Image Processing Techniques to Separate Linear and Curvilinear Features in Textures

Bhavani Pidaparthi

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IMAGE PROCESSING TECHNIQUES TO SEPARATE LINEAR
AND CURVILINEAR FEATURES IN TEXTURES

By

BHAVANI PIDAPARTHI

Thesis submitted to the Faculty of Rochester Institute of technology in partial fulfillment of the
requirements for the degree of

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Approved by

Thesis Advisor Dr. Raghuveer M. Rao

Thesis Committee Dr. Peter Paul

Thesis Committee Dr. Daniel B. Phillips

Thesis Committee Dr. Harvey E. Rhody

Department Head Dr. Robert J. Bowman

DEPARTMENT OF ELECTRICAL ENGINEERING, COLLEGE OF ENGINEERING
ROCHESTER INSTITUTE OF TECHNOLOGY, ROCHESTER, NEW YORK

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Author: ___________________________
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IMAGE PROCESSING TECHNIQUES TO SEPARATE LINEAR AND CURVILINEAR FEATURES IN TEXTURES

By

BHAVANI PIDAPARTHI
Master of Science in Electrical Engineering

Abstract

Varied image processing techniques have been developed to extract or detect linear features from images. However, these techniques are targeted at extracting or detecting linear features, and it has been shown in an existing technique that the Fourier transform can be used in conjunction with the polar transformation to essentially lift or separate linear features from the background image. Extracting or detecting linear features in images involves locating these features in the image while separating or lifting them involves separating them from the background image such that we get two images: one image containing the linear features and the other image containing the background. This thesis presents approaches to separate linear and curvilinear features from textured backgrounds. The problem of separating linear features from a textured background is of importance in applications such as lithography, layout design and pattern recognition.

The existing Fourier transform based approach of linear feature separation effectively separates randomly located lines that are spread throughout the entire image and is found to be ineffective when the linear features are of varied lengths and thickness. This thesis presents an approach to overcome this limitation of the Fourier transform based approach. This thesis presents two new window based techniques relying on the Fourier transform and the wavelet transform to lift randomly located lines of varied
lengths and thickness. The proposed techniques are built upon the existing Fourier transform approach. The performances of the proposed techniques are compared to the Fourier Transform approach through application to several images. It is observed that the proposed Fourier based block approach and wavelet based block approach consistently perform better than the existing approach. It is also observed that the proposed techniques effectively lift curvilinear features from textures too. The mathematical analysis and experimental results verifying this claim are presented.
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1. Introduction

1.1 Background

An important approach to region description is to quantify its texture content. Texture analysis is complicated by the fact that both the patterns and periodic repetition may show significant random fluctuation. Texture is an important aspect in image analysis as it involves a measure of both the spectral and spatial variation in the scene. Although no formal definition of texture exists, intuitively this descriptor provides measures of properties such as smoothness, coarseness and regularity. The three principal approaches used in image processing to describe texture of a region are statistical, structural and spectral [10, 13, 14, 23, 29-34]. Statistical approaches yield characterizations of textures as smooth, coarse, grainy and so on. Structural techniques deal with the arrangement of image primitives such as the description of texture based on regularly spaced parallel lines. The primitive of image texture is a collection of pixels, which share a common property and are geometrically connected. Spectral techniques are based on properties of the Fourier spectrum and are used primarily to detect global periodicity in an image by identifying high energy, narrow peaks in the spectrum. This thesis presents a spectral image processing technique to separate linear features in textured images.

Processing of linear features in textured images has been addressed using various approaches. These approaches are useful in detecting, extracting and separating linear features from images. They are discussed in detail in the next chapter. These techniques are aimed at detecting or extracting linear features in images and not targeted at lifting or separating the linear features from images. Beyerer and Leon [1] propose an elegant
image processing technique to separate linear features from a texture to give two images: one containing the linear features and the other containing the textured background. Separating linear features from textured backgrounds offers potential applications in lithography, layout design and pattern recognition. This thesis presents image processing techniques to separate linear and curvilinear features from textured backgrounds and builds upon the Beyerer - Leon approach.

