A Novel test bed and stochastic vibration diagnostics for assessing the condition of constant velocity joints

Gregory Kacprzynski

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A Novel Test Bed and Stochastic Vibration Diagnostics for Assessing the Condition of Constant Velocity Joints

by

Gregory J. Kacprzynski

A Thesis Submitted in Partial Fulfillment of the Requirements for a

MASTER OF SCIENCE

In

Mechanical Engineering

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Gregory J. Kacprzynski
4/24/02
ACKNOWLEDGEMENTS

I would like to thank my parents for dedicating to me so much of their lives and for that first Capsella set. Secondly, I thank Mike Roemer and Mark Redding for being great mentors and believers in my abilities. Most importantly, I thank my loving wife Karin who gave me the final incentive (named Caden) I needed to finish this work.
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1.0 Summary

Automotive constant velocity joints (commonly known as CV joints) are commonly remanufactured to "like new" conditions at the end of their useful life. It is desirable to have the ability to quickly and autonomously determine the condition or "health" of a CV joint slated for remanufacture as well as provide a set of metrics for comparing the quality of remanufactured joints in general. In this thesis, the design of a novel test bench for quickly and repeatedly testing CV joints under torque is provided along with vibration diagnostics software, developed in Matlab, for assessing the condition of a particular joint.

A prototype test bench was successfully fabricated at the National Center for Resource Recovery and Remanufacturing (NCRRR) and several healthy and damaged CV-joints were tested to evaluate the observability of time and frequency domain vibration features to various CV joint seeded faults. Furthermore, a series of experiments were performed to determine the sensitivity of these features to test bench variables, such as direction of rotation and torque, so that optimal testing conditions can be maintained. In the end, a considerable body of evidence has been amassed and presented in this work that validate that the unique features of the test bench design, coupled with vibration signature analysis in a condition monitoring software package, can reliably distinguish healthy joints from faulty joints and function as a remanufacturing quality control measure. At the time of writing, the test bench and diagnostic software are pending a U.S. patent under No. 10/116,630 entitled "System and Method for Testing Linkages" and the concept has been patented under U.S. Patent No. 6,378,374.

2.0 Objectives

The objective of this work was to design and fabricate a testing device and associated diagnostic software for quickly testing the condition of automobile CV joints. The testing device hardware had to improve upon previous designs in the ability to extract
vibration signals, apply high torque loads to the CV joint, and allow for quick testing turnover rates. The software had to effectively process vibration and speed signals and, at minimum, classify a particular CV joint as scrap or salvageable. Ideally, the software was to diagnose various CV joint faults and defects so that proper corrective actions could be taken by a re-manufacturing facility.

3.0 Motivation

The motivation for this work began with a Program Opportunity Notice (PON) through the National Center for Resource Recovery and Remanufacturing (NCRRR) at RIT as part of a program that addresses pollution avoidance/prevention processes, methodologies, and technology research. Pollution avoidance/prevention strategies are in contrast to those that reduce pollution through treatment or remediation (often referred to as “end of pipe” strategies) and much more cost effective [1].

Remanufacturing is viewed as an important pollution avoidance/prevention strategy because it addresses material substitution issues. In remanufacturing, devices at the end of their life cycle are recycled or reworked to create a like-new device [2]. To make this process as economically feasible as possible, it is important to accurately assess the condition or “health” of the device so that the proper remanufacturing can be performed without having to disassemble it. Furthermore, additional benefits of condition assessment to a remanufacturing facility include the ability to validate the quality of a re-work process and continual improvement of that process due to expanded knowledge of a device’s failure mechanisms.

The niche market for remanufacturing generally arises for relatively simple yet expensive assemblies produced in large volume. Automobile parts such as batteries, alternators and CV joints fall into this category. The remanufacture of automobile parts are the responsibility of the Automotive Parts Rebuilders Association (ARPA), which has worked closely with the NCRRR because both organizations are concerned with the
quality of remanufactured devices. Effective quality control through automated diagnostics of CV joints was the focus of this effort.

4.0 Background

Constant Velocity (CV) joints are mechanical devices designed to transmit constant torque and angular velocity when the drive end and driven end shafts are offset at an angle. CV joints are used in all front-wheel, all-wheel, and 4-wheel drive automobiles. These automobiles have either floating transaxles or split drive shafts that turns each wheel independently. A drive axle (commonly called a half shaft) consists of a pair of CV joints with a shaft between them and splines on either end. The half shaft allows uniform torque to be transferred from the transmission to a wheel even as the suspension and steering systems allow the transmission and wheels to move relative to each other.

The most common type of CV joint is the Rzeppa joint. It consists of an outer and an inner race mechanically coupled through balls positioned in what is known as a constant velocity plane by axially offset meridinally curved grooves [3]. The balls are maintained in this plane by a cage located between the two races and packed in grease. The entire assembly is hermetically sealed with a neoprene accordion/bellow boot used for keeping out debris and moisture while retaining the grease. A spline on each end of the half shaft is the method of connecting the half shaft to the wheel and transmission while allowing axial play due to changes in the articulation of the joint. A detail drawing of a CV joint along with pictures of the major components are shown in Figure 1 through Figure 4 [4].
Figure 1 – Cross Section of a Constant Velocity Joint

Figure 2 - The outer race of the CV Joint

Figure 3 – Inner race

Figure 4 – CV joint cage
Automobile constant velocity (CV) joints are frequently remanufactured and resold as an option to buying new CV joints. Part of the remanufacturing process is to inspect the joint prior to the reconditioning to determine if the joint is salvageable. A final inspection after remanufacturing must ensure that the joint will hold up in service. Unfortunately, the fact that the CV joints are sealed makes visual inspection of the joint impossible [5]. In lieu of this fact, alternate and economically feasible non-destructive evaluation (NDE) of CV joint condition such as temperature and vibration are worthwhile.

A study by Eckler, backed up this claim by performing a cost/benefit analysis of the CV joint remanufacturing process. He found that for a typical sized facility, re-working 400 joint per day, 2% of the re-manufactured joints were still defective. A defective joint costs the shop approximately $100 in warranty expenses. The net result is that a shop has the potential to recover $800 per day if they are able to diagnose defective joints before shipping [6]. This rate of return has the potential to offset the additional costs of a CV joint condition assessment device.

5.0 Prior Work

In 1999, a team of Mechanical Engineering students designed a CV joint test fixture capable of applying torque and articulating a half shaft for their senior design project. This test stand is shown in Figure 5.

This design consisted of a motor driving a torque brake with a half shaft mounted as a link in the drivetrain. By mounting the torque brake (a magnetic particle brake in this case) on a pivot, a range of operating angles could be induced in one of the CV-joints. The fixture was able to induce up to 100 lbf-ft of torque with a speed range of 240-1170 RPM and articulate the CV joint up to 30 degrees.
The condition of a CV-joint in this fixture was to be determined by sensing the torque, temperature and acoustic emissions from the joint. To accomplish this, the following instrumentation was installed on the fixture:

- 80-180 degree F, Omega infrared thermocouple (focused on the CV joint)
- ICP model #TMS13B10 microphone (housed in sound proof box)
- PCB model 352C66 Accelerometer (mounted on the frame)
- Honeywell model CP18LDNL2 photoelectric speed sensor
- Magna Corp. Particle Break controller (for Torque measurements)

Unfortunately, the both the functionality and condition assessment capability of the fixture appeared to be inadequate. In the proceeding sections, the shortcoming of this design's capabilities (both in terms of hardware and software) are compared with the features of the new test fixture design. This comparison is important because it provides valuable insight into lessons learned and why features of the new hardware and software designed for this project were developed.
6.0 **Test Fixture Design – Old and New**

In the previous design, the torque that could be induced in the half shaft was limited by the power of the motor and energy dissipation capacity of the brake. The CV-joint condition was determined by monitoring acoustic emissions from a rotating and articulated CV joint using an audio sensor (microphone). In this case, the microphone did not appear to be an effective “health” sensing device because of the considerable and unpredictable background noise that had to be filtered out. Hence, the preferred method of monitoring joint health is through a direct connection achieved with an accelerometer mounted on the joint case. Finally, the old design had three qualities that made it prohibitive as a commercial product. First, it was time consuming to replace the half shaft. Secondly, it was only designed to test one CV-joint at a time and third, the size and cost of the motor and dynamometer threatened to outweigh the benefits of the machine in a commercial environment. Due to these shortcomings, a new fixture was developed.

