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Principles and simulation of straight boomerang

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PRINCIPLES AND SIMULATION OF THE FLIGHT OF A STRAIGHT BOOMERANG

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PRINCIPLES AND SIMULATION OF THE FLIGHT OF A

STRAIGHT BOOMERANG

by:
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A Thesis Submitted
in
Partial Fulfillment
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in
Mechanical Engineering

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Nicolás Restrepo
To my family, Germán, Maud, and Isolda Restrepo, without whose enduring and unselfish support it could have never been undertaken
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ABSTRACT

There has been a lot of research done on the regular boomerang but little or none has ever been done on the straight boomerang. This not so popular toy does behave as a boomerang but the physics governing its flight are very different from those governing the regular boomerang’s flight. In order to understand what makes the straight boomerang come back, high speed photography was used. It was concluded that the autorotation phenomenon played an important role during the flight. A mathematical model was then developed and the flight dynamics of the boomerang incorporating its actual behavior was simulated using MATLAB. The simulation results obtained were very close to those obtained from experimentation.
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LIST OF VARIABLES

a  acceleration

$A$  largest surface area of the flat plate

$AR$  aspect ratio of the boomerang

b  span of the plate

c  chord length of the boomerang

$C_D$  coefficient of drag

$C_L$  coefficient of lift

$C_M$  coefficient of moment

$C_r$  coefficient of rotation

$F$  forces

$F_x$  $x_b$ component of the forces acting on the boomerang

$F_y$  $y_b$ component of the forces acting on the boomerang

$F_z$  $z_b$ component of the forces acting on the boomerang

$F_D$  drag force

$F_L$  lift force

$H$  angular momentum

$I$  moment of inertia

$m$  mass

$M$  moments

$p$  roll parameter or Strouhal Number
p  $x_b$ component of the body's angular velocity
q  $y_b$ component of the body's angular velocity
r  $z_b$ component of the body's angular velocity
S  Strouhal Number
t  thickness of the boomerang
t  time
T  torque
$T_{b-f}$  transformation matrix for body to fixed frame transformation
$T_x$  $x_b$ component of the torque acting on the boomerang
$T_y$  $y_b$ component of the torque acting on the boomerang
$T_z$  $z_b$ component of the torque acting on the boomerang
u  $x_b$ component of the body's linear velocity
v  $y_b$ component of the body's linear velocity
V  magnitude of the velocity of the fluid in the plane of the cross-section
$V_c$  velocity vector of the center of gravity
$V_p$  velocity at point P along the boomerang
$V_{rel}$  velocity of the wind relative to the airfoil
w  $z_b$ component of the body's linear velocity
$x_b$  x axis of the body coordinate system
$X_f$  x axis of the fixed coordinate system
$y_b$  y axis of the body coordinate system
$Y_f$  y axis of the fixed coordinate system
$z_b$  z axis of the body coordinate system \\
$Z_f$  z axis of the fixed coordinate system \\
$\alpha$  angle of attack \\
$\rho$  density of fluid (kg/m$^3$) \\
$\theta$  Euler angle \\
$\phi$  Euler angle \\
$\psi$  Euler angle \\
$\omega$  vector of angular velocity
SECTION 1

HISTORY
1 HISTORY

The term “boomerang” applies to an object that when thrown, returns to the launching point due to its aerodynamic properties. The word “boomerang” began to be used about two centuries ago when the English occupied the coast of New South Wales. The aborigines of this area had returning and non-returning sticks. The non-returning stick, also called “killer stick”, “flying stick”, “throw stick”, or “non-returning boomerang,” flies in a slightly curved path close to the ground, but does not return to the thrower. Both of these kinds of sticks had over three hundred names. Among these were: boumarang, nanjal,baranganj, kali, wangium, wilgi, bangeet, kylie, and tootgundy [1]. This confused the English who mistakenly used the term boomerang to refer to both returning and non-returning sticks even though each was used for a different purpose. Nowadays, ‘boomerang’ refers to a returning stick and ‘killer stick’ to a non-returning one [2].

The killer stick weighs about 700 grams, 60-90 cm. in length, and 7.5 cm. wide. It has a crescent shape, and has a small bend around the center of its length. It was used for hunting and as a war weapon due to its long range (140 m.) and accurate flight path. On the other hand, the boomerang weighs 110-350 grams, 30-75 cm. long, and 5 cm. wide. Boomerangs were used primarily as toys, games, rituals, and sometimes for fishing and hunting flocks of birds. It was also used to train the children before they could throw the
killer stick. The boomerang however comes in a wide variety of shapes ranging from the curved banana (known as the aboriginal) to the shapes made by the capital letters $C, H, I, S, T, U, V, X, Y, Z,$ to the shapes of animals and even people [3] (see Fig. 1.1). The boomerang does return to the thrower and it can have a range of up to 100 m.

![Various kinds of boomerangs](image.png)

**Fig. 1.1** Various kinds of boomerangs [2]

There are several theories on the origins of the boomerang. Even though most people think the boomerang was invented in Australia, archeologists have found both types of sticks in such diverse places as India, Egypt, Holland, Denmark, Germany, Sudan, Jutland, Tasmania, and Arizona. This means that the boomerang was probably developed in all these places independently. The oldest boomerang and killer stick
fragments were found by Roger Leubbers in January 1974 in South Australia. It was determined that these fragments were from 11,000 to 15,000 years old.

Some scholars agree that the boomerang developed from the killer stick while people experimented with different shapes in order to improve the accuracy and range of the flight. By coincidence, a reduction in the curvature of the killer stick gave birth to the boomerang. As time passed, the dimensions were varied until a returning stick was constructed.

It is interesting to note that the list of capital letters which boomerangs may resemble does not include the letter I. Even though its flight is not the same as that of the other boomerangs, it does return to its launching point because of its aerodynamic properties, hence it is a boomerang. This I-shaped boomerang is known as the “tumblestick” or “straight boomerang.” The origins of this straight boomerang are unknown but as mentioned by Hess[4], this device was seen first at Celebes which is an island in Indonesia. As described by W. Kaudern:

During my stay at the villages of Kelei and Taripa in Ondae in E. Central Celebes I saw children playing with flat pieces of split bamboo, which they called tela. These tela measure 20 cm. by 2.5 cm., they have square ends, and the edges of the long sides are slightly rounded off. One side of the tela is convex, the other side slightly concave in the middle. ... A clever player knew how to make his tela revolve in the air so as to describe an almost elliptic trajectory and return to the place whence it started. At Kelei, where this sport, motela, was a popular amusement...
According to Felix Hess in his work *Boomerangs, Aerodynamics, and Motion*, the development of the straight boomerang was thanks to the aborigines' careful observation of nature and not a matter of chance as in the case of the regular boomerang.

Mr. Hubert de Castilla has suggested that the aborigines derived the invention of the woguim from the observation of the shape and the peculiar turn of the leaf of the whitegum tree [eucalyptus]. As the leaves of this tree fall to the ground, they gyrate very much in the same manner as the wongium does; and if one of the leaves is thrown straight forward, it makes a curve and comes back [5].
SECTION 2

BOOMERANGS IN GENERAL
2.1 REGULAR BOOMERANG

2.1.1 Construction and Geometry

These boomerangs are mostly made out of birch ply wood, although fiber glass, fiber board, and plastic laminates have proven to be good materials but they are more expensive and more difficult to shape. The thickness of the material usually varies between 0.5 and 0.8 cm. The aboriginal boomerang can have many shapes but the most common of all is the L-shaped one.

The most important feature of a regular boomerang is its cross-section. The cross-section should be that of an airfoil as shown in Figure 2.1. The type of airfoil is not very important as long as one side is more convex than the other. The cross-section of each arm should be oriented as the ones shown in Figure 2.2 which allow the leading edge to always face the wind when the boomerang is rotating. The boomerang in Figure 2.2 [6] is

Fig. 2.1 Airfoil Cross-section
Fig. 2.2 Typical shape and geometry of an aboriginal boomerang [6]

a right-handed one, and it can be differed from the left-handed one simply because the airfoil in each arm is facing in the other direction. The tip to tip dimension is usually 50 cm, the angle between the arms may vary from $70^\circ$ to $110^\circ$, and the width of the arms can be around 5 cm.
2.1.2 Launch

According to Jacques Thomas [7], the regular boomerang is launched usually at a speed of 25 m/s and a rotational frequency of 10 Hz. Figure 2.3 [8] shows the appropriate way of throwing the boomerang. The tilt angle shown, should not vary by more than 30° from the vertical, and about 15° above the horizon. The best flight conditions are when there is no wind, but if there is a slight breeze, it should be thrown at an angle of 45° to the right of the direction of the wind (if the boomerang is left-handed, 45° to the left of the wind). The boomerang’s flight can be controlled only if the wind speed doesn’t exceed 5 m/s. If the boomerang is thrown properly, it should be very easy to catch because it will hover back to the thrower. It should be caught by clasping the hands together.
2.1.3 Description of Flight

When the boomerang is released, it follows a path similar to the one shown in Figure 2.4 [9]. It starts following a somewhat straight path and then it turns towards the left (if the boomerang is a right handed one) in an elliptic manner. The rotational axis of the boomerang is always pointing towards the center of the ellipse until it has traced about 3/4 of the ellipse. Then it starts lying down by slowly pointing its rotational axis upwards. This is called the boomerang’s “lying down.” As this happens the lift generated by its airfoils counteract the boomerang’s weight, lifting it up and also slowing it down while it follows the elliptic path. By the time the boomerang starts approaching the thrower, its rotational axis is perpendicular to the ground and then it slowly hovers to the hands of the thrower. It should be noted that even though the boomerang’s orientation varies, its rotational axis always remains to be its initial axis of rotation. Watching a boomerang fly is so fascinating that its 8 to 10 seconds long flight may seem like minutes.
Fig. 2.4 Results from the simulation of the flight of a regular boomerang at various tilt angles[9]
2.2 STRAIGHT BOOMERANG

2.2.1 Construction and Geometry

Of all the boomerangs, the straight boomerang is the easiest one to construct but at the same time, the production of a reliable one can sometimes be a matter of luck. Out of a dozen straight boomerangs with exactly the same shape, dimensions, and material, probably one will have a perfect performance. This delicacy and temperamental quality, however, adds interest to the Tumblestick. Since they are so easily made, constructing a returning straight boomerang should be no problem. Even though there are no specific dimensions and geometry guaranteed to work, there are some guidelines which will easily lead anyone to the successful construction of a good Tumblestick.

Material

The material should be very light and must have an evenly distributed density. Some good materials are balsa wood, foam, and sometimes plastic or wood. The best choices are foam and balsa wood. The density of balsa wood varies between 0.1 and 0.2 g/cm$^3$. If the material is too light, the boomerang will circle around the thrower, and if it is too heavy it will follow projectile motion. Another material that works well and is easily cut is foam sandwiched with thin cardboard. This material is very light and it is structurally stiff.
Aspect Ratio

The other important factor that should always be kept in mind is the aspect ratio (span / chord) of the boomerang. The aspect ratio which the boomerang should have is greatly influenced by the type of material chosen. For balsa wood and foam ideal aspect ratios are between 10 and 15. If a material like plastic or heavier balsa wood is going to be used, a higher aspect ratio (i.e. 15 - 20) should be implemented even though it is not suggested that these materials be used for good performance. For balsa wood and foam some recommended dimensions are lengths ranging from 20 to 60 cm., widths from 2 to 4 cm., and thickness from 0.3 to 0.8 cm.

Cross-section

The effects of using different cross-sections (see Fig. 2.5) was studied and it was found that the only requirement that should be met was symmetry. The cross-section of the boomerang has to be symmetric at least about its axis perpendicular to the width (Fig. 2.6).

Fig 2.5 Possible cross-sections for straight boomerangs
The cross-section of a straight boomerang must be symmetric.

In addition the cross-section need not be symmetric with respect to the axis perpendicular to its thickness but the cross-sections that have both symmetries, perform best. An airfoil NACA 0012 was also tested but aerodynamically, this shape is so stable that the boomerang just glides away. Also, since the tested airfoils (i.e. NACA 0012) do not have symmetry with respect to the axis perpendicular to its width, they do not autorotate which prevents them from returning. So in order to construct a straight boomerang, the cross-section chosen must be able to autorotate and for this to happen, it must be symmetric with respect to the axis perpendicular to its width. For better results the top and bottom edges of the boomerang must remain square and flat so that its largest area has a rectangular planform area.

