An Experimental Investigation into the Effect of Surfactants on Air-Water Two-Phase Flow in Minichannels

Nathan J. English

Follow this and additional works at: https://scholarworks.rit.edu/theses

Recommended Citation


This Thesis is brought to you for free and open access by RIT Scholar Works. It has been accepted for inclusion in Theses by an authorized administrator of RIT Scholar Works. For more information, please contact ritscholarworks@rit.edu.
An Experimental Investigation into the Effect of Surfactants on Air-Water Two-Phase Flow in Minichannels

by

Nathan J. English

A Thesis Submitted in Partial Fulfillment of the Requirements for the

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Approved by:

Dr. Satish G. Kandlikar
Department of Mechanical Engineering

Dr. Abhijit Mukherjee
Department of Mechanical Engineering

Dr. Jeffrey Kozak
Department of Mechanical Engineering

Dr. Edward C. Hensel
Department Head of Mechanical Engineering,

DEPARTMENT OF MECHANICAL ENGINEERING
ROCHESTER INSTITUTE OF TECHNOLOGY

FEBRUARY, 2005
PERMISSION OF DUPLICATION

Permission Granted

Two-phase Air-Water Flow and Pressure Drop in Minichannels

I, Nathan J. English, hereby grant permission to the Wallace Library of the Rochester Institute of Technology to reproduce my thesis in whole or in part. Any reproduction will be for personal, research, or educational purposes and will not be for commercial use or profit.

Date: 5/17/05 Signature:
ACKNOWLEDGMENTS

I would like to express my sincere appreciation to my advisor, Dr. S. G. Kandlikar, for the guidance and insight given throughout the course of this study. Without your invaluable help and advice this study would not have been possible.

Appreciation is extended to everyone at RIT that contributed to the project. The Thermal Analysis and Microfluidics group, the mechanical engineering administration, Bill Finch, and the machine shop staff. Special thanks to John Borrelli and Mark Steinke for your expert skills and advice in everything experimental. Dr. R. K. Shah for your proficiency with fuel cells. Tom Allston of the chemistry department for your help with surface tension measurements. Dr. James Taylor of the systems engineering department for your help with channel characterization. Linette Koren of the Wallace Library for research assistance.

Thanks to Amie Cressman, Brian Gillis, Benny Kandhiraj, Brian Towner, and Christy White for your support and friendship during long days of studying.

Finally, I am indebted to my parents Steve and Naomi English, my sister Melissa, and brother Paul for encouragement and multi-faceted support throughout my academic career.
ABSTRACT

The complex interfacial phenomena involved in two-phase gas-liquid flow have defied mathematical simplification and modeling. However, the systems are used in heat exchangers, condensers, chemical processing plants, and nuclear reactor systems. The present work considers a 1 mm square minichannel and adiabatic flows corresponding to practical PEM fuel cell conditions. Pressure drop data is collected in experimentation covering mass fluxes of 4.0-33.6 kg/m²s, which correspond to superficial gas and liquid velocities of 3.4-10 m/s and 0.001-0.02 m/s respectively. The experiments are repeated with water of reduced surface tension, caused by the addition of surfactant, in order to quantify the surface tension effects, as it is recognized that surface tension is an important parameter for two-phase flow in minichannels. The published models are evaluated for correct consideration of the surface tension effects and accurate prediction of pressure drop. The addition of surfactant is shown to have no discernable influence on pressure drop. Two models by Chen et al. are found to acceptably predict the experimental data within 20-25%, however a new model is proposed that matches the experimental data with deviations of less than 5%.
# TABLE OF CONTENTS

- Duplication Permission .................................................................................. 2
- Acknowledgements ....................................................................................... 3
- Abstract ........................................................................................................ 4
- Table of Contents .......................................................................................... 5
- List of Figures and Tables ............................................................................ 7
- List of Variables and Abbreviations .............................................................. 10
- 1. Introduction ............................................................................................... 13
- 2. Background Information ........................................................................... 15
  - 2.1 Single Phase Flow .................................................................................. 15
  - 2.2 Two-Phase Flow Terminology ............................................................. 17
- 3. Literature Review ....................................................................................... 19
  - 3.1 Large Diameter Channels ..................................................................... 20
  - 3.2 Small Diameter Channels ..................................................................... 24
  - 3.3 Low Aspect Ratio Channels .................................................................. 30
  - 3.4 Channels With Refrigerant Flow ........................................................... 32
  - 3.5 Flow Regime Analysis .......................................................................... 33
  - 3.6 Fuel Cell Application ............................................................................ 35
- 4. Objectives .................................................................................................. 40
- 5. Experimental Setup ................................................................................... 41
  - 5.1 Test Section .......................................................................................... 41
  - 5.2 Fluid Supply .......................................................................................... 42
  - 5.3 Data Acquisition ................................................................................... 43
- 6. Experimental Procedure ............................................................................ 45
  - 6.1 Surfactants ............................................................................................ 45
  - 6.2 Calibrations ........................................................................................... 48
  - 6.3 Data Acquisition ................................................................................... 50
- 7. Experimental Uncertainties ....................................................................... 52
- 8. Experimental Results ................................................................................. 55
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1 Single Phase Validation</td>
<td>55</td>
</tr>
<tr>
<td>8.2 Surfactant Results</td>
<td>56</td>
</tr>
<tr>
<td>8.3 Pure Water Results</td>
<td>59</td>
</tr>
<tr>
<td>8.4 High Speed Photography</td>
<td>64</td>
</tr>
<tr>
<td>9. Discussion and Analysis</td>
<td>67</td>
</tr>
<tr>
<td>9.1 Influence of Surfactant</td>
<td>67</td>
</tr>
<tr>
<td>9.2 New Model Development</td>
<td>69</td>
</tr>
<tr>
<td>9.3 Impact of Experimental Uncertainty</td>
<td>72</td>
</tr>
<tr>
<td>9.4 Data Repeatability</td>
<td>75</td>
</tr>
<tr>
<td>10. Conclusions</td>
<td>76</td>
</tr>
<tr>
<td>11. Suggestions for future work</td>
<td>77</td>
</tr>
<tr>
<td>12. References</td>
<td>78</td>
</tr>
<tr>
<td>Appendix A: Sample Calculations</td>
<td>82</td>
</tr>
<tr>
<td>Appendix B: Experimental Setup Pictures</td>
<td>88</td>
</tr>
<tr>
<td>Appendix C: Experimental Setup Specification Sheets</td>
<td>91</td>
</tr>
<tr>
<td>Appendix D: Uncertainty in Measurement and Predicting Models</td>
<td>95</td>
</tr>
<tr>
<td>Appendix E: Experimental Data</td>
<td>102</td>
</tr>
<tr>
<td>Appendix F: Extended Results Information</td>
<td>105</td>
</tr>
</tbody>
</table>
LIST OF FIGURES AND TABLES

Fig. 1 Cross-sectional view of a rectangular channel geometry.

Fig. 2 Cross-sectional view of a square minichannel with a possible annular flow configuration.

Fig. 3 Mishima and Hibiki’s correlation for Chisholm’s parameter C as a function of channel hydraulic diameter.

Fig. 4 Cross-sectional schematic of the test section, not to scale.

Fig. 5 Three phase contact angle and surface tensions.

Fig. 6 Sessile drop test device.

Fig. 7 Comparison of the single phase data for friction factor with conventional theory, including the uncertainty in measured quantities.

Fig. 8 Comparison of pressure drop data using surfactant solutions, as designated by concentration, Cₙ, for the case of a 6 m/s superficial air velocity.

Fig. 9 Comparison of pressure drop data using surfactant solutions, as designated by concentration, Cₙ, for the case of 10 m/s superficial air velocity.

Fig. 10 Two-phase pressure drop data as a function of mass quality for pure water at all air flow rates tested, G = Gₐ, J = Jₐ.

Fig. 11 Two-phase pressure drop predictions of the most relevant models over the full range of mass qualities for the case of a 6 m/s superficial gas velocity.

Fig. 12 Experimental data plotted with two-phase pressure drop predictions of the most relevant models for the case of a 6 m/s superficial air velocity.

Fig. 13 Experimental data plotted with two-phase pressure drop predictions of less relevant models for the case of a 6 m/s superficial air velocity.

Fig. 14 Experimental data plotted with the two-phase pressure drop predictions of the most relevant models for the case of a 10 m/s superficial air velocity.

Fig. 15 High speed photographs of pure water flow with (a) annular flow (b,c,d,g) induced plugs and slugs (e) wall wetting and (f) separated corner flow.

Fig. 16 High speed photographs of the 0.109% surfactant solution flowing in the test section with induced plugs and slugs.
Fig. 17 Pure water experimental data plotted with the two-phase pressure drop predictions of the most relevant models for the case of a 6 m/s superficial air velocity and comparing predictions for a reduced surface tension.

Fig. 18 Experimental data plotted with the two-phase pressure drop predictions of the most relevant models for the case of a 6 m/s superficial air velocity and compared with the proposed model.

Fig. 19 Comparison of pure water data with the proposed model’s predictions at different mass fluxes. The two are characterized by air mass flux \( G = G_a \) and superficial velocity \( J = J_a \).

Fig. 20 Comparison of the proposed model with the experimental data and including the uncertainty in measurement.

Fig. 21 Comparison of the Chen et al. (2002) model with the experimental data and including the uncertainty in measurement.

Fig. 22 Comparison of the Homogenous Flow model with the experimental data and including the uncertainty in measurement.

Fig. 23 Pressure drop data taken under similar test conditions for repeatability comparison.

Fig. 24 Photograph of assembled test section.

Fig. 25 Photograph of disassembled test section.

Fig. 26 Photographs of the Sessile drop test device used for the three-phase contact angle measurement.

Fig. 27 Screen-shot of the Labview virtual interface and data acquisition system.

Fig. 28 Flow rate chart for the Fl-5531st flow meter used to measure air flow rate.

Fig. 29 Specifications for the PX26 pressure transducers used in pressure measurements.

Fig. 30 Continued specifications for the PX26 pressure transducers used in pressure measurements.

Fig. 31 Calibration chart for the FL-120 flow meter used in liquid flow measurement.

Fig. 32 Calibration chart for the PX26-001GV pressure transducer used for differential pressure measurement along the test section.

Fig. 33 Calibration chart used for the PX26-005GV pressure transducer used to measure inlet pressure.

Fig. 34 Calibration chart for the thermocouple used to measure inlet air temperature.
Fig. 35 Calibration chart for the thermocouple used to measure ambient temperature.

Table 1 Values of Chisholm's Parameter.

Table 2 Feature summary of the most relevant published literature.

Table 3 Feature summary of related, but less relevant published literature.

Table 4 Properties of Triton DF-12 surfactant measured at $C_s = 0.1$ wt% and 25°C.

Table 5 Measured properties of the surfactant solutions, as designated by concentration, $C_s$, compared to those of pure water.

Table 6 Absolute mean deviation of the most accurate two-phase pressure drop models when averaged over the experimental runs.

Table 7 Average values of non-dimensional numbers for the pure water and characteristic surfactant solutions under the present conditions.

Table 8 Absolute mean deviation of the most accurate two-phase pressure drop models, including the proposed model, when averaged over the experimental runs.
LIST OF VARIABLES AND ABBREVIATIONS

A  Area, m²
Aₐ  Fuel cell active area over which reactions occur, m²
a  Channel width, m
B  Constant in Chisholm’s B equation (non-dimensional)
Bₑ  Bias error in measurement, it carries the units of the measured quantity
b  Channel height, m
Bo  Bond number (non-dimensional), \((\rho_L - \rho_G)gD_h^2/\sigma\)
C  Chisholm’s parameter (non-dimensional); also the circumference of the wire ring used in surface tension measurements, cm
Ca  Capillary number (non-dimensional), \(\mu G/\rho \sigma = We/Re\)
Cₕ  Concentration calculated on a % weight basis (non-dimensional)
D  Diameter, m
f  Friction factor (non-dimensional)
Fr  Froude number (non-dimensional), \(G^2/gD_h \rho^2\)
g  Gravitational acceleration, m/s²
G  Mass flux, kg/m²s, \(\dot{m}/A_{cs}\), \(G_L = xG_T, G_G = (1-x)G_T\)
J  Superficial velocity, m/s, \(\dot{V}/A_{cs}\)
L  Length, m
Lₑ  Entrance length, m, \(L_e \equiv 0.06 \text{Re } D_h\)
m  Mass, kg
\(\dot{m}\)  Mass flow rate, kg/s
N  The number of recorded measurements used to calculate a mean measurement (non-dimensional)
N_conf  Confinement number (non-dimensional), \([\sigma/g(\rho_L - \rho_G)]^{0.5}/D_h\)
P  Pressure, Pa
\(P_e\)  Electric power output, watts
\(P_w\)  Wetted perimeter, m
The radius of the wire ring used to measure surface tension, cm

The radius of the wire used to make the ring used to measure surface tension, cm

Reynolds number (non-dimensional), $GD_h/\mu$

Systematic error in measurement, it carries the units of the measured quantity

Experimental uncertainty in measurement to within a 95% confidence level, it carries the units of the measured quantity

Velocity, m/s

Operating voltage, volts

Volumetric flow rate, m$^3$/s

Weber number (non-dimensional), $D_hG^2/\rho\sigma$

Martinelli Parameter (non-dimensional)

Mass quality (non-dimensional), $m_G/(m_G + m_L)$, $G_G/(G_G + G_L)$

Greek Symbols

Aspect ratio (non-dimensional), $a/b$ assuming $a < b$

Chisholm’s version of the Lockhart-Martinelli parameter (non-dimensional)

Change in pressure, Pa

Least count of a measurement device, it carries the units of the measured quantity

Dynamic viscosity, kg/ms

Modification factor of Chen et al. (2002), (2001) (non-dimensional)

Density, kg/m$^3$

Surface tension, N/m; also standard deviation between measurements, it carries the units of the quantity measured

Two-phase friction factor, (non-dimensional)

Ratio of the flow volume of a single channel to that of the whole fuel cell, (non-dimensional)

Subscripts

Air
ab  Absolute
ap  Apparent
cs  Cross-sectional
G   Calculated with the properties of the gas phase
Go  Calculated as if the total mass flux has the fluid properties of the gas phase
H   Homogenous
h   Hydraulic
L   Calculated with the properties of the liquid phase
Lo  Calculated as if the total mass flux has the fluid properties of the liquid phase
S   Solid phase
T   Two-phase
w   Water
1.0 INTRODUCTION

The behavior of single-phase internal flow is both well understood and predictable over a wide range of operating conditions. Two-phase flows, while quite common, are not as well understood and involve significant error in predictability. As the technology develops, two-phase flows will be used more extensively, but more research is required to reduce the predictive uncertainty. The fundamental physics has proved too complicated to characterize by simplified mathematical models of the governing conservation equations. Likewise, the use of computers for computational fluid dynamics (CFD) analysis has produced less than satisfying results. Therefore, emphasis is placed on experimentation and empirical correlations.

The majority of experimentation in two-phase flow uses large diameter channels. It is recognized that as one moves to mini and microchannels the influence of the surface tension forces tends to increase, while that of the gravitational force decreases, which causes models intended for larger channels to inaccurately predict the two-phase pressure drop in minichannels. Such technologies as compact heat exchangers, refrigeration systems, and micro-tube condensers are driving the movement to smaller channels. Literature that is specific to minichannels often focuses on flow regime analysis, refrigerant flow, low mass quality flow and relatively high mass fluxes. Even for these, there is a dearth of quality data published with enough information for useful comparison and analysis. They are often based on limited ranges of operating conditions and are not externally verified. Furthermore, even though the effect of surface tension is recognized, it is not usually isolated in experimentation.

The present work focuses on an area with little published literature, adiabatic air-water flow with low mass fluxes \( G_T < 50 \text{ kg/m}^2\text{s} \) and high mass quality \( x > 0.1 \). For industrial relevance and to reduce the experimental scope, the flow conditions are taken from works done on proton exchange membrane (PEM) fuel cells. In a PEM fuel cell, minichannels (typically rectangular, trapezoidal, or semi-circular) are used as structural elements, as well as a means of reactant fuel delivery. In the fuel cell cathode, air flows in a minichannel and drives water that is diffusing into the channel through one of its walls. To optimize the system and ensure adequate reactant delivery, it is important to be able to
predict the channel pressure drop and flow conditions. There is inherent heat transfer, mass transfer, multiple materials, channel bends, and parallel channels fed through a manifold, all of which are not presently considered. In order to narrow the experimental scope, the system is simplified by focusing on the two-phase pressure drop and surface tension effects. The surface tension effects are isolated by decreasing the surface tension of the water while holding its other properties constant. The intent is to give insight into how surface tension influences the pressure drop in small diameter channels and not to produce a correlation that covers every channel diameter, channel geometry, or fluid property.

It should be noted that the term channel will be considered equivalent to and substituted for words such as pipe or tube. The channel classification developed by Kandlikar and Grande (2002) is used and considers minichannels to be within the range of $3 \text{ mm} \geq D_h > 200 \mu\text{m}$. 
2.0 BACKGROUND INFORMATION

2.1 Single Phase Flow

Single-phase flow theory is well established and can be taken from general works on fluid mechanics, such as Fox and McDonald (1998). The calculation of single-phase pressure drop is a simple function of the flow rate, channel geometry, friction factor, fluid density and fluid viscosity. The mathematical model can be written in terms of the fanning friction factor Eq. (1), and use the measured value of the flow rate to calculate mass flux Eq. (2). Hydraulic diameter is calculated as four times the cross-sectional area divided by the wetted perimeter Eq.(3). The measured channel width and height are given by a and b as depicted in Fig. 1.

\[ \Delta P = f \operatorname{Re} \frac{2L \mu G}{D_h^2 \rho} \]  
\[ G = \frac{\dot{V} \rho}{A_c} \]  
\[ D_h = \frac{4A_c}{P_w} = \frac{4ab}{2(a + b)} \]  

Fig. 1 Cross-sectional view of a rectangular channel geometry.

The friction factor values are usually tabulated or charted, however to enter the pressure drop equations into a computer it is easier to use an approximating equation. One such equation, by Kakac et al. (1987), approximates the combined value of the friction factor and the Reynolds number, \( f \operatorname{Re} \), to within 0.05\% of the tabulated values Eq. (4). The
equation is applicable to fully developed laminar flow in smooth rectangular channels, and is primarily dependant on the aspect ratio \( \alpha \), which is the ratio of the channel width and height Eq. (5). If the Reynolds number is desired separately, then Eq. (6) can be used.

\[
f \text{Re} = 24\left(1-1.3553\alpha+1.9467\alpha^2-1.7012\alpha^3+0.9564\alpha^4-0.2537\alpha^5\right)
\]

\[
\alpha = \frac{a}{b}
\]

\[
\text{Re} = \frac{GD_h}{\mu}
\]

One can see that the value of \( f \text{Re} \) is constant for a given aspect ratio. For a perfectly square channel, the value is 14.23 (compared to 16 for a circular channel). Therefore, the single phase pressure drop equation is only first order dependant on the viscosity, density, and velocity of the fluid, but second order dependant on the channel’s hydraulic diameter. Therefore, a slight change in the channel diameter significantly influences the single phase pressure drop, with the pressure drop increasing as the channel size decreases. This dependence is further explained in the uncertainties section.

