Dynamically loaded self-aligning journal bearings: A Mobility method approach

Nathan Mayer

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Dynamically Loaded Self-Aligning Journal Bearings: A Mobility Method Approach

by

Nathan Mayer

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Partial Fulfillment of the
Requirement for the

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IN
MECHANICAL ENGINEERING

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August 2004
Dynamically Loaded Self-Aligning Journal Bearings: A Mobility Method Approach

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First and foremost I would like to thank God for giving the strength and ability to complete this work.

I want to express my deep appreciation for the support of my loving wife. I don't know how I could have made it the past three years without her. I also want to thank my parents for helping me through my education and encouraging me to do my best.

I would like to thank my advisor, Dr. Boedo for sticking with me and supporting me through this process. I would also like to thank Dr. Török and Dr. Ghoneim for taking the time to review my thesis and serve on my committee.
Abstract

Journal bearings are a fundamental component in many pieces of machinery. A properly designed bearing is capable of supporting large loads in rotating systems with minimal frictional energy losses and virtually no wear even after years of services. These benefits make an understanding of dynamic behavior of these devices very valuable.

In applications where journal bearings are subject to misalignment (conditions where the journal and sleeve axes are not parallel), it has long been understood that bearing performance can be compromised. To eliminate this problem, self-aligning bearings have been designed. The sleeves of these bearings are capable of moving freely to accommodate misalignment. The aligning motion of these bearings under dynamic loading is, however, largely unexplored. Additionally, the impact of misalignment on journal midplane motion is unknown.

Currently, finite element and finite difference analyses are the only available tools for this kind of work. This requires a unique and computationally costly mathematical solution for every possible bearing configuration.

It is the goal of this thesis to develop a computationally efficient means of predicting journal motion within a self-aligning bearing. This is done by creating a set of “mobility” mapping functions from finite element bearing models that relate the velocity of the journal to the applied load. Such maps have previously been built and used successfully for the design of perfectly aligned bearings. This expansion of the mobility method to self-aligning bearings provides a valuable tool for the designer and gives valuable insight into the journal motion for these devices.
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<tbody>
<tr>
<td>c</td>
<td>radial clearance [L]</td>
</tr>
<tr>
<td>D</td>
<td>bearing diameter [L]</td>
</tr>
<tr>
<td>e</td>
<td>eccentricity [L]</td>
</tr>
<tr>
<td>F</td>
<td>bearing load [F]</td>
</tr>
<tr>
<td>h</td>
<td>minimum film thickness [L]</td>
</tr>
<tr>
<td>L</td>
<td>bearing length [L]</td>
</tr>
<tr>
<td>M</td>
<td>misaligning moment [FL]</td>
</tr>
<tr>
<td>$M_0$</td>
<td>aligned bearing translational mobility magnitude [-]</td>
</tr>
<tr>
<td>$M$</td>
<td>misaligned bearing translational mobility magnitude [-]</td>
</tr>
<tr>
<td>p</td>
<td>fluid film pressure [FL^-2]</td>
</tr>
<tr>
<td>R</td>
<td>bearing radius [L]</td>
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<td>t</td>
<td>time [t]</td>
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<td>S</td>
<td>spacing ratio [-]</td>
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<tr>
<td>T</td>
<td>rotational mobility [-]</td>
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<td>$\gamma$</td>
<td>direction of mobility vector relative to the force vector [-]</td>
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<td>$\varepsilon$</td>
<td>midplane eccentricity ratio [-]</td>
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<tr>
<td>$\kappa$</td>
<td>mobility magnitude adjustment factor [-]</td>
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<tr>
<td>$\mu$</td>
<td>fluid viscosity [FL^-2*T]</td>
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<td>$\rho$</td>
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<td>$\tau$</td>
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<tr>
<td>$\omega$</td>
<td>angular velocity [-]</td>
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Symbol Convention

- **A** vector A
- \( A^B \) The component of vector A in the B direction [-]
Chapter 1: Background

Lubrication is an effective means of reducing friction between moving surfaces. A common lubricated system used in rotating machinery is the journal bearing. A journal bearing consists of a lubricated cylindrical journal within a cylindrical sleeve. This device allows the journal to rotate about its axis at high speeds while the sleeve supports large radial loads imposed on the journal with minimal frictional losses.

The operation of a journal bearing is based on the fluid dynamics principle that when two surfaces, wet with a fluid, move relative to each other pressures can be created within the fluid. In journal bearings, these pressures serve to support loads applied to the surfaces, thus reducing the load carried by direct surface asperity contact. Reducing contact loads reduces frictional forces, energy loss, and wear on surfaces. The benefits of fluid film lubrication make the ability to predict the behavior of a lubricated system valuable to a designer.

The journal bearing’s lubricated surfaces are the outer diameter of the journal and inner diameter of the sleeve. The radius of the journal, \( R_j \), is machined slightly smaller than the radius of the sleeve, \( R_s \). The radial clearance between the two, \( c = R_s - R_j \), is typically on the order of a thousandth of the journal or sleeve diameter; therefore \( R_s \approx R_j \approx R \). The clearance gap is filled with lubricant. For analysis, the journal and sleeve surfaces are treated as parallel planes because their radii are orders of magnitude greater than the clearance between them.

Historically, journal bearing analyses have been performed under the assumption that the journal and sleeve axes are parallel (Khonsari and Booser 2001). The designer can then consider the location of the journal within the sleeve to vary circumferentially but not axially. This idealized scenario simplifies the calculations involved in such work.
Additionally, it reduces the number of parameters that must be considered. This assumption, however, is generally untrue.

Misalignment in bearings has been a matter of concern since modern bearing analysis began. Early automobile designers had experimentally determined that a misalignment as small as 0.002 inches at a bearing's end per 12 inches of bearing length could reduce a bearing's load capacity by over 30% (Pigott, 1941). This was followed by detailed experimental work on misaligned bearings with results in dimensionless form intended for bearing design (DuBois et al., 1955).

As computers became available, analysts began examining misaligned bearings using mathematical models. Studies modeling a variety of realistic bearing configurations were performed using finite difference based models (Pinkus and Bupara, 1979) and finite element based models (Goenka, 1984B). These studies focused primarily on the impact of misalignment on maximum film pressure, minimum film thickness, and frictional losses. This was done by imposing a constant misalignment angle to the bearing model and observing how these parameters differed from the equivalent aligned case. Recent analytical work by Boedo and Booker (2003) has investigated load and moment bearing capacity as a misaligned journal approaches the sleeve in edge point contact.

Some authors have suggested that, for bearings subject to loading that may result in significant amounts of misalignment, a self-aligning bearing should be used (Fuller, 1956). A self-aligning journal bearing is one in which the sleeve is free to rotate and accommodate misalignment. This can be achieved by installing the bearing sleeve within a low friction spherical socket (Falz, 1937; DuBois et al., 1955) as shown in Figure 1.1. The spherical socket is unable to support the moments that are created by the imbalance in fluid film.
Figure 1.1 Spherical Socket Self-Aligning Journal Bearing
pressures along the bearing axis caused by misalignment. As a result, the film pressure in a misaligned self-aligning bearing will move the bearing sleeve into an aligned condition.

Other designs for self-aligning bearings exist such as ones where the sleeve is supported by an elastomer connection or a pivoted point as shown in Figures 1.2 and 1.3, respectively (Fuller, 1956). These types of self-aligning bearings function differently from the spherical socket type and have not been evaluated in this study.

The nature of self-aligning bearings has often led analysts to assume that they are always in a perfectly aligned state. However, the dynamic behavior of a self-aligning bearing under varying load and speed is largely unexplored. The time needed for a self-aligning bearing to align itself and the effect misalignment has on journal midplane motion during the aligning process is of particular concern. It is the goal of this thesis to develop a computationally efficient means of determining the motion of a journal within a self-aligning sleeve. This is achieved by developing dimensionless “mobility” curve fits for finite element based journal motion calculations. Similar work has been successfully done for aligned bearings (Goenka, 1984A). These results expand this design tool for the analysis of misaligned self-aligning bearings.
Figure 1.2 Elastomer Supported Self-Aligning Journal Bearing

Figure 1.3 Pivoted Sleeve Self-Aligning Journal Bearing
Chapter 2: Problem Formulation

The objectives of this chapter are to describe the theory behind fluid film lubrication as it applies to journal bearings and to present the problem created by misalignment in self-aligning journal bearings.

2.1 Bearing Geometry

In a journal bearing, forces and moments are transmitted from journal to sleeve through the lubricant film. Figures 2.1 and 2.2 depict a grooveless journal bearing of diameter D and length L under general loading conditions with features exaggerated radial clearance c for clarity. Its geometry is explained below.

Figure 2.1 defines two coordinate frames. An (inertial) cartesian “computing” coordinate frame is fixed to the bearing sleeve center and defined by the x, y, and z axes. The inertial coordinate system’s origin is located on the bearing midplane with the z-axis coincident with the sleeve axis. The bearing midplane is defined as the plane equidistant from the ends of the bearing sleeve. The journal and sleeve rotate about the z-axis with angular velocities of $\omega_j$ and $\omega_s$, respectively. A moving $\xi$-$\eta$ cartesian coordinate has its origin at the bearing center with the $\xi$ axis defined by the direction of the force vector.

In Figure 2.1, the radial force vector $\mathbf{F}$ is the load transmitted from the journal to the sleeve through the lubricant film. Also shown is the journal radial displacement vector $\mathbf{e}$. The magnitude of the radial displacement vector is also known as bearing eccentricity. This vector measures the midplane displacement of the journal center, $O_j$, from the sleeve center, $O_s$. Bearing kinematics, for a complete 360° sleeve bearing, require that the bearing eccentricity always be less than the radial clearance.

In Figure 2.1, the journal misalignment vector $\phi$ is decomposed into x and y components. It is defined as the angle measured from the sleeve axis to the journal axis. As a result of misalignment, the moment $\mathbf{M}$ is transmitted from the journal to the sleeve through the lubricant film as shown in Figure 2.2.

