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Performance Analysis of a Black Liquid Solar Collector

Jackson Trentelman

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PERFORMANCE ANALYSIS OF A BLACK LIQUID 
SOLAR COLLECTOR

by

Jackson P. Trentelman

A Thesis Submitted 
in 
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Approved by:

Prof. Name Illegible 
(Thesis Advisor)

Prof. Name Illegible

Prof. Name Illegible

Prof. Name Illegible 
(Department Head)

DEPARTMENT OF MECHANICAL ENGINEERING 
COLLEGE OF ENGINEERING 
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Abstract

The performance of a black liquid, sheet flow solar collector has been investigated both analytically and experimentally. In addition, its performance has been compared to that of a baseline collector of the tube and fin design.

The black liquid solar collector was predicted to exhibit an instantaneous efficiency improvement of 10-15% over a well designed tube and fin collector. This result was confirmed experimentally. Long term efficiency calculations indicate similar performance improvements over long term use.

The major contribution to improved efficiency was found to be the direct absorption process whereby a) the collector efficiency, $F'$, is unity and b) the effective transmittance-absorptance, $(\tau_0)_{eff}$, is increased for most angles of incidence over that of a comparable tube and fin collector.

A significant advantage of the proposed collector was found to be reduced cost per unit energy collected since no metals are required in collector fabrication.
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Chapter 1

Introduction

The exploitation of various low grade, "free" energy resources to fulfill energy needs and requirements has enjoyed renewed interest in recent years. This has resulted largely from the increased costs associated with the procurement, as well as the environmentally acceptable utilization, of high grade energy resources which have played a dominant role in power production in the last half-century. Within this latter group are included coal, oil and its derivatives, as well as nuclear fission. The technology needed for harnessing the nuclear fusion process is at this time not sufficiently developed to offer short-term assistance in the energy area.

Low intensity energy sources are plentiful on earth. Solar, geothermal, tidal, to name some, represent vast untapped energy reserves. The problem has been to put these resources to work in a manner which can compete on an economic basis with fossil and nuclear energy generation. Historically, the development of the technology needed for solar, wind, etc., energy to achieve this economic parity has lagged behind, with greater emphasis being placed on fossil and nuclear conversion. However, the so-called alternative energies have often been successfully, though sometimes crudely, employed in a variety of applications. Solar evaporation of salt brines to recover salt has been used for centuries. Windmills to convert wind energy to useful work
date back to at least the 10th century.

Of various energy alternatives, solar energy is receiving wide attention. Yet it is not without its shortcomings. Its intermittent and unpredictable nature requires storage facilities for many applications. Its intensity is relatively low, about 1 kW/m² maximum on the earth’s surface. Thus to collect usable amounts, large collector and/or reflector areas are needed. Balancing the scales are its inexhaustable, non-polluting nature, and the fact that it’s free for the taking.

The heart of any solar system is the actual solar collection equipment. The basic process is one in which solar radiation is absorbed by a surface and converted into heat. All surfaces will absorb some degree of incident solar radiation, and black surfaces can be constructed with efficiencies approaching 100% for the absorption of solar electro-magnetic waves.

Solar-heat collectors, as distinguished from solar electric (photo voltaic) devices, generally fall into two broad catagories, flat-plate and focussing. Hybrids consisting of a flat-plate collector and reflectors can also be envisioned. The focussing collector is generally more complex in that a tracking action to follow the sun is required. These are appropriate for high temperature applications. The basic flat-plate collector, however, is inherently simple. A blackened surface absorbs sunlight and heat is produced. The heat is transferred to a (colder) fluid in thermal contact with the plate and is swept out of the collector by pumping or other means for storage or use. To prevent
convective and radiative heat loss from the absorber plate to the ambient, one or more transparent cover plates (glazings) with dead air spaces between them can be placed over the absorber surface. Insulation can be placed on the back side and possibly along the edges to retard heat flow from these areas. Other, more exotic means of reducing heat loss have been approached. These include evacuated regions in contact with the absorber to reduce convection losses, and so-called selective surfaces which are good absorbers in the solar spectrum but poor emitters in the thermal region. Working fluids generally used are water or water/antifreeze mix, because it is plentiful and has a high specific heat, or air because of its convenience.

The standard approach to the water type flat-plate collector has been a tube and fin geometry. This consists of tubes running the length of the collector bonded to the metal absorber plate. The plate area between tubes acts as a fin and energy collected there is conducted to the tube and then to the water along the temperature gradients that are set up by the energy absorption process. Many commercially available flat-plate collectors represent minor variations on this theme. It is possible for a collector of this type, operating under optimal circumstances, to convert up to about 90% of the incident radiation into useful heat. But a more realistic efficiency for practical operation, i.e., winter operation with an output temperature of 80-100°C, is about 25-30%. Clearly room for improvement exists.
Initial cost per unit area presents itself as a second major consideration. Commercially-constructed liquid flat-plate collectors can cost up to $10/ft². Other more sophisticated geometries producing small increases in efficiencies may cost substantially more. Thus for a collector area adequate to supply, say, a typical residence with 50% of its heating needs in Rochester, an initial investment of several thousand dollars is required.

Figure 1.1 depicts the cross-sectional view of a typical tube and fin solar heat collector.

The need for a low initial cost, simple, and reasonably efficient flat-plate solar collector provides the impetus for the present work. The concept of using a sheet of "black liquid", which absorbs solar radiation directly, is explored from the theoretical and experimental standpoints. Figure 1.2 illustrates the proposal. A heat transfer fluid (e.g., water appropriately dyed black or very dark) flows between two parallel plates. The top plate is transparent allowing solar radiation to enter the liquid and be absorbed by it. The liquid flows, in principle, uniformly between the plates. Glazings and the dead air spaces created tend to insulate the liquid from the ambient.

Surprisingly, the black liquid concept has received little attention to date. Studies of two geometries have appeared in the literature. Minardi and Chuang [1] have examined a black liquid flowing within a spiralled transparent tube. This work establishes the feasibility of the concept of direct absorption
Fig. 1.1 Cross-sectional view of a two cover tube and fin collector.
Fig. 1.2 A 3-dimensional perspective and a cross sectional side view of the proposed black liquid collector geometry.
and discusses several different black liquid candidates. However, quantitative assessment of their results is difficult since their measure of collector efficiency is ambiguous. Boeing Engineering and Construction has examined [2] a different black liquid collector configuration consisting of a glass panel structure containing vacuum cells (for insulating purposes) and liquid passageways. Details of this work were not available at the time of this writing.

This recent interest in a black liquid approach to the collection of solar radiation underlines the desirability of (a) avoiding strategic and costly materials in collector construction, and (b) exploiting the inherent simplicity and efficiency of the direct absorption process.

This work will deal with the theoretical and practical aspects of the sheet flow black liquid collector. In Chapter 2 general formulae are developed which describe the energy transfer processes of reflection, transmission, convection, conduction and radiation. In Chapter 3 these results are applied to predict the instantaneous performance of the proposed black liquid collector and compare it to that of a "baseline" tube and fin collector. Chapter 4 concerns itself with long-term collector performance. A method is developed to estimate average values of hourly solar radiation on a surface tilted towards the equator for Rochester. Collector performance is then determined theoretically for "realistic" orientations for typical winter and summer months. In Chapter 5 the prototype is described and
experimental determinations of instantaneous efficiency are presented. Chapter 6 contains a discussion of the results and recommendations for future work.
Chapter 2
General Theory

In order to model analytically the sheet-type black liquid solar collector, all modes of energy gain and energy loss for the system have been considered. These include absorption, reflection and transmission of both solar radiation and thermal reradiation, heat loss by convection and conduction from the point of absorption, and useful heat gain of the collector.

This chapter concerns itself with the development of analytical expressions necessary to describe these energy transfer processes which occur in the collector.

2.1 The Solar Constant and the Spectral Nature of Solar Radiation.

The solar constant, $I_{sc}$, is the amount of solar radiation received per unit time on a surface perpendicular to the sun's rays, in space, and located at the earth's mean distance from the sun. The proposed standard value of the solar constant is $1353 \text{ W/m}^2$ [3] (1.940 Langley/min., 428 BTU/ft$^2$hr., or 4871 kJ/m$^2$-hr.).

The energy spectrum of extraterrestrial solar radiation extends over the range of wavelengths $0.2 \rightarrow 3\mu$m ($1\mu$m = $10^{-6}$m). Of the total energy, 7.0% falls in the ultra-violet ($\lambda < 0.38\mu$m), 47.3% in the visible ($0.38\mu$m $\leq \lambda \leq 0.78\mu$m), and 45.7% in the infra-red ($\lambda > 0.78\mu$m).

The amount of solar energy received at the surface of the earth is nearly always less that $I_{sc}$. This is the result of
both scattering and absorption of the beam as it passes through the atmosphere. Ozone in the atmosphere absorbs most of the ultra-violet portion, while water vapor and CO₂ absorb in the infrared bands. Both water vapor and dust particles cause scattering and a further attenuation of the beam. The net result is that cloudless day solar radiation at sea level amounts to about 60-70% of the solar constant on a surface which is oriented perpendicular to the sun's direct rays. Cloud cover, of course, reduces the above further, as does any change of orientation of the solar collector from a position normal to the sun's rays. On earth, the solar spectrum extends from \( \lambda = 0.3 \) to 2.3\( \mu m \).

2.2 Transmission of Light Through Partially Transparent Material

The performance of a solar collector is intimately associated with the amount of solar radiation incident on the surface of the collector and the amount which ultimately impinges on the absorbing medium, the remainder having been either reflected or absorbed by the system of partially transparent cover plates.

Consider the case of a slab of partially transparent material of refractive index \( n_2 \), interfaced on either side by totally transparent media of refractive indices \( n_1 \) and \( n_3 \). The situation is depicted in Figure 2.1. The amount of radiation reflected at the interface of the two media of differing refractive indices is given by the Fresnel relation\(^4\) (valid for non-polarized radiation)
Fig. 2.1 Transmission of light through a partially absorbing medium.
\[ \frac{I_r}{I_o} = \rho = \frac{1}{2} \left[ \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} \right] + \left[ \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} \right]. \] (2.1)

where \( \theta_1 \) and \( \theta_2 \) are the angles of incidence and refraction (as measured from the normal to the surface) and are related through the indices of refraction \( n_1 \) and \( n_2 \) by Snell's Law

\[ \frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} \] (2.1a)

Equation 2.1 can be evaluated for \( \theta_1 = 0 \) by combination with Eq. 2.1a, yielding

\[ \rho_{\theta_1=0} = \left[ \frac{n_1 - n_2}{n_1 + n_2} \right]^2 \]

The absorption of light as it passes through a partially transparent medium can be described by Beer's Law, which states that each differential thickness, \( dx \), absorbs an amount proportional to the local radiation intensity. Algebraically this can be expressed as

\[ dI = IK_v \, dx \] (2.2)

where \( K_v \), the extinction coefficient for light of frequency \( v \), is a measure of the absorbing power of the medium. Integration of Eq. 2.2 over a path length, \( L \), yields

\[ \frac{I_L}{I_o} = a = e^{-KL} \]
where $I_0 = \text{intensity at } x = 0$, and it is assumed $K_v$ is the same for all frequencies and is thus set equal to $K$.

Now, consider light of unit intensity originating in medium 1 incident at some angle $\theta$, on the interface between media 1 and 2 (see Figure 2.1) a fraction, $\rho$, is reflected and a fraction, $(1 - \rho)$, is transmitted just past the interface. As the transmitted light passes through medium 2 of thickness $t$, a fraction, $a = e^{-KL}$, will be absorbed over the path length, $L$, where $L = \frac{t}{\cos \theta} = \text{actual distance traveled}$. Thus, the amount of light initially incident on the interface of media 2 and 3 is of intensity $a(l - \rho)$. If $\beta$ represents the fraction of reflected light $I_r/I_0$ at the interface of media 2 and 3, then $a(l - \rho)(1 - \beta)$ will succeed in initially passing both interfaces. Similar consideration of all subsequent reflected rays yields an expression for $T_1$, the total light energy transmitted past the second interface. Then, from Figure 2.1, we have

$$T_1 = a(l - \rho)(1 - \beta) + a^3 \rho \beta (l - \rho(1 - \beta) + a^5 \rho^2 \beta^2 (l - \rho)(1 - \beta)$$

or

$$T_1 = a(l - \rho)(1 - \beta) \sum_{n=0}^{\infty} (a^2 \rho \beta)^n$$

Recalling that the infinite series $\sum_{n=0}^{\infty} x^n = \frac{1}{1 - x}$ for $0 < x < 1$, the above becomes
\[ T_1 = \frac{a(1 - \rho)(1 - \beta)}{1 - a^2 \rho \beta}; \quad \text{where } a = e^{-Kt/cos \theta_2} \tag{2.3} \]

Similarly, the total reflected energy, \( R_1 \), amounts to:

\[ R_1 = \rho + a^2(1 - \rho)^2 \beta \sum_{n=0}^{\infty} (a^2 \rho \beta)^n \]

or

\[ R_1 = \rho + \frac{a^2(1 - \rho)^2 \beta}{1 - a^2 \rho \beta} \tag{2.4} \]

Equations 2.3 and 2.4 can be simplified somewhat by noting that \( 1 - a^2 \rho \beta \approx 1 - \rho \beta \) for \( \rho, \beta \ll 1 \) and \( a \approx 1 \). Thus, to an excellent approximation,

\[ T_1 = a \frac{(1 - \rho)(1 - \beta)}{1 - \rho \beta} \quad \text{and} \]

\[ R_1 = \rho + \frac{a^2(1 - \rho)^2 \beta}{1 - \rho \beta} \quad \text{(valid for } \rho, \beta \ll 1) \]

and for the special case of \( \rho = \beta \ll 1 \), we have

\[ T_1 = a \frac{(1 - \rho)}{(1 + \rho)} \quad \text{and} \quad R_1 = \rho + a^2 \frac{(1 - \rho) \rho}{1 + \rho} \]

To derive the expression for transmission of light through two plane parallel slabs of different material and separated by a non-absorbing medium (see Figure 2.2), we note that amount \( T_1 \) is initially transmitted through plate 1 and onto plate 2, which then transmits an amount \( T_1 T_2 \). Accounting for subsequent reflections, the total energy transmitted is:
Fig. 2.2  Transmission of light through two plates.
\[ T_{\text{tot}} = T_1 T_2 \sum_{n=0}^{\infty} (R_1 R_2)^n = \frac{T_1 T_2}{1 - R_1 R_2} \]

where \( T_1 \) and \( T_2 \) are given by expressions of the form of Eq. 2.3 and \( \rho \) and \( \beta \) are the appropriate reflectances at each interface. Likewise, \( R_1 \) and \( R_2 \) are given by Eq. 2.4.