The channel length required to reach fully developed laminar flow, \( L_e \), is an important consideration in single phase flow theory Eq. (7). For air flowing at an average velocity of 6 m/s in a 1 mm square minichannel, the entrance length would be about 23.2 mm, and for 10 m/s it would be 38.3 mm. Only laminar flow is considered experimentally.

\[
L_e \equiv 0.06 \text{Re} D_h
\]

There are several single and two-phase non-dimensional numbers that could give insight into the flow behavior and are discussed in Kandlikar (2004). The Froude number, \( \text{Fr} \), is the ratio of the inertia forces to gravity forces Eq. (8). The Weber number, \( \text{We} \), is the ratio of the inertia forces to the surface tension forces Eq. (9). The Capillary number, \( \text{Ca} \), is the ratio of the Weber number to the Reynolds number, or viscous to surface tension forces Eq. (10). The Bond or Eotvos number, \( \text{Bo} \), is the ratio of the buoyancy to surface tension force Eq. (11).
According to Kandlikar, the Bond number is not expected to be important for small channels due to the limited effect of gravity. Likewise, the Froude number is largely dependant on mass flux. Contrarily, Capillary number is expected to be very important, as both the viscous and surface tension forces play a role in capillary flow.

2.2 Two-Phase Flow Terminology

Two-phase flow involves a different set of concepts and vocabulary than single phase flow. Also, various researchers use the assorted concepts in sundry, and sometimes contradictory, manners. For consistency, those terms used in the present work are explained here:

**Flow regime** – A characterization of the shape of the interfacial interactions. As it is observational and subjective, different authors might identify the same flow regime by a variety of names. Typical regimes include annular, bubbly, intermittent, plug, slug, and stratified flow, though each regime might also be subdivided further.

**Annular flow** – The flow regime having the gas phase flowing in the middle of the channel and encircled by a flowing liquid. The present research is conducted primarily in this regime. The regime can be further divided by interface features, such as the presence of waves. Figure 2 gives a cross-sectional view of what annular flow in a square minichannel might look like. If a similar flow were established, but without the water fully encircling the air, then it is typically called stratified flow. Annular flow is usually considered a subset of
stratified flow and experimentally it is not always possible to determine if the flow is specifically in the stratified or annular flow regime.

![Diagram of cross-sectional view of a square minichannel with a possible annular flow configuration.](image.png)

**Fig. 2** Cross-sectional view of a square minichannel with a possible annular flow configuration.

**Flow map** – A plot of the superficial fluid velocities, flow rates, mass quality, or mass fluxes that is divided into sections based on flow regime.

**Superficial velocity** – The volumetric flow rate of the fluid divided by the channel cross-sectional area. For single phase flows, this is equal to the average velocity. For two-phase flows, the average velocity of either individual phase is greater, as the actual flow area is restricted by the other phase. It is used to calculate the superficial Reynolds number (usually termed Reynolds number here).

**Void fraction** – The local channel volume that is in the gaseous phase, as compared to the total volume. Depending on the flow conditions, different flow regimes are possible for the same void fraction.

**Mass flux** – The mass flow rate per unit cross-sectional area of the channel. The individual phase fluxes are calculated using the whole channel area, though the actual flow area is restricted by the other phase. They can also be calculated by using the mass quality. The two-phase mass flux is merely the addition of both single phase fluxes.
3.0 LITERATURE REVIEW

The last 50 years produced a wide variety of papers on two-phase flow, however very few included experiments under conditions similar to those presently considered. Those most pertinent to the current work can be divided into 5 major topical areas, conventional channels, minichannels, low aspect ratio rectangular minichannels, minichannels with refrigerant flow, and papers only considering flow regime. Some papers deal with several of these topical areas and the divisions are not always distinct. Literature dealing with the specific application of minichannels to fuel cell systems will also be discussed. The overwhelming majority of the research published in the scientific community focuses on large diameter conventional channels, using both air-water and other mixes, and generally for round geometries. Application of the developed correlations to conditions outside of their experimental scope is widespread, but not always justified. Research specific to two-phase flow in square minichannels is uncommon and typically employs either air-water or refrigerants, which have very different surface tension properties. Experiments using fluids with surface tensions between that of water and the refrigerants are practically nonexistent. Therefore, it is important to test correlations produced under different experimental conditions to see if they still apply to the present conditions and to glean insight from the methodology used.

Overall, there is a lack of acceptable models in the annular flow regime, at high mass qualities, and at low superficial liquid velocities, which is where the present work is targeted. Both agreement and contradictions are found in literature, but there is a general consensus that not enough experimental information and published data exists, and particularly that the influence of fluid properties and surface tension needs further investigation. There is agreement by Chen et al. (2002), Coleman and Garimella (1998), Fukano and Kariyasaki (1993), and Garimella (2004) that the surface tension force becomes important for channels of hydraulic diameter less than 10 mm (or rectangular channels with small gap widths) and dominates below 5 mm. It is agreed that the pressure drop characteristics change when going from conventional channels to minichannels, however it is not the diameter itself that leads to the change, but rather the presence of surface tension.
Typical two-phase pressure drop models follow either of two methods. The first correlates the two-phase pressure drop to the single-phase pressure drops. This is accomplished by either calculating the single-phase pressure drop as if one of the phases is flowing alone in the channel at its mass flux ($\Delta P_L$, $\Delta P_G$), or by using the total mass flux with the fluid properties of the one of the single phases ($\Delta P_{Lo}$, $\Delta P_{Go}$). The second method collects characteristic non-dimensional numbers and gives them correlated weight in predicting the pressure drop. Either method might refine the model by targeting a specific flow regime.

Unless specified, the operating fluids are assumed to be air and water. With exception, other geometries, such as heat exchanger plate arrays, triangular channels, or trapezoidal channels, will not be considered.

### 3.1 Large Diameter Channels

Research publications using large diameter channels are of the most historical importance. They are well established and validated models that one might describe as accurate, though the accuracy is not comparable to the accuracy of the single phase equations.

In 1949 Lockhart and Martinelli set the ground work for most of the following two-phase research by considering the flow of a variety of operational fluids such as benzene, kerosene, water and several oils. Their key contribution was to represent the two-phase pressure drop as a function of the single phase pressure drops, and express it in terms of a two-phase frictional multiplier. The single phase pressure drops are calculated on a superficial basis and related to each other by the parameter $X$ (note that mass quality is represented by $x$ and the two are different quantities), which is referred to as the Martinelli parameter Eq. (12). The single phase pressure drop calculation uses the traditional single phase formulas, but assumes that the given mass flux of either individual phase is flowing alone in the channel. The single phase pressure drops are then related to the two-phase pressure drop by a two-phase frictional multiplier ($\phi_g, \phi_l$). Lockhart and Martinelli tabulated values for the multipliers based upon the parameter $X$, and their research indicated that the single and two-phase pressure drops can be correlated with $X$ alone.
\[ X^2 = \frac{\Delta P_L}{\Delta P_G} \]  \hspace{1cm} (12)

\[ \Delta P_T = \phi_G^2 \Delta P_G \]  \hspace{1cm} (13)

\[ \Delta P_T = \phi_L^2 \Delta P_L \]  \hspace{1cm} (14)

In 1967 Chisholm recommended that an engineer might approximate the tabulated Lockhart-Martinelli values by using the parameter \( C \) Eq. (15). Equation (16) can then be similarly derived. In the present work, the gas phase is volumetrically dominant (also based on mass for the most part), therefore Eq. (16) is used. Re-arranging the Eqs. (12), (13) and (16) results in Eq. (17).

\[ \phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \]  \hspace{1cm} (15)

\[ \phi_G^2 = 1 + CX + X^2 \]  \hspace{1cm} (16)

\[ \Delta P_T = \left( 1 + C \sqrt{\frac{\Delta P_L}{\Delta P_G} + \frac{\Delta P_G}{\Delta P_G}} \right) \Delta P_G \]  \hspace{1cm} (17)

Chisholm designated values for \( C \) based upon the superficial Reynolds numbers of the two phases, or whether each would be in laminar or turbulent flow. Essentially, this gives a strong dependence on the superficial velocity of the fluids, as well as on how much of each is flowing in relation to the other, or mass quality \( x \). Chisholm’s values for \( C \) are summarized in Table 1. Hereafter, references to the Lockhart-Martinelli correlation assume the Chisholm approximation.

**Table 1** Values of Chisholm’s Parameter.

<table>
<thead>
<tr>
<th>2 Phase Flow Characteristics</th>
<th>Chisholm’s Parameter ( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar Liquid, Laminar Gas</td>
<td>5</td>
</tr>
<tr>
<td>Turbulent Liquid, Laminar Gas</td>
<td>10</td>
</tr>
<tr>
<td>Laminar Liquid, Turbulent Gas</td>
<td>12</td>
</tr>
<tr>
<td>Turbulent Liquid, Turbulent Gas</td>
<td>21*</td>
</tr>
</tbody>
</table>

*Chisholm (1973)
In 1973 Chisholm attempted to establish his own theory on two-phase flow and it resulted in Eq. (18). He replaced the Lockhart-Martinelli parameter, $X$, with his own version, $\Gamma$, which is the square root of the ratio of the single phase pressure drops calculated as if the total mass flux has the fluid properties of one of the fluids. However, he still only considered large diameter channels, and also focused on Turbulent-Turbulent flow. For the conditions presently considered, $B$ is a constant of value 4.8. Equation (19) is typically called the Chisholm B equation.

$$\Delta P_T = \varphi_{Lo}^2 \Delta P_L$$

$$\varphi_{Lo}^2 = 1 + \left(\Gamma^2 - 1\right)\left[Bx^{0.875}(1-x)^{0.875} + x^{1.75}\right]$$

$$\Gamma^2 = \frac{\Delta P_G}{\Delta P_L}$$

$$\Delta P_L = f \text{ Re} \frac{2L\mu_L G_T}{D_h^2 \rho_L}$$

$$\Delta P_G = f \text{ Re} \frac{2L\mu_G G_T}{D_h^2 \rho_G}$$

Friedel developed a more complicated method of predicting pressure drop in conventional channels in 1979, but he based his research on a large bank of data points. Friedel recognized the role of surface tension in two-phase flow and it became one of the correlation’s parameters, however it was given a weak influence while gravity was given a strong influence. Friedel’s model makes no adjustment for smaller channels. Therefore, it was a step in the right direction, but small channels were not considered. Friedel’s correlation is typically broken into sections, but can be assembled into Eq. (23). It uses a two-phase friction factor that assumes the total flow has the fluid properties of the liquid only ($\varphi_{Lo}$). The two-phase Froude number, Eq. (25), and Weber number, Eq. (26), are included in the correlation.

$$22$$
\[
\phi_{L_0}^2 = (1 - x)^2 + x^2 \frac{\rho_L f_{G_0}}{\rho_G f_{L_0}} + \frac{3.24 x^{0.78} (1 - x)^{0.224}}{F_r^{0.045} \rho_L^{0.035}} \left( \frac{\rho_L}{\rho_G} \right)^{0.91} \left( \frac{\mu_G}{\mu_L} \right)^{0.19} \left( 1 - \frac{\mu_G}{\mu_L} \right)^{0.7}
\]  
(23)

\[
\Delta P = \phi_{L_0}^2 \Delta P_{L_0}
\]  
(24)

\[
F_r = \frac{G^2}{g D \rho_H^2}
\]  
(25)

\[
We = \frac{G^2 D}{\rho_H \sigma}
\]  
(26)

Another popular model is the Homogenous Flow model. Various authors have personal versions of the model, but all of them consider the two-phase flow to have fluid properties that are an average of the two phases. The averaged fluid is then treated as a single phase flow. The density and viscosity of the fluid are averaged as described in Hewitt and Kawaji (1999) Eqs. (27), (28). Other models use the homogenous assumption in such calculations as the Froude and Weber number of Eqs. (25) and (26).

\[
\rho_H = \frac{\rho_L \rho_G}{x \rho_L + (1 - x) \rho_G}
\]  
(27)

\[
\mu_H = \frac{\mu_L \mu_G}{x \mu_L + (1 - x) \mu_G}
\]  
(28)

The above mentioned models are the most widely used for large diameter channels, however there is host of other literature published on them. Some seek to refine the above works, whereas others focus on a specific flow regime and develop either theoretical or empirical equations for certain flow conditions. They meet with more and less success, but it is widely accepted that they do not apply to small channel systems very well.

The work by Weisman et al. (1979) took a different approach than most other work and influenced the direction of the current project. The group primarily investigated flow regimes in large diameter pipes, but their methodology differed through the use of non-standard fluids. They sought to customize the fluid properties, so as to isolate their influence on flow regime transitions. The operational fluids where Freon, air-water, and air-water with
modified water properties. They used glycerol to change the viscosity of the water, while minimally influencing the surface tension and density. Likewise, they added a surfactant to change the surface tension of the water, while holding the other properties constant. Rather than looking at a range of surface tensions they only considered one reduced surface tension, and so could not show a progression of dependence. They also modified density, though with more of an impact on the other properties. The smallest diameter channel used was 120 mm, and they concluded that relative volumetric flow rates and channel diameter played a significant role in the flow regime transitions, but that fluid properties had relatively little effect. They noted a change in the flow regime transitions for different diameter channels, it is expected that extending the studies to minichannels would see a continuation of this trend, though with a more pronounced surface tension effect.

3.2 Small Diameter Channels

The strength of the Lockhart-Martinelli model lies in that it is a simple correlation with the single phase pressure drops. However, this becomes its weakness in regard to minichannels. Changing the surface tension of a single phase liquid flow does not alter the classical prediction of pressure drop, as the wall shear is adequately represented, and there is no gas-liquid interface for the surface tension to manifest itself. However, when one moves to two-phase flow, the surface tension contributes to fluid dynamics at both the two and three phase interfaces. As one progresses to smaller channels, the impact of the surface tension increases, and merely relating to the single phase pressure drops does not account for it. Likewise, the single phase pressure drop takes gravity into account, as there is a gravitational gradient present, even in a horizontal channel. However, in smaller channels, the surface tension force dominates the gravitational force. Therefore, if the Lockhart-Martinelli, Homogenous Flow, and Friedel models are applied to minichannels, the correlations place too much emphasis on gravity and not enough on surface tension.

In 1999 Triplett et al. tested the applicability of the large diameter models to smaller channels and found them quite inaccurate. The work focused on void fraction, but found similar results for pressure drop. Two circular test sections (with 1.1 mm and 1.45 mm diameters) were used to generate adiabatic air-water data, and the Lockhart-Martinelli, Friedel, and Homogenous Flow models were compared with the data. They found all of the
models to over predict pressure drop in the annular flow regime by greater than 100%, though the Homogenous Flow model was the most accurate. The models were found to be more accurate for bubble and slug flows, though more so at high Re_L than at low Re_L. They suggested that the interfacial momentum transfer and wall friction processes might differ significantly in small channels (as compared to larger channels), and that more refined correlations are needed. It was emphasized that the annular flow regime is particularly problematic and needs to be concentrated on. One cautionary note on the work is that, throughout the paper, the experimental setup is described as entailing precision bore circular channels, but at one point it states that all experiments were conducted with high aspect ratio channels. It is assumed that this is a typographical error, or meant as an assumption for the theoretical analysis.

A widely referenced work on minichannel two-phase flow was presented by Mishima and Hibiki (1996). The work sought to quantify the effect of channel diameter on pressure drop in minichannels. Their experiments used round channels with diameters of 1, 2, 3, and 4 mm. They modified Chisholm's parameter C to adjust for channel diameter in minichannel applications. The intention was to continue relating the two-phase pressure drop to the single phase pressure drops, but to bias it in lower diameter channels. The original Lockhart-Martinelli correlation did not include the channel diameter, outside of its inclusion in the single phase relationships. However, one can see Mishima and Hibiki's implication that a liquid with low surface tension will have the same pressure drop characteristics as a liquid with high surface tension. They proposed a version for round tubes Eq. (29), as well as, one for other geometries Eq. (30).

\[
C = 21\left(1 - e^{-333D}\right) \tag{29}
\]

\[
C = 21\left(1 - e^{-319D_h}\right) \tag{30}
\]

A plot of Mishima-Hibiki's value for C as a function of hydraulic diameter can be seen in Fig. 3. The value increases quickly over the range of 0-5 mm and is constant for diameters greater than 12 mm. The result is a direct relation of the two-phase prediction to the channel diameter, which they found to be within ±12%. However, they removed the consideration for whether the flow is laminar or turbulent, and there is no adjustment for surface tension.
The diameter effects are seen as primarily dependant on the surface tension, so this correlation is not be expected to predict the low surface tension cases well. Mishima and Hibiki considered both vertical and horizontal channels, and found there to be little difference between the two cases.

Mishima and Hibiki's intention seems to have been to reduce C for smaller diameters, and then raise it to the constant large diameter value for diameters greater than 12 mm. However, they made no mention of adjusting the multiplication value of 21 to account for Reynolds number. For a hydraulic diameter of 1 mm, their value of C would be 5.74, which is close to the Chisholm value of 5 for laminar-laminar flow, and which leads to similar predictions as the Lockhart-Martinelli correlation. Smaller values of C are needed for air-water flow in minichannels, but for laminar-laminar flow the Mishima-Hibiki model actually leads to larger values.

In 2000 Bao et al. considered a minichannel air-water system both adiabatically and with heat transfer, though no boiling. The test section was a round 1.95 mm channel with heaters and insulation surrounding it. They found little difference in the pressure drop data.

Fig. 3 Mishima and Hibiki’s correlation for Chisholm’s parameter C as a function of channel hydraulic diameter.
between the adiabatic cases and those with heat transfer, and in both cases the conventional two-phase models over predicted the pressure drop.

In 2000 Zhao and Bi considered minichannel systems of exotic geometry and attempted to correct the Lockhart-Martinelli model. Primarily, their experiments used triangular channels with hydraulic diameters close to 1 mm. They did not actually modify the Lockhart-Martinelli model itself, but rather the single-phase pressure drops that are entered into it, leading one to expect similar problems with surface tension. They used the single-phase friction factor defined by Churchill (1977) and performed experiments in the slug flow regime. The Churchill friction factor makes no modification for channel geometry, as it merely tries to reproduce the friction-factor plot for circular geometries, which makes Zhao and Bi’s use of it for triangular channels suspect. They found good agreement with the model, except at low water superficial velocities. The majority of their collected data is in the 0 to 0.1 mass quality range and therefore not very applicable to the present work. One point to note is that they were concerned about secondary liquid flow in the corners of the channel when the channel is not horizontal, despite the effects of surface tension.

In 1993 Fukano and Kariyasaki studied several minichannels, including a 1 mm channel. Their work was based on the underlying assumptions that flow in minichannels is axisymmetric, and that the air is moving relatively slow compared to the water. They observed that, due to the effect of surface tension, there is little or no stratified flow occurring. They observed no gravity induced difference in film thickness above and below air bubbles, but the films became smaller with decreasing diameter channels. Also, bubbles that did form in the liquid tended to not disperse in the liquid or in the liquid films, but coalesced into slugs. They looked at both horizontal and vertical orientations, with the same flow rates, to compare the relative effects of gravity and surface tension. They found the transitional diameters to be between 5 and 9 mm, with surface tension forces dominating under 5 mm and gravitational over 9 mm. They noted zero drift velocity for diameters less than 5.6 mm. Drift velocity is the velocity of an air bubble induced by buoyancy alone. They found the Lockhart-Martinelli model to be less accurate in the intermittent flow regime, and developed their own model from more of a theoretical standpoint. Their model requires information about bubble mean free path, and film thickness that might not be readily available in an engineering situation. This serves to make the model impractical for pressure
drop predictions, and inapplicable to the current work. Also, though the surface tension effects were observed and noted, they were not quantified in the model.