1.2 Objectives and Contributions

The objective of this research is to develop an image processing technique to separate linear features from textured backgrounds. The proposed technique uses the wavelet transform and the Fourier transform for image analysis. It is an extension of the algorithm proposed by Beyerer and Leon [1] to separate linear features in textured backgrounds.

1.2.1 Tasks involved

The tasks involved in this research are stated below.

- Identifying problems in the Beyerer – Leon approach.
- The algorithm suggested by Beyerer and Leon [1] was studied in detail and it was implemented.
- The limitations of the algorithm as proposed by Beyerer and Leon [1] are presented.
- The algorithm proposed in [1] is modified so as to overcome the limitations.
- A mathematical model verifying the observations is presented.
- A comparison of the performances of the proposed technique of linear feature separation and the existing technique is presented.
1.2.2 Contributions

The algorithm as proposed by Beyerer and Leon [1] effectively separates linear features that extend throughout the entire image. However, it is ineffective in extracting linear features of varied lengths and thickness. In that aspect the major contributions of this research include the following.

a) The proposed approach overcomes the limitations of the algorithm proposed by Beyerer and Leon [1] and effectively separates linear and curvilinear features of varied lengths and thickness.

b) The proposed approach is a modified form of the algorithm as proposed by Beyerer and Leon [1].

c) The proposed approach is tested on different kinds of images and results indicate that the proposed algorithm performs better than the existing technique proposed in [1].

d) A mathematical analysis verifying these observations is presented.

1.3 Outline of the Thesis

The following chapters describe the existing techniques of linear feature separation, the Beyerer - Leon approach to linear feature separation, the proposed approaches to separate linear features and the results of the three techniques of linear feature separation.

Chapter 2 contains a description of the existing techniques of linear feature extraction and detection. It also presents a detailed account of the technique of linear feature separation proposed by Beyerer and Leon [1]. The signal model and the Beyerer-Leon algorithm are also presented.
Chapter 3 gives a detailed account of the limitations of the Beyerer Leon approach. The proposed techniques are presented so as to overcome these limitations. A mathematical analysis is also presented.

Chapter 4 presents concluding remarks and thoughts on possible future work.
2. Linear Feature Separation in Textures

2.1 Introduction

This chapter presents different techniques of linear feature extraction. It also discusses in detail, the technique of linear feature separation from textured backgrounds as proposed by Beyerer and Leon [1]. The signal model and algorithm of the Beyerer Leon approach are also presented.

2.2 Background

A linear feature in an image is a feature which can be described or approximated by a straight line. Varied techniques to extract or detect linear features in images have been developed. The technique proposed by Nevatia and Price [3], describes an approach to extract linear features in aerial images. They present a segmentation based approach to extract linear features in aerial images. They use both edge and region segmentation techniques as some types of features are more readily detected by one than the other. Image descriptions consist of simple properties of the extracted regions and lines and also their geometrical relations to each other. An edge detector developed by Nevatia and Babu [4] and a region segmenter developed by Ohlander, Price and Reddy [5] are used to achieve edge and region segmentation. After edge and region segmentation, this technique uses features to isolate different regions in the image after the segmentation is done and linear features are extracted. Tupin et al, [6] present a technique to detect linear features in SAR images. The first part of their algorithm performs a local detection of linear structures. It is based on the fusion results from two line detectors. The responses from the two detectors are merged to obtain a unique response. Then segments,
corresponding to true roads, are connected. This method of connecting segments relies on a Markov Random Field based model of roads. Hence, linear features such as roads can be detected in SAR images. Zhou and Chellappa [7], present a linear feature extractor, which operates on the edge pixels using a 2-D autoregressive model. Once the linear features have been detected using the edge detector a segment finder is used to trace the edge points and link the lines. Hou and Bamberger [8] propose a structural approach for line detection based on the Hough transform. This approach consists of computing the Hough Transform of the texture and extracting features useful for classification. The Hough Transform maps lines in the image into points in the transform domain. Campos and Kasparis [9] propose a new morphological operator for linear feature enhancement, which is based on a preprocessing orientation analysis stage. In this technique a preprocessing stage returns the dominant orientation of local features. A feature enhancement algorithm is then applied to the linear features based on the local orientation information by suppressing components not in the dominant orientation so that linear features at different orientations can be detected.

