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# Modeling the MTF and noise characteristics of complex image formation systems

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SIMG-503

Senior Research

# Modeling The MTF and Noise Characteristics of Complex Image Formation Systems

Final Report

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Center for Imaging Science  
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May 15, 1998

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## Abstract

The intent of this research was to model sensor degradation effects using an image chain applied to a synthetic radiance image. The model is relatively basic so that many types of sensors can be modeled. The sensor effects were applied in the frequency domain by cascading modulation transfer functions (MTF) and phase transfer functions (PTF) from the different stages of the image chain. The stages consist of the optical transfer function due to the aperture effects of the optical system. Noise due to photon arrival, the signal loss due to the transmission of the optics, the MTF due to the detector size, the MTF and PTF due to the Charge Transfer Efficiency (CTE) of the detector/electronics, and the noise associated with the electrical system. Each component is a spatial averager where the input radiance field is smoothed as it travels through each component in the chain. The system is intended to make the synthetic image look real while still preserving the radiometry of the scene. The components were modeled through functions written in the interactive data language (IDL). Results of the component modules show that indeed each component acts as a spatial averager in the system.

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This report is accepted in partial fulfillment of the requirements of the course SIMG-503 Senior Research.

Title: Modeling the MTF and Noise Characteristics of Complex Image Formation Systems

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# Modeling the MTF and Noise Characteristics of Complex Image Formation Systems

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# Modeling the MTF and Noise Characteristics of Complex Image Formation Systems

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## 1 Introduction

Our goal in studying science is to try to explain what occurs in nature by studying and researching various phenomena. To demonstrate or predict what will happen in the physical world, mathematical models are developed. Complex image formation systems are a way of obtaining data about real world events and can be modeled. In the remote sensing community for example, people model the radiometry of a scene given certain characteristics such as time of day, material, weather conditions, etc. Using this information, a synthetic radiometrically correct real world scene can be generated. This means the radiance values for the different objects in the modeled scene will be similar to the radiance values measured in that same real world scene. Other factors that degrade the radiance in a scene include optical diffraction, noise due to photon arrival, and noise from the detector and its electronics.(1) If the previous effects can be modeled, we would have a basis for characterizing current or future complex image formation systems by utilizing these specific system characteristics. This research hypothesized that a suite of modules could be developed to model the various sources of MTF and noise of complex image formation systems. These systems include, some medical imaging systems, conventional and digital camera systems, and remote sensing systems. This research explores the specific case of a remote sensing based system.

## 2 Background

Throughout time we have used our senses to explore and learn about our environment. These senses include sight, touch, smell, taste, and hearing. Probably the most important of these senses is sight. Through our use of sight, we remotely sense the world around us.

### 2.1 Remote Sensing and the Image Chain

Remote sensing is defined as, the field of study associated with extracting information about an object without coming into physical contact with it.(1) Under this definition many things can be considered remote sensing. Vision, astronomy, satellite imaging, most medical imaging, digital imaging, photography, etc. This research looked at remote sensing through a digital imaging, satellite perspective and an image chain approach. By digital imaging it is specifically meant through an electro-optical system. In an image chain there is a sequence of steps or chain of events that leads to a final output.(1) In acquiring data remotely, our data, as the cliché goes, is only as strong as the weakest link. Most imaging chains consist of the components shown in figure 2.1.



[Figure 2.1. Typical Imaging Chain.\(3\)](#)

## 2.2 Synthetic Image Generation

Remote sensing satellite systems can cost anywhere from 50 million dollars to a billion dollars, not to mention the 100 million needed to put it in orbit.(3) Once the satellite is in orbit if there are any flaws, or problems they are difficult if not impossible to correct. The same situation holds true for less expensive aerial remote sensing systems. If there is a problem with any of the equipment you could lose some, if not all of your data, and the cost of flying the aerial system for that day. With the ever decreasing costs of computer systems and there ever increasing performance, computer modeling has become a viable, cost effective alternative. These synthetic image generation (SIG) models, use the same parameters that are used in the satellite or aerial remote sensing systems. SIG systems usually approach the image chain in one of two ways. The first is to model as much of the image chain as possible. The Digital Imaging and Remote Sensing Image Generation (DIRSIG) model follows this approach. It treats the source, atmosphere, target/background, optics, and sensor as links and models their interactions. It attempts to produce synthetic imagery that is radiometrically correct. The other method is to model one link in the chain with the greatest possible detail.

There are several groups that are interested in SIG for various reasons. Sensor designers, algorithm developers, image analysts, trainers, and operators all have things to gain with the use of SIG. Operators of remote sensing equipment for example, are interested in this technology so they can test a variety of parameters to find out if it is worth the cost and the effort required to gather the data they are looking for.(1) (i.e. Can they get the data they need on a cloudy day with the system they will use.) Sensor designers use SIG to evaluate tradeoffs between the various types of image fidelity parameters.(1) SIG models allow researchers to determine what impact one parameter might have on the whole model. This is practically impossible to do under real world conditions.

## 2.3 Analyzing the Image Chain

The various links in the chain each have an impact on the spatial image fidelity of the final image.(1) This means that each part of the chain is a potential source for image degradation. We can evaluate the system by evaluating the modulation transfer function (MTF) for each part of the chain. The system MTF is then the product of these individual MTF's.(1) If we assume that the world is composed of objects that are composed of all spatial frequencies, a perfect system would not attenuate any frequencies. This perfect system would have a MTF of unity.(1)

## 2.4 The Sensor System

An electro-optical sensor system is usually composed of several components. Optomechanical elements, detectors, preamplifier, and conditioning electronics. An optomechanical system is composed of the individual optical components and the mechanical elements that control what the focal plane "sees".(1) The purpose of the optomechanical elements is to collect, focus, and disperse the radiant flux from the source. This energy is then focused on the individual detectors. The preamplifier amplifies the low signals without adding a significant amount of noise to the signal. The conditioning electronics perform the signal conditioning and is done in order for the output signal to span the range needed for ease of recording, analog to digital conversion, or transmission to a data receiving station.(1) Detectors are used to detect the incoming signal. The sensor system can be thought of as its own chain.



Figure 2.2 The Sensor Chain



## 2.5 The Optical System

In the majority of electro-optical systems, reflective optics (mirrors) are used because they have the distinct advantage of reflecting all wavelengths relatively equally in comparison to refractive optics.. The disadvantage of reflective optics is the difficulty in seeing the full aperture because the optical elements are opaque.(1) The reflective optical system then leaves obscurations which affect the MTF of the sensor system, because the optical transfer function (OTF) is determined by the shape of the aperture. The OTF is calculated through the autocorrelation of the pupil function. In the majority of sensor systems, the aperture is circular (figure 2.3).



Figure 2.3. Circular Aperture In most satellite sensors, there is a secondary obscuration.

## 2.6 Detectors

Detectors are characterized by several figures of merit. The first and probably most basic is the spectral response. The spectral response of a detector describes in a relative sense the way in which a detector changes its output in response to a change in the input wavelength.(5) Second is the detector responsivity. The responsivity is a measure of the signal output of a detector due to the radiant flux.(5) The signal-to-noise ratio is also a very important characteristic of a detector, and is a measure of the fidelity or "cleanliness" of a signal pattern.(6) All of these figures of merit are important in understanding how and why a detector attenuates a signal. The detector attenuates the signal because it averages the input signal over the detector size in both the x, and y directions, and because there is noise associated with the electrical system.

## 2.7 Noise

Noise refers to any unwanted signal. There are several different types of noise associated with a detector. First, photons do not arrive at an absolutely constant rate, the rate actually fluctuates slightly. (6) This is called photon noise and can be modeled with a Poisson distribution. The noise associated with the detector is called dark or background noise, and is caused by the vibration of the molecules in the detector. Shot noise is electrical noise that arises from the random manor in which electrons are emitted. The noise that is caused by thermal fluctuations of the electrons is called Johnson noise.

## 3 Theory

The main objective of this research was to develop a suite of modules that could be used to model the various sources of MTF and noise in complex image formation systems. This section is devoted to the remote sensing application of this process. It should be noted that although this describes a remote sensing application in detail, the concepts remain the same for other complex image formation systems. The general procedure that will be used to model the degradations for a sensor system in a remote sensing application can be seen in the following flow chart (figure 3.1).



Figure 3.1. Complex Image Formation System Degradation Model

### 3.1 Noise

The photon noise can be modeled with a Poisson distribution. The input image is the radiance source and the noise is calculated by using a weighting function to determine a localized average for generating a statistical mean.(4)

$$P_n = e^{-\mu} \mu^n / n!$$

where:

- u is the mean and variance of the distribution,
- n is the number of photon arrivals

### 3.2 The Optical Effects

The model then attempts to make a generic correction for degradation of the imagery due to optical diffraction effects based on the autocorrelation of the pupil function. A properly scaled 2-D autocorrelation can be used to model the OTF for an incoherent system.(7) The OTF is the Fourier transform of the point spread function (PSF),  $h[x,y]$  of an incoherent illumination system.(7)

$$H[\xi,\eta] = F\{h[x,y]\}$$

Where:

$h[x,y]$  is the point spread function of the incoherent illumination system,

$H[\xi,\eta]$  is the optical transfer function (OTF).

Several exit pupils will be looked at. They are clear, "half-moon", cassegrain, and hinged (figure 3.2).(4) In this figure white represents the area where the signal passes, and black represents where no signal passes. The cassegrain aperture for example has a circular obscuration in the center of the aperture due to the design of the optics (figure 3.3).



Figure 3.2. Clear, "Half-Moon", Cassegrain, and Hinged Apertures

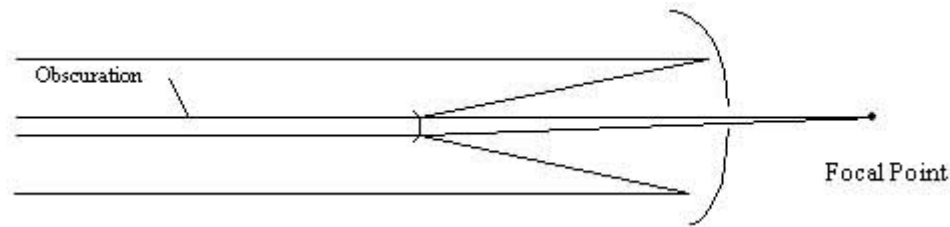


Figure 3.3. Cassegrainian Optical Configuration

Part of the reason for considering the aperture is because it controls the irradiance onto the focal plane. The parameters that are involved in generating these different types of apertures are outside radius, inside radius, and slit width.

### 3.3 Detector MTF

When a detector is used to sample a scene, it samples over finite detector elements which averages the input signal in the x and y directions. This is equivalent to convolution of the 2-D image with a 2-D Rect function, followed by sampling at infinite intervals.[\(8\)](#)

$$f_s(x,y) = [f(x,y) ** |1/bd| \text{Rect}(x/b,y/d)] |1/D_x D_y| \text{Comb}(x/D_x,y/D_y)$$

where:

b and d are the horizontal and vertical widths of the Rect, respectively

$D_x$  and  $D_y$  are the x-direction and y-direction sampling intervals, respectively

$f(x,y)$  is the input image,

$f_s(x,y)$  is the sampled image

The module that was developed asks the user for several parameters. The x and y sizes of the detector element and the x and y fill factor. In some detectors there is an area that is unable to register photons. This results in the detector having an effective area defined by the x and y fill factors.

### 3.4 Charge Transfer Efficiency

A detector absorbs photons at each detector element 'well'. For a CCD to output its values, it must measure the number of photons that were absorbed at each well. This is accomplished by reading the electron count across a horizontal path and then down a vertical path.[\(4\)](#) Not all of the electrons are transferred which results in image degradation if the array transfer efficiency is poor.

### 3.5 Electrical/Detector Noise

There are many sources of noise associated with the electrical system in an image formation system. These sources were discussed in the background section on noise. Another source of degradation in an image formation system is due to the type of material(s) used in the detector. Different materials respond differently to incoming photons.

## 4 Methods And Results

The methods and results sections are combined into one to enhance the readability of this thesis. All modules were written in the Interactive Data Language (IDL).

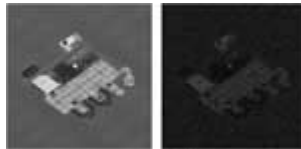
### 4.1 Poisson Methods and Results

The noise associated with the fluctuating arrival rate of photons over time was modeled with a Poisson distribution. The noise is based upon a local brightness and added to the synthetic input image in the spatial domain. The program calls for two inputs: the local blur standard deviation and the percent brightness. The local blur standard deviation is actually the size of the kernel that the user wants to use to determine the average about a pixel. Pixels located around the edges are dealt with by ignoring the area of the kernel that is outside the image boundaries. The percent Brightness factor is used to turn this localized average into a statistical mean, which is then used by the Numerical Recipe's Poisson noise generator.<sup>(4)</sup>



[Figure 4.1 Input image and Poisson Noise Image](#)

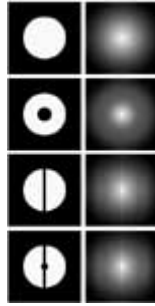
Figure 4.1 shows the original input image and its resulting photon noise image when given the inputs of local blur equal to 5 and a percent brightness of 1%. The input image was a uniformly illuminated image of Gray Value 127. The resulting noise image has a mean of 35.73 and a variance of 31.74. This indicates that the output is indeed Poisson. Analysis of a synthetic image and its resulting noise image further proved the output to be Poisson (Figure 4.2).



[Figure 4.2 Input Scene and Resulting Poisson Noise of Mean 27.37 and Variance 28.82](#)

### 4.2 Aperture Functions and Exit Pupil MTF's

As stated before in the theory the MTF due to the aperture can be calculated by the autocorrelation of the pupil function. This is accomplished by generating the exit pupil and taking the magnitude of the Fourier transform. The magnitude part is then squared and its inverse transform is taken resulting in the MTF due to the aperture function. The inputs for generation of the exit pupils include the radius of the primary, secondary, and slit width. Figure 4.2 shows all of the exit pupils and their corresponding MTF's. All pupil images were 512x512 with the primary radius defined to be 150 pixels, secondary of 25 pixels, while the slits were 10.

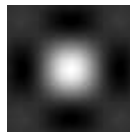


[Figure 4.2 Exit Pupils and Their Corresponding MTF's](#)

All the apertures act like low pass filters attenuating the higher frequency components of the image. Each aperture attenuates those frequencies slightly differently. The cassegrain aperture has a sharper attenuation curve for the low frequency signal than the clear aperture. The half-moon aperture has an asymmetrical MTF that acts like a clear aperture in the vertical direction, but with the horizontal MTF attenuating lower frequencies more severely than the vertical MTF. The hinged aperture is similar to the cassegrain aperture in the vertical direction, but attenuates the lower frequencies more in the horizontal direction.

### 4.3 Detector MTF

The detector MTF can be modeled in the frequency domain by a 2-D sinc function. The user is required to specify the x and y detector element sizes, the x and y fill factors, the horizontal and vertical Fields Of View (FOV), and the effective focal length. The fill factor is the fraction of the detector that is actually able to register photons. Figure 4.3 shows the output from the detector module with the following parameters: FOV = 25°, detector element size of 25mm, focal length of 200mm, and fill factors of 0.95.



[Figure 4.3 MTF Due To The Detector](#)

### 4.4 Charge Transfer Efficiency MTF and PTF

The Charge Transfer Efficiency (CTE) MTF and Phase Transfer Function (PTF) due to a CCD in the horizontal direction as described by LeBlanc and Contini are: [\(10\)](#) where:  $e$  is the transfer efficiency per pixel,  $N$  is the number of charge transfers, and  $xN_{yq}$  is the Nyquist frequency. Since the MTF and PTF are based on the horizontal direction the vertical direction is ignored. In a framing array type of detector, which is what is being modeled, the flow of the charge down the vertical should also be modeled. (A suggestion for future improvement.) The inputs for this module are; the number of detector elements per line ( $N$ ), and the transfer efficiency per pixel. Figure 4.4 shows the output for this module given the input parameters of,  $N$  being 512 while the transfer efficiency was 0.9999.



[Figure 4.4 CTE MTF and the CTE PTF](#)

## 4.5 Detector and Electronic Noise

The detector noise will be a Gaussian white-noise distributed over the image. The detector noise is based on the specific detectivity of the material type. The noise is computed as a Noise-Equivalent Radiance (NER) and is based on the detectivity.<sup>4</sup> The inputs for this module include: the specific detectivity, passband, detector time constant, F#, transmission of the filter, and the transmission of the optics. NER is a function of the Noise-Equivalent Power (NEP), effective detector area, and the optical throughput. The optical throughput or G# as it is more commonly referred to can be calculated with the following equation:

$$G\# = \frac{1 + 4F\#^2}{\tau_o \tau_f \pi}$$

where:

F# is the F-number of the system

$\tau_f$  is the transmission of the filter, and

$\tau_o$  is the transmission of the optics

The NEP is a function of the detector area, detector bandwidth, and the specific detectivity:

$$NEP = \frac{\sqrt{A_{eff}} \sqrt{\Delta f}}{D^*} \text{ [W]}$$

where:

$A_{eff}$  is the detector area

$\Delta f$  is the detector bandwidth computed as  $\frac{1}{2t_d}$  where  $t_d$  is the detector time

constant, and

$D^*$  is the specific detectivity [ $\text{W}^{-1} \text{cm Hz}^{1/2}$ ]

Finally we can compute the NER where:

$$NER = \frac{NEP}{A_{eff}} G\# \text{ [W/m}^2\text{]}$$

The calculated NER is used as the standard deviation for the noise image which results in an uncorrelated pixel-to-pixel Gaussian white-noise distribution as seen in Figure 4.5.<sup>(4)</sup>



[Figure 4.5 Gaussian noise image and Histogram](#)

The noise image in Figure 4.5 was generated with the following parameters;  $f\# = 2.0$ ,  $t_f = 0.9$ ,  $t_o = 0.9$ ,  $t_d = 1.695e-6$ , wavelength region of 3 - 5 mm, and a specific detectivity of  $3.0e13$ . With these parameters the above equations yield a  $G\#$  of 6.68057, Bandwidth of 294912 [Hz], NEP of  $4.299e-16$  [W], and NER of  $5.09e-6$ [W/m<sup>2</sup>]. How much noise was added to the image by the detector? This is computed as the Noise Equivalent Temperature Difference (NETD) where: [K] where  $DL$  is;  $DL = L(T_2) - L(T_1)$ , and  $DT$  is;  $DT = T_2 - T_1$  The NETD for the detector for the above listed parameters was  $6.511e-5$  [K]. Also incorporated into the noise module is the amplifier noise associated with the electronics. The quality of the electronics can be characterized by the Signal-to-Noise Ratio (SNR). The Noise module also allows the user to define a SNR for the electronics package. The model uses the relationship:

The noise is applied after the detector noise and then the above equation is applied to simulate the electronics noise.(4)

## Conclusions

This research hypothesized that a suite of modules could be developed to model the various sources of MTF and noise of complex image formation systems. The modules that were proposed were the exit pupil MTF, the detector MTF, the CTE MTF, and the noise associated with photon arrival, the detector, and the electronics. In the methods and results section it was demonstrated that indeed each of these components could be modeled. The research shows that each component in a complex image formation system has a negative impact on the output. All modules were written in the IDL programming language and have a Graphical User Interface (GUI) associated with them. Although all the modules have been verified to work correctly, much work still needs to be done to complete the project. Currently a GUI needs to be developed for the inputs of each module as well as the code to link all of the modules together so that a final output image can be obtained. At which point a complex imaging system needs to be characterized and modeled so that the real output and modeled output can be compared. The comparison can be done with an MTF analysis. Other suggested improvements include the addition of other modules such as the atmosphere and improvements to the types of detectors that can be modeled because the system is currently limited to a 2-D framing array detector. The more components that are added the more accurate the final output simulation will be. Since the model is based purely on linear systems theory, non-linear systems cannot be modeled which is yet another limitation. Overall, the goals of this research were met, to develop modules that could model the various sources of MTF in a complex imaging system consisting of optics, detector(s), and electronics.

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