6.1 **New Hardware Design Objectives**

The objectives of the new CV joint test fixture were as follows:

- Allow standard accelerometers to be attached directly to the joint casing with a magnet.
- Articulate the joints under high torque loads with low noise with an inexpensive drive system.
- Allow for a quick testing turnover rate due to both a quick half-shaft removal/replacement process and with the ability to test both CV-joints simultaneously.
- Allow flexibility to test different size half shafts.
- Build the fixture as safe as possible.
6.2 Drive System Concept

To achieve the design objectives of the 2nd generation CV joint testing machine, a novel drive system design was developed that, at the time of this writing, has been submitted for a US Patent under serial No. 10/116,630 entitled “A System and Method for Testing Linkages”.

Figure 6 and Figure 7 show front and side view schematics respectively of the half shaft drive system. One end of the half-shaft attaches to the center of a driving link as shown in Figure 6. This link is supported by crank arms that rotate on shafts, driven by a low powered, low noise motor. As these links translate in a circular path, they move the end of the CV joint assembly in a circular path through its entire operating angle range without rotating the joint. Because there is no rotation, the opposite end of the half-shaft is supported by a limited-pivot connection that loads the assembly with a torque (discussed in the next section).

To counteract the torque transmitted by the half shaft to the driving link, an identical link that is oriented 90 degrees out-of-phase with the driving link is installed on the opposite side of the rotating system as shown in Figure 6. These two links will alternately absorb the torque applied to the system, resulting in no net torque being transferred to the drive motor. This torque containment characteristic is generally referred to as circulating power and is associated with a class of mechanisms known as 4-square machines. Appendix I contains a free-body diagram and associated force equations that support the circulating power characteristic of the design. The CV-joint testing device is believed to be a novel application of a 4-square machine and is currently pending a US patent.

The significance of circulating power feature is that a large amount torque can be imposed on the half-shaft while it is in motion with the system being driven by a low-power (1 HP), low noise motor. The fact that there is little power required of the drive system reduces the acoustical and structural noise floor of the system. Testing different
articulation angles of the joints is performed by attaching the drive links in the different drive plate holes.

**Figure 6 – Front View Schematic of the Drive System**

**Figure 7 – Side View schematic of Drive System**
6.3 Torque Application System Concept

A constant torque is applied to one end of the half shaft with a pneumatic cylinder attached to a lever arm. The pneumatic cylinder was chosen because it would maintain a constant force and, when coupled with a pressure regulator and 110 psi supply, can safely provide over 600 lbs of force. When this force is applied to a six inch lever arm, 300 ft-lbs can be imparted on the half shaft.

A second feature of the “torque end” of the test fixture helps satisfy both the quick testing turnover rate and half shaft size design criteria. The entire torque application systems rests on a set of linear journal bearings. This feature allows the torque end to be moved back and forth to facilitate quickly changing out a half shaft or accommodating different size half shafts.

Under this project, a very common half shaft assembly was selected for testing that is can be found in the Honda Accord and similar sized automobiles. As per Car and Driver magazine, these vehicles can accelerate from 0-30 mph in 2.9 seconds [7]. With a mass of 3044 lbm (fully loaded) and a tire diameter of 24 inches, the each CV joint could see a maximum torque of 717 ft-lbf because:

\[ T = R \times ma \]  
\[ a = \frac{30 \text{ mph}}{2.9 \text{ sec}} \times \frac{5280 \text{ ft}}{\text{mile}} \times \frac{hr}{3600 \text{ sec}} = 15.17 \text{ ft/sec}^2 \]  
\[ \text{Torque/Tire} = \frac{24 \text{ in}}{2} \times 3044 \text{ lbm} \times 15.17 \text{ ft/sec}^2 \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ lbf}}{32.2 \text{ lbm/sec}^2} = \frac{1434 \text{ ft-lbf}}{2 \text{ tires}} = 717 \text{ ft-lbf} \]

It can be safely assumed that typical accelerations yield loading 30-70% of this value. With the new fixture able to load a half shaft at \((300/717) \times 100\) or 42% of the maximum expected torque, this amount was considered more than enough to reproduce realistic operating loads and extract adequate diagnostic information from the joints.
6.4 Prototype Test Fixture

Figure 8 and Figure 9 shows a prototype CV joint testing fixture that was developed. The mechanism rests on a steel frame approx. 3ft X 6ft in size. The drive system resides between the 2 sets of drive plates and consists of a 1 HP variable speed motor (not visible) connected to one shaft of the drive plate system through a V-belt drive. The articulation angle in the half-shaft is produced by changing the attachment point of the link on the front set of driving plates. A constant torque of up to 300 ft-lbs is applied to the half shaft with a pneumatic cylinder / offset arm arrangement attached to the non-articulating end of the half shaft. The force of the cylinder and the offset distance of the cylinder to the pivot produces the constant torque on the half shaft.

The fixture was incorporated with a movable Lexan© safety guard system, pneumatic and motor controls and safety shutoff system. Finally, a data acquisition system was incorporated which served as the link between the hardware and software objectives of this project.
Figure 8 – Side View of the CV joint test fixture

Figure 9 – Close up view of the Drive System
6.5 Sensed Parameters and Data Acquisition System

In this prototype phase, the system parameters that were sensed by the data acquisition system were:

1) CV joint and test fixture acceleration levels – processed from two 0-10 mV signals from model 352C66 PCB ICP vibration transducers (one on the CV-joint and one on the frame)

2) Drive shaft speed – processed from a Honeywell model CP18LDNL2 photoelectric speed sensor viewing a reflective strip on one of the drive shafts.

Many methods of monitoring the torque imposed on the half shaft in real-time were investigated. However, due to cost constraints and the potential that it might be an unnecessary feature, torque was simply inferred from the relationship between cylinder pressure and applied torque given in Table 1, to give an average (DC) level of torque not usable in signal processing. The expected uncertainty in this measurement due to system friction, regulator inaccuracy, and pneumatic system response characteristics was deemed acceptable.

<table>
<thead>
<tr>
<th>Pneumatic Pressure (psi)</th>
<th>Torque Imparted (ft-lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>104</td>
</tr>
<tr>
<td>30</td>
<td>156</td>
</tr>
<tr>
<td>40</td>
<td>208</td>
</tr>
<tr>
<td>50</td>
<td>260</td>
</tr>
<tr>
<td>60</td>
<td>312</td>
</tr>
</tbody>
</table>

Table 1 – Pressure vs. Torque
0-10 mV signals from the tachometer and accelerometers were acquired and controlled via the LabView software package and National Instruments Analog to Digital (AD) card. While the data acquisition system was not the focus of this project, there were a few system configuration elements that were essential to the software aspect of this project.

1) The sampling rate was set at 5000 Hz because it insured frequency content could be uniquely identified up to the Nyquist frequency of 2500 Hz. With operating speeds ranging from 0 to a maximum of 100 RPM and with 6 balls per CV joint, signal content less than 2500 Hz would contain all primary frequencies as well as the majority of harmonics. Details on the frequency analysis are given in Section 7.5.

2) The data was sampled in 4 seconds blocks thus generating 20000 data points per test.

7.0 Condition Assessment Techniques – Old and New

As stated previously, the original test fixture was designed to monitor acoustic signals and infrared temperature. The fundamental method of condition assessment was to investigate the content of an arbitrary CV joint acoustic emission as a function of articulation angle and torque and compare it to a known “healthy” or baseline CV joint at those conditions. A fundamental requirement for this technique to be successful was the ability to distinguish the portion of the acoustic signal generated from the CV joint from the noise created by the test fixture itself. This was attempted with an autoregressive (AR) technique used to correlate background noise in the vicinity of the joint. The premise was that the remaining acoustic signal was a function of the CV joints health, torque, and articulation angle. [1]
After a substantial amount of testing, it was discovered that the signal to noise ratio on the 1st generation testing machine was simply too low. Beyond the noise generated from the test fixture itself, ambient noise was an unpredictable factor that affected test repeatability. Conclusive evidence of a statistically significant acoustic signature could only be attained with a CV joint badly damaged with sand contamination. [4]

Because the 2nd generation fixture developed under this project made it simple to attach accelerometers directly on the CV joints, a much higher signal to noise ratio was guaranteed. This characteristic gave promise that more proven time and frequency techniques might be used to not only distinguish good joints from faulty ones, but also be sensitive enough to diagnose specific CV joint faults.