Transmission through 3 or more plates can be approached in a similar manner. For the case of identical plane parallel, non-absorbing plates, separated by layers of air, the expression for total transmitted energy reduces to:\[5\]:

\[ T = \frac{1 - \rho}{1 + (2n - 1)\rho} \] (2.5)

For the present analysis we are concerned with light transmission through 2 (or possible 3) 1/8" thick (double strength = DS) sheets of window glass, the final sheet being in contact with the black fluid. The situation is shown in Figure 2.3. Thus, two types of interfaces are involved, water-glass and air-glass. The reflectance, \( \rho \), for these two interfaces has been calculated from Eq. 2.1 and is shown in Figure 2.4. From Figure 2.4 it is seen that for angles of incidence less than about 50°, \( \rho \) is essentially constant and equals about 0.04 and 0.005 for air-glass and water-glass interfaces, respectively.

### 2.3 The Top Loss Coefficient, \( U_t \)

The energy which a solar collector loses to the atmosphere because its temperature is elevated with respect to the ambient...
Fig. 2.3  Double glazed black liquid system.
Fig. 2.4 The reflectance $\rho$, for an air-glass and a water-glass interface as a function of the angle of incidence.
can be expressed as:

\[ q_{\text{lost}} = U_L (T_f - T_{\text{amb}}) \]

where \( U_L \) is the overall heat transfer coefficient, \( T_f \) is the local fluid temperature, and \( T_{\text{amb}} \) (or \( T_a \)) the ambient temperature. The magnitude of \( U_L \) reflects the magnitude of losses due to both radiation and convection as well as conduction losses through the bottom and sides of the collector.

Of interest in the present case is the radiation energy exchange between two infinite, parallel plates. For two such plates at temperatures, \( T_1 \) and \( T_2 \), and emissivities, \( \varepsilon_1 \) and \( \varepsilon_2 \), the radiation heat transfer per unit time per unit area is given by\(^\text{[6]}\):

\[
\frac{Q}{A} = \frac{\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \quad (2.6a)
\]

where \( \sigma \) is the Stefan-Boltzmann constant and equals \( 5.6697 \times 10^{-8} \text{ W/m}^2\text{°K}^4 \). Also of interest is the case of a small convex object at temperature \( T_1 \) surrounded by a large enclosure at \( T_2 \). This corresponds to the top glazing of a solar collector exchanging radiation with the surroundings (mostly sky). This radiation exchange is given by\(^\text{[6]}\):

\[
\frac{Q}{A_1} = \varepsilon_1 \sigma (T_1^4 - T_{\text{sky}}^4) \quad (2.6b)
\]
Several relations between the actual air temperature and \( T_{\text{sky}} \) have been proposed for clear days. Swinback\(^7\) proposes \( T_{\text{sky}} = 0.0552 \ T_{\text{air}}^{1.2} \) where \( T_{\text{sky}} \) and \( T_{\text{air}} \) are in degrees Kelvin. Whillier\(^8\) has suggested the following relation: \( T_{\text{sky}} = T_{\text{air}} - 6^\circ \text{C} \).

In the absence of a detailed theory, this latter result is used in the present work to approximate \( T_{\text{sky}} \).

Eq. 2.6a can be expressed in the customary linearized form \( Q/A = h_r(T_2 - T_1) \) by defining the non-linear radiation heat transfer coefficient (for two plane, parallel surfaces) as

\[
h_r = \frac{\sigma(T_2^2 + T_1^2)(T_2 + T_1)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}
\]

Eq. 2.6b can be similarly modified. The concept of a radiation heat transfer coefficient facilitates expressing the top loss as a function of \( (T_{\text{fluid}} - T_{\text{ambient}}) \).

An equally important heat transfer consideration is the natural convection between parallel flat plates. The following correlations (valid for \( 10^4 < \text{Grashof No.} < 10^7 \)) can be used\(^9\) for horizontal, 45°, and vertical orientations respectively, and a mean temperature of 10°C.

\[
h_{10}^{(0^\circ)} = 1.613 \ \frac{\Delta T^{0.281}}{\varepsilon^{0.157}} \tag{2.7a}
\]

\[
h_{10}^{(45^\circ)} = 1.14 \ \frac{\Delta T^{0.310}}{\varepsilon^{0.070}} \tag{2.7b}
\]
\[ h_{10}(90°) = 0.82 \frac{\Delta T^{0.327}}{\lambda^{0.019}} \]  \hspace{1cm} (2.7c)

where \( h_{10} \) is in W/m\(^2\)°C, \( \Delta T \) is in °C, and \( \lambda \) is the plate spacing in centimeters. For temperatures other than 10°C the correction

\[ h_T = h_{10}[1 - .0018(T - 10)] \]

can be applied, where \( T \) is the average temperature between the plates in °C.

In general, the top loss coefficient for a multicover system can be written,

\[ U_t = \frac{1}{R_1 + R_2 + \ldots + R_n} \]

\( R_1 \) is the heat transfer resistance from the absorbing medium to the first cover, \( R_2 \) is that from the first to the second cover, \( R_{n-1} \) is that from the \((n-2)\) to the \((n-1)\) cover, while \( R_n \) is that from the \((n-1)\)th cover to the ambient.

As an example, a single-cover tube and fin collector has a top loss coefficient of the form

\[ U_t = [R_1 + R_2]^{-1} = \left[ \frac{1}{h_{p-c} + h_{r,p-c} + \frac{1}{h_w + h_{r,c-s}}} \right]^{-1} \]

Here, \( h_{p-c} \) = the convective coefficient between the absorber plate and the 1st cover.

\( h_{r,p-c} \) = the radiative coefficient between the absorber plate and the 1st cover.
the convective coefficient between the top cover and the wind.

\( h_{w, c-s} \) = the radiative coefficient between the top cover and the surroundings.

For non-selective absorbing surfaces, the convective term usually dominates.

Where back and side losses are to the ambient the overall heat transfer coefficient becomes

\[ U_L = U_t + U_{back} + U_{side} \]

The thermal network of this simple one-cover system is shown in Figure 2.5.

Alternatively, the top loss coefficient, \( U_t \), can be calculated according to the semi-empirical prescription developed by Klein [10]. This relationship is:

\[
U_t = \left[ \frac{N}{(344/T_p)\left[(T_p - T_a)/(N + f)\right]^{3.1} + \frac{1}{h_w}} \right]^{-1}
+ \left[ \frac{\sigma(T_p^4 + T_a^4)}{[\varepsilon_p + 0.0425N(1 - \varepsilon_p)]^{-1} + [(2N + f - 1)/\varepsilon_g] - N} \right]
\]

(2.8a)

where, \( N \) = number of glass covers
\( f = (1.0 - 0.04h_w + 5.0 \times 10^{-4}h_w^2)(1 + 0.058N) \)
\( \varepsilon_g \) = emittance of glass
\( \varepsilon_p \) = emittance of plate
\( T_a \) = ambient temperature (°K)
Fig. 2.5  Heat transfer coefficients and thermal network for a single cover solar heat collector.
\[ T_p = \text{average plate temperature (°K)} \]
\[ h_w = \text{wind heat transfer coefficient} \]

The quantity \( h_w \) can be calculated approximately from the expression given by McAdams\[11\], namely \( h_w = 5.7 + 3.8V \), where \( V \) is in m/s and \( h_w \) in W/m\textsuperscript{2}°C.

The relation 2.8a is calculated for plate spacings of 2.54 cm., but can be used for other spacings with little error. This can be seen from Eqs. 2.7a, b and c, where the convective heat transfer coefficient \( h_{10} \) is quite insensitive to \( l \). The radiation heat transfer coefficient is independent of \( l \).

\( U_t \), as given by Eq. 2.8a, is calculated for a tilt of 45°, but can be used for other tilt values through the correlation of Klein\[10\]:

\[
\frac{U_t(s)}{U(45)} = 1 - (s - 45)(0.00259 - 0.00144 \varepsilon_p) \tag{2.8b}
\]

The tilt, \( s \), is specified in degrees.

2.4 The Collector Efficiency Factor, \( F' \)

It is convenient to describe the instantaneous useful heat gain per unit area, \( q_u \), for a flat-plate solar collector as a function of the energy input and energy losses. The energy input is usually described in terms of the following quantities:

\[ H_h = \text{solar energy incident on a horizontal surface per unit area per unit time (W/m}^2 \text{ or BTU/hr-ft}^2 \text{).} \]
\[ H_t = \text{solar radiation incident on a surface tilted toward the equator.} \]
R = a geometric factor which is a function of the orientation of the collector with respect to the direct rays of the sun. For a given orientation receiving a radiation flux, $H_t$, then

$$R = \frac{H_t}{H_h}.$$  

$\tau$ = the transmissivity of the cover system (about 0.87 for a one-glass cover system at normal incidence).

$\alpha$ = the absorptance of the absorbing surface (about .96 for a flat black surface at normal incidence).

Thus, if the energy flux $H_t = H_h R$ at the outer surface of the cover system, an amount $H_t (\tau \alpha)$ is absorbed at the absorbing surface.

Similarly, the heat lost from the collector can be described by a term $U_L (T_f - T_{amb})$, where $U_L$ is the overall heat loss coefficient, which takes radiation, convection, conduction and back and side losses into account, and $T_f - T_{amb}$ represents the temperature difference between the fluid at a particular point and the ambient.

Heat transfer analysis of flat-plate solar collector geometries generally result in the instantaneous useful heat gain, $q_u'$, taking the form[12]

$$q_u' = F' [H_t (\tau \alpha) - U_L (T_f - T_a)]$$

The constant factor, $F'$, is called the collector efficiency factor and is $\leq 1$. $F'$ can be considered a measure of the heat transfer resistance from the point of absorption of solar energy
to ultimate absorption by the fluid passing through the collector.

As an example of a case where \( F' < 1 \), consider the solar collector depicted in Figure 2.6. Fluid flows between the absorber plate and the back of the collector. The heat transfer coefficient from the absorber to the fluid equals \( h_1 \), and from the fluid to the back equals \( h_2 \). Losses through the cover are accounted for by \( U_t \) (both radiation and convection-conduction; Eq. 2.8a), and back losses are assumed negligible. Further, the absorber and back exchange radiation through the radiative heat transfer coefficient, \( h_r \). Setting \( S \equiv H_t(\tau a) \), steady-state energy balances on the back surface, absorbing plate, and fluid yield the following three equations:

\[
\begin{align*}
    h_r(T_p - T_b) &= h_2(T_b - T_f) & \text{back surface} \quad (2.9) \\
    S &= h_1(T_p - T_b) + h_r(T_p - T_b) + U_t(T_p - T_a) & \text{absorber} \quad (2.10) \\
    q_u &= h_1(T_p - T_f) + h_2(T_b - T_f) & \text{fluid} \quad (2.11)
\end{align*}
\]

Rearranging Eq. 2.9 yields

\[
T_b = \frac{h_r T_p + h_2 T_f}{h_2 + h_r} \quad (2.12)
\]

Forming the quantity \( T_b - T_f \), we have

\[
T_b - T_f = \frac{h_r T_p + h_2 T_f}{h_2 + h_r} - T_f = \frac{h_r}{h_2 + h_r} (T_p - T_f)
\]
Fig. 2.6 Solar collector with fluid in thermal contact with absorber plate.
Now \( q_u \) can be expressed as a function of \( T_p \) and \( T_f \):

\[
q_u = h_1(T_p - T_f) + h_2(T_b - T_f) \\
= \left[ h_1 + \frac{h_2 h_r}{h_2 + h_r} \right] (T_p - T_f),
\]

(2.13)

Substituting Eq. 2.12 into 2.10 yields, after some rearrangement

\[
S = T_p \left[ h_1 + \frac{h_r h_2}{h_2 + h_r} + U_t \right] - h_1 T_f - \frac{h_2 h_r}{h_2 + h_r} - U_t T_a
\]

Solving the above for \( T_p \) gives

\[
T_p = \frac{S + h_1 T_f + \frac{h_2 h_r}{h_2 + h_r} T_f + U_t T_a}{[h_1 + \frac{h_2 h_r}{h_2 + h_r} + U_t]}
\]

(2.14)

Substitution of 2.14 into 2.13 eliminates \( T_p \):

\[
q_u = \frac{h_1 + \frac{h_2 h_r}{h_2 + h_r}}{h_1 + \frac{h_2 h_r}{h_2 + h_r} + U_t} \left[ S + h_1 T_f + \frac{h_2 h_r}{h_2 + h_r} + U_t T_a \right] - h_1 T_f - \frac{h_2 h_r}{h_2 + h_r} T_f - U_t T_f]
\]

\[
q_u = \frac{h_1 + \frac{h_2 h_r}{h_2 + h_r}}{h_1 + \frac{h_2 h_r}{h_2 + h_r} + U_t} \left[ S - U_t(T_f - T_a) \right]
\]
Thus, \[ q_u = F'[S - U_t(T_f - T_a)] \]

where, \[ F' = \frac{1}{l + \frac{U_t}{h_I + \frac{1}{h_r} + \frac{1}{h_2}}} \] (2.15)

It is evident from Eq. 2.15 that \( F' < 1 \), and that in the limit \( h_I \to \infty \), \( F' \to 1 \). From this consideration it is seen that for the black liquid concept, \( F' = 1 \) since solar radiation is absorbed directly, corresponding to the situation \( h_I \to \infty \) (heat transferred to the fluid without resistance).

