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Noise Power Ratio testing of multichannel single sideband communication equipment

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NOISE POWER RATIO TESTING OF MULTICHANNEL

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prepared

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ABSTRACT

It is shown, that Noise Power Ratio (NPR) testing of multichannel Single Sideband (SSB) transmitters should be performed loading all channels with identical white, gaussian, zero-mean noise signals in order to obtain the worst case result.

A theoretical analysis of bichannel equipment under the two different input conditions: (a) statistically independent noise signals, and (b) statistically dependent noise signals; shows that the intermodulation distortion on the output of the transmitter is the same for either case.

This result is extended for an arbitrary number of channels to the point of showing that the test results obtained for statistically dependent noise inputs will be either equal to or worse than those obtained for statistically independent noise inputs.

The theoretical conclusions made for a bichannel transmitter are verified by means of practical measurements performed on such a transmitter.

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NPR	=	Noise Power Ratio
SSB	=	Single Sideband
LSB	=	Lower Sideband
USB	=	Upper Sideband
$H[x(t)]$	=	$\hat{x}(t)$ = Hilbert Transform of $x(t)$
$\overline{x(t)}$	=	Average of $x(t)$
$\mathcal{R}_x(\tau)$	=	Time Autocorrelation Function of $x(t)$
$S_x(f)$	=	Power Density Spectrum of $x(t)$

CHAPTER 1: INTRODUCTION

1.1 Intermodulation due to Amplifier Nonlinearity.

Nonlinearities in Single Sideband exciters and power amplifiers give rise to intermodulation distortion of the output signal and, in the case of multichannel equipment, crosstalk. This distortion, along with noise introduced in the transmission channel, will greatly affect the intelligibility of the detected signal at the receiving end of the communication system. It is therefore important, not only to minimize nonlinearities by careful design, but also to perform an intermodulation distortion test as part of the performance inspection of the final equipment.

As an aide in understanding the following discussion a simplified block diagram of a typical multichannel Single Sideband transmitter using the filtering method* is shown in figure 1-1.

An amplitude modulated or Double Sideband-Suppressed Carrier signal is generated in each channel by mixing the audio signal and the carrier injection in a balanced modulator. The modulator output is then passed through a bandpass filter, which in the ideal case processes the desired sideband without attenuation and attenuates all other inputs infinitely. The SSB output from all the channels are then combined linearly (summed) and applied to the input of the high frequency bandpass amplifier. It is nonlinearities in this amplifier that cause inter-

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*See also chapter 2, section 1.

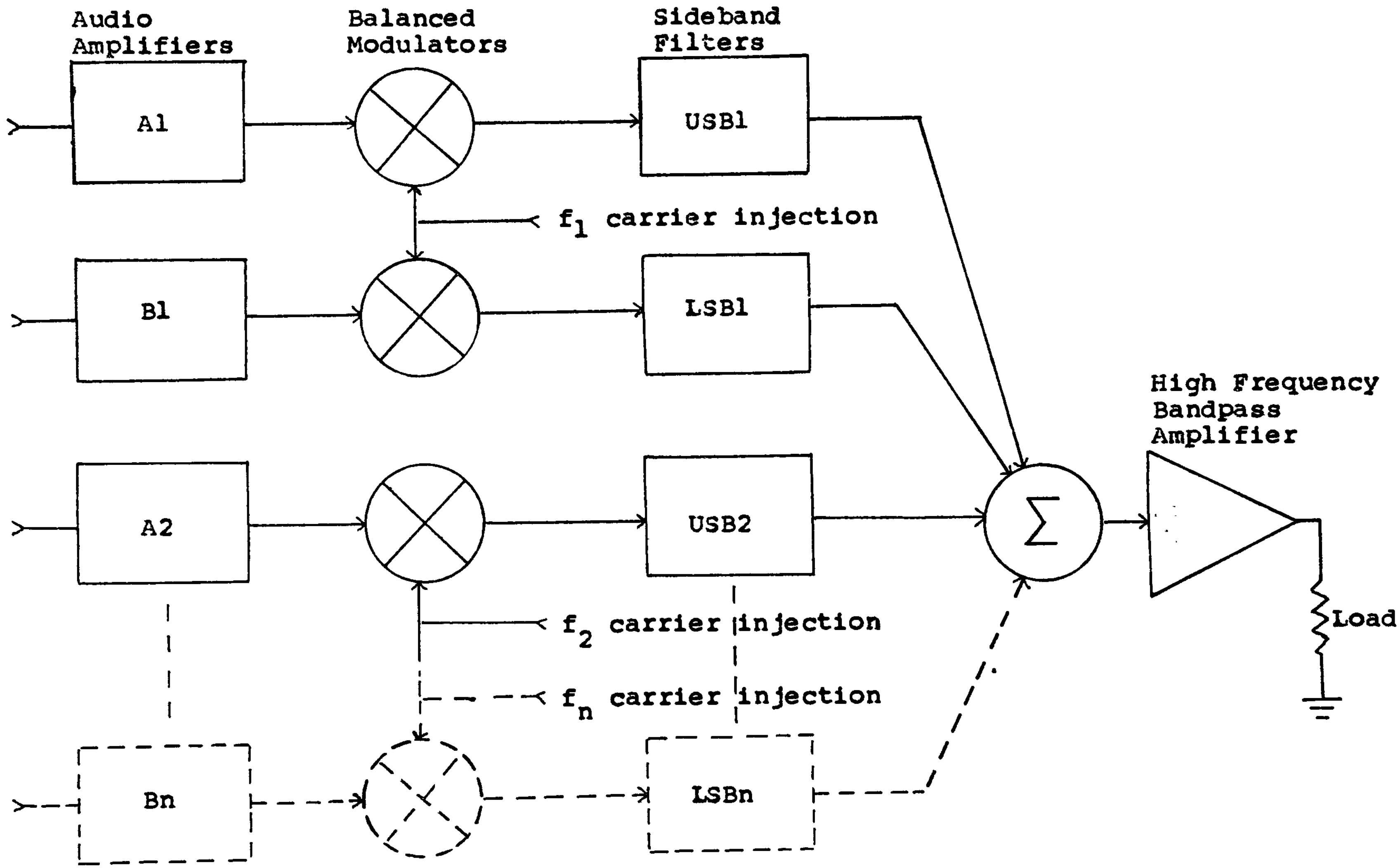

Figure 1-1: Multichannel Single Sideband Transmitter. Block Diagram.

modulation distortion of the output signal.

1.2 Noise Power Ratio Test.

So far, it has been the accepted standard to measure the intermodulation distortion by means of a two-tone test signal applied to the input of the transmitter (single channel) and then measure the amplitudes of the significant intermodulation products in the output signal with a spectrum analyzer. The two test tones were chosen so that at least thé 3rd and 5th order intermodulation products fell within the passband of the transmitter. The measure of the intermodulation distrotion was the ratio of the most significant intermodulation product and one of the desired tones.

The Radio Subcommittee of the MIL-STD-188 Committee* has recently recommended that the standard method of measuring nonlinearity in SSB communication equipment be a Noise Power Ratio Test, in which the individual channel is loaded with white, gaussian, zero-mean noise, instead of the two-tone signal mentioned above.

The complexity of the test equipment required to perform the Noise Power Test on multichannel equipment would depend on whether or not the input noise signals for the various channels should be statistically independent. The reason is, that if statistical independence between the noise signals were required, then a different noise generator for each channel would

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*The MIL-STD-188 Committee makes recommendations for inclusions and revisions of the MIL-STD-188, which is a general specification for all military equipment.

have to be used to test the multichannel equipment. On the other hand a single noise generator and a simple multicoupler could be used, if statistical dependence was required. The latter procedure offers considerable advantage in cost, maintainability and reliability.

For the reasons stated above it is necessary to investigate whether statistically dependent or independent noise inputs lead to the worst case of Noise Power Ratio reading for the Single Sideband equipment.

A theoretical analysis of the effect of statistical dependence or independence on Noise Power Ratio is made in this thesis.

The Noise Power Ratio Test is performed as follows: All channels of the transmitter are loaded simultanously with white, gaussian, zero-mean noise and the noise power output in a particular, narrow slot in the channel under test is measured. Then the input noise is removed from the corresponding slot by means of a notch filter, and the residual noise power is measured in the output slot. This is then repeated for every channel. These measurements are used to calculate the Noise Power Ratio, NPR(SSB), which is defined as

> "the ratio of the mean noise powers measured in the notch filter bandwidth for the notch in and the notch out conditions with total system mean power output equal for both conditions."*

Figure 1-2 shows a typical test set-up for a single channel.

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*This definition is quoted from paragraph 2.194 of SPECIFICATIONS RECOMMENDED FOR INCLUSION IN MIL-STD-188C, submitted by the Radio Subcommittee.