The techniques presented so far are useful in extracting or detecting linear features in images. However, Beyerer and Leon [1] present an elegant image processing technique to separate or lift linear features from textured backgrounds. Extracting or detecting linear features in images involves locating these features in the image while separating or lifting linear features involves separating them from the background image such that we get two images: one image containing the linear features and the other image containing the background. The Beyerer - Leon approach uses the Fourier Trans-
form for image analysis and is discussed in detail in the next section.

2.3 Overview of the Fourier Transform Technique Of Line Separation in Textures

This section describes an image processing method to separate randomly located straight line-like structures from an isotropic background texture. The lines are assumed to extend across the entire image, and the isotropy of the background means that the corresponding Fourier spectrum is approximately symmetric with respect to rotation so that on spatial average, the background does not contain predominant directional structures.

The separation of linear features can be achieved by filtering out the spectral components typical for line-like structures with heuristically defined linear filters. In the simplest case, the transfer functions of such filters are constructed using elementary formed regions as pass-bands and stop bands. This procedure yields good results, if the line-like structures are periodic and therefore well concentrated in the Fourier domain. However, if there are many lines at rather different angles, as on honed surfaces, the heuristic approach performs poorly. The reason for this is that the union of all the regions to be suppressed in the Fourier domain occupies a large area, and therefore many spectral components belonging to the background signal are also eliminated. Moreover, if the angles of the lines are unknown \textit{a priori}, it is necessary to measure them to arrange an adequate transfer function based on a heuristically found filter for line structures of one known direction [1, 2, 17-20].

The method of linear feature separation presented in this section enables a separation of the two image components by exploiting the spectral isotropy of the background and
the anisotropy of the line-like structures. For each image an individual binary Discrete Fourier transform transfer function is generated. The pass-bands and stop bands are defined by deciding whether a spectral component contributes more to the background or more to the line-like structures. The decision is based on filtering the magnitude of the DFT by means of a transform sequence that maps both signal components onto two nearly disjoint regions, The geometric constellation of the resulting regions is essentially independent of the parameters of the line-like structures and the background. In particular, no measurement of line angles is necessary, and the pass-bands and stop bands of the transfer function are defined in a point-by-point manner that is not arbitrary. The adaptive definition of a binary transfer function for each individual image can be regarded as an adaptive splitting of the DFT into two complementary spectra with disjoint support, where one is assigned to the background and the other to the line-like structures. The theoretical foundation of this technique is given in the next section.

2.3.1 Signal Model

Consider an image consisting of straight lines in a textured background such as the one shown in Figure 1. Notice that the lines are spread throughout the image.

![Figure 2.1 Texture with lines extending throughout the image](image)

Suppose we are interested in separating the image in Figure 2.1 into the line and
background textures. Following [1], the image could be mathematically represented as

\[ g(x) = \lambda(x^T n - d) + b(x) \]  

(1)

where \( g(x) \) is the texture with linear features, \( b(x) \) is the background texture, \( \lambda(.) \) is the cross profile of the line, \( \alpha \) is the line angle, \( n = (\cos \alpha, \sin \alpha)^T \) is the normal vector of the line, \( p = (-\sin \alpha, \cos \alpha)^T \) is a unit vector parallel to the line, \( x = (x_1, x_2)^T \in \mathbb{R}^2 \) is the location vector, and \( d \) is the distance between the line and the origin of the \( x_1, x_2 \)-domain. The isotropy of \( b(x) \) means that the magnitude of the corresponding Fourier transform is symmetric with respect to rotation as given in Eq. 2

\[ B(f) = \Im \{b(x)\} \]

\[ = B(||f||) \]  

(2)

where, \( ||f|| \) denotes the Euclidean norm of the spatial frequency vector \( f = (f_1, f_2) \in \mathbb{R}^2 \) and \( B(.) \) the radial profile of \( B(f) \).