2.5 The Collector Heat Removal Factor, \( F_R \)

The use of the equation \[ q_u = F'[H_t(T_\alpha) - U_L(T_f - T_{amb})] \] to predict solar collector performance will, in general, be inadequate, since \( T_f \) is a function of position in the flow direction in the collector. A more convenient representation would be to express \( q_u \) as a function of the fluid inlet temperature \( T_{f,i} \) (or \( T_{in} \)) and mass flow rate \( \dot{m} \), two quantities which can be readily measured.

Consider then an element of fluid of length \( \Delta y \), thickness \( \Delta z \), and width \( W \), flowing as indicated in Figure 2.7. The rate of energy gain or loss of this element is given by

\[ \frac{dE}{dt} = \rho C_p \frac{dT}{dt} (W\Delta y\Delta z) \] (2.16)
Fig. 2.7 Fluid element of width $W$, depth $\Delta z$, and length $\Delta y$ in the flow direction.

$$(q_{u}W\Delta y) = \text{energy in}$$
If the fluid is flowing uniformly at a velocity \( v (= \frac{dy}{dt}) \), we have

\[
\frac{dE}{dt} = \rho C_p \left[ \frac{dT}{dy} \frac{dy}{dt} \right] (W \Delta Z \Delta y)
\]

\[
= [\rho (W \Delta Z) v] C_p \frac{dT'}{dy} \Delta y
\]

If \( q_u \) is equal to the energy change per unit area per unit time, \( \frac{dE}{dt} = q_u W \Delta y \). Since \( \rho (W \Delta Z) v \) is seen to be \( \dot{m} \), the mass flow rate, the above reduces to

\[
q_u W \Delta y = \dot{m} C_p \frac{dT}{dy} \Delta y
\]

On a per unit area basis, this becomes

\[
q_u = \dot{m}_w C_p \frac{dT}{dy}
\]  

(2.17)

where \( \dot{m}_w \) is now the mass flow rate per unit width. The quantity \( q_u \) will, in general, be equal to \( F' [S - U_L (T_f - T_{amb})] \) which is a function of \( y \). Thus we are led to the differential equation

\[
\dot{m}_w C_p \frac{dT_f}{dy} - F' [S - U_L (T_f - T_{amb})] = 0
\]  

(2.18)

where it is assumed that \( F' \), \( U_L \), \( S \), and \( T_{amb} \) are constant with respect to position. The solution to Eq. 2.18 is

\[
T_f(y) = \left[ - \frac{U_L F'}{\dot{m}_w C_p} \right] (T_{f,i} - T_{amb} - \frac{S}{U_L}) + T_{amb} + \frac{S}{U_L}
\]  

(2.19)
where the fluid inlet temperature \((y = 0)\) is \(T_{f,i}\). Rearranging 2.19 yields

\[
\frac{T_f - T_{amb} - S}{U_L} = \frac{U_L F'y}{m_w C_p} - \frac{T_{f,i} - T_{amb} - S}{U_L}
\]

(2.20)

A quantity \(F_R\) can now be defined, which is the ratio of actual heat removed from the collector, \(\dot{m}C_p(T_{f,o} - T_{f,i})\), to the heat removed had the entire absorbing surface been at the fluid inlet temperature. Thus,

\[
F_R = \frac{G C_p (T_{f,o} - T_{f,i})}{S - U_L (T_{f,i} - T_{amb})} \quad \text{where } G = \dot{m}/A_c
\]

(2.21)

\(F_R\) can be rearranged to form

\[
F_R = \frac{G C_p}{U_L} \left[ \frac{(T_{f,o} - T_{amb} - S/U_L) - (T_{f,i} - T_{amb} - S/U_L)}{S/U_L - (T_{f,i} - T_{amb})} \right]
\]

or

\[
F_R = \frac{G C_p}{U_L} \left[ 1 - \frac{T_{f,o} - T_{amb} - S/U_L}{T_{f,i} - T_{amb} - S/U_L} \right]
\]

(2.22)

Substituting Eq. 2.20 with \(y = L = \text{collector length}\) (where \(T_f = T_{f,o}\)) we have, after setting \(G = \dot{m}_w/L = \text{mass flow rate}\)
Substituting Eq. 2.20 for the last term in Eq. 2.22, we have

\[ F_R = \frac{G C_p}{U_L} \left( 1 - e^{-\frac{U_L F'}{G C_p}} \right) \]  

(2.23)

From the definition of \( F_R \), Eq. 2.21, we see that the useful gain per unit area \( q_u \) (= \( G C_p [T_f, o - T_f, i] \)) can be expressed as

\[ q_u = F_R [S - U_L (T_f, i - T_{amb})] \]  

(2.24)

Eq. 2.24 represents the useful heat gain of the collector per unit area. \( F_R \) is called the collector heat removal factor and is given in general by Eq. 2.23. It is seen to be a function of the variables \( G, C_p, F' \) and \( U_L \), and is constrained to be \(< F' \) by virtue of its functional dependence on these variables. The formulation of collector gain in terms of the quantities \( F' \) and \( F_R \) is due to Hottel\[12]\, , Whillier\[8,12,13]\, and Bliss\[14]\.

Equation 2.24 is known as the Hottel-Whillier-Bliss (HWB) equation, and is a particularly useful form for examining useful gain from a collector since \( F' \), and hence \( F_R \), can be considered essentially constant for a given set of operating conditions (which establish \( U_L \) and \( \dot{m} \)). Equation 2.24 is used in the following chapters to compare theoretical performance of the black liquid collector with that of a "baseline" tube and fin model.

In addition, the predictions of Eq. 2.24 are compared with more sophisticated calculations including the variation of \( U_{top} \) with fluid temperature.
In Chapter 2 analytical expressions were developed which describe the heat transfer processes taking place within a flat-plate solar collector. This chapter concerns itself with the application of these general formulae to the description of the performance of the black liquid sheet-type solar collector with a single glass cover and its comparison to an equivalent "baseline" tube and fin type.

3.1 Qualitative Considerations

Consider the system of Figure 1.2. Incident solar radiation is partially transmitted through the glass cover as well as the glass plate which defines the upper boundary of the absorbing liquid. Of the radiation which reaches and enters the liquid, a portion in the infrared band is absorbed within a few microns of the surface\([15]\), the remainder being absorbed according to Beer's Law, \(I_\lambda(x) = I_0 e^{-K_\lambda x}\), where the range for \(\lambda\) is approximately given as \(.4\mu \leq \lambda \leq .79\mu\). The spectral extinction coefficient \(K_\lambda\) is determined by the "degree of blackness" of the fluid. For sufficiently turbulent flow, time averaged absorbed thermal energy/unit volume should be independent of the depth, \(Z\), within the fluid layer of thickness, \(Z_0\) (see Figure 1.2), to a good approximation. This assumption of thermally mixed flow, which
can be stated symbolically as \( \frac{dT}{dz} \) \text{time averaged} = 0, greatly simplifies the calculation of \( U_{\text{top}} \). For the case of laminar flow between the sheets the assumption can still retain validity. The actual degree to which temperature gradients are eliminated will depend upon the blackness of the liquid, which determines the absorption profile, and the flow velocity, which determines the degree to which the profile will "relax" due to the thermal conductivity of the water as the fluid passes through the collector.

The assumption that the fluid can be treated as fluid at some average local temperature is consistent with the approach used to determine tube and fin collector performance\(^5\). There, temperature gradients for flow within the tubes are ignored in the first approximation although the flow is generally assumed laminar\(^5\).

The top plate in thermal contact with the fluid exchanges heat with the fluid through the heat transfer coefficient, \( h_1 \). It may absorb solar energy as well \((1 - e^{-Kx} = 0.05 \) for DS glass\). For the assumed turbulent flow\(^5\), \( h_1 \sim 10^3 \) W/m\(^2\)-\( ^\circ \)C. The magnitude of \( h_1 \), together with the relatively high thermal conductivity of glass \((1.05 \text{ W/m} \cdot ^\circ \)C\) and the low heat transfer coefficient coupling the plate to the ambient \((\sim 10' \) W/m\(^2\)-\( ^\circ \)C for a one-cover system\)) suggest that the top plate will be very close to the local fluid temperature. Also, any solar radiation absorbed in the plate is not lost since the plate is in thermal contact with the fluid. This can be seen by considering the quantity \( F' \) for this "type" of collector (absorber in contact with fluid) derived in Chapter 2.
\[ F' = \frac{1}{\frac{1}{U_t} + \frac{1}{h_1} + \frac{1}{h_r} + \frac{1}{h_2}} \]

Taking \( U_t \) and \( h_1 \) as 10 and 1000 W/m\(^2\)-°C, respectively, and assuming \( h_2 = h_r = h_1 \), one sees that \( F' \approx 1 \) indicating that the energy absorbed by the glass plate results in useful gain.

From these considerations it is evident that the absorbing fluid and the confining plates on either side of it behave, from a heat transfer viewpoint, as does a tube and sheet absorbing plate, since in each case the heat transfer fluid and the surface from which heat is lost can be considered at the same local temperature. (This is not strictly true for the tube and fin design. Temperature gradients do exist between the tubes but are such that the absorber plate can be considered to be essentially at the local fluid temperature\[^5\].)

As a final consideration we note that for a black liquid, absorption of light within the fluid will be independent of the angle of incidence of light as long as the extinction coefficient of the black liquid is sufficiently large that all light at normal incidence is absorbed. Thus, \( \alpha \) can be set equal to unity for all \( \theta \) for the proposed black liquid collector geometry.

3.2 The "Effective Transmittance-Absorptance" \( (\tau \alpha)_{\text{eff}} \)

In treating the transmission of light through glass plates in Chapter 2, absorption in the glass was taken into account by the term \( \alpha = e^{-KL} \), where \( KL = 0.05 \) for DS window glass. Because
this absorbed radiation produces a temperature rise within the
glass, it tends to retard the loss of heat through the top of
the collector, i.e., it decreases \( U_{\text{top}} \). This gives rise to
the concept of "effective transmittance-absorptance", since some
of the energy absorbed in the glazing produces actual useful
gain.

Whillier[8] has analyzed the case of absorption in single
and multiple glazings and determined the "effective transmittance-
absorptance" to be given by:

\[
(\tau \alpha)_{\text{eff}} = (\tau \alpha) + (1 - a) \sum_{i=1}^{N} \beta_i \tau^{i-1}
\]

(3.1)

where the quantities \( \beta_i \) depend on the collector properties and
operating conditions, and the summation extends over \( N \), the
number of glazings. For a single glazed collector Eq. 3.1 reduces
to[8]

\[
(\tau \alpha)_{\text{eff}} = (\tau \alpha) + (1 - a) \frac{U_L}{U_2}
\]

(3.2)

where \( U_L/U_2 \) is the ratio of heat transfer coefficients between
the plate and ambient, and glazing and ambient. Equivalently,
it is seen that \( U_L/U_2 = (T_C - T_{\text{amb}})/(T_P - T_{\text{amb}}) \). A calculation[8]
of \( \beta_1 = U_L/U_2 \) for typical operating conditions shows that \( \beta_1 \approx 0.25
\) and is fairly independent of collector parameters. Thus the correc-
tion term \((1 - a) U_L/U_2 \approx 1.2 \times 10^{-2} \) since \( (1 - a) \approx 0.05 \) for 1/8"
window glass[16]. This indicates that about 1/5 of the 5% of
incident energy absorbed in the single glazing is reclaimed as useful heat. Whillier has shown\textsuperscript{[8]} that as more glazings are added, the fraction of reclaimed energy rises substantially for the innermost glazing.

Graphs of $(\tau \alpha)_{\text{eff}}$ for the black liquid and a typical blackened plate collector have been calculated and are shown in Figure 3.1. The absorptance, $\alpha$, is taken as unity for the black liquid for all angles of incidence. For a blackened plate, $\alpha(\theta)$ is taken from Figure 5.7.1 of Duffie and Beckman\textsuperscript{[5]}. The quantity $\beta_1 = U_L/U_2$ was assumed equal to 0.25, as suggested by the calculations of Section 3.5 (see Figure 3.4).