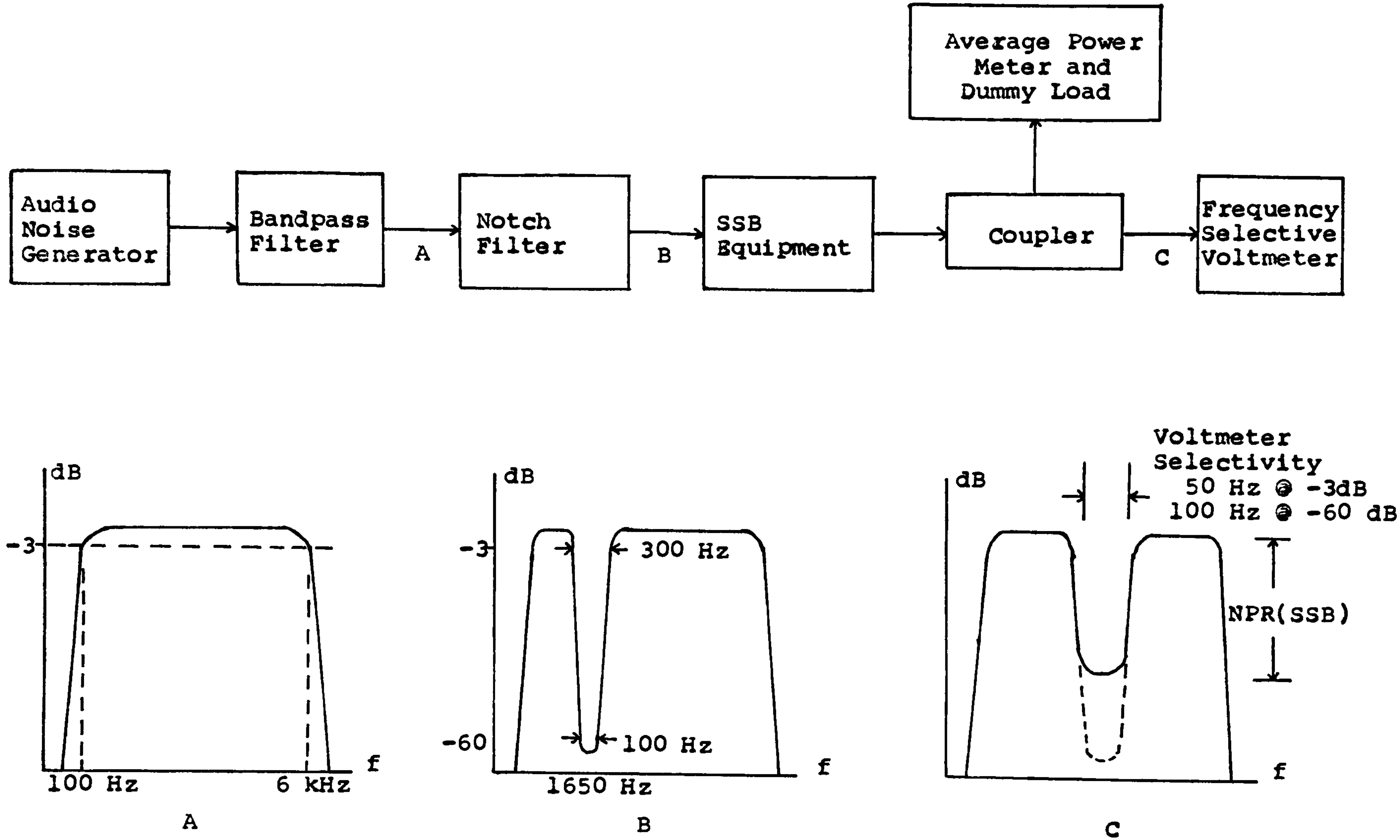

Figure 1-2: Typical Arrangement for Measurement of NPR(SSB).

The use of white noise as a test signal for measuring amplifier nonlinearity in SSB equipment was suggested in 1956 by Icenbice and Fellhauer[1], when common usage of SSB techniques in radio communication was still being debated. But this particular test method had been in use in checking out multiplex telephone equipment earlier than 1956. In those tests, the input noise signals were applied to several hundred channels simultanously and the quiet slot was obtained by notching out one complete channel[2]. An alternate approach was to measure the noise power level in an idle channel outside the band in normal use, and then compare this level to the power level of the normal traffic.

1.3 Nonlinear Interference.

It can be shown, that interference in bandpass amplifiers is due mainly to third order nonlinearity. This can be seen most readily by cubing a single frequency term

$$a_3(A \cos\omega t)^3 = a_3 \frac{3\ A^3}{4} \cos\omega t + a_3 \frac{A^3}{4} \cos 3\omega t$$

The first term on the right hand side of the above equation falls within the passband of the amplifier and will add to or subtract from the linear term depending on the sign of the coefficient a_3.

For a multichannel system this problem becomes even more grave since each cube law intermodulation product spreads over three channels. If we for instance consider three adjacent channels, A, B and C, with identical bandwidths and power density,

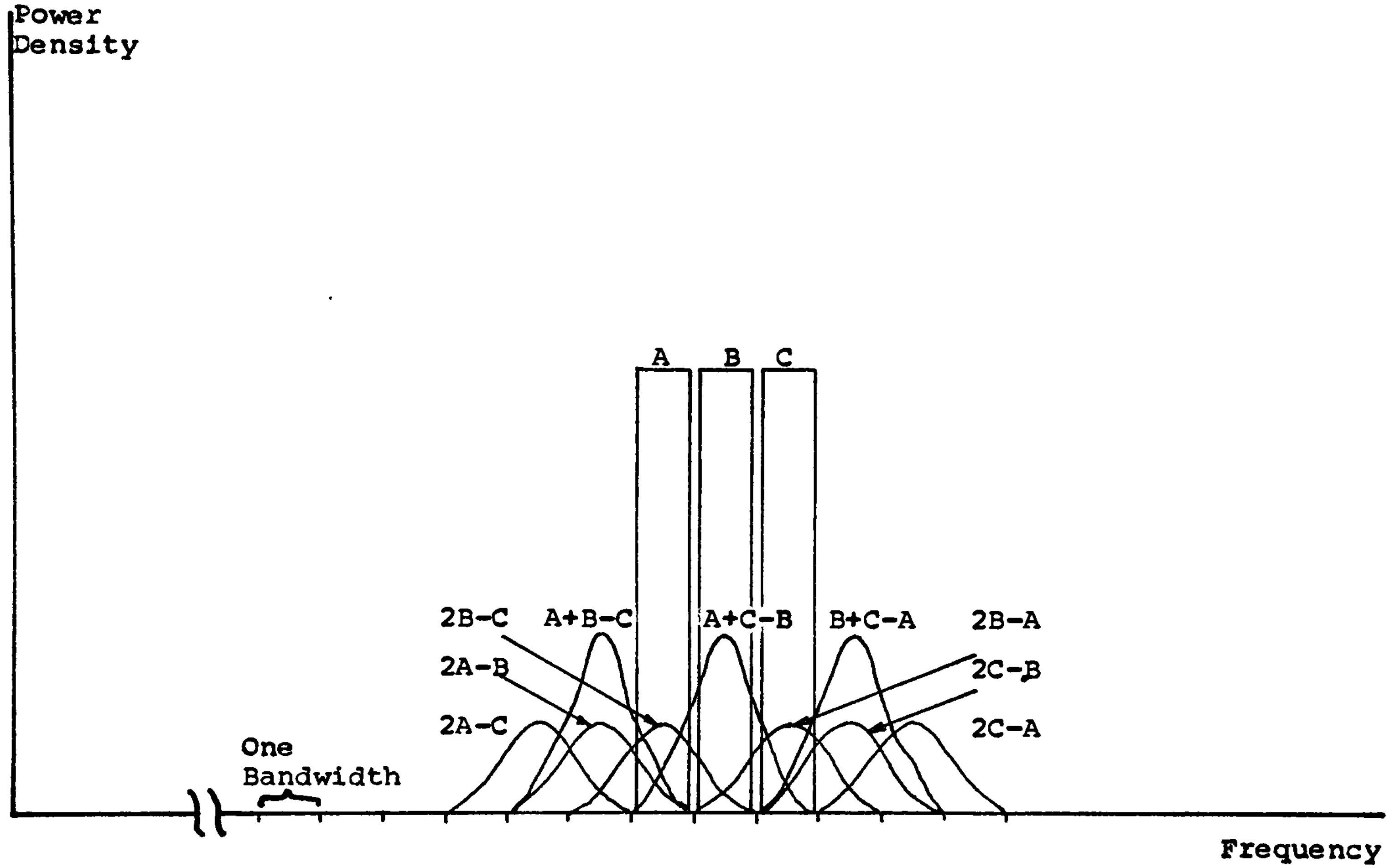

Figure 1-3: Third Order Intermodulation Products for Three Channels.

then the intermodulation products falling within the passband of the amplifier will be of the form, A+B-C, A+C-B, B+C-A and 2A-B, 2A-C, 2B-A, 2B-C, 2C-A, and 2C-B, where the three former are the predominant ones[3]. This is indicated in principle on the diagram shown in figure 1-3. From the diagram it appears as if the intermodulation distortion will be different for the different channels. This is true for a small number of channels, whereas for an "infinite" number of channels additional intermodulation products will cause even distribution of the distortion. Such interfering products greatly distort the amplifier output signal unless due consideration is given to the problem during the design phase.

An extensive mathematical treatment of interchannel interference due to amplifier nonlinearity is given for discrete frequencies in the literature[1,3]. The processing of a sine wave plus gaussian noise through a nonlinear device has also been covered in depth by several authors[4,5].