### 7.1 Software Design Objectives

The software design and functionality objectives encompassed under the scope of the project for the 2nd generation test fixture were as follows:

1) The software had to be capable of taking in data files containing vibration and speed signals and generating features that are indicative of the condition of a CV joint.

2) The software had to apply a rulebase to translate the features into meaningful diagnoses about the condition of the joint.

3) The software had to properly manipulate, process, and store large amounts of test data and produce consolidated results files

In this project, these design objectives were achieved entirely using Matlab Professional Version 5.3 R11. No special toolboxes were required. The design details and functionality of the CV joint diagnostic software are described in the proceeding sections.
7.2 **Software Implementation**

The CV-joint diagnostic software kernel and accessory code for file I/O, visualization and databasing was developed in Matlab. This section briefly supplies the layout of the software, which may be partitioned into four distinct modules. The software kernel is initiated through the Matlab m-file called MAIN.m. Other function calls initiated from this function handle I/O and produce the entire suite of diagnostic features described in detail herein. A visual summary of this module is given in Figure 10.

<table>
<thead>
<tr>
<th>Main.m</th>
<th>262 LOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Raw Data File</td>
<td></td>
</tr>
<tr>
<td>Calculate all Vibration Features</td>
<td></td>
</tr>
<tr>
<td>Generate Histograms</td>
<td></td>
</tr>
<tr>
<td>Save feature plots &amp; histograms</td>
<td></td>
</tr>
</tbody>
</table>

**1) Subroutines**

INITTHETA.m – initialization file – 152 LOC
GlobalFeature.m – calculates features – 195 LOC
- DCHP.m – remove DC offset – 37 LOC
- Preprocessing.m – calc RMS and Kurtosis for both standard and TSA data – 8 LOC
  - RMScalc.m – 22 LOC
  - Kurtosiscalc.m – 5 LOC
- Windowing.m – partitions based on Tach – 57 LOC
- INTERPOLATEDATA.m – creates equal size arrays – 33 LOC
- TSAcalc3.m – generates TSA – 49 LOC
- TSAcalc2.m – 53 LOC
- FFTcalc2.m – generate FFT for standard and TSA data – 74 LOC

---

Total LOC = 937

*Figure 10 - Features and subroutine of Main program*

The 2nd module is called Saved_File_Reader. This module is used solely for the purposes of plotting the features generated from the 1st module. The 3rd module is File_Limits_Reader. This module generates the elements required for statistical analysis of the features and generates the appropriate visualization. Finally the 4th module is Final_Diagnostic.m. This module is used for evaluating the statistical differences
between features from baseline “healthy” CV-joints and a particular CV-joint being tested. A visual summary of the 3rd and 4th modules is provided in Figure 11.

Figure 11 - Features and subroutines of Diagnostic programs

8.0 Vibration Features

The “features” mentioned in the previous section are based on the vibration and speed signals and are both time and frequency based.

Time-based or “temporal” signature analysis involves calculating and tracking features that are a function of time as opposed to frequency-based features that are functions of the periodic nature of a signal. In many cases, a complete signature analysis requires investigation in both the time and frequency domains. However, depending on the application, one form of analysis is more commonly used that another. Because of the periodic nature of events in rotating machinery, frequency-domain analysis is more traditional and generally more insightful as opposed to temporal analysis which must be
applied to highly non-periodic signal problems such to speech recognition and the like. Analysis of three-dimensional plots with the frequency content of a signal shown as a function of time (commonly known as spectral or waterfall plots) is also performed.

For this project, acceleration signals were investigated in both the time and frequency domain. In some cases, proper development of a vibration feature even required transforming back and forth between the domains. In the proceeding subsections the "hows" and "whys" of certain temporal and frequency analysis techniques employed in this project are given in addition to background development of techniques as well.

8.1 Development of Vibration Features

A substantial amount of research work has been done in developing various transformations to vibration data that yield that most information about the condition of the machinery being analyzed. In particular, the detection of cracks in rotating machinery has been approached by developing features that focus on the symptoms of an imminent mechanical failure. Cracking or pitting in precision machined surfaces can produce both localized changes in structural stiffness as well as high frequency impact events. Accordingly, features that focus on changes in mode shapes or fundamental frequencies and high frequency noise content have performed the best at identifying these type of conditions. [10,12]

From the onset of this project, it was suspected that time domain features would be most useful for CV joint condition assessment for the following reasons:

1) Unlike other rolling element bearings, the CV joint ball does not rotate at an angular velocity proportional to the half shaft speed. In fact, the balls may only slide or slide/roll due to the considerable grease packing and grooves in the outer race. This means that there are no definitive frequencies such as ball pass
frequency (BPI) used in traditional rolling element frequency analysis. The only repetitive event is the once-per-revolution.

2) A good indication of a poor CV joint is a “clicking” sound due to once-per-rev impact events as stress is placed on a damaged ball, concentrated area of the race, or the cage. While clicking can certainly be interpreted in the frequency domain as broadband excitations, it is more distinguishable as a non-stationary feature in the time-domain.

3) Due to both safety and design concerns, the maximum speed at which a CV joint can be articulated under load is about 180 RPM or 2 Hz. Frequencies below 10 Hz are not in the optimum performance regime for the small accelerometers required for this application. Furthermore, with low frequency events greater care must be taken in the data acquisition process and the signals can be diluted with ambient vibration and modal frequencies of the test fixture itself.

Though the focus was consequently on time domain analysis, two frequency domain applications of RMS and signal Kurtosis were investigated. Figure 12 shows a flowchart of the vibration feature extraction process which is explained briefly next. Details on each block in Figure 12 are provided in subsequent sections.
In the feature extraction process, minute displacements of a mass inside the accelerometer create small voltages in the piezoelectric crystal that are then conditioned to 0-10 mV and further manipulated into vibration (in g’s or in/sec^2) by a calibration function unique to each accelerometer. Sometimes, artificial gains or offsets in the signal can be present due to the data acquisition system so these are filtered out with a high pass filter. Next, the most basic statistical vibration features can be calculated for given sample sizes. With the presence of a tachometer signal, the exact time in which a given reference point have undergone 1 revolution can be determined. This feature enables a continuous time series signal to be averaged on a per revolution basis. The same statistical features can then be applied to the time synchronous averaged signal. Alternately, the raw signal can be enveloped and then transformed to the frequency domain with a Fast Fourier Transform. The enveloping (not shown in the chart) is a standard process for reducing the error produced in the time to frequency transform. Peak tracking of the frequency domain signal is analogous to measuring the amplitude of a distinct waveform. Generally, peak tracking takes place on frequencies of pre-identified physical significance such as modal, gear mesh or ball pass frequencies (which are currently not believed to be relevant to the CV-joint diagnostic problem). Many other
frequency domain features can be calculated as well including RMS, Kurtosis, power spectral density, peak amplitude, FM0, and NA4 which have been successful been applied by the author and researchers at Penn State Applied Research lab to detect gear tooth cracking at a very early stage [9].

8.2 Filtering

Figure 13 shows a typical conditioned accelerometer signal from the test bed after this stage of signal processing. Initially filtering was performed with an eighth order Butterworth digital highpass filter with a cutoff frequency of 10Hz. It was found however, that even with maximum attenuation levels the process tended to corrupt the filtered signal enough to show up in the features. Because the raw signals only exhibited a DC offset with a constant gain at worst, filtering was performed by simply subtracting out the signal’s trend line.

![Figure 13 – Time Domain Signal (DC offset removed)](image)

8.3 Basic Time Domain Features
Features such as RMS and Kurtosis can be calculated from the signal shown in Figure 13. These vibration features are explained herein.

8.3.1 RMS

RMS or Root Mean Square is a simple and commonly used technique for detecting anomalous machinery operation. The RMS based algorithm calculates the root mean square value of the time domain vibration signal. The RMS level of a signal $x$ consisting of $N$ samples is calculated using Equation 4 [11].