Chen et al. published a pivotal work in 2001. They focused on the influence of fluid properties on pressure drop in minichannels. They collected a relatively large amount of data, with a low estimated uncertainty. They tested 1 mm and 3 mm circular channels with air-water flow and a 3 mm channel with refrigerant R-410a. Their work only evaluated the traditional large diameter correlations, rather than the more recent small diameter correlations. They concluded that none of the models adequately predict the two-phase pressure drop, but that the Homogenous Flow model is consistently the best. Specifically, they considered the Lockhart-Martinelli, Friedel and Homogenous Flow models and found a 82.9%, 218.0%, and 53.7% mean deviation respectively. They emphasized lower mass qualities and collected little data at the high mass qualities considered in the present work. They found that at high mass qualities and fluxes the models over predicted the air-water data quite significantly while under predicting the refrigerant data. They attributed this difference to the large disparity between the surface tensions of water and the refrigerant, 0.073 N/m and 0.008 N/m respectively (temperature dependant), rather than to the other fluid properties. However, the refrigerants were also more wetting than the water and they were uncertain of the interaction between surface tension and wetting. According to the ideas presented in their work, two-phase pressure drop should increase for flows with the surface tension reduced by the addition of surfactant.

The work by Chen et al. is the most relevant consideration of the effect of surface tension in minichannels. They referred to the work of Barajas and Panton (1993) as potentially explaining the difference between the air-water predictions and the refrigerant predictions (discussed in the flow regime section). They noted that the refrigerants, with low contact angle, did not enter the rivulet flow regime, unlike the water which did. Rather, the more wetting refrigerant spread up the walls of the minichannel, increasing the area between the gas and the liquid. They hypothesized that this increase in surface area dissipates more energy, due to interfacial interaction and shear stress, and so increases the loss of pressure. Thus, with water’s high surface tension, its pressure drop was under predicted, and with practically no surface tension, the refrigerant data was over predicted. The surface tension and contact angle of the fluids they considered are at opposite ends of the spectrum, and no
information was gathered in between. Having found the Homogenous Flow model to be the most accurate, Chen et al. modified it to include the Bond and Weber numbers, which are surface tension dependant Eqs. (31), (32). They found the new correlation to match their data within 30.9%. However, they also attempted a modification of the Friedel equation that matched their data within 19.8% Eqs. (33), (34).

\[
\Omega = \begin{cases} 
1 + \left(0.2 - 0.9 e^{-Bo}\right) & \text{for } Bo < 2.5 \\
1 + \left(\text{We}^{0.2} / e^{Bo^{0.3}}\right) - 0.9 e^{-Bo} & \text{for } Bo \geq 2.5
\end{cases}
\]

(31)

\[
\Delta P = \Omega \Delta P_{\text{H}}
\]

(32)

\[
\Omega = \begin{cases} 
\frac{0.0333 \text{Re}_{\text{Lo}}^{0.45}}{\text{Re}_{\text{G}}^{0.09} (1 + 0.4 e^{-Bo})} & \text{for } Bo < 2.5 \\
\text{We}^{0.2} & \text{for } Bo \geq 2.5
\end{cases}
\]

(33)

\[
\Delta P = \Omega \Delta P_{\text{Friedel}}
\]

(34)

When they attempted to apply their correlation to other small channel data banks they found that there is not enough published, and what is published often does not include all of the necessary experimental information for comparison, such as total mass flux, quality, and operating pressure.

Chen et al. (2002) sought to expand and improve upon the group’s previous work, and it resulted in re-writing the proposed Homogenous flow adaptation of 2001. They collected as much published data as they could find, and compared their previous correlation with the Lockhart-Martinelli, Friedel, Homogenous Flow and several refrigerant pressure drop models. They were able to compare ten refrigerant and three air-water databases. The air-water bank primarily included their previous work and the work of Triplett et al. (1999), whereas the refrigerant database contained eight different refrigerants. The work upheld the findings of the previous work in the Homogenous Flow model is the most accurate traditional model for two-phase pressure drop prediction, but that it is still not acceptable. The group adjusted their version of the Homogenous Flow theorem and reported a mean deviation from
the collected data of 19.1%. This compares with the other models deviating by 35-95%. They observed that the refrigerant flow data was better approximated by the large diameter correlations and believed this to be due to the low surface tension.

\[
\begin{align*}
\Omega &= \frac{0.85 - 0.082Bo^{-0.5}}{0.57 + 0.004Re_G^{0.5} + 0.04Fr^{-1}} + \frac{80We^{-1.6} + 1.76Fr^{0.068} + \ln(Re_G) - 3.34}{1 + e^{(8.5-1000\rho_g/\rho_l)}} \\
\Delta P &= \Omega \Delta P_H
\end{align*}
\] (35)

Chang et al. (2000) established the ground work and test section for the research of Chen et al. They only considered a 5 mm channel, but did look at both air-water and refrigerant flow. They noted the over prediction of the air-water pressure drop and particularly in the high quality region and for low mass fluxes. They did establish a modification of the Friedel correlation, but did not mention how accurate they found it, nor compared it to published data. It is assumed that the work of Chen et al. (2002) is more accurate as it is results from the same research group, includes consecutive revisions of the pressure drop correlations, and goes into more depth in comparing published data.

Kawahara et al. (2002) studied nitrogen-water flow in a 0.1 mm circular Microchannel. They noted a continuation of the trends found in minichannels, such as the Lockhart-Martinelli model requiring even smaller values of C to accurately predict the data.

### 3.3 Low Aspect Ratio Channels

Much of the published minichannel research has focused on channels with low aspect ratios due to their usefulness in compact heat exchangers. These geometries are also known as parallel or infinite plate configurations of small gap width. Clearly, these configurations lead to a high ratio of wall surface area to cross-sectional flow area. In single-phase systems, this leads to more prominent boundary layer effects. Similarly, one expects the two-phase flow behavior for these channels to be quite different from square or circular channels. However, there is still insight to be gained from these published works, and particularly as most other minichannel research uses circular geometries.

Ide and Matsumura (1990) performed an extensive study into the influence of aspect ratio and channel orientation on the two-phase pressure drop in rectangular channels. They
recognized that the pressure drop in low aspect ratio channels is greater than that in other geometries, and were also concerned about inclination angle. They determined that the classical correlations were not adequate to take these into account and constructed a separated flow model. They even made an adjustment for hydraulic diameters less than 10 mm. The smallest hydraulic diameter they investigated was 7.3 mm, for which there was a gap width of 4 mm. They did not consider the surface tension effects, nor that separated flow is not expected to occur in square or circular minichannels.

Xu et al. (1999) studied the flow regimes of 12 mm wide channels with gaps of 0.3, 0.6, and 1.0 mm width. They found a large difference in the behavior of the 0.3 mm gap flow from the others. However, the flow in the other channels was consistent with what they found for larger channels. They did mention that typical studies are inaccurate in predicting annular flow as they do not take into consideration the effects of the rectangular geometry.

Lowry and Kawaji (1988) found the large diameter correlations to not acceptably predict the flow patterns in parallel plate type systems of small gap width. They did find acceptable agreement for pressure drop using the Lockhart-Martinelli correlation, however they did not think it made adequate consideration for the mass velocity effects. Their work concluded that the two-phase frictional multiplier is strongly dependant on dimensionless gas velocity, while the channel diameter and the liquid velocity have little influence. In order to test the correlations, the work recognized the need for two-phase research that modified the fluid properties.

Lee and Lee (2001) investigated air-water flow in low aspect ratio minichannels for the plug and slug flow regimes. They found the Chisholm value of C to require reduction in channels of small gap width, which agrees with Mishima and Hibiki’s findings for circular minichannels.

Ali et al. (1993) looked at very wide channels with small gap widths and found there to be little change in flow conditions between different orientations, except for horizontal flow in vertical plates where a stratified flow was established. They also found the gap width to have little influence, which disagrees with other researchers.

Wambsganss et al. (1992) studied air-water flow in low aspect ratio minichannels extensively, took a broad range of experimental data, and developed a modification of the Lockhart-Martinelli correlation. Equations (12), (13), and (16) are used with Eq. (37), which
is dependant on the Martinelli parameter and the two-phase Reynolds number that is calculated using the liquid fluid properties. However, the lowest two-phase mass flux considered was 50 kg/m²s, which is more than double that presently considered

\[ C = ( - 2.44 + 0.00939 \text{Re}_{Lo})X^{(-0.938+0.00432\text{Re}_{Lo})} \]  

(37)

3.4 Channels With Refrigerant Flow

Many practical systems do not use air or water, but utilize refrigerants in a single component two-phase flow. These might include various heating ventilating and air-conditioning (HVAC), heat pipe, or electronics cooling devices, but they use the properties of the refrigerants to maximize heat transfer through boiling and condensation. The fluid properties are quite different from those of an air-water mix, and the systems are not adiabatic. Generally, there is a change in mass quality along the flow path, whereas the air-water flow is assumed to be at a constant quality. However, the underlying physics and fluid dynamics are related, and it is worthwhile to consider such systems and test proposed correlations for applicability outside of their experimental conditions.

Tran et al. (1999) studied flow boiling of several refrigerants in minichannels. Finding both large diameter and state of the art correlations to be inadequate, they sought to establish their own. They focused on using the confinement number, which is a ratio of the surface tension to buoyancy forces Eq. (38), in a pressure drop model based on Chisholm’s B equation. One can see that the confinement number is similar to the Bond number, but with a slightly different arrangement. The resulting correlation leads to a larger pressure drop for smaller channels and for higher surface tensions. However, they are hesitant to apply their correlation outside of the flow conditions, fluids, and channel geometries discussed. It is applied to the experimental data to determine if it is capable of modeling the pressure drop of air-water and reduced tension air-water in a minichannel. Although the operational fluid is different, their emphasis on turbulent-turbulent flow is seen as having the greatest potential for causing deviation from the present case.
\[ N_{\text{conf}} = \left( \frac{\sigma}{g(\rho_L - \rho_G)} \right)^{0.5} \]  

\[ \Delta P = \left[ 1 + (4.31^2 - 1)N_{\text{conf}} x^{0.875}(1 - x^{0.875}) + x^{1.75} \right] \Delta P_0 \]  

Garimella (2004) wrote a summary of the ongoing work in his lab, which focuses on minichannel condensation two-phase flow, and includes rectangular and other more exotic geometries. Aside from a potentially changing mass quality, the flow is very similar to the system presently being investigated. Pressure drop was considered, but emphasis was placed on flow regime analysis. It was expressed that flow regime based pressure drop models will be necessary to make accurate two-phase pressure drop predictions. Garimella found hydraulic diameter to be the controlling parameter for flow regime transitions, with geometry or aspect ratio having little effect. He found increasing quality, increasing mass flux, or decreasing diameter to contribute to an increase in pressure loss. Garimella focused on lower mass qualities and did not present an acceptable flow map. A model for pressure drop is developed, but it is somewhat unpractical for engineering considerations as it requires difficult to find information in practical systems.

### 3.5 Flow Regime Analysis

Much of the past research effort has tried to develop flow maps for various two-phase systems, with the hope of being able to predict the flow regime under any given set of conditions. Not only is the flow behavior crucial to how much heat will be transferred by a diabatic system, but it also influences the pressure drop. The more interaction there is between the two phases, the greater the turbulent behavior and pressure drop. As the pressure drop is flow regime dependant, some researchers focus their pressure drop efforts on a particular flow regime. All of the research considers flow regime to some degree, but the following authors focus on it almost exclusively. Void fraction analysis is often studied concurrently.

Damianides produced an extensive work on two-phase flow regimes in circular minichannels in his doctoral thesis (1987). The objective of the thesis was to conclusively determine if channel diameter is a significant variable in two-phase flow. High speed
photography was used to analyze the flow patterns in 1, 2, 3, 4, and 5 mm minichannels. It was found that the Lockhart-Martinelli correlation did not adequately account for the size of the channel within this range of diameters. The flow regimes were classified in detail and compared to those found in large diameter channels. It was determined that there was a primary dependence on the superficial velocities or mass flux. The conclusion was that channel diameter is a very significant parameter when it comes to small channels, and the Lockhart-Martinelli model was found to deviate by 60-100%. Also, it was recommended that fluids of different surface tension be tested to determine the influence of surface tension. One important point to note is that many researchers, when talking about pressure drop, try to take the flow regime into consideration. Damianides took a different stance on this, and used the dynamic pressure readings to determine the transitions between the flow regimes. He found the form of the differential pressure oscillations to be quite indicative of the flow regime.

In 1992 Barajas and Panton investigated air-water flow regimes in minichannels with specific consideration of the three phase contact angle. The contact angle was adjusted by using different test section materials. Contact angle gives a quantification of the surface’s wettability, and altering the solid material leaves the surface tension of the water constant. Three of the systems used were partially wetting and one was partially non-wetting, and all used 1.6 mm circular minichannels. The three were found to yield similar results, while the non-wetting had very different characteristics. One key point of the work is that they noted a flow regime they labeled as rivulet flow, where a stream of liquid flows down the channel but does not cover the full channel diameter as in stratified flow. They also noted multiple rivulets at the same time. It was even noted that the rivulet stream might travel up the channel wall and the channel top. Establishing such a flow in rectangular channels is unlikely, as the walls form barriers to the rivulet travel and the corners form convenient travel routes, but they did not investigate rectangular channels. They found that as the contact angle increased the flow tended to rivulet flow rather than wavy annular flow. Likewise, they found the high contact angle case to inhibit annular flow, as the water did not want to spread up the channel walls. Overall, they noted little flow transition dependence on contact angle for the partially wetting systems, but a large dependence for the partially non-wetting case. For example, the transition from slug to annular flow occurred at lower gas and
liquid superficial velocities. The ramifications of the work for the mass fluxes presently considered are unclear, as relevant data was not recorded.

Coleman and Garimella (1998) studied the effects of channel diameter and geometry for air-water flow in minichannels. They found their work to generally agree with that of Damianides (1987), while showing that the traditional large channel correlations were not adequate. The found the correlation of Taitel and Dukler (1976), a popular large diameter flow regime transition model, to be based on assumptions that are inconsistent with minichannel flow. They found that the diameter, channel aspect ratio, and the surface tension play an important role in channels with hydraulic diameters of less than 10 mm. Particular to minichannels, they found the transition to annular flow to occur at a nearly constant value of superficial gas velocity that reaches a limiting value with decreasing channel diameter. The case plotted was for a square minichannel with hydraulic diameter of 5.36 mm, and the limiting value reached for transition to annular flow was 2.5 m/s, though the map was not extended to the low superficial liquid velocities presently considered. Also, they noted an increase in the size of the intermittent flow regime at the cost of the stratified flow regime.

Yang and Shieh (2001) studied both air-water and refrigerant flow in circular minichannels. They observed that the flow transitions are relatively sharp for the refrigerant flow, whereas for air-water it is more of a transition zone that is at least partially dependent on the flow's history. Also, the refrigerant shifts the slug to annular transition to lower gas velocities, and some other transitions were changed significantly as well. The refrigerant flow was seen to reach annular flow with a superficial gas velocity of 3 m/s whereas the air-water did not reach annular flow until a superficial velocity of over 10 m/s. The offset was speculated as being dependant on the large difference in surface tension.

3.6 Fuel Cell Application

In PEM fuel cells, air and hydrogen are reacted to produce electricity, water, and heat. The water either evaporates, or is entrained in the air flow and exhausted. The air is generally delivered to the cell via minichannels, though even smaller channels are being considered. The typical result is a two-phase mixture of air and water, having a changing mass quality, flowing throughout the cell. The quantities of air used and water produced are
governed by the related chemical reactions. One can use the equations representing these reactions to find preliminary flow rates for the operational fluids. However, the PEM might not be operating at a steady state. Also, the actual channels often involve bends where water can collect. Bends will not be considered experimentally, however the result is similar to local increases in the volumetric flow of water.

Larminie and Dicks (2000) gave the rate of water production and air consumption in a typical PEM electrochemical by reaction Eqs. (40), (41). An application of these formulas to calculate the required flow of air and water in one of the delivery minichannels can be seen in appendix A. The result of that sample calculation is a water flow rate of 0.0079 mL/min and an air flow rate of 65.1 mL/min, which is a flow with a quality of 0.905 and leads to superficial velocities of 1.32x10^{-4} m/s and 1.085 m/s respectively. Clearly, this is a low mass flux system of high quality flow. However, lower qualities are also tested using higher water flow rates, due to the accumulation mentioned earlier.

\[ \dot{m}_w = 9.34 \times 10^{-8} \times \frac{P_e}{V_c} \]  \hfill (40)

\[ \dot{m}_a = 3.57 \times 10^{-7} \times \lambda \times \frac{P_e}{V_c} \]  \hfill (41)

The fluid interactions in a fuel cell are far more complicated than will be considered presently. Oxygen is consumed by the reaction and water produced, so mass transfer is inherent to the problem. Likewise, the cell is not adiabatic, as it can operate over a range of temperatures, and produces heat as byproduct. Furthermore, there are necessarily two different solid materials involved, and thus several potential multiphase interfaces. Due to the lack of published information on the fundamental interactions present, that would give insight into the basic fluid behavior of such a system, and in order to increase applicability to other types of systems, the experimental setup is simplified to a basic adiabatic air-water flow with no heat transfer, mass transfer, or multiple materials. Additionally, the channels are generally not perfectly square. The corners are usually slightly rounded from manufacturing. The channels might be semi-circular, trapezoidal, or triangular depending on the process used to manufacture them. However, they are usually closer to a rectangular
shape than to a circular one. Furthermore, they often use a manifold and parallel channels, which leads to mal-distribution problems. For example, a liquid plug might fill one channel and redirect gas flow to other channels rather than being pushed through.

Trabold (2004) gave a good starting point from the perspective of PEM applications. It is noted that due to the need for water in the fuel cell reactions, but with the desire to avoid flooding, the water management issue is very important. For example, it is not wise to operate in the slug flow regime, as the slugs cause maldistribution problems, and inhibit air flow to catalyst sites. Trabold recommended that a superficial gas velocity of 5-6 m/s be maintained to keep the flow in the annular regime, which requires higher air flow rates than those calculated in appendix A. Also, it was recognized that up to 80% of the fuel cell’s operation might occur outside of this flow regime if operated at low power, which implies the existence of higher local water flows than those calculated in the appendix. From a mechanical standpoint, channels larger than 3 mm are not typically used. 1 mm square channels are commonly found, though occasionally with an aspect ratio of as little as 0.5.

Wheeler et al. (2001) considered a slightly different system than that described by Trabold. Theirs was a PEM system with porous channel walls that wick away some of the liquid. However, some water remains and they also emphasized that annular flow be maintained. A superficial air velocity of 6 m/s was recommended as a minimum, though the group investigated much larger velocities as well. Lower superficial velocities were considered to perform poorly.