Davenport and Root[5] have in chapters 12 and 13 indicated some general methods of arriving at the output autocorrelation function and power density spectrum for a ν-th law device. A ν-th law nonlinear device is based on the half-wave transfer characteristic

$$y(t) = \begin{cases} a\, x^{\nu}(t), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

where a is a constant and ν is a real, positive number. This basic characteristic can be used to describe a whole class of nomlinear devices.

For more simple devices such as a square-law detector, Davenport and Root indicate a "Direct Method", in which the output autocorrelation function is found by calculating the expected value or average of the product of the output signal taken at times t_1 and t_2.

The treatment in this thesis is an extension of this "Direct Method", such that the output autocorrelation function is determined for an input of two nonoverlapping, narrowband noise signals.

1.4 Scope of the Thesis.

The thesis consists of two major parts, a theoretical analysis and a practical measurement.

A mathematical model of a SSB transmitter is defined, in which certain idealizing conditions are established in order to simplify the mathematical analysis.

A theoretical analysis of this model is then carried out, arriving at an expression for the output autocorrelation function for two different input conditions, namely:

(a) statistically independent noise inputs; and

(b) identical noise inputs.

The resulting expressions for these two conditions are then compared in order to find which one of them causes the highest intermodulation distortion.

The practical measurement is carried out in an attempt to verify the theoretical conclusions. This measurement has only been performed on a bichannel transmitter, since highly specialized test equipment would be necessary to perform the test on a transmitter with more than two channels.

The reason for performing this investigation lies in the fact that, if the case of identical noise inputs does indeed result in the worst case Noise Power Ratio, then the test equipment required to perform the NPR test can be very simple and inexpensive.

Chapter 2 of the thesis contains the theoretical analysis, performed first for bichannel equipment and then extended to cover multichannel equipment in general. The practical test set-up and the test results are described in chapter 3, and in chapter 4 are given some concluding remarks relating to theoretical versus practical results.

CHAPTER 2: THEORETICAL ANALYSIS

2.1 Single Sideband Generation.

There are two principal methods of SSB generation, the filtering method and the phasing method. Of these two the filtering method is the most common in military radio equipment. However, the mathematical expression for a SSB signal is derived more easily by using a simplified block diagram of a phasing method SSB generator, as shown in figure 2-1.

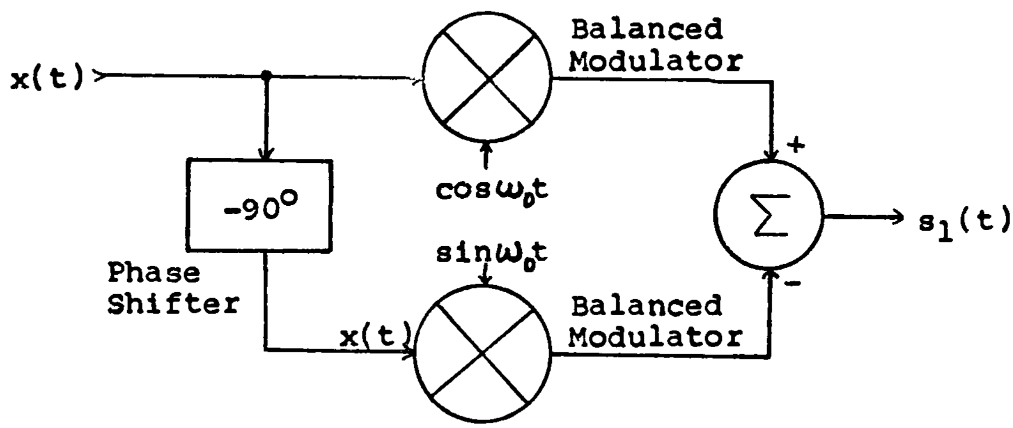

Figure 2-1: Phasing Method for Generation of SSB.

The output signal is an upper sideband (USB) signal[3]:

$$s_1(t) = x(t)\cos\omega_0 t - \hat{x}(t)\sin\omega_0 t \qquad (2\text{-}1)$$

where x(t) is the Hilbert Transform of x(t), defined as follows:

$$H[x(t)] \equiv \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau \,,$$

and f_o is the carrier frequency.

The lower sideband (LSB) signal can be obtained by adding the two modulator outputs

$$s_2(t) = x(t)\cos\omega_o t + \hat{x}(t)\sin\omega_o t \qquad (2\text{-}2)$$

x(t) can be any audio signal, single tone, voice or some kind of noise, the only requirement is, that the bandwidth is less than f_o.

If an analytic signal, z(t), is defined by the equation

$$Z(t) = x(t) \pm j\,\hat{x}(t) \qquad (2\text{-}3)$$

then $s_1(t)$ and $s_2(t)$ can be expressed in a more compact form as the real part of a complex signal

$$s_1(t) = Re\left\{ Z_1(t)\,e^{j\omega_o t} \right\}, \qquad (2\text{-}4)$$

where $z_1(t) = x(t) + j\,\hat{x}(t)$; and

$$S_2(t) = Re\left\{ Z_2(t)\,e^{j\omega_o t} \right\}, \qquad (2\text{-}5)$$

where $z_2(t) = x(t) - j\,\hat{x}(t)$.

It can be easily verified, that $s_1(t)$ and $s_2(t)$ are the upper and lower sideband signals respectively, simply by substituting $\cos\omega_1 t$ for x(t), where f_1 is an audio frequency, and performing a trigonometric contraction.

If x(t) is a gaussian variable, then $\hat{x}(t)$ is also a gaussian variable, since the Hilbert Transform is a linear operation on x(t). Further we have that x(t) and $\hat{x}(t)$ are statistically independent or

$$\overline{x(t)\hat{x}(t)} = 0 ,$$

whereas

$$\overline{x(t)\hat{x}(t+\tau)} \neq 0.^{3}$$

2.2 Noise Input to Bichannel Equipment.

Since the simplest form of multichannel equipment is the bichannel transmitter the treatment herein is based on such a transmitter. The results obtained can then be extended to cover transmitters with an arbitrary number of channels.

Certain idealizing assumptions have been made in order to simplify the mathematical analysis. It is assumed:

1) that the two sideband filters have a transfer function $H(\omega) = 1$ within the passband, infinite attenuation outside the passband, and a skirtfactor of 1, i. e. vertical flanges.
2) that the effects of mixing stages on the nonlinearities are identical to those of regular bandpass amplifiers.

The output, v(t), of a nonlinear device can be expressed as a function of the input, u(t), in terms of an infinite power series

$$v(t) = a_o + a_1u(t) + a_2u^2(t) + a_3u^3(t) + - -, \quad (2-6)$$

where a_o, a_1, a_2, - - can be obtained by Taylor Series expansion.

It can be shown[1], that only odd order distortion products appear in the output of a bandpass amplifier. Also, for high performance equipment, the ratio (a_i/a_j), $i<j$, between any two coefficients is several orders of magnitude. Therefore, equation (2-6) can be simplified to the form

$$v(t) = a_1u(t) + a_3u^3(t), \quad (2-7)$$

without introducing any significant error.

The mathematical analysis is carried out for the two cases:

(a) statistically independent input signals, and

(b) identical input signals.

The results for the two cases are then compared, to determine which case yields the worst Noise Power Ratio.

2.2.1 Independent Noise Inputs.

The mathematical model of the bichannel transmitter for this case is shown in figure 2-2.

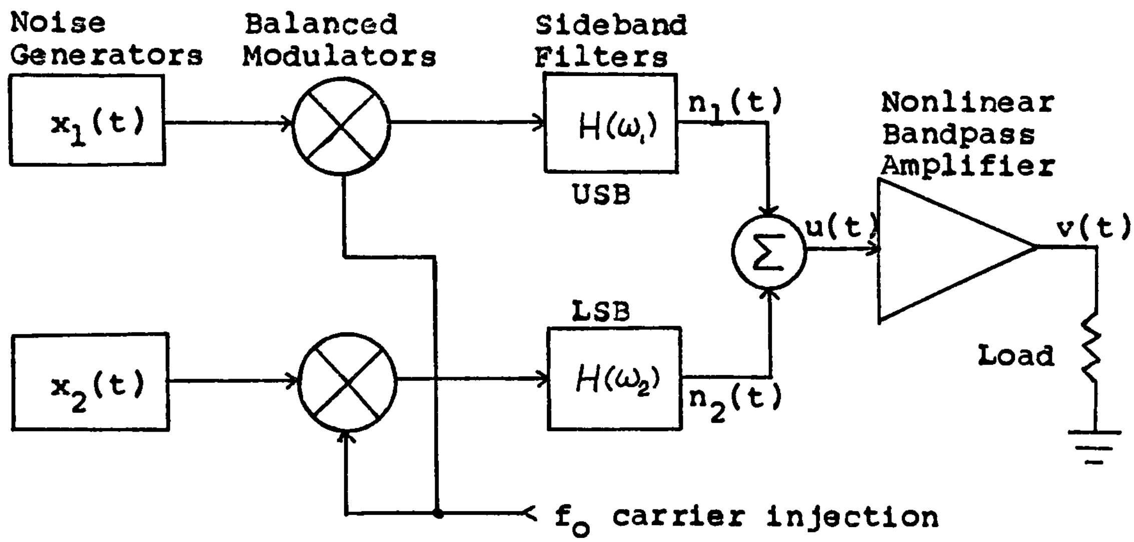

Figure 2-2: Mathematical Model. Independent Case.