Fine Tuning

As it was previously mentioned, most of the times straight boomerangs can become very tedious to make and that is why some fine tuning may be necessary. Sometimes the problem lies in the material which may not be very evenly balanced. Other times the geometry is the problem. If the Tumblestick doesn’t come back, first try bending the tips.
in the same direction at a third of the length away from both ends. If this does not improve the flight, try reducing some material by shaving wood off from either side. If this still doesn’t work, it probably means that another boomerang should be constructed.

If this seems too much work, the firm *The Goose Co.* of San Jose in California, produces the straight boomerang under the name of BOOMERANGAZANG. This foam boomerang flies very accurately up to a distance of 3 meters, weighs not more than 10 grams, and has the dimensions of a school ruler. This is the only commercial toy of this type which can be very safely thrown inside the house.

### 2.2.2 Launch

There are many differences between the way a regular boomerang and a straight boomerang are thrown. The straight boomerang does not require as much power. The boomerang should be grasped between the thumb and forefinger, with the flat sides touching both fingers. Swing it straight back so that the end is behind the shoulder as shown in Figure 2.7b [10], and then throw it forward and upward over the level of the thrower’s head (Fig. 2.7c). There is one important thing to consider when releasing it and that is to concentrate more on the movement of the wrist than on the movement of the arm. The wrist should be turned sharply downward and not much power should be put into the motion. This gives the boomerang a high rotational speed which is more important than its translational speed. The boomerang should be held loosely between the
fingers so that it rotates freely. Also, the thin side should point in the direction of the motion. If it is thrown with the flat side facing the air, the boomerang will not come back. And if it is thrown with too much translational speed and very small rotation, the Tumblestick will start autorotating and it will fly in a curved path if looked from the top. To get the boomerang to come back requires a lot of practice and it should be thrown in a room with no wind. Catching the boomerang is very easy because if it is thrown correctly, it will slowly come back. If the boomerang seems to drift to the right, it should be thrown with an initial leftwards tilt, and if it drifts to the left the initial tilt should be towards the right.
As it can be seen there are many differences between the way the regular and straight boomerang are thrown. The straight boomerang is considered to be more of a toy for playing indoors while the regular one needs to be thrown in a large field and it is considered a sport.

2.2.3 Description of Flight

The initial velocity should be of about 5 m/s and according to Vos [11], its rotational speed should be of 4 Hz. The straight boomerang should not be thrown with a large tilt angle and should be released at a 30° angle between the boomerang and the horizontal. Since this device is very light, it should be thrown indoors. After the boomerang has gone through 3/4 of a revolution, it experiences a transition through which it changes its spin axis to the its longitudinal axis. This spin of a body along its longitudinal axis is known as autorotation. The tumblestick then follows an elliptical path but instead of it being in the horizontal plane, it is on the vertical plane, as seen on Figure 2.8 [12]. The straight boomerang then slowly autorotates back to the hands of the thrower after traveling a distance between one half to four meters.
Fig. 2.8 Flight of the straight boomerang [12]
SECTION 3

PROBLEM DEFINED FOR STUDY

Hey Clumsy, how 'bout a boomerang? I bought one last year. Throw it away and get a new one! Throw it away...?
3.1 STUDIES DONE ON THE STRAIGHT BOOMERANG

There have been numerous studies done on the regular boomerang, the most extensive of these being the Ph.D. thesis of Dr. Felix Hess from the University of Groningen. On the other hand the straight boomerang has only been studied partially by Dr. Henk Vos in a paper titled “Straight Boomerang of Balsa Wood and its Physics” published in the *American Journal of Physics* of June of 1984. Dr. Vos never considered the straight boomerang for further study. The only other reference is by B. S. Mason in his book *Boomerangs, How to Make and Throw Them* published in 1974, under a chapter titled “Tumblesticks.” In this reference the straight boomerang is considered to be a mysterious toy which does fly back to the thrower but for unknown reasons. Probably one of the main reasons why this object has never been taken into consideration is because of its lack of popularity and the limited amount of research that has been done on autorotating plates which as will be seen later, is the main reason why the tumblestick acts like a boomerang.

In his study, Vos [13] noted that for a straight boomerang with moment of inertia $I_x$ (see Fig. 3.1 where the $x$, $y$, and $z$ axis are defined) and much greater than $I_z$ (about 200 times greater) and with initial angular rotation $\omega$ of 4 rev/s, conservation of angular momentum would estimate an angular rotation about its longitudinal axis during the return part of the flight, of 800 rev/s (200 times larger). According to Vos, who measured the pitch of the humming sound made by the boomerang during this final stage, the
rotational rate was of about 75 rev/s which means that there are some losses involved during the flight. This loss of angular momentum is mainly due to external aerodynamic forces. This means that in order to simulate the trajectory of the straight boomerang, not only the three-dimensional dynamics should be considered but also the aerodynamics. Section 4 will introduce the necessary physics to understand the motion of the boomerang.
3.2 DEFINITION OF THE COORDINATE SYSTEMS

Fig. 3.1 Body and Fixed Frame Coordinate Systems
In Figure 3.1 the body and fixed coordinate systems that will be used in this work, are shown. The fixed reference frame ($X_f - Y_f - Z_f$) is attached to the Earth with its positive $Z_f$ axis pointing down into the Earth and the $X_f$ axis pointing in the direction in which the boomerang is being initially thrown. The body coordinate system ($x_b - y_b - z_b$) is completely attached to the body and it is aligned with the boomerang’s principal axes.
3.3 GEOMETRY

For this problem, the boomerang chosen is a 450x35x5 mm piece of foam. The cross-section is rectangular and it weighs 9.36 gms. Experimentally, this boomerang proved to return to the thrower very effectively and the flights were very easily reproduced.

Fig 3.2 Shape and dimensions of the straight boomerang

For this geometry the moments of inertia obtained were calculating by simply assuming that the boomerang is a rectangular plate. The following were the results:

\[ I_{xx} = 0.000158 \text{ kg m}^2 \]
\[ I_{yy} = 0.000159 \text{ kg m}^2 \]
\[ I_{zz} = 0.000001 \text{ kg m}^2 \]
3.4 ASSUMPTIONS

1) Cross-section of the boomerang is a rectangle and constant throughout the length of the boomerang.

2) The model is symmetric with respect to the \( x_b-y_b \) plane.

3) The aerodynamic forces are on the plane of the cross-section

4) 3-Dimensional aerodynamic effects are ignored

5) The boomerang is flying in still air.

6) The aerodynamic center and the center of gravity coincide.

7) The principal axis of the boomerang and the body coordinate system coincide with each other.

8) The products of inertia of the body \( (I_{xy}, I_{xz}, I_{yz}) \) are negligible.

9) The center of gravity of the boomerang will be tracked.

10) The boomerang starts autorotating about its negative \( z_b \) axis.
"Hi! Just like every time, you'll get about 100 yards out before you start heading back."
4.1 AERODYNAMICS

4.1.1 Flat Plate Aerodynamics

Flat Plate Geometry

Even though the body studied in this work is not a wing but a flat plate, the terminology will be the same as that used in wing theory. The geometry of the flat plate is defined as shown in Figure 4.1 where \( t \) is the thickness of the plate, \( b \) its span, and \( c \) its chordlength. The Aspect Ratio \( AR \) of the wing is defined as

\[
AR = \frac{b}{c}
\]  

(4.1)

Lift & Drag

Lift is the component of resultant aerodynamic force perpendicular to the fluid motion [14]. The lift is caused by a higher velocity of the fluid flowing around the top than around the bottom of the plate or wing, or vice versa, depending on the angle of attack.
According to Bernoulli’s equation if the velocity is high, the corresponding pressure is low. So, for the case shown in Figure 4.2 [15], a higher velocity around the top of the wing will cause a lower pressure at the top than at the bottom. This difference in pressure gives rise to a lift force in the direction perpendicular to the wind velocity. The drag is the component acting on a body parallel to the relative wind [16]. Both of these forces are shown on Figure 4.3 acting at what is called the aerodynamic center.

![Fig. 4.2a](image1)

*Fig. 4.2a* Flat plate at angle of attack and generating lift [15]

![Fig. 4.2b](image2)

*Fig. 4.2b* Airfoil at angle of attack and generating lift [15]
The *angle of attack* $\alpha$ is the angle between the velocity of the wind seen by the plate and its chord. For a flat plate the *chord* goes across the center of its cross-section ($x$-axis of Figure 4.3). The lift and drag forces ($F_L$ and $F_D$ respectively) are

$$F_L = \frac{1}{2} \rho V^2 C_L A$$

$$F_D = \frac{1}{2} \rho V^2 C_D A$$

where $A$ is the planform area of the wing ($A = b \cdot c$), $\rho$ is the density of the fluid, $V$ is the relative velocity of the air. $C_L$ and $C_D$ are the coefficients of lift and drag respectively. Both of these coefficients depend on $\alpha$ which is defined as

$$\alpha = \tan^{-1}\left(\frac{V}{U}\right)$$

*Fig. 4.3* Plate in airstream with lift and drag forces and angle of attack
where u and v are the local x and y components of the air velocity as shown in Figure 4.3.

In the case of a rotating cylinder the terminology is the same but the lift force is in a different sense than the one defined for a wing or nonrotating plate. If a cylinder is rotating and translating in the sense shown in Figure 4.4, the lift force will be as shown.

![Diagram of rotating cylinder in airstream with lift and drag forces](image)

**Fig. 4.4** Rotating cylinder in airstream with lift and drag forces

What happens in a rotating cylinder is that the velocity of the air at the boundary layer is higher at the top than at the bottom. The velocity of the surface of the cylinder at the top is in the same direction as that of the air, and opposite at the bottom. This gives rise to a higher velocity of the air in the boundary layer at the top than at the bottom. Again, a higher velocity at the top gives rise to a lift force perpendicular to the fluid velocity. This concept is very important in the study of the straight boomerang because as it will be
stated later in the following section, when a body is autorotating, the lift generated is in the same direction as that of a rotating cylinder.

### 4.1.2 Autorotation

In fluid dynamics the subject of autorotation has not been considered a subject of serious study. Instead, it is seen as a curiosity and sometimes even labeled as 'toy aerodynamics.' Autorotation has gained interest in areas such as airfoil study, flight dynamics of aircraft, aeroballistics, ballistics, meteorology, and sports. Such a large diversity in its applications, makes any collection of literature very difficult which explains part of the reason for the lack of a uniform treatment. Another reason for the lack of understanding of this phenomenon is the difficulty of obtaining qualitative experimental and theoretical data. The experimental difficulty can be understood by imagining how challenging it would be to measure the surface pressure on a piece of balsa wood autorotating in air. And theoretically, the difficulty is indicated by the fact that autorotation is a non-linear phenomenon. There also exist analogies to vortex-induced vibrations of bodies which is an area that has developed more extensively. Up to now, all the work done on autorotation has been experimental providing only empirical correlations for the specific cases studied [17]. This introduced a difficulty into this work because sometimes the experiments performed by various aerodynamicists were not done on the geometry of the straight boomerang. When this happened, the best estimates possible were made.
The Autorotation Phenomenon

Autorotation is sometimes defined as any continuous rotation of a body in a parallel flow without any external sources of energy. In this case, the body’s geometry generates a torque which allow it to start rotating when released in a parallel flow. Examples of this ‘pseudo-autorotation’ are windmills, waterwheels, and anemometers (see Fig. 4.5) [18].

Fig. 4.5 Examples of pseudo-autorotation [18]

But the proper definition of autorotation is that one given by Riabouchinski, who first introduced the term in 1906. According to him, autorotation can only be present if a body has one or more stable positions in which the fluid flow exerts no torque on the resting body. In this case, an initial impulse is needed to move the body away from the stable position so that the fluid flow can sustain a continuous spinning of the body [19].

There are two kinds of autorotation that fall under Riabouchinski’s definition. The two cases are autorotation with the spin axis parallel to the flow, and autorotation with
the spin axis perpendicular to the flow. For the first case (see Fig. 4.6) [20], the body must be symmetric with respect to the flow. Due to this symmetry, an initial impulse must be applied to rotate the body away from the stable position in order to obtain autorotation.