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2}$$  \hspace{1cm} \text{Eq. 4}

Figure 14 shows a typical RMS feature from the time domain signal for a defective CV joint (PLS3PC with a 0.005” dimple on one of the balls). In this figure, 20000 samples were taken and the RMS was calculated for $N=25$ samples yielding 800 RMS samples. Note the apparent periodic nature of the larger amplitude values.
8.3.2 Kurtosis

The Kurtosis algorithm utilizes the fourth statistical moment, or kurtosis, of the time domain vibration signal. A kurtosis value greater than three indicates that the frequency of large spikes is greater than would be expected for normally distributed noise. This is because, for a normal distribution, +/- 3 standard deviations encompass 99.7% of a population. A signal x consisting of N samples with mean \( \mu \) and variance \( \sigma^2 \) has a kurtosis given by Equation 5.[11] Signals from mechanical elements with high Kurtosis are generally attributed to metal-to-metal contact such as a crack opening and closing or a metal particle or defect destroying the hydrodynamic film in a bearing.[12]

\[
Kurtosis = \frac{\sum_{i=1}^{N} (x_i - \mu)^4}{N\sigma^4}
\]

Eq. 5

Figure 15 shows a typical Kurtosis feature from the time domain signal for the same defective CV joint shown previously. Again, N=25 for a sample size of 20000. Note that Kurtosis values have a lower bound of zero because of the 4th power term. Kurtosis values of greater than 4 would be highly unlikely for a signal with only white noise.
8.4 Time Synchronous Averaging

As stated earlier, the presence of a tachometer signal providing the exact time in which a given reference point has undergone 1 revolution can enable time synchronous averaging. In theory, the time domain vibration signature from a CV joint in this fixture should be cyclostationary, that is, it should repeat for every revolution [13]. The 20000 data points per sample can represent any number of revolutions depending on the speed that the fixture is functioning at. Averaging the signal on the basis of each revolution is designed to highlight real physical characteristics of the CV joint and reduce the influence of other vibrational contamination. A notable source of error is time synchronous averaging is the presence of subharmonics of the once-per-rev frequency such as oil whirl for fluid film bearings. No physics-based subharmonics are currently known to exist for CV-joints.

Performing time synchronous averaging requires windowing and interpolating or decimating at signal. Windowing is partitioning of the accelerometer signal on the basis of the tachometer signal (shown in Figure 16). Since this signal is not perfect, and slight
RPM changes are likely, care must be taken to properly identify the revolutions and segment the data accordingly. For this application, a revolution was denoted when 2 consecutive tachometer voltage data points where below 0.2 volts (5 volts signified the nominal output). Secondly, only whole revolutions could be considered, so the first and last partial revolutions were discarded.

![Tachometer signal](image)

**Figure 16 – Tachometer signal**

After windowing, the accelerometer data was partitioned into N vectors of slightly varying length. The final average or TSA of the signal requires the N vectors to be interpolated to the same length the basis of which is the average length of a revolution for a particular data set. The number of revolutions “R” to be included in the TSA is a global variable assigned in the initialization file. The function calls for the windowing and interpolation routines are given next.

```matlab
function [WINDOWS,TIMESTAMP]=WINDOWING(TACHSIG,RAWDAT,INI)
function [INTERPDATA,INTERPTIME]=INTERPOLATEDATA(DATACELLS,TIMECELLS,INI)
```

where:
INI = initialization structure
RAWDAT = 2 X 20000 array (1 for each accelerometer)
TACHSIG = 1X 20000 vector of tachometer voltages
WINDOWS = R vectors of unequal length
TIMESTAMP = N X 1 vector of indeces for each revolution
DATACELLS = same as WINDOWS
TIMECELLS = same as TIMESTAMP
INTERPDATA = R data vectors of equal length
INTERPTIME = Indeces associated with INTERPDATA

After the TSA is processed, the sample size has been reduced by a factor of R. Both RMS and Kurtosis can be applied to the TSA signal but care must be taken to not reduce the sample size too much because these statistical vibration features further reduce the sample size by the number of data points per calculation (N).

Figure 17 and Figure 18 show the raw TSA signal of a healthy joint (#7042270521 from data set F2270521_2_30_F_F) and a damaged joint (PLS3PC from data set FPLS3PC_2_30_F_F) respectively on the basis of 4 independent runs each to insure good and repeatable data. The two vibration signals (in g’s) shown are from the accelerometers mounted on the frame (A01) and directly on the CV-joint (A02). Note that a significant signal anomaly occurs around index 1500 in Figure 18. To appear clearly in the TSA this anomaly must have occurred at the same phase angle for each revolution. In the case of PLS3PC as 0.005” dimple was “seeded” into one of the balls. It is believed this damage is this cause of the signal anomaly. Further details on this and other results are provided in subsequent sections.
Figure 17 – TSA signal – healthy joint - 7042270521

Figure 18 – TSA signal – faulty joint PLS3PC (dimple)
8.5 Frequency Analysis

When the periodic nature of a signal is of interest, frequency domain analysis is often used. Jean Fourier, a French mathematician (1768-1830) showed that a periodic continuous signal can be represented as an infinite series of the form [14]:

\[
x(t) = \frac{a_0}{2} + a_1 \cos \Omega_1 t + a_2 \cos \Omega_2 t + \ldots + b_1 \sin \Omega_1 t + b_2 \sin \Omega_2 t + \ldots \quad \text{Eq. 6}
\]

where

\[
\Omega_1 = \frac{2\pi}{T} \quad \Omega_n = n\Omega_1 \quad \text{Eqs. 7, 8}
\]

By multiplying both sides of equation Eq. 6 thru by \( \cos \Omega_n t \) or \( \sin \Omega_n t \) and integrating over the time period (T), the coefficients \( a_n \) and \( b_n \) can be found as shown in Eqs.9 and 10.

\[
a_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos \Omega_n t \cdot dt \quad \text{Eq. 9}
\]

\[
b_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin \Omega_n t \cdot dt \quad \text{Eq. 10}
\]

Because the Euler identity states that:

\[
e^{\pm j \theta} = \cos \theta \pm j \sin \theta \quad \text{Eq. 11}
\]

then Eq. 6 can be written as:

\[
x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\Omega_n t} \quad \text{Eq. 12}
\]

where
\[ c_o = \frac{1}{2} a_o \]  
\[ Eq. 13 \]

\[ c_n = \frac{1}{2} (a_n - j b_n) \]  
\[ Eq. 14 \]

When an analog signal (such as a 0-10 mV signal) is put through an analog to digital converter (A/D), it is sampled at discrete time intervals and is no longer continuous. As a result, a Fourier series for a discrete time signal must be employed.

Thus, the Discrete Fourier Series for a periodic signal \( x(n) \) with a period of \( N \) is:

\[ x(n) = \sum_{k=0}^{N-1} c_k e^{j 2 \pi k n / N} \]  
\[ Eq. 15 \]

where:

\[ c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j 2 \pi k n / N} \]  
\[ Eq. 16 \]

\( k = 0, 1, \ldots, N-1 \)

For this application, the Matlab 'fft' function was employed which uses a mixed-radix fast fourier transform algorithm. The function call is of the form \( Y = \text{fft}(X,n) \), where \( X \) is the data and \( n \) is the number of points in the FFT. If the length of \( X \) is less than \( n \), \( X \) is padded with trailing zeros to length \( n \). If the length of \( x \) is greater than \( n \), the sequence \( x \) is truncated. [15] For simplicity, \( n \) is forced to be equal to the sampling rate (for this application, sampling rate = 5000), though more efficient operation can be obtained by using only power of 2's. Finally, the redundant part of the FFT, contained above the Nyquist frequency (sampling rate/2) was removed.
The results of the FFT on the Time Synchronous Averaged (TSA) signal of both a healthy joint (7042270521) and damaged joint (PLS3PC) are shown in Figure 19 and Figure 20 respectively. In these plots, FFTTSA1 is based on the signal from the accelerometer mounted on the CV-joint housing and FFTTSA2 is from the accelerometer mounted on the test fixture frame. The Y-axis is the absolute magnitude of the FFT in g's.