3.3 Instantaneous Collector Efficiency

The instantaneous collector efficiency, $\eta$, is most conveniently defined as the ratio of useful heat output per unit area to the solar radiation incident on the collector per unit area. Symbolically, the efficiency can be written as

$$\eta = \frac{q_u}{H_t}$$

or

$$\eta = \frac{F_R[(\tau \alpha)_{\text{eff}} - U_L \frac{T_{in} - T_{amb}}{H_L}]}{H_t}$$

or

$$\eta = \frac{\dot{m} C_p (T_{f,o} - T_{f,i})}{H_t}$$

Eq. 3.3 can be applied to predict collector efficiency since both $(\tau \alpha)_{\text{eff}}$ and $U_L$ can be calculated knowing the material,
Fig. 3.1 The effective transmittance-absorptance vs. angle of incidence for a single glazed (DS glass) black-liquid and tube and fin collector. \( \alpha(\theta) \) for the tube and fin collector was taken from Fig. 5.7.1 of ref. 5.
spacing and number of glazings, the emittance, $\varepsilon_p$, of the plate, and angular distribution of incident radiation. While $F_R$ is sensitive to $U_L$ and $F'$, it also contains the dependence on $\dot{m}$ and $C_p$, the mass flow rate and specific heat of the fluid. For a given collector geometry, $\eta$ can be plotted as a function of 
\[
\frac{(T_f,i - T_{amb})}{H_R},
\]
using Eq. 3.3, once $(\tau\alpha)_{eff}$, $U_L$ and $F_R$ are determined. Alternatively, Eq. 3.4 is a useful form to determine collector efficiency experimentally since $\dot{m}$, $T_{amb}$, $(T_{f,0} - T_{f,i})$, and $H_t$ can be readily measured.

If $U_L$ is assumed constant everywhere on the collector surface, or equivalently an average $U_L$ is assumed, Eq. 3.3 predicts a straight line for a given $\dot{m}$ (and hence $F_R$). However, as $\dot{m}$ and thus $F_R$ change, so does $\eta$, so that each straight line is a line of constant $F_R$. Figures 3.2a and b show the effect of varying $F_R$ and $\dot{m}$ on the efficiency for a typical single-cover collector. Because of the functional dependence of $F_R$ on $\dot{m}$, the variation of $\eta$ with $\dot{m}$ is seen to be slight when $\dot{m}[C_p/U_A c]$ is greater than about 10.

3.4 Comparison with "Baseline" Tube and Fin Collector

The performance curves for the black liquid collector can be readily compared to those of a typical tube and fin collector by applying the simple (linear) approach offered by the Hottel-Whillier-Bliss equation (Eq. 2.24)
\[
Q_u = F_R A_c [H_t (\tau\alpha)_{eff} - U_L (T_{f,i} - T_a)]
\]
Fig. 3.2a The instantaneous collector efficiency plotted as a function of the parameter \( \frac{(T_{in} - T_{amb})}{H_t} \) (\( m^2 \cdot ^\circ C/W \)) for various values of \( F_R \).
The instantaneous collector efficiency plotted as a function of the parameter \( \frac{(T_{in} - T_{amb})}{H_t} \) for various values of the mass flow rate.
where

\[ F_R = \frac{G \, C_p}{U_L} \left( 1 - e^{-\frac{U_L F'}{G \, C_p}} \right) \]

and

\[ (\tau_\alpha)_{\text{eff}} = (\tau_\alpha) + (1 - a) \sum_{i=1}^{n} \beta_i \tau_i \]

A meaningful comparison can be established by assuming that \( \dot{m} \), \( A_c \) (hence \( G = \dot{m}/A_c \)), \( C_p \), and \( U_L \) are the same for each collector. Typical values for these parameters are:

\[ G = 0.0163 \text{ kg sec}^{-1} \text{ m}^{-2} = 12 \frac{\text{lb}_m}{\text{hr} \cdot \text{ft}^2} \]

\[ C_p = 4.187 \frac{\text{kJ}}{\text{kg} \cdot ^\circ \text{C}} = 1.0 \frac{\text{BTU}}{\text{lb}_m \cdot ^\circ \text{F}} \]

\[ U_L = 8 \frac{\text{W}}{\text{m}^2 \cdot ^\circ \text{C}} = 1.41 \frac{\text{BTU}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ \text{F}} \]

The value chosen for \( U_L \) corresponds closely to a one-cover system under typical operating circumstances. The back heat loss coefficient is contained in \( U_L \) and edge losses are ignored.

The quantity \( (\tau_\alpha)_{\text{eff}} \) can be taken from Figure 3.1 and is seen to be 0.84 at 0\(^\circ\) for the black liquid and 0.85 at 0\(^\circ\) for the tube and fin.

It is also necessary to calculate \( F' \) for each design. For the tube and fin geometry, Duffie and Beckman\([5]\) calculate \( F' \) to be

\[ F' = \left[ \frac{1}{U_L} \right] \left[ \frac{1}{W \left[ \frac{1}{U_L (D + (W-D)F)} + \frac{1}{C_p} + \frac{1}{\pi D_i h_{f,i}} \right]} \right] \]
where, \( W \) = center-to-center distance between tubes

\( D \) = ID of tube

\( C_b \) = bond conductance of tube to sheet

\( h_{f,i} \) = tube to fluid heat transfer coefficient

\( F \) = standard fin efficiency for straight fins of rectangular profile = \( \frac{\tanh \left[ \frac{m(W-D)}{2} \right]}{m(D-W)/2} \);

where \( m^2 = \frac{U}{k} \), and \( \delta = \) fin thickness, \( k = \) fin thermal conductivity.

For the comparison at hand it is assumed these quantities take on the following (typical) values:

\( W = 15 \text{ cm (6")} \)

\( D = 1 \text{ cm (0.4")} \)

\( h_{f,i} = 1500 \frac{W}{m^2-\circ C} \left( \frac{265}{\text{BTU/hr-ft}^2-\circ F} \right) \) - turbulent flow

\( \frac{1}{C_b} \to 0 \) (negligible heat transfer resistance between fin and tube - "best" case)

\( F' = 0.892 \) (assuming 0.5 mm thick Al fin)

\( = 0.935 \) (assuming 0.5 mm thick Cu fin)

It is apparent from Eq. 3.5 that \( F' \) is quite insensitive to \( h_{f,i} \) or \( C_b \) as long as they are large, but is particularly dependent upon \( W \). For the black liquid collector, \( F' \) is taken as unity.

The results of the comparison are shown in Figures 3.3a and 3.3b for angles of incidence of 0° and 60° respectively. It is
Fig. 3.3a Instantaneous collector efficiency for black liquid and tube and fin collectors calculated at 0 degrees angle of incidence. Results assume \( W = 15 \text{ cm} \), \( D = 1 \text{ cm} \), \( C_p = \infty \), and \( h_{f,i} = 1500 \text{ W/(m}^2\text{C)} \) for the tube and fin collectors.
Fig. 3.3b  Instantaneous collector efficiency for black liquid and tube and fin collectors calculated at 60 degrees angle of incidence. Results assume $W = 15$ cm, $D = 1$ cm, $C_b = \infty$, and $h_{f,i} = 1500$ W/(m$^2$°C) for the tube and fin collectors.
seen that the heat transfer resistance from the point of absorption to the fluid (proportional to \([F']^{-1}\)) results in a performance decrease of the tube and fin configuration compared to that of the black liquid. This difference is further enlarged at increasing angles of incidence by the strong dependence of \(\alpha\) on \(\theta\), the angle of incidence, for the tube and fin geometry.

Duffie and Beckman\(^5\) suggest that diffuse radiation can be treated as beam radiation with \(\theta = 60^\circ\), where it is assumed that diffuse radiation is uniformly distributed over the sky (valid for cloudy or hazy days). Thus Figure 3.3b also shows the performance difference of the two collector types under these conditions. For this particular example the black liquid collector output is about \(\frac{4.8 - 4.1}{4.5} \times 100 \approx 15\%\) greater than the tube and fin collector under typical operating circumstances (\([T_f,i - T_{amb}]/H_t \approx 0.03[\frac{W}{m^2 \cdot ^\circ C}]^{-1}\)).

3.5 The Computer Model

The single glazed, sheet flow, black liquid collector has been modelled by computer simulation in order to determine its performance curves. In the model the variation of the top loss coefficient with position was taken into account. This precluded the use of the simplified result given by Eq. 3.3 and tends to introduce curvature to the linear predictions of the Hottel-Whillier-Bliss equation. The assumptions of the model were:
a) uniform flow, i.e. flow rate independent of position.

b) thermally mixed flow resulting in a uniform temperature distribution throughout the depth of the liquid (Z direction in Figure 1.2).

c) fluid temperature independent of coordinate parallel to collector width (x-direction in Figure 1.2). Thus edge effects are neglected.

d) fluid is optically thick so that all incident light is absorbed within the fluid layer.

e) single glazing of 1/8" thick glass with KL = 0.05.

f) fluid is constrained by two parallel glass sheets (1/8" thick) separated by 1/4". Temperature of top sheet is at local fluid temperature.

g) \( U_L = U_{\text{top}} + U_{\text{back}} \)

h) negligible heat conduction in the direction co-linear to the flow direction (y-direction in Figure 1.2)

For these assumptions an algorithm was devised to calculate numerically the following quantities as a function of y, the coordinate co-linear with the flow direction.

\[
T_C = \text{cover temperature}
\]

\[
T_f = \text{fluid temperature} \quad (= T_p \text{ by assumption})
\]

\[
U_t = \text{top loss coefficient}
\]

The parameters \( V_{\text{wind}} \), \( (\tau_\alpha)_{\text{eff}} \), \( \tau_{\text{amb}} \), \( C_p \), \( U_{\text{back}} \) and \( \varepsilon_g \) were taken as 5 m/s, 0.84, 15\(^\circ\)C (= 59\(^\circ\)F), 4.187 \( \frac{\text{kJ}}{\text{kg} \cdot \text{\(\circ\)C}} \) (= 1.0 \( \frac{\text{BTU}}{\text{lb} \cdot \text{\(\circ\)F}} \)),

\[
0.5 \frac{W}{\text{m}^2 \cdot \text{\(\circ\)C}} \quad (= 0.9 \frac{\text{BTU}}{\text{hr} \cdot \text{ft}^2 \cdot \text{\(\circ\)F}}), \text{ and 0.95 respectively.}
\]
The iteration technique is as follows:

a) Establish values for $T_{f,i}$ and $\dot{m}$.

b) $T_{f,i} = T_f(y = 0)$; assume a value for $T_c(y = 0)$ such that $T_f > T_c > T_{amb}$.

c) Calculate the four heat transfer coefficients for $y = 0$ according to formulae of Section 2.3.

$$h_{r,p-c} = \frac{\sigma}{2/\varepsilon_g - 1} (T_p^2 + T_c^2)(T_p + T_c)$$

Radiation from plate to cover

$$h_{r,c-s} = \sigma \varepsilon_g (T_{sky}^2 + T_c^2)(T_{sky} + T_c)$$

Radiation from cover to sky

$$h_w = 5.7 + 3.8V_{\text{wind}} \quad (V_{\text{wind}} \text{ in m/s})$$

Convection from cover to ambient

$$h_{p-c} = \left[ \left\{ \frac{1.63 (T_p - T_c)^{.281}}{\lambda(cm)^{.157}} \right\} \times (1 - .0018[\overline{T} - 10]) \right]$$

Convection from plate to cover

d) Calculate $U_t = \left[ \frac{1}{h_{p-c} + h_{r,p-c}} + \frac{1}{h_w + h_{r,c-s}} \right]^{-1}$ at $y = 0$.

e) $U_t$ from part d) can now be used to calculate an improved $T_c(y = 0)$ from the relation

$$T_c = T_p - \frac{U_t(T_p - T_s)}{h_{p-c} + h_{r,p-c}}$$

which results from equating the heat transfer from the plate to the cover to the heat transfer from the plate to the sky. This gives $T_c(0)$ and $U_L(0)$. 
f) Repeat c) - e) until \( T_c(0) \) and \( U_L(0) \) converge.

\[ T_{p,\Delta y} = T_{p,\Delta y}^{(0)} + (T_{amb} + \frac{S}{U_L})[1 - \frac{U_L}{mC_p} \Delta y] \]

This is Eq. 2.19, but uses the best available value for \( U_L \), namely \( U_L(0) \).

h) Now calculate \( T_c, \Delta y \) and \( U_L, \Delta y \) using iteration of steps b) - f).

i) Use result of h) to produce a corrected value of \( T_{p,\Delta y} \).

j) Repeat h) and i) until \( T_c, \Delta y' \), \( T_{p,\Delta y} \) and \( U_L, \Delta y \) converge.

k) Repeat procedure for \( T_{p,2\Delta y} \), \( T_c,2\Delta y \) and \( U_L,2\Delta y \), etc.

l) When \( n\Delta y \) = length of collector, calculate

\[ \eta = \frac{mC_p}{H_t} (T_f,0 - T_{f,i}) \]

m) Repeat entire procedure for various \( m \)'s, \( T_{f,i} \)'s, etc.

A FORTRAN IV program was written employing the above iteration procedure. The results, displayed in Figures 3.4 through 3.7, are discussed below.

Figure 3.4 shows the quantity \( \beta_1 = \frac{U_L}{U_2} = \frac{T_c - T_{amb}}{T_p - T_{amb}} \) as a function of \( T_p - T_{amb} \). \( \beta_1 \) is seen to be \( 0.25 \pm 10\% \) when \( T_p - T_{amb} \) ranges between 28 and 78°C. These results are in agreement with previous calculations of \( \beta_1 \) and support the use of \( \beta_1 = 0.25 \) in the calculation of \((\tau \alpha)_{eff}\) (Section 3.2).
Fig. 3.4 The ratio $\beta_1 = \frac{U_L}{U_2} = \frac{\text{heat loss coefficient: plate to ambient}}{\text{heat loss coefficient: glazing to ambient}}$ as a function of the parameter $T_p - T_{\text{amb}}$. Results were calculated from computer model discussed in section 3.5.
The position dependence of $T_C$, $T_p$ and $U_t$ for two different values of the mass flow rate are shown in Figures 3.5 and 3.6. For the "realistic" flow rate of $1.5 \times 10^{-2} \text{kg/m}^2\text{s} = 11.1 \text{lb}_m/\text{ft}^2\text{hr}$. These three quantities are essentially linear with distance. However, for the very slow flow rate of $1.5 \times 10^{-3} \text{kg/m}^2\text{s}$, both $T_p$ and $U_L$ exhibit considerable non-linearity, as expected by qualitative energy-balance arguments.