![Fig. 4.6 Bodies autorotating parallel to the flow. (a) Lanchester Propeller; (b) airfoil [20]](image)

The second kind of autorotation is one in which the spin axis is perpendicular to the fluid flow. In this case the body also has to be symmetric with respect to the flow direction (see Fig. 4.7) [21]. Again, an initial impulse is required to help the body pick up the initial angular momentum to overcome the adverse torque present around the stable position. This autorotation is the one found in the straight boomerang's flight so it will be the only one mentioned here.
Fig. 4.7 Bodies autorotating perpendicular to the flow. (a) Rotating dumbbell; (b) rotating plate; (c) Magnus rotors [21].

It is important to mention that the fundamental difference between the two kinds of autorotation is that the one with the spin axis parallel to the flow presents a constant rate of stable autorotation while the kind with the spin axis perpendicular to the flow, has a periodic rate of stable autorotation.

As mentioned earlier, the areas in which autorotation theory is used are many. An example of the application of the first kind of autorotation is that of flight dynamics when analyzing spinning aircrafts. The second kind of autorotation can be observed in the free-fall of very light rectangular objects such as the straight boomerang, and in nature in the free-fall of some tree fruits and seeds. The theory of this kind of autorotation is used in aeroballistics to predict the dispersion patterns of bombs, in the study of the fragmentation of a flying body, in the calculation of autorotating reentry bodies such as
As mentioned earlier, the areas in which autorotation theory is used are many. An example of the application of the first kind of autorotation is that of flight dynamics when analyzing spinning aircrafts. The second kind of autorotation can be observed in the free-fall of very light rectangular objects such as the straight boomerang, and in nature in the free-fall of some tree fruits and seeds. The theory of this kind of autorotation is used in aeroballistics to predict the dispersion patterns of bombs, in the study of the fragmentation of a flying body, in the calculation of autorotating reentry bodies such as aircraft disposable nose sections, and in meteorology to determine the flight patterns of hailstones which usually grow in an oblate spheroid shape.

As mentioned earlier, the rate of stable autorotation for the second kind of autorotation, is periodic. So when executing any calculations of the torque $T$ the body is experiencing, a cyclic average $\overline{T}$ must be used instead. So

$$\overline{T} = \frac{1}{2\pi} \int_{0}^{2\pi} Td\alpha$$

where

$$T = I \frac{dr}{dt}$$

and $I$ is the moment of inertia, $r$ is the spin rate of the body, and $\alpha$ is the angle of attack [22]. This average calculation should be done because the body is undergoing a dynamic hysteresis effect. This means that the torque on the body cannot be calculated by adding the torques present assuming that the body is static at each one of these angles of attack.
(i.e. static hysteresis). If a static hysteresis analysis was to be done on a flat plate, a $T = 0$ would be the result, which is not the case. As the angle of attack increases the lift and drag increase. At some angle of attack less the $90^\circ$, the flat plate stalls and the lift starts to decrease. The period in which $\alpha$ goes from $0^\circ$ to $90^\circ$ is known as the supporting period because the center of gravity and aerodynamic center do not coincide which gives rise to a torque. When the angle of attack is $90^\circ$ the drag coefficient reaches its maximum and the lift becomes zero. As the angle of attack continues to increase, the drag decreases and the lift coefficient increases the opposite direction. The lift vanishes when the plate is parallel to the flow and the drag generated is only due to skin friction. The retreating edge then becomes the leading edge and the process repeats itself. The period in which $\alpha$ goes from $90^\circ$ to $180^\circ$ is known as the retarding period because the lift generated counteracts the rotation of the plate. Following this analysis leads to the conclusion that the drag force is very high and the lift force per cycle as well as the driving torque is zero. When a plate is autorotating the drag force generated is less than the one resulting from the quasi-steady process described above, and there is a considerable amount of lift since the plate stalls at a much larger angle of attack.

Not much is known about the causes of the autorotation phenomenon but some theories have been developed. The most accepted theory is one in which autorotation is considered to be essentially associated with vortex shedding. From all the research done in autorotation, the following people, Hans J. Lugt, E. H. Smith, A. C. Bustamante, J. D. Iversen, and D. P. Riabouchinski have influenced both the experimental and theoretical ideas. Riabouchinski was probably the first aerodynamicist to consider autorotation as a
area of research. He did a lot of experimental work by using wind tunnels. Smith, Bustamante, and Iversen have also done a lot of experimental work on this area. And finally Lugt concentrated on the theoretical aspect of autorotation and solved numerically the Navier-Stokes equation for an elliptic cylinder with a fixed axis perpendicular to a parallel flow. He analyzed the 2-Dimensional effects, 3-Dimensional effects are yet unknown.

One of the most important factors involved in the study of autorotating bodies is the Strouhal Number (S) which Lugt [23] called the roll parameter \( p \) and is defined as follows:

\[
S = p = \frac{V}{U}
\]  

(4.5)

where \( V = c \omega \) ( \( c \) is the chord length of the plate, and \( \omega \) is the angular velocity of the plate), and \( U \) is the constant speed of the parallel flow.

As shown by Lugt, viscous effects must be taken into account. After Lugt solved numerically the Navier-Stokes equations, the moment coefficient \( C_M \) was found to vary with respect to \( \alpha \) over half-revolution of an elliptical plate for various \( p \) and \( Re = 200 \), as shown in Figure 4.8 [24]. The maximum for \( C_M \) happens at about \( \alpha = 135^\circ \). It should be noticed how the \( C_M \) curve for \( p = 1 \) is shifted up. This shows that there is the presence of a supporting torque.
Fig. 4.8 Comparison of $c_M$ vs. $\alpha$ for various $p$ over one cycle for a thin elliptical cross-section [24]

If the value of $C_M$ is averaged and then plotted versus $p$, one obtains Riabouchinski’s curve (Fig. 4.9) [25]. This curve shows the torque $T$ required to keep a flat plate rotating at a constant $\omega$. This means that there are two values of $p$ at which no torque is required to maintain autorotation. The first value of $p$ at which $T = 0$ is unstable so the conditions for stable autorotation (position A in Fig. 4.5) are

\[
T = 0 \\
\frac{dT}{dp} > 0
\]
The shaded region showed in Figure 4.9 is known as the autorotative region.

During autorotation the wing stalls at a later angle of attack than that of a wing which is not autorotating. It is not known why the stall angle of attack increases, but it is thought to be due to a higher pressure acting on the boundary layer of the upper edge of a clockwise rotating body which prevents the flow from detaching so rapidly. In Figure 4.10 [27], a schematic of the flow pattern over an autorotating wing can be seen. When $\alpha = 135^\circ$ the lift reaches its maximum and a vortex is being shed and the flow around the edge looks similar to the flow past a plate parallel to the stream. This vortex shedding is what is thought to cause autorotation. When the values of $p$ are outside the autorotative region, the frequency of vortex shedding is either smaller (Fig. 4.11a, superharmonic
mode) or larger (Fig. 4.11c, subharmonic mode) [28] than the rate of rotation. A rapidly rotating plate traps vortices before it releases them after several revolutions. When the

![Diagram of vortex shedding](image)

*Fig. 4.10* Schematic flow pattern over an autorotating wing [27]

plate is rotating at a value of \( p \) lower than the autorotative \( p \), the vortex-shedding frequency approaches that of a nonrotating plate. When the frequency of vortex-shedding is the same as the frequency of rotation, the body is considered to be autorotating [29].

![Sketch of vortex shedding](image)

*Fig. 4.11* Sketch of vortex shedding. (a) \( p \) is greater than that for autorotation; (b) autorotative \( p \); (c) \( p \) is slower than that for autorotation [28].
**Important Considerations**

For this study, only some aspects of autorotation needed to be considered. These important aspects will now be stated.

1) For a body to start autorotating an initial disturbance is needed. This disturbance can be either an initial torque or a small angle of attack. If the initial angle of attack is larger than its stalled $\alpha$, the body will not autorotate, instead it will stabilize at $\alpha = 90^\circ$ after several oscillations. In the case of the boomerang, it is impossible to throw it with an $\alpha = 0^\circ$ because the hand cannot follow such a precise motion, which means that there will always be an induced $\alpha$ which is enough to allow autorotation to begin.

2) When a body autorotates, it is of no practical use to know the coefficients of lift and drag as a function of $\alpha$. Instead, average values of these coefficients have been found by Iversen [30], and Smith [31] as functions of the Reynolds Number. For aspect ratios similar to the one of the straight boomerang here analyzed, the lift and drag coefficients were determined from Figure 4.12 [32].

![Diagram](image-url)

*Fig. 4.12* Average lift and drag coefficients vs. Reynolds number for AR = 9 [32]
3) Autorotation begins when Re > 100. The flow over the straight boomerang is Re > 10,000.

4) If

\[ \frac{32}{\pi \rho c^5 A} < 0.1 \]

no autorotation will be present [30], where \( K \) is a moment of inertia parameter. The boomerang here analyzed has a value of \( K = 12.3 \).

5) According to Smith [33], the angular acceleration \( \dot{\theta} \) of a plate from zero to its stable autorotative speed is

\[ \dot{\theta} = C_r r^2 \]

(4.6)

where \( C_r \) is the coefficient of rotation and \( r \) is the angular speed of the body.

6) The Lift/Drag ratio can be determined from the cotangent of the glide angle of an autorotating body.

7) The Strouhal Number approaches a limit of 0.78 as the aspect ratio increases according to Iversen [34].

8) When a body starts autorotating, the lift increases and the drag decreases if compared to a non-autorotating body.

9) The lift generated by an autorotating body is in the same direction as that generated by a rotating cylinder.
4.2 DYNAMICS

4.2.1 Newton’s Laws

In order to study the motion and forces involved in the three dimensional rotation and translation of the straight boomerang, Newton’s second law equations of motion (Eq. 4.7) and Euler’s law (Eq. 4.8) for linear and angular motion of the center of mass must be developed.

\[ F = ma \]  \hspace{1cm} (4.7)

\[ M = \frac{dH}{dt} \]  \hspace{1cm} (4.8)

where \( F \) and \( M \) are the vector sum of the forces and moments acting on the body respectively; \( m \) is the mass of the boomerang; \( a \) is the acceleration of the body; \( \frac{dH}{dt} \) is the time rate of change of the angular momentum (\( H \)) of the boomerang.

These two equations have to be set up from an inertial coordinate system. For reasons stated later on, the equations were developed about an inertial system attached to the body of the boomerang. This coordinate system will be referred to as the body coordinate system (or frame) and it moves with respect to a fixed or inertial reference coordinate system which is fixed to the earth (see Fig. 4.13).
**Linear Acceleration**

Newton’s second law for linear motion (Eq. 4.7) represents the boomerang’s change in linear acceleration due to the forces acting on its body. In this case, the only forces acting are the gravitational and aerodynamic forces. The development of these forces will be discussed later. For now, only the acceleration component will be considered. As mentioned earlier, Newton’s equations of motion require that the forces and time rate of change of the velocity of
the center of gravity (Eq. 4.9) be calculated from a fixed

\[ \sum \mathbf{F} = m \left( \frac{d\mathbf{V}_c}{dt} \right)_{\text{fixed}} \]  

(4.9)

reference frame where \( \mathbf{V}_c \) is the velocity of the center of gravity. In this analysis the body frame is rotating with respect to the fixed reference frame because the boomerang is translating and rotating. It can be shown [35] then that the time rate of change of a vector with respect to a fixed coordinate system is

\[ \left( \frac{d\mathbf{A}}{dt} \right)_{\text{fixed}} = \left( \frac{d\mathbf{A}}{dt} \right)_{\text{rotating}} + \boldsymbol{\omega} \times \mathbf{A} \]  

(4.10)

where \( (d\mathbf{A}/dt)_{\text{rotating}} \) is the change in \( \mathbf{A} \) as seen from the body coordinates and \( \boldsymbol{\omega} \) is the angular velocity of the body coordinate system. So Newton's equation of linear motion becomes

\[ \sum \mathbf{F} = m \left[ \left( \frac{d\mathbf{V}_c}{dt} \right)_{\text{rot}} + \boldsymbol{\omega} \times \mathbf{V}_c \right] \]  

(4.11)

where

\[ \mathbf{V}_c = u \hat{i}_b + v \hat{j}_b + w \hat{k}_b \]  

(4.12)

\[ \left( \frac{d\mathbf{V}_c}{dt} \right)_{\text{rot}} = \dot{u} \hat{i}_b + \dot{v} \hat{j}_b + \dot{w} \hat{k}_b \]  

(4.13)

\[ \boldsymbol{\omega} = p \hat{i}_b + q \hat{j}_b + r \hat{k}_b \]  

(4.14)

Equations 4.12 and 4.14 are written in terms of the components of each vector as seen from the
body coordinate system. By expanding equation 4.11, the following is obtained:

\[
\sum \mathbf{F} = m \begin{bmatrix}
\hat{u} \mathbf{i}_b + \hat{v} \mathbf{j}_b + \hat{w} \mathbf{k}_b
\end{bmatrix} + \begin{bmatrix}
\hat{\mathbf{i}}_b & \hat{\mathbf{j}}_b & \hat{\mathbf{k}}_b
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\begin{bmatrix}
\mathbf{u} \\
\mathbf{v} \\
\mathbf{w}
\end{bmatrix}
\]

which leads to

\[
F_x = m(\dot{u} + qw - vr)
\]
\[
F_y = m(\dot{v} + ru - pw)
\]
\[
F_z = m(\dot{w} + pv - qu)
\]

Again, \(F_x\), \(F_y\), and \(F_z\) represent the gravitational and aerodynamic forces along the \(x\), \(y\), and \(z\) body axis, respectively. The dotted variables mean a time rate of change.