Figure 19 – FFT of TSA for good joint - 7042270521
Clearly, a significant difference is apparent between the results in the 500 to 1000 Hz range. As previously stated, it is unclear as to the physics behind the repeatable frequencies produced from a CV-joint on the test fixture designed. With an RPM of 142 (used in all the “fast” runs) a one-per rev frequency is only 142/60 or 2.3 Hz. With 6 balls, the ball rolling frequency (if the balls roll instead of sliding) would be 2.3 * 6 or 14.2 Hz. Even main harmonics of these frequencies are far from the 500 Hz range. It is possible that the frequencies in the 500-1000 Hz range are modal frequencies of the test fixture being excited. Another possibility is that rolling element impacts are producing attenuated broadband excitation or that the underlying ball motion is chaotic. In any case, and for the purposes here, the FFT clearly produced results that appeared to distinguish healthy joints from unhealthy joints. Thus, the features used in the time series analysis could again be employed in the frequency domain.

8.5.1 Frequency domain features

In this study, a RMS-based feature was generated from the frequency domain representation of the Time Synchronous Averaged accelerometer signals and was called
RMS of FFT of TSA. From the author’s experience, this is not done in the common practice of signature analysis (in contrast to peak tracking and spectral density measures) and while it may represent a novel approach, it is not much more than a statistical smoothing of the FFT of a TSA signal. As in the time domain analysis, 25 points were used per RMS calculation. Figure 21 shows the RMS of the TSA signal in the frequency domain averaged over 4 independent test runs for joint PLS3PC. Both signals are shown with the larger amplitude being from the CV-joint mounted accelerometer (green).

![Figure 21 – RMS of FFT of TSA for joint #PLS3PC](image)

### 9.0 Diagnosis

An autonomous diagnostic process was desired to discriminate between healthy and unhealthy CV-joints and to potentially isolate specific types of faults. One common method for diagnoses is to assign warning and alarm limits on the magnitude of features and then use a rulebase to classify the limit exceedance as a particular fault. However, considerable variation in vibration features is a well known phenomena for many mechanisms due to manufacturing variations, environmental conditions, accelerometer mounting etc. These uncertainties generally force the diagnostician to set alarm limits
quite high to avoid false alarms. As a consequence, fault conditions may be detected only after significant damage exists in the component of interest. Furthermore, it is rare for faults in components to manifest themselves in only one feature or that a simple limit is sufficient for uniquely classifying a fault. For example, the entire noise floor of a signal could be elevated due to a particular fault (one example is turbulence in pump vibration diagnostics), but a limit may not be exceeded. *In lieu of these observations, two statistical approaches to automated diagnostics were attempted that determined if statistically significant shifts existed between features generated from a candidate CV-joint and pre-determined “healthy” joint features in addition to standard limit exceedences.* A summary of this statistical process is shown in Figure 22 and details on each step in the process are provided in the proceeding sections.

![Figure 22 - Diagnostic Process](image-url)
9.1 Feature Histograms

After vibration features are generated in the processes already described, the 1st step in the diagnostic process was to develop histograms. A histogram is a statistical representation of data where the frequency of occurrence of data within a range of values is plotted versus the range of data. In the overall program’s Initialization file (Initial.m), each feature was assigned a vector of size 1X50 containing threshold values ranging from x to y where x represented the lowest bin for the histogram and y the highest bin selected on the basis of trial and error. An example of this process is shown below:

```matlab
% Kurtosis thresholds
INI.KurtTSAThresh=linspace(1,7,50);
INI.KurtosisThresh= linspace(1,7,50);

% RMS thresholds
INI.RMSThresh=linspace(0,0.2,50);
INI.RMSTSAThresh=linspace(0,0.15,50);
```

Matlab's HIST function was used to generate histograms of all features on the basis of the threshold vectors.

9.2 Probability Density Functions

The second step in the diagnostic process was to fit normal or lognormal probability density functions to the feature histograms. A Probability Density Function (commonly referred to as a PDF) f(x), has the mathematical property that:

$$\int_{-\infty}^{\infty} f(x) dx = 1$$  \hspace{1cm} \text{Eq. 17}

Two common PDFs observed in nature are the Gaussian (or Normal) and Lognormal distributions. These distributions are given in Eqs. 18 and 19 respectively as a function of the mean ($\mu$) and standard deviation ($\sigma$) of a data set [16].
\[ y = f(x|\mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]  
Eq. 18

\[ y = f(x|\mu, \sigma) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{-(\ln(x)-\mu)^2}{2\sigma^2}} \]  
Eq. 19

For a PDF to be fitted to a feature histogram, the histogram had to be area normalized so the sum of the areas of each of the bars equaled unity. The area normalization was performed with the following routine:

\[
\text{step1} = xdata(2)-xdata(1);
\text{area}\_\text{each} = ydata*\text{step1};
\text{Anormalized} = \left(\frac{ydata}{\text{sum(\text{area}\_\text{each})}}\right)';
\]

Where:

\text{xdata} = \text{a given bin center}
\text{ydata} = \text{original height of a histogram bar}
\text{area}\_\text{each} = \text{original area of a histogram bar}
\text{Anormalized} = \text{vector representing the area normalized histogram}

In this study, normal and lognormal PDF were fit to the histograms generated from the following vibration features:

RMS of the A02 (CV joint mounted accelerometer) raw signal over 4 runs – \text{RMSThresh}
Kurtosis of the A02 raw signal over 4 runs – \text{KurtosisThresh}
RMS of the TSA of A02 averaged over 4 runs – \text{RMSTSAThresh}
Kurtosis of the TSA of A02 averaged over 4 runs – \text{KurtTSAThresh}
RMS of the FFT of the TSA of A02 averaged over 4 runs – \text{RMS\_FFTTSAThresh}
In the program initialization file, INITIAL.m, the histogram bin centers were defined and are given below. The Matlab linspace command is of the form \((x_1,x_2,\text{num})\) where 'num' bin centers are created between 'x1' and 'x2'.

%Kurtosis thresholds
INI.KurtTSAThresh=linspace(1,7,50);
INI.KurtosisThresh= linspace(1,7,50);

%RMS thresholds
INI.RMSThresh=linspace(0,0.2,50);
INI.RMSTSAThresh=linspace(0,0.15,50);

%RMS FFTTSA thresholds
INI.RMS_FFTTSA Threshold = linspace(0.00025,0.025,20);

The feature histogram results and associated PDF fits are given in Figure 23 thru Figure 25 for the healthy joint #7042270521. All but the RMS of the raw signal can be adequately approximated by a normal or lognormal distribution. For comparison, the PDFs from the RMS of the FFT (both accelerometers) for the known damaged joint PLS3PC is given in Figure 26 with respect to the healthy joint. In both cases, the tail of the lognormal distribution is considerably greater for the faulty joint.

![Figure 23 – healthy joint (7042270521) - Standard features](image)
Figure 24 – Healthy joint (7042270521) – TSA features

Figure 25 – Histogram of RMS of FFT of TSA for 7042270521
9.3 Diagnostic Methods

The objective of fitting PDFs to feature histograms was to enable the comparison of a healthy (baseline) CV joint feature to a candidate’s feature on the basis of histogram shape. Two different methods for evaluating whether a change in the shape of a histogram was statistically significant were investigated. The first technique was the statistically robust T-test and the second was a simpler error-based approach.

9.3.1 Statistical T-test

In the T-test, the means of two normal distributions of different variances, representing samples from two populations, are compared on the basis of a confidence interval ($\alpha$). The null hypothesis that could be either accepted or rejected was $H_0 : \mu_1 = \mu_2$, or “With ($\alpha \times 100$)% confidence, is the mean of population X statistically significantly different from population Y?” The mathematical process for performing a T-test and graphical
representation of the comparison are given in Equations 20 and 21 and shown in Figure 27 respectively [16].

\[ T_0^* = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \]  

Eq. 20

Where:
\( \bar{X}_1 - \bar{X}_2 - \Delta_0 \) = the difference in means of the populations
\( \frac{S_i^2}{n_x} \) = the sample variance / number of data points taken
\( T_0^* \) = t-statistic

![Figure 27 – Representation of the Statistical T-test](image)

If the null hypothesis is true then the t-statistic given in Eq. 20 is distributed approximately as the t-distribution with the number of degrees of freedom given in Eq. 21.
\[ v = \frac{\left( \frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right)^2}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} - 2 \]  

Eq. 21

In contrast the null hypothesis would be rejected (i.e. the populations are not statistically different) if a value for \( T_0^* \) is greater than \( t_{\alpha,v} \) or less than \( -t_{\alpha,v} \). Tabulated values for the \( t \)-distribution can be readily found in statistics textbooks and incorporated as lookup tables in code. As the number of degrees of freedom increases, the \( t \) distribution approximates a normal distribution.

