difficult as a result. The first thing investigated is how much the thickness of each layer effects the functional of the device, as shown in Figure 9.9.

![Figure 9.9](image)

**Figure 9.9** Functional value at varying thickness of the beam layers, with beam lengths (100, 550, and 1000 microns) and width fixed at (50, 200 and 350 microns).

Figure 9.9 shows the device functional is heavily dominated by the device thickness and that the thicker that the device is the more the strain energy is stored in the device.
Figure 9.9 actually shows the results of several surfaces, at varying beam lengths and widths. These changes in length and width have a minimal effect on the value of functional for the beam. This is apparent in Figure 9.9 where the three surfaces are almost indistinguishable from one another, even though the range of the parameters held constant for each surface are shown at the most extreme values allowed in the optimization. All three of the surfaces show that the functional is minimized when both layers are near the maximum thickness. Figure 9.10 shows the beam functional when the thickness of each layer is held constant, while the beam length and width are varied.

\[ \begin{align*}
  &T=0.1 \text{ microns} \\
  &B=1.0 \text{ microns} \\
  &T=0.5 \text{ microns} \\
  &B=2.5 \text{ microns} \\
  &T=1.0 \text{ microns} \\
  &B=4.0 \text{ microns}
\end{align*} \]

**Figure 9.10 Beam functional value as a function of the length and width of the device**

Unlike the surfaces in Figure 9.9, Figure 9.10 exhibits three easily distinguished. These all three of these curves are relatively flat. This shows that the length and width of the device do not change the beam functional value much, over the range that these parameters are varied. The lack of curvature to these surfaces, along with the spacing of these surfaces also shows the beam thickness is dominated by the layer thickness of the function. Because the stiffness of the beam is so heavily dominated by the beam thickness, it is reasonable to say the optimization functional favors stiffer devices over beams which will consume less power.
The surfaces shown in Figure 9.9 and Figure 9.10 do not show the effect of the constraints that are placed on the optimization of the beam; they simply report the value of the functional under certain conditions. The constraints in the beam subsystem will further limit these design sets by disallowing sets exceeding the yield strength of the beam layers. The optimization of the beam when subjected to stress constraints is shown in Figure 9.11.

![Figure 9.11 Beam functional values for at varying layer thickness with physical constraints shown](image)

Figure 9.11 shows the feasible and infeasible sets based solely on the stresses seen in the device. There are many infeasible sets shown in this graph, and this indicates the high stresses in these devices are critical to determining which set of parameters for the device is the best set for the micro robotics application. The other restriction on the devices is the manufacturability of the devices. When the manufacturability constraints are applied to the optimization results shown in Figure 9.11 the number of feasible design sets is further decreased.
Figure 9.12 Beam functional values for at varying layer thickness with physical and design constraints

Figure 9.12 demonstrates how much of an effect the constraints placed on the device have on the optimization process for the beam. While there are many sets with lower functional values than the set chosen, none of these sets meet the constraints that are placed on the system.

Development of a heater for this device will complete this optimization. The heater will need to supply the amount of energy to the device, required for the device to perform at the level it did during the optimization phase. Since heat dissipates quickly around the beam from the heater, the specifics of placing the heater are ignored, and focus is given to designing a heater that will supply the required heat to the device. Since the micro robotics application is the focus of this optimization, it is unlikely the power for the device will be much greater than 1.5 volts. In order to draw 1.19 mW from the device the heater will need to draw only 0.79mA to heat the device. Based on these values the resistance of the heater is determined to be about 1900 ohms. To produce a resistance this high with a deposited heater is fairly simple.
A six-leg heater can be made with TiW that will have the necessary resistance. The thickness of the heater layer is 0.25 microns, which is around the thickness of previously patterned resistive elements. The dimensions are shown below in Figure 9.13.

![Figure 9.13 Dimensions of the deposited heater](image)

To conclude the optimization portion of this documentation it is clear the overall goal of the device optimization has been met. The device has had a significant improvement in the behavior. Listings of all of the meaningful results of the optimization are listed in Table 9.5. From this table and it is clear there is a significant increase in device performance over the results of the beam that has been previously manufactured.