Both eccentricity and misalignment vectors can be normalized. For eccentricity, the dimensionless parameter known as eccentricity ratio is defined by
Figure 2.1 Bearing Geometry
Figure 2.2 Bearing Loads and Kinematics
\[ \varepsilon = \frac{e}{c} \]  \hspace{1cm} (2.1)

The eccentricity ratio magnitude varies from a value of 0, where the journal and sleeve axes share the same point at the bearing midplane, to a value of 1, where the journal and sleeve surfaces contact at the bearing midplane.

A normalized misalignment angle \( \Phi \) is defined by

\[ \Phi = \frac{\phi}{\tan^{-1}\left(\frac{2c}{L}\right)} \approx \frac{L\phi}{2c} \]  \hspace{1cm} (2.2)

since for all realistic bearings \( c \ll L \). For a bearing with zero eccentricity, the normalized misalignment angle has a value of 1 when the journal is in contact with the ends of the bearing sleeve, and it is zero when the journal and sleeve are perfectly aligned.

2.2 Mobility Formulation: Aligned Bearings

The mobility method of solution is a technique developed to generalize the solution to a dynamically loaded bearing problem where the load history is specified. The method employs dimensionless maps (generated from bearing solutions) that relate the given force on the journal to the rate of journal displacement within the sleeve. This greatly reduces calculation time for the bearing designer. The mobility method for aligned bearings was introduced by Booker (1965) and is summarized below.

To simplify the derivation, a bearing with zero journal and zero sleeve angular velocity, known as a squeeze film bearing, will be examined first. The squeeze film solution can be expanded to account for journal and sleeve rotation using methods described in section 2.4. Given a perfectly aligned bearing with incompressible lubricant where variations in viscosity can be ignored, the film pressure \( p \) is governed by the Reynolds equation

\[
\frac{\partial}{\partial \theta} \left[ h^3 \frac{\partial p}{\partial \theta} \right] + R^2 \frac{\partial}{\partial z} \left[ h^3 \frac{\partial p}{\partial z} \right] = 12\mu R^3 \frac{\partial h}{\partial t} \]  \hspace{1cm} (2.3)
where \( h \) is the fluid film thickness, \( \mu \) is the fluid viscosity, and \( \theta \) is the circumferential fluid film coordinate measured from the force vector. For a circumferentially symmetric bearing, the film thickness is given by

\[
h = c - e^c \cos \theta - e^n \sin \theta
\]  

(2.4a)

and the rate of change of film thickness is given by

\[
\frac{\partial h}{\partial t} = -e^c \cos \theta - e^n \sin \theta
\]  

(2.4b)

The boundary conditions on the Reynolds equation (2.3) come from oil feed grooves, the open ends of the bearing, and fluid film cavitation. For a grooveless bearing with the bearing ends at ambient (zero) pressure, the boundary equation at the bearing ends is given by

\[
p(\theta, \pm L/2) = 0
\]  

(2.5)

Cavitation boundary conditions capture the phenomena of fluid film rupture, which occurs because fluids are unable to support negative pressures of a significant magnitude. Due to this cavitation boundary condition, there is no known closed form solution of the Reynolds equation. This has led to the development of a number of approximate solutions. These approximate solutions are based on assumptions about lubricant flow and do not completely satisfy both the Reynolds equation and the boundary conditions. Examples include the Ocvirk short bearing solution, the Sommerfeld long bearing solution, and the Warner finite bearing solution. Each solution gives film pressure as a function of the bearing length to diameter ratio \( L/D \) and the eccentricity ratio vector \( e \). Formulation details for these approximate solutions can be found in Booker (1965).

Mobility formulations for these approximate solutions to the Reynolds equation have been calculated and used to create two-dimensional maps of mobility, as a function of \( e \), for a fixed bearing \( L/D \) ratio. In addition to these approximations, a more accurate mobility
solution, based on finite element analysis, was developed by Goenka (1984), and is included in Appendix A.

The film pressures that satisfy the Reynolds equation (2.3) and boundary conditions (2.5) also need to satisfy

\[ F^\xi = F = \int_A p \cos \theta \, dA \] \hspace{1cm} (2.6a)

\[ F^\eta = 0 = \int_A p \sin \theta \, dA \] \hspace{1cm} (2.6b)

where \( F \) is the given force. These constraints arise from the need for the fluid film pressures to transmit the specified load. To account for cavitation, the area of integration, \( A \), is limited to the areas of positive pressure. Justification for this method is based on the fact that numerical results obtained using this method agree reasonably well with experimental data where inertial effects can be neglected (Booker, 1965).

The equations presented up to this point can be cast into dimensionless form by defining the following parameters for film pressure and axial film coordinate:

\[ \bar{p} = \frac{LDp}{F} \] \hspace{1cm} (2.7)

\[ \bar{z} = \frac{2}{L} z \] \hspace{1cm} (2.8)

Film thickness and its time rate of change can be represented in dimensionless form as follows:

\[ \bar{h} = h/\epsilon = 1 - \epsilon \xi \cos \theta - \epsilon \eta \sin \theta \] \hspace{1cm} (2.9a)

\[ \frac{\partial \bar{h}}{\partial t} = -\dot{\epsilon} \xi \cos \theta - \dot{\epsilon} \eta \sin \theta \] \hspace{1cm} (2.9b)

When non-dimensionalized, the Reynolds equation for axially aligned bearings under pure squeeze can be written as
The boundary conditions and constraints for this partial differential equation, given in equations 2.5-2.6, are also non-dimensionalized as follows:

\[
\bar{p}(\theta \pm L/2) = 0 \quad (2.11)
\]

\[
1 = \frac{1}{4} \int \bar{p} \cos \theta \, d\theta \, d\bar{x} \quad (2.12a)
\]

\[
0 = \int \bar{p} \sin \theta \, d\theta \, d\bar{x} \quad (2.12b)
\]

The solution to the dimensionless Reynolds equation under these pressure constraints takes the following form:

\[
\frac{\mu LD \dot{e}_\xi}{F(c/R)^2} = M_0^\xi (\varepsilon_\xi, \varepsilon_\eta, L/D) \quad (2.13a)
\]

\[
\frac{\mu LD \dot{e}_\eta}{F(c/R)^2} = M_0^\eta (\varepsilon_\xi, \varepsilon_\eta, L/D) \quad (2.13b)
\]

where \( M_0^\xi \) and \( M_0^\eta \) are dimensionless ratios of journal translational velocity to force in the \( \xi \) and \( \eta \) directions, respectively. These ratios are henceforth denoted as the aligned bearing mobility relations.
2.3 Mobility Formulation with Journal Misalignment

The mobility formulations described in the previous section ignore the effects of misalignment. A common practice in bearing analysis is to assume a perfectly aligned bearing. This assumption is almost always not true in practice, but, as discussed in Chapter 1, little work has been done to quantify the errors it introduces. In this section, the theory that explains journal motion is expanded to account for misalignment in a self-aligning bearing.

The problem presented by a misaligned self-aligning journal bearing is one where the following parameters are known at a given instant:

\( F \) - force transmitted from journal to sleeve
\( e \) - journal displacement (eccentricity)
\( \phi \) - journal misalignment
\( \omega_j \) - journal angular velocity
\( \omega_s \) - sleeve angular velocity

and the following parameters need to be found at that instant:

\( \dot{\phi} \) - time rate of change of journal misalignment
\( \dot{e} \) - time rate of change of journal eccentricity

subject to the additional constraint that there is no misaligning moment, i.e. \( M = 0 \). Figure 2.2 shows the physical representation of these parameters.

The Reynolds equation for a misaligned journal bearing with incompressible lubricant, constant viscosity, and zero journal and sleeve angular velocities can once again be written in the following form:

\[
\frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right) = \frac{\partial h}{\partial t}
\]  

(2.14)

where the film thickness, \( h \) is now defined as
\[ h = c - e^c \cos \theta - e^n \sin \theta - z \phi^\eta \cos \theta + z \phi^\zeta \sin \theta \] (2.15a)

and its time derivative is

\[ \frac{\partial h}{\partial t} = -\dot{e}^c \cos \theta - \dot{e}^n \sin \theta - z \phi^\eta \cos \theta + z \phi^\zeta \sin \theta \] (2.15b)

The pressure boundary condition at the bearing ends is the same as for the aligned case:

\[ p(\theta, \pm L/2) = 0 \] (2.16)

The pressure across the film due to radial forces is the same as for the aligned case.

\[ F^\xi = F = \int_A p \cos \theta \, dA \] (2.17a)

\[ F^\eta = 0 = \int_A p \sin \theta \, dA \] (2.17b)

Further constraints on pressure are needed to represent the situation of a self-aligning bearing. The moments transmitted through the fluid film must be zero and are constrained with the following equations:

\[ M^\xi = 0 = \int_A p z \sin \theta \, dA \] (2.18a)

\[ M^\eta = 0 = -\int_A p z \cos \theta \, dA \] (2.18b)

As with the aligned formulation, the areas of pressure integration are the portions of the fluid film where the pressure is greater than ambient to account for cavitation.