As stated previously, the T-test is only applicable for normal distributions while the majority of the population examined under this study are expected to be lognormal. Therefore to enable the T-test for lognormal distributions, the transformation \( x^* = \ln(x) \) must be first made.

The sole number used for accepting or rejecting the null hypothesis is called the p-value. The p-value is the \( \alpha \) probability at which the null hypothesis can be rejected. In this diagnostic algorithms p-values of 0-0.05 are considered definitely different, 0.05-0.15 as probably different and >0.15 as not statistically different. Results are provided in section 12.1.

### 9.3.2 Error-based Diagnosis

The second approach taken to classifying differences in two distributions was to compare the Sum of the Differences Squared (hereafter referred to as SDS) between “healthy” joint feature histograms and associated PDFs to candidate feature histograms. In this approach, the SDS built from the healthy joint features (referred to as baseline) is mapped to 0 (hereafter referred to as the SDS number). This means that any candidate joint’s feature histogram that produced a SDS less than the baseline SDS would also get mapped
to an SDS number of zero. Similarly, the SDS produced from the most damaged joint gets mapped to an SDS number of 1. The assumptions with this approach are that:

1) The PDFs fitted to healthy feature histograms are representative of entire populations.
2) The SDS between candidate feature histograms as baseline PDFs is directly related to joint condition.
3) The condition of a particular joint is directly proportional to its SDS number.

10.0 Testing

In the fall of 2000, a comprehensive CV-joint testing plan and testing convention was developed jointly by the author, Tim Cook, and Joel Berg of NCRRR. The test plan was designed to ensure repeatability of a particular CV-joint by requiring 4 identical tests at condition for each joint. Furthermore, the joints were tested at 2 different speeds in both forward and reverse directions and at 2 different torque loads to attempt to identify the dependency of the vibration features on these variables.

10.1 Naming Convention

The standard naming convention that was developed is shown in Figure 28. The following section briefly describes the reasoning behind each of the elements in the naming convention:
**Filename Convention**

- Data Set (1 thru 4)
- RPM (S = 88, F = 142)
- Direction of Rotation (Forward or Reverse based on Test Stand setup)
- Pressure (in psi as indicated on Test Stand)
- Offset (Holes 1 thru 4, inner most is 1, outer most is 4)
- Last 7 Characters of Joint ID (ID as indicated on Joint)

**Figure 28 – Naming Convention**

**Joint ID:** Last 7 characters of ID number stamped on each half shaft. Only the tire-end CV joint of each half shaft was tested.

**Offset:** The drive plates where designed to accommodate 4 different articulation angles for each half shaft. For the particular half shafts tested, setting 1 was too shallow to ensure the full range of motion of the rolling elements while setting 4 placed each joint at risk of over-extension. All of the test data evaluated in this study was performed at setting 2.

**Pressure:** This value is the pressure in psi of the supply air allowed through the pressure regulator to the pneumatic piston. A direct relationship was found between this pressure and the constant torque imparted to the half shaft was given in Table 1.

The majority of testing was performed at 20 and 30 psi (104 and 156 ft-lbs respectively). This was because 1) it was believed that 100-150 psi was sufficient to pre-load the joint and 2) higher torques put the drive bearings at risk of pre-mature failure in the prototype fixture design.
Direction of rotation: Testing was performed with the half shaft rotating both clockwise or counterclockwise. The torque imposed on the half-shaft is always in the same direction.

RPM: 2 speed settings were selected as standards for joint-to-joint comparison. The seemingly arbitrary selection of 88 RPM as “slow” and 142 RPM as “fast” in fact corresponded to the values of 2 and 3 on the AC motor speed control dial. The majority of the data evaluated in this study was collected in “fast” mode because more revolutions per time synchronous average could be obtained in a fixed sample size.

Data Set: As previously stated, each CV-joint was tested 4 times under identical conditions to insure test repeatability. The diagnostic software automatically accounts for the Data Set number.

10.2 Test Log

In addition to the naming convention, an electronic test log was developed. After an initial fixture testing period, the majority of the testing and upkeep of the electronic logs was performed by Tom Cook and Joel Berg of NCRRR. In essence, the test log states the known seeded faults in each joint and all of the testing performed on each joint. An example of a test log is shown in Figure 29.
Joint ID: PLS3PC  
Defect: .008 - .010 Dimple

Span (inch): 7-5/16"  
Offset (Hole #): 2

Number of Data Points: 20,000  
"Lab View Setting"

Scan Rate: 5000  
"Lab View Setting"

<table>
<thead>
<tr>
<th>File Name</th>
<th>Pressure (psi)</th>
<th>Torque (ft-lbs)</th>
<th>Direction of Applied Torque</th>
<th>Target RPM</th>
<th>Actual RPM</th>
<th>Direction of Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLS3PC_2_20_F_S_1</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>88</td>
<td>88.4</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_S_2</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>88</td>
<td>88.2</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_S_3</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>88</td>
<td>88.2</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_S_4</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>88</td>
<td>88.3</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_F_1</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>142</td>
<td>142.7</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_F_2</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>142</td>
<td>142.4</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_F_3</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>142</td>
<td>142.8</td>
<td>Forward</td>
</tr>
<tr>
<td>PLS3PC_2_20_F_F_4</td>
<td>20</td>
<td>104</td>
<td>Rev (piston out)</td>
<td>142</td>
<td>142.1</td>
<td>Forward</td>
</tr>
</tbody>
</table>

Figure 29 – Test Log for Joint #PLS3PC

### 10.3 Test Plan

The test plan for this project consisted of 4 basic steps:

1) Evaluate the test fixture and data acquisition system by:
   a. Testing the fixture for durability and effectiveness.
   b. Testing the feasibility of the test fixture/data acquisition system to enable quick and consistent testing of a half shaft (i.e. devise best practice)

2) Develop baseline feature histograms by testing the population of healthy CV-joints.

There were 8 initial half shafts analyzed. Of this population, only 2 were known to be healthy. The half shaft ID numbers and associated seeded faults are given in Table 2.
<table>
<thead>
<tr>
<th>Joint ID</th>
<th>Seeded Fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>7042270521</td>
<td>None</td>
</tr>
<tr>
<td>7042270543</td>
<td>None</td>
</tr>
<tr>
<td>PLS3PC</td>
<td>0.008-0.01” dimple</td>
</tr>
<tr>
<td>PL00900</td>
<td>0.008” dimple</td>
</tr>
<tr>
<td>PL00800</td>
<td>0.008” dimple</td>
</tr>
<tr>
<td>PL00300</td>
<td>0.003” dimple</td>
</tr>
<tr>
<td>PLSP1A</td>
<td>unknown</td>
</tr>
<tr>
<td>7042270588</td>
<td>unknown</td>
</tr>
</tbody>
</table>

*Table 2 – Condition of half-shaft population*

3) Test damaged joints. Test feature histograms on the most damaged CV-joints (PLS3PC, PL00900, and PL00800) to determine effectiveness and threshold levels for the features. Down select from feature set or improve features as required.

4) Perform blind tests and attempt to automatically diagnose whether the candidate joint is healthy or damaged and, if damaged, the severity of the damage.

**11.0 Sensitivity Analysis**

Several initial tests were performed on the data to 1) insure that a given diagnostic approach was validated and 2) that the effects of the significant number of process variables (speed, torque, damage level, direction) could be understood. Specifically, statistical tests were performed to either accept or reject the following hypotheses:

1) H0: Direction (Forward or Reverse) does not affect the vibration features
2) H0: The two “Baseline” CV-joints can be considered the same (feature-wise)
3) H0: Vibration features are dependent on torque
The results of these hypotheses tests were an important step in selecting test data with which to answer a primary question of this study: Is a candidate CV-joint healthy or damaged?

