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Application of S-parameter techniques to amplifier design

Frank Sulak

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APPLICATION OF S-PARAMETER TECHNIQUES TO AMPLIFIER DESIGN

by

Frank Sulak

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of MASTER OF SCIENCE in Electrical Engineering

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List of Symbols

- $s_{ij}$: Scattering parameters.
- $K$: Rollett's Stability factor.
- $r_{o1}$: The center of constant gain circle on the input plane.
- $R_{o1}$: The radius of constant gain circle on the input plane.
- $r_{o2}$: The center of constant gain circle on the output plane.
- $R_{o2}$: The radius of constant gain circle on the output plane.
- $r_{s1}$: The center of stability circle on input plane.
- $R_{s1}$: The radius of stability circle on input plane.
- $r_{s2}$: The center of stability circle on output plane.
- $R_{s2}$: The radius of stability circle on output plane.
- $G_{\text{max}}$: Maximum power gain possible.
- $R_{ms}$: Reflection coefficient of that source impedance required to conjugately match the input of the transistor.
- $R_{ml}$: Reflection coefficient of that load impedance required to conjugately match the output of the transistor.
- $G_T$: Transducer power gain.
- $G_P$: Desired total amplifier gain (numeric).
I Abstract

1. Discussion of s parameters and their applicability to high frequency design.


3. Synthesis of high frequency transistor circuitry with the aid of scattering parameter design equations.

4. Verification of design theory by evaluating the performance of bread-board models.
II Introduction

Improved high frequency performance of semiconductor devices has made their use practical into the microwave frequency range. The measurement of commonly accepted amplifier design parameters, such as y, h or z parameters, becomes difficult over 100 MHz due to short and open circuit port termination requirements. Since the scattering, or s parameters, are related to the traveling waves on a transmission line and their measurement can be referenced to the characteristic impedance of the line, their practicality quickly becomes evident to a designer.

Another one of the major advantages is that the matching networks are also measured in terms of s parameters. Thus, once the scattering parameters of both the active and passive circuits are determined, the design of circuitry can proceed in a simplified manner.
III Scattering Parameter Theory

The interest of this author lies in two port amplifier design. Thus, after a brief introduction to n-port scattering matrices, the remaining discussion will be confined to two-port devices only.

Consider the n-port device

where the incident and reflected power waves are defined as

\[
a_i = \frac{V_i + Z_i I_i}{\sqrt{|\text{Re}Z_i|}} \quad \quad \quad \quad \quad b_i = \frac{V_i - Z_i I_i}{\sqrt{|\text{Re}Z_i|}}
\]

\(V_i\) and \(I_i\) are the voltage and current respectively entering the \(i\)-th port, whereas \(Z_i\) is the impedance seen by the \(i\)-th port.

Using Kurokawa's simpler notation throughout the discussion of n-port devices, the power wave vectors can be written:
\[ a = F(v + Gi) \]
\[ b = F(v - G^+i) \]

Where \( F \) and \( G \) are diagonal matrices with \( \frac{1}{2} \sqrt{R_e Z_i} \) and \( Z_i \) being the \( i \)-th component respectively, the \( + \) sign designates a complex transpose matrix.

Since \( a \) and \( b \) are linear transformations of \( v \) and \( i \), and since

\[ v = Z_i \]

there must be a linear relation between \( a \) and \( b \). This is expressed by Kurokawa as

\[ b = sa \]

Now using these relations, the generalized form of scattering matrix can be found. Eliminating \( a \), \( b \), and \( v \) from

\[ a = F(v + Gi) \text{ and} \]
\[ b = F(v - G^+i) \text{ we have} \]
\[ F(Z + G)i = SF(Z - G^+)i \]

which can be arranged as

\[ S = F(Z + G)(Z - G^+)^{-1}F^{-1} \]

after dropping \( i \) and post multiplying both sides by \((Z - G^+)\) and \( F \). In practical design one must consider the behavior of the circuit with any arbitrary source and load impedances. This is equivalent to replacing all \( Z_i \)'s seen by the n-port with arbitrary impedances \( Z_i \)'s, keeping in
mind that there will be a reflection coefficient given as

\[ r_i = \frac{(z_i' - z_i)}{(z_i' + z_i)} \]

As Kurokawa has shown the new set of scattering parameters for this case are

\[ s' = A^{-1} \quad (S = r^{-1}) \quad (I = r^{-1} S)^{-1} A^+ \]

where \( r \) and \( A \) are diagonal matrices with \( r_i \) and \( (1 - r_i^*) \frac{|1 - r_i|^2}{|1 - r_i|} \) being their \( i \)-th diagonal components.

When one substitutes the appropriate values, the two port \( s_{ij} \) parameters can be derived in the following manner:

\[
A = \begin{bmatrix}
(1 - r_i^*) & 1 - r_i^2 & 0 \\
1 - r_i & 0 & (1 - r_2^*) & 1 - r_2^2 & 0 \\
0 & 1 - r_2 & 0 & 1 - r_2 & 0 \\
\end{bmatrix}
\]

\[
s = \begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22} \\
\end{bmatrix}
\]

\[
\Gamma = \begin{bmatrix}
r_1 & 0 \\
0 & r_2 \\
\end{bmatrix}
\]

\[
\Gamma S = \begin{bmatrix}
s_{11} r_1 & s_{12} r_1 \\
s_{21} r_2 & s_{22} r_2 \\
\end{bmatrix}
\]

\[
A^{-1} = \frac{1}{a_{11} a_{22}} \begin{bmatrix}
a_{22} & 0 \\
0 & a_{11} \\
\end{bmatrix}
\]
(S - r^+) = \begin{bmatrix}
(s_{11} - r_1^*) & s_{12} \\
 s_{21} & (s_{22} - r_2^*)
\end{bmatrix} = \begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}