The entire literature review detailed above may be summarized in Tables 2 and 3 on the following pages. The most pertinent research is separated from the more distantly related and data characteristic of the works’ experimental conditions is presented.
Table 2 Feature summary of the most relevant published literature.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm)</td>
<td>1.488-25.8</td>
<td>n/a</td>
<td>12-50</td>
<td>1.1, 1.45, 1.09, 1.49</td>
<td>1.05, 2.05, 3.12, 4.08</td>
<td>1.02, 3.17, 5.05,</td>
<td>1, 2, 3, 4, 5</td>
</tr>
<tr>
<td>Cross-Section</td>
<td>Circular</td>
<td>n/a</td>
<td>Circular</td>
<td>Circular Triangular</td>
<td>Circular</td>
<td>Circular</td>
<td>Circular</td>
</tr>
<tr>
<td>$J_D$ (m/s)</td>
<td>$Re_D = 880 - 76,000$</td>
<td>n/a</td>
<td>0.02 - 3.7</td>
<td>0.02-80</td>
<td>0.08-30</td>
<td>$Gr_T = 283 - 3279$</td>
<td>0.015-125.34</td>
</tr>
<tr>
<td>$J_L$ (m/s)</td>
<td>$Re_L = 382 - 122,000$</td>
<td>n/a</td>
<td>0.06 - 30</td>
<td>0.02-80</td>
<td>0.01-1</td>
<td>$x = 0.01 - 0.93$</td>
<td>0.0024-5.72</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>713-15,000</td>
<td>n/a</td>
<td>6100</td>
<td>200</td>
<td>650</td>
<td>150-995</td>
<td>160D</td>
</tr>
<tr>
<td>Orientation</td>
<td>n/a</td>
<td>n/a</td>
<td>Horizontal</td>
<td>Horizontal</td>
<td>Vertical Horizontal</td>
<td>Horizontal</td>
<td>Horizontal</td>
</tr>
<tr>
<td>Materials</td>
<td>n/a</td>
<td>n/a</td>
<td>Glass</td>
<td>n/a</td>
<td>Glass</td>
<td>Copper</td>
<td>Glass</td>
</tr>
<tr>
<td>Operating Fluids</td>
<td>Air, Several Liquids</td>
<td>n/a</td>
<td>Air, Altered Water</td>
<td>Air, Water</td>
<td>Air, Water</td>
<td>Air, Water Refrigerant</td>
<td>Air, Water</td>
</tr>
<tr>
<td>Flow Patterns</td>
<td>Several</td>
<td>n/a</td>
<td>Several</td>
<td>Slug Bubbly Annular</td>
<td>Several</td>
<td>Several</td>
<td>Annular Intermittent Bubbly</td>
</tr>
<tr>
<td>Notes</td>
<td>Developed a simple pressure drop model that relates the two-phase pressure drop to the single phase superficial pressure drops. Their experiments did not include air-water data and focused on large diameter channels. Chisholm developed an approximation of their tabulated values.</td>
<td>For optimal fuel cell operation, Trabold suggested that the annular flow regime be maintained by using a minimum superficial gas velocity of 6 m/s, which is supported by the work of Wheeler et al. (2001).</td>
<td>Used water additives to modify the water's surface tension, viscosity and density, in order to isolate their effects and determine their influence on flow regime transitions. They only considered large diameter channels and found surface tension to have little effect.</td>
<td>Collected experimental data and used it to evaluate two-phase pressure drop prediction models. They determined that models for large diameter channels typically over-predict the pressure drop. They found the Homogenous Flow model to be the most accurate.</td>
<td>In order to adjust for flow in mini-channels, they modified the Lockhart-Martinelli model with a channel diameter dependant value for Chisholm's parameter, C. They did not take surface tension into account and they focused on turbulent-turbulent flow.</td>
<td>Found the Homogenous Flow model to most accurately fit the air-water data and the Friedel model to most accurately fit the refrigerant data. They added correction factors to these models that rely on non-dimen-sional numbers, some of which involve surface tension.</td>
<td>Performed extensive research into two-phase flow patterns in small diameter channels. He only used circular glass channels and did not focus on pressure drop. Considering the present focus on annular flow, his circular data is expected to match the present square channel well.</td>
</tr>
</tbody>
</table>

38
Table 3 Feature summary of related, but less relevant published literature.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm)</td>
<td>1.95</td>
<td>0.866, 1.443, 2.886</td>
<td>1.0, 2.4, 4.9</td>
<td>1.6</td>
<td>0.4, 1, 2, 4</td>
<td>0.778, 1.465</td>
<td>0.3, 0.6, 1.0</td>
<td>1.59</td>
<td>0.5, 1, 2</td>
<td>1.0, 2.0</td>
<td>1.0, 2.0</td>
<td>0.4 - 4.91</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Cross-Section</td>
<td>Circular</td>
<td>Triangular</td>
<td>Circular</td>
<td>20 mm wide rectangles</td>
<td>80 mm wide rectangles</td>
<td>12 mm wide rectangles</td>
<td>9.52 mm wide rectangles</td>
<td>50 mm wide rectangles</td>
<td>20, 15 mm wide rectangles</td>
<td>Circular</td>
<td>Wide Variety</td>
<td>Circular</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Air Superficial Velocity (m/s)</td>
<td>0.1 - 50</td>
<td>0.1 - 100</td>
<td>0.1 - 11</td>
<td>0.1 - 100</td>
<td>0.05 - 18.7</td>
<td>0.15 - 16</td>
<td>0.1 - 14</td>
<td>0.2 - 18</td>
<td>0.1 - 14</td>
<td>0.043 - 71.4</td>
<td>0.0095 - 1.6</td>
<td>x = 0 - 1</td>
<td>0.1 - 60</td>
<td>n/a</td>
</tr>
<tr>
<td>Liquid Superficial Velocity (m/s)</td>
<td>0.08 - 0.5</td>
<td>0.08 - 6</td>
<td>0.03 - 1.2</td>
<td>0.003 - 2</td>
<td>0.03 - 2.39</td>
<td>0.019 - 6</td>
<td>0.1 - 1</td>
<td>0.2 - 5</td>
<td>0.2 - 5</td>
<td>0.0095 - 1.6</td>
<td>x = 0 - 1</td>
<td>0.02 - 4</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>870</td>
<td>270</td>
<td>260</td>
<td>300</td>
<td>640</td>
<td>240</td>
<td>260</td>
<td>171</td>
<td>80</td>
<td>630</td>
<td>200, 400, 600</td>
<td>22.5 - 508</td>
<td>64.5</td>
<td>n/a</td>
</tr>
<tr>
<td>Orientation</td>
<td>Horizontal</td>
<td>Vertical</td>
<td>Vertical</td>
<td>Horizontal</td>
<td>Horizontal</td>
<td>Vertical</td>
<td>Horizontal</td>
<td>Vertical</td>
<td>Horizontal</td>
<td>Horizontal</td>
<td>Horizontal</td>
<td>Horizontal</td>
<td>Vertical</td>
<td>Horizontal</td>
</tr>
<tr>
<td>Materials</td>
<td>Copper</td>
<td>Lucite</td>
<td>n/a</td>
<td>Pyrex polyethylene</td>
<td>lucite</td>
<td>Acrylic</td>
<td>Lucite</td>
<td>Pyrex</td>
<td>PMMA</td>
<td>Lucite</td>
<td>Plexiglass</td>
<td>Fused silica</td>
<td>n/a</td>
<td></td>
</tr>
<tr>
<td>Flow Patterns</td>
<td>Several</td>
<td>Slug</td>
<td>Slug</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
<td>Several</td>
</tr>
<tr>
<td>Notes</td>
<td>Examined both adiabatic and non-boiling diabatic air-water flow, and found little difference in pressure drop between the two cases.</td>
<td>Used a non-standard geometry, and found the Lockhart-Martinelli model adequate when used with the Churchill friction factor.</td>
<td>Observed a large surface tension effect, and small orientation effect, for diameters less than 5 mm. Surface tension is not included in their model.</td>
<td>Analyzed the effect of wetting on flow regime using test sections of different material, to produce different three-phase contact angles, and also found wetting to increase pressure loss.</td>
<td>Developed a pressure drop model that decreases the value of C for small diameter channels, but only applies to slug flow.</td>
<td>Verified that mini-channel orientation has little effect on pressure drop and flow pattern. However, also found the effect of gap width to be small.</td>
<td>Developed high aspect ratio channels for flow regime and pressure drop.</td>
<td>Developed a presently inapplicable pressure drop model.</td>
<td>Examined flow patterns of air-water flow between parallel plates and saw need of experiments isolating the fluid properties. Found Taitel inaccurate.</td>
<td>Determined flow regime transition criteria and found them to agree fairly well with other research, though not very well with that of Taitel.</td>
<td>Found refrigerant condenstate flow in mini-channels of various geometries and found hydraulic diameter effects much more significant than geometry.</td>
<td>Developed an empirical correlation with many data points that were taken from large diameter channel research. Chen modifies the model for mini-channels.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.0 OBJECTIVES

The long term goal is to be able to predict and reduce system pressure loss as accurately as possible over a wide range of operating fluids, channel geometries, and flow conditions. The more concentrated focus of the present work may be summarized as follows:

1. Use adiabatic two-phase air-water flow to collect pressure drop data under conditions that are outside those typically reported in research and that are relevant to PEM fuel cells.
2. Use a surfactant to sequentially reduce the water’s surface tension and collect additional pressure drop data under the same flow conditions.
3. Attempt to correlate any observed change in pressure drop to the changing surface tension or surfactant concentration.
4. Evaluate published models for adequate prediction of the pressure drop and accurate representation of the surface tension effects.
5. Make suggestions for improvement of the published models.
6. Consider the three phase contact angle for additional clarification of the wetting effects.
7. Use high speed photography to supplement the pressure drop analysis by giving insight into the two-phase interfacial phenomena.
8. Provide a better predictive base for two-phase flow of fluids with reduced surface tension and establish a foundation for further, and more refined, two-phase experimental investigation.
5.0 EXPERIMENTAL SETUP

5.1 Test Section

The test section is constructed of lexan, which is selected for its machineability and optical clarity. It allows one to customize the channel’s geometry, while being transparent for flow photography. The channel is milled into a piece of lexan using a 1 mm diameter bit to produce a nominally square channel. The fourth channel wall is formed by a second piece of lexan compressed on the bottom. Due to the influence of surface characteristics on flow phenomena (i.e. the contact angle might change with surface roughness characteristics), all of the internal surfaces were carefully machined, and then lightly and evenly sanded with 600 grit sandpaper. The sanding removes slight irregularities without changing the channel geometry. In order to prevent leakage (mass loss), a groove is machined along the outer edge of the lexan and strips of rubber are glued in. When the two lexan pieces are clamped together the rubber compresses and prevents any leakage of the operating fluids. Photographs of the test section may be seen in appendix B and a cross-sectional schematic is given in Fig. 4. The whole assembly is compressed between two additional pieces of lexan using a series of 17 clamps.

![Cross-sectional schematic of the test section, not to scale.](image)

The channel is 321 mm long and has two pressure taps that are centered 177.8 mm apart. The first pressure tap is 110 mm downstream from the air inlet and 100 mm from the water inlet. Typical pressure tap lengths are between 200 – 300 mm long in published literature (Mishima and Hibiki 1996, Bao et al. 2000, Zhao and Bi 2000, Barajas and Panton 1992, and Yang and Shieh 2001), however the present section is longer than Chen et al. (2002) at 150 mm and Damianides (1987) at 60 mm. These two works are very related to the present work, and the length is close to the range found in the other works. The entrance and
exit lengths are more of a concern, as the flow pattern might be developing, or liquid might be held up at the exit. However, the viscous damping, due to the small channel and low flow rates, reduces the entrance length. Even for a superficial gas velocity of 10 m/s the single phase entrance length would be only 37.7 mm. Damianides found 100-Dh to be an acceptable entrance length for minichannels, despite the construction of the entrance mixing section, and references a personal communication from Taitel as asserting that even a 20-Dh entrance length would be acceptable. Therefore, there is 100-Dh distance for entrance effects and flow calming, and a 33.2 mm exit section. The last 2 mm of the channel floor are removed and lead to an expanded chamber below the channel. From there, the fluids drain from the test section and any liquid buildup is in the chamber, rather than the channel.

The pressure taps are 0.396 mm holes drilled into the top of the channel. The taps expand to fit 3.18 mm aluminum tubing that is epoxied into place. Since the pressure is constant perpendicular to the flow and the two tap tubes terminate on opposite sides of a differential pressure transducer, the transducer’s output is essentially the frictional pressure drop in the channel. Due to the horizontal orientation and as the flow is fully accelerated when it reaches the taps, the gravitational and acceleration pressure drops are assumed to be negligible.

5.2 Fluid Supply

The experiments use distilled water that is degassed with the method of Kandlikar et al. (2002). The water flow rates are quite low and a gravity feed system is adequate for establishing them. A plastic bag, designed for the intravenous drip delivery of medical fluids, is suspended two meters above the test section and a 1.6 mm internal diameter tube feeds an Omega FL-120 flow meter. The flow meter is a low flow precision rotameter of typical variable area design, that is capable of 0-1.2 mL/min of flow, which is adequate for mass qualities of greater than 0.1 in the system. Many of the data points were taken at flows in the lower half of the flow meter’s range. It would have been preferable to find a flow meter fitting this range more closely, unfortunately conventional rotameters are not typically available in this range and alternatives are more expensive. One that was available commercially required too large a pressure head to establish flow and was unpractical for experimental use.
The building’s compressed air supply was determined to be unacceptable, due to a lack of purity and the possibility of entrained moisture (water would increase the superficial velocity of the water in the test section). Instead, ultra zero grade compressed air from a 18 MPa storage tank is used. The following properties are required for ultra zero grade air: moisture < 5 ppm, hydrocarbons < 0.1 ppm, CO\(_2\) < 1 ppm, and CO < 1 ppm. To reduce the pressure to 200 kPa a two stage regulator is used. The regulated air flows through an Omega FL-5531st flow meter that is capable of 0-870 mL/min of flow. The flow rates used lie in the middle of this range.

Leak testing is performed on every section of the flow path, following the flow meters, by spraying a soapy water solution over all potential leak sites (i.e. the edges of the test section, all fittings, and all valves). Any leakage is readily apparent as soap bubbles form. Likewise, any drops of escaping water are noted and terminated.

A Fisher “Surface Tensiomat” Model 21, Catalog No. 14-814, is used to measure the surface tension of the surfactant solutions. Essentially, the unit is an accurate balance that measures the surface tension force on a suspended wire ring of known dimensions. Distilled water is used to clean the ring between measurements, and each measurement is the average of several readings. The actual measurement is an apparent surface tension and it must be converted to the absolute surface tension Eq. (42).

\[
\sigma_{ab} = \sigma_{ap} \left[ 0.7250 + \sqrt{\frac{0.01452 \cdot \sigma_{ap}^2}{C^2 (\rho_L - \rho_G)} + 0.04534 - \frac{1.679 \cdot r}{R}} \right]
\] (42)

5.3 Data Acquisition

K type thermocouples are used to measure the ambient temperature, as well as the inlet temperature of the air. The water is assumed to be at ambient, as it is held at room temperature.

Two Omega pressure transducers are used in the experiments. The first is a PX26-005GV transducer that is placed after the air flow meter, to monitor the air pressure entering the channel, and operates within a 0-34 kPa range. The second transducer is a PX26-001GV that operates between 0-7 kPa and is positioned between the test section pressure taps. The
differential reading from the second transducer is the pressure drop along that section of channel. The two transducers are separately excited by a constant voltage source at 10V.

A National Instruments SCXI-1000 data acquisition module with a SCXI-1303 card are used to capture the signals coming from the thermocouples and pressure transducers. The data is acquired every 0.2s, and is fed to the Labview software and a virtual user interface, where it can be viewed, manipulated, and recorded. A screen-shot of the virtual interface is presented in appendix B.
6.0 EXPERIMENTAL PROCEDURE

6.1 Surfactants

Surfactant science is not widely addressed in standard academic chemistry programs, however there exists a vast amount of surfactant information in specialized publications. The textile, soap, paint, and other chemical industries have focused much effort on the refinement of surfactant science, so much of the information is proprietary. Also, the same chemicals might be known by different house names or brands. Myers (1988) gives a good overview of surfactants for the non-specialist. The term surfactant is the short name for all chemicals characterized as surface active agents. These chemicals are often soluble in a variety of liquids, and they migrate to the interfacial surfaces and change the surface properties (especially the free energy). Typically, they are applied to increase wetting or detergency. Interfacial surfaces exist throughout both nature and industry, and modifying these surfaces influences the chemical interaction dynamics. Therefore, it is quite important to understand how the interfaces will act. For the present experiments, it is desired to change the surface tension of the air-water interface occurring in a two-phase flow. As the surfactant effects only influence the surface, it is possible to use a very low concentration of surfactant and significantly lower the surface tension of the liquid, with a negligible change to the density or viscosity. Increasing the surface tension would also be instructive, but doing so usually requires the addition of significant amounts of other chemicals, which then changes the other fluid properties.

One obstacle to applying surfactants to two-phase experiments is that the surfactant solutions tend to hold bubbles or produce foam. This can be seen in the foaming action of most hand soaps. In the test section, air is pushing past the water and it is easy for small amounts of air to be entrapped by the water and produce foam. Therefore, it is very important to choose a low-foaming surfactant. The foaming action of a surfactant is typically tested by the Waring blender test, where the liquid is disrupted in a controlled manner and the foam height is measured. However, there is also a temperature effect involved in the foaming, and it is important that the surfactant be used at temperatures just above a given temperature, called the cloud point. Therefore, low foam/low temperature surfactants are
appropriate for the current experiments. Another negative quality of surfactants is that, even in very low concentrations, they can change the optical qualities of the operational fluid. Typically, the fluid becomes milky and opaque. The change occurs at the cloud point and does not influence the pressure drop, but potentially inhibits video imaging of the fluid flow.

Essentially, the desire is to modify the water's properties in such a manner that it acts like water in every way, but for the surface tension effects. Shurell (2004) recommended three nonionic surfactants made by Dow Chemical, Triton™ EF-19, Triton™ DF-12, and Tergitol™ Min-Foam 2x. All three were tested external to the test section and the Min-Foam 2x was found to produce a low, but still undesirable, level of foaming. The other two surfactants produce almost no foaming. The DF-12 has more of an effect on opacity, but less technical data is available for the EF-19. Therefore, DF-12 is the surfactant used in the experiments. The characteristic properties of a DF-12 solution at a 0.1% concentration by weight are listed in Table 4.

<table>
<thead>
<tr>
<th>Density (kg/m³)</th>
<th>Viscosity (kg/ms)</th>
<th>Equilibrium Surface Tension (N/m)</th>
<th>Dynamic Surface Tension (N/m)</th>
<th>Critical Micelle Concentration (ppm)</th>
<th>Cloud Point (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,029</td>
<td>0.06</td>
<td>0.034</td>
<td>0.046</td>
<td>290</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 4 Properties of Triton DF-12 surfactant measured at Cₛ = 0.1 wt% and 25°C.