**Angular Acceleration**

The same procedure is now applied to the angular momentum equation. In this case Euler’s law for angular momentum (Eq. 4.8) is applied. \(\mathbf{M}\) is the sum of torques acting on the body, which in the boomerang’s case are all due to the aerodynamic lift and drag forces along the cross-section. The gravitational force does not cause any torque on the body because the body axis is located at the center of gravity. These torques will be discussed later on. For now, only the time rate of change of the angular momentum (\(\mathbf{H}\)) will be considered.
It can be shown that [36]

\[ \mathbf{H} = \mathbf{I} \cdot \mathbf{\omega} \]  \hspace{1cm} (4.16)

where \( \mathbf{I} \) is the moment of inertia tensor and is equivalent to

\[
\mathbf{I} = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz!}
\end{bmatrix}
\]  \hspace{1cm} (4.17)

and \( \mathbf{\omega} \) is the angular rotation of the boomerang which is defined by equation 4.14. So:

\[
\mathbf{H} = \mathbf{I} \cdot \mathbf{\omega}
\]

\[
H_x = I_{xx} p + I_{xy} q + I_{xz} r
\]

\[
H_y = I_{yx} p + I_{yy} q + I_{yz} r
\]

\[
H_z = I_{zx} p + I_{zy} q + I_{zz} r
\]  \hspace{1cm} (4.18)

In order to calculate the time rate of change of the angular momentum, it must be noted that the angular momentum must be measured from the fixed reference frame. So the same method used for the linear acceleration applies in this case and the rate of change of the angular momentum from the fixed frame can be calculated.

\[
\left( \frac{d\mathbf{H}}{dt} \right)_{\text{fixed}} = \left( \frac{d\mathbf{H}}{dt} \right)_{\text{rotating}} + \mathbf{\omega} \times \mathbf{H}
\]

Thus, the equation of rotational motion can be rewritten as

\[
\mathbf{M} = \left( \frac{d\mathbf{H}}{dt} \right)_{\text{rotating}} + \mathbf{\omega} \times \mathbf{H}
\]  \hspace{1cm} (4.19)
where

\[
\left( \frac{d\mathbf{H}}{dt} \right)_{\text{rotating}} = \mathbf{I} \cdot \omega + \mathbf{I} \cdot \dot{\omega} \tag{4.20}
\]

Since the time rate of change of the angular momentum is measured from the body axis, the moment of inertia tensor remains constant with time, which allows equation 4.20 to reduce to

\[
\dot{\mathbf{H}} = \mathbf{I} \cdot \dot{\omega} \tag{4.21}
\]

Equation 4.19 can now be rewritten as

\[
\mathbf{M} = \mathbf{I} \cdot \dot{\omega} + \omega \times (\mathbf{I} \cdot \omega) \tag{4.22}
\]

If the time rate of change was to be measured from the fixed axis, the second part of the right hand side of equation 4.19 would not be needed, but the inertia tensor would change with time. In this case, the moment equation would be

\[
\mathbf{M} = \dot{\mathbf{I}} \cdot \omega + \mathbf{I} \cdot \dot{\omega}
\]

This equation introduces the difficulty involved in calculating the time rate of change of the inertia tensor. That is one of the reasons why Newton’s equations of motion are solved from the body axis. Another reason is because the lift and drag forces are determined from the air’s relative velocity around the boomerang.

Before expanding equation 4.22, another simplification must be made. The coordinate system may be attached to the boomerang in such a way that the products of inertia (I_{xy}, I_{xz}, I_{yz}) from the moment of inertia tensor may be ignored. This can be done because the Euler angles for both the symmetry axes and the principal axes will be the same since both coordinate systems are attached to the body. The axes of this coordinate
The system are said to be the principal axes of the body (see Fig. 4.13). The moment of inertia tensor reduces to

\[
I = \begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}
\]

Since the shape of the boomerang is that of a uniform thin rectangular lamina, the principal axes point parallel each side of the body. This is very convenient because it is about these axis that the rotations of the boomerang can be more easily visualized. Therefore, the principal axes of the body coincides with the body axes (see Fig. 4.13).

Now the moment equation can be rewritten as

\[
M = \begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} + \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} \times \begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

\[
M_x = I_{xx}p + qr(I_{zz} - I_{yy})
\]
\[
M_y = I_{yy}q + pr(I_{xx} - I_{zz})
\]
\[
M_z = I_{zz}r + pq(I_{yy} - I_{xx})
\]

Now, equations 4.15 and 4.25 can be used to calculate the effect that the forces and moments applied to the body have on the linear and angular accelerations.
4.2.2 Precession

The boomerang (both straight and aboriginal) behave as gyroscopes because they rotate at high angular velocities about an instantaneous center passing through a fixed point. The gyroscope has the property that if it is spinning say about its z-axis (see Fig. 4.14 [37]) and a torque about its x-axis is applied, a rotation about the y-axis will occur. This skewing around of the spin axis is called precession. Precession can be understood in one of two ways. It can be seen to arise as in obedience to the fundamental relation

\[ M = \frac{dH}{dt} \]

where \( M \) is the sum of the moments applied to the body, and \( H \) the angular momentum. So a torque in the x-axis applied to a body that is rotating about the z-axis, will generate a rate of change in the angular momentum along the x-axis. This causes the \( \omega \) vector to precess about the y-axis.

Fig. 4.14 Gyroscope spinning about its z-axis [37]
The other way of looking at precession is in a more physical form. A body spinning about its x-axis (see Fig. 4.15) with a torque applied to its z-axis will experience coupled forces at locations 1 and 3 in the y direction. The torque applied to the body can be thought of two forces (F) opposite in direction at locations 1 (+F) and 3 (-F). As point A, which is fixed to the body, rotates to location 2, the torque component due to the force at 1, diminishes to zero. Since point A experiences a negative force between 1 and 2, it achieves its maximum velocity in the -x direction, at location 2. As point A enters the region between locations 2 and 3, the component of the applied force at location 3, starts decelerating point A until it reaches a velocity equal to zero. As A moves from 3 to 4, the equivalent +F has more effect on the A’s

![Diagram of body spinning about its x-axis with a torque applied about its z-axis](image)

*Fig. 4.15* Body spinning about its x-axis with a torque applied about its z-axis
velocity and accelerates it until it reaches a maximum velocity at location 4 in the \(+x\) direction. Between 4 and 1 the force decreases to \(-F_{\text{max}}\) and point \(A\) is decelerated to a velocity of zero. As it can be seen, a moment about the \(z\)-axis for a body rotating about its \(x\)-axis, generates a net angular velocity about the \(y\)-axis. And again, this is what is known as precession.

Two examples of precession can be observed when riding a bicycle or motorcycle (see Fig. 4.16). When the person is riding on a bicycle at a low speed, the wheel spins about the \(x\)-axis slowly. If the rider wants to turn to his/her left, he/she only needs to tilt

\[\text{Fig. 4.16 Bicycle or motorcycle with } z\text{-axis coming out of the page}\]

his/her body to the left. This generates a torque about the negative \(z\)-axis. Since the wheel is spinning, it will precess about the positive \(y\)-axis which will turn the bike towards the left. This is known as the no-hands turning. The second example of precession which can be experienced more easily while riding a motorcycle than a bicycle, is one which most riders are not aware of. When riding at a faster speed (i.e. high \(x\)-axis angular velocity), a
simple shift in the body position towards the side where the rider wants to go, does not
generate a torque sufficiently high enough to cause the wheel to precess about the $y$-axis.
In this case, if the rider wants to turn left, he/she must apply a torque to the handle-bar in
the negative $y$-axis direction. This generates a torque that will cause the wheel and
therefore the whole motorcycle to precess about the negative $z$-axis. Since the torque
applied to the handle-bar is not very high, the wheel doesn’t turn a considerable amount
about the $y$-axis. It is important to understand the difference between these two cases
because they will serve later on as an analogy that will clarify one of the basic differences
between the physics governing the straight and the regular boomerang.

4.2.3 Eulerian Angles

The equations previously developed will generate results (i.e. position, velocity,
acceleration) expressed on the body coordinate system. Since this coordinate system is
rotating, it is very difficult to visualize these results. The vectorial results must be
transformed to the fixed axes which is where an outside observer would be. To allow
these results to be transformed into the fixed reference frame, the Eulerian angles are
used.

The Eulerian angles ($\theta$, $\phi$, $\psi$) connect the moving axes to the fixed axes as shown in
Figure 4.17 [38].
At time $t = 0$ the body axes is at position $X_f$, $Y_f$, $Z_f$, which coincides with the orientation of the fixed reference frame. The orientation of the boomerang can be described by rotating the body axes through the angles $\psi$, $\theta$, $\phi$ in that same order. Imagine the body axes to coincide with the fixed axes and then apply the following rotations:

1) Rotate the $x_f$, $y_f$, $z_f$ system about $oz_f$ through the yaw angle $\psi$ to the frame $x_1$, $y_1$, $z_1$.

2) Rotate the frame $x_1$, $y_1$, $z_1$ about $oy_1$ through the pitch angle $\theta$ bringing the frame to $x_2$, $y_2$, $z_2$.

3) Rotate the $x_2$, $y_2$, $z_2$ frame about $ox_2$ through the roll angle $\phi$ to bring the frame to $x_3$, $y_3$, $z_3$, the actual orientation of the body frame relative to the fixed frame.

This procedure outlines the definition of the Eulerian angles which should be followed in the specified order of rotation [39].
Velocities

After defining the Eulerian angles, the velocity vector as viewed from the body axes can be expressed in terms of the fixed axes and vice versa. To do this, a transformation matrix that takes any vector whose components on the body axes are known and expresses it in the fixed reference frame in terms of the Eulerian angles, must be developed. The following transformation matrix will be developed by referring to Figure 4.17. The unit vector in the fixed axes will be rotated to the axes $I$ through the Eulerian angle $\psi$, then rotated to axes 2 through the Eulerian angle $\theta$, and finally to the body frame by rotating axes 2 through the angle $\Phi$.

Rotation from axes 1 to the fixed axes:

\[
\begin{bmatrix}
\hat{i}_f \\
\hat{j}_f \\
\hat{k}_f
\end{bmatrix} =
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{i}_1 \\
\hat{j}_1 \\
\hat{k}_1
\end{bmatrix}
\]

(4.26)

The transformation matrix in Eq. 4.26 transforms any vector with known components in the fixed reference frame, into its respective components of axes $I$. At this point it is convenient to introduce a change in notation. From now on, cosine, sine, and tangent, will be written as C, S, T, respectively for clarity reasons.
Rotation from axes 2 to axes 1:

\[
\begin{align*}
\begin{bmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix} &= \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} \hat{i}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{bmatrix} \\
\end{align*}
\]  
(4.27)

Rotation from body axes to axes 2:

\[
\begin{align*}
\begin{bmatrix} \hat{i}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} \hat{i}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix} \\
\end{align*}
\]  
(4.28)

Now, by back substitution of equation 4.28 into equation 4.27 and then into 4.26, the following is obtained:

\[
\begin{align*}
\begin{bmatrix} \hat{i}_f \\ \hat{j}_f \\ \hat{k}_f \end{bmatrix} &= \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} \hat{i}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix} \\
\end{align*}
\]  
(4.29)

After carrying out the previous multiplication, the transformation matrix from body to fixed coordinate frame \( T_{b \cdot f} \) is obtained.

\[
T_{b \cdot f} = \begin{bmatrix} C\theta C\psi & S\phi S\theta C\psi - C\phi S\psi & C\phi S\theta C\psi + S\phi S\psi \\ C\theta S\psi & S\phi S\theta S\psi + C\phi C\psi & C\phi S\theta S\psi - S\phi C\psi \\ -S\theta & S\phi C\theta & C\phi C\theta \end{bmatrix}
\]  
(4.30)

**Angular Velocities**

Since the equations of motion previously developed involve the knowledge of the body rotations as viewed from the body \( (p, q, r) \), then these must also be expressed in terms of the Euler rates \( (\dot{\psi}, \dot{\theta}, \dot{\phi}) \). From Figure 4.17 these can be readily developed by...
projecting the Euler rates into the body coordinate system. The following are the results of this relationship:

\[
\begin{align*}
\begin{bmatrix} p \\ q \\ r \end{bmatrix} &= \begin{bmatrix} 1 & 0 & -S\theta \\ 0 & C\phi & C\theta S\phi \\ 0 & -S\phi & C\theta C\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}
\end{align*}
\]  
(4.31)

Equation 4.31 can also be solved for the Euler rates in terms of the body angular velocities and is given in Equation 4.32

\[
\begin{align*}
\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} &= \begin{bmatrix} 1 & S\phi T\theta & C\phi T\theta \\ 0 & C\phi & -S\phi \\ 0 & S\phi \sec\theta & C\phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}
\end{align*}
\]  
(4.32)

So with the aid of the Euler angles, any known vector can be expressed in either the body or the fixed coordinate system.
SECTION 5

EXPERIMENTAL OBSERVATIONS
5.1 EXPERIMENTAL OBSERVATIONS

Two different kinds of experiments were done. One required the design of a machine that would throw the boomerang in a reproducible manner. The other was using a video camera to capture the motion in a detailed manner.

5.1.1 Boomerang Throwing Machine

After manually throwing the boomerang many times it was determined that the way in which it was thrown affected the flight dramatically. These initial conditions were too difficult to control manually. The initial conditions included the orientation of the boomerang upon release, and the initial angular and linear velocity. This lead to the conclusion that a machine that would control these initial conditions had to be designed. The machine had to be as versatile as possible so that all the initial conditions could be varied. One of the problems encountered was that it was of great importance that the machine would imitate the human arm because the snap of the wrist is necessary for the boomerang to return. The wrist increases the angular velocity of the boomerang without increasing its linear velocity too much. The machine, which can be seen in Figure 5.1 is operated by rubber bands. The machine can control the angular velocity of the arm, the angular velocity of the wrist, and the release angle in a reproducible matter. The machine was calibrated by making it throw an object which would follow parabolic motion and from the distance it traveled, the velocity at which it was launched was determined. Also,
the spring coefficients and the amount by which the rubber bands were stretched was also known which allowed the arm’s velocity to be calculated. This machine was only designed to determine what the boomerang does under certain conditions and what parts of the flight are common to any combination of initial conditions. It was not used to provide information like time of flight, or distance of flight.

![Boomerang throwing machine](image)

*Fig. 5.1* Boomerang throwing machine

**5.1.2 Video Capture**

The pictures of the flight of the boomerang used in this work were taken from a TV which played a video taken at the Rochester Institute of Technology’s High Speed Photography Lab under the supervision of Dr. Andrew Davidhazy. The video camera captured images at a frequency rate of 30 Hz and 60 Hz. This frequency was sufficient to determine some aspects of the flight. The most important information captured in this
video is the fact that the straight boomerang starts autorotating as soon as it leaves the hands of the thrower (see Figs. 5.2 and 5.3)
Fig. 5.2 Side-view of the flight of the straight boomerang

Fig. 5.3 Flight of the straight boomerang from the thrower's point of view
5.2 OBSERVATIONS

After conducting many throws with the machine, it was concluded that even though the flight of the boomerang is greatly influenced by the initial conditions (i.e. initial linear and rotational velocity, initial orientation), there are some aspects of the flight that can be predicted. The most important of these observations are:

1) The boomerang needs to leave the thrower with an angular velocity. If there is no initial angular velocity the boomerang will not return. Instead it will follow a curved path if the trajectory is projected onto the $x_f - y_f$ plane.

2) If the initial Euler angle $\phi$ is in between $\pm 30^\circ$, the boomerang returns as long as the other initial conditions are not greatly varied.

3) If the initial Euler angle $\theta$ is between $0^\circ$ and $60^\circ$, the boomerang returns as long as the other initial conditions are not greatly varied.

4) If the absolute value of the initial Euler angle $\psi$ is greater than $10^\circ$, the boomerang will not return.

5) The boomerang always tips after it completes at least $1/2$ of a revolution about its $y_b$ axis and at the most $3/4$ of a revolution.

6) The boomerang can return to the thrower with either its $z_b$ axis pointing to the right or to the left of the thrower. What determines the final orientation of the $z_b$ axis is the initial $\psi$ given.
7) If the initial $0^\circ > \psi > 15^\circ$, the $z_b$ axis will point towards the left of the thrower during its return flight.

8) If the initial $-15^\circ < \psi < 0^\circ$, the $z_b$ axis will point towards the right of the thrower during its return flight.

*Note:* For right hand throwers it seems as though observation 7 is always true and for left hand throwers observation 6 seems to always hold. The reason for this is because right hand throwers give the boomerang an initial $\psi < 0^\circ$ without noticing it and the opposite happens with left hand throwers.

9) If the angular and linear velocities are not carefully balanced, it is very easy to obtain a return flight in which the boomerang starts going in a corkscrew during Stage III (see next section) and does not return to the thrower.

10) If the initial $-15^\circ < \psi < 0^\circ$ and $\phi$ is negative and large (i.e. $-15^\circ$), the initial angular velocity has to be higher than if $\phi$ is closer to $0^\circ$. 
5.3 STAGES OF FLIGHT

In order to analyze the flight of the straight boomerang, its trajectory was divided into three different stages (see Fig. 5.4).

Stage I:
The boomerang leaves the hand of the thrower and it completes about 1/2 of a revolution about its $y_b$ axis.

Stage II:
The boomerang ‘tumbles’ and it changes its rotation axis from it being the $x_b$ and $y_b$ axis to the $z_b$ axis.

Stage III:
The boomerang only presents rotation about its $z_b$ axis and it returns to the thrower.
Fig. 5.4 Stages of Flight
SECTION 6

MODEL DEVELOPMENT
6.1 DESCRIPTION

After examining the pictures obtained, and studying the results of many throws it was concluded that the boomerang flies back to the thrower thanks to a combination of aerodynamics and dynamics. The aerodynamics involved was that of autorotation. Now, a detailed explanation of what happens in each one of the three stages of the flight, will follow. Since the boomerang can start autorotating in either direction about its $z_b$ axis, for the following explanation assume that the initial conditions are the following:

- $\psi < 0^\circ$ and small
- $\theta \approx 0^\circ$
- $\phi \approx 0^\circ$
- $V = \hat{x} \hat{i}_f$
- $\omega = -q \hat{j}_b$

In order to make the following explanation easier to follow, assume that the body axes are attached to the boomerang but not forced to rotate about the $z_b$ axis. That means that the $x_b$ axis will trace a circle in the plane during Stage I. This assumption was not made for the mathematical model developed.

**Stage I:**

In this stage the boomerang leaves the hands of the thrower with angular and linear velocity. As soon as the air hits the boomerang in the air, autorotation begins to develop ($-r$, because the initial $\psi < 0^\circ$). Even though at this stage the boomerang is not yet autorotating, the air applies an external torque about the $-z_b$ axis which starts accelerating
the body's angular velocity about this axis. During this stage, only the drag needs to be considered because since the body is not autorotating, no lift is generated (see Section 4.1.2). The initial \( q \) rotation is being reduced and the body's \( r \) angular velocity continues to accelerate to its autorotating speed. The boomerang undergoes a \( 1/2 \) to \( 3/4 \) of a revolution about its \( y_b \) before it achieves its autorotating angular speed. When a body reaches its autorotating speed, there is no external torque required for it to maintain this angular speed.

\[
\begin{align*}
\text{Note: Since the axes are assumed not to be autorotating with the body, after 1/2 of a revolution, the} \; x_b \; \text{axis will be pointing towards the ground and the} \; y_b \; \text{axis to the right of the thrower.}
\end{align*}
\]
Stage II:

This stage begins as soon as the body starts autorotating. At this point the magnitude of the drag force acting on the body decreases and the lift force starts being generated. This lift force now causes a torque that will eventually tumble the boomerang into its final position. At this stage is where one of the main differences between the regular and the straight boomerang can be appreciated. The regular boomerang is traveling so fast through the air with such a small drag force opposing its rotational motion that the torque caused by the lift force will make it precess in the same way that it happens when riding a motorcycle at a high velocity. For an explanation of why the aerodynamic forces cause a torque on the body, see Sections 6.2.2 and 6.2.3.

On the other hand, the straight boomerang experiences the torque due to the lift force when it has a very low \( q \) rotation. The effect of this is similar to the one of the no-hands turning on a bicycle. The body’s angular momentum is not enough to make it precess about its third axis, instead it just tilts in the same direction in which the torque is being applied (see Fig. 6.2). For the initial conditions previously established, the lift force will cause a torque about the \(-x_b\) axis (see Fig. 6.2). This result agrees with the experimental observation #7. At this point the higher rotational velocity on the \(+y_b\) side of the boomerang (Fig. 6.4), causes an aerodynamic torque in the \(-x_b\) direction as seen in Figure 6.3. A similar result will be obtained if the initial \( \psi \) is chosen to be positive and small.
**Stage III:**

During this stage the boomerang still has some horizontal velocity and most of its initial angular momentum has damped out due to aerodynamic effects and its axis of rotation is now $z_b$. Now the boomerang starts to return thanks to the Magnus Effect. This Magnus Effect can be seen in Figure 6.5 which is a result of the autorotation of the boomerang and of the gravitational pull on the body. Gravity now starts changing the
flight path because the horizontal velocity is small and the drag and lift forces are high. The drag force is always acting in the direction opposite to the velocity and the lift force is always pointing towards the center of the curved path. The resultant of the lift force acting perpendicular to the body’s velocity is a curved path (Fig. 6.4 [39]). The boomerang follows this curved path until the lift force is not strong enough to make it do a loop. The gravitational force starts pulling the boomerang down which makes it glides back to the thrower following a constant glide angle.

*Fig. 6.4* The Magnus Effect on the boomerang with lift force and rotation [39]

If the boomerang has a low aspect ratio (i.e. < 10), it will not come back as accurately. The reason being that for such a low aspect ratio the fluid velocity needed to make the boomerang autorotate has to be very high. If such a high speed is provided initially, the resultant drag force after release is high enough to dampen out the initial rotation about the $y_b$ axis. The boomerang then starts gliding away from the thrower.
6.2 MATHEMATICAL MODEL

From equations 4.15 and 4.21 derived in sections 4.2.1 and 4.2.2 respectively, shown below, the mathematical model will be developed.

\[ F_x = m(\dot{u} + qw - vr) \]
\[ F_y = m(\dot{v} + ru - pw) \]
\[ F_z = m(\dot{w} + pv - qu) \]

\[ T_x = I_{xx} \dot{p} + q r (I_{zz} - I_{yy}) \]
\[ T_y = I_{yy} \dot{q} + p r (I_{xx} - I_{zz}) \]
\[ T_z = I_{zz} \dot{r} + p q (I_{yy} - I_{xx}) \]

In order to develop the mathematical model, the left hand side of the two previous equations must be calculated. The forces \((F_x, F_y, F_z)\) involved are due to aerodynamics and gravity. The torques \((T_x, T_y, T_z)\) involved are only due to aerodynamics. Since the body axis is located at the center of gravity of the boomerang, its weight does not cause any torque.

6.2.1 Gravity

From Figure 6.5 [40], the force due to gravity in terms of the Euler angles can be determined.
The components of the gravitational force on the body coordinate system are:

\[ F_{x\text{ gravity}} = -mg \sin \theta \]

\[ F_{y\text{ gravity}} = mg \cos \theta \sin \phi \]

\[ F_{z\text{ gravity}} = mg \cos \theta \cos \phi \]  

(6.1)

6.2.2 Aerodynamic Forces

Firstly, the lift and drag forces, will be developed, and secondly, their \( x_b \) and \( y_b \) components will be calculated.
The aerodynamic forces are:

\[
\begin{align*}
F_L &= \frac{1}{2} \rho C_L V^2 A \\
F_D &= \frac{1}{2} \rho C_D V^2 A
\end{align*}
\]  

(6.2)

where \( \rho \) is the density of air (kg/m\(^3\)), \( C_L \) and \( C_D \) are the coefficients of lift and drag respectively, \( V \) is the magnitude of the local velocity of the air, and \( A \) is the blockage area which in this case is span x chord length.

In the case of the boomerang, since it’s center of gravity is translating and the body is rotating, the velocity varies along the its span (Fig. 6.6). If the boomerang is divided into differential airfoils of width \( dz \) located at a distance \( z \) away from the center

![Fig. 6.6 Variation of the air's velocity along the boomerang](image_url)
of gravity, then the relative velocity of each airfoil section P is calculated as follows:

\[ \mathbf{V}_p = \mathbf{V}_{eg} + \omega \times \hat{z} + \begin{vmatrix} \hat{i}_b & \hat{j}_b & \hat{k}_b \\ i & j & k \\ 0 & 0 & z \end{vmatrix} \]

\[ \mathbf{V}_p = (u + qz)\hat{i}_b + (v - pz)\hat{j}_b + w\hat{k}_b \] (6.3)

Since 2-Dimensional aerodynamic effects are being considered, only the relative velocity components on the \( x_b-y_b \) plane will be taken into account. So:

\[ \mathbf{V}_{rel}^2 = (u + qz)^2 + (v - pz)^2 \]

\[ \mathbf{V}_{rel}^2 = (p^2 + q^2)z^2 + (2uq - 2vp)z + (u^2 + v^2) \] (6.4)

The area \( A \) in Eq. 6.2 is

\[ A = c \, dz \]

Now the equation for the differential force at a distance \( z \) can be written as:

\[ dF_L = \frac{1}{2} \rho C_L \left[ (p^2 + q^2)z^2 + (2uq - 2vp)z + (u^2 + v^2) \right] c \, dz \] (6.5)

Equation 6.5 is now integrated over the entire span \( b \) of the boomerang.

\[ F_L = \frac{1}{2} \rho C_L c \int_{-b/2}^{b/2} [(p^2 + q^2)z^2 + (2uq - 2vp)z + (u^2 + v^2)] \, dz \] (6.6)

The second term in Eq. 6.6 is odd so it vanishes. The result of this integration is:

\[ F_L = \frac{1}{2} \rho C_L c \left[ (u^2 + v^2)b + (p^2 + q^2) \frac{b^3}{12} \right] \] (6.7)

Equation 6.7 is the magnitude of the total of all the differential lift forces acting along the
surface of the boomerang. Similarly, the magnitude of the drag force is

\[ F_D = \frac{1}{2} \rho C_D c \left[ (u^2 + v^2)b + \left( p^2 + q^2 \right) \frac{b^3}{12} \right] \]

(6.8)

Now that the magnitudes have been determined, their \( x_b \) and \( y_b \) components must be found. For a body autorotating in the \(-z_b\) direction, the forces can be found by analyzing Figure 6.7.

![Diagram](image)

**Fig. 6.7** Forces acting on the autorotating boomerang

The components are then

\[ \begin{align*}
F_{x\text{ aerodynamic}} &= F_L \sin \alpha - F_D \cos \alpha \\
F_{y\text{ aerodynamic}} &= -F_L \cos \alpha - F_D \sin \alpha
\end{align*} \]

(6.9)

where \( \alpha \) is the angle of attack. These aerodynamic force components were derived assuming that the boomerang is autorotating in the negative \( z_b \) direction. If the boomerang was assumed to start autorotating about the positive \( z_b \) axis, the lift force would have to
be rotated by 180°. Since the body is flying in still air the wind velocity is the same as the relative velocities. Based on this assumption it can be determined from Fig. 6.7 that

\[
\sin \alpha = \frac{v}{\sqrt{u^2 + v^2}} \\
\cos \alpha = \frac{u}{\sqrt{u^2 + v^2}}
\]  

(6.10)

Finally, the \(x_b\) and \(y_b\) components of the aerodynamic and gravitational forces can be simplified to

\[
F_x = -mg \sin \theta + \left\{ \frac{1}{2} \rho c \left[ (u^2 + v^2)b + (p^2 + q^2) \frac{b^3}{12} \right] \frac{C_L v - C_D u}{\sqrt{u^2 + v^2}} \right\}
\]

\[ \tag{6.11} \]

\[
F_y = -mg \cos \theta \sin \phi + \left\{ \frac{1}{2} \rho c \left[ (u^2 + v^2)b + (p^2 + q^2) \frac{b^3}{12} \right] \frac{C_L u - C_D v}{\sqrt{u^2 + v^2}} \right\}
\]

\[ \tag{6.12} \]

\[
F_z = mg \cos \theta \cos \phi
\]

\[ \tag{6.13} \]

### 6.2.3 Torques

The torques can be added to the model in either one of two ways. One way is by applying the forces previously determined forces at a resultant distance \(z\). The results would be as follows:

\[
\vec{T} = \vec{r} \times \vec{F}
\]

\[
\begin{vmatrix}
\hat{i}_b & \hat{j}_b & \hat{k}_b \\
0 & 0 & z \\
F_x & F_y & 0
\end{vmatrix}
\]

\[
\vec{T} = -F_y z \hat{i}_b + F_x z \hat{j}_b
\]

\[ \tag{6.14} \]
It should be noted that as the boomerang rotates, the appropriate \( z \) will change so its value in Eq. 6.14 would have to be shifted accordingly. This would provide a model more difficult to model and less accurate.

The other way of computing the torques due to the aerodynamic forces is by calculating the torque due to every differential force applied at a distance \( z \) from the center of gravity.

\[
\mathbf{d\tau} = \mathbf{\vec{r}} \times \mathbf{d\vec{F}}
\]

\[
\begin{vmatrix}
\hat{i}_b & \hat{j}_b & \hat{k}_b \\
0 & 0 & z \\
dF_x & dF_y & 0
\end{vmatrix}
\]

\[
d\mathbf{\tau} = -dF_y z \hat{i}_b + dF_x z \hat{j}_b
\]  

(6.15)

where

\[
dF_x = dF_L \sin \alpha - dF_D \cos \alpha
\]

\[
dF_y = -dF_L \cos \alpha - dF_D \sin \alpha
\]

The differential lift force was previously defined in Eq. 6.5 and the drag force can be similarly defined, and the \( \cos \alpha \) and \( \sin \alpha \) were defined in Eq. 6.10.

Each component of the torque from equation 6.15 will be integrated separately.

\[
T_x = \frac{1}{2} \rho c \frac{C_L u + C_D v}{\sqrt{u^2 + v^2}} \int_{-b/2}^{b/2} \left[ (p^2 + q^2)z^2 + (2uq - 2vp)z + (u^2 + v^2) \right] dz
\]

(6.16)

The first and third terms of equation 6.16 are odd so they vanish after the integration. The
total torque about the \( x_b \) axis is then

\[
T_x = \frac{\rho c b^3 (C_L u + C_D v) (uq - vp)}{12\sqrt{u^2 + v^2}} \quad (6.17)
\]

Similarly, \( T_y \) is found to be

\[
T_y = \frac{\rho c b^3 (C_L v - C_D u) (uq - vp)}{12\sqrt{u^2 + v^2}} \quad (6.18)
\]

Finally, the autorotation torque needs to be computed. This torque is in charge of accelerating the body’s angular velocity to its autorotational speed. According to Eq. 4.6, as mentioned previously, it was determined experimentally that

\[
\dot{\rho} = C_r r^2 \quad (6.19)
\]

where \( C_r \) is a constant. By using equation 4.5, and equation 6.4, equation 6.19 becomes

\[
\dot{\rho} = C_r \left( \frac{S}{c} \right)^2 \left[ (u^2 + v^2) + (2uq - 2vp)z + (p^2 + q^2)z^2 \right] \quad (6.20)
\]

Since Eq. 6.20 is a function of \( z \), an average over the entire span of the boomerang must be found.

\[
\dot{\rho} = C_r \left( \frac{S}{c} \right)^2 \frac{1}{b} \left[ (u^2 + v^2) + (2uq - 2vp)z + (p^2 + q^2)z^2 \right] dz \quad (6.21)
\]

The second integrand of equation 6.21 is odd so it vanishes, leaving

\[
\dot{\rho} = C_r \left( \frac{S}{c} \right)^2 \left[ (u^2 + v^2) + (p^2 + q^2) \frac{b^2}{12} \right] \quad (6.22)
\]
Finally, the torque about the \( z_b \) axis is

\[
T_z = I_z C_r \left( \frac{S}{c} \right)^2 \left( u^2 + v^2 \right) + \left( p^2 + q^2 \right) \frac{b^2}{12}
\]  

(6.23)

The final mathematical model is then

\[
m(\dot{u} + qw - vr) = -mg \sin \theta + \left\{ \frac{1}{2} \rho c \left[ (u^2 + v^2)b + (p^2 + q^2) \frac{b^3}{12} \right] \frac{C_L v - C_D u}{\sqrt{u^2 + v^2}} \right\}
\]

\[
m(\dot{v} + ru - pw) = -mg \cos \theta \sin \phi + \left\{ \frac{1}{2} \rho c \left[ (u^2 + v^2)b + (p^2 + q^2) \frac{b^3}{12} \right] \frac{-C_L u - C_D v}{\sqrt{u^2 + v^2}} \right\}
\]

\[
m(\dot{w} + pv - qw) = mg \cos \theta \cos \phi
\]

\[
I_{xx} \dot{p} + qr \left( I_{zz} - I_{yy} \right) = \frac{\rho c b^3 (C_L u + C_D v) (uq - vp)}{12 \sqrt{u^2 + v^2}}
\]

\[
I_{yy} \dot{q} + pr \left( I_{xx} - I_{zz} \right) = \frac{\rho c b^3 (C_L v - C_D u) (uq - vp)}{12 \sqrt{u^2 + v^2}}
\]

\[
I_{zz} \dot{r} + pq \left( I_{yy} - I_{xx} \right) = I_{zz} C_r \left( \frac{S}{c} \right)^2 \left[ (u^2 + v^2) + (p^2 + q^2) \frac{b^2}{12} \right]
\]

(6.24)

Equations 6.24 represent the six degrees of freedom the straight boomerang has including all the gravitational and aerodynamic forces and torques.
6.3 MATLAB

The simulation of the flight of the straight boomerang was done using *MATLAB* which is a very powerful numeric computation, data analysis, and visualization software. In order to run the simulation three files were created. One of the files (*throw.m*, see Appendix) was in charge of posing the initial conditions. The function *ode23* was used in this file which executes a Runge-Kutta-Fehlberg integration method and solves simultaneously the non-linear differential equations found in the file *simavel.m* [41]. These differential equations will be developed from the equations of motion of the boomerang later on. The third file, *simcol.m*, plots the results obtained from the simulation.

*MATLAB* requires that the non-linear differential equations be expressed as a set of first order differential equations:

\[
\frac{dx}{dt} = f(t,x) \tag{6.25}
\]

where \( t \) is usually time, \( x \) is the state vector, and \( f \) is a function that returns the state derivatives as a function of \( t \) and \( x \).

In the case of the straight boomerang, its flight is governed by six equations of motion (Eqs. 6.24) and 9 variables (\( u, v, w, p, q, r, \psi, \theta, \phi \), and their derivatives) which need to be expressed as first order differential equations. In order to put these equations in the form of equation 6.25, each one of the variables has to be solved for. Also, three more
equations need to be used in order to have 9 unknowns and 9 equations. The three equations missing are Eqs. 4.32 which express the Euler rates in terms of the body rotations. After rearranging all these equations, the results are as follows:

\[
\dot{u} = -qw + rv - g \sin \theta + \left\{ \frac{1}{2m} \rho c \left[ (u^2 + v^2) b + (p^2 + q^2) \frac{b^3}{12} \right] \frac{C_L v - C_D u}{\sqrt{u^2 + v^2}} \right\},
\]

\[
\dot{v} = -ru + pw + g \cos \theta \sin \phi + \left\{ \frac{1}{2m} \rho c \left[ (u^2 + v^2) b + (p^2 + q^2) \frac{b^3}{12} \right] \frac{-C_L u - C_D v}{\sqrt{u^2 + v^2}} \right\},
\]

\[
\dot{w} = -pv + qw + mg \cos \theta \cos \phi
\]

\[
\dot{\rho} = -q \rho \left( \frac{I_{zz} - I_{yy}}{I_{xx}} \right) + \frac{\rho c b^3 (C_L u + C_D v)(uq - vp)}{12 I_{xx} \sqrt{u^2 + v^2}}
\]

\[
\dot{q} = -p \rho \left( \frac{I_{xx} - I_{zz}}{I_{yy}} \right) + \frac{\rho c b^3 (C_L v - C_D u)(uq - vp)}{12 I_{yy} \sqrt{u^2 + v^2}}
\]

\[
\dot{r} = -pq \left( \frac{I_{yy} - I_{xx}}{I_{zz}} \right) + C_f \left( \frac{S}{c} \right) \left[ (u^2 + v^2) + (p^2 + q^2) \frac{b^2}{12} \right]
\]

\[
\dot{\theta} = q C \phi - r S \phi
\]

\[
\dot{\phi} = p + q S \phi T \theta + r C \phi T \theta
\]

\[
\dot{\psi} = (q S \phi + r C \phi) \sec \theta
\]

\[
\begin{align*}
\dot{x} &= u (C \theta C \psi) + v (S \phi S \theta C \psi - C \phi S \psi) + w (C \phi S \theta C \psi + S \phi S \psi) \\
\dot{y} &= u (C \theta S \psi) + v (S \phi S \theta S \psi + C \phi C \psi) + w (C \phi S \theta S \psi - S \phi C \psi) \\
\dot{z} &= u (-S \theta) + v (S \phi C \theta) + w (C \phi C \theta)
\end{align*}
\]

The last three equations of equation set 6.26 are the time rate of change of the boomerang’s center of gravity as seen from the fixed reference frame, which are developed by transforming the velocity vector from the body the fixed reference frame as shown in Section 4.2.3. The variable names have to be changed in the following way.
The simulation output will be a matrix of the integrated values of the *zdot* variables which will be plotted.
6.4 SIMULATION RESULTS

With the mathematical model already developed, the simulation is will be controlled by the initial conditions and the coefficients of lift and drag. One of the problems involved in this simulation is that the coefficients such as $C_L$, $C_D$, $C_r$, Lift/Drag ratio, and Strouhal number, for the particular geometry of the boomerang, were not determined experimentally. This was beyond the scope of this thesis. Instead, the results obtained by others for similar geometries were used. Since the experiments already published were done on a wide range of geometries, estimates of the values for the geometry of the straight boomerang were made. After studying the works of Smith [32], and Iversen [41] and Figure 4.12, the following ranges were established:

\[
1.5 < C_L < 2.2 \\
1.0 < C_D < 1.5 \\
0.05 < C_r < 0.08 \\
0.78 < S < 0.86
\]

The Lift/Drag ratio was obtained by analyzing the glide angle of the boomerang during Stage III of the flight. This was determined as follows:

\[
\frac{\text{Lift}}{\text{Drag}} = \tan (\text{glide angle})
\]

from Figure 6.8.
The glide angle was measured to be in between 55° and 65°, so

$$1.43 < \frac{\text{Lift}}{\text{Drag}} < 2.15$$

The $C_D$ before the boomerang started autorotating was determined from Figure 6.9 [42] which is a plot of the coefficient of drag for plat plates facing a stream of air as a function of aspect ratio. For an aspect ratio of 12, the $C_D$ in Stage I was a value somewhat less than 1.3 because the boomerang's largest area isn't always facing the air.

\textbf{Fig. 6.8} Glide angle of the straight boomerang during the final part of \textit{Stage III}
Fig. 6.9 Variation of drag coefficient with aspect ratio for a flat plate normal to the flow (Re > 1000) [42]

Also, it can be seen from experimental observations, that the system being studied may be a chaotic system because its flight is greatly influenced by the initial conditions. So these were varied in between some ranges which were determined from the pictures taken. The coefficients of lift and drag and of autorotation were chosen to be in between the ranges established earlier and held fixed while the initial conditions were changed.

The initial conditions are:

\[
3.5 \text{ m/sec} < u < 5 \text{ m/sec} \\
v = 0 \text{ m/sec} \\
w = 0 \text{ m/sec} \\
p = 0 \text{ rad/sec} \\
-15 \text{ rad/sec} < q < -25 \text{ rad/sec} \\
r = 0 \text{ rad/sec} \\
0^\circ < \theta < 60^\circ \\
\phi = 0^\circ \\
\psi < 0^\circ \text{ and small}
\]

The simulation results are analyzed by studying the plots obtained from MATLAB. The plots obtained for analysis are

1) local velocities (u, v, w) vs. time

2) body angular rates (p, q, r) vs. time
3) center of gravity locations in the fixed reference frame (x, y, z) vs. time

4) trajectory trace of the center of gravity in 3-Dimensions as seen from the fixed reference frame

5) projection of the trajectory on the $x_f - y_f$ coordinate system

6) projection of the trajectory on the $x_f - z_f$ coordinate system

7) projection of the trajectory on the $y_f - z_f$ coordinate system

These plots have proven to provide enough information to determine the boomerang’s position and orientation.

In the following section, an example of the first simulation is given, followed by the analysis of results, and finally the necessary changes were made for the simulation to mimic reality to its best.
6.4.1 Simulation #1

Initial conditions:

\[
\begin{align*}
    u &= 4 \text{ m/sec} \\
    v &= 0 \text{ m/sec} \\
    w &= 0 \text{ m/sec} \\
    p &= 0 \text{ rad/sec} \\
    q &= -15 \text{ rad/sec} \\
    r &= 0 \text{ rad/sec} \\
    \theta &= 30^\circ \\
    \phi &= 0^\circ \\
    \psi &< 0^\circ \text{ and small}
\end{align*}
\]

Coefficients:

\[
\begin{align*}
    C_{D1} &= 1.2 \\
    C_{D2} &= 1.0 \\
    C_L &= 2.0 \\
    C_r &= 0.08 \\
    \text{Lift/Drag} &= 2.0
\end{align*}
\]

Analysis of Results

Since the body coordinate system is autorotating with the body, the curves of \( u, v, p, \) and \( q \) are going to be of sinusoidal shape. When the boomerang leaves the thrower with an angular velocity \( q \), since it starts to rotate about its \( z_b \) axis, after \( 1/4 \) of a turn about this axis, the initial \( q \) given will be seen by the body frame as a \( p \) rotation. The same effect can be observed from Fig. 6.10.1. Since the boomerang has an \( r \) rotation, at one moment in time the \( x_b \) axis will be pointing in the direction of flight, but after a \( 1/4 \) of a revolution about \( z_b \), the \( y_b \) axis will be pointing in the direction of flight. Also since \( r \) is increasing, the frequency of oscillation of \( u, v, p, \) and \( q \) is the same as the rotational rate about \( z_b \) divided by \( 2\pi \) (\( r / 2\pi \)). As \( r \) increases, the frequency of oscillation of these curves also increase.
From Figure 6.10.1 it can be seen that at about 0.1 sec the value for the local velocity $w$ is at a maximum. It is negative and maximum at this point because the boomerang has undergone $1/4$ of a rotation about $y_b$, so the $z_b$ axis is pointing at the thrower (see Fig. 6.11) and the boomerang is moving in the $-z_b$ direction. A decrease in the magnitude of $w$ is noticed which is due to the $q$ rotation which changes the direction in which $z_b$ is points. At about 0.2 seconds, $w$ reaches a minimum magnitude and then starts to grow again in the negative direction. What this means is that the drag on the body was too high to allow the boomerang to continue its rotation. Since at this point the lift force is already acting, the body is tumbled but not to a horizontal position. Also, since the body is not rotating a lot, $p$ and $q$ are small so the torque due to the lift is small (see Eqs. 6.17, and 6.18). The boomerang then starts to act like an autorotating body that has been dropped at an angle not parallel to the $X_f$ - $Y_f$ plane which has its $z_b$ axis tilted and pointing up (i.e. in the $-Z_f$ direction). As it can be seen from Figures 6.10.4, 6.10.5, 6.10.6, the boomerang does start to come back, but the boomerang’s position in Stage III is so tilted that the lift force generated has a very small component acting against gravity. This effect can be appreciated in Figure 6.10.6 in which the center of gravity just drops at the end of the flight.
For the next simulation, the initial \( q \) has to be increased in order to make the body overcome the drag force. Since the initial angular rotation is being increased and the body is being held from one of the tips, the velocity of the center of gravity is also going to change to be at least \( q^*b/2 \).
Local Velocities $u$, $v$, $w$ in (m/s) vs Time (s)

**Figure 6.10.1**
Figure 6.10.2
Fixed Coordinate System Body Location $x$, $y$, $z$ in (m) vs Time (s)

Figure 6.10.3
Trajectory of the Boomerang as Viewed From the Fixed Reference Frame

Figure 6.10.4
Projection of the Trajectory on the X - Y Plane of the Fixed Reference Frame

Figure 6.10.5
Projection of the Trajectory on the X - Z Plane of the Fixed Reference Frame

Figure 6.10.6
Projection of the Trajectory on the Y - Z Plane of the Fixed Reference Frame

Figure 6.10.7
6.4.2 Simulation #2

**Initial Conditions**

\[ u = 4.5 \text{ m/sec} \]
\[ v = 0 \text{ m/sec} \]
\[ w = 0 \text{ m/sec} \]
\[ p = 0 \text{ rad/sec} \]
\[ q = -23 \text{ rad/sec} \]
\[ r = 0 \text{ rad/sec} \]
\[ \theta = 30^\circ \]
\[ \phi = 0^\circ \]
\[ \psi < 0^\circ \text{ and small} \]

**Coefficients:**

\[ C_{D1} = 1.2 \]
\[ C_{D2} = 1.0 \]
\[ C_L = 2.0 \]
\[ C_r = 0.08 \]
\[ \text{Lift/Drag} = 2.0 \]

**Analysis of Results**

In this case, the initial \( q \) was increased to \(-23 \) rad/sec in order to allow the boomerang to rotate more. The velocity \( w \) goes to zero about 0.45 seconds and then it starts increasing in the positive direction (Fig. 6.12.1). The fact that \( w \) reached zero means that the boomerang acquired a vertical position in which the \( z_b \) axis points straight up. The velocity \( w \) continued growing which means that the body continued rotating about the \( y_b \) axis. So the initial rotation was too high and prevented the boomerang from tumbling to an orientation in which the \( z_b \) axis pointed to the left of the thrower. Instead, as it can be seen in Figure 6.12.4, the boomerang starts falling down in a corkscrew motion. This
motion is very commonly seen during real flights and it happens when the boomerang does not end up laying flat but tries to return when it has a tilted position. This motion is also observed in nature when some long and slender objects, such as eucalyptus leaves and Samsara seeds, fall to the ground. In fact this is an area of research which helps biologists study the way in which fields are fertilized by the wind spreading tree seeds.

It can be concluded then that the initial $q$ rotation has to be somewhere in between -15 rad/sec and -23 rad/sec so that the boomerang will try to fly back to the thrower.
Local Velocities $u, v, w$ in (m/s) vs Time (s)

Figure 6.12.1
Body Angular Velocities (rad/s) $p$, $q$, $r$ in (rad/s) vs Time (s)

Figure 6.12.2
Fixed Coordinate System Body Location $x, y, z$ in (m) vs Time (s)

Figure 6.12.3
Trajectory of the Boomerang as Viewed From the Fixed Reference Frame

Figure 6.12.4
Projection of the Trajectory on the X - Y Plane of the Fixed Reference Frame

Figure 6.12.5
Projection of the Trajectory on the X - Z Plane of the Fixed Reference Frame
Projection of the Trajectory on the Y - Z Plane of the Fixed Reference Frame

Figure 6.12.7
SECTION 7

FINAL MATHEMATICAL MODEL
7.1 FINAL SIMULATION

**Initial Conditions:**

\begin{align*}
    u &= 4.0 \text{ m/sec} \\
    v &= 0 \text{ m/sec} \\
    w &= 0 \text{ m/sec} \\
    p &= 0 \text{ rad/sec} \\
    q &= -20 \text{ rad/sec} \\
    r &= 0 \text{ rad/sec} \\
    \theta &= 30^\circ \\
    \phi &= 0^\circ \\
    \psi &= < 0^\circ \text{ and small}
\end{align*}

**Coefficients:**

\begin{align*}
    C_{D1} &= 1.2 \\
    C_{D2} &= 1.0 \\
    C_L &= 2.0 \\
    C_r &= 0.08 \\
    \text{Lift/Drag} &= 2.0
\end{align*}

In this final simulation it can be observed that the boomerang returns to a distance 0.2 m away from the thrower. The final orientation of the boomerang can be determined again from Figure 7.1. Since the w velocity is somewhat negative, it means that the z_b axis is tilted in the -Z_f direction. As it can be determined experimentally, this situation is easily reproducible. It can be observed also from Figure 7.2 that the rotational energy of the system is being transferred from the x_b and y_b axis to the z_b axis.
7.1.1 Energy

The energy of the system is not going to be conserved because there are external torques being applied. By looking at Figure 7.8 it can be seen that the energy decreases from 12.12 J to 11.94 J. The drop in energy is very high at the beginning because the drag force is higher due to the high relative velocities and orientation of the boomerang. At 0.1 seconds, the boomerang has rotated by 1/4 of a revolution about the \( y_b \) axis, which is the position of least drag. The torque applied on the body is very low at this point, that is why the energy remains almost constant. After this point the lift torque starts to act and the energy continues to decrease. After the boomerang has tumbled and \( p \) and \( q \) rotations are minimized, the loss in energy will only be due to the loss of potential energy because the torques being applied will be negligible. In this simulation, this effect cannot be observed because the \( p \) and \( q \) rotations are still present.

![Energy of the system vs time](image_url)

*Fig. 7.8 Energy of the system vs time*
7.1.2 Results

From Figure 7.9, it can be seen how close the simulation and experimental results are. The simulation results are being overlapped with the experimental results for a more detailed comparison. This picture shows how close both $x_b - z_b$ paths are at the beginning. The picture was taken at a rate of one frame every 1/30 seconds. The total flight time can be determined to be 27/30 seconds (0.9 sec) which is in agreement with the simulation timing. Again, since the dimension of the boomerang is known, the picture can be scaled and the initial conditions and flight distance can be determined. The following are the results obtained from this picture, compared with the simulation results:

<table>
<thead>
<tr>
<th></th>
<th>Experimental Results</th>
<th>Simulation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$ (m/s)</td>
<td>$3.5 &lt; u &lt; 5.2$</td>
<td>4.0</td>
</tr>
<tr>
<td>$q$ (rad/s)</td>
<td>$-15 &lt; q &lt; -23$</td>
<td>-20</td>
</tr>
<tr>
<td>$\theta$ (°)</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>max. horizontal distance of flight (m)</td>
<td>0.95</td>
<td>1.01</td>
</tr>
<tr>
<td>time of flight (s)</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The final rotation rate is of 40 rev/s. This value is lower than the one measure by Vos (75 rev/s) but this difference is justified by the fact that the geometry of the boomerang used in Vos' work was different (600x25x3 mm) from the one used in this work. If the Strouhal Number (Eq. 4.5) is used to calculate the final autorotation rate, a value of 90 rad/s is obtained. From the simulation the final autorotation rate is seen to be about 250 rad/s when the boomerang is moving at a speed of 2 m/s. This large difference from the
theoretical is attributed to the fact that the model used does not include an autorotational damping coefficient. This damping exists as mentioned by Smith and Iversen but its magnitude and mathematical behavior is unknown.

As it can be seen there is a close relationship between the experimental and the simulation results. The main factor that affected the experimental part was that the room in which these pictures were taken had an air inlet coming from the ceiling. The differences present at the end of the flight (Stage III) can be attributed to the wind coming from the ceiling, which increased the boomerang's glide angle. The flight shown in this picture is a returning one, but the field of view of the camera used was not wide enough to capture the entire flight.
Fig. 7.9 Comparison between experimental and simulation results
Local Velocities $u$, $v$, $w$ in (m/s) vs Time (s)

Figure 7.1
Body Angular Velocities (rad/s) $p$, $q$, $r$ in (rad/s) vs Time (s)

Figure 7.2
Fixed Coordinate System Body Location $x$, $y$, $z$ in (m) vs Time (s)

**Figure 7.3**
Trajectory of the Boomerang as Viewed From the Fixed Reference Frame

Figure 7.4
Projection of the Trajectory on the X - Y Plane of the Fixed Reference Frame

Figure 7.5
Projection of the Trajectory on the X - Z Plane of the Fixed Reference Frame

Figure 7.6
Projection of the Trajectory on the Y - Z Plane of the Fixed Reference Frame

Figure 7.7
CONCLUSIONS
CONCLUSIONS

Other than this work and that done by Dr. Henk Vos, there has been no more research done on the straight boomerang. On the other hand the regular boomerang has been studied extensively. One of the reasons behind the straight boomerang’s obscurity is that it has never been as popular as the regular L-shaped one. Another reason why not much research has been done on the straight boomerang is because the aerodynamics governing its flight is autorotation. After conducting many throws and examining the video capture it was determined that wing theory could not be applied, instead autorotation was of primordial importance. Autorotation has never been considered as a primary area of research which reduced the amount of literature available and made the problem more difficult to solve.

Since not much is known about why autorotation happens, the mathematical model was developed by only taking into consideration the most important known behaviors of this phenomenon (listed in pg. 43). As it can be seen from the comparison between the simulation and experimental results (Fig. 7.9), these considerations lead to very good results. Also it can be concluded that the $C_L$, $C_D$, and $C_t$ coefficients chosen were accurate for the initial conditions analyzed. Since the system here analyzed is a chaotic one, these coefficients would have to be adjusted if the initial conditions were to be changed.
RECOMMENDATIONS

In order to obtain better results the autorotation characteristics of the geometry of the boomerang should be studied. This should include the way in which the coefficients depend on the air velocity. This will enable the simulation to work better for a broader range of initial conditions. Also a video capture can be done at a higher sampling rate. This will allow for a more detailed adjustment and comparison between the simulation and the experimental results.
BIBLIOGRAPHY


2. Ibid., 80-83

3. Ibid., 85

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5. Ibid., 70


7. Ibid., 21.

8. Ibid., 24.


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13. Ibid., 525.


18. Ibid., 124.

19. Ibid., 125.

20. Ibid., 125.

21. Ibid., 126.

22. Ibid., 129.

23. Ibid., 127.

24. Ibid., 136.

25. Ibid., 129.

26. Ibid., 129.


29. Ibid., 137.


32. Ibid., 525.
33. Ibid., 531.
36. Ibid., 296-299.
40. Ibid., 91.
41. *MATLAB* User’s Guide
APPENDIX
This file imposes the initial conditions and solve simultaneously the equations of motion from file simfinal.m

```matlab
t0 = 0;
tf = 1;
z0 = [4, 0, 0, 0, -20, 0, 30*pi/180, 0, 0.02, 0, 0, 0]';
[t, z] = ode23('simfinal', t0, tf, z0);
simcol
```
This file has the equations that describe the flight of the straight boomerang.

```matlab
function zdot=simfinal(t,z)
    zdot=zeros(12,1);
    t
    if abs(z(6))<80,
        Cd=1.2;
        Cl=0;
    end;
    if abs(z(6))>80,
        Cd=1;
        Cl=2;
    end;
    zdot(1)=-(9.81*sin(z(7)))-(z(5)*z(3))+((0.00909*((z(1)^2)+(z(2)^2)))+((0.00014*((z(4)^2)+(z(5)^2)))+(-(Cl*z(2))-(Cd*z(1)))/(sqrt((z(1)^2)+...
        (z(2)^2)*0.00936));
    zdot(2)=(9.81*cos(z(7))*sin(z(8)))-(z(6)*z(1))+(0.00014*((z(4)^2)+(z(5)^2)))*(-Cl*z(2))-(Cd*z(1)/(sqrt((z(1)^2)+...
        (z(2)^2)*0.00936));
    zdot(3)=(9.81*cos(z(7))*cos(z(8)))-(z(4)*z(5))+(z(5)*z(1));
    zdot(4)=(z(5)*z(6))+(0.00028*((z(1)*z(5))-(z(2)*z(4)))*(-Cl*z(1)+...
        (Cd*z(2)))/(sqrt((z(1)^2)+(z(2)^2))));
    zdot(5)=-(0.987*z(4)*z(5))-(0.00028*((z(1)*z(5))-(z(2)*z(4)))*((Cl*z(2))+...
        (Cd*z(1)))/(sqrt((z(1)^2)+(z(2)^2))));
    Cr=0.08;
    if abs(z(6))>250,
        Cr=0;
    end;
    zdot(6)=-(Cr*((0.78/0.035)^2)*((z(1)^2)+(z(2)^2)+(0.0154*(((z(4)^2)+(z(5)^2)))-...
        (z(4)*z(5)));
    zdot(7)=(z(5)*cos(z(8)))-(z(6)*sin(z(8)));
    zdot(8)=(z(4)+(z(5)*sin(z(8)))*tan(z(7)))+(z(6)*cos(z(8))*tan(z(7)));
    zdot(9)=(z(5)*sin(z(8)))+(z(6)*cos(z(8)))*sec(z(7));
    zdot(10)=(z(1)*cos(z(7))*cos(z(9)))+(z(2)*((sin(z(8))*sin(z(7))*cos(z(9)))-(cos(z(8))*sin(z(7))*cos(z(9)))+...
        (cos(z(8))*sin(z(9)))+((cos(z(8))*sin(z(7))*cos(z(9)))+(sin(z(8))*sin(z(9))));
    zdot(11)=(z(1)*cos(z(7))*sin(z(9)))+(z(2)*((sin(z(8))*sin(z(7))*sin(z(9)))+(cos(z(8))*sin(z(7))*sin(z(9))))+...
        (cos(z(8))*cos(z(9)))+((cos(z(8))*sin(z(7))*sin(z(9)))-(sin(z(8))*cos(z(9)));
    zdot(12)=-(z(1)*sin(z(7)))+(z(2)*sin(z(8))*cos(z(7)))+(z(3)*cos(z(8))*cos(z(7)));
end;
```

% This file plots the variables
u = z(:,1);
v = z(:,2);
w = z(:,3);
p = z(:,4);
q = z(:,5);
r = z(:,6);
theta = z(:,7);
phi = z(:,8);
psi = z(:,9);
x = z(:,10);
y = z(:,11);
ze = z(:,12);
figure
plot(t, u, 'r-', t, v, 'b-', t, w, 'g-'), title('Local Velocities u, v, w in (m/s) vs Time (s)'), ... 
xlabel('Time (s)'), ylabel('Local Velocity (m/s)'), grid, gtext('u'), gtext('v'), gtext('w')
figure
plot(t, p, 'r-', t, q, 'b-', t, r, 'g-'), title('Body Angular Velocities (rad/s) p, q, r in (rad/s) vs Time (s)'), ... 
xlabel('Time (s)'), ylabel('Body Angular Velocities (rad/s)'), grid, gtext('p'), gtext('q'), gtext('r')
figure
plot(t, theta, 'r-', t, phi, 'b-', t, psi, 'g-'), title('theta, phi, psi'), ... 
xlabel('t')
figure
plot(t, x, 'r-', t, y, 'b-', t, z, 'g-'), title('Fixed Coordinate System Body Location x, y, z in (m) vs Time (s)'), ... 
xlabel('Time (s)'), ylabel('Body Location (m)'), grid, gtext('x'), gtext('y'), gtext('z')
figure
plot3(x, y, ze, 'r-'), xlabel('X'), ylabel('Y'), zlabel('Z'), ... 
title('Trajectory of the Boomerang as Viewed From the Fixed Reference Frame'), ... 
view(-37.5, -210.5), axis('ij'), axis('equal'), axis([0 1 0.5 -0.5 1])
figure
plot(x, ye, 'r-'), xlabel('X (m)'), ylabel('Y (m)'), title('Projection of the Trajectory on the X - Y Plane of the Fixed Reference Frame'), ... 
axis('ij'), axis('equal'), axis([0 1 0.5]), grid
figure
plot(x, ze, 'r-'), xlabel('X (m)'), ylabel('Z (m)'), title('Projection of the Trajectory on the X - Z Plane of the Fixed Reference Frame'), ...
axis('ij'), axis('equal'), grid
figure
plot(y, ze, 'r-'), xlabel('Y (m)'), ylabel('Z (m)'), title('Projection of the Trajectory on the Y - Z Plane of the Fixed Reference Frame'), ...
axis('equal'), grid