The characteristics of the two sideband filters are shown in figure 2-3.

Using equation (2-1), we get

$$n_1(t) = x_1(t) \cos \omega_o t - \hat{x}_1(t) \sin \omega_o t \qquad (2-8)$$

and using equation (2-2), we get

$$n_2(t) = x_2(t)\cos\omega_o t + \hat{x}_2(t)\sin\omega_o t \qquad (2\text{-}9)$$

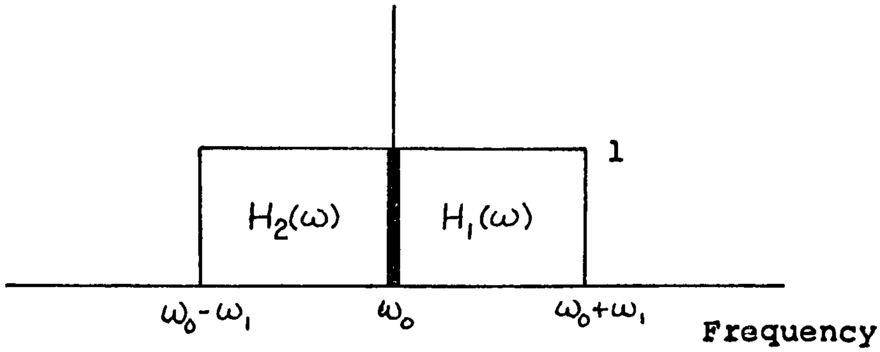

Figure 2-3: Sideband Filter Characteristics.

Since the audio noise generators $x_1(t)$ and $x_2(t)$ are white, gaussian and zero-mean, we also have

$$\overline{n_1(t)} = \overline{n_2(t)} = 0,$$

and by definition the variances are equal:

$$\sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_n^2$$

The amplifier input, u(t), is

$$u(t) = n_1(t) + n_2(t)$$

and the amplifier output, v(t), is from equation (2-7)

$$v(t) = a_1\left[n_1(t) + n_2(t)\right] + a_3\left[n_1(t) + n_2(t)\right]^3,$$

Since it can be shown, that the amplifier output is Ergodic for statistically independent input signals, the Ensemble Average and the Time Average are identical. Considering then the Time Average, the output autocorrelation function is

$$\mathcal{R}(t_1, t_2) = a_1^2 \, \overline{\{[n_1(t_1) + n_2(t_1)][n_1(t_2) + n_2(t_2)]\}}$$

$$+ a_3^2 \, \overline{\{[n_1(t_1) + n_2(t_1)]^3 [n_1(t_2) + n_2(t_2)]^3\}} ,$$

Let $n_{11} = n_1(t_1)$,

$n_{12} = n_1(t_2) = n_1(t_1 + \tau)$,

$n_{21} = n_2(t_1)$, and

and $n_{22} = n_2(t_2) = n_2(t_1 + \tau)$.

Then we obtain

$$\mathcal{R}_V(\tau) = a_1^2 \, \overline{[(n_{11} + n_{21})(n_{12} + n_{22})]}$$

$$+ a_3^2 \, \overline{[(n_{11} + n_{21})^3 (n_{12} + n_{22})^3]} , \qquad (2\text{-}10)$$

$$= a_1^2 \, \overline{[n_{11}n_{12} + n_{11}n_{22} + n_{21}n_{12} + n_{21}n_{22}]}$$

$$+ a_3^2 \, \overline{[(n_{11}^3 + 3n_{11}^2 n_{21} + 3n_{11}n_{21}^2 + n_{21}^3)(n_{12}^3 + 3n_{12}^2 n_{22} + 3n_{12}n_{22}^2 + n_{22}^3)]}$$

and as the average of a sum equals the sum of the averages, this leads us to

$$\mathcal{R}_V(\tau) = a_1^2 [\, \overline{n_{11}n_{12}} + \overline{n_{11}n_{22}} + \overline{n_{21}n_{12}} + \overline{n_{21}n_{22}} \,]$$

$$+ a_3^2 [\, \overline{n_{11}^3 n_{12}^3} + 3\,\overline{n_{11}^3 n_{12}^2 n_{22}} + 3\,\overline{n_{11}^3 n_{12} n_{22}^2}$$

$$+ \overline{n_{11}^3 n_{22}^3} + 3\,\overline{n_{11}^2 n_{12}^3 n_{21}} + 9\,\overline{n_{11}^2 n_{12}^2 n_{21} n_{22}}$$

$$+ 9\,\overline{n_{11}^2 n_{12} n_{21} n_{22}^2} + 3\,\overline{n_{11}^2 n_{21} n_{22}^3} + 3\,\overline{n_{11} n_{12}^3 n_{21}^2}$$

$$+ 9\,\overline{n_{11} n_{12}^2 n_{21}^2 n_{22}} + 9\,\overline{n_{11} n_{12} n_{21}^2 n_{22}^2} + 3\,\overline{n_{11} n_{21}^2 n_{22}^3}$$

$$+ \overline{n_{21}^3 n_{12}^3} + 3\,\overline{n_{12}^2 n_{21}^3 n_{22}} + 3\,\overline{n_{12} n_{21}^3 n_{22}^2} + \overline{n_{21}^3 n_{22}^3} \,] , \qquad (2\text{-}11)$$

The first term on the right hand side of equation (2-11) is the linear part, and is of no further interest in evaluation of the intermodulation distortion. The second term on the right hand side of the equation is the intermodulation part, and it can be further split up into three parts:

$$\begin{aligned}
\mathcal{R}_V'(\tau) = {} & a_3^2\, \overline{n_{11}^3 n_{12}^3} \quad + \quad a_3^2\, \overline{n_{21}^3 n_{22}^3} \\
& a_3^2 \left[3\overline{n_{11}^3 n_{12}^2 n_{22}} + 3\overline{n_{11}^3 n_{12} n_{22}^2} + \overline{n_{11}^3 n_{22}^3} \right. \\
& + 3\overline{n_{11}^2 n_{12}^3 n_{21}} + 9\overline{n_{11}^2 n_{12}^2 n_{21} n_{22}} + 9\overline{n_{11}^2 n_{12} n_{21} n_{22}^2} \\
& + 3\overline{n_{11}^2 n_{21} n_{22}^3} + 3\overline{n_{11} n_{12}^3 n_{21}^2} + 9\overline{n_{11} n_{12}^2 n_{21}^2 n_{22}} \\
& + 9\overline{n_{11} n_{12} n_{21}^2 n_{22}^2} + 3\overline{n_{11} n_{21}^2 n_{22}^3} + 3\overline{n_{12}^2 n_{21}^3 n_{22}} \\
& \left. + 3\overline{n_{12} n_{21}^3 n_{22}^2} + \overline{n_{21}^3 n_{12}^3} \right] ,
\end{aligned} \qquad (2\text{-}12)$$

or

$$\mathcal{R}_V'(\tau) = \mathcal{R}_{n_1 x n_2}(\tau) + \mathcal{R}_{n_2 x n_2}(\tau) + \mathcal{R}_{n_1 x n_2}(\tau) ,$$

where

the $\mathcal{R}_{n_1 x n_1}(\tau)$ term is due to $n_1(t)$ interacting with itself, and it would be present in the output even if $n_1(t)$ alone was applied to the input, and

the $\mathcal{R}_{n_2 x n_2}(\tau)$ term is due to $n_2(t)$ interacting with itself, and it would be present in the output even if $n_2(t)$ alone was applied to the input.

The remaining $\mathcal{R}_{n_1 x n_2}(\tau)$ term is due to interaction between the two noise signals. The individual terms of $\mathcal{R}_{n_1 x n_2}(\tau)$ can then be evaluated separately. (Refer to the appendix for details). Each average can be expanded to a product with factors of the following form:

$$\overline{n_{11}n_{12}}\ ,\ \overline{n_{11}n_{21}}\ ,\ \overline{n_{21}n_{12}}\ ,\ \overline{n_{21}n_{22}}\ \text{and}\ \delta_n^2 .$$

Since $n_1(t)$ and $n_2(t)$ are statistically indepent with zero mean, factors of the form

$$\overline{n_{11}n_{22}}\quad \text{and}\quad \overline{n_{12}n_{21}}$$

equal zero, making several of the products vanish.