The system feed water is contained in a plastic intravenous drip bag and is gravity fed to the test section. When the surfactant solution is used, the bag is half-filled with water, the appropriate amount of surfactant is added, the bag is shaken to mix it, and the rest of the water is added. Since the surfactant will migrate to any air-water surface, complete air removal from the bag is important. Care is taken to seal air out of the bag while not losing water from the surface of the bag, which would mean a relatively high loss of surfactant. The solution is bled through the flow control loop before being allowed into the test section. The desire is to ensure that the static surface tension measured is as close to the surface tension of the operational solution as possible.

Once the water is introduced into the channel, the surfactant migrates to the air-water interface, where it depletes the surface tension. In order to guarantee that there is enough surfactant in the bulk of the fluid, the critical micelle concentration must be met. A micelle
is a group of surfactant molecules clustering together in the bulk of the fluid. The critical concentration is defined as an adequate amount of these micelles present for cleaning purposes. Here, it is used to gauge if there is enough surfactant present to reduce the surface tension during dynamic two-phase flow, and for the chosen surfactant relates to a concentration of 0.030% by weight. According to Langner (2004) a typical surfactant molecule can travel 1 micron in 1 \(\mu s\) by diffusion. This is sufficient as in a fully loaded solution a molecule should never have to travel more than the channel diameter to reach an interfacial surface, or 1 ms. In the worst case, the air takes 18 ms to travel the distance between the pressure taps. Furthermore, the water is usually moving a lot slower than the air and a 18 ms time frame would be unlikely. However, it was noted by Weisman et al. (1979) that a static surface tension measurement might not be a true indication of interface behavior. The dynamic surface tension is typically higher than the static surface tension. Currently, the laboratory is not equipped to measure dynamic surface tension, so the results are analyzed using static surface tension and concentration. The reported value of dynamic surface tension for a DF-12 solution is given in Table 4. As the concentration of surfactant is so low, it is difficult to hit a particular concentration exactly. Therefore, a general distribution of concentrations between the critical micelle concentration and that producing the lowest surface tension was attempted.

Contact angle is intimately connected to surface tension, and it is somewhat difficult to quantify their interactions and differentiate their influence on wettability. Figure 5 depicts the forces acting on a liquid droplet resting on a flat solid surface. There exists a circular line of contact between the three-phases, and three dimensional surfaces between any two phases. The forces are typically related by Young’s Equation, given in Eq. (43).

![Diagram of three phase contact angle and surface tensions.](image-url)

**Fig. 5** Three phase contact angle and surface tensions.
\[ \sigma_{LG} \cos \theta = \sigma_{SG} - \sigma_{SL} \] (43)

The apparatus constructed for contact angle measurement by the Sessile drop method uses gravity to push water through a small hole in a lexan plate Fig. 6. The plate surface is finished with the same method as the channel for the same three-phase interaction characteristics. Tubing leads away from the bottom of the hole and up the outside, to form a fluid column. A micrometer spindle changes the height of the tube, to slightly increase or decrease the head of the fluid, and so establishes flow to or from the fluid droplet forming on the plate surface. A picture of the apparatus can be found in appendix B. As water feeds to the droplet, the contact angle increases until the three phase interface is pushed outward. The maximum angle achieved is called the advancing contact angle. Similarly, if water is removed from the droplet, the contact angle diminishes until the three phase interface recedes, at which point the smallest contact angle is termed the receding contact angle. Both the advancing and receding contact angle are recorded for all liquid solutions used.

![Fig. 6 Sessile drop test device.](image)

6.2 Calibrations

The pressure transducers are calibrated with an Omega DPI 610 pressure calibrator. The calibrator establishes an absolute pressure on the high pressure side of the differential transducer, and the voltage reading from the transducer is recorded with the data acquisition.
Ten of these readings are recorded over the range of the transducer. A linear regression equation is fit to the data points, so that any voltage reading from the transducer can be used to calculate the actual differential pressure. The data points and regression equations for all calibrations are reproduced in appendix D.

The thermocouples are calibrated with an Omega Hot Point Calibration Cell. The cell establishes a known temperature within a metal block. The thermocouples are inserted into the block, and their changing electrical resistance is recorded as the block changes temperature. The thermocouples are used at the ambient temperature, but the calibrator is only capable of establishing temperatures greater than ambient. Therefore, nine test points are recorded between 28 and 50 °C, and a zero point is taken with a slushy mix of water and ice. The ten points are collectively fit to a regression line.

The manufacturer specified calibration equation is accepted for the air flow meter. However, as the liquid flow rates are so low, the FL-120 flow meter is calibrated on site. The flow rate is set and stabilized, and the water is allowed to flow into a small cup for a recorded period of time. The cup is weighed on a Ohaus AR2140 scale (accurate to 0.0001 g) before and after the liquid flow. The weight difference allows the calculation of liquid flow volume. Typically, the higher flows where recorded for a minute, the middle for two minutes, and the low flows for five to ten minutes. Using the water volume and fill time, the flow rate is calculated and the scale on the flow meter is calibrated. The data should fit a polynomial curve, but there proved to be a range between the 40 and 50 mm positions where the flow rate is nearly constant. Therefore, this region is avoided, and the upper and lower regions are calibrated separately. The calibration chart is presented in appendix D. To ensure accuracy, 28 data points were taken for the calibration, with at least 10 in each of the high and low flow regions. Due to the time required for such low flow measurements, there is some evaporation. This means there is slightly more flow than measured, but it is assumed to be negligible, and is balanced by the fact that there is some evaporation in the channel as well.

It should be noted that there is some instability in the Fl-120 flow meter, due to the nature of its design and the low flow rates used. Though uncommon, the float might remain at a level for a few minutes, but then suddenly drop. Therefore, the calibration points and experimental data points are only taken after careful observation and assurance of float
stability. More importantly, it is not always possible to achieve a specific flow rate stably. Rather, one has to find stable points near to the desired flow. Essentially, this means that the data points might not be perfectly dispersed over the operational range, but still acceptably. A positive displacement system is recommended for future work, as it would improve the accuracy of the liquid flow measurement.

6.3 Data Acquisition

In order to record acceptable data, the following procedure for data acquisition is carefully followed:

1. The test section and channel are cleaned with methanol and distilled water.
2. The test section pieces are assembled and evenly compressed with the distributed clamps.
3. The inlet, outlet, and pressure tap tubing are attached and positioned.
4. The test section is mounted to a holding bracket, for stability, and is leveled.
5. The data acquisition equipment is initiated and tested.
6. The air pressure vessel is opened and regulated to 240 kPa.
7. The air flow rate is set to the desired level.
8. Once the system has reached equilibrium, the single-phase gas pressure and temperature signals are recorded for one minute along with the flow meter reading.
9. The water flow is set to the desired starting point.
10. Once the system has reached equilibrium, the pressure and temperature signals are recorded for one minute along with the flow meter readings.
11. The water flow rate is slowly increased to the next desire level and step nine is repeated.

At a particular air flow rate, data is recorded for a complete set of water flow rates in a single run. If the next liquid is of a different surface tension (has a different concentration of surfactants), then the test section is disassembled, cleaned, and the procedure is begun again. However, if the same operating liquid is used, then the test section is not disassembled and the following procedure is followed:
12. The air flow rate is maximized for a few seconds to blow out any remaining water.
13. The air flow is reduced to 100 mL/min and allowed to flow alone in the channel for an hour, to make certain that it is fully dried out.
14. The outlet and channel are observed to ensure no residual water.
15. Step 7 of the previous procedure is begun again.

The cleanliness of the channel is important as dust, fingerprint oils, or other contaminants would influence the surface tension properties of the water. Therefore, care is taken in handling the test section, and touching the channel is avoided. If allowed to sit for more than a few days, the channel is disassembled and cleaned. The test section is stored in an assembled configuration, though not usually clamped.

The flow rate of the water is increased slowly and steadily, in order to avoid the introduction of large flow transients to the system. It should be noted that this is not necessarily representative of an industrial application, where there might be quick and drastic changes in either of the fluid flow rates. Also, there might be more localized effects or maldistribution due to a parallel flow system. One would expect a wider scattering of data for those cases and a larger degree of predictive error. The goal is to record more consistent data that can be quantitatively compared.

Using the above method, the signal from the pressure transducer is fairly level. However, there is still some dynamic behavior and slight fluctuations. The mean values used in the calculations are time averages of one minute recordings, which is generally over 300 data points. This is done to guarantee that the pressure drop recorded is actually representative of the equilibrium case, and not a local time dependant fluctuation, and to reduce the uncertainty in measurement. The flow meters are carefully observed to make sure they are holding a constant flow rate. If there is more than a half gradation’s fluctuation in the air flow, or more than a quarter in the liquid flow, then the data point is re-taken.
7.0 EXPERIMENTAL UNCERTAINTIES

The uncertainty in experimental measurement can be ascertained to within 95% using Eq. (44), which takes into account both the systematic and random error found in measurement. All of the uncertainties are calculated with this method and it is found that the calibration of the test equipment essentially makes the bias error, $B_e$, negligible or reduces it to the uncertainty found in the calibration equipment. This is then combined with the error inherent in reading the equipment to find the total uncertainty.

\[ U = 2 \sqrt{\left( \frac{B_e}{2} \right)^2 + (S_x)^2} \]  

(44)

\[ B_e = \Delta R \]  

(45)

\[ S_x = \frac{\sigma}{\sqrt{N}} \]  

(46)

For the pressure transducers, the calibration equation produced has an $R^2$ value of 1, implying that any uncertainty in the recorded pressure is due to the uncertainty in the calibrator alone. The individual data points are an average of 300 recorded points (typically). The result is that uncertainties of calibration and standard deviation are negligible compared to the uncertainty in the pressure calibrator. The pressure calibrator has an uncertainty of 6.89 Pa. Therefore, the pressure measurements have an uncertainty of ± 14 Pa, which is 0.7% of the lowest pressure reading and 0.2% of the largest.

The calibration equations produced for the thermocouples have $R^2$ values of 0.9997 and 1. Similar to the pressure reading at least 300 points are recorded for each data point, and as for the pressure transducers the uncertainty is primarily dependant on the uncertainty of the calibration unit. The calibration unit has an uncertainty of ± 0.9°C, which is about 4% of the average temperature reading.

The air flow meter is used with the manufacturer issued flow charts, which espouse an $R^2$ value of 0.99998669. There is also a ± 0.5 mm uncertainty in the reading of the flow meter. The result is an uncertainty in flow of ± 3 cc/min over the whole flow range. This is
a 1.5% uncertainty for the lowest air flow rate used and a 0.5% uncertainty for the highest flow rate.

The liquid flow meter is calibrated separately for the gradations below 40 mm and those above 50 mm, in order to best fit the test points and avoid the flat mid-section. The lower range has an $R^2$ value of 0.9981 and the higher range has an $R^2$ value of 0.9979. There is also a ± 0.5 mm uncertainty in reading the flow meter. The result is an uncertainty in flow rate of ± 0.003 cc/min (8.6%) at the lowest used position of 11 mm, which becomes a ± 0.01 cc/min (0.82%) uncertainty at a position of 95 mm.

There is relatively little uncertainty in the measurement of the pressure drop. Assuming that there are no gravitational, acceleration, entrance, exit, or mass transfer effects, the pressure drop can be taken as the actual frictional pressure drop in the channel. However, the correlations being compared to the experimental data rely on the measured values of channel width, channel depth, air flow rate, water flow rate, and temperatures. Once the uncertainty in these measurement is determined, its propagation through those correlations must be evaluated to see if they make broader predictions based on those uncertainties. One way of doing this is to take the mean measured values and add or subtract the various uncertainties before calculating the models, to determine the “worst case scenarios” where there is the largest difference in prediction. One can see the considerable potential impact of the uncertainty by examining the single phase pressure drop equation. If one takes Eqs. (1) and (3) and re-writes them in terms of experimentally measured values one can get Eqs. (47) and (48).

\[
\Delta P = f \frac{Re}{12} \frac{L \mu Q (a + b)^2}{2a^3 b^3} \quad (47)
\]

\[
f \frac{Re}{12} = 24 \left( 1 - 1.3553 \left( \frac{a}{b} \right) + 1.9467 \left( \frac{a}{b} \right)^2 - 1.7012 \left( \frac{a}{b} \right)^3 + 0.9564 \left( \frac{a}{b} \right)^4 - 0.2537 \left( \frac{a}{b} \right)^5 \right) \quad (48)
\]

The value of $\mu$ is calculated from a linear approximation using temperature. The values of $L$ and $Q$ are measured quantities. Equation (47) is single order dependent on $\mu$, $L$, and $Q$, however one can see that the channel dimensions $(a,b)$ permeate the calculation, and even a slight change in these values can significantly impact the pressure drop prediction. Most of
the two phase prediction models rely on Eq. (47) in some way and the uncertainty in channel dimensions spreads uncertainty throughout the models. Therefore, even if a model does not match the data based upon the mean measured values, the data might be within its predicting range once uncertainties are considered, or visa-versa.

A depth micrometer is used to measure the channel depth and it is found to be 0.93 mm ± 0.02 mm. For the width measurement, a Mitutoyo “Height Master” gauge measurement device is used, and the width is found to be 1.124 mm ± 0.008 mm. Though not unmanageable, the uncertainty is undesirably high, however any more precise measurement would require destructive testing, and preservation of the test section is desired for future use. Therefore, the “worst case” diameters are used to calculate bounding predictions for the various models.

The milling machine used to drill the pressure taps is accurate to ±0.0508 mm, and the diameter of the pressure tap holes are 0.508 mm. Therefore, the uncertainty in length of the pressure drop section is ± 0.6 mm (0.34% of the length).

The contact angle is measured using the sessile drop test device and a digital camera. Pictures of the droplet on the surface are taken and the contact angle is measured off of printed copies of the pictures. There is uncertainty between the droplets, the pictures, and in the measurement of the angle. Therefore, for the most accurate measurement of contact angle multiple shots of multiple droplets are taken for each of the solutions. For any particular solution there are standard deviations of up to 7°, but taking 25-30 measurements of each reduces the uncertainty to ± 0.5° as seen in Eq. (46).
8.0 EXPERIMENTAL RESULTS

8.1 Single Phase Validation

![Graph showing comparison of friction factor with conventional theory and uncertainty in measured quantities.]

Fig. 7 Comparison of the single phase data for friction factor with conventional theory, including the uncertainty in measured quantities.

The experimental setup is validated by measuring the single phase pressure drop, using it to calculate a value for friction factor, $f$, and comparing the results to the predictions of conventional theory. Kakac et al. (1987) gave a constant value of $f_{Re}$ for laminar flow in rectangular channels that is used to calculate the theoretical friction factor and is represented by the solid line in Fig. 7, where the theoretical and experimental values are compared as a function of Reynolds number. There is an average 10.7% deviation between the experimental data and the mean theoretical prediction, however the experimental results rely on the measured channel geometry, temperatures, and flow rate. The uncertainty in these measurements is included in Fig. 7, where it can be seen that the experimental and theoretical results are in fair agreement and the test setup is considered to be performing as expected. It
is likely that any remaining discrepancy is due to slight imperfections in test section machining and minor fluctuations in channel size resulting from clamping the fourth channel wall to the other three. In any following plot of experimental two-phase data, the single phase data will be included as the value at a mass quality of one.

In order to be concise, the test runs are usually referred to here by the target nominal superficial air velocities of 4, 6, 8 and 10 m/s, however the actual superficial velocities are listed in Table 5.

Table 5 Actual mass fluxes and superficial velocities of the air flow during experimentation.

<table>
<thead>
<tr>
<th>Target Superficial Velocity (m/s)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Flux (kg/m²s), Gₘ</td>
<td>3.78</td>
<td>6.75</td>
<td>9.03</td>
<td>11.33</td>
</tr>
<tr>
<td>Actual Superficial Velocity (m/s), Jₘ</td>
<td>3.19</td>
<td>5.66</td>
<td>7.58</td>
<td>9.51</td>
</tr>
</tbody>
</table>

8.2 Surfactant Results

Fig. 8 Comparison of pressure drop data using surfactant solutions, as designated by concentration, Cₛ, for the case of a 6 m/s superficial air velocity.
Figure 8 plots the data collected using various surfactant solutions and compared to pure water for the 6 m/s superficial gas velocity case. Figure 9 is a similar plot for the 10 m/s superficial gas velocity case. Error bars are presented on the pure water data and show that any difference between the pure water and surfactant cases is within the experimental error. There is error along the x-axis due to uncertainty in the liquid flow rate measurement, however there is also some scatter in that direction due to the difficulty in reproducing exact liquid flow rate settings. Consideration of the error bars on the pressure drop reading is more important, and is primarily due to uncertainty in the channel dimensions and air flow rate, but also to slight fluctuations in the ambient temperature and air flow rate settings between runs. The error amounts to an average of less than ± 3%, and one can see that the surfactant data falls within this range. Furthermore, there is no pattern or progression in the surfactant pressure drop data that is surface tension (or otherwise) dependant. Therefore, either the experimental uncertainty must be reduced, or the pressure drop is not dependant on surface tension under the experimental conditions tested. The only data point that this does not hold true for is in the 10 m/s superficial gas velocity case at the lowest mass quality, which raises the question of whether lower mass qualities should be investigated with surfactant flow.

The actual experimental surfactant concentrations are 0.021, 0.037, 0.072, and 0.109% by weight, which yielded surface tensions and three-phase contact angles as listed in Table 6. The contact angle listed is the advancing contact angle, as for all cases the receding contact angle proved to be so low as to be impossible to measure with the available methods. Due to the low concentrations of surfactant required, it is difficult to get surface tensions between that of pure water and the minimum, however several different cases are still considered. Clearly, as the concentration of the surfactant increases the surface tension and contact angle decrease.

Table 6 Measured properties of the surfactant solutions, as designated by concentration, C_s, compared to those of pure water.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Water</th>
<th>0.021</th>
<th>0.037</th>
<th>0.072</th>
<th>0.109</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Tension (N/m)</td>
<td>0.073</td>
<td>0.048</td>
<td>0.041</td>
<td>0.035</td>
<td>0.034</td>
</tr>
<tr>
<td>Three-phase Contact Angle (deg.)</td>
<td>72</td>
<td>48</td>
<td>40</td>
<td>35</td>
<td>28</td>
</tr>
</tbody>
</table>
Fig. 9  Comparison of pressure drop data using surfactant solutions, as designated by concentration, $C_s$, for the case of 10 m/s superficial air velocity.
8.3 Pure Water Results

![Graph of pressure drop vs mass quality for pure water](image)

**Fig. 10** Two-phase pressure drop data as a function of mass quality for pure water at all air flow rates tested, $G = G_a$, $J = J_a$.

