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SIMG-503

Senior Research

Algorithm for MTF Estimation by Histogram Modeling of an Edge

Final Report

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Center for Imaging Science
Rochester Institute of Technology
October 20

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I. Abstract

The Modulation Transfer Function, or MTF, is a property of an imaging system that describes the effect that the system has on the sharpness of an object. It is an important image quality metric that has applications in almost every major Imaging Science application. The traditional method of determining the MTF, however, relies on aligning an edge perpendicular to the scan line that will be used to take the measurement. This may not always be a convenient orientation for your experiments. It is hypothesized that there is a relationship between the histogram of the scan line (regardless of its position) and the MTF of the system. This research project will explore this relationship and determine if it will be a useful alternative to the traditional method of calculation.

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This report is accepted in partial fulfillment of the requirements of the course SIMG-503 Senior Research.

Title: Algorithm for MTF Estimation by Histogram Modeling of an Edge.
Author: Brian Perry
Project Advisor: Dr. Jonathan Arney
SIMG 503 Instructor: Anthony Vodacek

III. Acknowledgment

I want to extend a special thanks to Dr Jonathan Arney for his outstanding help and guidance in this Research Project in his role as my Research Advisor. I would also like to thank him for his support and guidance in my studies here at the Center for Imaging Science.

IV. Background

The traditional method of calculating the MTF involves taking an edge scan of an image perpendicular to the edge as illustrated in Figure 1. The scan averages all the pixels in the vertical direction parallel to the edge, and the result is a graph of reflectance versus position as shown in Figure 2.

Figure 1: Scan of an Edge

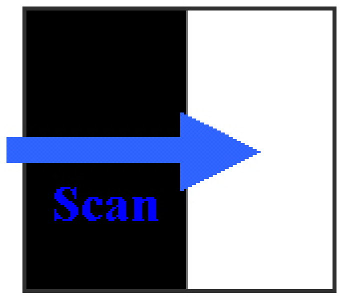


Figure 2: Edge Scan Function

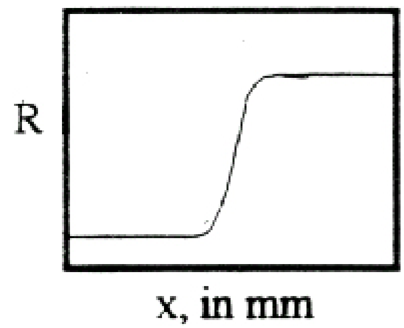


Figure 2 is also referred to as the edge spread function. This function gives a very good description of what the line scan looks like. In this case you can see that the edge doesn’t appear to be an instantaneous transition, but in fact it is a gradual transition from Black through the grays to white. This begins to give us some information about how the imaging system treats edges.

The next step is to take the derivative of the edge scan function.

$$Line\,Spread\;Function(x) = \frac{dR}{dx} \tag{1}$$

The resultant Line Spread Function, illustrated in figure 3, gives us a function that describes how the sharp edge of the original image is spread out by the imaging process.

The line spread function is the probability density function for the location of the edge in the output image. In an imaging system with perfect resolution, this function would be a delta function of zero width.

Figure 3: Line Spread Function

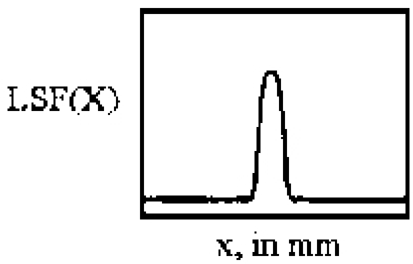
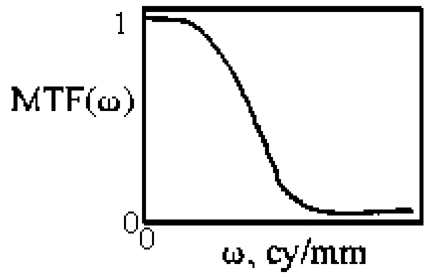


Figure 4: Modulation Transfer Function



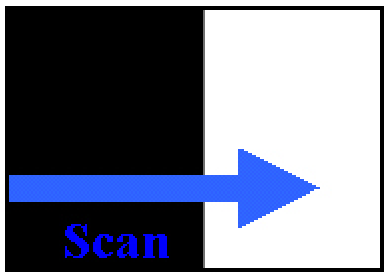
The Modulation Transfer function, MTF(ω), is the line spread function represented in the frequency domain, ω. In this new space the MTF provides us information on what frequencies are attenuated by the imaging system. In order to get from the Line Spread Function to the MTF you have to use a Fourier Transform function as illustrated in equation 2.

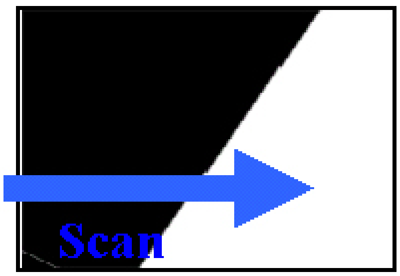
$$\mathrm{MTF}(\omega) = \mathrm{FFT}\{\mathrm{LSF}(x)\} \qquad (2)$$

The value of MTF(ω) in Figure 4 is the fraction of the contrast of the image that is attenuated by the imaging system for image features at each spatial frequency, ω. In this particular graph the low frequencies get through with little attenuation but quickly drop off at higher frequencies. In this way the MTF is an important tool in describing how an imaging system preserves or looses resolution in an image.

V. Experimental Design and Methods

The main problem that this project was designed to solve involves finding the MTF of images in which the edge is not perfectly perpendicular.





If the MTF were taken of these two images, one would find that the MTF of the second image appears lower, even though both images are from the same system and therefore must have the same MTF. The challenge is to find a technique that will produce the same MTF regardless of the orientation of the edge. One might, for example, try to develop a way to locate the perpendicular direction and perform a slant scan, or to adjust the image to make the edge perpendicular to the direction of the scan. The approach taken in this project was to abandon the scan process and to show that the edge scan function, R(x) vs x, can be calculated directly from the gray level histogram of the image regardless of the orientation of the edge.

Traditional wisdom in imaging science holds that the gray level histogram of an image contains no spatial information. This, however, is not entirely true. The histogram by itself is not sufficient to give spatial information, but with additional information about the spatial characteristics of an image, it is often possible to extract quantitative spatial information from the histogram. The additional information needed to constrain the histogram and allow calculation of spatial information is simple in this experiment.

The additional information is that the image is an edge. This means the gray values increase monotonically across the image in the direction perpendicular to the edge.

Figures 5 and 6 demonstrate how the histograms of a sharp and a smooth edge must differ. The gray levels increase in the histogram from left to right. This also represents a monotonic spatial motion across the edge in the perfectly orthogonal scan direction. The differences between the histograms of the sharp and soft edges is clearly shown in Figures 5 and 6. There are far fewer mid-tone gray values in the sharp edge. Indeed, as will be shown below, the shape of the mid-tone gray portion of the histogram is a direct measure of the edge sharpness and the line spread function.

Figure 5: Histogram of a Sharp Edge

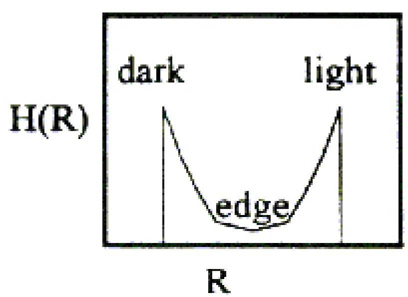
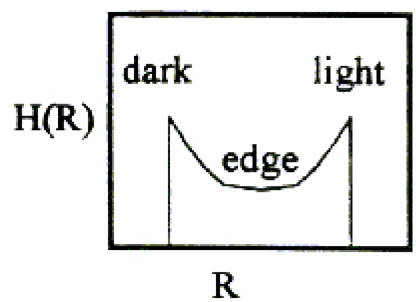


Figure 6: Histogram of a Soft Edge



The histogram of an edge image can be interpreted as being proportional to the rate of change of position with respect to the change in R.

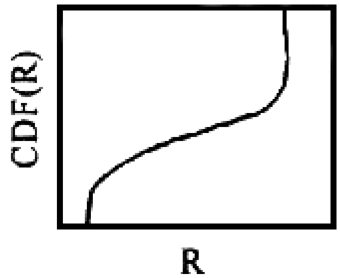
$$Histogram(R) = \frac{dx}{dR} \tag{3}$$

By integrating the Histogram we find the Cumulative Distribution Function.

$$CDF(R) = \int_{R=0}^{R} Histogram(R)dR \tag{4}$$

A CDF(R) function is illustrated in Figure 7.

Figure 7: The Cumulative Distribution Function



The CDF(R) function is the location along the edge according to equation (5), and the inverse of Figure 7 is the edge function, R(x) versus position, x.

$$x(R) = CDF(R) \tag{5}$$

Experimentally, we rotate Figure 7 ninety degrees clockwise and obtain the line spread function of Figure 2. This is the line scan function for x as the scan direction that is orthogonal to the edge. We should then be able to derive the MTF using the steps outline in the Background section of this paper.

VI. Results

We designed 7 different geometries to test our theory. All of the images contained approximately the same amount of light and dark space. Figure 8 shows the 7 different targets: Single Vertical Edge, Double Vertical Edge, Single Horizontal Edge, Double Horizontal Edge, Single Diagonal Edge, Light Circle on a Dark Background and Dark Circle on a light background.

Figure 8: The different test images

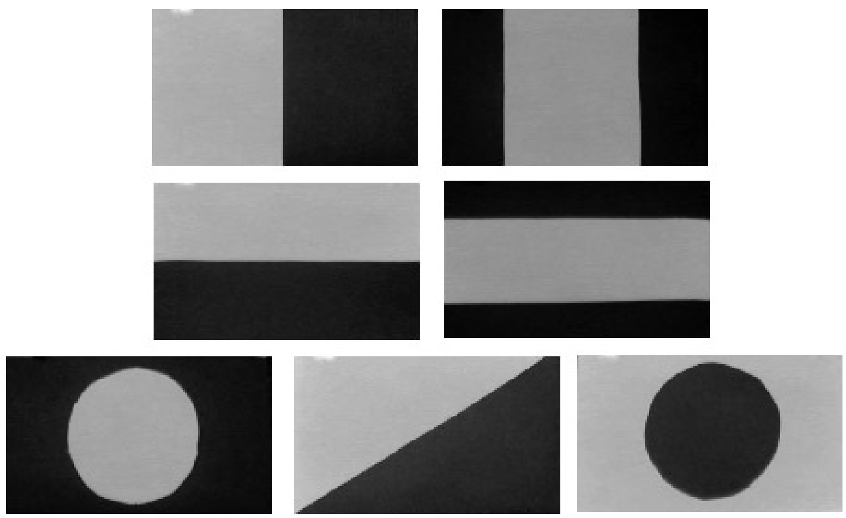


Figure 9 shows the results of our histogram analysis for the single vertical edge. Our histogram method produced line (A), and line (B) was produced by the traditional method. This shows that the histogram method produces a remarkably similar result to the traditional edge method. We would expect this to happen considering that our theory is based on the assumption that the image is increasing monotonically in the horizontal direction.

Figure 9: Comparison of the histogram analysis, Line (A), with the traditional edge analysis, Line (B). Responsivity is the MTF for the traditional edge analysis, Line (B), but it is the response function F(ω) for the histogram analysis technique. The frequency at 1/2 response is the same for both.

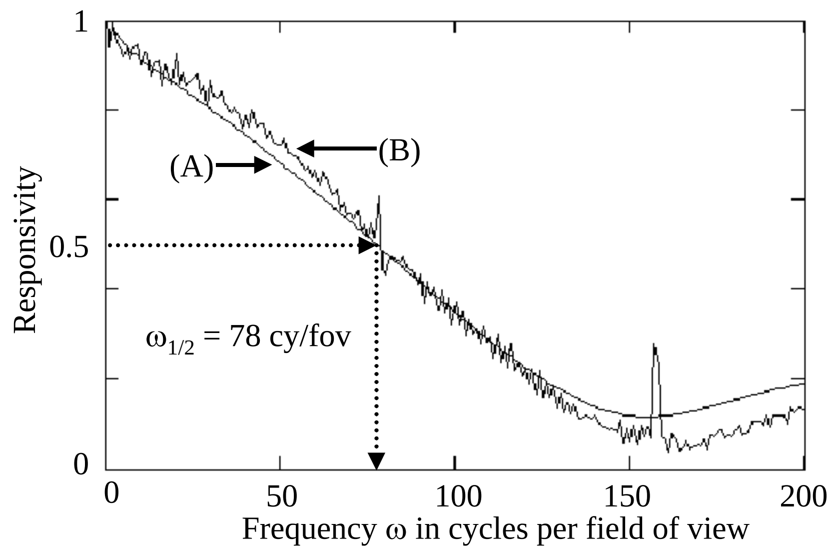


Figure 10: The MTF for the traditional edge scan technique (a), and the response functions for the histogram technique applied the images in Figure 1. Lines (b) through (h) are respectively the single vertical, double vertical, diagonal, single horizontal, double horizontal, dark circle, and light circle.

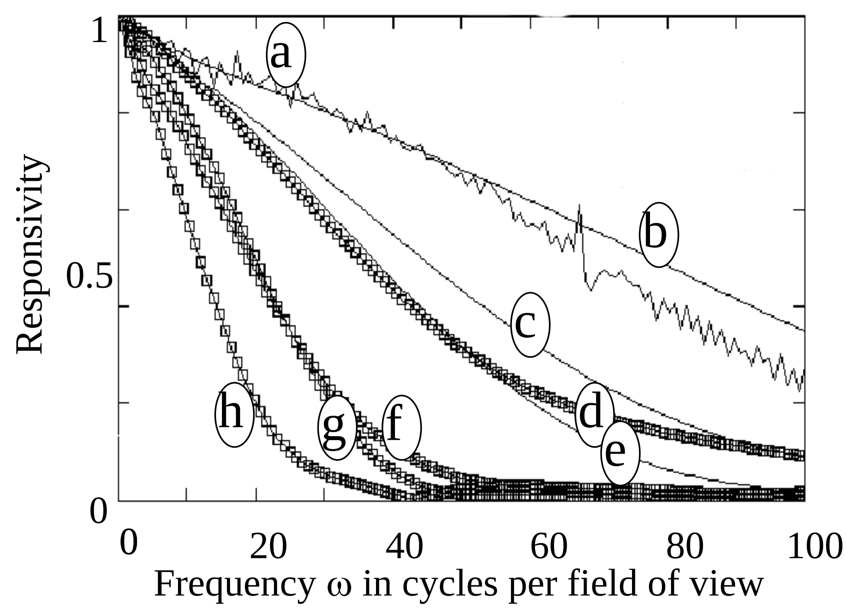


Figure 11 shows the results of the histogram analysis applied to all of the test edges of Figure 8, and clearly the nature of the edge has a significant effect on the results of the analysis. Figure 11 indicates that as the length of the edge increases, the response curve shifts to lower frequency. Table I summarizes the effect of edge length on the wavelength (1/frequency) at a response of 0.5.

Table I: Summary of edge length and 50% wavelength (L and $\lambda_{1/2}$) for the images in Figure 8.

Type of Edge Image	**Edge Length (pixels)**	**λ_{half} (mm)**
One Vertical	374	7.0
Two Vertical	635	16.2
One Horizontal	748	15.5
Two Horizontal	1270	45.4
One Diagonal	737	12.5
Dark Circle	1000	28.9
White Circle	1000	28.9

Figure 11: Wavelength at 50% response, $\lambda_{1/2}$, versus a power function of the edge length, $K{\cdot}L^p$, from the data in Table I.

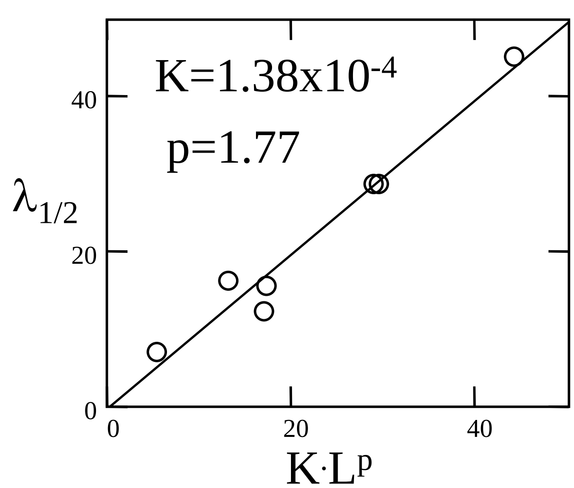


Figure 11 illustrates the data in Table I. Empirically it was found that the data fit well with equation (6) with constants K = 0.000138 and p = $\sqrt{\pi}$.

$$L = K \cdot (\lambda_{1/2})^p \tag{6}$$

Using this equation, we can derive a general expression for converting the histogram response function, F(ω), to an estimate of the MTF(ω) function. We do this by assuming, based on experimental evidence, that F(ω) = MTF(ω) for the single vertical edge of length L_o. Then it can be shown from equation (6) that the MTF(ω) function is given by equation (7).

$$MTF(\omega) = F\left[\varpi \cdot \left(\frac{L_o}{L}\right)^P\right] \tag{7}$$

The advantage of using a test target with multiple edges is in the increased data available for estimation of the shape of the MTF. By using equation (7), the multiple edge target can be used with the histogram analysis technique to estimate the MTF. However, it is important to recognize that equation (7) is an empirical expression found by fitting experimental data. It would be very useful to derive a justification for equation (7) from fundamental theory. This, however, remains to be done.

VII. Discussion and Conclusion

In this experiment we demonstrated that it is possible to arrive at the modulation transfer function by having only a histogram and an assumption about image composition. We know that this technique is not sensitive to small errors in the edge orientation. We were able to derive an expression to describe the relationship between wavelength and edge length in this set of images. Our data can be calibrated to a wide range of edge types ranging from a single vertical edge to a circle. In addition, this project has exposed some issues that deserve more research attention.

- **The Theoretical Origin of Equation 6**

We are still attempting to understand the theory that dictates equation 6. The constant K depends on the experimental field of view and must be determined by calibration every time the instrument is set up. It seems logical that the power p should be a universal characteristic, and we hope that future research will help to derive the theory that gives us the value of p.

- **Noise**

Experimental noise seems to manifests itself differently in this analysis, and we still don't have a full evaluation of the effect of noise. The histogram analysis technique assumes that gray level changes monotonically with location orthogonal to the edge. Random fluctuations in gray level are treated experimentally as monotonic gray level change. This manifests itself in a slope to line spread function, R versus CDF(R), before and after the location of the edge. Presumably it will be possible to correct for such random noise by a simple slope adjustment, but further research will be needed to fully develop this analytical technique of MTF estimation.

- **What are the Boundaries of this Technique**

We know already that there will be limitations to this technique. For example, if the edges are enhanced in a way that results in a non-monotonic change in gray level with location, this technique will incorrectly reconstruct the MTF. We need to know what other limits may exist in this analysis.

Although a number of uncertainties remain regarding the extraction of spatial information form the histogram, this project has demonstrated that histograms, with limits, do have the potential of providing useful spatial information. Thus, the major contribution of this project is to demonstrate that additional research in this area would be worthwhile.

VII. References

Arney, J.S and Wong, Yat-Ming, “Histogram Analysis of the Microstructure of Halftone Images.” IS&T’s 1998 PICS Conference

Some images taken from a MathCad document prepared by J.S Arney