The Fourier transform of \( g(x) \) is given as,

\[ \Im \{g(x)\} = G(f) \]

\[ = \Lambda(f^T n) \delta(f^T p) \times \exp(-j2\pi d f^T n) + B(||f||) \]  

(3)

where,

\[ \Lambda(\eta) = \int \lambda(\xi) \exp(-j2\pi \xi \eta) d\xi \]

\[ \eta \in \mathbb{R} \]

Since, the \( \delta \)-function dominates \( G(f) \) for \( f^T p = 0 \) and is zero elsewhere, the magnitude of Eq.3 is

\[ |G(f)| = |\Lambda(f^T n)\delta(f^T p)| + |B(||f||)| \]

(4)
The removal of the phase eliminates the dependence on the distance \( d \). The background \( b(x) \) is transformed into a rotationally symmetrical component centered around the origin and \( \lambda(x^Tn-d) \) is concentrated on a straight line with normal vector \( p \) through the origin of the \( f_1,f_2 \) domain.

The polar co-ordinates can be given as,

\[
f = \rho (\cos \varphi, \sin \varphi)^T
\]

where \( \varphi \in [0, \pi) \) is the angle of the spatial frequency vector and \( \rho \in \mathbb{R} \) is the signed, radial frequency co-ordinate.

Given,

\[
\delta(f^T \rho) = \delta(\varphi-\alpha)/|\rho|
\]

Then,

\[
|G(\rho \cos \varphi, \rho \sin \varphi)| = \left| \Lambda(\rho) \right| \left| \frac{\delta(\varphi-\alpha)}{|\rho|} \right| + |B(\rho)|
\]

Multiplying Eq.6 by \( |\rho| \) and taking the Fourier transform on both sides.

\[
\mathbb{F}_{\rho, \varphi}[|G(\rho \cos \varphi, \rho \sin \varphi)|] = \mathbb{F}_{\rho, \varphi}[\Lambda(\rho)] \sum_{k=-\infty}^{\infty} \delta(f_\rho - \frac{k}{\pi}) \exp(-j2\pi f_\varphi \alpha)
\]

\[
+ \mathbb{F}_{\rho, \varphi}[|B(\rho)|] \delta(f_\varphi)
\]

Hence, linear features are mapped to vertical \( \delta \)-lines in the \( \rho, \varphi \) domain and will be concentrated around the \( f_\varphi \) axis in the \( f_\rho, f_\varphi \) domain. These delta functions are suppressed by applying a mask filter to bring about separation of the line and background textures.
2.3.2 Algorithm

A Fourier transform is first performed on the input image. Then, a rectangular to polar co-ordinate transform is performed. To keep the algorithm independent of the knowledge of $|\Lambda(\rho)|$, its dynamics are attenuated by applying a logarithm. The value of 1 is added before this to avoid the singularity of the logarithm at zero. A Fourier transform is performed on the resulting image, such that all lines are transformed to delta functions along the x-axis. The transformed image is then mask filtered to suppress the delta-functions. The filtered image is then passed through a sequence of transformations: the inverse Fourier transform, an exponential transform and a co-ordinate transform. The resultant spectrum, $B'(f)$ is an estimate of the background spectrum and is used to separate the spectrum of the input image into line and background spectrums.