It is beneficial to express film thickness in dimensionless form using the relationship:
\[ h = h/c = 1 - \epsilon \cos \theta - \epsilon \sin \theta - \bar{\Phi} \cos \theta + \bar{\Phi} \sin \theta \] (2.19)

\[ \frac{\partial \bar{h}}{\partial t} = -\dot{\epsilon} \cos \theta - \dot{\epsilon} \sin \theta - \bar{\Phi} \cos \theta + \bar{\Phi} \sin \theta \] (2.20)

The dimensionless Reynolds equation is then written as

\[ \frac{\partial}{\partial \theta} \left[ (h)^3 \frac{\bar{\partial} \bar{p}}{\bar{\partial} \theta} \right] + \left( \frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left[ (h)^3 \frac{\bar{\partial} \bar{p}}{\bar{\partial} \bar{z}} \right] = \frac{12 \mu LD}{F(c/R)^2} \frac{\partial \bar{h}}{\partial t} \] (2.21)

The pressure \( \bar{p} \) is subject to the (dimensionless) integral constraints:

\[ 1 = \frac{1}{4} \int \bar{p} \cos \theta \, d\bar{z} \, d\theta \] (2.22a)
\[ 0 = \int \bar{p} \sin \theta \, d\bar{z} \, d\theta \] (2.22b)
\[ 0 = \int \bar{p} \bar{z} \sin \theta \, d\bar{z} \, d\theta \] (2.22c)
\[ 0 = \int \bar{p} \bar{z} \cos \theta \, d\bar{z} \, d\theta \] (2.22d)

To determine bearing motion, each of the following four quantities must be found which solve the Reynolds equation (2.21) and satisfy both the pressure and cavitation boundary conditions:

\[ \frac{F(c/R)^2}{\mu LD \dot{\epsilon} \zeta} \] (2.23a)
\[ \frac{F(c/R)^2}{\mu LD \dot{\epsilon} \eta} \] (2.23b)
\[ \frac{F(c/R)^2}{\mu LD \Phi \zeta} \] (2.23c)
\[ \frac{F(c/R)^2}{\mu LD \Phi \eta} \] (2.23d)
The goal of this thesis is to construct translational mobility mapping functions

$$\frac{\mu LD\dot{e}^\xi}{F(c/R)^2} = M^\xi(\varepsilon, \Phi, L/D)$$  \hspace{1cm} (2.24a)

$$\frac{\mu LD\dot{e}^\eta}{F(c/R)^2} = M^\eta(\varepsilon, \Phi, L/D)$$  \hspace{1cm} (2.24b)

and rotational mobility mapping functions

$$\frac{\mu LD\dot{\Phi}^\xi}{F(c/R)^2} = T^\xi(\varepsilon, \Phi, L/D)$$  \hspace{1cm} (2.25a)

$$\frac{\mu LD\dot{\Phi}^\eta}{F(c/R)^2} = T^\eta(\varepsilon, \Phi, L/D)$$  \hspace{1cm} (2.25b)

from repeated solution of (2.21) for a given range of L/D, \(\varepsilon\), and \(\Phi\). The rationale in constructing mobility mapping functions \(M\) and \(T\) is that computation of eccentricity rates and misalignment rates directly from (2.21) at each instant in time is computationally costly.

Maps for aligned solutions have only three independent variables: eccentricity ratio, eccentricity direction, and bearing length to diameter ratio. When L/D is fixed, the map can be displayed graphically in two dimensions. Figure 2.3 is an example of one of these aligned bearing mobility maps.

Creating a graphical map is not possible for misaligned bearings because the mobility equations are functions of five independent variables. Mapping functions, however, can still be made. The creation of these mapping functions is discussed below.

Translational mobility components \(M^\xi\) and \(M^\eta\) form the vector \(M\) that can also be written in terms of magnitude and direction. Translational mobility magnitude is found using the following equation:

$$M = \sqrt{(M^\xi)^2 + (M^\eta)^2}$$  \hspace{1cm} (2.26)
Figure 2.3 Example Mobility Map: Sort bearing model (Booker, 1971)
The direction of the mobility vector is measured as an angle $\gamma$ with respect to the force vector as shown in Figure 2.4.

As mentioned in the previous section, many translational mobility functions for aligned bearings have already been developed. The development of translational mobility functions for misaligned bearings relies heavily upon these aligned-based mobility functions. Specifically, two mobility adjustment functions have been created which, when applied to aligned bearing mobility functions, account for the effects of misalignment.

The first adjustment function is the magnitude magnification factor $\kappa$. It is applied to the aligned-based mobility magnitude equation using the following relationship

$$M(\varepsilon, \Phi, L/D) = \kappa(\varepsilon, \Phi, L/D)M_0(\varepsilon, L/D) \quad (2.27)$$

where

$$M_0 = \sqrt{(M_0^\varepsilon)^2 + (M_0^\gamma)^2} \quad (2.28)$$

is the translational mobility magnitude for aligned bearings.

The change in translational mobility direction from misalignment is accounted for using the mobility direction adjustment angle $\delta$. This angle is added to the aligned-based mobility direction angle $\gamma_0$ to account for misalignment using the equation

$$\gamma(\varepsilon, \Phi, L/D) = \gamma_0(\varepsilon, L/D) + \delta(\varepsilon, \Phi, L/D) \quad (2.29)$$

Proper curve fits for $\kappa$ and $\delta$ should result in having no impact on mobility calculations for aligned bearings. For the aligned case, $\Phi = 0$, the following statements must be true:

As $\Phi \to 0$, $M \to M_0$ so $\kappa \to 1$ for any $\varepsilon$

As $\Phi \to 0$, $\gamma \to \gamma_0$ so $\delta \to 0$ for any $\varepsilon$
Figure 2.4 Translational Mobility Vector
This serves as a check to confirm these curve fits are reasonable. The fits given in Appendix B satisfy these criteria.

There are no known previously existing rotational mobility mapping functions. Therefore, two new functions, \( T^x \) and \( T^n \), have been developed to determine misalignment rates using the relationships

\[
T^x (\varepsilon, \Phi, L/D) \frac{F(c/R)^2}{\mu LD} = \Phi^x
\]  
(2.30a)

and

\[
T^n (\varepsilon, \Phi, L/D) \frac{F(c/R)^2}{\mu LD} = \Phi^n
\]  
(2.30b)

2.4 Generation of Mobility Mapping Functions

The mobility formulations described in the previous section require a solution to the Reynolds equation. The finite element method was used to develop the adjustment functions presented in this paper. The lubricant film was represented by a uniform array of four-noded two-dimensional elements. For visualization purposes, the cylindrical fluid film mesh is shown unwrapped in Figure 2.5. A three dimensional representation of the mesh is given in Figure 2.6.

The finite element method, as it applies to journal bearings, was developed by Booker and Huebner (1971). In this application, the boundary variable applied to the nodes is film pressure. For a grooveless bearing, the nodes on the bearing ends are fixed to ambient pressure. Nodes on the interior of the bearing are constrained to have no net outward flow. Film thickness and its rate of change must be known to solve the Reynolds equation. Eccentricity, misalignment, and their rates are specified and are used to calculate film thickness and its rate at each node using equations 2.19 and 2.20.

The finite element solution yields pressures at each node. Nodal pressures can be summed across the film to determine resultant forces and moments transmitted from the journal to the sleeve. Unfortunately, when the finite element method is applied in this way
Figure 2.5 3-D Unwrapped Example Elemental Mesh

Figure 2.6 3-D Representation of Example Elemental Mesh
the forces and journal positions (eccentricity and misalignment) are known, but the journal velocities (eccentricity rate and misalignment rate) are unknown. This problem is solved by taking advantage of the fact that the Reynolds equation is linear with respect to the journal velocities. Therefore, the finite element mesh is solved for five trials. In the first trial, all velocities are set to zero. For the remaining four trials each of the velocity components, \( \varepsilon^e \), \( \varepsilon^n \), \( \phi^e \), and \( \phi^n \), are individually set to unit velocities while the others are set to zero. The results of these five trials form a linear set of equations that can be solved to find the required combination of velocities to produce the desired net forces and moments on the sleeve.

The velocity combination determined from the linear set of equations is then run through the finite element solver. Fluid film cavitation, as discussed in section 2.2, is then evaluated. Because incompressible fluids cannot sustain negative pressures, nodes that yield negative pressures are physically impossible. Through an iterative process, these nodes are given a boundary condition of zero pressure and the mesh is recalculated. The iterations continue until no nodes yield negative pressures. This cavitation calculation process is discussed in detail by LaBouff and Booker (1985).

To create the mobility maps, the results from the finite element solutions were used to calculate bearing mobility for various combinations of the eccentricity vector, the axial misalignment vector, and the length to diameter ratio. With a wide range of data from all independent variables, the data was curve fit using the following process. The technique described here was developed specifically for the purpose of this study.

The process taken to create both of the adjustment functions began by examining the behavior of the adjustment function with respect to one variable at a time. One variable was then chosen to be curve fit first, referred to here as the active variable. A simple family of functions that could approximate the adjustment function behavior with respect to the active variable, across the ranges of the remaining variables was sought. The adjustment function was then fit with respect to the active variable to this function family for every combination of inactive variable. This created a set of empirical values corresponding to every combination of inactive variable. The fit of the adjustment data resulting in one set of empirical values for each combination of inactive variable comprises what will be referred to here as the first generation fit.
With one of the variables accounted for, a series of second-generation curve fits followed the first. In the second generation, the empirical values from the first fit, rather than the adjustment data points, were each curve fit. A different variable was chosen as the active variable for each of the second generation fits. Like the first generation, families of functions were chosen that could describe the data for all combinations of inactive variables. When completed, these fits produced more empirical values. This process was followed by a third and fourth generation fit to account for all the variables.

The fit results from each generation were then compiled to form functions to approximate the original adjustment function data. A mathematical representation of the general multi-generation curve fitting process is shown in Appendix D.

In some instances, a suitable family of functions of the active variable could not be found during the curve fitting process. For these cases one of the following alternative approaches were taken:
1. The empirical value was fit with respect to two active variables at a time.
2. A function family was scaled by a function of several variables before fitting. This resulted in the use of infinite series in many of the curve fits.
3. A selectively inaccurate function family was used. This was done only when the resulting error in the final adjustment function fit would be insignificant.

The resulting adjustment functions are given in Appendix B.