11.1 Baseline and Directional Analysis

Hypotheses #1 and #2 were simultaneously tested using the t-test already described in a matrix format. 4 sets of 4 tests from the 2 “baseline” CV-joints were compared for each of the 5 features with no assumptions made as to how accurate or sensitive a particular features was. The test cases selected were:

1) 7042270521_2_30_F_F  (Setting 2, Torque Setting = 30, Clockwise, Speed = Fast)
2) 7042270521_2_30_R_F  (Setting 2, Torque Setting = 30, CounterCW, Speed = Fast)
3) 7042270543_2_30_F_F
4) 7042270543_2_30_R_F

A T-statistic of 1.645, representing an α/2 error of 5% was selected as the criteria for rejection of a H0 hypothesis that any given average feature level was equal to any other average of the same feature. The results of this analysis are compiled in Table 3.

For a given feature undergoing the first test (Difference due to direction?), if test case #1 is statistically different than #2 and #3 is statistically different from #4 then “Diff. Due to Direction” would be “No”. If the two possibilities contradict each other than result was “Inconclusive”. If neither are significantly different than the result was “Yes”. The second test had 4 possibilities 1 vs. 3, 1 vs. 4, 2 vs. 3, 2 vs. 4. In this case, high T-statistic for all cases was “Yes”, 1 vs. 3 or 4 and 2 vs. 3 or 4 was “Likely”, 1 or 2 vs. 3 and 4 was “Unlikely” and no significant differences was “No”.

The test results indicated that is it safe to assume that direction has no effect on the features (as would be intuitively expected) but that there is a definitive difference between the two “healthy” CV-joints.
Table 3 – Baseline and Speed Test Results

<table>
<thead>
<tr>
<th></th>
<th>Diff. Due to Direction?</th>
<th>Diff. Between (1,2) and (3,4)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>No</td>
<td>Unlikely</td>
</tr>
<tr>
<td>RMS TSA</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Kurtosis TSA</td>
<td>Inconclusive</td>
<td>Likely</td>
</tr>
<tr>
<td>FFT RMS TSA</td>
<td>Inconclusive</td>
<td>Likely</td>
</tr>
</tbody>
</table>

As a result of these conclusions, the analysis continued using the following conjunctures:

1) Data from tests run in the clockwise direction may be assumed to be the same as those run in the counterclockwise direction with all other things equal (i.e. torque, speed, setting)

2) Assuming that CV-joint 7042270521 and 7042270543 are genuinely “healthy” joints, the differences in their features represents the differences that may be seen amongst a population of “healthy” joints.

11.2 Effect of Torque

An Analysis of Variance (ANOVA) test was conducted on the effect of torque on each of the vibration features. With this type of study, total corrected sums of squares is broken up into the contribution due to the treatment (torque) and that due to error. With this information another test statistic, distributed as the F-distribution, is created.

\[
F_0 = \frac{MS_{Treatments}}{MS_{Error}} \quad \text{Eq. 22}
\]

This statistic may be thought of as representative of a signal-to-noise ratio. If the F0 statistic is greater than an appropriate upper-percentage point of the F distribution, with
α error, v₁ and v₂ the degrees of freedom of treatments and samples respectively, the effect of the treatment is statistically significant [17].

Torque testing was performed on CV-joint #PL00800. A total of 4 tests at 4 different torque settings (all other things equal) were performed on this joint labeled as:

PL00800_2_20_F_S  (Setting 2, Torque Setting = 20, Clockwise, Speed = Slow)
PL00800_2_30_F_S  (Setting 2, Torque Setting = 30, Clockwise, Speed = Slow)
PL00800_2_40_F_S
PL00800_2_50_F_S

Based on the conclusions of the Directional Analysis, this data was expanded upon to include identical tests performed in the “Reverse” or counterclock-wise direction enabling an 4X2 matrix for each feature. An example of this matrix is given in Table 4 for RMS.

<table>
<thead>
<tr>
<th>Torque Setting</th>
<th>Avg. of 4 “Forward” tests</th>
<th>Avg. of 4 “Reverse” tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.028682</td>
<td>0.034971</td>
</tr>
<tr>
<td>30</td>
<td>0.038953</td>
<td>0.041489</td>
</tr>
<tr>
<td>40</td>
<td>0.055659</td>
<td>0.076221</td>
</tr>
<tr>
<td>50</td>
<td>0.147975</td>
<td>0.139228</td>
</tr>
</tbody>
</table>

Table 4 – Results for RMS

This analysis was performed using Minitab V12.21 for quick results and because this was a one-time analysis. A typical Minitab output [18] for a One-way ANOVA is shown for the RMS feature next along with a Boxplot of the data in Figure 30. In the Boxplot, a gray region represents the +/- 1 σ band.
One-way Analysis of Variance

Analysis of Variance for RMS

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque</td>
<td>3</td>
<td>0.015541</td>
<td>0.0051847</td>
<td>76.06</td>
<td>0.001</td>
</tr>
<tr>
<td>Error</td>
<td>4</td>
<td>0.0002727</td>
<td>0.0000682</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>0.0158267</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Individual 95% CIs For Mean Based on Pooled StDev

<table>
<thead>
<tr>
<th>Level</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>2</td>
<td>0.03183</td>
<td>0.00445</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>0.04022</td>
<td>0.00179</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.06594</td>
<td>0.01454</td>
</tr>
<tr>
<td>40</td>
<td>2</td>
<td>0.14360</td>
<td>0.00619</td>
</tr>
<tr>
<td>50</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pooled StDev = 0.00826

Boxplots of RMS by Torque

(means are indicated by solid circles)

Figure 30 – Boxplot of RMS vs. Torque

Table 5 provides a summary of these results for all the features.
<table>
<thead>
<tr>
<th>Feature</th>
<th>Torque Effect Significant?</th>
<th>20/30</th>
<th>30/40</th>
<th>40/50</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>RMS TSA</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Kurtosis TSA</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>FFT RMS TSA</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 5 – Torque Results

The conclusions that may be drawn from this analysis are as follows:

1) Both the Kurtosis and Kurtosis TSA are statistically unaffected by torque
2) FFT RMS TSA is the most sensitive to torque effects
3) In general, torque affects all the features in dissimilar non-linear fashions
4) In general, the effects of torque become more apparent after setting 40 or 208 ft-lbs.

As a result of the conclusions from the Torque analysis and the Baseline & Direction analysis, the consistent test setup at which vibration features performance were assessed was torque setting = 30 and indiscriminately between forward and reverse. Furthermore, all analysis was performed at CV-joint angle setting #2 because no data was taken at setting #1 or #4 and observational evidence suggests that, for the particular CV-joint type tested, notch #3 may have been exceeding the maximum operating angle of the joint. Finally, the analysis was performed using “Fast” tests (142 RPM) only. This choice in no way reflects that “slow” testing (88 RPM) is inadequate however, from a practical standpoint, more Time Synchronous Averages can be performed on 5000 samples from a “Fast” test and one-per-rev frequencies are higher. Further discussion on test setup effects is provided in the “Recommendations” section 14.0.
12.0 Feature Results

As previously stated, two distinct diagnostic techniques were employed in this study to evaluate the performance and effectiveness of the vibration features. The first technique, employed a statistical t-test on each features Gaussian mean and standard deviation with respect to the average baseline test statistics. The second technique, referred to as the Error Pattern technique, compared feature histograms to baseline Probability Density Functions with the degree of difference indicative of the CV-joint condition.

12.1 T-test Approach results

Recall that the sensitivity study found that Forward and Reverse tests were not statistically different and could therefore be pooled. The averaged results of the vibration feature for a total of 16 tests on the 2 baseline CV-joints run at 142 RPM and at 156 ft-lbs is given at the top of Figure 31. The average features from 8 runs (4 forward, 4 reverse) for each of the candidate CV-joints are given in Figure 31 and Figure 32. Green in the “p-value” column means the feature distribution was not significantly different from the average baseline. Yellow means the feature was probably different (p-value of 0.05-0.15), while red means the feature was definitely statistically different from the baseline population.

Examination of the features yielded a subjective “observability” strength for each feature to faulty joints. This was observability was applied in a weighted average of the form:

\[ 1 - \omega_1(1-P_1) \times \omega_2(1-P_2) \times \omega_3(1-P_3) \ldots \] = Weighted Average

Where:

\( P_X = \) p-value of feature X
\( \omega X = \) weight on the p-value of feature X
The weights must sum to one. These subjective weights used are given below:

RMS TSA – 0.3  
RMS – 0.3  
Kurtosis TSA – 0.2  
RMS FFT TSA – 0.15  
Kurtosis – 0.05  

By applying this approach, individual features are fused together to form a single robust diagnostic feature.