The ratio $|G(f)| / |B'(f)|$ is compared to a threshold $\gamma$ for each discrete frequency. If the ratio exceeds $\gamma$, the corresponding spectral component is assigned to the line pattern and if it is lesser than $\gamma$ it is assigned to the background texture.

$$L'(f) = G(f) \quad \text{if} \quad |G(f)| / |B'(f)| > \gamma$$
$$= 0 \quad \text{otherwise} \quad \quad \quad \quad \quad (8)$$

$$B''(f) = G(f) \quad \text{if} \quad |G(f)| / |B'(f)| \leq \gamma$$
$$= 0 \quad \text{otherwise} \quad \quad \quad \quad \quad (9)$$

It has been found mathematically by a mean squared optimization criterion that when $\gamma = \sqrt{2}$, good separation of the line and background textures is observed.
3. A New Approach to Linear Feature Separation in Textures

3.1 Introduction

In this chapter the limitations of the existing Fourier transform approach are presented. Two block based approaches to overcome the limitations of the existing technique are proposed. A mathematical analysis for the block based approach is also presented. The performances of the Fourier based block approach, the wavelet based block approach and the existing Fourier transform approach are compared and the results are presented.

3.2 Limitations of the Fourier transform approach

As shown in the earlier chapter, the Fourier transform approach proposed by Beyerer – Leon assumes that the randomly located linear features are spread throughout the entire image. As we demonstrate, this results in it being ineffective in separating linear features of varied lengths and thickness. This is a limitation of the Fourier transform approach in applications since textures of wood and other surfaces have linear features, which are of random lengths on the surface and are also of varied thickness. This thesis presents two approaches, one based on a windowed Fourier transform and the other based on a windowed wavelet transform, to effectively separate linear features of varied lengths and thickness. A mathematical analysis as well as experimental results, are provided in support of this claim.

Unlike the development in [1], where the signal model is in continuous space, we develop the signal model in a bounded discrete space consistent with actual image representation [12, 15, 21, 22, 24-26, 28].
Consider a straight line embedded in an image, \( g[n_1, n_2] \) defined as,

\[
g[n_1, n_2] = \delta[n_1 - an_2] \sum_{k_1=0}^{M-1} \sum_{k_2=0}^{M-1} \delta[n_1 - k_1, n_2 - k_2]
\]

(10)

where, \( a \) is an integer. The function, \( g[n_1, n_2] \) is mostly zero except for a straight line contained in a \( M \times M \) window.

Taking the Fourier transform on both sides,

\[
G(\omega_1, \omega_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} g[n_1, n_2] \exp(-j(\omega_1 n_1 + \omega_2 n_2))
\]

\[
= \sum_{n_2=0}^{M-1} \exp(-j\omega_2 n_2) \sum_{n_1=0}^{M-1} \delta[n_1 - an_2] \exp(-j\omega_1 n_1)
\]

(11)

Since,

\[
\sum_{n_1=0}^{M-1} \delta[n_1 - an_2] \exp(-j\omega_1 n_1) = \exp(-j\omega_1 an_2) \quad \text{for } n_1 = an_2 \text{ & } 0 \leq n_2 \leq M-1
\]

(12)

\[
= 0 \quad \text{otherwise}
\]

From Eq.11 and Eq.12,

\[
G(\omega_1, \omega_2) = \sum_{n_2=0}^{M-1} \exp(-jn_2(\omega_2 + a\omega_1))
\]

(13)

Therefore,

\[
G(\omega_1, \omega_2) = \exp\left(-j(\omega_2 + a\omega_1)M\right) \frac{\sin\left(\frac{(\omega_2 + a\omega_1)M}{2}\right)}{\sin\left(\omega_2 + a\omega_1\right)}
\]

(14)

Since,

\[
\sum_{n=0}^{N-1} \alpha^n = \frac{1 - \alpha^N}{1 - \alpha}
\]
If \( g[n_1,n_2] \) is a square region of size \( N \times N \), the Discrete Fourier Transform of \( g[n_1,n_2] \) is given as,

\[
\therefore G(k_1,k_2) = G(\omega_1,\omega_2)
\]

\[
\sin\left(\frac{\pi(k_2 + ak_1)M}{N}\right) \sin\left(\frac{\pi(k_2 + ak_1)}{N}\right)
\]

\[
\therefore G(k_1,k_2) = \exp(-j\pi(k_2 + ak_1)) \frac{\sin\left(\frac{\pi(k_2 + ak_1)M}{N}\right)}{\sin\left(\frac{\pi(k_2 + ak_1)}{N}\right)}
\]

\[
M \leq N
\]

\[
|G(k_1,k_2)| = \frac{\sin\left(\frac{\pi(k_2 + ak_1)M}{N}\right)}{\sin\left(\frac{\pi(k_2 + ak_1)}{N}\right)}
\]

Eq.16 shows that the \( N \)-point Discrete Fourier Transform of \( g[n_1,n_2] \) when the straight line is of length \( M \), where \( M < N \), is a two dimensional sinc function.

If \( M = N \), Eq.16 becomes,

\[
|G(k_1,k_2)| = \frac{\sin\left(\frac{\pi(k_2 + ak_1)}{N}\right)}{\sin\left(\frac{\pi(k_2 + ak_1)}{N}\right)}
\]

where,

\[
|G(k_1,k_2)| = N\delta(k_2 + ak_1) \quad \text{for } k_2 + ak_1 = 0
\]

\[
= 0 \quad \text{elsewhere}
\]

Eq.18 shows that the \( N \)-point Discrete Fourier Transform of \( g[n_1,n_2] \) when, the straight line of length \( N \) is a sheet impulse function.

Figures 3.2, 3.3, 3.4, 3.5 and 3.6 indicate the transformations \( g[n_1,n_2] \) undergoes.
when \( M = 40 \) and when \( 0 \leq n_1, n_2 \leq 99 \), i.e. \( N = 100 \) & \( M < N \). In conjunction with the Beyerer – Leon approach, a Fourier transform, polar transform and Fourier transform are performed successively on \( g[n_1, n_2] \). Figure 3.1 shows a short length line in an image.

![Figure 3.1 Short line in an image](image1)

![Figure 3.2 Fourier Transform of image shown in Figure 3.1](image2)
Figure 3.3 Surface plot of Fourier Transform of image in Figure 3.1

Figure 3.4 Polar Transform of the Fourier Transform of the image in Figure 3.1
Figure 3.5 Fourier Transform of the Polar Transform of the image in Figure 3.1

Figure 3.6 Surface plot of image in Figure 3.5

As shown in Figure 3.2, the Fourier transform of a short line results in a two-dimensional sinc function. Figure 3.4 shows that the polar transform of the Fourier transform results in a spread of values in the polar domain. When a Fourier transform is done on the polar transform the line is not transformed to a sequence of delta functions along the x-axis. Results obtained when the straight line fits the entire image are shown in figures 3.8, 3.9, 3.10, 3.11 and 3.12.
Figures 3.8, 3.9, 3.10, 3.11 and 3.12 indicate the transformations $g[n_1,n_2]$ undergoes when $M = 100$ and when $0 \leq n_1, n_2 \leq 99$, i.e. $N = 100$ & $M = N$. In conjunction with the Beyerer–Leon approach, a Fourier transform, polar transform and Fourier transform are performed successively on $g[n_1,n_2]$. Figure 3.7 shows a line extending throughout the image.

**Figure 3.7 Image with line extending throughout**

**Figure 3.8 Fourier Transform of image shown in Figure 3.7**
Figure 3.9 Surface plot of Fourier Transform of image in Figure 3.7

Figure 3.10 Polar Transform of the Fourier Transform of the image in Figure 3.7
As shown in Figure 3.8, the Fourier transform of a line extending throughout the image is a sheet impulse function. Figure 3.10 shows that the polar transform of the Fourier transform results in a sheet impulse along the \( \rho \)-axis in the polar domain. As shown in Figure 3.11, when a Fourier transform is done on the polar transform the line is transformed to delta functions along the \( x \)-axis. Thus, the existing Fourier Transform
approach will fail in separating short lines. As shown when $g[n_1,n_2]$ contains a line of length fitting the entire image the line can be conveniently removed by applying a mask filter after going through all the transformations. These results are used as the building block for the window based approach.

3.3 Handling Lines of Varied Lengths

To overcome the limitations of the Beyerer – Leon technique of separation of linear features, it is modified to a block based approach, which is later extended to wavelets. These techniques also extract curved features in a texture since in a block, a curved feature can be approximated using a straight line. A mathematical analysis as well as experimental results are provided in support of this claim.

3.3.1 The Fourier Transform Block Approach

As shown in the earlier section, the Fourier transform of a short line in an image is a two-dimensional sinc function. The Fourier transform of a straight line extending across the entire image is a sheet impulse function. In an image having varied lengths and thickness, when the size of a block is made to fit the length of a line it results in delta functions which can be removed by mask filtering after going through the other transformations. Hence, when the image is split into blocks a better separation of linear features should be observed.

In this technique the input image is split into blocks. The algorithm as proposed in [1] is then applied to every block. This results in two blocks; one block containing the linear features and one block containing the textured background. Composite images of the line and background textures are created from all the blocks of the image. The results
of this algorithm are shown in Section 3.4.

3.3.2 The Wavelet Based Block Approach

We can take advantage of certain properties of the wavelet transform. In the wavelet transform of images, edges in the image show up prominently at all levels of the wavelet transform as discussed by Rao and Bopardikar [11]. The background texture on the other hand typically takes the appearance of noise in the detail coefficients. One can therefore attempt to extract edges at all levels of detail from the background noise. Hence, lifting linear features in the detail coefficients against the noisy background along with the approximation coefficients will likely result in better separation of linear features from the texture [16, 35-37]. The block based approach gives better separation of linear features from background textures when extended to wavelets due to the property of the block based approach to lift lines of varied lengths and thickness as shown in Section 3.2.
A block diagram of the proposed wavelet based algorithm is given in Figure 3.13.

As shown in the block diagram, a level one wavelet decomposition is done on the image resulting in the LL, LH, HL and HH sub-images. Each sub-image is split into blocks. Every block is then subjected to the existing Fourier transform algorithm. This results in $L'(\mathbf{f})$ and $B''(\mathbf{f})$ for each block. An inverse Fourier transform of $L'(\mathbf{f})$ and $B''(\mathbf{f})$ gives the separated line and background textures of each block respectively. The
composite images of the approximation and detail coefficients of the line and background textures are formed using the blocks. The line and background textures are then reconstructed from the approximation and detail coefficients through an inverse wavelet transform.

The results of the wavelet based block approach, the block based Fourier Transform approach and the existing Beyerer – Leon approach are presented in the next section. It is observed that the wavelet transform based block approach performs at least as good as and sometimes as well as the Fourier transform based block approach. This results in an effective extraction of linear features of varied lengths and thickness from the background texture.
3.4 Results

The wavelet based and Fourier transform based algorithms are tested on an image of wood, barcode and a customized image. The Daubechies wavelet of order 8 is used in testing of the wavelet based approach.

Set I - Wood Texture

(a)
Figure 3.14 (a) Wood texture sample, Fourier transform approach: (b) Background Texture, (c) Line Texture, Fourier transform block approach: (d) Background Texture, (e) Line Texture and Wavelet based block approach: (e) Background Texture, (f) Line Texture

Set II – Texture with a short and a long line

Figure 3.15 (a) Texture showing a short and long line, Fourier transform approach: (b)
Background Texture, (c) Line Texture, Fourier transform block approach: (d) Background Texture, (e) Line Texture and Wavelet based block approach: (e) Background Texture, (f) Line Texture

Set III  Image of a barcode

Figure 3.16 (a) Image of a barcode, Wavelet based block approach: (b) Background Texture, (c) Line Texture
Figure 3.17 (a) Texture showing a wood sample, Fourier transform approach: (b) Background Texture, (c) Line Texture, Fourier transform block approach: (d) Background Texture, (e) Line Texture and Wavelet based block approach: (e) Background Texture, (f) Line Texture
Set V – Texture of silk cloth

(a)

(b)

(c)

(d)

(e)
Figure 3.18 (a) Texture of silk cloth, Fourier transform approach: (b) Background Texture, (c) Line Texture, Fourier transform block approach: (d) Background Texture, (e) Line Texture and Wavelet based block approach: (e) Background Texture, (f) Line Texture

Set VI – SAR image
Figure 3.19 (a) SAR image with linear features, Fourier transform approach: (b) Background Texture, (c) Line Texture, Fourier transform block approach: (d) Background Texture, (e) Line Texture and Wavelet based block approach: (e) Background Texture, (f) Line Texture
Set VII - Image of Copperwire

(a)

(b)

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The results of the wavelet based block approach, the Fourier based block approach and the existing Fourier transform approach are compared. From Set I we can conclude that linear features lifted using the wavelet based block approach have more continuity than that observed when the Fourier block approach is used. From Set I we can also conclude that the block based techniques separate curvilinear features in images since a curved feature can be approximated to a straight line in a small region. This is an important property of the block based techniques. Set II shows that the wavelet based block approach is the most effective when applied to a background texture with linear features. The wavelet based block approach retains the background texture features better.
than the other techniques in this case. When the techniques are applied to a barcode as shown in Set III, the gray background is lifted from the barcode. This results in a sharper image containing the barcode. From Set IV and Set V we can conclude that linear features which, have not been lifted effectively by the Fourier based block approach are lifted by the wavelet based block approach. As observed in these sets of images, linear features lifted using the wavelet based block approach are stronger and more prominent than those lifted using the Fourier transform based block approach. Hence, using the results of the Fourier based block approach and the wavelet based block approach we can obtain separated linear features from the textured background. The three approaches are further tested on SAR images, which are shown in Set VI and Set VII where linear features such as roads, canals or rivers in SAR images are best lifted from the image using the wavelet based block approach. Hence, the background image contains regions which do not contain linear features like fields or other regions. When the proposed techniques are tested on an image of a Copperwire with curvilinear features as shown in Set VIII, it is observed that the block based approaches effectively separate curvilinear features in textures unlike the Beyerer – Leon approach. It is observed that the wavelet based block approach performs better than the Fourier based block approach to lift the curvilinear features from the textured background.
5. Conclusion and Future Work

The problem of separating linear features from textured backgrounds is of importance in certain applications. This thesis presents window-based techniques to separate or lift linear features from textured backgrounds. The window based techniques separate linear features of varied lengths and varied thickness, which are ineffectively separated by the existing Fourier transform approach. Results from the sets of images demonstrate that the Fourier and wavelet block approaches are more effective in lifting linear features of varied lengths and thickness as compared to the existing approach. It is observed that the wavelet based block approach sometimes performs better than, or as well as, the Fourier based block approach in lifting linear features of varied lengths and thickness. The linear features separated by the block based techniques are more prominent in the line texture. It is also observed that the background texture features are better retained when the block based techniques are used. An important property of the block based techniques is the ability to separate curvilinear features in textures since a curvilinear feature can be approximated or described by a straight line over a small region. Since the proposed approaches are block based techniques, blocking artifacts are observed at the edges of the blocks. The performance of the block based approaches have been proved and verified both experimentally and mathematically.

Since the window based techniques lead to blocking artifacts, work could be done so as to remove these artifacts to improve the results of these techniques. This could be achieved by using averaging at the block edges. The wavelet based technique relies on the existing Fourier Transform approach. Hence, more study could be done to develop a
pure wavelet based technique to achieve linear feature separation. The proposed
techniques of linear feature separation have been tested on gray scale images, which
could be extended to color images for further analysis.
Bibliography


[27] http://engineering.rowan.edu/~polikar/WAVELETS