2.5 Application of the Mobility Mapping Functions

Mobility maps are constructed in a coordinate “map” frame fixed to the bearing load. This frame is helpful for the development of mobility maps, but it is impractical for design uses. Bearing designers typically need to work in an inertial “computing” coordinate frame fixed to the sleeve center like the x-y frame given in Figure 2.1. At a given instant in time, \( F^x, F^y, e^x, e^y, \phi^x, \phi^y, \omega_j \), and \( \omega_s \) are specified with respect to this computing frame. Translation of the force, eccentricity, and misalignment vectors between the map and computing frames is achieved by a coordinate rotation:

\[
F^z = |F| = \sqrt{(F^x)^2 + (F^y)^2} \tag{2.31a}
\]
\[ F^n = 0 \]  
(2.31b)

\[ \begin{cases} 
\dot{e}^x \\
\dot{e}^y 
\end{cases} = \begin{bmatrix} 
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta 
\end{bmatrix} \begin{bmatrix} 
\dot{e}^x/c \\
\dot{e}^y/c 
\end{bmatrix} \]  
(2.31c)

\[ \begin{cases} 
\dot{\Phi}^x \\
\dot{\Phi}^y 
\end{cases} = \begin{bmatrix} 
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta 
\end{bmatrix} \begin{bmatrix} 
L\dot{\Phi}^x/(2c) \\
L\dot{\Phi}^y/(2c) 
\end{bmatrix} \]  
(2.31d)

The dimensionless eccentricity and misalignment rates can then be calculated using equations 2.32a through 2.32d where a subscript \( s \) denotes the squeeze film contribution:

\[ \dot{\varepsilon}^x_s = \frac{F(c/R)^2}{\mu LD} M^x(\varepsilon, \Phi, L/D) \]  
(2.32a)

\[ \dot{\varepsilon}^\eta_s = \frac{F(c/R)^2}{\mu LD} M^\eta(\varepsilon, \Phi, L/D) \]  
(2.32b)

\[ \dot{\Phi}^x_s = \frac{F(c/R)^2}{\mu LD} T^x(\varepsilon, \Phi, L/D) \]  
(2.32c)

\[ \dot{\Phi}^\eta_s = \frac{F(c/R)^2}{\mu LD} T^\eta(\varepsilon, \Phi, L/D) \]  
(2.32d)

Once the dimensionless squeeze film eccentricity and misalignment rates are known they must be converted back into dimensional form and translated back into the inertial coordinate frame using

\[ \begin{cases} 
\dot{e}^x_s \\
\dot{e}^y_s 
\end{cases} = \begin{bmatrix} 
\cos(\beta) & -\sin(\beta) \\
\sin(\beta) & \cos(\beta) 
\end{bmatrix} \begin{bmatrix} 
\dot{e}^x_s/c \\
\dot{e}^y_s/c 
\end{bmatrix} \]  
(2.33a)

\[ \begin{cases} 
\dot{\phi}^x_s \\
\dot{\phi}^y_s 
\end{cases} = \begin{bmatrix} 
\cos(\beta) & -\sin(\beta) \\
\sin(\beta) & \cos(\beta) 
\end{bmatrix} \begin{bmatrix} 
2c\dot{\Phi}^x_s/L \\
2c\dot{\Phi}^y_s/L 
\end{bmatrix} \]  
(2.33b)

Eccentricity rate and misalignment rate in the computing frame accounting for journal and sleeve angular velocity can then be found using the following relationships based on Booker (1971):
\[
\begin{align*}
\begin{bmatrix}
\dot{e}^x \\
\dot{e}^y 
\end{bmatrix} &= \begin{bmatrix}
\dot{e}^x \\
\dot{e}^y 
\end{bmatrix} + \bar{\omega} \begin{bmatrix}
0 & -1 \\
1 & 0 
\end{bmatrix} \begin{bmatrix}
e^x \\
e^y 
\end{bmatrix} \\
\begin{bmatrix}
\dot{\phi}^x \\
\dot{\phi}^y 
\end{bmatrix} &= \begin{bmatrix}
\dot{\phi}^x \\
\dot{\phi}^y 
\end{bmatrix} + \bar{\omega} \begin{bmatrix}
0 & -1 \\
1 & 0 
\end{bmatrix} \begin{bmatrix}
\phi^x \\
\phi^y 
\end{bmatrix}
\end{align*}
\] (2.34)

where

\[
\bar{\omega} = \frac{1}{2} (\omega_j + \omega_i) 
\] (2.36)

With the eccentricity and misalignment rates known in the computing coordinate frame they can be used to step the journal orientation forward in time. This is done using a time integration solving routine. The solutions generated in Chapter 3 employ an Euler routine. This process is repeated for each time step to determine the transient motion of a bearing. A flowchart depicting this logic is given in Appendix E.
Chapter 3: Application and Validation

The purpose of this chapter is to verify the accuracy of the curve fit solutions developed in this study and to demonstrate their usefulness. A number of classical transient and steady state problems are analyzed using both the curve fit solutions presented in this paper as well as full finite element solutions. The map results are compared with those from the finite element analyses and assessed for accuracy.

Two meshes of different densities were used to create the finite element solutions used for comparison in this chapter. These meshes are shown unwrapped in Figures 3.1 and 3.2. Their parameters are given in Table 3.1. Both meshes are finest at the bearing ends and decrease in density towards the middle. The axial spacing of the nodes is determined by a spacing ratio defined by the equations given in Appendix F and shown in Figure 3.3. Both meshes are for a bearing with L/D = 1. This ratio was chosen for all the analyses in this chapter because data for such bearings is readily available in existing work, and the ratio is representative of bearings currently in practice.

Table 3.1: Mesh Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coarse Mesh</th>
<th>Fine Mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Nodes</td>
<td>14</td>
<td>20</td>
</tr>
<tr>
<td>Circumferencial Nodes</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Spacing Ratio (S)</td>
<td>1.25</td>
<td>1.15</td>
</tr>
</tbody>
</table>

3.1 Pure Squeeze

The problem of pure squeeze is one where a constant force is applied to the journal with no rotation of either the journal or the sleeve. This problem highly approximates the motion of a wrist-pin bearing found in automotive engines. In such a situation the journal continually approaches the sleeve at an ever-decreasing rate. For an aligned bearing, a steady state solution is never achieved because fluid film pressures go to infinity as the minimum film thickness approaches zero resulting in an infinite load capacity. Therefore, these problems are presented to demonstrate the transient behavior of a self-aligning bearing.
Figure 3.1 Coarse Mesh: 14 axial by 40 circumferencial nodes, spacing ratio of 1.25

Figure 3.2 Fine Mesh: 20 axial by 60 circumferencial nodes, spacing ratio of 1.15
Figure 3.3 Axial Node Spacing Scheme  
(Boedo and Booker, 2003)
The pure squeeze analyses in this section employ the dimensionless time parameter $\tau$ defined by

$$\tau = \frac{F(C/R)^2 t}{\mu L D}$$

(3.1)

A number of cases with various initial conditions are presented here to demonstrate motion in a self-aligning bearing. The initial parameters used for each of these studies are listed in Table 3.2 and are discussed below.

**Table 3.2: Initial Conditions for Pure Squeeze Simulations**

<table>
<thead>
<tr>
<th>Case</th>
<th>$\varepsilon^c$</th>
<th>$\varepsilon^n$</th>
<th>$\Phi^c$</th>
<th>$\Phi^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2a</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>2b</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>3a</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>3b</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>4a</td>
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<td>0.282843</td>
<td>0.282843</td>
</tr>
<tr>
<td>4b</td>
<td>0</td>
<td>0</td>
<td>0.565685</td>
<td>0.565685</td>
</tr>
<tr>
<td>5</td>
<td>-0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**3.1.1 Case 1: Pure Squeeze, Zero Initial Misalignment**

The misaligned mobility equations developed in this study require the use of an existing aligned bearing mobility map. Because of its high degree of accuracy, the map developed by Goenka (1984A) has been chosen for all the analyses presented in this chapter. This first case compares this perfectly aligned mobility map solution to a full finite element solution. It serves to demonstrate the accuracy of the aligned mobility map that is modified for the misaligned solutions.
In this simulation, as with each of the first four pure squeeze calculations, the bearing begins at an eccentricity of zero. No initial misalignment was applied in this simulation. The results of this case are plotted in Figure 3.4. They show a strong agreement between this aligned bearing mobility map and the finite element solutions for journal motion along the load line.

3.1.2 Cases 2a and 2b: Pure Squeeze, Initial Misalignment about the \( \eta \)-axis

These cases are intended to demonstrate the simplest possible misalignment. An initial misalignment is imposed on the bearing solely about the \( \eta \)-axis. Physically, this means that the journal axis, when projected onto the bearing midplane, is parallel to the applied load. The simulation was run with two different magnitudes of initial misalignment, \( \Phi = 0.4 \) and \( \Phi = 0.8 \).

The results for eccentricity ratio and normalized misalignment for these cases are plotted in Figures 3.5 through 3.8. For both magnitudes of initial misalignment very strong agreement between the map solution and the finite element solutions is seen in the plots of eccentricity ratio time histories. The plots of normalized misalignment also show agreement with the map curves trending slightly lower than the finite element curves. As expected, the misalignment goes to zero simulating the motion of a self-aligning bearing.

Due to the symmetry of this problem, eccentricity in the \( \eta \) direction and misalignment about the \( \xi \)-axis should be zero for all time. As expected, the analyses yielded results numerically equivalent to zero for these two parameters; therefore they are not graphed.

Figures 3.9 and 3.10 show map results for the midplane eccentricity ratio for Cases 2a and 2b plotted against the eccentricity ratio calculated for Case 1. This is to directly compare the midplane motion of the misaligned bearing to the aligned one. These figures show a small increase in eccentricity rate as a result of misalignment. This increase is much more pronounced Figure 3.10 which demonstrates the impact of an initial misalignment ratio of 0.8 to the equivalent aligned case.
Figure 3.4 Time History of Journal Eccentricity:
Pure Squeeze, zero initial misalignment (Case 1)
Figure 3.5 Time History of Journal Eccentricity:
Pure Squeeze, specified misalignment about the $\eta_1$-axis
(Case 2a)

Figure 3.6 Time History of Journal Misalignment:
Pure Squeeze, specified misalignment about the $\eta_1$-axis
(Case 2a)
Figure 3.7 Time History of Journal Eccentricity:
Pure Squeeze, Specified misalignment about the $\eta$-axis
(Case 2b)

Figure 3.8 Time History of Journal Misalignment:
Pure Squeeze, Specified misalignment about the $\eta$-axis
(Case 2b)
Figure 3.9 Time History of Journal Eccentricity: Pure Squeeze, comparison between aligned case and initial misalignment about the η-axis.

Figure 3.10 Time History of Journal Eccentricity: Pure Squeeze, comparison between aligned case and initial misalignment about the η-axis.
3.1.3 Cases 3a and 3b: Pure Squeeze, Initial Misalignment about the $\xi$-axis

Similar to Cases 2a and 2b, these cases demonstrate the effects of initial misalignment about only one axis. The same initial misalignment magnitudes of $\Phi = 0.4$ and $\Phi = 0.8$ are used, but unlike the previous cases the misalignment is about the $\xi$-axis only.

