Measurement of thermal conductivities of polymer films with the TC-1000 thermal comparator

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To my parents
MEASUREMENT OF THERMAL CONDUCTIVITIES OF POLYMER FILMS WITH THE TC-1000 THERMAL COMPARATOR

by
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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of MASTER OF SCIENCE in Materials Science and Engineering at Rochester Institute of Technology

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ABSTRACT

The single-probe, direct reading thermal comparator has been used extensively to measure the thermal conductivity of bulk solids, liquids, and gases. With the use of an analytical heat flow model based on the work of Dryden\(^1\) and the work of Carslaw and Jeager\(^2\), the comparator can also be used to obtain an in-situ measurement of the thermal conductivity of a film while attached to a substrate.

The purpose of this paper is to test this model by measuring the thermal conductivities of commercially available polymer films (DuPont Kapton and Teflon). The values obtained with the comparator were found to be \(0.24 \pm 0.02\) (W/m K) for the Kapton film and to be \(0.22 \pm 0.03\) (W/m K) for the Teflon film. Our results are consistent with the conductivity values issued by DuPont. The systematic uncertainty on our measurements is \(\pm 45\%\) and is mainly due to an inability to accurately estimate a model parameter called the "heat flow radius". This is the radius of the effective heat flow contact area between the probe tip and the film.

The TC-1000 thermal comparator technique is also reviewed in this thesis. An attempt is made to explain in detail both the calibration and operating procedures for the thermal comparator.


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1 Introduction

1.1 The Objective of the Thesis

Some research reports [Lambropoulos et al, 1989; Decker, 1984] show that the thermal conductivity of a thin film can be much lower than the corresponding conductivity of the bulk material. The thermal conductivity of a thin film is a very significant factor when the film is used in a laser device. This fact implies that it is very important to understand the thermal conductivity of a thin film in order to design more powerful lasers and to prevent catastrophic damage to the coated device because of heat accumulation.

The basis of this thesis is a thin film thermal conductivity research project which is being done as a collaboration between the RIT Thin Films Lab and the Optical Materials Group of the Laboratory for Laser Energetics at the University of Rochester. The project is centered on the use of a single-thermocouple probe or "thermal comparator" to measure the thermal conductivity of a film attached to a substrate with a known conductivity which is significantly different from that of the film. Since the measurement of thin film conductivity with the TC-1000 thermal comparator is a relatively new technique, it is important to understand the uncertainties of this technique in order to obtain meaningful results. As a basic test of this technique, some commercially available free standing polymer films with the known thermal conductivities (DuPont Teflon and Kapton) were used.

The comparative method by using the TC-1000 thermal comparator has the following features: (1) the measurement is relatively fast compared with other methods — each run requires only 10 to 15 seconds; (2) the measurement is "non-destructive" — the overall specimen is not destroyed during the measurement; and
(3) the measurement is "in-situ"—the specimen can be part of a conventional "film-on-substrate" system. Because of these unique features, the thermal comparator technique may become a fast and practical method which is extensively used in the measurement of thin film thermal conductivity.

1.2 The Definition of the Thermal Conductivity

According to the Fourier's law of heat conduction, the following relationship can be obtained for a heat flow rate $\dot{Q}$ (heat energy per unit time) through a bar with a cross sectional area $A$ and length $L$.

$$\frac{\dot{Q}}{A} = K\frac{\Delta T}{L}$$

(1.1)

where $\Delta T$ is the temperature difference between the two ends of the bar. The coefficient of proportionality, $K$, is called the thermal conductivity of the bar. (See Figure 1.1)

The thermal conductivity $K$ is a basic physical quantity which depends on the nature of the material itself. $K$ can also depend on the temperature of the material. Recent research [Lambropoulos et al. 1989; Amsden 1988; Nath and Chopra 1973] has shown that the thermal conductivity of a film depends on its thickness. The value of the thermal conductivity of a thin film is one or two orders of magnitude lower than that for the corresponding bulk material.

1.3 The Methods for the Measurement of Thermal Conductivity

Several different measurement techniques were developed during the past several decades, such as the pulse method [Kelemen 1975], the steady state method
\[ \frac{\dot{Q}}{A} = K \frac{\Delta T}{L} \]

\[ \Delta T = (T_{high} - T_{low}) \]

Figure 1.1: The sketch of a conductor with different temperature at two ends
and the transient method [ Nath and Chopra 1973 ], the guarded-comparative longitudinal heat flow method [ ASTM E1225-87 ], and the evaporation-calorimetric method [ ASTM D4351 ]. These methods can be mainly divided into two big groups. One of these is called the absolute method and the other is named the comparative method.

The absolute method just requires that we measure the heat flow rate $\dot{Q}$ and the temperature difference of two ends of the specimen to get the thermal conductivity $K$ using the equation 1.1. This method sometimes is very inconvenient and time consuming. In 1981, Hoosung Lee in IBM Corporation developed an absolute method [ Lee, 1982 ] using common laboratory equipment to measure the thermal conductivity of polymer film. He derived an expression for the thermal conductivity of a specimen and the temperature variation with time according to Fourier’s law of heat conduction and Newton’s law of cooling. This method can achieve rapidity in the measurement and simplicity in instrumentation. However, Lee’s method is not “in-situ”. It is not possible to measure the conductivity of a film when it is attached to a substrate. The comparative method will be introduced in the next chapter.
2 The Thermal Comparator Technique

2.1 Idea of a Thermal Comparator

The thermal comparator technique was developed between 1957 and 1969 by R.W. Powell. [Powell 1957; Powell et. al 1969] The idea of a thermal comparator is based on Powell’s observation that it is possible to obtain “a quantitative assessment of [thermal conductivity]... by merely handling a range of materials and arranging them in order of increasing [thermal conductivity], according to their relative coldness to the touch.” [Powell 1969] For instance, two spheres, one metal and the other ceramic, are at the same temperature. The metal sphere will “feel” cooler than the ceramic sphere. It is because the thermal conductivity of the metallic sphere is larger than that of the ceramic one.