A total of 36 pure water data points were collected at the 4 selected air flow rates and at 9 typical water flow rates. The pressure drops recorded at these points (on a per unit length basis) are plotted in Fig. 10 as a function of the mass quality. The mass quality range is from 0.15 to 0.98 and is typical of the flow conditions in PEM fuel cells. The relationship between pressure drop and quality is almost linear, though it rises more rapidly in the low quality region. Clearly the pressure drop is primarily dependant on the air and water flow rates as is expected from the single phase theory.
Figure 11 plots the most relevant two-phase pressure drop prediction models using pure water over the full range of mass qualities. The Friedel (1980), Tran et al. (2000), and Chisholm B (1973) models offer such different predictions that they are not included. The model developed by Wambsganss et al. (1992) produced unfeasible results under the flow conditions considered. The particular sample case is at an air mass flux of 7.14 kg/sm², or nominally for a superficial air velocity of 6 m/s. The other cases show similar trends, though with higher and lower predictions, as the predicted pressure drops increase with air flow rate. The models exhibit similar behavior to each other and appear to predict similar values, however for mass qualities of over 0.5 one can see that there is as much as a 300% difference between the model’s predictions. The present work focuses on this high quality region, where the relationship between pressure drop and quality is relatively linear.
Figure 12 represents the actual data taken for the same case as in Fig. 11 and compares the closest predictions. None of the models accurately predict the experimental data for this case or any other tested. The Chen et al. (2002) and Chen et al. (2001) Friedel modification are consistently the closest to the data. Although they have low percent errors, it can be seen that the curves do not fit the trend of the experimental data well. The other models increase in accuracy at higher air flow rates, but the same is not true of the Chen et al. models. Furthermore, as scaling modifications of other models, they are intrinsically more complicated. The Homogenous Flow model is very close to the data in the highest mass quality region, but it quickly diverges at decreasing qualities. The Lockhart-Martinelli and Mishima-Hibiki models over predict the data significantly, though the Mishima-Hibiki more so than the Lockhart-Martinelli.
Fig. 13 Experimental data plotted with the two-phase pressure drop predictions of less relevant models for the case of a 6 m/s superficial air velocity.

The Friedel (1979), Tran et al. (2000), and Chisholm B (1973) correlations are not included in Fig. 12 as they over predict the data significantly and they are not included in future plots for the same reason. Their predictions are presented in Fig. 13.

Table 7 Absolute mean deviation of the most accurate two-phase pressure drop models when averaged over the experimental runs.

<table>
<thead>
<tr>
<th>Model</th>
<th>Lowest %</th>
<th>Highest %</th>
<th>Average %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lockhart-Martinelli (1949)</td>
<td>43.5</td>
<td>58.6</td>
<td>50.3</td>
</tr>
<tr>
<td>Homogenous Flow</td>
<td>19.70</td>
<td>55.9</td>
<td>29.8</td>
</tr>
<tr>
<td>Friedel (1979)</td>
<td>1602</td>
<td>1944</td>
<td>1746</td>
</tr>
<tr>
<td>Mishima-Hibki (1996)</td>
<td>53.2</td>
<td>72.2</td>
<td>61.4</td>
</tr>
<tr>
<td>Chen et al. (2002)</td>
<td>16.10</td>
<td>39.7</td>
<td>22.2</td>
</tr>
<tr>
<td>Chen et al. Homogenous (2001)</td>
<td>48.9</td>
<td>61.0</td>
<td>55.5</td>
</tr>
<tr>
<td>Chen et al. Friedel (2001)</td>
<td>22.5</td>
<td>27.6</td>
<td>24.3</td>
</tr>
</tbody>
</table>
The same trends of the 6 m/s superficial air velocity case presented in Fig. 12 are seen in the 10 m/s case presented in Fig. 14. Similar plots for the 4 and 8 m/s cases are included in appendix F. Ultimately, the mean absolute deviations between the predicted and experimental values are averaged over each run, and the minimum, maximum, and average of these averages are presented in Table 7. It is possible to have higher or lower deviations at any individual point during the run. The Chen et al. (2002) and Chen et al. (2001) Friedel modification are the most accurate models at 22.2% and 24.3% respectively, and clearly work much better than the large diameter correlations. The Homogenous Flow and Lockhart-Martinelli models follow at 29.8% and 50.3% respectively, making the Homogenous Flow model the most accurate of the large diameter correlations, and which agrees with the findings of both Chen et al. (2002) and Triplett et al. (1999). For two-phase flow, an adequate model is capable of predicting within ± 50% and a good model is within the range of ± 20%. However, this is not agreeable from an engineering perspective. Furthermore, although some of the models predicted within 20-30%, it can be seen that their predictions pass through the data and do not match its curve. Also, as the conditions are fairly narrowly focused, it is reasonable to attempt targeting a model more closely. Therefore, a new model is proposed in the discussion section.
Fig. 14 Experimental data plotted with the two-phase pressure drop predictions of the most relevant models for the case of a 10 m/s superficial air velocity.

8.4 High Speed Photography

An Olympus Encore PCI 8000s high speed digital video camera is used to capture images of the two-phase flow at a frame rate of 1,000 frames per second and with a shutter speed of 1/20,000 s. Bright halogen lights are required for enough illumination to achieve high quality images. Single frames of the video are not as informative as the video itself, but a collage of pure water flow frames is presented in Fig. 15. There is not much to observe during annular flow Fig. 15a, but with the video one is able to confirm that the flow is indeed annular, and has liquid film thicknesses on both the top and bottom of the channel. The liquid films are observed to both dynamically change thickness and transport small air bubbles, though typically they are even and concentric. It is also possible to achieve liquid flow in the corners, with dry areas along the walls Fig. 15f. The flow is sometimes restricted to the lower corners, but more often is in both the lower and upper corners. Also, there is sometimes a spray or mist deposited on the walls, and both wetting and drying of the wall are
observed. Figures 15e, and 15f show how the pure water has distinct lines separating the wetted portions of the wall and the dry portions.

![Images](a, b, c, d, e, f, g)

**Fig. 15** High speed photographs of pure water flow with (a) annular flow (b,c,d,g) induced plugs and slugs (e) wall wetting and (f) separated corner flow.

As interpretation of the annular flow videos is subjective and they show little of importance, plug and slug flows were induced for most of the frames in Fig. 15. Some of this is accomplished outside of the experimental range, and some is obtained by quickly changing one of the flow rates to induce transient situations. However, the images give insight into how surface tension is actually influencing the two-phase interface. The front of most all of the air bubbles is a clean bullet shape Fig. 15b, whereas the rear is less defined and often includes entrained bubbles Fig. 15g. One can see the influence of gravity in that many of the rear interfaces are angled, with slightly more water on the bottom than on top Figs. 15c, 15d.

Due to the optical change caused by surfactant addition, the photographs of the surfactant flows in Fig. 16 show less contrast and detail than those of pure water. Though it is not as clear in the still images, the videos make it clear that the surfactant solutions are more wetting than the pure water flow, which agrees with the lower surface tensions and
contact angles measured for the surfactant solutions. The water flow exhibits sharp lines along the two and three-phase interfaces Fig. 15e,f, whereas the surfactant solutions are less defined Fig. 16c. Typically, the air bubbles in the surfactant solutions have bullet shaped fronts and rears Fig. 16a, though sometimes with entrained bubbles Fig. 16d, but they show no sign of foaming.

Fig. 16 High speed photographs of the 0.109% surfactant solution flowing in the test section with induced plugs and slugs.
9.0 DISCUSSION AND ANALYSIS

9.1 Influence of Surfactant

The non-dimensional numbers discussed in the literature review section give insight into the two-phase flow conditions. Typical two-phase values for the Froude, Weber, Capillary, and Bond numbers under the present experimental conditions are given in Table 8. Only the cases of a 6 m/s and 10 m/s superficial air velocity are considered so that the pure water can be compared to the surfactant cases. All but the Bond number make the homogenous assumption for density and the Capillary number also makes it for viscosity. The Froude number shows that the inertial forces clearly dominate the gravitational in all cases and especially at higher air flow rates. The Weber number shows that for lower mass fluxes and pure water the surface tension dominates the inertial forces, but for most cases the inertial forces dominate the surface tension. The Capillary number shows that in all cases the surface tension force dominates that of viscosity. The Bond number shows that the surface tension force dominates that of buoyancy, though slightly less in the surfactant solutions.

Table 8 Average values of non-dimensional numbers for the pure water and characteristic surfactant solutions under the present conditions.

<table>
<thead>
<tr>
<th></th>
<th>Fr</th>
<th>We</th>
<th>Ca</th>
<th>Bo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Water</td>
<td>3206-9100</td>
<td>0.589-4.25</td>
<td>0.001-0.007</td>
<td>0.0353</td>
</tr>
<tr>
<td>0.021% Solution</td>
<td>3235-9054</td>
<td>0.901-5.98</td>
<td>0.002-0.0092</td>
<td>0.0527</td>
</tr>
<tr>
<td>0.109% Solution</td>
<td>3206-9055</td>
<td>1.252-8.60</td>
<td>0.003-0.013</td>
<td>0.0745</td>
</tr>
</tbody>
</table>

The fact that surface tension influences the flow characteristics of two-phase flow is supported by other research and is seen in its dominance over the buoyancy, viscosity, and gravitational effects. The objective was not to try and prove this, but to quantify the surface tension effects on pressure drop. The addition of surfactant proved inadequate in ascertaining this under the present test conditions. It is possible that the surface tension was not reduced enough for an observable change considering the experimental uncertainties. It is more likely that the impact of the inertial forces, as well as the annular flow conditions do
not lend themselves to the disclosure of the surface tension effects. In annular laminar flow, there is less of an interaction between the two phases than in bubble, plug, or slug flow, and therefore less opportunity for surface tension to exhibit itself. The effects might be more discernable at lower mass qualities or under turbulent flow conditions. Certainly, the fluid flow rates appear to contribute much more significantly to the pressure drop than surface tension. It is possible that the particular surfactant chosen was inappropriate, however a run with the surfactant EF-19 at a concentration of 0.097% by weight was performed and yielded similar results as those of Fig. 13. As surfactant behavior is known to be temperature dependant, and the ambient temperature is not precisely controllable, it is possible that slight deviations in the temperature impacted the results as well.

Fig. 17 Pure water experimental data plotted with the two-phase pressure drop predictions of the most relevant models for the case of a 6 m/s superficial air velocity and comparing predictions for a reduced surface tension.
It was expected that some of the models that incorporated surface tension into their calculations would predict the low surface tension data better. The Lockhart-Martinelli, Homogenous Flow, and Mishima-Hibki are the only models considered that do not take surface tension into account. However, as there was little difference between the surfactant and pure water data, the models that become more accurate with surface tension adjustment do so incidentally, as can be seen in Fig. 17. The Friedel (1979) and Tran et al. (2000) models also adjusted for surface tension, but were still very inaccurate. The results indicate that it is more appropriate for the models to not take surface tension into account, at least under the experimental conditions considered.

9.2 New Model Development

The Mishima-Hibiki model modifies the Lockhart-Martinelli model for flow in minichannels under the turbulent-turbulent flow condition, therefore in Figs. 13 and 14 the model is being applied outside of its intended range of operability. Indeed, it leads to a $C$ value of 5.74, very close to the Lockhart-Martinelli value of 5 though actually higher when it needs to be lower for minichannels, which causes over prediction. It is proposed that the following model is applicable to laminar-laminar flow in minichannels:

$$C = 5\left(1 - e^{-319D_h}\right)$$  \hspace{1cm} (49)

It includes Chisholm’s value for $C$ under laminar-laminar flow and Mishima-Hibiki’s channel diameter adjustment. This causes the value of $C$ to go to Chisholm’s value of $C$ for channels of hydraulic diameter greater than 12 mm. No experiments were performed with circular channels, but extension to circular geometries is reasonable as water fills the rectangular channel’s corners and they do not interact much with the flow:

$$C = 5\left(1 - e^{-333D}\right)$$  \hspace{1cm} (50)
Likewise, it is theorized that Eq. (51) is applicable for minichannels under any flow conditions, though no experiments were conducted in the laminar-turbulent and turbulent-laminar ranges:

\[ C^* = C \left( 1 - e^{-319D_n} \right) \]  
\[ \phi_G^2 = 1 + C^* X + X^2 \]  
\[ \Delta P_T = \phi_G^2 \Delta P_G \]

The modification results in a 2.5-4.5% deviation from the experimental data when averaged over the runs and with a greatest local deviation of 7.4%. This is a significant improvement over the other models, as can be seen in Table 9. The prediction is very accurate at high mass qualities, and the curve matches the data more closely than any other model.

**Table 9** Absolute mean deviation of the most accurate two-phase pressure drop models, including the proposed model, when averaged over the experimental runs.

<table>
<thead>
<tr>
<th>Model</th>
<th>Lowest %</th>
<th>Highest %</th>
<th>Average %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lockhart-Martinelli (1949)</td>
<td>43.5</td>
<td>58.6</td>
<td>50.3</td>
</tr>
<tr>
<td>Homogenous Flow</td>
<td>19.70</td>
<td>55.9</td>
<td>29.8</td>
</tr>
<tr>
<td>Friedel (1979)</td>
<td>1602</td>
<td>1944</td>
<td>1746</td>
</tr>
<tr>
<td>Mishima-Hibki (1996)</td>
<td>53.2</td>
<td>72.2</td>
<td>61.4</td>
</tr>
<tr>
<td>Chen et al. (2002)</td>
<td>16.10</td>
<td>39.7</td>
<td>22.2</td>
</tr>
<tr>
<td>Chen et al. Homogenous (2001)</td>
<td>48.9</td>
<td>61.0</td>
<td>55.5</td>
</tr>
<tr>
<td>Chen et al. Friedel (2001)</td>
<td>22.5</td>
<td>27.6</td>
<td>24.3</td>
</tr>
<tr>
<td>Proposed Model</td>
<td>2.4</td>
<td>4.5</td>
<td>3.3</td>
</tr>
</tbody>
</table>
Fig. 18 Experimental data plotted with the two-phase pressure drop predictions of the most relevant models for the case of a 6 m/s superficial air velocity and compared with the proposed model.

Figure 18 demonstrates how well the proposed model fits the data for a specific case, however the other cases produce very similar results, as demonstrated in Fig. 19. The deviation between the data and prediction is within the experimental uncertainty, particularly that inherent in the flow readings and the channel dimensions. It is recognized that very focused experimental conditions were used, however under similar conditions it is expected that the proposed model will accurately predict the pressure drop. Caution is recommended for different channel diameters or geometries, flow rates, qualities, or fluids.
Fig. 19 Comparison of pure water data with the proposed model’s predictions at different mass fluxes. The two are characterized by air mass flux \((G = G_a)\) and superficial velocity \((J = J_a)\).

### 9.3 Impact of Experimental Uncertainty

The uncertainty involved in the pressure drop measurement is small enough that if error bars of it were included in the various plots the points themselves would be larger than the error bars, therefore they have not been included. The uncertainty in the liquid flow rate measurement has very little impact on the pressure drop prediction models, however it has a significant impact on the calculated value of mass quality. Therefore, it will not be included in calculating the prediction models’ uncertainty, but as x-axis (mass quality) error bars on the data. The uncertainty in channel dimension measurement, as well as the air flow rate uncertainty, drastically influences the pressure drop prediction models and is included in the calculation of those models. Figure 20 plots the proposed model against the experimental data and includes the above mentioned uncertainties. The uncertainty in the prediction is
included as dotted lines indicating minimum and maximum potential predictions based on those uncertainties.

![Graph showing the comparison between the proposed model and the experimental data.](image)

**Fig. 20** Comparison of the proposed model with the experimental data and including the uncertainty in measurement.

The Chen et al. 2002 model is reasonably close to the experimental data if the uncertainty is considered, as can be seen in Fig. 21. The curve does not quite match the data, however a mean error of 23.7% is usually satisfactory for two-phase flow and therefore it is a viably acceptable model for two-phase pressure drop prediction under the conditions considered. The Chen et al. 2001 Friedel modification model is the second most accurate of the models tested, but the curve matches the data worse than the 2002 model. The uncertainty plot for the Friedel modification model, as well as for the other models, can be seen in appendix D. The uncertainty plot for the Homogenous Flow model can be seen in Fig. 22 as it is the most accurate of the large diameter prediction models. The model matches the data well in the very high mass quality region, but quickly diverges for lower mass qualities.
Fig. 21 Comparison of the Chen et al. (2002) model with the experimental data and including the uncertainty in measurement.

Fig. 22 Comparison of the Homogenous Flow model with the experimental data and including the uncertainty in measurement.
9.4 Data Repeatability

In order to verify that the preparation and assembly of the test section, or slight variations in ambient conditions are not significantly influencing the pressure drop data, the same experimental conditions are re-produced on two different days and compared. The test section is fully disassembled, cleaned and reassembled in between days. Figure 23 presents the data for both days, and one can see that there is only slight variation between them. Repeating exact mass qualities is not possible, but the values for pressure drop and the overall trends match within ±3.0%. This particular case is at a superficial gas velocity of 10 m/s and uses a surfactant solution of 0.072% concentration by weight.

Fig. 23 Pressure drop data taken under similar test conditions for repeatability comparison.
10.0 CONCLUSIONS

1. The test setup was validated by producing single phase data that agrees with the friction factor predicted by conventional theory to within an average of 3.3% once the experimental uncertainties are considered.

2. Adiabatic two-phase pressure drop data was collected for mass fluxes of 4.0-25 kg/m^2s and mass qualities of 0.15-0.98 for both pure water and reduced surface tension aqueous solutions.

3. The Chen et al. (2002) model and Chen et al. (2001) Friedel modification are the most accurate of the published models, having average mean deviations of 22.25 and 24.3% respectively.

4. The Homogenous Flow and Lockhart-Martinelli models are fairly accurate at 29.8% and 50.3% respectively, while the other models are inaccurate by 56-1700%.

5. A new model is proposed for the laminar-laminar flow case and predicts the pressure drop more accurately than the other models for the specified flow conditions, with an average mean deviation of 3.3%.

6. The addition of surfactant to the water to reduce its surface tension and three-phase contact angle produced no quantifiable changes to the pressure drop results.

7. High speed videos of the two-phase flows show that the surfactant did increase the wetting of the liquid flowing in the channel.

8. Application to PEM fuel cells was considered, and the air flow rates were found to be acceptable for reaching the desired annular flow, also the data collected provides a predictive base for pressure drop in PEM fuel cells.
11.0 SUGGESTIONS FOR FUTURE WORK

The surfactant experimental data proved inconclusive, however further exploration of the concept is worthwhile. Perhaps, different surfactant could be used to achieve lower surface tensions, but more importantly the range of experimental conditions could be expanded. To further understand the influence of surface tension, it would be worthwhile to repeat the present experiments at lower mass qualities, under more turbulent conditions, and/or at higher mass fluxes. One could also study contact angle and wetting effects separately by using different test section materials. Furthermore, one could include other fluids, such as refrigerants, oils, and fuels, or extend the research to even smaller Microchannels. If a greater focus on fuel cells is desired, then there is great room for expansion of the experiments. One could consider the multiple materials present, changing mass quality (mass transfer), channel bends, heat transfer, and multi-channel systems.

If further experimentation is performed, then it is recommended that a more refined setup and better equipment be used for ease of use and decreased uncertainty. The manufacturing process of the channel itself proved adequate for the present work, however it is recognized that the channel dimensions are critical to the flow calculations and even a small variance can have significant impact. Therefore, precision bored glass, or other tightly tolerated channels are recommended. At a minimum, an accurate means of determining the actual channel dimensions needs to be established. Glass is also recommended for high speed photographic investigations as the bright lights needed for the camera produces significant amounts of heat and tend to melt plastic test sections. For more control in the flow rate measurements, it is recommended that a venturi style flow meter or a positive displacement device such as a syringe pump be used. If a similar two piece channel is used, then a more refined clamping and gasket system is advised. It would be useful to have a data acquisition system capable of a higher collection rate, so that the instantaneous pressure signature can be collected and analyzed.
12.0 REFERENCES


APPENDIX A: SAMPLE CALCULATIONS

For insight into the analytical process and to verify the computer calculated results, the basic calculations used in the thesis are performed here by hand. The nominal values of the channel dimensions are used with fluid properties at a typical room temperature. The calculations initiate with consideration of a typical fuel cell operating under standard conditions, and the mass fluxes of the air and water are determined for a single minichannel within the flow field. Two and a half times the stoichiometric flow rate of air is used, as this is generally the lower limit of functional operation. The stoichiometric flow rate is the amount actually consumed by the chemical reaction. For the experiments, the equations are used in the same manner, but the mass fluxes are modified. Here, the given mass fluxes are used to calculate the single phase properties, such as quality and pressure drop. Some of the most accurate prediction models are then calculated for demonstration purposes.