The eccentricity in the $\xi$ direction and the misalignment about the $\xi$-axis for both initial misalignment magnitudes are plotted in Figures 3.11 through 3.14. Again, strong agreement between map and finite element solutions is found for the eccentricity ratio. The map solution also follows the finite element predictions for misalignment about the $x$-axis closely. For the larger initial misalignment, the map solution errs slightly higher than the finite element results, while the opposite is true for the smaller initial misalignment.

The map results for the eccentricity ratio in these cases are also plotted against the eccentricity ratio calculated for the aligned squeeze film case (Figures 3.15 and 3.16). Though smaller than the change in eccentricity ratio rate resulting from misalignment about the $x$-axis, an increase in eccentricity ratio rate can also be observed in these squeeze film cases.

Although this problem is not symmetric, a balance of forces results in no change in misalignment about the $\eta$-axis and no change in eccentricity in the $\eta$ direction. As a result, both $\epsilon^\eta$ and $\Phi^\eta$ remain equal to zero for the duration of the analysis. This is due to the fact that each end of the journal experiences equal film pressure forces. The pressures in the $\eta$ direction oppose each other and align the bearing while the pressures in the $\xi$ direction counter the applied load $F$.

A significant point of interest in these cases is that the misalignment dies out at approximately half of the rate as in the previous cases. This is most likely due to the fact that, for equivalent magnitudes of misalignment and eccentricity, smaller minimum film thicknesses occur when the projection of the journal axis is parallel to the eccentricity vector. The smaller film thickness results in greater maximum film pressures. Higher film pressures result in greater rates of alignment.
Figure 3.11 Time History of Journal Eccentricity:
Pure Squeeze, specified misalignment about the $\zeta$-axis
(Case 3a)

Figure 3.12 Time History of Journal Misalignment:
Pure Squeeze, specified misalignment about the $\zeta$-axis
(Case 3a)
Figure 3.13 Time History of Journal Eccentricity:
Pure Squeeze, specified misalignment about the $\xi$-axis
(Case 3b)

Figure 3.14 Time History of Journal Misalignment:
Pure Squeeze, specified misalignment about the $\xi$-axis
(Case 3b)
Figure 3.15 Time History of Journal Eccentricity:
Pure Squeeze, comparison between aligned case and initial misalignment about the $\xi$-axis

Figure 3.16 Time History of Journal Eccentricity:
Pure Squeeze, comparison between aligned case and initial misalignment about the $\xi$-axis
3.1.4 Cases 4a and 4b: Pure Squeeze, Initial Misalignment 45° from the Force Vector

To compare the effects of combining misalignment about both the $\xi$ and $\eta$ axes, this simulation was initialized with equal misalignment about each axis. The same two initial normalized misalignment magnitudes were used in these cases as with the previous two.

The results from these analyses are plotted in Figures 3.17 through 3.24. Similar behaviors can be seen for both initial misalignment magnitudes. The map curves for both $\xi$ and $\eta$ are nearly indistinguishable from the finite element results. The values for misalignment about the $\xi$-axis predicted by the map also agree reasonably well with the finite element solutions. In these cases, the misalignment resulted in some journal motion perpendicular to the force vector. This motion is captured in Figures 3.18 and 3.22, the plots of $\eta$. In both cases the maximum eccentricities perpendicular to the force caused by misalignment were very small. A noticeable relative error in the map model is observed near the peak in $\eta$ for the curves generated by both initial misalignment magnitudes. The absolute magnitude of these errors is very small.

Figures 3.25 and 3.26 demonstrate the impact of this type of misalignment on eccentricity ratio. As with misalignment purely about the $\xi$ and $\eta$ axes, a small increase in eccentricity ratio rate can be observed. The increase appears to be of a magnitude greater than that occurring as a result of misalignment about the $\xi$-axis only but less than what was caused by misalignment solely about the $\eta$-axis. This is a logical trend considering misalignment in Case 4 is a combination of the misalignments in Cases 2 and 3.

3.1.5 Case 5: Pure Squeeze, Arbitrary Initial Journal Location

In order to validate the curve fits created in this study, they need to be tested for an arbitrary set of initial conditions. This final pure squeeze case uses the initial conditions listed in Table 3.2.

The transient response of the journal is plotted in Figures 3.27 through 3.30. Excellent agreement between finite element results and the map solutions is seen for all parameters with the exception of $\Phi^\xi$. This parameter, while still following the finite element results closely, trends slightly lower than the accepted values.
Figure 3.17 Time History of Journal Eccentricity:
Pure Squeeze, specified misalignment 45° from the load
(Case 4a)

Figure 3.18 Time History of Journal Eccentricity:
Pure Squeeze, specified misalignment 45° from the load
(Case 4a)
Figure 3.19 Time History of Journal Misalignment: Pure Squeeze, specified misalignment 45° from the load (Case 4a)

Figure 3.20 Time History of Journal Misalignment: Pure Squeeze, specified misalignment 45° from the load (Case 4a)
Figure 3.21 Time History of Journal Eccentricity: Pure Squeeze, specified misalignment 45° from the load (Case 4b)

Figure 3.22 Time History of Journal Eccentricity: Pure Squeeze, specified misalignment 45° from the load (Case 4b)
Figure 3.23 Time History of Journal Misalignment: Pure Squeeze, specified misalignment 45° from the load (Case 4b)

Figure 3.24 Time History of Journal Misalignment: Pure Squeeze, specified misalignment 45° from the load (Case 4b)
Figure 3.25 Time History of Journal Eccentricity:
Pure Squeeze, comparison between aligned case and initial misalignment 45° from the load η-axis

Figure 3.26 Time History of Journal Eccentricity:
Pure Squeeze, comparison between aligned case and initial misalignment 45° from the load η-axis
Figure 3.27 Time History of Journal Eccentricity: Pure Squeeze, specified misalignment and eccentricity (Case 5)

Figure 3.28 Time History of Journal Eccentricity: Pure Squeeze, specified misalignment and eccentricity (Case 5)
Figure 3.29 Time History of Journal Misalignment: Pure Squeeze, specified misalignment and eccentricity (Case 5)

Figure 3.30 Time History of Journal Misalignment: Pure Squeeze, specified misalignment and eccentricity (Case 5)
It is evident from this comparison that curve fits presented in this study are capable of calculating journal motions comparable to those found using finite element solutions under a variety of journal orientations.

3.2 Steady Rotation

While pure squeeze solutions have academic (and some practical) value, they do not completely capture the conditions experienced by journal bearings in real world operation. This section presents transient solutions for bearings with constant rotational speeds that reach a steady-state journal position.

Because journal rotation has a time component, the dimensionless time constant, \( \tau \), used for the constant rotational speed problems is defined by

\[
\tau = \omega_j t
\]  

The addition of rotation increases the number of independent variables that must be considered. The squeeze film solutions were valid for all bearings with equal L/D ratios because the dimensions of the bearing are accounted for in the definition of the time constant. Here the dimensional parameters are combined to create a new dimensionless parameter \( f \) as defined by

\[
f = \frac{F(C/R)^2}{\muLD\omega_j} \]  

A variation in \( f \) can result from a change in applied force or journal speed because of its linear relationship with these two parameters.

Similar to the pure squeeze analyses, a number of cases with various initial conditions have been run to compare the map results to those found using a complete finite element analysis. The value of \( f \) and the initial journal orientations used in each of these cases are listed in Table 3.3. These cases are discussed in detail in the following sections.
Table 3.3: Initial Journal Position and Constant value of $f$ for Steady Rotation Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>$\varepsilon^x$</th>
<th>$\varepsilon^h$</th>
<th>$\Phi^x$</th>
<th>$\Phi^h$</th>
<th>$f$</th>
</tr>
</thead>
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<tr>
<td>6a</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>6b</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>7a</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>7b</td>
<td>0</td>
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<td>0</td>
<td>0.8</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>7c</td>
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<td>0</td>
<td>0</td>
<td>0.4</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>7d</td>
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<td>0</td>
<td>0</td>
<td>0.8</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>8a</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>8b</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>8c</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>8d</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>9a</td>
<td>0</td>
<td>0</td>
<td>0.282843</td>
<td>0.282843</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>9b</td>
<td>0</td>
<td>0</td>
<td>0.565685</td>
<td>0.565685</td>
<td>$15/\pi$</td>
</tr>
<tr>
<td>9c</td>
<td>0</td>
<td>0</td>
<td>0.282843</td>
<td>0.282843</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>9d</td>
<td>0</td>
<td>0</td>
<td>0.565685</td>
<td>0.565685</td>
<td>$15/(4\pi)$</td>
</tr>
<tr>
<td>10</td>
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<td>-0.6</td>
<td>0.2</td>
<td>0.6</td>
<td>$15/\pi$</td>
</tr>
</tbody>
</table>

3.2.1 Case 6a and 6b: Steady Rotation, Zero Initial Misalignment

As in Case 1, the bearing begins with no initial misalignment or eccentricity. These cases demonstrate the accuracy of the aligned bearing mobility map modified to account for misalignment. Figures 3.31 through 3.34 show the eccentricities predicted by the map model as it compares to the finite element models. The map curve follows the finite element curve closely with little error.
Figure 3.31 Time History of Journal Eccentricity:
Steady Rotation, zero initial misalignment (Case 6a)

Figure 3.32 Time History of Journal Eccentricity:
Steady Rotation, zero initial misalignment (Case 6a)
Figure 3.33 Time History of Journal Eccentricity: Steady Rotation, zero initial misalignment (Case 6b)

Figure 3.34 Time History of Journal Eccentricity: Steady Rotation, zero initial misalignment (Case 6b)
3.2.2 Cases 7a-8d: Steady Rotation, Initial Misalignment about the $\xi$-axis or $\eta$-axis

Cases 7a through 8d are the rotational equivalent of cases 2a through 3b. Cases 7a through 7d have initial misalignment only about the $\eta$-axis while cases 8a through 8d are initially misaligned only about the $\xi$-axis. The rotational component in these cases causes the journal to move off of the load line towards its steady state location. This motion results in eccentricity in the $\eta$ direction and the misalignment vector to change direction. These behaviors were not present in the squeeze film cases where a balance of forces prevented the misalignment vector from changing direction. The data for these cases is graphed in Figures 3.35 through 3.66.