The effect of including a position dependent $U_L$ in the calculation of $\eta$ is illustrated in Figure 3.7. The efficiency curve falls off more rapidly with this taken into account than the Hottel-Whillier-Bliss equation predicts. However, the actual point of zero efficiency is difficult to predict unless operating conditions are specified exactly. This can be seen by noting that different (non-linear) performance curves are generated by assuming different values of $H_t$ (see Figure 3.7), a result of the fact that $U_L$ is not a single-valued function of $(T_{in} - T_{amb})/H_t$.

For example, $(T_{in} - T_{amb})/H_t$ can be made small by holding $H_t$ constant and decreasing $T_{in} - T_{amb}$ ($U_L$ then decreases) or by holding $T_{in} - T_{amb}$ constant and increasing $H_t$ ($U_L$ increases). Thus $H_t$ cannot be entirely factored out by plotting $\eta$ against the parameter $(T_{in} - T_{amb})/H_t$. Fortunately, divergence in the performance curves occurs to a significant extent only when $(T_{in} - T_{amb})/H_t \lesssim 0.04 [W/(m^2\cdot ^\circ C)]^{-1}$. Since most useful collector heat output for flat-plate collectors occurs during operation when $(T_{in} - T_{amb})/H_t \gtrsim 0.04 [W/(m^2\cdot ^\circ C)]^{-1}$, any reasonable value of $H_t$ can be chosen.
Fig. 3.5 Fluid and glazing temperature calculated as a function of position in the flow direction along the black liquid collector for two different values of the mass flow rate.
Fig. 3.6 The top loss coefficient calculated as a function of position in the flow direction along the black liquid collector for two different values of the mass flow rate.
Fig. 3.7 Comparison of instantaneous efficiency for the black liquid collector calculated using linear (HWB) and non-linear (iteration) methods. Non-linear results are calculated for two different values of $H_t$. Linear prediction using the HWB eq. assumes $U_L = 7.1 \ W/(m^2\cdot ^\circ C)$. 

$$10^2 \times \frac{(T_{in} - T_{amb})}{H_t} \quad (m^2\cdot ^\circ C/W)$$

non-linear: $H_t = 632 \ W/m^2 = 200 \ BTU/ft^2\cdot hr$

linear: assuming $U_L = 7.1 \ W/m^2\cdot ^\circ C$
in the Hottel-Whillier-Bliss equation
Indeed, the Hottel-Whillier-Bliss equation presents an adequate representation in this regime, as is evidenced by the comparison of Figure 3.7.

The results of the non-linear numerical calculation then suggest that \( q_u = F_R [H(\tau x) \text{eff} - U_L (T_{in} - T_{amb})] \) can be accurately applied in most practical situations if \( U_L, \text{avg} \) can be determined. The value of \( (T_{in} - T_{amb})/H_t \) which it predicts for the point of zero efficiency must, however, be interpreted as an upper limit, the actual value being determined by the exact operating conditions.
Chapter 4
Long-Term Performance Analysis

The instantaneous collector efficiency can be a useful concept when comparing efficiencies of similar collectors under similar meteorological circumstances. But because of the large number of variables involved in collector performance (e.g., $H_t$, $(\tau \alpha)_{\text{eff}}$, $U_L$, $\dot{m}$, $T_{f,i}$, $T_{\text{amb}}$, and $C_p$), recourse to a more general measure of efficiency is sometimes necessary. By defining the overall efficiency, denoted $\eta'$, as

$$\eta' = \frac{\int q(t)\,dt}{\int H_t(t)\,dt}$$

it becomes possible to incorporate the variation of $(\tau \alpha)_{\text{eff}}$ and collector losses with time into a performance evaluation.

To accurately calculate or measure overall collector efficiency one needs, ideally $H_t = H_t(t)$, or more practically, hourly values for $H_t$, the radiation on a tilted surface, and indeed the direct and diffuse components thereof. Since these measurements are rarely available, it becomes necessary to estimate them from measurements which are available, namely $\bar{H}_h$, the monthly average daily radiation on a horizontal surface.

In this chapter semi-empirical means are developed to estimate average hourly beam and diffuse solar radiation on an arbitrarily tilted south-facing surface. These methods are then applied to actual Rochester insolation data to produce tilted surface insolation values and determine optimum angles of tilt for each month.
Using this as input, collector performance is calculated for both the black liquid collector and the "baseline" tube and fin collector. Results are presented and compared for average days in January and August. These months presumably typify winter and summer months of operation in Rochester.

4.1 Insolation on Horizontal and Tilted Surfaces

The solar constant, $I_{sc}$, was discussed in Section 2.1 and seen to equal 4871 kJ/m$^2$hr = 428 BTU/ft$^2$hr. Knowledge of $I_{sc}$ allows the total daily extraterrestrial radiation on a horizontal surface, $H_0$, to be calculated$^{[17]}$. $H_0$ is shown in Figure 4.1 for four latitudes as a function of time of year. The variation is seen to be quite severe. For instance, at 50°N latitude, $H_0$ in January is about 19% of the June value. For comparison, the total cloudless day solar radiation on a horizontal surface at sea level is given in Figure 4.2. The cloudless day values are observed to represent about 60-70% of the respective extraterrestrial values.

Daily total solar radiation on a horizontal surface in the Rochester area has been recorded for the years 1970-73, 1975 at SUNY at Brockport. These data, in the form of monthly average daily totals, $\bar{H}_h$, are presented in Figure 4.3 along with the horizontal extraterrestrial and total cloudless values for Rochester, New York.

To obtain tilted insolation it is necessary to first decompose $\bar{H}_h$ into its direct and diffuse components. This can be
Fig. 4.1  Total daily extraterrestrial radiation $H_o$ on a horizontal surface for various latitudes (from Table 1.1, ref. 17)
Fig. 4.2 Total cloudless day solar radiation on a horizontal surface at sea level for various latitudes (from Table 3, Ref. 18)
done by forming the quantity \( K_T = \frac{\bar{H}_h}{H_0} \) for a given month. Then from Figure 7 of Liu and Jordan\(^{[19]}\), the ratio \( K_D = \frac{\bar{D}}{\bar{H}_h} \), and hence \( \bar{D} \) can be found, where \( \bar{D} \) is defined as the monthly average diffuse radiation on a horizontal surface. Thus \( \bar{D} \) and \( \bar{H}_h \) can each be determined for each month.

The daily distribution of \( \bar{D} \) and \( \bar{H}_h \) is determined by again referring to Liu and Jordan\(^{[19]}\). From Figure 15 of their work the ratio \( \frac{\text{hourly diffuse radiation}}{\text{daily diffuse radiation}} \) for each hour is found once \( \bar{D} \) and the number of hours from sunrise to sunset have been specified. Similarly, from Figure 16 of Liu and Jordan\(^{[19]}\), the ratio \( \frac{\text{hourly total radiation}}{\text{daily total radiation}} \) for each hour can be determined with knowledge of \( \bar{H}_h \). Calculation of the hourly distribution of horizontal beam radiation follows directly. As an example, average hourly diffuse and total radiation for the month of January in Rochester is presented in Figure 4.4. The similarity of these distributions to that of a sine function is quite close as indicated in Figure 4.4.

We are now in a position to transform the average hourly values of beam and diffuse radiation on a horizontal surface to corresponding values on a tilted south-facing surface. The geometrical transformation factor for direct (beam) radiation, \( R_b = \frac{H_{t,b}}{H_b} \), is a function of tilt angle, time of day, time of year and location. According to Hottel and Woertz\(^{[20]}\), this scalar transformation takes the form

\[
R_b = \frac{\cos(\phi - S) \cos \delta \cos \omega + \sin(\phi - S) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta} \quad (4.1)
\]
Fig. 4.4 The hourly distribution of horizontal diffuse and total radiation for the month of January in Rochester compared to a sine distribution.
where, \( \phi = \) latitude
\[
\delta = \text{declination} \approx 23.45 \sin \left( 360 \frac{284 + n}{365} \right); \quad n = \text{day of year}
\]
\( \omega = \) hour angle; solar noon = 0, and each hour = 15° longitude with mornings positive and afternoons negative.

\( S = \) tilt towards the equator, degrees.

Thus \( R_b \) can be calculated for each hour for a given location from which \( H_{t,b} = R_b H_b \) is determined. In Figure 4.5 is shown \( R_b \) at solar noon as a function of day of year for five angles of tilt. The results are valid for 43°N latitude.

The diffuse component can be handled in several different ways. Some diffuse radiation is associated with small angle scattering and thus appears to originate from an area near the sun, and might therefore be most appropriately transformed by the same prescription as beam radiation. Other diffuse radiation is the result of back scattered radiation from clouds, sky, surroundings, etc., and is more uniformly distributed over the sky. The intensity of this radiation should be approximately independent of surface orientation. The measurements of Threlkeld\textsuperscript{[21]} suggest that on clear days the uniform component is about 40-50% of the total diffuse incident on a surface facing the sun. Any presence of clouds or haze would increase the uniform component until under totally cloudy skies no forward scattered component would be present.

Because of the difficulty in estimating the relative magnitudes of the forward scattered and isotropic components on an
Fig. 4.5 Plot of the instantaneous geometric factor $R_B$ for Rochester, N.Y., through the year for $s = 0^\circ$ (solar noon). Curves are shown for $s = 0^\circ$, 28°, 43°, 58°, and 90°.
average basis the procedure used by Liu and Jordan\textsuperscript{[22]} to transform diffuse radiation is adopted here. Accordingly, a surface tilted at an angle, $S$, will see a portion of the sky given by $[1 + \cos (S)]/2$. Assuming a uniform distribution of diffuse radiation (that is, an isotropic angular distribution but not necessarily constant with respect to time), this then gives the transformation factor for sky-originating radiation.

However, the tilted surface sees the ground or other surroundings as well. This solid angle is $[1 - \cos (S)]/2$, and radiation amounting to $(H_b + D)$ is reflected with some average diffuse reflectance, $\rho$, from it to the tilted surface. Thus, the total solar radiation detected on a tilted surface for a given time is:

$$H_t = H_b R_b + D \left[ \frac{1 + \cos S}{2} \right] + \rho (H_b + D) \left[ \frac{1 - \cos S}{2} \right] \quad (4.2)$$

Liu and Jordan\textsuperscript{[22]} recommend ground reflectances of 0.7 and 0.2 for snow cover and no snow, respectively.

It is to be noted that the transformation factors $(1 - \cos S)/2$ and $(1 + \cos S)/2$ are independent of time, unlike the quantity $R_b$.

To compute the monthly average daily totals of radiation on a tilted surface, Eq. 4.2 must be summed over the hours of daylight. $R_b = R_b (\text{hour})$ is calculated from Eq. 4.1 and the hourly percentages of $H_b$ can be deduced from Figures 15 and 16 of Liu and Jordan\textsuperscript{[16]}. Denoting these hourly percentages by $a_i$, Eq. 4.2 becomes:
\[ \overline{H}_{t, \text{day}} = \overline{H}_b \left[ \alpha_1 R_{b_1} + \ldots + \alpha_n R_{b_n} \right] + \overline{D} \left[ \frac{1 + \cos S}{2} \right] + \rho \left( \overline{H}_b + \overline{D} \right) \left( \frac{1 - \cos S}{2} \right) \]  

(4.3)

where the summation extends over the hours of daylight and it is assumed that \( R_{b_i} \) represents an average hourly value and does not vary significantly over the period of a month. Thus Eq. 4.3 provides a means of calculating monthly average daily values of radiation on a tilted surface from knowledge of \( \overline{H}_h \) and empirical information contained in Liu and Jordan[19].

We now define \( \overline{R} = \frac{\overline{H}_t}{\overline{H}_h} \). From Eq. 4.3 it is seen that \( \overline{R} \) is

\[ \overline{R} = \frac{\overline{H}_b}{\overline{H}_h} \left[ \alpha_1 R_{b_1} + \ldots + \alpha_n R_{b_n} \right] + \frac{\overline{D}}{\overline{H}_h} \left[ \frac{1 + \cos S}{2} \right] + \rho \left[ \frac{1 - \cos S}{2} \right] \]  

(4.4)

The use of \( \overline{R} \) enables tilted radiation to be calculated from horizontal radiation by means of a single scalar quantity.

\( \overline{R} \) has been calculated for Rochester insolation for the 12 months of the year and is plotted as a function of tilt angle in Figure 4.6. The presence of significant amounts of diffuse radiation reduce \( \overline{R} \) to well below the transformation factor for pure beam radiation, since diffuse radiation is assumed to transform essentially in a one-to-one correspondence (aside from small ground reflectance effects).

From Figure 4.6 it is seen that for each month \( \overline{R} \) can be optimized by the appropriate choice of tilt angle. For the months
Fig. 4.6  Plot of $\bar{R}$, the number which transform "monthly average daily total radiation" on a horizontal surface to that on a tilted surface for latitude $= 43^\circ$. 

$s = \text{the angle of tilt towards the equator}$
of November, December, January and February, \( \bar{R}_{\text{max}} \) occurs at about \( L + 15^\circ = 58^\circ \); whereas for summer months, \( \bar{R} \) is maximized at \( S_{\text{tilt}} \leq L - 15^\circ = 28^\circ \). However, \( \bar{R} \) is not a rapidly varying function of \( S_{\text{tilt}} \) and small (5°) deviations from the optimum tilt angle do not result in large changes in \( \bar{R} \).