Thus the intermodulation term $\mathcal{R}_{n_1xn_2}(\tau)$ takes on the relatively simple form

$$\mathcal{R}_{n_1xn_2}(\tau) = a_3^2\,[\,27\delta_n^4\mathcal{R}_{n_1}(\tau) + 27\delta_n^4\mathcal{R}_{n_2}(\tau) + 18\mathcal{R}_{n_1}^2(\tau)\mathcal{R}_{n_2}(\tau) + 18\mathcal{R}_{n_1}(\tau)\mathcal{R}_{n_2}^2(\tau)\,]\,, \tag{2-13}$$

Taking the Fourier Transform of equation (2-13), the power density spectrum is obtained

$$S_{n_1xn_2}(f) = 27a_3^2\delta_n^4\int_{-\infty}^{\infty}[\mathcal{R}_{n_1}(\tau) + \mathcal{R}_{n_2}(\tau)]e^{-j\omega\tau}\,d\tau + 18\,a_3^2\int_{-\infty}^{\infty}\mathcal{R}_{n_1}(\tau)\mathcal{R}_{n_2}(\tau)[\mathcal{R}_{n_1}(\tau)+\mathcal{R}_{n_2}(\tau)]e^{-j\omega\tau}\,d\tau,$$

which leads to[5]

$$\begin{aligned} S_{n_1xn_2}(f) = {} & 27a_3^2\delta_n^4\,[\,S_{n_1}(f) + S_{n_2}(f)\,] \\ & + 18\,a_3^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S_{n_2}(f'')\,S_{n_1}(f'-f'')\,S_{n_1}(f-f')\,df''df' \\ & + 18a_3^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S_{n_1}(f'')\,S_{n_2}(f'-f'')\,S_{n_2}(f-f')\,df''df' \end{aligned}$$

where $S_{n_1}(f)$ and $S_{n_2}(f)$ are the spectral densities of the input noise signals $n_1(t)$ and $n_2(t)$ respectively.

Equation (2-14) could now be further evaluated by performing the two multiple convolutions of S_{n_1} with S_{n_1} with S_{n_2}, and S_{n_2} with S_{n_2} with S_{n_1} respectively, but first we will consider the case of statistically dependent input signals.

2.2.2 Dependent Noise Inputs.

The mathematical model for the dependent case is shown in figure 2-4.

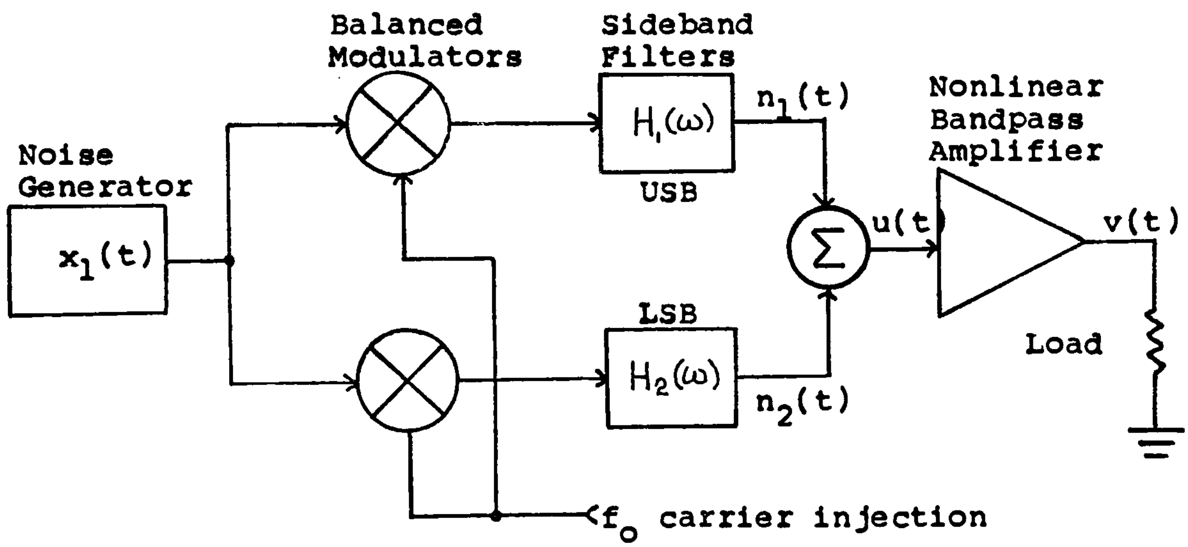

Figure 2-4: Mathematical Model. Dependent Case.

The characteristics of the two sideband filters are again as shown in figure 2-3.

Utilizing equations (2-1) and (2-2), $n_1(t)$ and $n_2(t)$ can be expressed as follows

$$n_1(t) = x_1(t)\cos\omega_o t - \hat{x}_1(t)\sin\omega_o t \qquad (2\text{-}15)$$

and $$n_2(t) = x_1(t)\cos\omega_o t + \hat{x}_1(t)\sin\omega_o t \qquad (2\text{-}16)$$

Further we have, as for the independent case

$$\overline{n_1(t)} = \overline{n_2(t)} = 0$$

and equal variances

$$\delta_{n_1}^2 = \delta_{n_2}^2 = \delta_n^2$$

The input to the amplifier, u(t), is

$$u(t) = n_1(t) + n_2(t)$$

and from equation (2-7), the amplifier output, v(t), is

$$v(t) = a_1 \left[n_1(t) + n_2(t) \right] + a_3 \left[n_1(t) + n_2(t) \right]^3,$$

In this case the output function can be shown not to be Ergodic, but the practical behaviour of the transmitter can be considered to be that of a time averager, due to filter delays, etc., so if we now calculate the time average of the product of the output signals at times t_1 and t_2 by a procedure similar to the one used in the independent case, we obtain the following expression

$$\begin{aligned} \mathcal{R}_v(\tau) = {} & a_1^2 \left[\overline{n_{11}n_{12}} + \overline{n_{11}n_{22}} + \overline{n_{21}n_{12}} + \overline{n_{21}n_{22}} \right] \\ & a_3^2 \left[\overline{n_{11}^3 n_{12}^3} + 3\overline{n_{11}^3 n_{12}^2 n_{22}} + 3\overline{n_{11}^3 n_{12} n_{22}^2} \right. \\ & + \overline{n_{11}^3 n_{22}^3} + 3\overline{n_{11}^2 n_{12}^3 n_{21}} + 9\overline{n_{11}^2 n_{12}^2 n_{21} n_{22}} \\ & + 9\overline{n_{11}^2 n_{12} n_{21} n_{22}^2} + 3\overline{n_{11}^2 n_{21} n_{22}^3} + 3\overline{n_{11} n_{12}^3 n_{21}^2} \\ & + 9\overline{n_{11} n_{12}^2 n_{21}^2 n_{22}} + 9\overline{n_{11} n_{12} n_{21}^2 n_{22}^2} + 3\overline{n_{11} n_{21}^2 n_{22}^3} \\ & \left. + \overline{n_{21}^3 n_{12}^3} + 3\overline{n_{12}^2 n_{21}^3 n_{22}} + 3\overline{n_{12} n_{21}^3 n_{22}^2} + \overline{n_{21}^3 n_{22}^3} \right] \end{aligned}$$

which can be seen to be identical to equation (2-11).

Again, we disregard all terms not due to interaction between the two input noise signals. The procedure of the appendix is again used to evaluate the general terms of $\mathcal{R}_{n_1 x n_2}(\tau)$ where

$$\begin{aligned}
\mathcal{R}_{n_1 x n_2}(\tau) = a_3^2 \Big[& 3\overline{n_{11}^3 n_{12}^2 n_{22}} + 3\overline{n_{11}^3 n_{12} n_{22}^2} + \overline{n_{11}^3 n_{22}^3} \\
& + 3\overline{n_{11}^2 n_{12}^3 n_{21}} + 9\overline{n_{11}^2 n_{12}^2 n_{21} n_{22}} + 9\overline{n_{11}^2 n_{12} n_{21} n_{22}^2} \\
& + 3\overline{n_{11}^2 n_{21} n_{22}^3} + 3\overline{n_{11} n_{12}^3 n_{21}^2} + 9\overline{n_{11} n_{12}^2 n_{21}^2 n_{22}} \\
& + 9\overline{n_{11} n_{12} n_{21}^2 n_{22}^2} + 3\overline{n_{11} n_{21}^2 n_{22}^3} + 3\overline{n_{12}^2 n_{21}^3 n_{22}} \\
& + 3\overline{n_{12} n_{21}^3 n_{22}^2} + \overline{n_{21}^3 n_{12}^3} \Big]
\end{aligned}$$

is recognized as the third part of the right hand side of the equation (2-12). During this evaluation the following terms appear:

$$\overline{n_{11} n_{22}},\ \overline{n_{12} n_{21}},\ \overline{n_{11} n_{21}} \text{ and } \overline{n_{12} n_{22}}.$$

Expressing n_{11}, n_{12}, n_{21} and n_{22} in terms of equations (2-4) and (2-5), we get

$$n_{11} = Re\left\{ z_1(t) e^{j\omega_0 t} \right\}$$

$$n_{12} = Re\left\{ z_1(t+\tau) e^{j\omega(t+\tau)} \right\}$$

$$n_{21} = Re\left\{ z_2(t) e^{j\omega_0 t} \right\}$$

$$n_{22} = Re\left\{ z_2(t+\tau) e^{j\omega_0(t+\tau)} \right\}$$

where $\quad z_1(t) = x_1(t) + j\,\hat{x}_1(t)$

and $\quad z_2(t) = x_1(t) - j\,\hat{x}_1(t).$

We also have the following relation

$$\mathrm{Re}(y_1)\mathrm{Re}(y_2) = \tfrac{1}{2}\mathrm{Re}(y_1 y_2) + \tfrac{1}{2}\mathrm{Re}(y_1^* y_2)$$

where y_1^* is the complex conjugate of y_1.

Then we obtain the product

$$n_{11} n_{22} = \mathrm{Re}\left\{\frac{1}{2} z_1(t) z_2(t+\tau) e^{j\omega_0(2t+\tau)}\right\} + \mathrm{Re}\left\{\frac{1}{2} z_1^*(t) z_2(t+\tau) e^{j\omega_0\tau}\right\}$$

and the time average is

$$\overline{n_{11} n_{22}} = \mathrm{Re}\left\{e^{j\omega_0\tau} \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2} z_1(t) z_2(t+\tau) e^{j\omega_0 2t}\,dt\right\} + \mathrm{Re}\left\{e^{j\omega_0\tau} \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2} z_1^*(t) z_2(t+\tau)\,dt\right\}, \quad (2\text{-}17)$$

but since $z_1(t)$ and $z_2(t)$ are varying slowly compared to $e^{j\omega_0 2t}$, then the first integration on the right hand side of equation (2-17) equals zero[6]. Substituting in the second integral for $z_1(t)$ and $z_2(t)$ we obtain

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2} z_1^*(t) z_2(t+\tau)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2}[x_1(t) - j\hat{x}_1(t)][x_1(t+\tau) - j\hat{x}_1(t+\tau)]\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2} x_1(t) x_1(t+\tau)\,dt - j\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x_1(t)\hat{x}_1(t+\tau)\,dt$$

$$-j\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2}\hat{x}_1(t) x_1(t+\tau)\,dt - \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \hat{x}_1(t)\hat{x}_1(t+\tau)\,dt,$$

In accordance with the definition of the Hilbert Transform, we have:

$$\overline{x_1(t)\,x_1(t+\tau)} = \overline{\hat{x}_1(t)\,\hat{x}_1(t+\tau)}$$

and $$\overline{x_1(t)\,\hat{x}_1(t+\tau)} = -\overline{\hat{x}_1(t)\,x_1(t+\tau)}$$

which means, that also the second integration on the right hand side of equation (2-17) equals zero:

$$Re\left\{e^{j\omega_0\tau} \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{1}{2}\, z_1^*(t)\, z_2(t+\tau)\,dt\right\} = 0 .$$

Therefore

$$\overline{n_{11}\,n_{22}} = 0 .$$

From this it is deduced, that

$$\overline{n_{11}\,n_{21}} = 0 ,$$

$$\overline{n_{12}\,n_{21}} = 0$$

and $$\overline{n_{12}\,n_{22}} = 0 .$$

The remaining terms of $\mathcal{R}_{n_1xn_2}(\tau)$ are found to be

$$\mathcal{R}_{n_1xn_2}(\tau) = a_3^2\,[27\,\sigma_n^4\,\mathcal{R}_{n_1}(\tau) + 27\,\sigma_n^4\,\mathcal{R}_{n_2}(\tau) + 18\,\mathcal{R}_{n_1}^2(\tau)\,\mathcal{R}_{n_2}(\tau) + 18\,\mathcal{R}_{n_1}(\tau)\,\mathcal{R}_{n_2}^2(\tau)\,]$$

which is seen to be identical to the expression obtained for the independent case, as shown in equation (2-13).

It has therefore been proven, that for bichannel equipment, it makes no difference whether the input signals are statistically dependent or not.

2.3 Multichannel Equipment.

In the preceding sections it was shown that the statistical relationship between two input noise signals is of no consequence to the intermodulation distortion on the output of the nonlinear device.

The purpose of this section is to extend this result to an arbitrary number of channels. Figure 2-5 shows the idealized characteristics of a multichannel transmitter of the type shown in figure 1-1. Nominal carriers are included in figure 2-5 for reference only.

Again we will discuss separately the two cases: statistically independent noise inputs and identical noise inputs.

2.3.1 Independent Noise Input.

Let the audio noise generators be designated as follows

x_{1L} and x_{1U},

x_{2L} and x_{2U},

x_{3L} and x_{3U},

- - - - - -

x_{nL} and x_{nU}.

where the first subscript refers to the nominal carrier frequency and the second subscript refers to lower (L) or upper (U) sideband.

Then the various sideband signals applied to the nonlinear device will be

$$n_{KL}(t) = x_{KL}(t)\cos\omega_K t + \hat{x}_{KL}(t)\sin\omega_K t$$

$$n_{KU}(t) = x_{KU}(t)\cos\omega_K t - \hat{x}_{KU}(t)\sin\omega_K t$$

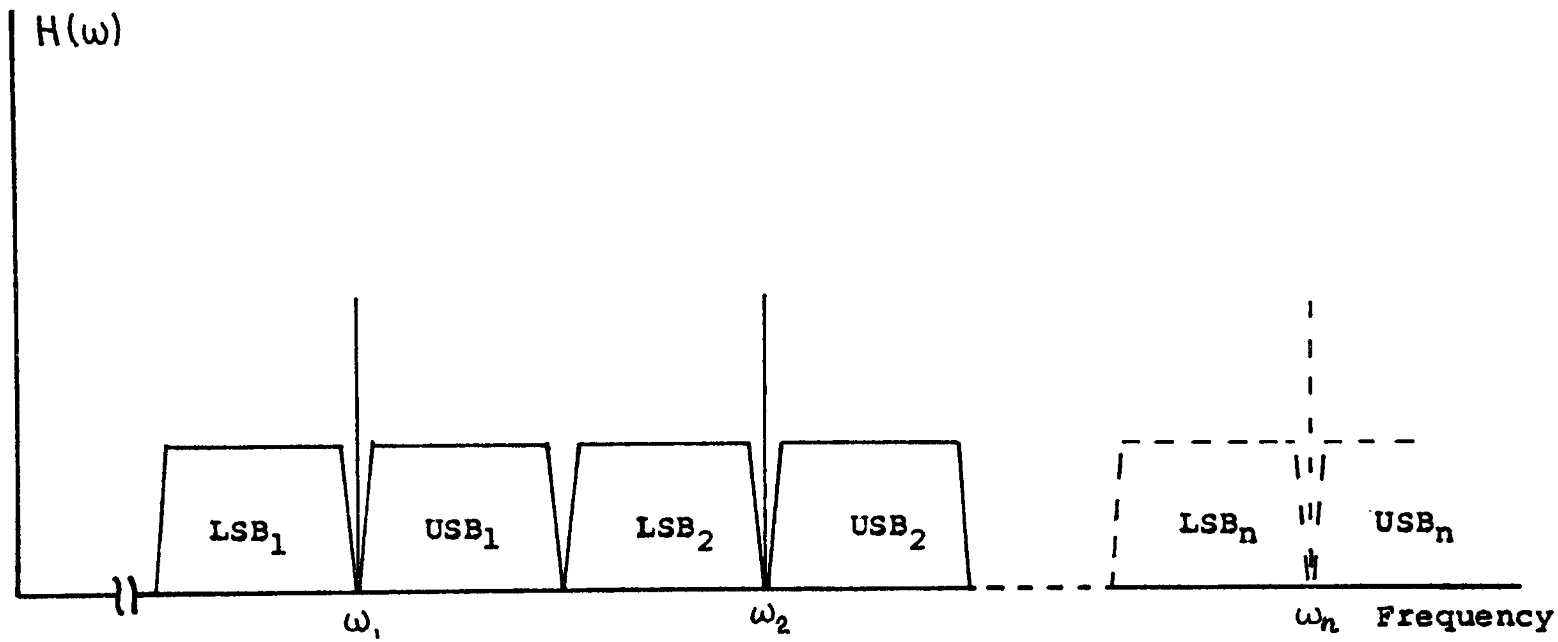

Figure 2-5: Multichannel Filter Characteristics.

where k = 1, 2, 3, - - -.

From the results obtained for the bichannel equipment, we know that the interaction between two channels with the same nominal carrier frequency will result in an intermodulation distortion term of the form

$$\mathcal{R}_{n_{KL} \times n_{KU}}(\tau) = 27 a_3^2 \sigma_n^4 \left[\mathcal{R}_{n_{KL}}(\tau) + \mathcal{R}_{n_{KU}}(\tau)\right] + 18 a_3^2 \mathcal{R}_{n_{KL}}(\tau) \mathcal{R}_{n_{KU}}(\tau)\left[\mathcal{R}_{n_{KL}}(\tau) + \mathcal{R}_{n_{KU}}(\tau)\right],$$

which is obtained by rewriting equation (2-13) for the k-th carrier frequency. The additional autocorrelation terms resulting from interaction between each pair of channels will have products in which the factors are of the general form

$$\overline{n_{i1} n_{j2}},$$

$$\overline{n_{i1} n_{j1}},$$

$$\overline{n_{i2} n_{j2}}$$

and $$\overline{n_{i2} n_{j1}}. \qquad (2\text{-}18)$$

These factors are arrived at by applying the method outlined in the appendix. In equation (2-18) the subscripts i and j refer to the nominal carrier frequency and the 1 and 2 subscripts refer to times t_1 and t_2 respectively.

Since it has been shown in section 2.2.2, that the time average of the product of an upper and a lower sideband signal equals zero, the terms of interest will only be the ones with either two lower sideband signals or two upper sideband signals together. In the trivial case, i = j, the time average is the same whether or not the input noise signals are dependent.