As with the squeeze film cases, the eccentricity ratio for each case with initial misalignment is graphed against the equivalent aligned case. Figures 3.67 through 3.74 show a greater eccentricity rate early on in the transient when misalignment magnitudes were greatest. This is consistent with the observations made about the squeeze film cases. From these graphs it is also apparent that the eccentricity ratios in simulations with smaller values of $f$ are more heavily influenced by misalignment.

For all of these cases, the map-based results follow the finite element results closely for $\varepsilon^\xi$, $\varepsilon^\eta$. The value of $\Phi^n$ in cases 7a through 7d, which initially have misalignment only about the $\eta$-axis, also matches the finite element results. The misalignment about the $\xi$-axis shows a large relative error near its peak for these four cases. The absolute magnitude of this error is, however, quite small, and the map curve does follow the same shape as the finite element curves.

Similarly, cases 8a through 8d show strong agreement between the map results and the finite element results for values of $\Phi^\xi$. In these cases a noticeable relative error occurs at the peak value of $\Phi^n$, which had an initial value of zero.

In the squeeze film solutions it was observed that the rate of alignment in cases 2a and 2b was significantly greater than in cases 3a and 3b respectively. This difference in alignment rate is less pronounced in bearings with constant rotation. Additionally, it appears that the higher the value of $f$, the closer the alignment rates about the two axes become to equal. This is a reasonable conclusion because the rotation term in the mobility rate equation for bearings with rotation, Equation 2.33, becomes dominant as $f$ increases.
Figure 3.35 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the \( \eta \)-axis
(Case 7a)

Figure 3.36 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the \( \eta \)-axis
(Case 7a)
Figure 3.37 Time History of Journal Misalignment: Steady Rotation, specified misalignment about the $\eta$-axis (Case 7a)

Figure 3.38 Time History of Journal Misalignment: Steady Rotation, specified misalignment about the $\eta$-axis (Case 7a)
Figure 3.39 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the η-axis
(Case 7b)

Figure 3.40 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the η-axis
(Case 7b)
Figure 3.41 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\eta$-axis
(Case 7b)

Figure 3.42 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\eta$-axis
(Case 7b)
Figure 3.43 Time History of Journal Eccentricity: Steady Rotation, specified misalignment about the $\eta$-axis (Case 7c)

Figure 3.44 Time History of Journal Eccentricity: Steady Rotation, specified misalignment about the $\eta$-axis (Case 7c)
Figure 3.45 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\eta$-axis
(Case 7c)

Figure 3.46 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\eta$-axis
(Case 7c)
Figure 3.47 Time History of Journal Eccentricity: Steady Rotation, specified misalignment about the η-axis (Case 7d)

Figure 3.48 Time History of Journal Eccentricity: Steady Rotation, specified misalignment about the η-axis (Case 7d)
Figure 3.49 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\eta$-axis
(Case 7d)

Figure 3.50 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\eta$-axis
(Case 7d)
Figure 3.51 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8a)

Figure 3.52 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8a)
Figure 3.53 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8a)

Figure 3.54 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8a)
Figure 3.55 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the $\zeta$-axis
(Case 8b)

Figure 3.56 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8b)
Figure 3.57 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8b)

Figure 3.58 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\zeta$-axis
(Case 8b)
Figure 3.59 Time History of Journal Eccentricity: Steady Rotation, specified misalignment about the \( \xi \)-axis (Case 8c)

Figure 3.60 Time History of Journal Eccentricity: Steady Rotation, specified misalignment about the \( \xi \)-axis (Case 8c)
Figure 3.61 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8c)

Figure 3.62 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\zeta$-axis
(Case 8c)
Figure 3.63 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8d)

Figure 3.64 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8d)
Figure 3.65 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8d)

Figure 3.66 Time History of Journal Misalignment:
Steady Rotation, specified misalignment about the $\xi$-axis
(Case 8d)
Figure 3.67 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and misalignment about the $\eta$-axis

Figure 3.68 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and misalignment about the $\eta$-axis
Figure 3.69 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and misalignment about the $\eta$-axis

Figure 3.70 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and misalignment about the $\eta$-axis
Figure 3.71 Time History of Journal Eccentricity:
Steady Rotation, comparison between aligned case and initial misalignment about the $\xi$-axis.

Figure 3.72 Time History of Journal Eccentricity:
Steady Rotation, comparison between aligned case and initial misalignment about the $\xi$-axis.
Figure 3.73 Time History of Journal Eccentricity: 
Steady Rotation, comparison between aligned case and 
misalignment about the $\zeta$-axis

Figure 3.74 Time History of Journal Eccentricity: 
Steady Rotation, comparison between aligned case and 
misalignment about the $\zeta$-axis
3.2.3 Cases 9a-9d: Steady Rotation, Initial Misalignment 45° from the Force Vector

These cases demonstrate the effect of misalignment about both axes on a bearing with constant rotation. The transient behavior of the bearing is plotted in Figures 3.75 through 3.90. These graphs show agreement between the map curves and the finite element solution curves.

Figures 3.91 through 3.94 show the midplane eccentricity ratios of these cases as compared to the matching aligned cases (6a and 6b). Again, an increase in eccentricity rate is observed as a result of misalignment.

3.2.4 Case 10: Steady Rotation, Arbitrary Initial Journal Location

To supplement the validation begun in Case 5, a constant rotation problem with arbitrary initial conditions is tested in this case. The initial parameters chosen for this case are listed in Table 3.3.

The map-based results for $\varepsilon^s$, $\varepsilon^n$, and $\Phi^n$ compare very favorably with the finite element-based results. The map calculated curves of $\Phi^s$ follow the same trend as the finite element curves but do not carry the same degree of accuracy as the other parameters. Plots of these quantities are given in Figures 3.95 through 3.98.

3.2.5 Cases 11a and 11b: Steady Rotation, Variable Load

The final steady rotation cases differ from the others in that the load varies with time. They are intended to mimic the cyclical loading conditions experienced by many real world bearings, such as those found in combustion engines. These cases are presented in a dimensional manner because the load is not constant.

The bearing parameters used for these analyses are given in Table 3.4. The periodic load history applied to the bearings is plotted in Figure 3.99. The initial conditions used in these cases are listed in Table 3.5. The initial eccentricity that was used was chosen because it falls on the orbital path of the bearing when it reaches its cyclical steady state condition.
Figure 3.75 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment 45° from the load (Case 9a)

Figure 3.76 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment 45° from the load (Case 9a)
Figure 3.77 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9a)

Figure 3.78 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9a)
Figure 3.79 Time History of Journal Eccentricity: Steady Rotation, specified misalignment 45° from the load (Case 9b)

Figure 3.80 Time History of Journal Eccentricity: Steady Rotation, specified misalignment 45° from the load (Case 9b)
Figure 3.81 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9b)

Figure 3.82 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9b)
Figure 3.83 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment 45° from the load
(Case9c)

Figure 3.84 Time History of Journal Eccentricity:
Steady Rotation, specified misalignment 45° from the load
(Case9c)
Figure 3.85 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9c)

Figure 3.86 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9c)
Figure 3.87 Time History of Journal Eccentricity: Steady Rotation, specified misalignment 45° from the load (Case 9d)

Figure 3.88 Time History of Journal Eccentricity: Steady Rotation, specified misalignment 45° from the load (Case 9d)
Figure 3.89 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9d)

Figure 3.90 Time History of Journal Misalignment:
Steady Rotation, specified misalignment 45° from the load
(Case 9d)
Figure 3.91 Time History of Journal Eccentricity:
Steady Rotation, comparison between aligned case and initial misalignment 45° from the load

Figure 3.92 Time History of Journal Eccentricity:
Steady Rotation, comparison between aligned case and initial misalignment 45° from the load
Figure 3.93 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and initial misalignment 45° from the load.

Figure 3.94 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and initial misalignment 45° from the load.
Figure 3.95 Time History of Journal Eccentricity: Steady Rotation, specified misalignment and eccentricity (Case 10)

Figure 3.96 Time History of Journal Eccentricity: Steady Rotation, specified misalignment and eccentricity (Case 10)
Figure 3.97 Time History of Journal Misalignment: Steady Rotation, specified misalignment and eccentricity (Case 10)

Figure 3.98 Time History of Journal Misalignment: Steady Rotation, specified misalignment and eccentricity (Case 10)
Table 3.4: Dimensional Parameters for Cases 11a and 11b

<table>
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<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>Bearing Diameter</td>
<td>40 mm</td>
</tr>
<tr>
<td>Bearing Length</td>
<td>40 mm</td>
</tr>
<tr>
<td>Radial Clearance</td>
<td>20 μm</td>
</tr>
<tr>
<td>Journal Rotational Speed</td>
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</tr>
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Table 3.5: Initial Conditions for Cases 11a and 11b

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 11a</th>
<th>Case 11b</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.718375</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.024480</td>
<td>0.024480</td>
</tr>
<tr>
<td>$\Phi_\xi$</td>
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<td>0.65</td>
</tr>
<tr>
<td>$\Phi_\eta$</td>
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<td>0</td>
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</table>

Figures 3.100 through 3.103 show the midplane motion of the bearing for these two cases. The bearing misalignment ratios are plotted in Figures 3.104 and 3.105. The eccentricity magnitudes generated by the map functions for the aligned and misaligned cases are compared in Figure 3.106.