4.2 Comparison of Overall Efficiencies.

From Figure 3.1 it is evident that the dependence of \( (\tau\alpha)_{\text{eff}} \) on angle of incidence is different for the tube and fin collector and the black liquid collector studied in this work. To properly account for this in a comparison of calculated efficiencies, the performance during the course of an entire day must be considered. Thus, it becomes necessary to know, in addition to the hourly amounts of beam and diffuse radiation on a tilted surface, the average angle of incidence during each hour for each of these components.

We can calculate the daily performance of a given collector for an "average" day with knowledge of the monthly average daily total radiation on a horizontal surface. By use of Eq. 4.2, hourly values of the monthly average daily total on a tilted surface can be obtained, as well as the beam and diffuse components thereof. Next, it is necessary to calculate the angle of incidence of the beam radiation for each hour. This is computed from the relation\(^{[23]}\)

\[
\cos \theta_z = \sin \delta \sin \phi \cos S - \sin \delta \cos \phi \sin S \\
+ \cos \delta \cos \phi \cos S \cos \omega \\
+ \cos \delta \sin \phi \sin S \cos \omega \tag{4.5}
\]
where $\theta_z$ is the angle of incidence measured from the normal to the tilted surface. It will be assumed, for simplicity, that diffuse radiation is incident at some average angle of incidence taken as $60^\circ$ as suggested by Duffie and Beckman[5]. We are thus in a position to calculate the overall efficiency.

$$
\eta' = \frac{\int_{\text{day}} q_u(t) \, dt}{\frac{\text{day}}{H_t}} = \frac{F_R}{H_t} \int_{\text{day}} [H_t(t)(\tau\alpha)_{\text{eff}}(t) - U_L(T_{f,i} - T_a)]
$$

$$
= \frac{F_R}{H_t} \sum_{i} [H_t^i(\tau\alpha)_{\text{eff}}^i - U_L(T_{f,i} - T_a)^i]^+ \quad (4.6)
$$

Here, $i$ indicates that the summation extends over the hours of daylight and $+$ indicates that only positive values will be included to more accurately simulate the operation of a real system. Considering beam and diffuse components, Eq. 4.5 becomes

$$
\eta' = \frac{F_R}{H_t} \sum_{i} [H_b^i(\tau\alpha)_{\text{eff}}^i + D^i(\tau\alpha)_{\text{eff}}^i, 60^\circ - U_L(T_{f,i} - T_a)] \quad (4.7)
$$

From Eq. 4.2 the reflectance, $\rho$, was arbitrarily adjusted so that $\rho = D/(H_b + D)$ for each hour. This simplifies Eq. 4.7 and represents only a small correction to Eq. 4.2. Furthermore, $U_L(T_{f,i} - T_a)$ is assumed constant throughout the day.

As indicated previously, Eq. 4.7 should provide a meaningful relative comparison of overall collector performance for the "average" day deduced from the monthly average daily totals. However, the absolute efficiencies predicted by Eq. 4.7 will be an under-estimate of the monthly average overall efficiency since
they are based on use of a month of average days, a procedure which does not properly account for the statistical distribution of insolation intensity.

Tables 4.1 and 4.2 show the results of comparison of the black liquid collector and the copper tube and fin collector described in Section 3.4. The comparison is hourly for average days for the months of January and August and it was assumed that \( S = L + 15° = 58° \) for the former and \( L - 15° = 28° \) for the latter. These tilt angles represent approximately optimum orientations for these months. A single glazing was assumed present in each case. The quantities \( \Omega_{BL} \) and \( \Omega_{TF} \) are the calculated amounts of energy absorbed in the absorbing medium in each collector before losses \( [U_L(T_f,i - T_{amb})] \) are subtracted.

Figure 4.7 shows \( \eta'_{day} \) for the average January and August days as a function of \( (T_f,i - T_{amb}) \). Here \( U_L \) was taken as 8 W/m\(^2\)-°C (= 1.4 BTU/ft\(^2\)-hr.-°F) for each collector and \( F'' = F_R/F' \) was set at 0.95. Unlike instantaneous efficiency, when plotted as a function of \( (T_f,i - T_{amb}) \), as in Figure 3.2a and b, \( \eta'_{day} \) exhibits an upper curvature as \( (T_f,i - T_{amb}) \) increases. This results from the definition of \( \eta'_{day} \), where only positive terms in the summation are included, a procedure which neglects losses and the associated negative efficiency when \( U_L(T_f,i - T_{amb}) > \Omega \). Subsequently, \( \eta' \) is increased somewhat over the value obtained when these losses are taken into account.

The curves of Figure 4.7 reveal two interesting features. First, the January curves are higher than those of August for
January: $s = 58^\circ$, 9 hr. from sunrise to sunset

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<th>$D$</th>
<th>$H_b$</th>
<th>$H_{bR}$</th>
<th>$\theta_z$</th>
<th>$(\tau\alpha)_{BL}$</th>
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† in units of kJ/m$^2$-hr

Table 4.1 The hourly values of the quantity $\Omega = H_{t,b}(\tau\alpha)_{eff} + D(\tau\alpha)_{eff}$, $60^\circ$ for the proposed black liquid collector and the baseline tube and fin collector. Results are based on average January insolation for Rochester, N.Y. $R_b$ is calculated for a tilt angle of $58^\circ$. Columns 2-4 are based on $H_h = 6494$ kJ/m$^2$-day and the daily distributions given in Figures 15 and 16 of ref. 19 for a 9 hour period of daylight. Columns 7 and 8 were deduced from Fig. 3.1 of this work. $\theta_z$ was calculated using eq. 5.5.
August: \( s = 28^\circ, 13.8 \) hr. from sunrise to sunset

<table>
<thead>
<tr>
<th>hour (solar time)</th>
<th>( H_h )</th>
<th>( D )</th>
<th>( H_b )</th>
<th>( H_{b}R_{b} )</th>
<th>( \Theta_z )</th>
<th>( (\tau\alpha)_{BL} )</th>
<th>( (\tau\alpha)_{TF} )</th>
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<td>.85</td>
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+ in units of kJ/m²-hr

Table 4.2 The hourly values of the quantity \( \Omega = H_{t,b} (\tau\alpha)_{eff} + D(\tau\alpha)_{eff, 60^\circ} \) for the proposed black liquid collector and the baseline tube and fin collector. Results are based on average August insolation for Rochester, N.Y. \( R_{b} \) is calculated for a tilt angle of 28°. Columns 2-4 are based on \( H_h = 19318 \) kJ/m²-day and the daily distributions given in Figures 15 and 16 of ref. 19 for a 13.8 hr. period of daylight. Columns 7 and 8 were deduced from Fig. 3.1 of this work. \( \Theta_z \) was calculated using eq. 5.5.
Fig 4.7 $\eta'$ as a function of $T_{in} - T_{amb}$ for "average days" during the months of January and August in Rochester. $U_L$ was taken as 8 W/m²°C for each collector and $F'' \equiv F_R/F'$ was set at 0.95.
This results from $Z_{f,i} - T_{amb}$ being, on the average, smaller in January than in August. Thus, less radiation is lost to reflection resulting in enhanced efficiency. Secondly, the January curves trail off more rapidly than those of August. This is a consequence of a larger value of $\bar{H}_h$ in August which allows the collector to operate at positive efficiency for large values of the loss term $\sim (T_{f,i} - T_{amb})$.

The same mechanisms which were seen in Chapter 3 to be responsible for a 10-15% increase in instantaneous efficiency for the black liquid collector over the tube and fin collector are seen in Figure 4.7 to provide about the same degree of enhancement in long-term efficiency. These two effects are (a) $F' = 1$ for the black liquid collector, as opposed to about 0.9 for a tube and fin design; and, (b) $(\tau \alpha)_{eff}$ is larger, on the average, for the black liquid collector (Figure 3.1).

To increase the accuracy of the long-term performance predictions of this section a number of refinements could be incorporated in the procedure. Primarily, these are: (a) relaxation of the assumption that $T_{f,i} - T_{amb}$ = constant throughout the day by using (say) actual hourly data; (b) accounting for heat capacity effects during times when the collector presumed not in operation; and, (c) use of actual hourly insolation data, $H_L$, instead of monthly average values.
5.1 The Prototype

In order to experimentally test the analytical results predicted in Chapter 3, a working prototype was constructed and measurements were carried out to determine its efficiency under various meteorological circumstances.

The prototype consisted of two glass (DS window glass - 1/8" thick) plates held parallel to one another by a frame and separated by 1/4". The plates were about 18" x 36", giving a collector surface area of 4.40 ft². The frame was constructed of 5/8" thick acrylic sheet (Plexiglas) appropriately grooved to accommodate the two sheets. The grooves were 3/8" deep, and to prevent leakage, were sealed with "G.E. Silicone Glue and Seal". This product was chosen since it bonded well, was unaffected by the temperature range involved (≤ 200°F), and was sufficiently elastic to allow for expansion or flexure of the plates and/or frame.

To approximate a uniform flow within the collector (i.e., flow such that \( \dot{m} \)/unit width was independent of the position along the width) both the inlet and outlet were manifolded. These manifolds consisted of a cavity 1/4" x 1/2" x 18" along both the top and bottom of the collector. Each cavity was connected to the main collector chamber (the 18" x 36" x 1/4" volume) by a
multiple of 0.1" diameter holes spaced approximately 0.25" apart, center to center, in a linear array. The side of the manifold opposite the holes was connected to a header pipe (1/2" ID copper) via connections at three locations, near both ends and in the center. These connections were spaced 5" apart and helped insure an even flow distribution to the manifold.

Preliminary testing indicated that the flow distribution was quite uniform when the inlet header was parallel to the horizontal plane. This was observed by allowing clear water to be pumped through the collector, then introducing a dye to the water and observing the resulting flow pattern.

A photograph of the prototype is shown in Figure 5.1.

For the black absorbing fluid, different coloring agents were tested. First India Ink was added to water. It was hoped that India Ink, being particles of carbon in suspension, would present itself as an adequate black solution, as indicated by a previous investigation[1]. However, a problem arose due to particles precipitating out during stagnant periods and forming a scale on the glass and clogging some of the holes leading to the body of the collector. This was unsatisfactory.

A better darkening agent proved to be "Cyanamid Cycal 1452" dissolved in water. This chemical, NaSO₃, is used in boilers to reduce scale deposits. It has, in dry form, the consistency of dirt or sand, and about 2 ozs. is adequate to darken about 2 gallons of water. This results in a suspension of very fine particles and a very dark red-colored liquid. During prolonged
Fig. 5.1 Photograph of the prototype black liquid solar collector without fluid. Dimensions are approximately 1.5' x 3'.
stagnant periods, some precipitation of the chemical did occur, but minor agitation was generally sufficient to adequately remix the fluid.

For initial testing the necessary amount of "darkness" within the liquid was determined when a bright light passing through a 1/4" thickness of liquid could not be seen. Since about 46% of sunlight is in the infrared spectrum, and this is absorbed very quickly in the liquid, only visible light will pass through. (The 7% of energy in the ultra-violet region is mostly absorbed in the atmosphere.) A reflector of aluminum foil was placed at the back side of the collector so any light passing through was reflected back into the liquid. The reflector also served to reduce radiation losses through the back of the collector.

A single glazing (DS window glass) was added to the collector arrangement after initial testing. It was separated from the top plate by 0.6" and held in the Plexiglas frame. It was not sealed to facilitate removal if necessary. For testing, the collector was supported by a frame of 2" x 4" studs mounted on a plywood sheet. Fiberglass insulation 3-1/2" deep was added to the back of the collector to reduce heat loss through the back. Styrofoam insulation 3/4" thick was placed along either side to reduce edge losses.

5.2 Experimental Procedure

To determine the instantaneous efficiency of a solar collector several variables must be monitored simultaneously. These
are \( H_t \), the solar radiation incident on the collector surface; \( \dot{m} \), the mass flow rate of fluid through the collector; \( T_{in} \), the fluid inlet temperature; \( T_{out} \), the fluid outlet temperature; and \( T_{amb} \), the ambient air temperature.

Solar radiation incident on a horizontal surface was measured by means of an Eppley pyranometer (Model 8-48) mounted on the roof of the R.I.T. Engineering building. A Model 8-48 S/N 14352 integrator and a recorder (Model 292) integrated the voltage developed by the pyranometer and recorded each Langley (= 1 cal/cm\(^2\)) as a mark on a chart advancing forward uniformly with time. In order to determine instantaneous values of \( H_t \), it was necessary to measure the time interval between strikes of the recorder. This was done with a stopwatch. As collector efficiency data was taken only on clear, cloudless days, \( H_t \) (instantaneous) was measured at approximately 15 minute intervals and plotted accordingly. This resulted in a smooth functional representation of \( H_t \) throughout the period of interest. The actual variation of \( H_t \) during the period of two hours before and after solar noon on a clear day is quite small, about 5-10\%, depending on time of year. Where possible, tests were conducted near solar noon.

The accuracy of the pyranometer is estimated by the manufacturer as about \( \pm 2-3\% \).

The mass flow rate was measured with a stopwatch and graduated cylinder once flow through the collector achieved steady state. The flow rate was periodically checked throughout the course of a test to insure it remained constant. This means of measurement
allowed \( \dot{m} \) to be determined to within \( \pm 1-2\% \) in most cases.

Inlet and outlet temperatures were recorded from Hg thermometers located in the inlet and outlet headers, such that the bulb was in the flow. Insulation around the headers minimized heat loss from the liquid as it emerged from the collector and proceeded to the point where its temperature was recorded. The graduations of the thermometers allowed \( T_{\text{in}} \) and \( T_{\text{out}} \) to be determined to within approximately \( \pm 0.2^\circ \text{F} \approx \pm 0.1^\circ \text{C} \). Ambient temperature was determined by an Hg thermometer placed out of direct sunlight and above ground. Relative differences between readings of the three thermometers when placed in identical environments was accounted for in the analysis.