(1 - rS)^{-1} = \frac{1}{(1 - r_1 s_{11})(1 - r_2 s_{22}) - r_1 r_2 s_{12} s_{21}} \begin{bmatrix}
(1 - r_2 s_{22}) & r_1 s_{12} \\
 r_2 s_{21} & (1 - r_1 s_{11})
\end{bmatrix}

(1 - rS)^{-1} = c \begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}

s' = \frac{c}{a_{11} a_{22}} \begin{bmatrix}
a_{22} & 0 \\
0 & a_{11}
\end{bmatrix} \begin{bmatrix}
b_{11} & b_{12} \\
 b_{21} & b_{22}
\end{bmatrix} \begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix} \begin{bmatrix}
a_{11} & 0 \\
0 & a_{22}
\end{bmatrix}

Performing the multiplication we have

s' = \frac{c}{a_{11} a_{22}} \begin{bmatrix}
a_{11} a_{22} (b_{11} c_{11} + b_{12} c_{21}) & a_{22} a_{22} (b_{11} c_{12} + b_{12} c_{22}) \\
a_{11} a_{11} (b_{21} c_{11} + b_{22} c_{21}) & a_{22} a_{11} (b_{21} c_{12} + b_{22} c_{22})
\end{bmatrix}

Substituting the appropriate values for the constants

s' = \begin{bmatrix}
s_1 & s_2 \\
s_{12} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix}

Where
\[ S'_{11} = \frac{a_{11}^* (s_{11} - r_1^*)(1 - r_2 s_{22}) + r_2 s_{12} s_{21}}{a_{11} (1 - r_1 s_{11})(1 - r_2 s_{22}) - r_1 r_2 s_{12} s_{21}} \]

\[ S'_{12} = \frac{a_{22}^* s_{12} (1 - r_1^2)}{a_{11} (1 - r_1 s_{11})(1 - r_2 s_{22}) - r_1 r_2 s_{12} s_{21}} \]

\[ S'_{21} = \frac{a_{11}^* s_{21} (1 - r_2^2)}{a_{22} (1 - r_1 s_{11})(1 - r_2 s_{22}) - r_1 r_2 s_{12} s_{21}} \]

\[ S'_{22} = \frac{a_{22}^* (s_{22} - r_2^*)(1 - r_1 s_{11}) + r_1 s_{12} s_{21}}{a_{22} (1 - r_1 s_{11})(1 - r_2 s_{22}) - r_1 r_2 s_{12} s_{21}} \]
The behavior of a circuit for arbitrary load and source impedances is indicated by the $s'_{11}$ equations. Prevention of oscillation is of great importance in amplifier design. Depending on their ability to resist oscillation, amplifiers are generally separated into two distinct categories: the absolutely stable case, and the potentially unstable case. An amplifier is absolutely stable if all passive source and load impedances will insure oscillation free operation. A conditionally stable amplifier is likely to oscillate if the load and source impedances are not selected with particular care.

Examining the generalized scattering parameter equations of $s'_{11}$ and $s'_{22}$ one can gain insight into the different cases of stability since an oscillation free device has to satisfy the conditions:

$$|s'_{11}| < 1 \quad \text{and} \quad |s'_{22}| < 1$$

Hence, the design equations concerning circuit stability are derived from $s'_{11}$ and $s'_{22}$. A two port, for example, can be conjugately matched simultaneously at both ports if $s'_{11} = 0$ and $s'_{22} = 0$ can be satisfied with a given set of load and generator impedances.

The references given should be consulted for detailed discussion of two port network stability.

The stability and the design equations are expressed in terms of the $s_{1j}$ device parameters. Thus, the first step in amplifier design consists of the evaluation of scattering parameters of a selected semiconductor.
IV Measurement of S Parameters

Figure #1 shows the two port model with voltages, currents, load and generator impedances and power waves.

\[
\begin{align*}
I_1 & \quad Z_1 & \quad S_{11} & \quad S_{12} & \quad V_1 & \quad V_L & \quad Z_2 \\
E_0 & \quad b_2 & \quad V_s & \quad S_{21} & \quad S_{22} & \quad b_1 & \quad I_L
\end{align*}
\]

Figure #1

The linear equations describing the two port are:

\[
\begin{align*}
b_1 &= s_{11} a_1 + s_{12} a_2 \\
b_2 &= s_{21} a_1 + s_{22} a_2
\end{align*}
\]

Thus, the parameters \( s_{11} \), \( s_{12} \), \( s_{21} \), and \( s_{22} \) are:

\[
\begin{align*}
s_{11} &= \left. \frac{b_1}{a_1} \right|_{a_1 = 0} \quad \text{Input reflection coefficient with the output port terminated by a matched load.} \\
&\quad (Z_1 = Z_c \implies a_2 = 0) \\

s_{22} &= \left. \frac{b_2}{a_2} \right|_{a_1 = 0} \quad \text{Output reflection coefficient with the input port terminated by a matched load.} \\
&\quad (Z_1 = Z_c) \\

s_{21} &= \left. \frac{b_2}{a_1} \right|_{a_2 = 0} \quad \text{Forward transmission gain with the output port terminated in a matched load.} \\

s_{12} &= \left. \frac{b_1}{a_2} \right|_{a_1 = 0} \quad \text{Reverse transmission gain with the input port terminated in a matched load.}
\end{align*}
\]
The $s_{ij}$ parameters of a semiconductor thus can be determined over the desired frequency range on the broad band basis using a 50 ohm system. These measurements can be performed in a reasonably simple manner with the use of a carefully constructed test jig. Since the reflection and transmission coefficients of a transistor are to be measured with respect to a 50 ohm system, the construction of three identical lines on the same board is helpful. (See Figure #2)

Line A is straight through 50 ohms reference transmission line. (See Appendix I for a calculation of line width) Line B is short circuited at the center of the board. Transmission line C is cut at the center and appropriately modified to serve as a mount for the device to be measured.

A block diagram showing an $s$ parameter test set up is illustrated in Figure #3.

The basic procedure to determine $s_{11}$ and $s_{22}$ is to measure the magnitude and the phase angle of the incident and reflected voltages. The value of $s_{12}$ and $s_{21}$ is determined by measuring the magnitude and the phase angle of the incident and the transmitted voltages. For the $s_{11}$ and $s_{22}$ measurements, the system is calibrated with the shorted section. The reflected wave at the short circuit is

\[ V_{\text{refl.}} = -V_{\text{incident}} \quad \text{since} \]

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{and} \quad Z_L = 0 \]
Figure #2a and 2b
S-parameter test setup block diagram

Figure #3
The reflected voltage wave measured with the B channel of the vector voltmeter will be in the form

\[ V_{\text{refl.}} = -V_{\text{incident}} \cdot e^{j\beta s} \]

This phase delay of \( \beta s \) is due to the additional path length that the reflected signal must travel. Thus, inserting a line stretcher in the path of the incident wave a delay can be introduced which will compensate for the path difference. Placing the transistor in the line and adjusting the signal source to register a unity reading on channel A gives the magnitude and phase of \( s_{11} \) reflection coefficient on channel B.

The output reflection coefficient \( s_{22} \) can be measured with the same test setup when the transistor jig is reversed, i.e. the output becomes the input and vice versa.

For the measurement of the forward and the reverse transducer gains \( s_{21} \) and \( s_{12} \), the test setup can be calibrated with the through transmission line section of the jig when probe B is in B_2 position. The line stretcher is utilized in the path of the incident wave to compensate for the phase delay of the transmitted wave. After the phase compensation, \( s_{21} \) can be measured when the transistor jig is inserted into the signal path. If the signal source is set to register unity reading on channel A, the magnitude and phase of \( s_{21} \) can be read directly on channel B.

The reverse parameter \( s_{12} \) is measured by exchanging the input and output ports of the transistor jig.
The above outlined technique was employed in the evaluation of some semiconductor devices in the 100 MHz to 400 MHz range. System calibration was done in the middle of the frequency band for both reflected and transmitted waves. This method yielded better than 5° phase accuracy throughout the measured frequency band. Experimental data is listed in Table #1.

Knowing the $s_{ij}$ parameters of a transistor at the desired operating level, one can proceed with the evaluation of the design equations and obtain stability, transducer gain, and matching condition information. (See Table #2) Since these design equations are algebraic combinations of vector quantities, their manual evaluation could be unenlightening and lengthy. To avoid these calculations, a computer program was developed as a design aid. See Appendix #II for the program. This program accepts $s$ parameter data in vector form as they are read from the vector voltmeter. The computer output data is listed in Table #4.

The following design examples are based on the evaluated $s$ parameter data and the applicable computer output data.
<table>
<thead>
<tr>
<th>Transistor: 2N3866</th>
<th>Bias: $V_{CE} = 24\text{v}$ $I_E = 50\text{ma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. MHz.</td>
<td>$</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>100</td>
<td>.265</td>
</tr>
<tr>
<td>150</td>
<td>.260</td>
</tr>
<tr>
<td>200</td>
<td>.255</td>
</tr>
<tr>
<td>250</td>
<td>.245</td>
</tr>
<tr>
<td>300</td>
<td>.24</td>
</tr>
<tr>
<td>350</td>
<td>.24</td>
</tr>
<tr>
<td>400</td>
<td>.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transistor: 2N4429</th>
<th>Bias: $V_{CE} = 24\text{v}$ $I_E = 50\text{ma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. MHz.</td>
<td>$</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>260</td>
<td>.7</td>
</tr>
<tr>
<td>350</td>
<td>.7</td>
</tr>
<tr>
<td>400</td>
<td>.71</td>
</tr>
<tr>
<td>500</td>
<td>.75</td>
</tr>
</tbody>
</table>
Table #2 S-Parameter Design Equations

\[ \begin{align*}
\Delta &= s_{11}s_{22} - s_{12}s_{21} \\
K &= \frac{1 + |\Delta|^2 - |s_{11}|^2 - |s_{22}|^2}{2|s_{12}s_{21}|} \\
B_1 &= 1 + |s_{11}|^2 - |s_{22}|^2 - |\Delta|^2 \\
B_2 &= 1 + |s_{22}|^2 - |s_{11}|^2 - |\Delta|^2 \\
C_1 &= s_{11} - \Delta s_{22} \\
C_2 &= s_{22} - \Delta s_{11} \\
G_{\text{max}} &= \left| \frac{s_{21}}{s_{12}} \right|^2 (K + \sqrt{K - 1}) \text{ If } B > 0 \text{ then the + sign applies} \\
R_{\text{rms}} &= C_1^* \frac{B_1 + \sqrt{B_1^2 - 4|C_1|^2}}{2|C_1|^2} \\
R_{m1} &= C_2^* \frac{B_2 + \sqrt{B_2^2 - 4|C_2|^2}}{2|C_2|^2} \text{ If } B_1 \text{ and } B_2 \text{ are negative, then the + signs apply.} \\
D_1 &= |s_{11}|^2 - |\Delta|^2 \\
D_2 &= |s_{22}|^2 - |\Delta|^2 \\
G_t &= |s_{21}|^2 \\
G &= G_p/G_t \\
r_{s1} &= C_1^*/D_1 \\
R_{s1} &= \left| s_{12}s_{21} \right|/D_1
\end{align*} \]
\[ r_{s2} = \frac{c_2^*}{D_2} \]
\[ R_{s2} = \left| s_{12} s_{21} \right| / D_2 \]
\[ r_{o1} = \frac{G}{1 + D_1 G} c_1^* \]
\[ R_{o1} = \frac{(1 + 2K \left| s_{12} s_{21} \right| G + \left| s_{12} s_{21} \right|^2 G^2)^{1/2}}{1 + D_1 G} \]
\[ r_{o2} = \frac{G}{1 + D_2 G} c_2^* \]
\[ R_{o2} = \frac{(1 + 2K \left| s_{12} s_{21} \right| G + \left| s_{12} s_{21} \right|^2 G^2)^{1/2}}{1 + D_2 G} \]
Design Case #1

A driver amplifier is required for the 170 MHz to 190 MHz frequency range. From the computer output data listed in Table #4, the 2N3866 should provide an absolutely stable operation at 200 MHz \( (K > 1) \) with a maximum gain of 14.8 db when both input and output are conjugately matched. To achieve conjugate match at both input and output, we need

\[
R_{ms} = 0.736 / 178.9^\circ \\
R_{m1} = 0.819 / 23.9^\circ
\]

By plotting these reflection coefficients on an Immittance Chart, one can obtain the required impedances directly from Figure #4.

\[
Z_s \simeq 8 \text{ ohms} \\
Z_1 = (100 + j 190) \text{ ohms}
\]

Assuming that the amplifier is driving a 50 ohms load and is being driven by a 50 ohms source at the same time, we need matching networks to transform 50 ohms to \( R_{ms} \) and \( R_{m1} \) to 50 ohms. Since shunt susceptances or series reactances move along constant admittance or constant impedance circles respectively, an L section will match \( R_{m1} \) to 50 ohms.

From Figure #4.

\[
X_{12} = \frac{1}{12 \times 10^{-3}} = 82 \text{ ohms} \\
L_2 = 70 \text{ nhy} \\
X_{c2} = 140 \text{ ohms} \\
C_2 = 5.5 \text{ pf}
\]
IMMITTANCE CHART

IMPEDEANCE COORDINATES — 50 OHM CHARACTERISTIC IMPEDANCE
ADMITTANCE COORDINATES — 20 MILLIMHO CHARACTERISTIC ADMITTANCE
The input matching can be accomplished with a bifilar wound impedance transformer and a small capacitance in series with the base to tune out the slight amount of inductance introduced by the transformer winding. The final configuration shown below illustrates the complete circuit.

![Circuit Diagram]

\[ R_1 = 3.6 \, \text{k} \Omega \quad C_1 = 5 - 25 \, \text{pf} \]
\[ R_2 = 560 \, \Omega \quad C_2 = 0.8 - 10 \, \text{pf} \]
\[ R_3 = 100 \, \Omega \quad C_3 = 1000 \, \text{pf} \]
\[ T_1 = 4 \text{ turns bifilar} \quad C_4 = 1000 \, \text{pf} \]
\[ L_2 = 70 \, \text{nH} \]

Amplifier performance is illustrated in Figure #5.

If other than maximum gain is required, one can construct constant gain circles and design appropriate matching networks that match source and load to a given gain circle.
Design Case #2

A medium power amplifier is required in the 340 MHz to 390 MHz frequency range. The low priced 2N3866 was investigated again for possible application.

S parameter data and the computer output data from tables 1 and 4 respectively indicate the following:

\[ K < 1 \]

Therefore, \( G_{\text{max}} \) is undefined; i.e. simultaneous complex conjugate matching at both input and output ports would require \( R_{\text{ms}} = R_{\text{ml}} = 1 \). The potentially unstable regions are:

<table>
<thead>
<tr>
<th>Impedances</th>
<th>Angles</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{s1} )</td>
<td>25.08</td>
<td>25.97 ( /-158^\circ )</td>
</tr>
<tr>
<td>( R_{s2} )</td>
<td>31.33</td>
<td>57.95 ( /-145^\circ )</td>
</tr>
<tr>
<td>( R_{s2} )</td>
<td>22.72</td>
<td>23.66 ( /-164^\circ )</td>
</tr>
<tr>
<td>( R_{s2} )</td>
<td>17.33</td>
<td>31.61 ( /-147^\circ )</td>
</tr>
<tr>
<td>( R_{s2} )</td>
<td>45.68</td>
<td>46.63 ( /-170^\circ )</td>
</tr>
<tr>
<td>( R_{s2} )</td>
<td>10.88</td>
<td>18.66 ( /-150^\circ )</td>
</tr>
</tbody>
</table>

Figure #6 shows the unstable regions which occur on the input plane only.

To achieve approximately 15 db amplifier gain between 50 ohms source and 50 ohms load, an attempt was made to match all impedances as nearly as possible to \( s_{11}^* \) of the appropriate stage.
Figure # 6

IMMITANCE CHART

IMPEDANCE COORDINATES — JOHNS CHARACTERISTIC IMPEDANCE
ADMITTANCE COORDINATES — 20 MILLIMHOMES CHARACTERISTIC ADMITTANCE
Amplifier configuration is shown below.

Matching networks can be determined from Figure #6 in the following manner:

\[ X_{L1} = \frac{1}{17 \times 10^{-3}} = 57 \text{ ohms} \]

\[ L_1 = 23 \text{ nhy} \]

\[ X_{C1} = 15 \text{ ohms} \]

\[ C_1 = 30 \text{ pf} \]

\[ X_{L2} = \frac{1}{18 \times 10^{-3}} = 55 \text{ ohms} \]

\[ L_2 = 22 \text{ nhy} \]

\[ X_{C2} = 62 \text{ ohms} \]

\[ C_2 = 7 \text{ pf} \]

\[ X_{L3} = \frac{1}{15 \times 10^{-3}} = 67 \text{ ohms} \]

\[ L_3 = 26 \text{ nhy} \]

\[ X_{C3} = 80 \text{ ohms} \]

\[ C_3 = 5 \text{ pf} \]
The circuit was bread-boarded and tested with the indicated values. Amplifier performance is shown in Figure #7.

Although a single stage 2N3866 amplifier @ 400 MHz did oscillate as was anticipated when being tuned at both ports, the two stage amplifier did not show oscillation any place in the range of tuning elements. This indicates that the $s_{ii}$ parameters of the composite stage must have satisfied the following condition:

$$|s_{ii}| < 1$$
Figure #7 Two Stage Amplifier Frequency Response
Design Case #3

This example is intended to illustrate the utilization of strip-line techniques in the design of high frequency amplifiers. S parameter data is listed in Table #1 for the 2N4429 transistor which is operated with a dc grounded emitter. The grounded emitter operation (R_e = 0) would result in thermal instability, however, this is compensated for with the pnp transistor and the zener diode in the biasing stage.

The list of the computer output in Table #4 contains the desired design information. It was determined by using tuning stubs and s parameter techniques that the amplifier will yield the desired response, illustrated in Figure #10, with the following impedance conditions:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Reflection coefficient presented at the input p_s</th>
<th>Reflection coefficient presented at the output p_l</th>
</tr>
</thead>
<tbody>
<tr>
<td>350 MHz</td>
<td>p_s = .56 /-164°</td>
<td>p_l = .48 / 36°</td>
</tr>
<tr>
<td>400 &quot;</td>
<td>p_s = .56 / 175°</td>
<td>p_l = .38 / 28°</td>
</tr>
</tbody>
</table>

The strip-line matching-element values which will transform the load and source impedances to the above indicated reflection coefficients can now be determined. Depending on the designer's preference, these values can be calculated from the appropriate design equations, or they can be determined by graphical techniques.
Since the mathematical approach is illustrated in the sighted references\textsuperscript{5}, graphical methods will be utilized in this paper.

Output matching.

We want to match the transistor output impedance of $90 + j38$ ohms (See Figure #8) at 400 MHz to a 50 ohm load impedance. A shorted inductive stub parallel with the load will transform the load on the 20 mmho line. The intersection of the constant SWR circle, drawn through the source impedance point on the immittance chart, and the 20 mmho admittance circle determines the proper value of the shunt stub. (See Figure #8) The value of the stub should be $-j17$ mmhos. Starting from the shorted end of the admittance chart, it can be seen that the required stub length is 0.138 of a wavelength. Since

$$\lambda = \frac{300 \times 10^6 \text{ m/sec}}{400 \times 10^6 \text{ Hz/sec}} = 75 \text{ cm/HZ} \quad \text{and} \quad E_R = 2.16$$

$$\lambda(E_R) = 51 \text{ cm}$$

Therefore

$$L_{\text{shunt}} = 7.1 \text{ cm}$$

This new load admittance can be transformed to the source impedance on the constant SWR circle with a 50 ohm transmission line. The proper length of this line is

$$L_{\text{series}} = (0.211 - 0.094) \lambda(E_R)$$

$$= 0.117 \times 51 = 6 \text{ cm}$$
Figure #8

IMMITTANCE CHART

IMPEDANCE COORDINATES — 50 OHM CH.  TERISTIC IMPEDANCE

ADMITTANCE COORDINATES — 20 MILLIMHO CH.  ERISTIC ADMITTANCE

(continued)
Input Matching.

We want to match the 50 ohm source impedance to the 14 + j2 ohms transistor input impedance at 400 MHz. A capacitive parallel stub is chosen to transform the source impedance clockwise on the 20 mmho line. The intersection of the 20 mmho line and the constant SWR circle, drawn through the 14 + j2 ohms point, indicates that the required capacitive susceptance is + j27 mmhos.

Therefore, the length of the open end stub is

\[ L_{\text{parallel}} = (0.398 - 0.250) \lambda (E_R) \]

\[ = 7.58 \text{ cm} \]

The required length of the series 50 ohm transmission line is

\[ L_{\text{series}} = (0.078 + 0.007) \lambda (E_R) \]

\[ = 0.085 \times 51 = 4.33 \text{ cm} \]

The complete circuit is shown in Figure #9.

The circuit was bread-boarded on a 1/32 inch single-sided teflon board. The strip-line was constructed using adhesive copper tape cut to the proper dimensions. Amplifier performance is illustrated in Figure #10.
Figure # 9 Strip-line Amplifier
Figure #10  Strip-line Amplifier Frequency Response
VI Conclusions

The accuracy of measured s parameters is influenced by several factors since their measurements are based on the evaluation of incident and reflected voltage waves. In view of this, the existence of multiple reflections, due to mismatches in the path of the signal, should be avoided in the test set-up. This problem was minimized by the employment of low VSWR bias elements, tee's, connectors, and loads. The directivity and the coupling factor of the dual-directional couplers also influence the measurement accuracy; however, if required, a calibration curve can be made up for the system.

The uniformity and the accuracy of the 50 ohm lines on the test jig, the proper grounding of the shorted reference line, and the quality of the RF connections will also affect s parameter data accuracy. These experimental errors were minimized with the careful construction of the transistor test jigs. With all these influencing factors minimized, measurement accuracies can be held well within engineering design requirements. This is evident from the close correlation of the theoretical and experimental results.

In Design Case #1, a maximum gain of 14.8 db was predicted at 200 MHz. Experimental results yielded 14.6 db at 187 MHz, but response could be tuned to peak at 200 MHz.

G\text{max} is undefined for Design Case #2 because K<1. One could predict approximately 8 db gain per stage at 400 MHz on the basis of design equations. A two stage bread-board model yielded 17.2 db at 400 MHz.
Theoretical and experimental values agree to within 1 db for Design Case #3 with 12 db being the desired gain and 12.8 db the obtained gain at 363 MHz.

The understanding and utilization of s parameter design techniques can lead to predictable and efficient amplifier design. Their importance cannot be over emphasized, and s parameter design techniques should be given equal value with those utilizing y parameters (as described by Linvill\(^7\)). As semiconductor devices are making their way into higher frequency ranges, the work of a designer is being somewhat lessened since data sheets for these devices are now appearing with both s and y parameter values\(^8\). This also indicates the rapid rate of recognition that s parameters are gaining amongst design engineers.
VII References


3. J. Lange, "Microwave Transistor Characterization Including S-Parameters" -- Hewlett-Packard Application Note No. 95.


Appendix I

The characteristic impedance of a microstrip transmission line, which has a single ground plane, can be calculated by an iterative technique. Table #3 lists the microstrip parameter equations. The cross-sectional view with dimensions is indicated in the figure below.

The design steps are as follows:

1. The line is assumed to be completely embedded in a dielectric substrate with dielectric constant $E_R$. The free-space impedance is then

$$Z_{01} = \sqrt{\frac{E_R}{Z_0}}$$

2. The filling fraction $q$ is determined from $Z_{01}$ vs. $q$ graph. (Figure #11)

3. The effective dielectric constant is calculated from

$$E'_R = 1 + q (E_R - 1)$$

4. $Z_{01}$ is recalculated with $E'_R$ replacing $E_R$. Steps 2 through 4 are repeated until $E_R$ in Step 3 is within 1% of the previous value.

5. The shape ratio $\frac{W}{h}$ is read from $Z_{01}$ vs. $\frac{W}{h}$ plot.
For a .063" teflon board, where \( \varepsilon_r = 2.6 \) and the desired \( Z_0 = 50 \) ohms, the calculations proceed:

1. \( Z_{01} = \sqrt{\frac{E'}{E_r} Z_0} = 80.5 \)
2. \( q = .705 \)
3. \( E'_r = 1 + q (E_r - 1) = 2.13 \)
   1a) \( Z_{01} = 1.46 \times 50 = 73 \)
   2a) \( q = .725 \)
   3a) \( E'_r = 1 + .725 (2.6 - 1) = 2.16 \)
1b) \( Z_{01} = 73.5 \)
2b) \( q = .72 \)
3b) \( E'_r = 2.15 \)

The shape ratio from \( Z_{01} \) \( vs. W/h \) graph is 2.8 making the width of the 50 ohm line 0.176".

The microstrip lines of the test jig were constructed in accordance with these calculations by etching technique and were measured to be 50 ± 2 ohms on a Time Delay Reflectometer.
Table #3 -- Microstrip Parameter Equations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic impedance (ohms)</td>
<td>$Z_o = \frac{Z_{01}}{\sqrt{E_R}}$</td>
</tr>
<tr>
<td>Relative effective dielectric constant</td>
<td>$E'_R = 1 + q \left( E_R - 1 \right)$</td>
</tr>
<tr>
<td>Guide wavelength</td>
<td>$\lambda = \lambda_o / E'_R$</td>
</tr>
<tr>
<td>Free-space characteristic impedances (ohms)</td>
<td>$Z_{01}$</td>
</tr>
<tr>
<td>Relative dielectric constant of substrate</td>
<td>$E_R$</td>
</tr>
<tr>
<td>Filling fraction</td>
<td>$q$</td>
</tr>
<tr>
<td>Free-space wavelength</td>
<td>$\lambda_o$</td>
</tr>
<tr>
<td>Effective dielectric constant</td>
<td>$E'_R$</td>
</tr>
</tbody>
</table>
So too > 5.

Free-space impedance ($Z_{01}$) - ohms

Shape Ratio (W/A)

Fig. 11
Appendix II

START

Read:
Freq., Sy
Write:
Freq.

Convert: Degree to Radians, Polar to rectangular coordinates

Compute: Δ, K, C1, C2, B1, B2, D1, D2

Compute: GT
P MISMATCH INPUT,
P MISMATCH OUTPUT

K > 1

B1 > 1

Compute:
Rs1, Rs2
Rs1, Rs2
Compute: \( G_{\text{max}}, \) \( R_{\text{M}} \)

Compute: \( G_{\text{max}}, \) \( R_{\text{M}} \)

\[ BZ \geq 1 \]

Compute: \( R_{\text{ML}} \)

Compute: \( R_{\text{ML}} \)

Compute: \( G_{\text{max}}, \) \( R_{\text{M}} \)

Convert: \( R_{\text{ML}}, R_{\text{M}} \) to polar coordinates

Write: \( K, G_{T}, P_{\text{M input}}, P_{\text{M output}}, G_{\text{max}}, R_{\text{M}}, R_{\text{ML}} \)

CLEAR: \( S_{ij} \) matrix
C AMPLIFIER DESIGN WITH S-PARAMETERS

C A(I) AND AA(I) ARE THE MAGNITUDE AND PHASE ANGLE OF S(I,J)

C WHERE 0)A(I)1 -180)AA(I)180

DIMENSION A(10),AA(10),BR(10),S(20)

REAL A,AA,BR,AK,B1,B2,D1,D2,RS1,RS2,FREQ,GMAX,FRMS,FRMSA,FRML,
1FRM1A,GT,FML,FM2

COMPLEX S,DELC1,C2,CRS1,CRS2,RIN,ROT

READ(5,101)FREQ, (A(I),AA(I),I=1,4)

WRITE(6,102)FREQ

FORMAT(1X,'FREQUENCY = ',F10.0,' MHz')

PI = 3.14159265

DO 105 I = 1,4

BR(I) = 0.0

S(I) = CMPLX(0.0,0.0)

BR(I) = AA(I) * (PI/180.0)

S(I) = CMPLX(A(I)*COS(BR(I)),A(I)*SIN(BR(I)))

105 CONTINUE

C COMPUTE ALL NECESSARY VARIABLES

DEL=S(1)*S(4)-S(2)*S(3)

AK=(1.0+(CABS(DEL))**2-(CABS(S(1)))**2-(CABS(S(4)))**2)/(2.0*
1CABS(S(2)*S(3)))

C1=S(1)-DEL*CONJG(S(4))

C2=S(4)-DEL*CONJG(S(1))
B1=1.0+(CABS(S(1)))**2-(CABS(S(4)))**2-(CABS(DEL))**2
B2=1.0+(CABS(S(5)))**2-(CABS(S(1)))**2-(CABS(DEL))**2
D1=(CABS(S(1)))**2-(CABS(DEL))**2
D2=(CABS(S(2)))**2-(CABS(DEL))**2
GT=10.0*ALOG10(CABS(S(3)))**2
PM1=ABS(10.0*ALOG10(1.0-CABS(S(1)))**2))
PM2=ABS(10.0*ALOG10(1.0-CABS(S(2)))**2))
IF(AK.LE.1.0) GO TO 160
IF(B1.LT.0.0) GO TO 200
GMAX=CABS(S(3)/S(2))*AK-SQRT(AK**2-1.0))
RIN=CONJG(C1)*(B1-SQRT(B1**2-4.0*CABS(C1)**2))/(2.0*CABS(C1)**2)
GO TO 210
200 GMAX=CABS(S(3)/S(2))*(AK+SQRT(AK**2-1.0))
RIN=CONJG(C1)*(B1+SQRT(B1**2-4.0*CABS(C1)**2))/(2.0*CABS(C1)**2)
210 CONTINUE
IF(B2.LT.0.0) GO TO 220
ROT=CONJG(C2)*(B2-SQRT(B2**2-4.0*CABS(C2)**2))/(2.0*CABS(C2)**2)
GO TO 230
220 ROT=CONJG(C2)*(B2+SQRT(B2**2-4.0*CABS(C2)**2))/(2.0*CABS(C2)**2)
230 CONTINUE
GMAX=10.0*ALOG10(GMAX)
RMS=CABS(RIN)
RMSA=57.29578*ATAN2(AIMAG(RIN),REAL(RIN))
RML=CABS(ROT)
RMLA=57.29578*ATAN2(AIMAG(ROT),REAL(ROT))
WRITE(6,250) AK, GT, PM1, PM2

250 FORMAT(1X, 'STABILITY FACTOR K= ', 1PE14.5, ' ', 'TRANSUCER GAIN =
1', 1PE10.3, ' DB', ':', 'MISMATCH LOSS INPUT = ', 1PE10.3, ' DB',
2/ ':', 'MISMATCH LOSS OUTPUT = ', 1PE10.3, ' DB')

WRITE(6,450) GMAX, RMS, RMSA, RML, RMLA

450 FORMAT(1X, 'GMAX = ', 1PE10.3, ' DB', ':', 'RMS = ', 1PE10.3, ' AT AN
1ANGLE OF ', 1PE10.3, ':', 'RML = ', 1PE10.3, ' AT AN ANGLE OF ', 1PE10.33)

151 DO 155 I=1,4
    A(I)=0.0
    AA(I)=0.0
155 CONTINUE

GO TO 100

160 CRS1=CONJG(C1)/D1
    RS1=CABS(S(2)*S(3)/D1)
C CALCULATE MAGNITUDE AND PHASE OF CRS1 AND CRS2
    CRS2=CONJG(C2)/D2
    RS2=CABS(S(2)*S(3)/D2)
    ARS1=CABS(CRS1)
    PH1=57.29578*ATAN2(AIMAG(CRS1), REAL(CRS1))
    ARS2=CABS(CRS2)
    PH2=57.29578*ATAN2(AIMAG(CRS2), REAL(CRS2))

WRITE(6,150) AK, GT, PM1, PM2

43
150 FORMAT(1X,'STABILITY FACTOR K= ',1PE14.5 /* ',TRANSUDER GAIN = \\n1',1PE10.3,' DB','MISMATCH LOSS INPUT = ',1PE10.3,' DB') \\
2/* ',MISMATCH LOSS OUTPUT = ',1PE10.3,' DB') \\
WRITE(6,170) RS1,ARS1,PH1,RS2,ARS2,PH2 \\
170 FORMAT(1X,'RAD1 = ',1PE10.3," DIST1 = ',1PE10.3,' AT AN ANGLE 0 \\
1F ',1PE10.3/','RAD2 = ',1PE10.3," DIST2 = ',1PE10.3,' AT AN \\
2ANGLE OF ',1PE10.3) \\
GO TO 151 \\
180 CONTINUE \\
STOP \\
END
<table>
<thead>
<tr>
<th>Frequency</th>
<th>Stability Factor K</th>
<th>Transducer Gain</th>
<th>Mismatch Loss Input</th>
<th>Mismatch Loss Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 MHz</td>
<td>9.58951E-01</td>
<td>1.784E 01</td>
<td>3.162E-01</td>
<td>1.249E 00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rad1 = 1.931E 02</td>
<td>Dist1 = 1.940E 02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>At an angle of 1.654E 02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rad2 = 5.936E 00</td>
<td>Dist2 = 8.625E 00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>At an angle of -1.610E 02</td>
</tr>
<tr>
<td>150 MHz</td>
<td>1.04527E 00</td>
<td>1.398E 01</td>
<td>3.040E-01</td>
<td>1.249E 00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gmax = 1.724E 01</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rms = 7.523E-01</td>
<td>At an angle of 1.738E 02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rml = 8.268E-01</td>
<td>At an angle of 2.130E 01</td>
</tr>
<tr>
<td>200 MHz</td>
<td>1.05305E 00</td>
<td>1.160E 01</td>
<td>2.920E-01</td>
<td>1.308E 00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gmax = 1.485E 01</td>
<td></td>
</tr>
</tbody>
</table>
RMS = 7.361E-01 AT AN ANGLE OF 1.789E 02
RML = 8.196E-01 AT AN ANGLE OF 2.393E 01
FREQUENCY = 250. MHZ
STABILITY FACTOR K = 1.00660E 00
TRANSDUCER GAIN = 1.010E 01 DB
MISMATCH LOSS INPUT = 2.688E-01 DB
MISMATCH LOSS OUTPUT = 1.432E 00 DB
GMAX = 1.414E 01 DB
RMS = 8.945E-01 AT AN ANGLE OF -1.759E 02
RML = 9.331E-01 AT AN ANGLE OF 2.659E 01
FREQUENCY = 300. MHZ
STABILITY FACTOR K = 9.49056E-01
TRANSDUCER GAIN = 8.787E 00 DB
MISMATCH LOSS INPUT = 2.576E-01 DB
MISMATCH LOSS OUTPUT = 1.634E 00 DB
RAD1 = 4.568E 01 DIST1 = 4.663E 01 AT AN ANGLE OF -1.700E 02
RAD2 = 1.088E 01 DIST2 = 1.866E 01 AT AN ANGLE OF -1.500E 02
FREQUENCY = 350. MHZ
STABILITY FACTOR K = 9.39729E-01
TRANSDUCER GAIN = 7.235E 00 DB
MISMATCH LOSS INPUT = 2.576E-01 DB
MISMATCH LOSS OUTPUT = 1.781E 00 DB
RAD1 = 2.272E 01 DIST1 = 2.366E 01 AT AN ANGLE OF -1.641E 02
RAD2 = 1.733E 01 DIST2 = 3.161E 01 AT AN ANGLE OF -1.473E 02

46
FREQUENCY = 400. MHz

STABILITY FACTOR K = 8.91743E-01
TRANSUDER GAIN = 5.801E 00  DB
MISMATCH LOSS INPUT = 2.576E-01  DB
MISMATCH LOSS OUTPUT = 1.938E 00  DB
RAD1 = 2.508E 01  DIST1 = 2.597E 01  AT AN ANGLE OF -1.580E 02
RAD2 = 3.133E 01  DIST2 = 5.795E 01  AT AN ANGLE OF -1.459E 02

2N4429  Data Card Code  2N4429-50,...53

FREQUENCY = 260. MHz

STABILITY FACTOR K = 1.69296E 00
TRANSUDER GAIN = 8.943E 00  DB
MISMATCH LOSS INPUT = 2.924E 00  DB
MISMATCH LOSS OUTPUT = 3.910E-03  DB
GMAX = 1.484E 01  DB
RMS = 8.035E-01  AT AN ANGLE OF 1.797E 02
RML = 7.242E-01  AT AN ANGLE OF 2.750E 01

FREQUENCY = 350. MHz

STABILITY FACTOR K = 1.31973E 00
TRANSUDER GAIN = 8.131E 00  DB
MISMATCH LOSS INPUT = 2.924E 00  DB
MISMATCH LOSS OUTPUT = 6.954E-03  DB
GMAX = 1.466E 01  DB
RMS = 8.457E-01  AT AN ANGLE OF -1.648E 02
RML = 7.879E-01  AT AN ANGLE OF 3.091E 01
FREQUENCY = 400. MHZ
STABILITY FACTOR K = 1.12946E 00
TRANSDUCER GAIN = 7.044E 00 DB
MISMATCH LOSS INPUT = 3.046E 00 DB
MISMATCH LOSS OUTPUT = 1.087E-02 DB
GMAX = 1.435E 01 DB
RMS = 8.954E-01 AT AN ANGLE OF -1.603E 02
RML = 8.508E-01 AT AN ANGLE OF 3.390E 01
FREQUENCY = 500. MHZ
STABILITY FACTOR K = 8.21120E-01
TRANSDUCER GAIN = 4.861E 00 DB
MISMATCH LOSS INPUT = 3.590E 00 DB
MISMATCH LOSS OUTPUT = 2.133E-02 DB
RAD1 = 2.718E-01 DIST1 = 1.233E 00 AT AN ANGLE OF -1.530E 02
RAD2 = 1.146E 00 DIST2 = 3.220E 00 AT AN ANGLE OF -1.432E 02