The midplane motion in this case is effectively captured by the map equations. The map-based misalignment ratio curves follow the same trends as the finite element curves but err high in magnitude. The maximum error is approximately 0.025 which is dimensionally about half a micron. The misalignment magnitude drops rapidly around 0.24 seconds corresponding to the load increase at the beginning of the second load cycle. The difference in eccentricity magnitude between cases 11a and 11b also decreases significantly at that point in the load history as seen in Figure 3.106.
Figure 3.99: Time History of one Load Cycle (Case 11)
Figure 3.100 Time History of Journal Eccentricity:
Steady Rotation, variable load with zero initial misalignment
(Case 11a)

Figure 3.101 Time History of Journal Eccentricity:
Steady Rotation, variable load with zero initial misalignment
(Case 11a)
Figure 3.102 Time History of Journal Eccentricity: Steady Rotation, variable load with initial misalignment (Case 11b)

Figure 3.103 Time History of Journal Eccentricity: Steady Rotation, variable load with initial misalignment (Case 11b)
Figure 3.104 Time History of Journal Misalignment: Steady Rotation, variable load with initial misalignment (Case 11b)

Figure 3.105 Time History of Journal Misalignment: Steady Rotation, variable load with initial misalignment (Case 11b)
Figure 3.106 Time History of Journal Eccentricity: Steady Rotation, comparison between aligned case and initial misalignment with variable load.
Chapter 4: Conclusions and Future Work

This study has shown that the dynamic behavior of a self-aligning journal bearing can be effectively approximated with the closed form equations developed in Chapter 2 and provided in Appendix B. This method can significantly reduce computation time without sacrificing a large degree of accuracy.

Table 4.1 shows the time required to process Case 4b given in Chapter 3. These calculations were run for 2500 fixed time steps using an Euler integration routine. The calculations were performed on a machine with a 1.3 GHz processor speed and 128 Mb of RAM. It is apparent that processing time is reduced by orders of magnitude when this approximate solution is used instead of the finite element method.

Table 4.1 Comparison of Computing Times (Case 4b)

<table>
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<tr>
<th>Calculation Method</th>
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</tr>
</thead>
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<tr>
<td>Map Solution</td>
<td>0.47 sec</td>
</tr>
<tr>
<td>Coarse Finite Element Mesh</td>
<td>450 sec</td>
</tr>
<tr>
<td>Fine Finite Element Mesh</td>
<td>1555 sec</td>
</tr>
</tbody>
</table>

The effect of misalignment on the midplane motion of a self-aligning bearing is generally small. This is primarily due to the fact that the self-aligning property of these bearings continuously reduces the magnitude of misalignment. From careful examination of the equations presented in Appendix B, it can be observed that an increase in misalignment magnitude will generally result in a greater modification to the aligned bearing mobility map. Therefore, if these misalignments did not dissipate but were sustained (as would be the case for an applied moment), one could conclude their impact on midplane motion would be greater.

Comparisons between the midplane motion of perfectly aligned bearings and of misaligned self-aligning bearings reveals increased eccentricity rates. This can be interpreted as a decrease in load capacity. The knowledge that load capacity is decreased during the
aligning process raises concerns about the performance of these bearings under such conditions.

It is clear from the results in Chapter 3 that the realignment process is not instantaneous and that normalized alignment rates can be on the same order of magnitude as eccentricity ratio rates. This leads to the conclusion that the minimum film thickness in a self-aligning bearing can be significantly impacted by misalignment. This is of paramount concern because small film thickness can result in journal to sleeve contact and destructive wear of bearing surfaces.

From the steady rotation cases it is apparent that alignment rates are significantly impacted by the value of the dimensionless parameter $f$. Greater values of this parameter result in faster rates of alignment. For a given bearing, an increase in $f$ can be caused by an increase in applied force or a decrease in journal rotational speed. This behavior can also be seen in the dimensional problem: Case 11, where the load varies cyclically. A change in alignment rate can be observed around the spikes in the load cycle (Figure 4.1). Therefore, the factors of bearing load and journal speed need to be considered carefully in evaluating alignment rates of self-aligning bearings.

A natural extension of this work is to consider the effects of misalignment in conventional bearings, that is to say bearings that are not designed to be self-aligning. For such a study the moments transmitted through the fluid film would vary in magnitude and direction instead of being constrained to zero. This increases the number of factors that impact journal motion. One approach to handing this type of problem would be to consider the velocities imposed on the journal by the forces and the moments independently. This decoupled approach would be based on the assumption that the velocities imposed by radial forces and the velocities imposed by misaligning moments can be calculated separately and summed to find the velocities for combined loading. Evidence of such decoupling can be found in Boedo and Booker (2003).

To develop this method a set of dimensionless parameters, similar to those found in Eq. 2.24a-2.25b, would have to be derived for a bearing with an applied moment but no applied force. The fundamental difference between this derivation and the one presented in Chapter 2 would be changing the pressure constraints imposed on the bearing film from Eq. 2.17a-2.18b to
Figure 4.1 Time History of Journal Misalignment and Loading: Variable Loading (Case 11)
\[ F^\xi = 0 = \int p \cos \theta \, dA \]  
\[ (4.1a) \]
\[ F^\eta = 0 = \int p \sin \theta \, dA \]  
\[ (4.1b) \]
\[ M^\xi = \int p R z \sin \theta \, dA \]  
\[ (4.1c) \]
\[ M^\eta = \int p R z \cos \theta \, dA \]  
\[ (4.1d) \]

where

\[ M = \sqrt{(M^\xi)^2 + (M^\eta)^2} \neq 0 \]  
\[ (4.2) \]

Map solutions would then need to be created for these new parameters. Finally, squeeze film journal motion for a misaligned bearing subject to both radial loading and misaligning moments could then be calculated as follows:

\[ \dot{\xi}^\xi = M^\xi (\varepsilon, \Phi, L/D) \frac{F(c/R)^2}{\mu LD} + V_1(M, \varepsilon, \Phi, L/D) \]  
\[ (4.3a) \]
\[ \dot{\eta}^\eta = M^\eta (\varepsilon, \Phi, L/D) \frac{F(c/R)^2}{\mu LD} + V_2(M, \varepsilon, \Phi, L/D) \]  
\[ (4.3b) \]
\[ \Phi^\xi = T^\xi (\varepsilon, \Phi, L/D) \frac{F(c/R)^2}{\mu LD} + V_3(M, \varepsilon, \Phi, L/D) \]  
\[ (4.3c) \]
\[ \Phi^\eta = T^\eta (\varepsilon, \Phi, L/D) \frac{F(c/R)^2}{\mu LD} + V_4(M, \varepsilon, \Phi, L/D) \]  
\[ (4.3d) \]

where \( V_i \) are the appropriate velocity components calculated from the maps for a bearing with applied moment but no applied force.

Another possible extension of this work would be to consider the dynamic effects of the sleeve. For these studies, the sleeve, mounted within a frictionless spherical bearing, was assumed to have no mass. In a real self-aligning bearing, inertial effects would have to be overcome during the alignment process. This effect would take the form of a moment that opposes changes in the aligning velocity. Accounting for this factor would slow alignment rates, increase the impact of misalignment on journal midplane motion, and possibly result in oscillations at the end of the alignment process.
The spherical socket, in which the sleeve is mounted, could also be modeled more accurately. In reality it is a spherical bearing that is governed by fluid dynamic principles similar to those used to model the journal bearing. Viscous forces in this bearing would impose small moments on the journal bearing that oppose the aligning motion. This enhancement to the model could also be expected to slow the aligning process.
Appendix A

Finite Element Based Aligned Bearing Mobility Map from Goenka (1984)

Define the aligned mobility approximation equations as:

\[ a \]

\[ \text{where} \]

\[ A = \gamma_{0.24}(L/D) \gamma_{0.4} \]

\[ M_l = -\gamma_{0.3}(L/D) \gamma_{0.4} + F + G \]

\[ \text{where} \]

\[ F = -0.3(1/0) \]

\[ G = -0.016 \gamma_{0.3}(L/D) \gamma_{0.4} + 1.7 \gamma_{0.3}(L/D) \gamma_{0.4} + 0.034 \]

\[ \gamma_{0.3}(L/D) \gamma_{0.4} = 1.03f \]

\[ J = 96 \]
Appendix B
Misalignment Adjustment Functions

Range of Validity
The following adjustment functions are valid under these criteria:

\[ 0 \leq \theta_e \leq \pi \]
\[ \Phi < 0.93 \]
\[ \varepsilon < 0.93 \]
\[ 0.25 < L/D < 1 \]

Convergence Criteria
Several adjustment functions contain infinite series of the form:

\[ \sum_{j=1}^{\infty} [f_1(\lambda, \sigma)] f_2(\theta_e, i) \]

where
\[ f_1 < 1 \text{ and } |f_2| \leq 1 \]

These are convergent series and need only to be evaluated to a point where dropping the remaining terms will not significantly impact the total of the series. The analyses performed in Chapter 3 employ a convergence limit for the counter \( i \) that is satisfied when the following statement is true:

\[ [f_1(\lambda, \sigma)] < 10^{-4} \sum_{j=1}^{\infty} [f_1(\lambda, \sigma)] f_2(\theta_e, j) \]