Circulation of the liquid through the collector was achieved by a pump. Two different types of pumps were used in the tests: a piston-type positive displacement pump with its characteristic pulsating flow; and a centrifugal pump with smooth flow. Because the centrifugal pump had greater capacity than was needed, a portion of the output was redirected to the main reservoir and only a small amount bled off and directed through the collector. A valve in the collector loop thus allowed this portion of the flow to be varied.

Actual readings were taken only after it was determined that the collector was operating in a steady-state or near steady-state condition. During some tests the inlet temperature was held constant and the outlet temperature was allowed to reach and maintain a steady value. This characteristically occurred
after 3-4 fluid mass changes in the collector (collector fluid capacity = 2.7 kg) or about 20-30 minutes for a typical flow rate of 0.4 kg/min. Once $T_{out}$ and $T_{in}$ were determined, an average value of $H_t$ was determined for period of the last mass change through the collector. This represented the incident energy on the collector fluid which resulted in the temperature rise from $T_{in}$ to $T_{out, final}$.

For some tests it was more convenient to allow $T_{in}$ to rise more slowly as recirculation occurred. $T_{in}$ was then monitored as a function of time, as were $T_{out}$ and $H_t$, and the variation $\frac{dT_{in}}{dt}$ was accounted for in the analysis.

Data were recorded for different values of $T_{in}$, $T_{in} - T_{amb}$, $H_t$ and fluid blackness. Because tests were run at different times of day, $(\tau a)_{eff}$ also varied. In each test the collector was maintained in the horizontal plane as was the pyranometer.

For comparison, instantaneous efficiency was tested for a commercially-produced aluminum roll-bond collector. The test procedures were identical to those used in the black liquid collector tests.

Unfortunately, no device was available to measure wind velocity at the test location. For any given test the wind velocity during the test was estimated as either negligible, slight, moderate, or strong. A moderate wind was assumed about 5m/S = 11 mph.

The specific heat of the black fluid was taken as that of water = 1.00 BTU/lb$^{-\circ C}$ throughout, since the fluid represented
a relatively small concentration of particles in suspension in water, which did not change the chemical properties of the water. This was confirmed experimentally by comparing the specific heat of the black fluid with that of water. A 1% uncertainty was associated with $C_p$ in the analysis to account for temperature variation and any small departure from $C_p = 1$ due to fluid blackening.

The total error in any instantaneous efficiency measurement was found by summing the percentage errors of the individual measurements. Typically, these might be

$$\eta = \frac{(\dot{m} \pm 2\%) (C_p \pm 1\%) (\Delta T \pm 4\%)}{H_t \pm 3\%} = \eta \pm 10\%$$

5.3 Experimental Results

The relative degree of fluid darkness or blackness was investigated to determine its role in collector performance. A value of "200% blackness" was assigned to that fluid which transmitted no visible light through 1/4" thickness as detected by the eye. The number 200% is of no consequence other than it fixes a relative scale of darkness, and is in anticipation of the result that 100% black fluid will represent the optimum blackness, since for it no visible light will be transmitted by a 1/2" layer which is the effective physical path length for the prototype (i.e., initial plus reflected paths). Tests were run at five different concentrations on the relative scale of 0-200% darkness. These were 0%, 6%, 12%, 18% and 200%. The
experimental results are given in Table 5.1. The quantities \( \dot{m}, V_{\text{wind}}, T_{\text{in}} - T_{\text{amb}}, (\tau\alpha)_{\text{eff}}, \) and \( H_t \) could not be held exactly constant throughout the series of tests, but the variation is sufficiently small that meaningful comparisons can be made.

A number of runs were made using "200% black fluid". These results are presented in Table 5.2. Unfortunately, circumstances precluded obtaining as great a variation in \( (T_{\text{in}} - T_{\text{amb}})/H_t \) as was desired. A large value for \( (T_{\text{in}} - T_{\text{amb}})/H_t \) implies either a large \( T_{\text{in}} - T_{\text{amb}} \), difficult to obtain in summer, or small \( H_t \), available only at early morning or late evening hours where \( (\tau\alpha)_{\text{eff}} \) is substantially different from its value at 0° angle of incidence and changes rapidly with time.

Finally, as a test of experimental procedure, tests were conducted on a commercially-produced roll-bond aluminum collector with one glass cover (DS window glass). This particular tube and fin design has a surface area of 5.44 ft\(^2\), and \( F' = 0.97 \). The tests were conducted in the same physical location as the black liquid collector tests and the results are given in Table 5.3.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time (solar)</th>
<th>$T_{amb}$ (°C)</th>
<th>$\dot{m}/A_C$ ($\frac{kg}{m^2\cdot°C}$)</th>
<th>$F_R$</th>
<th>$H_t$ (W/m$^2$)</th>
<th>$\frac{T_{in} - T_{amb}}{H_t}$ (m$^2\cdot°C/W$)</th>
<th>% blackness</th>
<th>$\eta_{exp}$</th>
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<tr>
<td>7/6/76</td>
<td>1:45</td>
<td>30.5</td>
<td>90.4</td>
<td>.98</td>
<td>833</td>
<td>=0</td>
<td>0.0</td>
<td>38.7</td>
</tr>
<tr>
<td>7/9/76</td>
<td>9:25</td>
<td>19.4</td>
<td>63.0</td>
<td>.97</td>
<td>747</td>
<td>.019</td>
<td>0.0</td>
<td>38.3</td>
</tr>
<tr>
<td>7/6/76</td>
<td>2:10</td>
<td>30.5</td>
<td>83.7</td>
<td>.98</td>
<td>810</td>
<td>.005</td>
<td>6.0</td>
<td>48.1</td>
</tr>
<tr>
<td>7/9/76</td>
<td>9:55</td>
<td>19.4</td>
<td>61.5</td>
<td>.97</td>
<td>804</td>
<td>.020</td>
<td>6.0</td>
<td>48.1</td>
</tr>
<tr>
<td>7/6/76</td>
<td>2:35</td>
<td>30.5</td>
<td>82.0</td>
<td>.97</td>
<td>787</td>
<td>.011</td>
<td>12.0</td>
<td>53.1</td>
</tr>
<tr>
<td>7/9/76</td>
<td>10:30</td>
<td>20.0</td>
<td>56.4</td>
<td>.96</td>
<td>950</td>
<td>.021</td>
<td>18.0</td>
<td>54.5</td>
</tr>
<tr>
<td>7/9/76</td>
<td>12:20</td>
<td>29.5</td>
<td>91.0</td>
<td>.98</td>
<td>938</td>
<td>.013</td>
<td>200.0</td>
<td>65.9</td>
</tr>
</tbody>
</table>

* centrifugal pump used in test

Table 5.1 Collector efficiency for various concentrations of blackness of the radiation absorbing fluid.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time (solar)</th>
<th>θz</th>
<th>(τα)_{eff}</th>
<th>Tamb (°C)</th>
<th>\dot{m}/A_c (kg/m²-hr)</th>
<th>FR</th>
<th>H_t (W/m²)</th>
<th>\frac{T_{in} - T_{amb}}{H_t} (m²°C/W)</th>
<th>V_{wind}</th>
<th>η_{exp}</th>
<th>η_{calc}</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/13/76</td>
<td>10:35‡</td>
<td>28°</td>
<td>.835</td>
<td>15.0</td>
<td>60.5</td>
<td>.96</td>
<td>678</td>
<td>.020</td>
<td>mod.</td>
<td>66.0±5.0</td>
<td>66.1</td>
</tr>
<tr>
<td>5/27/76</td>
<td>8:00‡</td>
<td>54°</td>
<td>.77</td>
<td>12.5</td>
<td>63.4</td>
<td>.97</td>
<td>516</td>
<td>.019</td>
<td>neglig.</td>
<td>62.8±4.0</td>
<td>64.8</td>
</tr>
<tr>
<td></td>
<td>8:45‡</td>
<td>47°</td>
<td>.81</td>
<td>14.2</td>
<td>61.6</td>
<td>.97</td>
<td>634</td>
<td>.021</td>
<td>neglig.</td>
<td>65.7±5.0</td>
<td>67.7</td>
</tr>
<tr>
<td></td>
<td>9:25‡</td>
<td>38°</td>
<td>.83</td>
<td>15.0</td>
<td>50.0</td>
<td>.95</td>
<td>760</td>
<td>.023</td>
<td>neglig.</td>
<td>57.1±5.0</td>
<td>67.7</td>
</tr>
<tr>
<td></td>
<td>10:30‡</td>
<td>28°</td>
<td>.835</td>
<td>17.2</td>
<td>56.5</td>
<td>.96</td>
<td>836</td>
<td>.019</td>
<td>slight</td>
<td>73.3±6.0</td>
<td>68.5</td>
</tr>
<tr>
<td></td>
<td>12:30‡</td>
<td>22°</td>
<td>.84</td>
<td>20.2</td>
<td>28.5</td>
<td>.93</td>
<td>930</td>
<td>.008</td>
<td>mod.</td>
<td>73.1±5.0</td>
<td>72.6</td>
</tr>
<tr>
<td>2/28/76</td>
<td>9:30‡</td>
<td>38°</td>
<td>.83</td>
<td>17.5</td>
<td>51.2</td>
<td>.96</td>
<td>763</td>
<td>.005</td>
<td>slight</td>
<td>78.3±5.0</td>
<td>76.7</td>
</tr>
<tr>
<td>7/6/76</td>
<td>10:45*</td>
<td>26°</td>
<td>.835</td>
<td>24.4</td>
<td>67.5</td>
<td>.97</td>
<td>700</td>
<td>.013</td>
<td>slight</td>
<td>60.0±6.0</td>
<td>72.9</td>
</tr>
<tr>
<td></td>
<td>11:20*</td>
<td>22°</td>
<td>.84</td>
<td>29.5</td>
<td>91.0</td>
<td>.98</td>
<td>772</td>
<td>.013</td>
<td>slight</td>
<td>65.9±6.0</td>
<td>73.7</td>
</tr>
<tr>
<td></td>
<td>11:50*</td>
<td>21°</td>
<td>.84</td>
<td>29.5</td>
<td>76.3</td>
<td>.97</td>
<td>853</td>
<td>.013</td>
<td>slight</td>
<td>66.3±6.0</td>
<td>73.4</td>
</tr>
<tr>
<td></td>
<td>12:30*</td>
<td>22°</td>
<td>.84</td>
<td>29.5</td>
<td>90.8</td>
<td>.98</td>
<td>889</td>
<td>.013</td>
<td>slight</td>
<td>71.2±6.0</td>
<td>74.8</td>
</tr>
</tbody>
</table>

‡ positive displacement pump used in test
* centrifugal pump used in test

Table 5.2 Black liquid collector efficiencies measured at various times of day and under various meteorological conditions. \( \eta_{calc} \) is calculated from the HWB equation where U_t is found from eqs. 2.8a and 2.8b.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time (solar)</th>
<th>Θs</th>
<th>(τa) eff</th>
<th>T_{amb} (°C)</th>
<th>(\dot{m}/A_c) (kg/m²-hr)</th>
<th>FR</th>
<th>Ht (W/m²)</th>
<th>(\frac{T_{in} - T_{amb}}{H_t}) (m²-°C/W)</th>
<th>V_{wind}</th>
<th>(\eta_{exp})</th>
<th>(\eta_{calc})</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/22</td>
<td>8:35*</td>
<td>48°</td>
<td>.77</td>
<td>18.3</td>
<td>41.4</td>
<td>.91</td>
<td>571</td>
<td>.0259</td>
<td>slight</td>
<td>51.1±3.0</td>
<td>55.0</td>
</tr>
<tr>
<td>7/22</td>
<td>9:00*</td>
<td>43°</td>
<td>.79</td>
<td>19.0</td>
<td>63.4</td>
<td>.92</td>
<td>642</td>
<td>.0414</td>
<td>slight</td>
<td>40.7±3.0</td>
<td>48.4</td>
</tr>
<tr>
<td>7/22</td>
<td>9:30*</td>
<td>37°</td>
<td>.81</td>
<td>21.1</td>
<td>75.8</td>
<td>.93</td>
<td>699</td>
<td>.0159</td>
<td>slight</td>
<td>60.4±3.0</td>
<td>65.9</td>
</tr>
</tbody>
</table>

* centrifugal pump used in test

Table 5.3  Aluminum roll bond collector (tube and fin design) efficiency measurements. \(\eta_{calc}\) is calculated from HWB equation where \(U_t\) is found from eqs. 2.8a and b.
6.1 Discussion of Results

The data of Table 5.1 are presented in graphical form in Figure 6.1. In some instances, where more than one determination of collector efficiency was made for a given fluid blackness, averaged values are used.

The most striking feature of Figure 6.1 is the "clear water" efficiency of about 38%, a result not expected from naive considerations. A qualitative understanding of this can be reached by inspection of Table 6.1, which reveals that in a thin layer of water strongest attenuation of incident light occurs in the infrared wavelengths 0.9→3.0 μm. The fraction of total energy of the beam contained in this band is about 40%, and because the glass is also absorbing in the infrared, it is reasonable to assume that most incident infrared radiation will either be absorbed by the glass or the water in its initial or reflected pass through these materials in the prototype. The glass on either side of the layer of water will transfer most of its absorbed heat to the water as discussed in Section 3.1, thus resulting in useful gain. In the absence of collector losses, then, an efficiency of about 40% is expected, in agreement with the measured values (where the $U_L(T_{in} - T_{amb})$ term was small).
Fig. 6.1  The experimental instantaneous efficiency of the black liquid prototype collector as a function of the relative degree of fluid blackness. The solid line represents a fit to the data calculated from equation 6.1 assuming $\eta_{i.r.} = 0.38$, $\beta = 0.28$, and $K_{vis} x_o = 5.0$ at 100% blackness. The extinction coefficient scale assumes $x_o = 0.5''$, the optical path of the collector.
Another interesting feature of Figure 6.1 is the apparent rapid rise in efficiency as the relative blackness of the fluid is increased from 0%. A relative blackness of 6% made the water appear only slightly dirty, yet produced efficiency measurements of 48%, an increase of about 25% over the clear water value. This is presumably a result of: a) absorption taking place in the ultra-violet and visible wavelengths due to suspended particles in the water, the infrared absorption already being complete or near complete; and, b) scattering of light by suspended particles which acts to increase the overall average path length of light.