Conditions:

Experimental Properties

\[ D_h = 1.0 \, mm = 1.0 \times 10^{-3} \, m \]
\[ a = 1.0 \, mm = 1.0 \times 10^{-3} \, m \]
\[ b = 1.0 \, mm = 1.0 \times 10^{-3} \, m \]
\[ L = 177.8 \, mm = 0.1778 \, m \]
\[ A_{cs} = (1.0 \, mm)^2 = 1.0 \times 10^{-6} \, m^2 \]

Physical Properties

\[ \rho_a = 1.177 \, \frac{kg}{m^3} \]
\[ \rho_w = 996.1 \, \frac{kg}{m^3} \]
\[ \mu_a = 1.8531 \times 10^{-5} \, \frac{Ns}{m} \left( \frac{kg}{m^2} \right) \]
\[ \mu_w = 8.328 \times 10^{-4} \, \frac{Ns}{m} \left( \frac{kg}{m^2} \right) \]

Fuel Cell Specific Equations:

\[ P_e = 1400 \, W \]
\[ V_c = 0.6 \, V \]
\[ A_a = 0.588 \, m^2 \]
\[ \lambda = 2.5 \]

\[ \dot{m}_w = 9.34 \times 10^{-8} \times \frac{P_e \, kg}{V_c \, s} = 9.34 \times 10^{-8} \times \frac{1400 \, kg}{0.6 \, s} = 2.179 \times 10^{-4} \, \frac{kg}{s} \]
\[
\dot{m}_a = 33.57 \times 10^{-7} \times \lambda \times \frac{P_k \ s}{V_c} = 33.57 \times 10^{-7} \times 2.5 \times \frac{1400 \ s}{0.6} \ s = 2.082 \times 10^{-3} \ s
\]

\[
\zeta = \frac{aL}{A_a/2} = \frac{(0.001 m)(0.1778 m)}{(0.588 m^2)/2} = 6.048 \times 10^{-4}
\]

\[
\dot{V}_w = \frac{m_w}{\rho_w \ zeta} = \frac{(2.179 \times 10^{-4} \ s)(6.048 \times 10^{-4})}{997 \ s/m^3} = 1.322 \times 10^{-10} \ m^3 = 7.93 \times 10^{-3} \ cm^3/\text{min}
\]

\[
\dot{V}_a = \frac{m_a}{\rho_a \ zeta} = \frac{(2.082 \times 10^{-3} \ s)(6.048 \times 10^{-4})}{1.161 \ s/m^3} = 1.085 \times 10^{-6} \ m^3 = 65.1 \ cm^3/\text{min}
\]

Single Phase Equations:

Air only:

\[
j_a = \frac{\dot{V}_a}{A_{cs}} = \frac{1.085 \times 10^{-6} \ m^3}{1.0 \times 10^{-6} \ m^2} = 1.085 \ m/\text{s}
\]

\[
G_a = \frac{\dot{V}_a \rho_a}{A_{cs}} = \frac{(1.085 \times 10^{-6} \ m^3)(1.161 \ s/m^3)}{1.0 \times 10^{-6} \ m^2} = 1.260 \ s/m\cdot s
\]

\[
Re_a = \frac{G_a D_h}{\mu_a} = \frac{(1.260 \ s/m\cdot s)(1.0 \times 10^{-3} \ m)}{1.846 \times 10^{-5} \ s/m} = 68.2
\]

\[L_{ea} \equiv 0.06 \ Re_a D_h = 0.06(68.2)(1.0 \times 10^{-3}) = 0.00409 \ m = 4.09 \ mm\]

\[f \ Re = 24(1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5)
\]

\[= 24(1 - 1.3553 + 1.9467 - 1.7012 + 0.9564 - 0.2537) = 14.23\]

\[\Delta P_a = f \ Re \frac{2L\mu_a G_a}{D_h^2 \rho_a} = 14.23 \frac{2 \times (0.1778 \ s)(1.846 \times 10^{-5} \ s/m)(1.260 \ s/m\cdot s)}{(1.0 \times 10^{-3} \ m)^2(1.161 \ s/m)} = 101.4 \ Pa\]

Water only:

\[
j_w = \frac{\dot{V}_w}{A_{cs}} = \frac{1.322 \times 10^{-10} \ m^3}{1.0 \times 10^{-6} \ m^2} = 1.322 \times 10^{-4} \ m/\text{s}
\]

\[
G_w = \frac{\dot{V}_w \rho_w}{A_{cs}} = \frac{(1.322 \times 10^{-10} \ m^3)(997 \ s/m^3)}{1.0 \times 10^{-6} \ m^2} = 0.1318 \ s/m\cdot s
\]
Reₜ = \frac{G_w D_h}{\mu_w} = \frac{(0.1318 \ \text{kg/m}^3) \times (1.0 \times 10^{-3} \text{m})}{8.55 \times 10^{-4} \ \text{kg/ms}} = 0.1541

L_{ew} = 0.06 \ \text{Re} \cdot D_h = 0.06(0.1541)(1.0 \times 10^{-3}) = 9.25 \times 10^{-6} \text{m} = 9.25 \times 10^{-3} \text{mm}

\Delta P_w = \frac{f \ \text{Re} \cdot 2L \mu_w G_w}{D_h^2 \rho_w} = 14.23 \times \frac{2 \times (0.1778 \text{m}) \times (8.55 \times 10^{-4} \ \text{kg/ms}) \times (0.1318 \ \text{kg/m}^3)}{(1.0 \times 10^{-3} \text{m})^2 \times (997 \ \text{kg/m}^3)} = 0.5719 \text{Pa}

x = \frac{G_a}{G_a + G_w} = \frac{1.260}{(1.260 + 0.1318)} = 0.905

Two-phase Equations:

Lockhart and Martinelli:

\[ C = 5 \]
\[ X^2 = \frac{0.5719}{101.4} = 5.64 \times 10^{-3} \]
\[ \phi^2_c = 1 + (5) \sqrt{5.64 \times 10^{-3} + 5.64 \times 10^{-3}} = 1.381 \]
\[ \Delta P_T = (1.381)(101.4 \text{Pa}) = 140.0 \text{Pa} \]

Homogenous Flow:

\[ \rho_T = \frac{x \rho_w + (1 - x) \rho_a}{x \rho_w + (1 - x) \rho_a} = \frac{(1.161 \ \text{kg/m}^3) \times (997 \ \text{kg/m}^3)}{(0.905) \times (997 \ \text{kg/m}^3) + (1 - 0.905) \times (1.161 \ \text{kg/m}^3)} = 1.283 \ \text{kg/m}^3 \]
\[ \mu_T = \frac{x \mu_w + (1 - x) \mu_a}{x \mu_w + (1 - x) \mu_a} = \frac{(1.846 \times 10^{-5} \ \text{Ns/m}^2) \times (8.55 \times 10^{-4} \ \text{Ns/m}^2)}{(0.905) \times (8.55 \times 10^{-4} \ \text{Ns/m}^2) + (1 - 0.905) \times (1.846 \times 10^{-5} \ \text{Ns/m}^2)} = 2.055 \times 10^{-5} \ \text{Ns/m}^2 \]
\[ \Delta P_T = f \ \text{Re} \cdot \frac{2L \mu_w G_T}{D_h^2 \rho_T} = 14.23 \times \frac{2 \times (0.1778 \text{m}) \times (2.055 \times 10^{-5} \ \text{kg/m}^3 \ \text{s}) \times (1.3918 \ \text{kg/m}^3)}{(1.0 \times 10^{-3} \text{m})^2 \times (1.283 \ \text{kg/m}^3)} = 111.7 \text{Pa} \]

Friedel:

\[ Fr_T = \frac{G_T^2}{g D_h \rho_T} = \frac{(1.260 + 0.1318)^2 \ \text{kg}^2 \ \text{m}^5 \ \text{s}^3}{(9.81 \ \text{m/s}^2) \times (1.0 \times 10^{-3} \text{m}) \times (1.283 \ \text{kg/m}^3)^2} = 120.0 \]
\[
We_T = \frac{G^2 D_h}{\rho_H \sigma} = \frac{(1.260 + 0.1318)^2 \frac{kg}{m^2} (1.0 \times 10^{-3} m)}{(1.283 \frac{kg}{m^2}) (71.7 \times 10^{-3} \frac{N}{m})} = 0.0210
\]

\[
f_{\text{Go}} = \frac{\text{Re}_{\text{Go}}}{\text{Re}_{\text{Lo}}}
\]

\[
\text{Re}_{\text{Lo}} = \frac{G_T D_h}{\mu_w} = \frac{(1.3918 \frac{kg}{m^2})(1.0 \times 10^{-3} m)}{8.55 \times 10^{-4} \frac{N s}{m^2}} = 1.628
\]

\[
\text{Re}_{\text{Go}} = \frac{G_T D_h}{\mu_a} = \frac{(1.3918 \frac{kg}{m^2})(1.0 \times 10^{-3} m)}{1.846 \times 10^{-5} \frac{N s}{m^2}} = 75.4
\]

\[
\phi_{\text{Lo}}^2 = (1 - x)^2 + x^2 \rho_L \frac{\text{Re}_{\text{Lo}}}{\rho_G \text{Re}_{\text{Go}}} + \frac{3.24 x^{0.78} (1 - x)^{0.224}}{Fr_x^{0.0454} We_T^{0.035} \left(\frac{\rho_L}{\rho_G}\right)^{0.91} \left(\frac{\mu_G}{\mu_L}\right)^{0.19} \left(1 - \frac{\mu_G}{\mu_L}\right)^{0.7}}
\]

\[
= (1 - 0.905)^2 + (0.905)^2 \left(\frac{997}{1.161}\right) \left(1.628\right) + \frac{3.24 (0.905)^{0.78} (1 - 0.905)^{0.224}}{(120.0)^{0.0454} (0.0210)^{0.035}} \left(\frac{997}{1.161}\right)^{0.91} \left(1.846 \times 10^{-5}\right)^{0.19} \left(8.55 \times 10^{-4}\right)^{0.7} = 377
\]

\[
\Delta P_{\text{Lo}} = f \text{Re} \frac{2L \mu_w G_T}{D_h \rho_w} = 14.23 \frac{2 (0.1778 m) (8.55 \times 10^{-4} \frac{kg}{m s}) (1.392 \frac{kg}{m^2})}{(1.0 \times 10^{-3} m)^2 (997 \frac{kg}{m^3})} = 6.04 \text{Pa}
\]

\[
\Delta P_T = (377)(6.04 \text{Pa}) = 2280 \text{Pa}
\]

**Mishima and Hibiki:**

\[
C = 21(1 - e^{-0.319(1)}) = 5.735
\]

\[
X^2 = \frac{0.572}{101.4} = 5.64 \times 10^{-3}
\]

\[
\phi_C^2 = 1 + C \sqrt{\frac{\Delta P_L}{\Delta P_G}} = 1 + 5.735 \sqrt{5.64 \times 10^{-3}} + 5.64 \times 10^{-3} = 1.436
\]

\[
\Delta P_T = (1.436)(101.4 \text{Pa}) = 145.6 \text{Pa}
\]

**Chen et al. (2002):** Note that the Bond number used by Chen is different than what is found elsewhere in literature. Chen uses half the hydraulic diameter as the characteristic length rather than the hydraulic diameter.
\[ Bo = \frac{(\rho_L - \rho_G)g\left(\frac{D_h}{2}\right)^2}{\sigma} = \frac{(997 - 1.161 \frac{kg}{m^3})(9.81 \frac{m}{s^2}) \left(\frac{1.0 \times 10^{-3} m}{2}\right)^2}{0.0717 \frac{N}{m}} = 0.0341 \]

\[ \Omega = \frac{0.85 - 0.082Bo^{-0.5}}{0.57 + 0.004Re^{0.5}_{Go} + 0.04Fr^{-1}} + \frac{80We^{-1.6} + 1.76Fr^{0.068} + \ln(Re_{Go}) - 3.34}{1 + e^{(8.5 - 1000\rho_a/\rho_w)}} \]

\[ = \frac{0.85 - 0.082(0.0341)^{-0.5}}{0.57 + 0.004(75.4)^{0.5} + 0.04(120.0)^{-1}} + \frac{80(0.021)^{-1.6} + 1.76(120.0)^{0.068} + \ln(75.4) - 3.34}{1 + e^{(8.5 - 1000\times1.161/997)}} = 25.9 \]

\[ \Delta P = \Omega \Delta P_h = (25.9)(111.7 \text{ Pa}) = 2890 \text{ Pa} \]

Chen et al. (2001) Homogenous Adaptation:

\[ \Omega_{\text{Hom}} = \begin{cases} 1 + \left(0.2 - 0.9e^{-Bo}\right) & \text{for } Bo < 2.5 \\ 1 + \left(We^{0.2}/e^{Bo^{0.1}}\right) - 0.9e^{-Bo} & \text{for } Bo \geq 2.5 \end{cases} \]

\[ = 1 + 0.2 - 0.9e^{-0.0341} = 0.3302 \]

\[ \Delta P = \Omega \Delta P_h = (0.3302)(111.7 \text{ Pa}) = 36.9 \text{ Pa} \]

Chen et al. (2001) Friedel Adaptation:

\[ \Omega = \begin{cases} \frac{0.0333Re^{0.45}_{Go}}{Re^{0.09}_Go(1 + 0.4e^{-Bo})} & \text{for } Bo < 2.5 \\ \frac{We^{0.2}}{(2.5 + 0.06Bo)} & \text{for } Bo \geq 2.5 \end{cases} \]

\[ = \frac{0.0333(1.628)^{0.45}}{(68.2)^{0.09}(1 + 0.4e^{-0.0341})} = 0.0204 \]

\[ \Delta P = \Omega \Delta P_h = (0.0204)(2280 \text{ Pa}) = 46.6 \text{ Pa} \]

Tran et al.:

\[ \Delta P_{Go} = f \text{Re} \left(\frac{2L\mu_G G_T}{D_h \rho_a}\right) = 14.23 \frac{2*(0.1778m)(1.864 \times 10^{-5} \frac{kg}{m s})(1.392 \frac{kg}{m^3})}{(1.0 \times 10^{-3} m)^2 (1.161 \frac{kg}{m^3})} = 113.1 \text{ Pa} \]

\[ \Gamma^2 = \frac{\Delta P_{Go}}{\Delta P_{Lo}} = \frac{113.1}{6.04} = 18.72 \]
\[ N_{\text{conf}} = \left( \frac{\sigma}{g(\rho_L - \rho_s)} \right)^{0.5} = \left( \frac{0.0717 \frac{N}{m}}{9.81 \frac{m}{s^2} \left(997 - 1.1614 \frac{kg}{m^3} \right)} \right)^{0.5} = 2.71 \]

\[ \Delta P = \left[ 1 + (4.3 \Gamma^2 - 1) N_{\text{conf}} x^{0.875} (1 - x^{0.875}) + x^{1.75} \right] \Delta P_{Lo} \\
= \left[ 1 + (4.3 \times 18.72 - 1)(2.71(0.905)^{0.875}(1 - 0.905)^{0.875} + 0.905^{1.75}) \right] (6.04 Pa) = 561 Pa \]
APPENDIX B: EXPERIMENTAL SETUP PICTURES

**Fig. 24** Photograph of assembled test section.

1. Inlet thermocouple
2. Water inlet
3. Air inlet
4. Inlet pressure transducer
5. Clamps
6. Differential pressure transducer
7. Holding bracket
8. Mutual outlet
9. Pressure taps
Fig. 25 Photograph of disassembled test section.

Fig. 26 Photographs of the Sessile drop test device used for the three-phase contact angle measurement.
Fig. 27 Screen-shot of the Labview virtual interface and data acquisition system.
APPENDIX C: EXPERIMENTAL SETUP
SPECIFICATION SHEETS

The following is a compilation of the specification sheets that accompany some of the lab equipment used in the experiments.
Fig. 28 Flow rate chart for the Fl-553 flow meter used to measure air flow rate.
GENERAL INFORMATION

OMEGA's Pressure sensors are four-active piezoresistive bridge devices. When pressure is applied, a different output voltage proportional to that pressure, is produced.

Wet/Wet Differential Pressure Sensors simultaneously accept independent pressure sources. Gage Pressure Sensors provide a form of differential pressure measurement in which atmospheric pressure is used as a reference.

The PX26 is available in variety of PSI ranges and as differential and gage sensors:

<table>
<thead>
<tr>
<th>MODEL NUMBER (GAGE)</th>
<th>RANGE</th>
<th>MODEL NUMBER (DIFFERENTIAL)</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PX26-001GV</td>
<td>0-1 PSIG</td>
<td>PX26-001 DV</td>
<td>0-1 PSID</td>
</tr>
<tr>
<td>PX26-005GV</td>
<td>0-5 PSIG</td>
<td>PX26-005DV</td>
<td>0-5 PSID</td>
</tr>
<tr>
<td>PX26-015GV</td>
<td>0-15 PSIG</td>
<td>PX26-015DV</td>
<td>0-15 PSID</td>
</tr>
<tr>
<td>PX26-030GV</td>
<td>0-30 PSIG</td>
<td>PX26-030DV</td>
<td>0-30 PSID</td>
</tr>
<tr>
<td>PX26-100GV</td>
<td>0-100 PSIG</td>
<td>PX26-100DV</td>
<td>0-100 PSID</td>
</tr>
<tr>
<td>PX26-250GV</td>
<td>0-250 PSIG</td>
<td>PX26-250DV</td>
<td>0-250 PSID</td>
</tr>
</tbody>
</table>

UNPACKING

Remove the packing list and verify that all equipment has been received. If there are any questions about the shipment, please call OMEGA Customer Service Department at 1-800-622-2358 or 203-359-1600. We can also be reached on the Internet at www.omega.com e-mail: info@omega.com.

Upon receipt of shipment, inspect the container and equipment for any signs of damage. Take particular note of any evidence of rough handling in transit. Immediately report any damage to the shipping agent.

The carrier will not honor any claims unless all shipping material is saved for their examination. After examining and removing contents, save packing material in event reshipment is necessary.

MEDIA COMPATIBILITY

Input media are limited to those media which will not attack polyester, flurosilicon, or silicon, such as oils, lacquer thinner, hydraulic fluid, most petroleum products, water and salt water. Not recommended for freons.

SOLDERING

Limit soldering temperature to 600°F (315°C) for 10 seconds duration maximum.

**WARNING! READ BEFORE INSTALLATION**

Fluid hammer and surges can destroy any pressure transducer and must always be avoided. A pressure snubber should be installed to eliminate the damaging hammer effects.