Derived Parameters

\[ \Phi = \frac{L|\phi|}{2c} \]
\[ \sigma = (1 - \Phi) \]
\[ \lambda = \frac{\varepsilon}{(1 - \Phi)} \]
Translational Mobility Magnitude Adjustment Function

\[ M(\varepsilon, \Phi, L/D) = \kappa(\varepsilon, \Phi, L/D) \cdot M_0(\varepsilon, L/D) \]
\[ \kappa(\varepsilon, \Phi, L/D) = 1 + A_\kappa + B_\kappa \cos(2\theta - C_\kappa) \]

where

\[ A_\kappa = (1.21 - 0.43 L/D)\left[ A_\kappa + A_\kappa B_\kappa \cos \theta + A_\kappa B_\kappa C_\kappa \right] \]

where

\[ a_\kappa = (0.585 - 0.0277 \lambda^{3.5}) \Phi^{(1.830.275 \lambda^{0.5})} \]
\[ b_\kappa = 1.75 \lambda^{0.13} \]
\[ c_\kappa = \sum_{i=1}^{\infty} \left[ (0.96 \lambda \sigma)^i \cos((i + 1)\theta) \right] \]

and

\[ B_\kappa = (1.28 - 0.57 L/D)\left[ d_\kappa + d_\kappa e_\kappa \cos \theta + d_\kappa e_\kappa f_\kappa \right] \]

where

\[ d_\kappa = (0.343 - 0.0184 \lambda^{2.26}) \Phi^{(1.70.423 \lambda^{2.15})} \]
\[ e_\kappa = 1.97 \lambda \sigma^{1.15} \]
\[ f_\kappa = \sum_{i=1}^{\infty} \left[ (0.87 \lambda \sigma^{1.26})^i \cos((i + 1)\theta) \right] \]

and

\[ C_\kappa = \pi + g_\kappa \sin \theta + g_\kappa h_\kappa \sin(2\theta) + g_\kappa h_\kappa j_\kappa \cdot \sin(3\theta) + g_\kappa h_\kappa j_\kappa l_\kappa \]

where

\[ g_\kappa = -0.937 \lambda^{0.92} \sigma^{(0.970.13 \lambda^{2})} \]
\[ h_\kappa = 0.54 \lambda \sigma \]
\[ j_\kappa = 0.741 \lambda \sigma \]
\[ l_\kappa = \sum_{i=1}^{\infty} \left[ (0.98 \lambda \sigma)^i \sin((i + 3)\theta) \right] \]
Translational Mobility Direction Adjustment Function

\[ \gamma(\varepsilon, \Phi, L/D) = \gamma_0(\varepsilon, L/D) + \delta(\varepsilon, \Phi, L/D) \]
\[ \delta(\varepsilon, \varphi, L/D) = A_\delta + B_\delta \cos(2\theta_\delta - C_\delta) + D_\delta \sin(4\theta_\delta) \]

where

\[ A_\delta = (1.3 - 0.58 L/D) \left[ a_\delta \sin \theta_e + a_\delta b_\delta \sin(2\theta_e) + a_\delta b_\delta c_\delta \right] \]

where

\[ a_\delta = \left[ 0.341(\Phi^{1.56} - \Phi^{2.24}) - 0.0024 \sin(3\pi \Phi) \right] \lambda \]
\[ b_\delta = 1.48 \lambda \sigma^{1.22} \]
\[ c_\delta = \sum_{i=1}^{\infty} \left[ (0.99 \lambda \sigma)^i \sin((i+2)\theta_e) \right] \]

and

\[ B_\delta = (1.27 - 0.53 L/D) \left[ d_\delta + d_\delta e_\delta \cos \theta_e + d_\delta e_\delta f_\delta \right] \]

where

\[ d_\delta = \left( 0.425 \Phi^2 - 0.217 \Phi^{3.63} \right) \left( 1 + 3.27 \lambda^2 \sigma^{5.85} \right) \]
\[ e_\delta = 1.92 \lambda \sigma^{1.3} \]
\[ f_\delta = \sum_{i=1}^{\infty} \left[ (0.917 \lambda \sigma^{1.125}) \cos((i+1)\theta_e) \right] \]

and

\[ C_\delta = -\pi/2 + g_\delta \sin \theta_e + g_\delta h_\delta \]

where

\[ g_\delta = -0.976 \lambda \sigma^{1.3} \]
\[ h_\delta = \sum_{i=1}^{\infty} \left[ (0.539 \lambda \sigma^{1.7}) \sin((i+1)\theta_e) \right] \]

and

\[ D_\delta = (1.36 - 0.76 L/D) \left[ j_\delta + j_\delta p_\delta \cos \theta_e + j_\delta p_\delta q_\delta \right] \]

where
\( j_\delta = l_\delta - \left(0.0346 \Phi^{1.68} + l_\delta\right)\lambda^{4(\sigma+1)} \)

where
\( l_\delta = -0.0506 \Phi^{3.1} + 0.0167 \Phi^6 \)
\( p_\delta = 5\lambda \sigma^{-1.37} - 3.18 \lambda^2 \sigma^{2.68} \)
\( q_\delta = \sum_{i=1}^{\infty} \left(0.44 p_\delta\right)^i \cos\left((i+1)\theta_\varepsilon\right) \)

**Rotational Mobility Functions in the \( \xi \)-direction**

\( T^\xi = A_\xi \left[B_\xi \sin \theta_\varepsilon + C_\xi \cos \theta_\varepsilon\right] \)

where
\( A_\xi = 0.132 (L/D - 0.0924)^{-1.52} - \sin \theta_\varepsilon \left[0.0269 + 0.0461(1 - L/D)^{2.25}\right] \)

and
\( B_\xi = a_\xi \sin \theta_\varepsilon + b_\xi \sin(2\theta_\varepsilon) \)

where
\( a_\xi = -7.19 \left(\sigma - \sigma^{2.5}\right)\lambda \)
\( b_\xi = 5.09 \left(\sigma^2 - \sigma^{2.5}\right)\lambda^2 \)

and
\( C_\xi = c_\xi + d_\xi \cos \theta_\varepsilon + d_\xi e_\xi \cos(2\theta_\varepsilon) \)

where
\( c_\xi = -8.68 \Phi + 3.25 \Phi^{1.15} - 0.4 \left(\Phi - \Phi^2\right)\lambda^{(2.3+2.3\sigma)} \)
\( d_\xi = 7.56 \left(\Phi - \Phi^2\right)\lambda \)
\( e_\xi = \left(-0.14 \lambda + 0.058 \lambda^2\right)\sigma^2 \)
Rotational Mobility Functions in the $\eta$-direction

$$T^\eta = (A_\eta + B_\eta) \left[ \sin \theta_\phi + C_\eta \cos \theta_\phi \right]$$

where

$$A_\eta = 0.126(L/D - 0.037)^{-1.81} \left[ a_\eta + b_\eta \cos(2\theta_e) \right]$$

where

$$a_\eta = -13.9 \phi + 3.88 \Phi^2 - 5.02 (\sigma^{2.25} - \sigma^3) \lambda^2$$

$$b_\eta = -3.77 (\sigma^2 - \sigma^{2.7}) \lambda^2$$

and

$$B_\eta = 3.7325(L/D)^{-2.03} (\sigma^{1.2} - \sigma^{1.9}) \lambda \cos \theta_e$$

and

$$C_\eta = d_\eta \sin \theta_e + d_\eta e_\eta \sin(2\theta_e) + d_\eta e_\eta f$$

where

$$d_\eta = (0.171 \sigma + 0.052 \sigma^2 - 0.062 \sigma^3) \lambda$$

$$e_\eta = 0.215 \lambda$$

$$f_\eta = \sum_{i=1}^{\infty} (0.75 \lambda \sigma)^i \sin((i+2)\theta_e)$$
Appendix C

Reference Tables of Results

These values are provided for the reader's reference.

Translational Mobility Adjustments

<table>
<thead>
<tr>
<th>L/D</th>
<th>Φ</th>
<th>ε</th>
<th>θe</th>
<th>θφ</th>
<th>κ</th>
<th>δ</th>
</tr>
</thead>
<tbody>
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<td>0.785398</td>
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Rotational Mobility Equations

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<tr>
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<td>-0.38117</td>
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</tbody>
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Appendix D
Multi-Generational Curve Fitting Process

\( f = \) Adjustment Function
\( x_j = j^{th} \) generation active variable
\( g_i^A = i^{th} \) member of the function A fit function family
\( c_i^1 = i^{th} \) empirical values from the \( j^{th} \) generation fit
\( m = \) number of members in the fit function family
\( n = \) number of independent variables

1\(^{st}\) Generation Fit

\[ f(x_1, x_2, \ldots, x_n) = \sum_{k=1}^{m} g_k \left( x_1, c_k^1(x_2, x_3, \ldots, x_n) \right) \]

2\(^{nd}\) Generation Fit

\[ c_i^1(x_2, x_3, \ldots, x_n) = \sum_{k=1}^{m} g_k \left( x_1, x_2, c_k^2(x_3, x_4, \ldots, x_n) \right) \]

\( j^{th}\) Generation Fit for \( j > 1 \)

\[ c_i^{j-1}(x_j, x_{j+1}, \ldots, x_n) = \sum_{k=1}^{m} g_k \left( x_1, x_2, \ldots, x_j, c_k^j(x_{j+1}, x_{j+2}, \ldots, x_n) \right) \]
Appendix E
Flowchart of Logic for Bearing Orbit Calculation using Mobility Maps

Given at an instant $(t_i)$
$e^x_i, e^y_i, \phi^x_i, \phi^y_i$
(computation frame)

Transform force and journal position vectors into
$F^x, F^y, e^x, e^y, \phi^x, \phi^y$
(map frame)

Convert journal position vectors to dimensionless form:
$e^x, e^y, \Phi^x, \Phi^y$

Calculate $M^x, M^y, T^x, T^y$
using mobility maps
(Appendices A and B)

Calculate dimensionless squeeze film journal velocities:
$e^x_s, e^y_s, \phi^x_s, \phi^y_s$
(Equations 2.24a-2.25b)

Convert squeeze film journal velocities into dimensional form:
$e^x_s, e^y_s, \phi^x_s, \phi^y_s$

Given for all time $t$
$F^x, F^y, \omega_j and \omega_s$
(computation frame)

Calculate journal position at the next time step $(t_{i+1})$
$e^x, e^y, \phi^x, \phi^y$
(Euler's method)

Account for journal rotation to get
$e^x_s, e^y_s, \phi^x_s, \phi^y_s$
(Equations 2.34-2.36)

Transform vectors into
$e^x_s, e^y_s, \phi^x_s, \phi^y_s$
(computation frame)

Continue to another time step or end?

End

End

Continue

Appendix F
Axial Spacing for Non-Uniform Meshes

From Boedo and Booker (2003)

The physical layout of a mesh created using this spacing technique is given in Figure 3.3. Given a mesh with m axial divisions at each end, a dimensionless length of $\bar{z}$ as defined in Eq. 2.8, and a spacing ratio of $S$, the axial nodal coordinates are given in the following equations:

$$z_0 = 0$$

$$z_{i+1} = z_i + \sum_{j=0}^{i} \frac{\Delta}{S^j}$$

where

$$\Delta = \left( \sum_{j=0}^{m-1} S^{j+m-1} \right)^{-1}$$

so that

$$z_1 = \Delta$$

$$z_2 = \Delta(1 + S^{-1})$$

$$z_3 = \Delta(1 + S^{-1} + S^{-2})$$
References