<table>
<thead>
<tr>
<th>Average Baseline</th>
<th>2_2_30_F/R_F</th>
<th>Distributed as</th>
<th>Normal Mean</th>
<th>Normal Stdv</th>
<th>P-value (alpha/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>Normal</td>
<td>0.06687</td>
<td>0.02890</td>
<td>Baseline</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Lognormal</td>
<td>0.86779</td>
<td>0.25980</td>
<td>Baseline</td>
<td></td>
</tr>
<tr>
<td>RMS TSA</td>
<td>Normal</td>
<td>0.04944</td>
<td>0.03057</td>
<td>Baseline</td>
<td></td>
</tr>
<tr>
<td>Kurtosis TSA</td>
<td>Lognormal</td>
<td>0.78064</td>
<td>0.26190</td>
<td>Baseline</td>
<td></td>
</tr>
<tr>
<td>RMS FFT TSA</td>
<td>Normal</td>
<td>-7.57357</td>
<td>0.86067</td>
<td>Baseline</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PLS3PC-Accel #2</th>
<th>0.008-0.01 dimple</th>
<th>2_2_30_F/R_F</th>
<th>Distributed as</th>
<th>Normal Mean</th>
<th>Normal Stdv</th>
<th>P-value (alpha/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>Normal</td>
<td>0.12679</td>
<td>0.32935</td>
<td>0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Lognormal</td>
<td>0.86225</td>
<td>0.35488</td>
<td>0.073</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS TSA</td>
<td>Normal</td>
<td>0.06111</td>
<td>0.13411</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis TSA</td>
<td>Lognormal</td>
<td>0.73394</td>
<td>0.31974</td>
<td>0.136</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS FFT TSA</td>
<td>Lognormal</td>
<td>-5.54616</td>
<td>0.71287</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PL00800-Accel #2</th>
<th>0.008 dimple</th>
<th>2_2_30_F/R_F</th>
<th>Distributed as</th>
<th>Normal Mean</th>
<th>Normal Stdv</th>
<th>P-value (alpha/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>Normal</td>
<td>0.10350</td>
<td>0.19610</td>
<td>0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Lognormal</td>
<td>0.93109</td>
<td>0.49908</td>
<td>0.043</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS TSA</td>
<td>Normal</td>
<td>0.07023</td>
<td>0.07691</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis TSA</td>
<td>Lognormal</td>
<td>0.80364</td>
<td>0.42082</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS FFT TSA</td>
<td>Lognormal</td>
<td>-5.89672</td>
<td>0.63377</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PL00900-Accel #2</th>
<th>0.009 dimple</th>
<th>2_2_30_F/R_F</th>
<th>Distributed as</th>
<th>Normal Mean</th>
<th>Normal Stdv</th>
<th>P-value (alpha/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>Normal</td>
<td>0.07880</td>
<td>0.04780</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Lognormal</td>
<td>0.89470</td>
<td>0.44630</td>
<td>0.131</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS TSA</td>
<td>Normal</td>
<td>0.06760</td>
<td>0.03846</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis TSA</td>
<td>Lognormal</td>
<td>0.72113</td>
<td>0.33830</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS FFT TSA</td>
<td>Lognormal</td>
<td>-7.19424</td>
<td>0.86533</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 31 – T-test results for 1st three faulty joints vs. Average baseline

58
Application of the weighted average formulation to the T-test results yields the overall diagnosis given in Table 6. Warnings or Alarms issued on the CV joints known to be damaged was 100% accurate with no false alarms. In actual implementation, the CV-joints labeled as “Warning” could be set aside for further inspection and the acceptance/rejection criteria could be easily adjusted.

<table>
<thead>
<tr>
<th>Joint ID</th>
<th>Seeded Fault</th>
<th>Diagnosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>7042270521</td>
<td>None</td>
<td>NA</td>
</tr>
<tr>
<td>7042270543</td>
<td>None</td>
<td>NA</td>
</tr>
<tr>
<td>PLS3PC</td>
<td>0.008-0.01” dimple</td>
<td>0.083</td>
</tr>
<tr>
<td>PL00900</td>
<td>0.008” dimple</td>
<td>0.039</td>
</tr>
<tr>
<td>PL00800</td>
<td>0.008” dimple</td>
<td>0.015</td>
</tr>
<tr>
<td>PL00300</td>
<td>0.003” dimple</td>
<td>0.017</td>
</tr>
<tr>
<td>PLSP1A</td>
<td>unknown</td>
<td>0.308</td>
</tr>
<tr>
<td>7042270588</td>
<td>unknown</td>
<td>0.074</td>
</tr>
</tbody>
</table>

Table 6 – T-test Diagnostic results
13.0 Conclusions

This paper has described the design and functionality of a novel test fixture, vibration data acquisition methodology and associated diagnostic software for determining the health of automotive constant velocity joints. The following objectives were fulfilled:

1) A novel CV-joint test fixture was designed and fabricated where high signal-to-noise ratios and fault observability were demonstrated due to the dynamic motion and high torque that can be applied to non-rotating half shafts being tested. The test fixture showed several additional improvements over prior designs including quick testing turnover rates, adaptability to different CV-joint types and relatively low hardware cost due to minimal motor horsepower requirements and absence of a dynamometer.

2) Five stochastic time and frequency domain features were developed and evaluated over the course of several repeatable tests on both healthy and faulty joints.

3) ANOVA analysis of several experiments yielded important information about the effects of testing variables. Specifically, the experiments determined that torque levels of 156-208 ft-lbs were optimum for pre-loading the joints while not introducing drive system non-linearities and that direction of rotation is unimportant.

4) The stochastic vibration features were developed into a diagnostic scheme through the implementation a t-test and weighted average. The diagnostic scheme was 100% effective in diagnosing CV-joints with known rolling element defects and yielded no false alarms.

14.0 Recommendations

The following recommendations are made if follow-on analysis is to be performed with the CV-joint test bench:
1) Different CV-joint types (size and design) should be tested in the machine with the vibration features developed to determine their performance and effectiveness over a broader population and to determine the adaptive limits of the fixture itself.

2) Further seeded fault tests should be performed with a broader variety of faults such as mis-sized grooves, flipped cages, contaminated grease, etc. to evaluate diagnostic capabilities

3) A run-to-failure test should be performed as known defects are systematically added to a healthy CV joint.

4) Endurance tests should be performed on the test fixture to determine weak components in the design subject to fatigue failures.

5) A modal analysis should be performed on the test fixture to determine natural frequencies that could be dampened or extracted out from the frequency spectrum to further raise signal to noise ratios.

6) Experiments should be devised to better classify the characteristic frequencies in CV-joints. If and when these frequencies are identified, more robust vibration features may be implemented that focus on residual signal content or peak tracking

15.0 Acknowledgements

I would like to thank the following individuals for helping with the testing, design and fabrication of the CV joint testing machine - Timothy Cook, William Morris, Scott Nichols, Josh Eckler, and Joel Burg. I also thank Nabil Nasser of NCRRR for funding this project and Kevin Kochersburger for being my advisor.
16.0 References


[7] Car and Driver, August 1997


[18] MINITAB Version 12.21
Appendix I – Free body diagram of 4-square machine
Method of proof: \( T_3 = T_9 \)

If this is true, Fr or the torque imposed on the motor must be 0.

From (3) \( T_9 = (R_x \sin \theta)(L) + (R_y \sin \theta)(L) \)

From (6) \( T_3 = (R_x \sin \theta)(L) - (R_y \sin \theta)(L) - Fr \)

Recall:
\[
R_x = R_{xy} \\
R_y = R_{xy}
\]

\[
T_9 = (R_x \sin \theta)(L) + (R_y \sin \theta)(L)
\]
\[
T_3 = (R_x \sin \theta)(L) - (R_y \sin \theta)(L) - Fr
\]

\[
T_9 + T_3 = 0 \\
T_9 = -T_3
\]

\( T_9 = T_3 \) with \( Fr = 0 \)

System is out of equilibrium with \( Fr \neq 0 \) for any given
\( T, x_1, \) or \( L \)