For statistically independent inputs all the terms of (2-18) will be zero. Therefore, if the same holds true for the case of identical inputs, the results for bichannel equipment will in fact be valid for 2k-channel equipment (k = 1, 2, 3, ---).

2.3.2 Dependent Noise Inputs.

If we try to evaluate the first expression of equation (2-18) we shall first have to repeat the equation (2-4) in a more general form

$$n_{i1} = Re\left[z_i(t_1) e^{j\omega_i t_1} \right]$$

and $$n_{j2} = Re\left[z_j(t_2) e^{j\omega_j t_2} \right]$$

where as before

$$z_i(t) = x(t) - j\,\hat{x}(t) = z_j(t),$$

with $z_i(t)$ and $z_j(t)$ being identical due to identical noise inputs. We are also assuming that both narrowband noise signals are the upper sideband.

Utilizing the relation

$$Re(y_1)Re(y_2) = \tfrac{1}{2}Re(y_1 y_2) + \tfrac{1}{2}Re(y_1{}^* y_2)$$

we get

$$n_{i1} n_{j2} = Re\left\{ e^{j\omega_j \tau} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2} z_i(t) z_i(t+\tau) e^{j(\omega_i+\omega_j)t} dt \right\}$$

$$+ Re\left\{ e^{j\omega_j \tau} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2} z_i^*(t) z_j(t+\tau) e^{j(\omega_i-\omega_j)t} dt \right\}, \quad (2\text{-}19)$$

Since, in general, equation (2-19) does not equal zero[6], it can be concluded that the intermodulation distortion will be higher, and in turn the Noise Power Ratio less when identical noise inputs are used as compared to when statistically independent noise inputs are used.

CHAPTER 3: EXPERIMENTAL TEST

A test was performed on a laboratory model of a bichannel exciter in an attempt to verify the results obtained in the theoretical analysis of the previous chapter.

3.1 Test Set-Up.

The experimental set-up used is shown in figure 3-1. The test equipment is all standard type except for the notch filter and the bandpass filter, the characteristics of which are shown in figures 3-2 and 3-3 respectively.

The 4:1 peak to average limiters used in each test channel serve the purpose of establishing some correlation to the old two-tone test method, since the ratio of peak envelope power (PEP) to average power of a two-tone single sideband signal is 4:1. Such a correlation is not required to serve the purpose of this thesis, but is included because it has been made a general requirement for the Noise Power Ratio Test.

The audio amplifier and average power adjustment in the one channel is used to compensate for the insertion loss of the audio notch filter, in order to keep the average noise power constant under both the notch in and the notch out condition. The notch in and the notch out conditions are (as explained in chapter 1, section 2) describing the input noise signal with a narrow band notched away and the complete input noise signal, respectively.

3.2 Test Results.

The test results shown in table 3-1 seem to verify the theoretical conclusion, that the statistical relationship be-

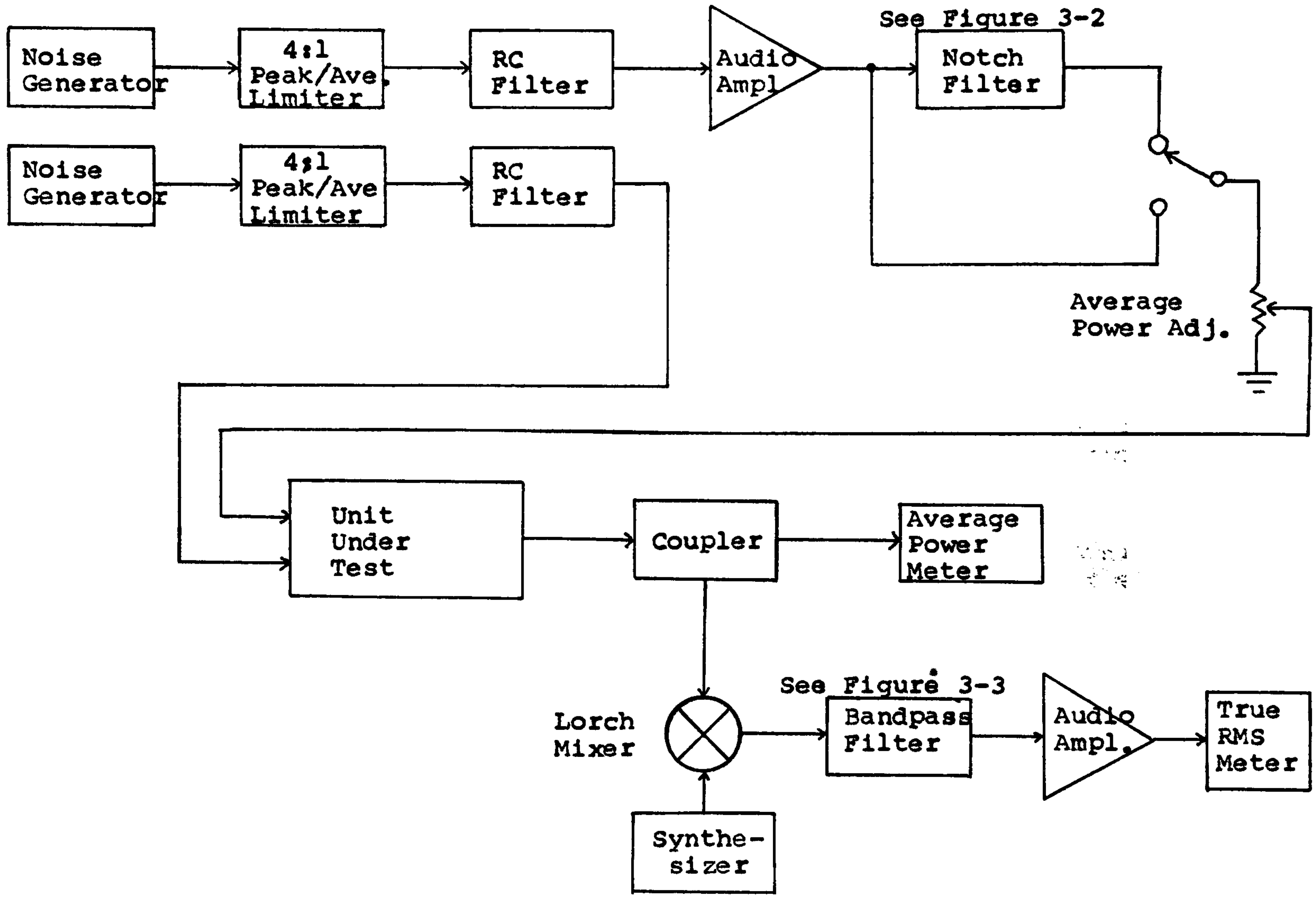

Figure 3-1: Test Set-Up for Experimental Noise Power Ratio Test.

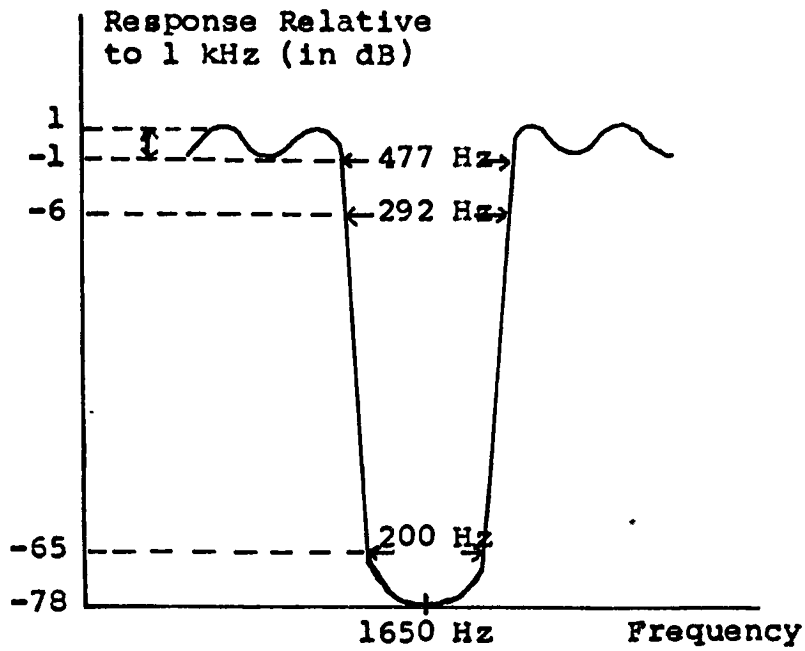

Figure 3-2: Response of Notch Filter.

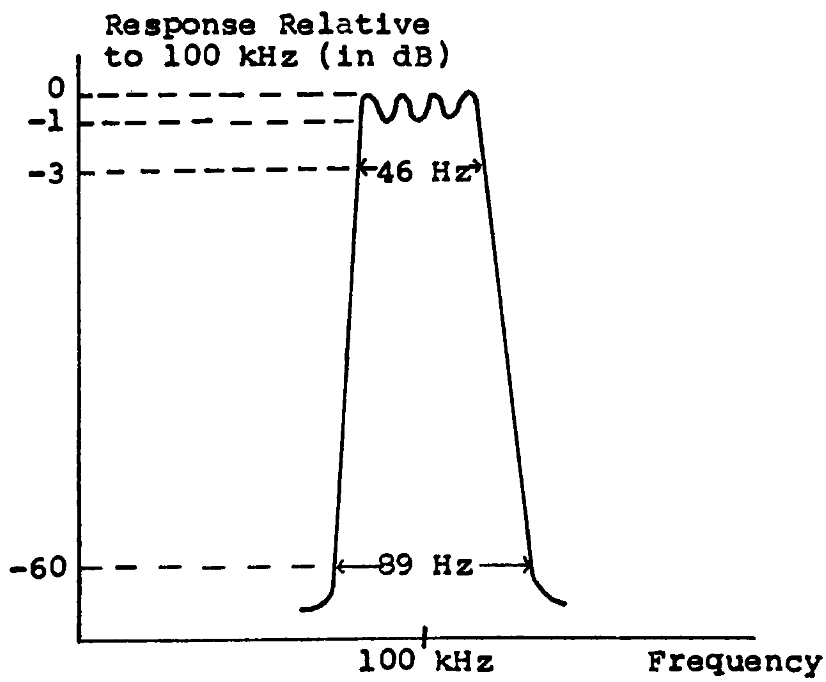

Figure 3-3: Response of Bandpass Filter.

Table 3-1: NPR Test Results.

Relationship Between Input Noise Signals	Audio Input Level	Transmit Output /50 Ohms	True RMS Meter Reading		NPR(SSB)
			Notch Out	Notch In	
Statistically Dependent.					
200 mV Mixer Injection:	10 mV	1.0 V	~ 500 mV	~ 22 mV	27 dB
100 mV Mixer Injection:	10 mV	1.0 V	~ 50 mV	~ 6.3 mV	18 dB
Statistically Independent.					
200 mV Mixer Injection:	10 mV	1.0 V	~ 500 mV	~ 22 mV	27 dB
100 mV Mixer Injection:	10 mV	1.0 V	~ 50 mV	~ 6.3 mV	18 dB

tween the noise inputsdoes not influence the intermodulation distortion of the transmitter output signal and in turn the Noise Power Ratio.

The test was performed using various audio input levels and various injection levels to the mixer stage in the output "frequency selective voltmeter" set-up. None of these variations altered the principal result, but the actual NPR reading was affected. For variations in the audio input level, the change in NPR could be due to harder loading of the amplifier stages, thereby increasing the nonlinearities, and for variations in the mixer injection level, the change in NPR is most probably due to the influence of intrinsic noise.

In order to avoid errors during testing, great care was exercised in keeping the injection levels the same for the two test conditions. However, due to heavy fluctuations of the output meter needle of 2 to 3 db, whenever noise peaks were experienced, an average reading was estimated for 5 to 10 second intervals. This is the only possible area of error in the test procedure. It is however believed that since both readings display fluctuations of the same magnitude, the error would be on the NPR reading and not on the conclusion, that the two readings are identical for dependent and independent noise inputs. It should perhaps be pointed out, that the emphasis during the test was on ratios rather than absolute values.

CHAPTER 4: CONCLUSIONS

It has been shown, that the autocorrelation function of the output signal from a third order nonlinear device contains the same intermodulation distortion terms whether the input is the sum of two statistically dependent, narrowband noise signals, or the sum of two statistically independent, narrowband noise signals. The reason for this is that the time average of the product of the two dependent input noise signals, derived as the lower and upper sidebands from the same audio noise source, equals zero.

The assumption was then made that the time average of the product of any one lower sideband signal and any one upper sideband signal of a multichannel system in which all channels are loaded with identical noise signals, equals zero. This reduces the analysis for multichannel systems to the evaluation of the time average of the product of any two lower sideband signals or of the product of any two upper sideband signals. Since such time averages equal zero for statistically independent input signals the intermodulation distortion is worst for statistically dependent input signals.

As a result, the Noise Power Ratio, as defined in section 1.2, will be lower for statistically dependent noise inputs than for independent inputs. Since we want to measure the performance of the equipment under the most adverse conditions, dependent noise inputs should be used as test signals. This implies that the testing will not require the more expensive and complicated system of independent audio sources, but only the simple and inexpensive system of a single noise source feeding all channels.

A practical measurement performed on a bichannel transmitter verified the theoretical conclusion within a reasonable degree of accuracy. The measurements also show that when Noise Power Ratios are specified for individual systems, exact injection levels should be established and due consideration should be given to interfering factors, such as the influence of automatic power level control loops and of the intrinsic noise power level of the equipment.

APPENDIX

In order to evaluate the general terms of equation (2-12) in section 2.2.1 the following equation from Laning and Battin[7] has been used

$$\overline{X_1^{r_1} X_2^{r_2} X_3^{r_3} X_4^{r_4}} = \frac{r_1! \times r_2! \times r_3! \times r_4!}{2^p \times p!} \sum \left(\overline{X_{i_1} X_{k_1}} \times \overline{X_{i_2} X_{k_2}} \times \cdots \right)$$

where the summation is to be carried out over all sets of indices $(i_1, k_1, \ldots, i_p, k_p)$ such that r_1 of the indices are unity, r_2 of them are two, etc. and where p is defined by the following equation

$$2p = r_1 + r_2 + r_3 + r_4$$

Laning and Battin give a further explanation on how to manage the large number of products resulting from evaluation of this summation. This explanation is the basis of the following manipulations.

There are three types of general terms in equation (2-12)

(a) $\overline{V_1^3 V_2^3}$

(b) $\overline{V_1^3 V_2^2 V_3}$

(c) $\overline{V_1^2 V_2^2 V_3 V_4}$

We will consider these one by one:

(a) $$\overline{V_1^3 V_2^3} = \frac{3! \times 3!}{2^3 \times 3!} \left[a\, \overline{V_1 V_1} \times \overline{V_2 V_2} \times \overline{V_1 V_2} + b\, \overline{V_1 V_2}^{\,3} \right]$$

where the coefficients a and b are determined by the number of possible permutations and the number of combinations in connection with each permutation:

$$a = 2 \times 3! = 12$$
$$b = 2^3 \times 3!/3! = 8$$

so that we get

$$\overline{V_1^3 V_2^3} = 9\sigma_v^4\,\overline{V_1V_2} + 6\,\overline{V_1V_2}^{\,3}.$$

(b)
$$\overline{V_1^3 V_2^2 V_3} = \frac{3! \times 2!}{2^3 \times 3!}\Big[c\,\overline{V_1V_2}^{\,2} \times \overline{V_1V_3} + d\,\overline{V_1V_1} \times \overline{V_1V_2} \times \overline{V_2V_3} + e\,\overline{V_1V_1} \times \overline{V_2V_2} \times \overline{V_1V_3}\Big]$$

where c, d and e are determined as under (a)

$$c = 2^3 \times 3!/2! = 24$$
$$d = 2^2 \times 3! = 24$$
$$e = 2 \times 3! = 12$$

so that this expression is obtained

$$\overline{V_1^3 V_2^2 V_3} = 6\,\overline{V_1V_2}^{\,2} \times \overline{V_1V_3} + 6\sigma_v^2 \times \overline{V_1V_2} \times \overline{V_2V_3} + 3\sigma_v^4 \times \overline{V_1V_3}$$

(c)

$$\overline{V_1^2 V_2^2 V_3 V_4} = \frac{2! \times 2!}{2^3 \times 3!} \Big[f \times \overline{V_1 V_1} \times \overline{V_2 V_2} \times \overline{V_3 V_4}$$

$$+ g \times \overline{V_1 V_1} \times \overline{V_2 V_3} \times \overline{V_2 V_4}$$

$$+ h \times \overline{V_1 V_2}^2 \times \overline{V_3 V_4}$$

$$+ k \times \overline{V_2 V_2} \times \overline{V_1 V_3} \times \overline{V_1 V_4}$$

$$+ l \times \overline{V_1 V_2} \times \overline{V_1 V_3} \times \overline{V_2 V_4}$$

$$+ m \times \overline{V_1 V_2} \times \overline{V_2 V_3} \times \overline{V_1 V_4} \Big],$$

where f, g, h, k, l and m are

$$f = 2 \times 3! = 12$$
$$g = 2^2 \times 3! = 24$$
$$h = 2^3 \times 3!/2! = 24$$
$$k = 2^2 \times 3! = 24$$
$$l = 2^3 \times 3! = 48$$
$$m = 2^3 \times 3! = 48$$

which results in

$$\overline{V_1^2 V_2^2 V_3 V_4} = \sigma_V^4 \times \overline{V_3 V_4} + 2\sigma_V^2 \times \overline{V_2 V_3} \times \overline{V_2 V_4}$$

$$+ 2\sigma_V^2 \times \overline{V_1 V_3} \times \overline{V_1 V_4} + 2\,\overline{V_1 V_2}^2 \times \overline{V_3 V_4}$$

$$+ 4\,\overline{V_1 V_2} \times \overline{V_2 V_3} \times \overline{V_1 V_4}$$

$$+ 4\,\overline{V_1 V_2} \times \overline{V_1 V_3} \times \overline{V_2 V_4} .$$

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