Fluid hammer occurs when a liquid flow is suddenly stopped, as with quick closing solenoid valves. Surges occur when flow is suddenly begun, as when a pump is turned on at full power or a valve is quickly opened.

Liquid surges are particularly damaging to transducers if pipe is original empty. To avoid damaging surges, fluid lines should remain full (if possible), pumps should be up to power slowly, and valves opened slowly. To avoid damage from both fluid hammer and surges, a surge chamber should be installed, and a pressure snubber should be installed on every transducer.

Symptoms of fluid hammer and surges damaging effects:
- a) Pressure transducer exhibits an output at zero pressure (large zero offset). If offset is less than 10% FS, user can usually re-zero meter, install proper snubber and continue monitoring pressures.
- b) Pressure transducer output remains constant regardless of pressure.
- c) In severe cases, there will be no output.

**ELECTRICAL CONNECTION**

Fig. 29 Specifications for the PX26 pressure transducers used in pressure measurements.
SPECIFICATIONS

EXCITATION:
OUTPUT:
10VDC, 16VDC max @ 2mA
100mV, 1.10mV (16.7mV, 1.67mV/V for 1 PSI range)
(50mV, 5mV/V for 5PSI range)

ACCUACY:
LINEARITY:
HYSERESIS & REPEATABILITY:
ZERO BALANCE:
SPAN TOLERANCE:
STORAGE TEMPERATURE:
OPERATING TEMPERATURE:
COMPENSATED TEMPERATURE:
THERMAL EFFECTS:
(25 to 50°C, 25 to 0°C)
PROOF PRESSURE:

INPUT RESISTANCE:
OUTPUT RESISTANCE:
RESPONSE TIME:
SHOCK:
VIBRATION:
GAGE TYPE:
WETTED PARTS:
MATING CONNECTOR:
PRESSURE PORT:
ELECTRICAL CONNECTION:

WEIGHT:

Fig. 30 Continued specifications for the PX26 pressure transducers used in pressure measurements.
APPENDIX D: UNCERTAINTY IN MEASUREMENT AND PREDICTING MODELS

The following charts compare all of the two-phase pressure drop prediction models with the experimental data and include bounding predictions that result from the experimental uncertainty. There are also calibration charts for the pressure transducers, thermocouples, and flow meters used.
Fig. 31 Calibration chart for the FL-120 flow meter used in liquid flow measurement.
Fig. 32 Calibration chart for the PX26-001GV pressure transducer used for differential pressure measurement along test section.
Fig. 33 Calibration chart used for the PX26-005GV pressure transducer used to measure inlet pressure.
Fig. 34 Calibration chart for the thermocouple used to measure inlet air temperature.
Ambient Temperature Thermocouple Calibration

Fig. 35 Calibration chart for the thermocouple used to measure ambient temperature.

\[ y = 1.0257x - 4.356 \]

\[ R^2 = 1 \]
APPENDIX E: EXPERIMENTAL DATA

The following calculations are performed using the mean values of $a = 1.124$ mm, $b = 0.930$ mm, $L = 177.8$ mm, and room temperature.

Table 9 Experimental data collected using pure water.

<table>
<thead>
<tr>
<th>J_G</th>
<th>G_G</th>
<th>G_L</th>
<th>Re_G</th>
<th>Re_L</th>
<th>Air Theoretical</th>
<th>Water Theoretical</th>
<th>Two-Phase Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>m/s</td>
<td>kg/m²s</td>
<td>kg/m²s</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
</tr>
<tr>
<td>3.212</td>
<td>3.804</td>
<td>0.666</td>
<td>203.320</td>
<td>0.774</td>
<td>1.693</td>
<td>0.016</td>
<td>2.109</td>
</tr>
<tr>
<td>3.212</td>
<td>3.804</td>
<td>0.923</td>
<td>203.320</td>
<td>1.073</td>
<td>1.693</td>
<td>0.022</td>
<td>2.124</td>
</tr>
<tr>
<td>3.212</td>
<td>3.804</td>
<td>1.602</td>
<td>203.320</td>
<td>1.861</td>
<td>1.693</td>
<td>0.039</td>
<td>2.168</td>
</tr>
<tr>
<td>3.189</td>
<td>3.777</td>
<td>2.130</td>
<td>201.870</td>
<td>2.475</td>
<td>1.681</td>
<td>0.052</td>
<td>2.217</td>
</tr>
<tr>
<td>3.122</td>
<td>3.804</td>
<td>3.016</td>
<td>203.320</td>
<td>3.504</td>
<td>1.693</td>
<td>0.073</td>
<td>2.302</td>
</tr>
<tr>
<td>3.122</td>
<td>3.804</td>
<td>4.360</td>
<td>203.320</td>
<td>5.065</td>
<td>1.693</td>
<td>0.106</td>
<td>2.400</td>
</tr>
<tr>
<td>3.189</td>
<td>3.777</td>
<td>6.168</td>
<td>201.870</td>
<td>7.165</td>
<td>1.681</td>
<td>0.150</td>
<td>2.502</td>
</tr>
<tr>
<td>3.189</td>
<td>3.777</td>
<td>10.624</td>
<td>201.870</td>
<td>12.341</td>
<td>1.681</td>
<td>0.259</td>
<td>2.734</td>
</tr>
<tr>
<td>3.189</td>
<td>3.777</td>
<td>18.217</td>
<td>201.870</td>
<td>21.161</td>
<td>1.681</td>
<td>0.443</td>
<td>2.992</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>0.577</td>
<td>374.403</td>
<td>0.642</td>
<td>2.873</td>
<td>0.015</td>
<td>3.303</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>1.231</td>
<td>374.403</td>
<td>1.368</td>
<td>2.873</td>
<td>0.031</td>
<td>3.385</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>1.483</td>
<td>374.403</td>
<td>1.649</td>
<td>2.873</td>
<td>0.038</td>
<td>3.404</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>2.167</td>
<td>374.403</td>
<td>2.409</td>
<td>2.873</td>
<td>0.055</td>
<td>3.506</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>3.018</td>
<td>374.403</td>
<td>3.354</td>
<td>2.873</td>
<td>0.077</td>
<td>3.600</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>4.421</td>
<td>374.403</td>
<td>4.914</td>
<td>2.873</td>
<td>0.112</td>
<td>3.730</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>6.545</td>
<td>374.403</td>
<td>7.274</td>
<td>2.873</td>
<td>0.166</td>
<td>3.872</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>10.868</td>
<td>374.403</td>
<td>12.079</td>
<td>2.873</td>
<td>0.276</td>
<td>4.186</td>
</tr>
<tr>
<td>5.658</td>
<td>6.748</td>
<td>18.225</td>
<td>374.403</td>
<td>20.256</td>
<td>2.873</td>
<td>0.463</td>
<td>4.498</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>0.469</td>
<td>500.890</td>
<td>0.522</td>
<td>3.854</td>
<td>0.012</td>
<td>4.371</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>0.998</td>
<td>500.890</td>
<td>1.112</td>
<td>3.854</td>
<td>0.025</td>
<td>4.427</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>1.483</td>
<td>500.890</td>
<td>1.653</td>
<td>3.854</td>
<td>0.038</td>
<td>4.476</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>2.278</td>
<td>500.890</td>
<td>2.539</td>
<td>3.854</td>
<td>0.058</td>
<td>4.554</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>3.297</td>
<td>500.890</td>
<td>3.674</td>
<td>3.854</td>
<td>0.084</td>
<td>4.694</td>
</tr>
<tr>
<td>7.559</td>
<td>9.005</td>
<td>4.602</td>
<td>499.250</td>
<td>5.129</td>
<td>3.841</td>
<td>0.117</td>
<td>4.758</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>6.276</td>
<td>500.890</td>
<td>6.995</td>
<td>3.854</td>
<td>0.159</td>
<td>4.897</td>
</tr>
<tr>
<td>7.584</td>
<td>9.035</td>
<td>11.110</td>
<td>500.890</td>
<td>12.383</td>
<td>3.854</td>
<td>0.282</td>
<td>5.259</td>
</tr>
<tr>
<td>7.559</td>
<td>9.005</td>
<td>18.335</td>
<td>499.250</td>
<td>20.436</td>
<td>3.841</td>
<td>0.465</td>
<td>5.796</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>0.622</td>
<td>628.387</td>
<td>0.696</td>
<td>4.835</td>
<td>0.016</td>
<td>5.384</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>0.998</td>
<td>628.387</td>
<td>1.117</td>
<td>4.835</td>
<td>0.025</td>
<td>5.450</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>1.603</td>
<td>628.387</td>
<td>1.794</td>
<td>4.835</td>
<td>0.040</td>
<td>5.513</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>2.204</td>
<td>628.387</td>
<td>2.467</td>
<td>4.835</td>
<td>0.056</td>
<td>5.621</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>3.017</td>
<td>628.387</td>
<td>3.377</td>
<td>4.835</td>
<td>0.076</td>
<td>5.725</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>4.245</td>
<td>628.387</td>
<td>4.752</td>
<td>4.835</td>
<td>0.107</td>
<td>5.828</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>5.767</td>
<td>628.387</td>
<td>6.455</td>
<td>4.835</td>
<td>0.146</td>
<td>6.010</td>
</tr>
<tr>
<td>9.514</td>
<td>11.334</td>
<td>20.048</td>
<td>628.387</td>
<td>22.440</td>
<td>4.835</td>
<td>0.506</td>
<td>6.990</td>
</tr>
</tbody>
</table>
Table 10 Experimental data collected using surfactant solutions at a nominal superficial gas velocity of 6 m/s.

<table>
<thead>
<tr>
<th>( J_G )</th>
<th>( G_G )</th>
<th>( G_L )</th>
<th>( Re_G )</th>
<th>( Re_L )</th>
<th>Air Theoretical</th>
<th>Water Theoretical</th>
<th>Two-Phase Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>m/s</td>
<td>kg/m²s</td>
<td>kg/m²s</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0.0208% Concentration by weight

| 5.684 | 6.764 | 0.712 | 374.744 | 0.801 | 2.890 | 0.018 | 3.324 |
| 5.684 | 6.764 | 1.257 | 374.744 | 1.416 | 2.890 | 0.032 | 3.406 |
| 5.684 | 6.764 | 1.695 | 374.744 | 1.909 | 2.890 | 0.043 | 3.437 |
| 5.684 | 6.764 | 2.315 | 374.744 | 2.606 | 2.890 | 0.058 | 3.506 |
| 5.684 | 6.764 | 2.713 | 374.744 | 3.054 | 2.890 | 0.068 | 3.552 |
| 5.684 | 6.764 | 4.075 | 374.744 | 4.587 | 2.890 | 0.102 | 3.713 |
| 5.684 | 6.764 | 4.788 | 374.744 | 5.391 | 2.890 | 0.120 | 3.770 |
| 5.709 | 6.795 | 7.112 | 376.428 | 8.008 | 2.903 | 0.179 | 3.928 |
| 5.684 | 6.764 | 11.355 | 374.744 | 12.784 | 2.890 | 0.285 | 4.189 |
| 5.709 | 6.795 | 18.334 | 376.428 | 20.641 | 2.903 | 0.460 | 4.495 |

0.0369% Concentration by weight

| 5.582 | 6.654 | 0.469 | 369.111 | 0.522 | 2.835 | 0.012 | 3.236 |
| 5.582 | 6.654 | 1.177 | 369.111 | 1.311 | 2.835 | 0.030 | 3.357 |
| 5.582 | 6.654 | 1.759 | 369.111 | 1.958 | 2.835 | 0.045 | 3.401 |
| 5.582 | 6.654 | 2.315 | 369.111 | 2.578 | 2.835 | 0.059 | 3.458 |
| 5.582 | 6.654 | 3.018 | 369.111 | 3.360 | 2.835 | 0.077 | 3.556 |
| 5.582 | 6.654 | 4.075 | 369.111 | 4.538 | 2.835 | 0.103 | 3.608 |
| 5.582 | 6.654 | 6.276 | 369.111 | 6.988 | 2.835 | 0.159 | 3.815 |
| 5.582 | 6.654 | 10.238 | 369.111 | 11.401 | 2.835 | 0.260 | 4.081 |
| 5.582 | 6.654 | 18.559 | 369.111 | 20.666 | 2.835 | 0.471 | 4.446 |

0.0719% Concentration by weight

| 5.607 | 6.679 | 0.804 | 370.246 | 0.899 | 2.850 | 0.020 | 3.306 |
| 5.607 | 6.679 | 1.048 | 370.246 | 1.171 | 2.850 | 0.026 | 3.381 |
| 5.607 | 6.679 | 1.603 | 370.246 | 1.792 | 2.850 | 0.041 | 3.430 |
| 5.607 | 6.679 | 2.131 | 370.246 | 2.383 | 2.850 | 0.054 | 3.483 |
| 5.607 | 6.679 | 3.017 | 370.246 | 3.373 | 2.850 | 0.076 | 3.599 |
| 5.607 | 6.679 | 4.602 | 370.246 | 5.145 | 2.850 | 0.116 | 3.666 |
| 5.607 | 6.679 | 6.824 | 370.246 | 7.625 | 2.850 | 0.172 | 3.820 |
| 5.607 | 6.679 | 10.008 | 370.246 | 11.189 | 2.850 | 0.253 | 4.097 |
| 5.607 | 6.679 | 18.224 | 370.246 | 20.374 | 2.850 | 0.461 | 4.470 |

0.1089% Concentration by weight

| 5.658 | 6.748 | 0.644 | 374.403 | 0.716 | 2.873 | 0.016 | 3.310 |
| 5.658 | 6.717 | 1.099 | 372.714 | 1.221 | 2.860 | 0.028 | 3.398 |
| 5.658 | 6.748 | 1.759 | 374.403 | 1.955 | 2.873 | 0.045 | 3.442 |
| 5.633 | 6.717 | 2.278 | 372.714 | 2.532 | 2.860 | 0.058 | 3.531 |
| 5.633 | 6.717 | 4.726 | 372.714 | 5.252 | 2.860 | 0.120 | 3.691 |
| 5.658 | 6.748 | 6.545 | 374.403 | 7.274 | 2.873 | 0.166 | 3.827 |
| 5.633 | 6.717 | 10.708 | 372.714 | 11.901 | 2.860 | 0.272 | 4.148 |
| 5.633 | 6.717 | 18.225 | 372.714 | 20.256 | 2.860 | 0.463 | 4.463 |
Table 11 Experimental data collected using surfactant solutions at a nominal superficial gas velocity of 10 m/s.

<table>
<thead>
<tr>
<th>J_G</th>
<th>G_G</th>
<th>G_L</th>
<th>Re_G</th>
<th>Re_L</th>
<th>Air Theoretical</th>
<th>Water Theoretical</th>
<th>Two-Phase Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>m/s</td>
<td>kg/m²</td>
<td>kg/m²</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
<td>kPa/m</td>
</tr>
<tr>
<td>0.0208% Concentration by weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.538</td>
<td>11.355</td>
<td>0.622</td>
<td>629.207</td>
<td>0.698</td>
<td>4.849</td>
<td>2.781</td>
<td>5.407</td>
</tr>
<tr>
<td>9.561</td>
<td>11.382</td>
<td>1.048</td>
<td>630.756</td>
<td>1.177</td>
<td>4.861</td>
<td>4.687</td>
<td>5.479</td>
</tr>
<tr>
<td>9.561</td>
<td>11.382</td>
<td>1.856</td>
<td>630.756</td>
<td>2.084</td>
<td>4.861</td>
<td>8.304</td>
<td>5.538</td>
</tr>
<tr>
<td>9.561</td>
<td>11.382</td>
<td>2.277</td>
<td>630.756</td>
<td>2.558</td>
<td>4.861</td>
<td>10.189</td>
<td>5.593</td>
</tr>
<tr>
<td>9.491</td>
<td>11.299</td>
<td>0.018</td>
<td>626.107</td>
<td>20.589</td>
<td>4.825</td>
<td>82.022</td>
<td>6.769</td>
</tr>
<tr>
<td>0.0369% Concentration by weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.491</td>
<td>11.308</td>
<td>0.758</td>
<td>627.043</td>
<td>0.846</td>
<td>4.822</td>
<td>3.411</td>
<td>5.362</td>
</tr>
<tr>
<td>9.444</td>
<td>11.252</td>
<td>0.948</td>
<td>623.935</td>
<td>1.058</td>
<td>4.798</td>
<td>4.268</td>
<td>5.394</td>
</tr>
<tr>
<td>9.491</td>
<td>11.308</td>
<td>1.603</td>
<td>627.043</td>
<td>1.788</td>
<td>4.822</td>
<td>7.215</td>
<td>5.494</td>
</tr>
<tr>
<td>9.467</td>
<td>11.280</td>
<td>2.391</td>
<td>625.489</td>
<td>2.669</td>
<td>4.810</td>
<td>10.766</td>
<td>5.597</td>
</tr>
<tr>
<td>9.467</td>
<td>11.280</td>
<td>4.075</td>
<td>625.489</td>
<td>4.547</td>
<td>4.810</td>
<td>18.345</td>
<td>5.749</td>
</tr>
<tr>
<td>0.0719% Concentration by weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.538</td>
<td>11.358</td>
<td>0.712</td>
<td>629.556</td>
<td>0.798</td>
<td>4.848</td>
<td>3.188</td>
<td>5.445</td>
</tr>
<tr>
<td>9.538</td>
<td>11.358</td>
<td>0.998</td>
<td>629.556</td>
<td>1.119</td>
<td>4.848</td>
<td>4.468</td>
<td>5.487</td>
</tr>
<tr>
<td>9.538</td>
<td>11.358</td>
<td>1.572</td>
<td>629.556</td>
<td>1.764</td>
<td>4.848</td>
<td>7.042</td>
<td>5.551</td>
</tr>
<tr>
<td>9.538</td>
<td>11.358</td>
<td>2.204</td>
<td>629.556</td>
<td>2.472</td>
<td>4.848</td>
<td>9.870</td>
<td>5.647</td>
</tr>
<tr>
<td>0.1089% Concentration by weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.538</td>
<td>11.374</td>
<td>0.712</td>
<td>631.089</td>
<td>0.789</td>
<td>4.843</td>
<td>54.750</td>
<td>5.410</td>
</tr>
<tr>
<td>9.538</td>
<td>11.374</td>
<td>1.204</td>
<td>631.089</td>
<td>1.335</td>
<td>4.843</td>
<td>56.979</td>
<td>5.482</td>
</tr>
<tr>
<td>9.538</td>
<td>11.374</td>
<td>2.167</td>
<td>631.089</td>
<td>2.403</td>
<td>4.843</td>
<td>61.345</td>
<td>5.595</td>
</tr>
<tr>
<td>9.538</td>
<td>11.374</td>
<td>4.916</td>
<td>631.089</td>
<td>5.451</td>
<td>4.843</td>
<td>73.796</td>
<td>5.806</td>
</tr>
</tbody>
</table>
APPENDIX F: EXTENDED RESULTS INFORMATION

The following plots compare the two-phase pressure drop prediction models with the pure water experimental data for the 4 m/s and 8 m/s nominal superficial gas velocity cases respectively.
The following plots compare the two-phase pressure drop prediction models with the surfactant experimental data as designated by concentration for 6 m/s nominal superficial gas velocity case.
The following plots compare the two-phase pressure drop prediction models with the surfactant experimental data as designated by concentration for the 10 m/s nominal superficial gas velocity case.