**Table 9.5 Important optimization results**

<table>
<thead>
<tr>
<th></th>
<th>Before Optimization</th>
<th>After Optimization</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (mW)</td>
<td>1.0752</td>
<td>1.1892</td>
<td>10.64</td>
</tr>
<tr>
<td>Input Energy (mJ)</td>
<td>18.06</td>
<td>12.13</td>
<td>-32.8</td>
</tr>
<tr>
<td>Deflection (µm)</td>
<td>105</td>
<td>63.5</td>
<td>-39.5</td>
</tr>
<tr>
<td>Rise Time (s)</td>
<td>16.8</td>
<td>10.2</td>
<td>-39.29</td>
</tr>
<tr>
<td>Relaxation Time (s)</td>
<td>70.3</td>
<td>43.6</td>
<td>-37.85</td>
</tr>
<tr>
<td>Output Work (µJ)</td>
<td>0.02374</td>
<td>0.07911</td>
<td>233.2</td>
</tr>
<tr>
<td>Device Efficiency</td>
<td>$1.31 \times 10^4$</td>
<td>$6.52 \times 10^4$</td>
<td>396</td>
</tr>
</tbody>
</table>
The mechanical work of the system has been dramatically increased and there is a minimal increase in the power needed to obtain this increase. The increased work in the system comes at the cost of reducing the deflection of the devices significantly. The cycle time of the device has been reduced by nearly forty percent. This reduction in cycle time will in the case of the micro robot lead to faster walking motion. The mechanical work done during the actuation cycle of each leg shows a large improvement from the initial beam device. While it is clear the devices are still very inefficient in producing the motion there is nearly a four hundred percent increase in the efficiency of these devices.

The success of the device optimization performed in this study can be applied to the manufacture of these devices for the micro robotics application. The principals presented in this study can be applied to micro cantilevers and other MEMS devices. By validating the model of the device against working devices, and checking the assumptions influencing the devices, major simplifications can be made to the model, and these simplifications can lead to reduced time and increased accuracy during device optimization. The dimensions of the overall optimized device are listed in Table 9.6:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
<th>Optimized Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Length ( L_{Beam} )</td>
<td>100</td>
<td>100</td>
<td>388</td>
</tr>
<tr>
<td>Beam Width ( W_{Beam} )</td>
<td>50</td>
<td>350</td>
<td>98.6</td>
</tr>
<tr>
<td>Top Layer Thickness ( D_{top} )</td>
<td>0.1</td>
<td>1</td>
<td>0.9465</td>
</tr>
<tr>
<td>Bottom Layer Thickness ( D_{Bot} )</td>
<td>1</td>
<td>4</td>
<td>3.9155</td>
</tr>
<tr>
<td>Etch Length ( L_{etch} )</td>
<td>100</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Etch Width ( W_{etch} )</td>
<td>100</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Etch Depth ( D_{etch} )</td>
<td>0</td>
<td>140</td>
<td>140</td>
</tr>
</tbody>
</table>
Chapter 10 Conclusions

The objectives of this study have been thoroughly studied and the results section of this documentation describes in detail the results from each portion of this study. The overall objectives of this study have been met. The model of the device has predicted the mechanical deflection of the device within 10 microns of the experimental devices presented in Ataka [21]. From the mechanical deflection a conclusion was also drawn about the thermal gradients. Correct mechanical deflections require correct values for the thermal gradients; therefore these gradients must be predicted within an acceptable range to give these correct mechanical deflections. The modeling techniques used for thermal bimorph actuator optimization yield several conclusions about the appropriate device model, and subsequent optimization results.

- Finite element models of MEMS devices can be reduced in complexity by applying the thermal boundary conditions as tabular boundary conditions, instead of modeling the heaters and the surrounding fluid. A simplified model of material properties, and single point thermal boundary conditions will not work in these models because the each heavily dependant on the temperature of the device.

- Radiation is a factor in the behavior of the devices. The temperatures in the device are much higher than the temperatures of the surroundings; the device will radiate heat to the environment. The cooling of these devices due to radiation cannot be ignored when the device is not in direct surface-to-surface contact with another solid. When convection and radiation effects are combined the radiation from the device is greater than 50% of the total cooling at operating temperature. The condition where the device does not have another solid body to directly conduct heat to is the load case for the micro robotics application of the MEMS device. With out condition to cool the device, radiation cannot be neglected regardless of the surrounding environment. Radiation is important to the accurate simulation of the model, and must be included in the models.

- Free convection is studied and the CFD results matched the results for flow developing over a flat plate to a high degree of accuracy. When the beam is heated
flow of air around the device develops from the buoyancy forces acting on the fluid around an arrayed device. The free convection resulting from this buoyancy driven flow has a significant effect on the devices at the array level. The cooling time for a device when free convection is applied is reduced by as much as 30% depending on input conditions.

- The deflections caused by fluid cooling the device, have little effect on the deflection of the legs. Because this deflection is so low, under three microns at fluid speeds in excess of 20 m/s the fluid structural interactions can be ignored, and the focus placed on the thermal interactions of the fluid cooling the device.

- The bulk substrate has a substantial cooling effect on the device. The models without substrate were several orders of magnitude faster in response, than the models with appropriate device spacing. This shows that each leg device is not easily thermally isolated from the rest of the wafer. Therefore, it will interact in the thermal domain with anything patterned near it on the wafer. Subsequently, the modeling of the substrate of the device is crucial to understanding the device behavior. The models are highly inaccurate without including the device spacing in the array. Also of importance is the input function for each device in the array. When reporting on thermal devices it is critical it include the substrate, other powered devices on the wafer, and heat sources which should be included in the model.

- A lumped capacitance model is developed using the boundary conditions developed for the device, and a simplification of the fabricated device geometry. The lumped capacitance model predicts the thermal behavior of the arrayed device when radiation, and free convection are applied to the device array within 14% for all the cases studied.

- Finite element models were developed to utilize the coupled field solvers. This allowed for the determination of the temperature distribution and device deflections at various temperatures in the device. This finite element model produced results similar to those from experimental data, presented by several Ataka, who reported temperatures in the beam device. The device deflections match within 10% at the normal operating temperature. Both the static and dynamic deflections have been compared, and show good correlation to the published experimental results.
• Based on the success of the finite element model when compared to the experimental model available, it is concluded that the finite element model could be successfully optimized for thermal actuation. Optimizing for maximum device efficiency, the results of the optimization method produced a beam similar in size to the original but with an output power 233% higher than the initial beam. This favorable comparison has yielded a high degree of confidence in recommendations about the appropriate beam geometry for the micro robotics applications.

• Optimization results have improved the device efficiency nearly 400% from the initial beam geometry proposed. The trends in the optimization predict that the limiting factor in improving the device for the micro robotics application was imposed by the size and manufacturing restrictions on the devices. Increasing this efficiency further could be accomplished by use of different materials, or improved manufacturing capability.
Chapter 11 Recommendations for Future Work

Based on the work presented in this document there are several suggestions for future work on the thermal bimorph actuators. These suggestions are presented in this section for consideration in future works on the thermal bimorph actuator.

- Develop a reliable set of thermal and mechanical properties, via testing for MEMS devices, or an accurate method of determining these properties. Currently there is work being undertaken at RIT to simplify the process used in the characterization of the material properties used in the MEMS devices.

- Attempt to build a device incorporating thermoelectric effects into the bimorph structure. Thermoelectric devices have one hot material and one cold material when current is run through them, they could possibly provide for more efficient heating and cooling of the bimorph actuators. Since thermoelectric devices use connected layers of dissimilar materials, a logical step would be to attempt to manipulate the mechanical properties of these layers as well.

- Attempt to use the deposited aluminum layer to heat the device and cause deflection in it. Aluminum has been successfully used as a heating element in the devices, and it has been used as a structural layer. A logical step would be to use an aluminum layer to perform both functions by designing the layer to act as both a heater and a structural layer.

- Study the effects of applying a thin layer of a less emissive material to the bulk substrate surfaces. By applying this layer unwanted radiation losses can be reduced in these devices. This coating should allow for higher steady-state beam temperatures at lower input power levels. These layers should not be applied in devices with high frequency forcing, due to increased cooling times.
References


Appendix A: Material Properties

Table A.1 Properties of air at standard pressure

<table>
<thead>
<tr>
<th>Table (K)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$C_p$ (J/kg · K)</th>
<th>$\mu \cdot 10^7$ (N·s/m$^2$)</th>
<th>$\nu \cdot 10^6$ (m/s$^2$)</th>
<th>$k \cdot 10^3$ (W/m·K)</th>
<th>$\alpha \cdot 10^6$ (m/s$^2$)</th>
<th>Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>1.3947</td>
<td>1006</td>
<td>159.6</td>
<td>11.44</td>
<td>22.3</td>
<td>15.9</td>
<td>0.720</td>
</tr>
<tr>
<td>300</td>
<td>1.1614</td>
<td>1007</td>
<td>184.6</td>
<td>15.89</td>
<td>26.3</td>
<td>22.5</td>
<td>0.707</td>
</tr>
<tr>
<td>350</td>
<td>0.9950</td>
<td>1009</td>
<td>208.2</td>
<td>20.92</td>
<td>30.0</td>
<td>29.9</td>
<td>0.700</td>
</tr>
<tr>
<td>400</td>
<td>0.8711</td>
<td>1014</td>
<td>230.1</td>
<td>26.41</td>
<td>33.8</td>
<td>38.3</td>
<td>0.690</td>
</tr>
<tr>
<td>450</td>
<td>0.7740</td>
<td>1021</td>
<td>250.7</td>
<td>32.39</td>
<td>37.3</td>
<td>47.2</td>
<td>0.686</td>
</tr>
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The data in this table is from [29]
Table A.1 Properties of selected materials at various temperatures

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<tr>
<th>Material</th>
<th>( T ) (K)</th>
<th>( E ) (GPa)</th>
<th>( v )</th>
<th>( C_{ote} ) (( \mu \text{m/m-K} ))</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( k ) (W/m-K)</th>
<th>( C_p ) (J/kg·K)</th>
<th>( r ) (( \Omega \cdot \text{cm} ))</th>
<th>( \varepsilon )</th>
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<td>Al</td>
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<td>2702</td>
<td>237</td>
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<td>400</td>
<td>30.5</td>
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<td>25.5</td>
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<td>232</td>
<td>1033</td>
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<td>745</td>
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<tr>
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<td></td>
<td>2.22</td>
<td>1.72</td>
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<tr>
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<tr>
<td>crystalline)</td>
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<td></td>
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<td>14300</td>
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<td>44 x 10(^{-6})</td>
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</tr>
<tr>
<td></td>
<td>600</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>65 x 10(^{-6})</td>
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<tr>
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All material properties in this table are obtained from [38] for consistency unless noted below:

Aluminum Young’s Modulus is obtained from [24]

TiW properties obtained from: [11]

Polyimide PIX-L100SX and Polyimide PIX-1400 Young’s Modulus and Coefficient of Thermal Expansion obtained from: [39]
Appendix B: Comparison of Beam Correlations

This appendix contains the equations presented in literature that are used to calculate the deflection of multi-layer cantilever beams. Also included are the basic assumptions used to derive these equations.

Timoshenko thermal beam curvature equation [1]:

\[
r = \frac{7}{24} \left( \frac{D_{\text{top}} - D_{\text{bot}}}{E_{\text{top}} W_{\text{top}} D_{\text{top}}^2} - 2D_{\text{top}}D_{\text{bot}} + \frac{E_{\text{top}} W_{\text{top}} D_{\text{top}}^3}{E_{\text{bot}} W_{\text{bot}} D_{\text{bot}}^2} + \frac{E_{\text{bot}} W_{\text{bot}} D_{\text{bot}}^3}{E_{\text{top}} W_{\text{top}} D_{\text{top}}^2} \right) \frac{\Delta \alpha \Delta T (D_{\text{top}} + D_{\text{bot}})}{3}
\]  

(B.1)

Major assumptions:
Uniform heat distribution in the beam

Modified Timoshenko thermal beam curvature equation [5]:

\[
r = \frac{D_{\text{top}} + D_{\text{bot}}}{6\Delta \alpha \Delta T} \left( 5 + \frac{1 + \omega^2}{\omega} \right)
\]  

(B.2)

Where \(\omega\) is given by :

\[E_{\text{bot}} = \omega E_{\text{top}}\]  

(B.3)

Major assumptions:
Uniform heat distribution in the beam
Geometries of each layer are roughly the same
Young’s Modulus for the material is on the same order of magnitude for each material

Chu thermal beam curvature equation [8]:

\[
r = \frac{2(D_{\text{top}} + D_{\text{bot}})}{3\Delta T} \left[ 1 + \frac{\left( E_{\text{top}} D_{\text{top}}^2 - E_{\text{bot}} D_{\text{bot}}^2 \right)^2}{4 E_{\text{top}} D_{\text{top}} E_{\text{bot}} D_{\text{bot}} \left( D_{\text{bot}} + D_{\text{top}} \right)^2} \right]
\]  

(B.4)
Burgreen thermal beam curvature equation [10]:

\[
r = \frac{E_{\text{top}} I_{\text{top}} + E_{\text{bot}} I_{\text{bot}} + E_{\text{top}} A_{\text{top}} \bar{y}_1 H}{E_{\text{top}} A_{\text{top}} \bar{y}_1 \left( \alpha_{\text{bot}} D_{\text{bot}} - \alpha_{\text{bot}} D_{\text{top}} \right) + E_{\text{top}} \alpha_{\text{top}} M_{\text{top}} + E_{\text{bot}} \alpha_{\text{bot}} M_{\text{bot}} + M}
\]  

(B.5)

Where \( H \) is the distance between the center of each layer \( M \) is the sum of the external forces on the beam and the equations below describe the other variables which were not previously defined in this work.

\[
\bar{y}_1 = H \frac{E_{\text{top}} A_{\text{top}}}{E_{\text{top}} A_{\text{top}} + E_{\text{bot}} A_{\text{bot}}}
\]  

(B.5)

\[
M_{\text{top}} = \int_{A_1} T_{\text{top}} y_{\text{top}} dA_{\text{top}}
\]  

(B.6)

\[
M_{\text{bot}} = \int_{A_2} T_{\text{bot}} y_{\text{bot}} dA_{\text{bot}}
\]  

(B.7)

Shown below are the beam shapes when a constant 300K temperature difference is applied across the beam. The ANSYS model is shown with large deformation effects included in the model.

![Graph showing beam deflection](image)

**Figure B.1 Device deflection for 300 degree temperature change**
Appendix C: ANSYS Optimized Beam Code

/clear
/PREP7
/TITLE,SYSTEM COMPAIRISON
ANTYPE,TRANS
ET,1,SOLID5
ET,2,SOLID98
MP,EX,1,163e9
MP,PRXY,1,0.28
MP,ALPX,1,2.4E-6
MP,KXX,1,1,124
MP,C,1,745
MP,DENS,1,2220
MP,EX,2,2,68e9
MP,PRXY,2,0.35
MP,ALPX,2,2,24E-6
MP,KXX,2,2,210
MP,C,2,905
MP,DENS,2,2,2720
PD=140E-6
PW=150E-6
PL=150E-6
BASE=PW-.644*PD
K,l,-175E-6,0,175E-6
K,2,-175E-6,0,-175E-6
K,3,-175E-6,-150E-6,175E-6
K,4,-175E-6,-150E-6,-175E-6
K,5,175E-6,0,175E-6
K,6,175E-6,0,-175E-6
K,7,175E-6,-150E-6,175E-6
K,8,175E-6,-150E-6,-175E-6
K,9,PW,0,PL
K,10,PW,0,-PL
K,11,-PW,0,PL
K,12,-PW,0,-PL
K,13,BASE,-PD,BASE
K,14,BASE,-PD,-BASE
K,15,-BASE,-PD,BASE
K,16,-BASE,-PD,-BASE
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L,1,3
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L,1,11
L,2,4
L,2,6
L,2,12
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L,13,9
L,13,14
L,13,15
L,15,16
L,16,12
L,16,14
L,14,10
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AL,1,2,5,8
AL,8,9,10,16
AL,7,6,15,19
AL,11,17,15,13
AL,3,18,4,13
AL,24,27,25,23
AL,5,6,14,10
AL,26,19,28,27
AL,26,20,21,25
AL,24,21,18,22
AL,22,17,28,23
AL,16,14,11,12
AL,2,9,12,3
VA,ALL
BW=98.9E-6/2
BL=388E-6
MAXX=BL+175E-6
block,175E-6,MAXX,0,-4E-6,-BW,BW
block,175E-6,MAXX,0,1E-6,-BW,BW
VGLUE,2,3
VGLUE,1,2
VSEL,S,,3
VATT,1,,1
VSWEEP,3
VSEL,S,,5
VATT,1,,2
vmesh,5
VSEL,S,,4
VATT,2,,1
VSWEEP,4
STEF,5.67E-8
TOFFSET,273.15
SPCTEMP,1,20
Vsel,all
NSEL,all
IC,all,temp,20
TREF,320
SFA,1,,RDSF,,87,1
SFA,4,,RDSF,,87,1
SFA,6,,RDSF,,87,1
SFA,7,,RDSF,,87,1
SFA,9,,RDSF,,87,1
SFA,10,,RDSF,,87,1
SFA,11,,RDSF,,87,1
SFA,12,,RDSF,,87,1
SFA,15,,RDSF,,87,1
SFA,16,,RDSF,,87,1
SFA,17,,RDSF,,87,1
SFA,20,,RDSF,,87,1
SFA,22,,RDSF,,87,1
SFA,23,,RDSF,,87,1
SFA,27,,RDSF,,87,1
SFA,28,,RDSF,,87,1
SFA,24,,RDSF,,05,1
ASEL,S,,2,14,6
DA,ALL,UX,0
DA,ALL,UY,0
DA,ALL, UZ,0
ASEL,ALL
TIME,100
DELTIM,5
kbc,1
OUTRES,ALL,ALL
LSWRITE,2

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TIME,25
KBC,1
DELTIM, 2.5
OUTRES, ALL, ALL
LSWRITE, 1
FINISH
/SOLU
OUTPR, BASIC, 1
## Appendix D: Optimization Results

### Table D.1 Bulk substrate optimization results

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Grey cells denote infeasible design sets
Yellow cells denote optimum set
### Table D.2 Beam optimization results

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Grey cells denote infeasible design sets
Yellow cells denote optimum set