The effect can be understood by assuming a simple model of the absorption process. Neglecting the ultra-violet radiation (most of which is absorbed in the atmosphere), absorption and scattering of visible light can be described as proportional to $-K_{\text{vis}}X$, where $K_{\text{vis}}$ is the extinction coefficient for

<table>
<thead>
<tr>
<th>Wavelength (μm)</th>
<th>1 mm</th>
<th>1 cm</th>
<th>10 cm</th>
<th>1 m</th>
<th>10 m</th>
<th>100 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 - 0.6</td>
<td>100.0</td>
<td>100.0</td>
<td>99.7</td>
<td>99.7</td>
<td>72.6</td>
<td>5.9</td>
</tr>
<tr>
<td>0.6 - 0.9</td>
<td>99.8</td>
<td>98.2</td>
<td>84.8</td>
<td>35.8</td>
<td>2.6</td>
<td>0.0</td>
</tr>
<tr>
<td>0.9 - 3.0</td>
<td>65.3</td>
<td>34.7</td>
<td>2.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
visible light and \( X_0 \) is a constant chosen to be \( 1/2'' \), the maximum optical path of the prototype collector for normal incidence. For simplicity, \( K_{\text{vis}} \) is assumed independent of wavelength. The collector efficiency as a function of \( K_{\text{vis}}X_0 \) is then seen to take the form

\[
\eta = \eta_{\text{i.r.}} + \beta(1 - e^{-K_{\text{vis}}X_0})
\]

(6.1)

where \( \eta_{\text{i.r.}} \) is a constant proportional to the absorbed infrared energy and \( \beta \) is a proportionality constant which limits the maximum value which \( \eta \) may assume. This model neglects any second order processes such as absorption of visible light by the glass or changes in total reflected light as \( K_{\text{vis}} \) varies, making \( \beta \) independent of \( K_{\text{vis}} \) in the first approximation.

The prediction of Eq. 6.1 is shown as the solid line in Figure 6.1. \( \eta_{\text{i.r.}} \) was taken as 0.38, as suggested by the measured clear water efficiency, and \( \beta \) was assumed to be 0.28 as suggested by the measured efficiency of 200% black fluid under typical conditions (i.e., \( F_R = 0.98, (T_{\text{in}} - T_{\text{amb}})/H_t = 0.013 \text{ m}^2\text{C/W} \)). Further, it was assumed \( K_{\text{vis}}X_0 = 5.0 \) at 100% blackness, a choice which provides an excellent fit to the available data and establishes the relation between relative blackness and extinction coefficient. The scale for \( K_{\text{vis}} \) is indicated on Figure 6.1. The range \( 0 < K_{\text{vis}} < 10 \text{ in}^{-1} \) is indicated as "gray water" since a significant portion of radiation may be transmitted through the fluid for these values of \( K_{\text{vis}} \). The region \( K_{\text{vis}} > 5 \) is labeled "black water" since the fluid appears optically thick.
for these values of fluid blackness.

From Figure 6.1, the conclusion is apparent that a degree of 200% blackness contained about twice the concentration of darkening agent as was necessary to obtain near optimum efficiency. Even a relative blackness of 20%, which appears as dirty water, provides about 85% of the maximum efficiency. These results could have important implications in determining the balance point between the amount of precipitable matter and collector efficiency in a black liquid collector system.

In Figure 6.2 the experimental results presented in Tables 5.2 and 5.3 are plotted as a function of the parameter \((T_{in} - T_{amb})/H_t\). The solid line represents the best fit, in the sense of least squares, to the black liquid data, and the dashed line is the best fit to the tube and fin data. These best fits support the predictions of Chapter 3 (see Figures 3.3a and b). At \((T_{in} - T_{amb})/H_t = 0.02 \text{ m}^2\cdot\text{C/W}\), for example, the instantaneous efficiency of the black liquid collector is enhanced by 
\[
\frac{.64 - .565}{.60} \times 100 = 12.5\%
\]
over that of the aluminum tube and fin design.

From the best fits, it is possible to extract an indirect experimental determination of \((\tau a)_{eff}\) and \(U_L\) for each collector design. From the Hottel-Whillier-Bliss equation

\[
\eta = F_R[(\tau a)_{eff} + U_L \frac{(T_{in} - T_{amb})}{H_t}]
\]
the y-intercept is seen to be \(F_R(\tau a)_{eff}\) and the slope is \(F_R U_L\).
Fig 6.2 Instantaneous efficiency data for the black liquid and aluminum tube and fin collectors. Straight lines indicate linear least squares fits to the data.
Taking $F_R$ as an average of the $F_R$ values of different runs, we find from Tables 5.2 and 5.3:

$$F_{R,\text{avg}} = 0.964 \quad \text{black liquid}$$

$$F_{R,\text{avg}} = 0.920 \quad \text{tube and fin}$$

Thus for the black liquid collector, $(\tau \alpha)_\text{eff} = \frac{\text{intercept}}{F_{R,\text{avg}}} = 0.810$. This experimental $(\tau \alpha)_\text{eff}$ can be compared with a value deduced from Figure 3.1 assuming 10% of the incident radiation is diffuse with $\theta_Z = 60^\circ$ and the remaining 90% is direct with $\theta_Z \approx 30^\circ$ (as suggested by Table 5.2). The theoretical $(\tau \alpha)_\text{eff}$ is then

$$0.90(\tau \alpha)_{\text{eff},30^\circ} + 0.10(\tau \alpha)_{\text{eff},60^\circ} = 0.90(0.835) + 0.10(0.73) = 0.816.$$ 

Likewise, $U_L$ can be deduced from the fits of Figure 6.2 and compared to predictions using the Klein equations 2.8a and b.

The results of the comparisons are summarized in Table 6.2.

<table>
<thead>
<tr>
<th></th>
<th>$(\tau \alpha)_\text{eff}$</th>
<th>$U_L\left(\frac{W}{m^2{\degree}C}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black liquid</td>
<td>0.810</td>
<td>0.816</td>
</tr>
<tr>
<td>Tube and fin</td>
<td>0.782</td>
<td>0.780</td>
</tr>
</tbody>
</table>
The calculated loss coefficients are given for the range of wind velocities \(0 \rightarrow 5\) m/S, since \(U_L\) is very sensitive to \(V_{\text{wind}}\) for a one-cover system.

The agreement between calculated and experimental values of \((\tau a)_{\text{eff}}\) is seen to be excellent for both collectors. The measured loss coefficients, however, are higher in both cases than calculated values assuming \(V_{\text{wind}} = 5\) m/S. The average wind velocities during experimental runs was estimated at about 2-3 m/S. It is interesting to note that the measured \(U_L\) for the black liquid collector is in better agreement than that for the tube and fin design, though it is to be cautioned that the experimental values are accurate to only about 10-15%.

The results of the comparison of Table 6.2 lend tentative support to the assumption that the black liquid and glass plates in contact with it can be considered at one average temperature. Were this not the case with, say, a top plate temperature of twice the average fluid temperature, \(U_L\) would be substantially higher than values calculated from the assumption of an average local temperature.

The results of this investigation have brought to light several advantages of the black liquid collector over the tube and fin design. These are:

a) A 10-15% enhancement in both instantaneous and overall efficiency compared to a well-designed tube and fin collector.
b) Metals can be avoided in construction which both reduces cost considerably and eliminates corrosion problems associated with fluids in contact with metals.

c) When the collector is drained (in summer, for example) high, potentially destructive temperatures will not be reached since the absorbing medium is not present.

Some disadvantages were also noted. These are:

a) The fluid capacity per unit collector area is generally larger than for a tube and fin collector, creating the possible need for added structural support for, say, roof-mounted systems.

b) Care must be taken in collector design to supply sufficient structural support to withstand the hydrostatic pressure of the contained column of water within the collector.

6.2 Recommendations for Future Work

The present investigation has laid groundwork in several areas where further work might be justified. These are discussed below:

**Fluid blackness.** The question of optimum fluid blackness appears worthy of greater attention. This work has established that for the prototype used, optimum darkness was achieved when no visible light could be detected by the eye in passing through a 1/2" fluid layer. However, no second order effects were included in the analysis, most
important of which is the absorption profile which changes with \( K_{\text{vis}} \). For laminar flow, this profile is responsible for temperature gradients within the fluid layer. How do these gradients affect \( U_{\text{top}} \) and \( U_{\text{back}} \), and how can they best be minimized?

The question, "Which black fluid is best?" was not addressed in this work, yet remains an important area. Numerous candidates exist. The discussion of Section 6.1 suggests that they need not be indeed black since visible light absorption is responsible for only about half of the total useful gain. Further investigation is needed as to which black liquid candidate best fulfills the requirements. To be considered are:

a) cost and availability  
b) specific heat  
c) toxicity  
d) boiling and freezing points  
e) chemical properties

Performance. A thorough test of long-term performance compared to that of a tube and fin collector would be useful. The calculations of Chapter 4 indicate a 10-15% performance gain for black liquid collector for typical summer and winter months. An actual test of this result was beyond the scope of the investigation, but could hopefully establish experimentally the result.
Work is also needed to determine the sensitivity of collector performance to actual types of flow. How uniform must the flow be to retain good performance characteristics?

**Design Improvements.** The prototype used in the present work fulfilled the needs of the experiment, but would probably not survive long term use. Needed is a better method of sealing to eliminate the problem of leaks. Also, the headers could be incorporated into the collector itself, thereby reducing complexity.

In future designs an effort must be made to minimize cost and complexity while maximizing efficiency. It certainly appears that a design which adequately achieves these goals is within technological reach.

**New Designs.** The success of the black liquid collector geometry investigated in this work indicates other geometries and designs may be worthy of examination. One such related configuration would be to retain essentially the same design but make the back plate (see Figure 1.2) black and use clear water as the heat transfer fluid. Such a design should retain $F' = 1$, while relaxing the requirement of black liquid.
Nomenclature

\( \lambda \) \quad \text{wavelength}

\( I_{\text{sc}} \) \quad \text{solar constant}

\( \rho \) or \( \beta \) \quad \text{fraction of reflected light at an interface}

\( K_v \) \quad \text{monochromatic extinction coefficient}

\( I_r \) \quad \text{amount of light reflected at an interface}

\( I_o \) \quad \text{total incident light}

\( T \) \quad \text{temperature}

\( T_{f,i} \) \quad \text{fluid inlet temperature}

\( T_{f,o} \) \quad \text{fluid outlet temperature}

\( \epsilon \) \quad \text{emissivity}

\( \sigma \) \quad \text{Stefan-Boltzmann constant}

\( A \) \quad \text{collector surface area}

\( q \) \quad \text{heat transfer per unit area}

\( h \) \quad \text{heat transfer coefficient}

\( U \) \quad \text{top loss coefficient}

\( F' \) \quad \text{collector efficiency factor}

\( \tau \) \quad \text{cover system transmissivity}

\( \alpha \) \quad \text{absorptance of absorbing surface}

\( F_R \) \quad \text{collector heat removal factor}

\( \dot{m} \) \quad \text{mass flow rate}

\( G \) \quad \text{mass flow rate per unit collector area}

\( H_o \) \quad \text{total daily extraterrestrial radiation on a horizontal surface}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{H}_h$</td>
<td>monthly average daily total radiation on a horizontal surface at a position on earth</td>
</tr>
<tr>
<td>$\bar{D}$</td>
<td>monthly average diffuse radiation on a horizontal surface</td>
</tr>
<tr>
<td>$K_t$</td>
<td>$\bar{H}_h/H_0$</td>
</tr>
<tr>
<td>$K_D$</td>
<td>$\bar{D}/\bar{H}_h$</td>
</tr>
<tr>
<td>$H_b$</td>
<td>instantaneous beam (direct) radiation on a horizontal surface</td>
</tr>
<tr>
<td>$H_{t,b}$</td>
<td>instantaneous beam (direct) radiation on a tilted surface</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$H_{t,b}/H_b$</td>
</tr>
<tr>
<td>$s$</td>
<td>angle of tilt towards the equator</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>ratio of monthly average daily total radiation on a tilted surface to that on a horizontal surface</td>
</tr>
<tr>
<td>$\eta$</td>
<td>collector efficiency</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>angle direct radiation makes with respect to normal to the collector surface</td>
</tr>
</tbody>
</table>
References

1) Minardi, J.E., Chuang, H.N., Solar Energy 17, 179 (1975) "Performance of a 'Black Liquid' Flat Plate Solar Collector."


Vita

Name: Jackson Phelps Trentelman

Place and Year of Birth: Amsterdam, New York, 1948

Education:

Union College, 1966-68
Eidgenossische Technische Hochschule (Zurich, Switzerland), 1968-69
Union College (Schenectady, N.Y.) 1969-70
B.S. Physics 1970
Queen's University (Kingston, Ontario, Canada) 1970-73
M.S. Physics 1972

Awards:

National Research Council of Canada Bursary 1971-72
National Research Council of Scholarship 1972-73

Publications:


