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Increasing Visual Detectability in Reduced Exposure Mammography by Contrast Adjustment

Stuart Richer

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INCREASING VISUAL DETECTABILITY
IN REDUCED EXPOSURE MAMMOGRAPHY
BY CONTRAST ADJUSTMENT

by

Stuart P. Richer

A thesis submitted in partial fulfillment of the requirements for the Bachelor of Science Degree in Photographic Science and Instrumentation at the Rochester Institute of Technology, Rochester, New York

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Thesis Advisor: Professor Gerhard Schumann
ABSTRACT

The radiobiologic level from an average direct-exposure, fine-grain mammograph approaches 4000 millirads. Copying reduced-exposure mammographs onto high gradient films yields a method of reducing radiation to the patient by a factor of 2. A mathematically based tone-reproduction system was developed in an attempt to produce acceptable facsimile images from a series of reduced-exposure mammographs. Acceptability was tested by radiologists using a sensitometric phantom. The radiologists were asked to quality rank a full exposure mammograph, facsimile mammographs, and a film-screen mammograph in terms of detectability of size and number of calcifications. Based on evidence, there is reason to believe that a facsimile mammograph produced with 50 percent normal radiation is diagnostically equivalent to a film/screen mammograph. However, even with this reduction in photographic and radiobiologic exposure, film/screen combination mammographs result in approximately 500 millirads, thus making them superior in terms of patient irradiation.
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Introduction

The first step in mammography, the radiographic analysis of the breast, is the production of a high quality image with a minimum of exposure. An x-ray exposure may be carcinogenic and it is mandatory to avoid any unnecessary irradiation. In this thesis, a system approach will be presented which attempts to produce high quality mammographs with minimum radiation. The aim of the experimental work was to produce facsimile images of a diagnostically acceptable mammography by copying reduced exposure mammographs on high gradient films.

Theory

It is known that net density of direct x-ray films is an approximately linear function of exposure. That is, the lack of a toe in a D vs. H plot gives the material the characteristic of recording information linearly, although not always at contrast levels perceptible to human vision.

Let \( \bar{\epsilon} \) be the average thickness of an area of the breast, and \( t = \bar{\epsilon} + \epsilon \) a spot in this area slightly deviating in thickness. Then the exposures on the film are:

\[
\begin{align*}
\bar{H} &= H_0 e^{-k\bar{\epsilon}} \\
H &= H_0 e^{-kt} \\
\text{let } \Delta H &= H - H_0 = H_0 (e^{kt} - e^{-k\bar{\epsilon}}) \\
&= H_0 e^{-k\bar{\epsilon}} (e^{-kt} - 1) \\
&= H_0 (e^{-k(t-\bar{\epsilon})} - 1)
\end{align*}
\]

let \( \Delta t = t - \bar{\epsilon} \)
\[ H \left( e^{-K\Delta t} - 1 \right) \approx H K \Delta t \quad \text{for} \quad K \Delta t \ll 1 \]

Now let the density, \( D \), be proportional to \( H \). Then:

\[ \Delta D = a\Delta H = a H K \Delta t \quad a = \text{constant} \]

If the total exposure \( H_\sigma \), and hence \( H \) are reduced by a factor \( b \), the contrast \( \Delta D \) is also reduced, and we have:

\[ \Delta^*D = \frac{a}{b} H K \Delta t \]

The resulting density differences in a reduced exposure mammograph may be diagnostically important, but due to the limitation of the human eye, not directly perceptible. A means of recovering the original contrast is copying on a suitable material of high gradient. The question is whether or not this procedure yields diagnostically acceptable images or whether degrading influences are seen?

Medical Considerations

Mammographs are viewed on a diffuse illuminator, often with the aid of a magnifying glass. Processing streaks, dust or the film grain itself is upsetting to the radiologist who must make a diagnostic decision by viewing the image.
The radiologist is looking for the following abnormalities in a mammograph:

1. Clusters of irregularly-bordered specks of CaCO$_3$ which are precursors of the disease.

2. Retraction of peripheral regions of the skin caused by a tightening effect of cancer.

3. Areas of increasing tissue density that would indicate a tumor.

4. Enlarged blood vessels.

In addition, a malignant breast will be denser throughout compared to the unaffected breast. Therefore, an important technique the radiologist uses is side by side comparison of both breasts as shown in Figure 1.

The tiny irregularly-bordered specks mentioned above occur roughly in one third of all cases. The size of these specks is usually less than one millimeter in diameter. It is important to note that calcium occurs naturally in the breast as well. The bigger and denser, the more circular and/or elliptical these specks appear, the less likely they are to be signs of cancer.$^2$

This discussion illustrates the expert photointerpretive abilities needed by the radiologist and the demands that are placed on the system he chooses to use.
Figure 1
Clinical Mammograph With Carcinoma Kodak Type AA X-Ray Film
Radiographic Techniques

Double-sided, fine grain, direct exposure x-ray films provide the best image quality but are susceptible to unsharpness caused by patient movement during long exposure. Due to the reciprocal relationship between grain size and speed, the limiting factor in this method is high patient dosage. A typical direct-exposure mammograph using Kodak type AA film at 34.3 cm. focal-skin distance (F.S.D.) with 0.4 mm aluminum filtration is: 26 Kv; 8 ma; 6.4 sec. This corresponds to 3888 millirads of radiation.  

Single-sided emulsion/screen combinations allow use of lower Kv levels with short exposure times: 25 Kv; 4 ma; 2 sec.; 34.3 cm. F.S.D.; 0.4 mm aluminum filtration. This corresponds to 536 millirads of radiation. There is less motion unsharpness due to lower exposure times. High definition screens with resolution in excess of 10 lp/mm are providing adequate radiographic quality. However, an element of "noise" or radiographic mottle inherent in all radiographs is accentuated when screens are used. Fine grain and screen structure are insignificant when compared to quantum mottle. This is an inherent problem in reduced exposure radiography due to the effect of local variations in the distribution of the x-ray quanta absorbed by the screen. Also of importance is the maintaining of a close film/screen contact and removal of potential screen artifacts.
Xeroradiography involves the recording of mammographs on a charged selenium plate. Xeroradiography's edge enhancement allows easier visualization of calcifications. The process also has a wider lattitude. The sensitivity of the selenium plate increases with increases in tube Kv over the range used in mammography. A typical exposure is high, being 40 Kv; 5 ma; 3 sec.; 1 mm Al filtration. This corresponds to 8064 millirads of radiation. The sensitivity can be increased by using a lower photon energy or increased thickness of the selenium plate which is difficult to do. Hevezi and Harle have increased the sensitivity by a factor of 2 by using a high atomic gas between the charged Se surface and the conductive subsurface of the xerographic cassette.

In an effort to lower patient exposure to radiation, many radiologists are using single-sided emulsion/screen combinations such as Dupont Low Dose or Kodak Minar. However, double-sided, fine grain films still provide best image quality. Producing a facsimile fine grain mammograph from a reduced exposure double-sided fine grain film might be a means of producing high quality images with minimum irradiation.
EXPERIMENTAL

1. Determination of crucial information regions of a typical direct-exposure mammograph.

2. Determination of ma, Kv, F.S.D., filtering and exposure time to produce a typical mammograph using a sensitometric phantom. The resulting useable region of the characteristic curve will be reconstructed in the experiment and will be referred to as the "aim curve."

3. Three mammographs will be made exactly as in (2) but with 1/2, 1/4 and 1/8 of the original time of exposure.


5. Starting with each reduced exposure characteristic curve, the computation of a tone-reproduction cycle will determine the required curve shape, gradient and exposure of both internegative and positive films needed to achieve facsimiles of the aim curve.

6. Production of Facsimile Mammographs.

7. Radiologists will be asked to rank image quality of the facsimile mammographs, full exposure mammograph and a film/screen combination mammograph with respect to detectability of calcifications (size and number) and with respect to general diagnostic acceptability.
Determination of Crucial Information Regions

From the clinical teaching files of Strong Memorial Hospital, typical mammographs on Kodak type AA x-ray film were sensitometrically analyzed in cooperation with a radiologist. That is, the radiologist was asked to identify what were considered the crucial regions of the mammograph. Transmission density was measured with a TD-102 Macbeth diffuse densitometer fitted with a 1 mm sample aperture. The following observations resulted:

- Calcifications larger than 1 mm imaged between 1.05 and 1.15 density units.

- A tumor imaged at 1.25 to 1.40 density units while the spectrum of fatty tissue surrounding the tumor imaged from 1.90 density units toward the center, to 1.30 density units a distance away from the center.

- Other diagnostic information occurred in the peripheral regions of the breast where skin thickening tends to hide information due to its higher density (2.0 density units and higher).

Due to unknown variables of exposure and processing which could have affected the randomly selected mammographs, these measurements only indicate a range of density where
crucial information is found. Moreover, absorption of x-rays in general is determined by thickness as well as by atomic number of materials. The breast can contain varying amounts of glandular tissue and be of differing size thus making it statistically difficult to obtain the macro-contrast of a typical mammograph.

From the photographic standpoint, the significant detail in the mammograms examined recorded in a density range extending from 1.05 to 2.00 density units. This range is critical and will be preserved in the facsimile mammographs.

**Determination of the Aim Curve**

The aim curve of this experiment is the characteristic curve of a typical mammograph. Once determined, all reductions in exposure are made relative to it, and all facsimile images are an attempt to match it, especially along the crucial areas mentioned above.

The Kodak Pathe I.T.O. sensitometric phantom was used for all experimental work. The I.T.O.'s components (see Figure 2) are embedded in a polyester resin whose x-ray absorbing power and scattering power are closest to that of soft human tissue. The various wedges, screen patterns and chalk are fixed in position assuring test to test reproducibility.
Figure 2

Kodak Pathe I.T.O.

A–F Stainless Steel Meshes
G Vinyl Capillary
H Steel Wool
K 2mm alum. step
L 1mm air step
M 2mm vinyl step
N–O Ground and Powdered CaSO₄
Exposures of the phantum were made at Elmwood Medical Center using a Radiological Sciences RSI x-ray machine having a nominal focal spot of 90 microns.

The aim curve was determined by first making a series of radiographs of the phantum with a geometric series of exposure times (3.2, 1.6, .8, .4, .2, seconds), all other exposure parameters the same. Then the density range of the calcifications of each of the radiographs was compared to the density range of calcifications in the clinical mammographs and the closest match was found. The x-ray exposure parameters for the aim curve are: 26 Kv; 4 ma; 1.6 sec.; 34.3 cm F.S.D.; 0.4 mm Al filtration. Three additional exposures at 0.8 sec., 0.4 sec. and 0.2 sec. are the reduced exposures for the experiment.

One x-ray machine with replicate runs was used to reduce machine to machine variability and within machine variability. Other x-ray exposure parameters such as ma or Kv could have been varied alone or in combination, but time was chosen since it is advantageous to reduce unsharpness due to patient motion.

The radiographs were processed in an X-OMAT processor under manufacturers recommended conditions.

The aim characteristic curve was determined in terms of relative log exposure by first plotting the density of one step for five different exposure times (0.1, 0.2, 0.4, 0.8, 1.6 sec. respectively). As there is no reciprocity
failure, if two or more steps are plotted side by side on one graph, by horizontally shifting and then superimposing the curves, a better approximation to a characteristic curve results. This process is illustrated in Figure 3. Taking the density of each step of the aim radiograph through the final curve generated determines the relative log exposure values of the aim curve. See Figure 4.

When the densities of the corresponding steps of the reduced exposure mammographs are plotted against these same relative log exposures, characteristic curves of all mammographs involved are obtained. See Figure 5.

Note that the density range of each reduced exposure mammograph is progressively narrower as exposure is decreased.

Tone Reproduction

The tone reproduction cycle needed is illustrated in Figure 6. It involves (4) quadrants, the first three of which contain data about one of the phases of the system, and the last which displays the net effect; that is, the aim curve or objective tone reproduction curve of the mammograph we wish to match.

Quadrant I: The density range of a given reduced exposure mammograph is found here and serves as input to Quadrant II.
Figure 3

Determination of the Characteristic Curve
CHARACTERISTIC CURVE
Kodak Type AA X-Ray Film

Figure 4
Characteristic Curve
Figure 5
Aim and Reduced Exposure Curves
4 QUADRANT TONE RÉPRODUCTION

Figure 6
Four Quadrant Tone Reproduction
**Quadrant II:** The internegative is produced by copying the reduced exposure mammograph on to a thin based, high gradient, fine grain material. The logarithmic exposure on the internegative is a constant minus the density of the reduced-exposure mammograph.

**Quadrant III:** The final positive image (facsimile mammograph) is produced by copying the internegative on to a fine grain blue base x-ray film using white light. The logarithmic exposure to the positive is a constant minus the density of the internegative.

**Quadrant IV:** The resultant and aim curve.

The following system parameters can be varied:

**Quadrant II**

1. The shape of the internegative characteristic curve through film selection. The following three films were considered:
   - KODAK type Fine Grain Positive
   - KODAK type Commercial
   - KODAK type SO-243 (Holographic)
2. The gradient of the internegative through development. As seen in Figure 7, the gammas of the above films span a range of 1.2 to 7.8.

3. The exposure of the internegative or the section of the characteristic curve we wish to use. That is, the reduced exposure mammograph can be copied onto the toe, straight line portion or shoulder of the internegative characteristic curve. This can be accomplished graphically by shifting Quadrant II vertically with respect to Quadrant I as shown in Figure 8. Decreases in exposure move the internegative to the toe end of the H and D curve while exposure increases shift the internegative toward the shoulder.

Quadrant III

1. The shape of the positive characteristic curve can also be varied through film selection. The following three films were considered:

   KODAK type Minar (w/o screen)
   KODAK type M
   KODAK type MA
INTERNEGATIVE TIME DEVELOPMENT SERIES

Figure 7
Internegative Time Development Series
Negative Exposure Shift:

Positive Exposure Shift:

Figure 8

Utilization of Different Regions of the Internegative Curve Through Exposure
2. The gradient of Minar can be varied somewhat with development as illustrated in Figure 9. The gamma of these films span a range from 1.5 to 3.7.

3. Similarly, the exposure of the positive can be varied resulting in utilization of different sections of the characteristic curve. Quadrant III is shifted horizontally with respect to Quadrant II. An increase in exposure utilizes more of the shoulder region while a decrease in exposure utilizes more of the toe region of the H and D positive curve.

Mathematical Tone Reproduction

Since tone reproduction calculations are essentially algebraic as well as graphic, the use of a computer enables higher accuracy and more versatility with less work. Information about each film's characteristics when time of development is varied will allow us to choose among various combinations of curve shapes, gradients and exposure shifts. The specific parameters which minimize the mean square error over the crucial areas of the aim curve can be ascertained by interactive computing.

The following five equations enable us to handle the tone reproduction cycle algebraically:
Figure 9

Positive Time Development Series
\[
\begin{align*}
HN(RH) &= H_i - SD(RH) + NS \\
DN(RH) &= F(HN(RH)) \\
DP(RH) &= H_p - DN(RH) + PS \\
DP(RH) &= F(HP(RH)) \\
RD(RH) &= DP(RH)
\end{align*}
\]

To understand the program, follow a point on the reduced exposure curve through the internegative and positive curves. The resulting point is one point of the resultant curve. The computer merely takes many points on the reduced exposure curve (21 points from 21 steps) and generates their final position. See Figure 10.

Before beginning, the start curve (reduced exposure curve), internegative, positive and aim curves are spline fit. Looking at the relative log exposure data of the start curve, we define our minimum value as 0.0. The range of all available steps on the wedge is now from 0.0 to 0.8 relative log H units. RH is a reference point and represents equal intervals of rel. log. H. RH is carried throughout so we may compare exposures and densities at this reference point throughout the tone reproduction program. SD(RH) is the density of the start curve at position (RH) as shown in Quadrant I of Figure 10. Looking at the internegative (Quadrant II):
Figure 10

Mathematical Tone-Reproduction Model
$H_{IN}$ is the original exposure to the internegative material and is modulated by SD(RH).

$DN(RH)$ is the density of the internegative at reference point RH.

$HN(RH)$ is the exposure responsible for $DN(RH)$ since density is a function of exposure.

$NS$ is the internegative (exposure) shift.

The following equation represents one point on the internegative:

$$HN(RH) = H_{IN} - SD(RH) + NS$$

This equation says that the exposure on the internegative curve at RH is equal to the original log exposure on the internegative material minus the density of the start curve at position RH plus any desired internegative shift in exposure. The second equation,

$$DN(RH) = F(HN(RH))$$

says that the density on the internegative curve at RH is a function of the $HN(RH)$ or log exposure on the internegative curve. See Quadrant II, Figure 10. Looking at the positive (Quadrant III):
$H_{ip}$ is the original exposure to the positive material and is modulated by $DN(RH)$.

$DP(RH)$ is the density of the positive at reference position $RH$.

$HP(RH)$ is the exposure of the positive at reference position $RH$.

$PS$ is the positive (exposure) shift.

This implies:

$$HP(RH) = H_{ip} - DN(RH) + PS$$

This equation says the exposure on the positive curve at $RH$ is equal to the original log exposure on the positive material minus the density of the internegative curve at position $RH$ plus any desired positive shift in exposure. See Quadrant III, Figure 10.

Finally, equation five,

$$RD(RH) \equiv DP(RH)$$

says that the density at point $RH$ on the positive is the same as the final resultant density at $RD(RH)$. Figure 10 shows the completed tone reproduction cycle with the resultant curve taking its position in Quadrant IV.
Once the resultant curve is calculated, the mean square error between the resultant and aim curve is calculated for the diagnostically significant range (1.0 to 2.5 density units) of the mammograph as shown in Figure 11.

RESTR

The determination of the final internegative and positive curves and exposures for the best tone reproduction is an iterative procedure. A Fortran program (RESTR) to perform the above mentioned calculations and graphically display the results on a Tektronix terminal was developed. The flow chart describing the major decisions and operations is shown in Figure 12.

Sensitometry of Duplicating Films

The sensitometry for the experiment involves two parts: First, the determination of the duplicating film characteristics, and secondly, the production of facsimile mammographs after the exposure parameters have been obtained.

Time development series determined the times of development resulting in a useful difference in gradient for each of the six films considered.

Once this information was obtained, three sensi-strips were made from each film for each time of development.
Figure 11

Aim and Resultant Curves
Figure 12

Flow Chart of Tone-Reproduction Program (RESTR)
Two sensi-strips were exposed by one person during one sitting using a Kodak model 101 sensitometer. All density readings, as well, were read by one person during one sitting with a stable Macbeth diffuse densitometer fitted with a 3 mm aperture. A 21 step control chart was used to assure stability across a density range from 0.25 to 3.86.

For fine grain positive, commercial, SO-243, and minqr, the sensi-strips were taped down on four corners in the middle of an 11" x 14" tray. The double-sided x-ray films, M and MA, were elevated off the bottom of the tray by four film clips. No more than four strips were processed during a single processing run and the R.I.T. tray rack method of agitation was employed.

Kodak developer D-11 was used to process all six films at 20 ± 0.5°C. An SB-1 stop bath was used for twenty seconds. The strips were then fixed, washed and dried.

The density readings from three sensi-strips (within tray and intertray) were averaged together by step, yielding an average characteristic curve for each time of development of each film. The data of these "average" characteristic curves are shown in figures 6 and 8 and were entered into the computer. (See Appendix 1). Utilization of the tone reproduction program resulted in film, exposure and development parameters for each of the reduced exposure mammographs as shown in Figure 13.
### Film, Exposure and Development Parameters

<table>
<thead>
<tr>
<th>REDUCED EXPOSURE</th>
<th>INTERNEG</th>
<th>POSITIVE</th>
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<tr>
<td>0.8</td>
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<td>Minar</td>
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<td>8.3 lux-sec</td>
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<td>Develop:</td>
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<td>5 min.; D-11; 20°C</td>
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Figure 13

Film, Exposure and Development Parameters
Production of Facsimile Mammographs

Production of a facsimile mammograph as illustrated in Figure 13, involves exposure and development of the internegative followed by exposure and development of the positive.

A contact printing frame with pressure back is used and films are inserted emulsion to emulsion to achieve maximum contact. The reduced exposure mammograph, unexposed films and contact frame glass were pulled through a Kodak antistatic and dust removal unit before exposure.

An Omega D2 enlarger with 75 mm f/4.5 Ektar lens was used as a source. The enlarger head was positioned high enough off the baseboard to assure uniform illumination across an 8" x 10" contact printing frame. Gross exposure was measured with a United Detector Technology photometer calibrated to read absolute lux against a standard lamp.

Exposure is determined by first adding the log H shift to the base log H of the sensi-strip and then taking the antilog. The resulting exposure (in lux sec) is read directly on the meter when set in integrating or energy mode.

Once gross exposure is determined, a trial processing run is made to determine whether given steps on the reduced exposure mammograph are reproduced at correct densities on the internegative. By using the film as a radiometer, final corrections in exposure can then be determined from
Production Of Facsimile Mammographs

![Diagram showing the process of producing facsimile mammographs](image)

Figure 14
Production of Facsimile Mammographs
the average characteristic curve by finding $\Delta H$ as a function of $\Delta D$ as shown in Figure 14. The degree of accuracy was set by 90 percent confidence intervals placed around each point or step of the given internegative characteristic curve. Confidence intervals around point locations are preferred to a confidence interval on the curve itself due to greater variability in density toward $D_{max}$.

Once the desired internegative is obtained, the above procedures are replicated for the positive, resulting in a final facsimile mammograph.

**Evaluation**

With the original sensitometric phantom as a reference, radiologist W.L. was asked to rank order the facsimile mammographs, full exposure mammograph and a film/screen mammograph in terms of detectability of size and number of calcifications. She was also asked to rank order all mammographs in terms of resolving power or the ability to see line separations on the highest frequency screen pattern. The results were:

<table>
<thead>
<tr>
<th>Calcifications</th>
<th>Resolving Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1.6 sec. Aim</td>
<td>1 1.6 sec. Aim</td>
</tr>
<tr>
<td>2 Minar film/screen</td>
<td>2 0.8 sec. facsimile</td>
</tr>
<tr>
<td>Tie 2 0.8 sec. facsimile</td>
<td>3 0.4 sec. facsimile</td>
</tr>
<tr>
<td>3 0.4 sec. facsimile</td>
<td>4 minor film/screen</td>
</tr>
<tr>
<td>4 0.2 sec. facsimile</td>
<td>5 0.2 sec. facsimile</td>
</tr>
</tbody>
</table>
90 Percent Confidence around Internegative Step Densities

Figure 15
90 Percent Confidence Around Internegative Step Densities
A second radiologist, S.B., was asked to rank the images using the same criteria:

<table>
<thead>
<tr>
<th>Tie</th>
<th>Calcifications</th>
<th>Resolving Power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 1.6 sec. Aim</td>
<td>1 1.6 sec. Aim</td>
</tr>
<tr>
<td>2</td>
<td>0.8 sec. facsimile</td>
<td>2 Minar film/screen</td>
</tr>
<tr>
<td>2</td>
<td>Minar film/screen</td>
<td>3 0.8 sec. facsimile</td>
</tr>
<tr>
<td>3</td>
<td>0.4 sec. facsimile</td>
<td>4 0.4 sec. facsimile</td>
</tr>
<tr>
<td>4</td>
<td>0.2 sec. facsimile</td>
<td>5 0.2 sec. facsimile</td>
</tr>
</tbody>
</table>

Radiologist, W.L., thought the contrast of the aim curve should be higher. When a new 3.2 sec. aim curve, along with a facsimile mammograph produced from a 0.8 sec. reduced exposure were shown to her, she found the facsimile image diagnostically unacceptable, because "calcifications on the limit of detectability could not be seen or gave only a slight impression of their existence." However, when S.B. viewed both of these images he believed that they were "both about the same, maybe a little better in E (the aim)."

In terms of detectability of calcifications, both radiologists quality ranked the images in the same order. In terms of resolving power, their quality ranks differed. It was found that radiologists in general are not used to speaking in terms of this image analysis parameter for information sought is of a low frequency nature.
Although not an exhaustive analysis, there is reason to believe that a facsimile mammograph produced with 50 percent normal radiation is diagnostically equivalent to a film/screen combination mammograph. However, a facsimile mammograph produced with 25 percent normal radiation was found to be acceptable only to radiologist S.B.

Conclusion

It is encouraging to find images equal in quality to those being used in the field today. Two additional observers from the photographic science faculty subjectively evaluated images using the same criteria as radiologists. Both arrived at the same quality rank as radiologist W.L.

With this method, direct-exposure mammographs can be made with 26 Kv; 8 ma; 3.2 sec.; 34.3 cm F.S.D.; 0.4 mm Al filtration, corresponding to a radiobiologic level of 1944 millirads. However, film/screen combination mammographs such as Kodak minar result in radiobiologic levels of 536 millirads (for minar). This corresponds to a factor of 3.63 less than the radiation reduction achieved in this experimentation. That is:

Reduced exposure mammography through contrast adjustment would be preferable at this time only if a factor of 8 could be achieved.
In photographic terms, this corresponds to a 0.9 reduction in logarithmic exposure to the film or 3 stops. Experimentally, this means acceptability of the 0.2 sec. mammograph. The inherent advantage of this system would be the use of fine grain films which provide best image quality.

Facsimile mammographs have been produced by conventional rules of objective tone reproduction where the product of the slopes at different points is equal to unity. An area of further study is that of manipulating tone reproductions of the final image. For instance, the local gradient near calcifications can be maintained, while the total macrocontrast increased. Such manipulations have been accomplished by electronic techniques and are equally possible with RESTR. Further, all measurements were based on subjective or large scale measurement. The study of noise and how it obscures the information the radiologist sees is an important area of further work which could lead to optimization of this system.
FOOTNOTES


2. Dr. Wende Logan, Personal Interview, 12/76.

3. RSI Tube Radiation data obtained from Dr. Wende Logan, Elmwood Medical Center, Rochester, New York, 5/77.

4. RSI Tube Radiation data obtained from Dr. Wende Logan, Elmwood Medical Center, Rochester, New York, 5/77.

5. RSI Tube Radiation data obtained from Dr. Wende Logan, Elmwood Medical Center, Rochester, New York, 5/77.

REFERENCES

MAJOR


MINOR


ACKNOWLEDGEMENTS

I would like to give special recognition and thanks to the following individuals who were instrumental throughout the development of this thesis.

To Dr. Wende Logan of Strong Memorial Hospital for her authoritative knowledge of mammography and radiographic techniques.

To Mr. Robert D. Anwyl, Mr. John E. Cullinan, and Mr. Warren S. Meyers of Eastman Kodak Company for technical support with regard to film selection.

To Mr. John Blakney, friend and fellow photoscientist, who made RESTR a reality and who provided countless contributions to the project.

To Mr. Daniel Siens, freshman photoscientist, for darkroom assistance.

Finally, a note of gratitude to Professor Gerhard Schumann for his counsel, advice and enthusiasm.

I would also like to acknowledge the Central Intelligence Agency for their financial support of this project.
Tone reproduction for 0.2 second reduced exposure
Tone reproduction for 0.4 second reduced exposure
Tone reproduction for 0.8 second reduced exposure
APPENDIX II
**PUT AFTER LIMIT CARD IN YOUR DECK**

**MESSAGE PLEASE MOUNT LTP SPSE RING IN**

**SPE LPS PSE**

COPY ALL .66WFSNH TO LT#SPSE

CANNOT COPY FILE

$ 1400

NUMALG 0700

27 FILES COPIED

**NEW LT#SPSE**

COPY L#SPSE/X'998983888599' TO UC/X'998983888599'

COPY L#SPSE/RESTRSRE TO LP

C *** MAIN PROGRAM RESTR - TONE REPRODUCTION FOR STU RICHER

C *** WRITTEN BY JOHN BLAKNEY JAN 26, 1977

C *** MUST BE USED ON A TECNIXRON Graphics Terminal

**IMPLICIT INTEGER (N)**

INTEGER AN,SN,NN,PN,RN,PI,P1,PI2

REAL AH(I),AD(I),SH(I),SD(I),NH(I),NO(I)

REAL PH(I),PD(I),AC(I),NC(I),PC(I)

REAL NID,NT,PID,PI,H(I),HN(I),HP(I),DP(I)

REAL DT(I),NTX(I,2),PIDT(I),PTX(I,2)

INTEGER L(5)

CALL INITT(-120)

L(1)=1

L(2)=2

L(3)=3

L(4)=4

L(5)=0

NS=PS=0.0

I=1

C *** INPUT THE AIM AND STARTING DATA

READ(10,900) AN,AN,AT

READ(10,901) AH(I),AH(I),I=1,AN

READ(10,900) SN,SN,ST

READ(10,901) SH(I),SH(I),I=1,SN

C *** CALCULATE SPLINE FIT COEFFICIENTS FOR A + S CURVES

CALL SPLICO(AH,AD,AN,AC)

CALL SPLICO(SH,SD,SN,SC)

C *** READ INFORMATION ABOUT THE NEG AND POS CURVES

READ(10,907) I1,I2

READ(10,902) NIDT(I),NIDT(I),I=1,I1

READ(10,902) PIDT(I),PIDT(I),I=1,I2

C *** HERE IS WHERE USER QUERY STARTS

C *** FIRST FIND OUT WHAT IS TO BE CHANGED OR WHERE TO GO

10 LI=L(I3)

L(I3)=0

GOTO (100,300,200,400), LI

GOTO 500

C *** THIS IS SECTION TO GET NEG CURVE

100 CALL NEWPAG

OUTPUT "INPUT THE NUMBER OF THE DESIRED NEG CURVE"*

OUTPUT "THE FOLLOWING ARE VALID"*

WRITE(108,903) I1,NIDT(I1),NIDT(I2),I=1,I1

READ(108,913) NI

OUTPUT "CALLING CURVT"*

CALL CURVT(NI,NH,NI,NN,NID,NT)

CALL SPLICO(NH,N0,NN,NC)

OUTPUT "BACK FROM CURVT"*

GOTO 10

C *** HERE IS SECTION TO GET POS CURVE

200 CALL NEWPAG

OUTPUT "INPUT THE NUMBER OF THE DESIRED POS CURVE"*

OUTPUT "ONLY THE FOLLOWING ARE VALID"*

WRITE(108,903) I1,PIDT(I),PTX(I1,1),PTX(I2,1),I=1,I2

READ(108,913) PI

PI2=PI+1

OUTPUT "CALLING CURVT POSITIVE"*

CALL CURVT(PI2,PH,PD,PN,P0,PT)

CALL SPLICO(PH,PD,PN,PC)

OUTPUT "BACK FROM CURVT"*

GOTO 10

C *** HERE IS NEG. EXPOSURE SHIFT SECTION

300 CALL NEWPAG
RESTR - SOURCE LISTING
OUTPUT "INPUT THE EXPOSURE SHIFT FOR THE NEGATIVE"
OUTPUT "FORMAT F5.2"
WRITE(108,908)NS
READ(108,904)NS
GOTO 10
C *** AND NOW THE SHIFT FOR THE POSITIVE
400 CALL NEWPAG
OUTPUT "INPUT THE EXPOSURE SHIFT FOR THE POSITIVE"
OUTPUT "FORMAT F5.2"
WRITE(108,908)PS
READ(108,904)PS
GOTO 10
C *** ALLOW CHANGES AND THEN PROCEED
500 CALL NEWPAG
OUTPUT "SUMMARY OF ACTIONS"
WRITE(108,909)NL,Nl,NI,NS
WRITE(108,910)PL,PI,PS
OUTPUT "IF YOU WISH TO CHANGE ANYTHING, TYPE C,"
OUTPUT "OTHERWISE TYPE RETURN"
READ(108,911)R
DATA R1=",C",
IF (R NE R1) GOTO 540
OUTPUT "GOING INTO 530"
530 CONTINUE
IS=1
OUTPUT "CHANGE OPTIONS ARE:
OUTPUT "0 DO NOTHING"
OUTPUT "1 INPUT NEW NEGATIVE CURVE"
OUTPUT "2 INPUT NEW NEG EXPOSURE SHIFT"
OUTPUT "3 INPUT NEW POSITIVE CURVE"
OUTPUT "4 INPUT NEW POSITIVE EXPOSURE SHIFT"
OUTPUT "5 INPUT NEW POSITIVE CURVE"
OUTPUT "TYPE YOUR SELECTION, 4 DIGITS PLEASE,"
OUTPUT "I.E. 2500"
READ(108,912) L(I), I=1,4
GOTO 10
C *** HERE IS WHERE CURVE IS DRAWN
540 CONTINUE
X OUTPUT "CALLING TRDRAW"
CALL TRDRAW
CALL PAWSER
550 CALL NEWPAG
OUTPUT "TO SEE AN INDIVIDUAL CURVE, TYPE 1, 2, 3, 4, OR 5,"
OUTPUT "FOR THE NEGATIVE POSITIVE, AIM, RESULTANT,"
OUTPUT "ELSE TYPE RETURN"
READ(108,913) I
IF (I.LT.1) GOTO 600
CALL CORAW(I)
CALL PAWSER
GOTO 550
600 OUTPUT "IF YOU WISH TO MODIFY THE DATA, TYPE C,"
OUTPUT "ELSE TYPE RETURN"
READ(108,914)R
IF (R.EQ.R1) GOTO 530
CALL FINIT
CALL EXIT
900 FORMAT(I2,F5.2,A4)
901 FORMAT(2I5,2)
902 FORMAT(10(F6.2,2A4)), MIN ID = ,2A4)
903 FORMAT(X,12,3X,F5.2, MIN ID = )
904 FORMAT(F5.2)
905 FORMAT(2I2)
906 FORMAT(X,"LAST SHIF T W AS ",F4.2, LOG H )
907 FORMAT(2I2)
908 FORMAT(X,"POS. CURVE # ",F5.2, MIN, ID = ",1 A4, SHIFT= ",F5.2, LOG H )
909 FORMAT(X,"NEG. CURVE # ",F5.2, MIN, ID = ",1 A4, SHIFT= ",F5.2, LOG H )
910 FORMAT(X,"POS. CURVE # ",F5.2, MIN, ID = ",1 A4, SHIFT= ",F5.2, LOG H )
911 FORMAT(A1)
912 FORMAT(11)
913 FORMAT(11)
914 FORMAT(411)
END
CALL DRAW(X,Y,I,NID,NT,PID,P,R,N)
RETURN
END

SUBROUTINE SPLICO

DIMENSION X(21),Y(21),D(21),P(21),E(21),C(4,21),A(21,3),B(21)
1,2(21)
MM=M-1
DO 2 K=1,MM

2 D(K)=X(K+1)-X(K)
P(K)=D(K)/6
E(K)=(Y(K+1)-Y(K))/D(K)
DO 3 K=2,MM

3 B(K)=E(K)-E(K-1)
A(1,2)=-1,-D(1)/D(2)
A(1,3)=D(1)/D(2)
A(2,3)=P(2)-P(1)*A(1,3)
A(2,2)=2.*(P(I)+P(2))-P(1)*A(1,2)
A(2,3)=A(2,3)/A(2,2)
B(2)=B(2)/A(2,2)
DO 4 K=3,MM

4 B(K)=2.*P(K-1)*P(K)-P(K-1)*A(K-1,3)
A(K,3)=P(K)/A(K,2)
A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)
A(K,3)=A(K,3)/A(K,2)
B(K)=B(K)/A(K,2)
Z(K)=B(K)/A(K,2)
U=D(M-2)/D(M-1)
A(M,1)=1,+U*Z(K,3)
A(M,2)=U-A(M,1)*A(M-1,3)
H(M)=B(M-2)-A(M,1)*B(M-1)
Z(M)=H(M)/A(M,2)
MN=M-2
DO 6 I=1,MN

6 Z(K)=H(K)-A(K,3)*Z(K+1)
Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)
DO 7 K=1,MM

7 U=1./6.*D(K)
C(1,K)=Z(K)+U
C(2,K)=Z(K+1)+Q
L(3,K)=Y(K)/D(K)-Z(K)*P(K)
C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
RETURN

SUBROUTINE CRVGT

SUBROUTINE DRAW

DIMENSION X(100),Y(100),L(3),D(12,2)
INTEGER I,N
REAL ID
REWIND 10
READ(10,905) A(L),L=1,9
50 IF (I=1) 300, 400
200 DO 100 K=1,(I-1)*3
100 CONTINUE
400 READ(10,906) N, ID, T
READ(10,901) H(J), U(J) ,J=1,N
RETURN
300 OUTPUT I
OUTPUT 'INCORRECT CHOICE, * ASSUMED
I=I
GOTO 50
900 FORMAT(12,F5.2,A4)
901 FORMAT(21F5.2)
905 FORMAT(4A)
906 FORMAT(F5.2)
RETURN
END

SUBROUTINE SPLICO

C**** FROM INTRODUCTORY COMPUTER METHODS AND NUMERICAL ANALYSIS ****
C**** 2ND EDITION R.H.PENNINGTON, MACMILLAN CO., 1970 ****
RESTR - SOURCE LISTING

D(1,2) = 1.
D(2,1) = 100.
D(2,2) = 1.
D(3,1) = FLOAT(INT(X(1)*2.5-0.9999))/2.5
D(3,2) = 0.0
D(4,1) = D(3,1) + 3.2
D(4,2) = 3.2
D(5,1) = 162.
D(5,2) = 25.
D(6,1) = 700.
D(6,2) = 700.
D(7,1) = 8.
D(7,2) = 8.
D(8,1) = 1.
D(8,2) = 1.
D(9,1) = 0.
D(9,2) = 0.
D(10,1) = 1.
D(10,2) = 1.
D(11,1) = 0.
D(11,2) = 0.
D(12,1) = 2.
D(12,2) = 2.
IF (N.EQ.2) GOTO 100
CALL GRAPH(X,Y,D)
GOTO 200

100 D(1,1) = 0.
D(2,2) = -74
D(8,1) = 0.
D(8,2) = 0.
D(10,1) = 0.
D(10,2) = 0.
D(12,1) = 2.
D(12,2) = 2.
CALL MOVEABS(380,740)
CALL ANMODE
WRITE (108,901)
901 FORMAT(X, "RESULTANT TONE REPRODUCTION")
CALL GRAPH(X,Y,D)
RETURN

200 L(1) = 4HDFNS
L(2) = 3HITY
D(1,1) = 0.
D(1,2) = 1.
D(2,1) = 7.
D(2,2) = 1.
D(3,1) = 100.
D(3,2) = 400.
D(4,1) = 1.
D(4,2) = 0.
D(5,1) = 0.
D(5,2) = 0.
CALL TITLE(L,D)
IF (RELATIVE.EQ.1.) GOTO 500

501 L(1) = 4HLUG
L(2) = 4HEXP0
L(3) = 4HSURE
D(2,1) = 12.
D(3,1) = 380.
D(3,2) = 15.
D(4,1) = 0.
CALL TITLE(L,D)
CALL DATIM(9,F)
90 FORMAT(X,2A4,2X,"SPR/JTB")
CALL MOVEABS(750,10)
CALL ANMODE
WRITE(108,90) F
900 FORMAT(X,"SPR/JTB")
RETURN
500 CALL MOVEABS(290,15)
WRITE(108,99)
99 FORMAT(X,"RELATIVE")
GOTO 501 END

C *** SUBROUTINE PIGET
C *** WRITTEN BY JOHN BLAKNEY JAN 26, 1977
C *** TO BE USED IN RESTR PROGRAM
C *** START AND STOP ARE THE BEGINNING AND ENDING
C *** VALUES FOR WHICH EXPOSURES ARE CALCULATED.
C *** SUBROUTINE PIGET(X,Y,H,D,N,C)
C *** DIMENSION X(100),Y(100),H(21),D(21),C(4,21)
C CALL SPLICO(H,D,N,C)
RESTRICT SOURCE LISTING

X(I)=H(I)-(H(N)-H(I))/100.
RO=X(I)
DO 10 I=1,100
X(I)=RU+(H(N)-H(1))/100.
RO=X(I)
IF (X(I).LE.H(N)) GOTO 15
Y(I)=D(I)
GOTO 10
15 IF (X(I).GE.H(I)) GOTO 20
Y(I)=D(I)
GOTO 10
20 CALL SPLINE(H,D,N,C,X(I),Y(I))
10 CONTINUE
RETURN
END

C *** SUBROUTINE RCAL
C *** WRITTEN BY JOHN BLAKNEY JAN 26, 1977
C *** TO BE USED WITH RESTR PROGRAM

SUBROUTINE RCAL
INTEGER AN,SN,NN,PN,RN
REAL AH(21),AD(21),SH(21),SD(21),NH(21),ND(21)
REAL PH(21),PD(21),AC(4,21),NC(4,21),PC(4,21),NS,PS
REAL NIO,NT,PID,PT,H(21),HN(21),HP(21),DP(21)
REAL DN(21),RH(21),RC(4,21),V,SC(4,21)
COMMON AN,AH,AD,SN,SH,SD,NN,NH,ND,PN,PH,PD,AC,NC
CALL SPLINE0(NH,N{),NN,MC)
CALL SPLINE0(PH,PD,PN,PC)
DO 100 IH=1,21
IAH=(IH*15)-15
RH(IH)=Fj qAT(IAH)/333.33
C *** FIND D OF VAL RH ON START CURVE.
C *** GREATER THAN SH(SN), RETURN A D OF 9.99
C *** IF RH(IH) IS
C *** HN(IH)=9.99
GOTO 20
5 IF(RH(IH).GE.SH(1)) GOTO 10
HN(IH)=9.99
GOTO 20
C *** RH IS VALID SO FIND IT’S D ON START CURVE
C *** HN(IH) IS THE START OF EXPOSURE ON NEGATIVE MATERIAL
C *** ACTUAL EXPOSURE WILL BE HORGN-HN(IH)+NS
HORGN=NH(NN)-0.15
HN(IH)=HORGN-HN(CN)+NS
C *** NOW, FOR AN EXPOSURE HN(IH), FIND D OF NEG. ... BUT,
C *** FIRST FIND OUT IF DATA IS VALID,
20 IF(HN(IH).GE.NH(1)) GOTO 30
HP(IH)=0.0
DN(IH)=9.99
GOTO 50
30 IF (HN(IH).GE.NH(1)) GOTO 40
HP(IH)=9.99
DN(IH)=0.00
GOTO 50
40 CALL SPLINE(NH,ND,NN,NC,HN(IH),Y)
DN(IH)=Y
C *** DN(IH) LIKewise IS TO BECOME EXPOSURE ON POS MATERIAL
HORG=PH(PN)-0.15
HP(IH)=HORG-DO(IH)+NS
C *** AGAIN A VALIDITY CHECK IS MADE AND D ON POS FOUND
50 IF (HP(IH).LE.PH(PN)) GOTO 60
DP(IH)=PD(PN)
GOTO 100
60 IF (HP(IH).LE.PH(1)) GOTO 70
DP(IH)=PD(1)
GOTO 100
70 CALL SPLINE(PH,PD,PN,PC,HP(IH),DP(IH))
C *** DP(IH) IS THE RESULTANT DENSITY ON THE PRINT MATERIAL
100 CONTINUE
HN=21
CALL SPLINE0(RH,DP,RN,RC)
RETURN
END

C************ SUBROUTINE SPLINE ************
C************ FROM INTRODUCTORY COMPUTER METHODS AND NUMERICAL ANALYSIS****
C************ 2ND EDITION R.H.PENNINGTON, MACMILLAN CO., 1970****
C************  SUBROUTINE SPLINE(X,Y,M,C,XINT,YINT)
RESTR - SOURCE LISTING
DIMENSION X(21),Y(21),C(4,21)
1 YINT=Y(1)
RETURN
2 K=1
3 IF(XINT-X(K+1))<0,4,5
4 YINT=Y(K+1)
RETURN
5 K=K+1
6 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))
7 YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))
RETURN
101 FORMAT(31HOUT OF RANGE FOR INTERPOLATION)
END
C *** WRITTEN BY JOHN BLAKNEY JAN 26, 1977
C *** TO BE USED WITH RESTR PROGRAM
SUBROUTINE TRDRW
INTEGER AN,SN,NN,PN,RN
REAL AH(21),AD(21),SH(21),SD(21),NH(21),ND(21)
REAL PH(21),PD(21),AC(4,21),NC(4,21),PC(4,21),NS,PS
REAL NID,NT,PID,PT,H(21),HN(21),HP(21),DP(21)
REAL DN(21),RH(21),RC(4,21),V,SC(4,21)
COMMON AN,AH,AD,SN,SH,SD,NN,NH,ND,PN,PH,PD,AC,NC
CALL NEWPAG
CALL RCAL
CALL RESCL
CALL CDRA(7)
CALL CDRA(6)
CALL VPUT(V,NS,PS)
RETURN
END
C *** WRITTEN BY JOHN BLAKNEY JAN 26, 1977
C *** TO BE USED WITH RESTR PROGRAM
SUBROUTINE VPUT
REAL NS,PS
CALL MOVABS(820,500)
CALL ANMODE
WRITE (108,901)NS
901 FORMAT(X,"NS=".,F4.2)
CALL MOVABS(820,475)
CALL ANMODE
WRITE(108,902)PS
902 FORMAT(X,"PS=".,F4.2)
CALL MOVABS(820,200)
CALL ANMODE
WRITE(108,900) V
900 FORMAT(X,"V=".,F8.2)
RETURN
END
C *** WRITTEN BY JOHN BLAKNEY JAN 26, 1977
C *** TO BE USED IN RESTR PROGRAM
SUBROUTINE RESCL
INTEGER AN,SN,NN,PN,RN
REAL AH(21),AD(21),SH(21),SD(21),NH(21),ND(21)
REAL PH(21),PD(21),AC(4,21),NC(4,21),PC(4,21),NS,PS
REAL NID,NT,PID,PT,H(21),HN(21),HP(21),DP(21)
REAL DN(21),RH(21),RC(4,21),V,SC(4,21)
COMMON AN,AH,AD,SN,SH,SD,NN,NH,ND,PN,PH,PD,AC,NC
CALL SPLICO(AH,AD,AN,AC)
CALL SPLICO(RH,DP,RN,RC)
LOW=1.000
HIGH=2.50
V=0.00
H1=0.00
50 CALL SPLINE(AH,AD,AN,AC,H1,DA)
IF (DA.GE.LOW) GOTO 70
H1=H1+0.01
GOTO 50
70 IF (DA.GT.HIGH) GOTO 80
CALL SPLINE(RH,DP,RN,RC,H1,DR)
V=V+(DA-DR)**2
RESTR - SOURCE LISTING

H1 = H1 + 0.02
GOTO 50
CONTINUE
OUTPUT V
RETURN
END