2.2 Principle of the Thermocouple

The main component of a thermal comparator is the thermocouple which is very effective for the precision measurement of temperature. It is used extensively in industry, science research, and in the home.

In 1821, T.J. Seebeck discovered that if two dissimilar metals or alloys, A and B, are in a closed circuit, there is a continuous electric current which flows as long as the two junctions are at different temperatures. \(T_1 \neq T_2\). This current is produced by a thermoelectric EMF and this phenomenon is called the Seebeck Effect which is basic principle of the thermocouple. (See Figure 2.1)

The simplest thermocouple circuit is shown in Figure 2.2.
"Seebeck" Thermoelectric EMF $= \mathcal{E}_{AB}^{(1)} - \mathcal{E}_{AB}^{(2)}$

Figure 2.1: The conceptual sketch of a thermocouple
Figure 2.2: The simplest thermocouple with a potentiometer (Zemansky, et.al, 1966)
Table 2.1: The Thermal EMF of Important Thermocouple Materials Relative to Platinum (at 100°C)

<table>
<thead>
<tr>
<th>Material</th>
<th>Voltage (mV)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromel P</td>
<td>2.81</td>
</tr>
<tr>
<td>Iron</td>
<td>1.89</td>
</tr>
<tr>
<td>Copper</td>
<td>0.76</td>
</tr>
<tr>
<td>Alumel</td>
<td>-1.29</td>
</tr>
<tr>
<td>Constantan</td>
<td>-3.51</td>
</tr>
</tbody>
</table>


One of the junctions, called test junction, comes into thermal contact with a test sample at some temperature. The other junction, called the reference junction, is usually maintained at a constant temperature, such as the temperature of the melting point of ice or the triple-point temperature of water. It is known that the thermal EMF depends upon the sample temperature $T_{test}$, the reference temperature $T_{ref}$, and upon the metallic materials A and B. The EMF is measured with a potentiometer.

What kinds of metallic materials can be used in a thermocouple? Generally speaking, the main factors are their thermoelectric powers, stabilities, reproducibilities, melting points, mechanical and chemical properties, and cost. To make a sensitive thermocouple, a pair of materials should have a relatively large difference in their respective thermal electronegativities. For example, each metallic material listed in Table 2.1 is thermoelectrically negative with respect to all those above it or positive with respect to all those below it.
From the Table 2.1, we find that the Chromel and Constantan comprise the most sensitive pair of materials with a relative thermal EMF of 6.32 mV at 100°C; this corresponds to a sensitivity of about 63.2 μm/°C.

2.3 Direct-Reading Thermal Comparator

To improve on the two-ball thermal comparator (See Appendix A), Powell and his colleagues designed and constructed a direct-reading comparator around 1963 [Powell 1969]. In the mid-1970's, the Lafayette Instrument Company made a direct-reading thermal comparator, TC-1000 [Lafayette Instrument Co.] which was based on the design of Powell and his colleagues.

The probe of the direct-reading thermal comparator actually is a sensitive thermocouple formed by constantan and chromel. (See Figure 2.3) The chromel wire is welded inside of a constantan tube to form the test junction of the thermocouple. The constantan block is maintained at a constant temperature of 56°C and can be considered the reference junction. Two chromel wires in place of copper wires are used to send the signal to a voltage amplifier. (See Figure 2.4)

When we do the measurement, the sensing tip makes contact with the test specimen. The heater, together with the control thermocouple, maintains the temperature of the thermal reservoir (copper heating block) at a temperature of about 56°C. (20°C above ambient.) This is also the initial temperature of the sensing tip. A chromel wire is connected to the inside of this sensing tip. It combines with the constantan to form a sensitive thermocouple whose thermal EMF is of the order of 60 mV/°C. This thermocouple will translate the difference of
Figure 2.3: The probe tip of the TC-1000 thermal comparator and the sample (Lambropoulos, et.al, 1989)
Figure 2.4: The sketch of the TC-1000 thermal comparator and the voltage amplifier
the temperatures between sensing tip and the thermal reservoir into the EMF displayed on the digital voltmeter.

The Lafayette comparator is shown schematically in Figure 2.5. The probe is mounted upside-down on the end of a counter-balanced arm. There is a counter weight on the other end of the arm. Adjusting the weight sets the force exerted by the probe tip on the specimen. In our experiments, the force was set at the weight corresponding to 5 grams.

The millivolt reading generated by the constantan-chromel thermocouple is amplified one thousand times, from microvolts to millivolts, by a voltage amplifier, and then is sent through a special shielded cable to a control and readout module. Finally, the reading is displayed on a digital voltmeter.

To maintain the ambient temperature precisely for the samples, an enclosure was built by the Laboratory for Laser Energestics. It is a sealed box with outer dimensions of about three feet long, three feet wide and two feet high. There are two "glove box" holes on its front side through which the operator can handle test specimens and can access the comparator. Against the rear wall of the enclosure there is a 480 Watt strip heater. The temperature of the enclosure is controlled through a thermistor probe by a temperature controller located outside the enclosure. In order to maintain a uniform temperature throughout the enclosure there is a mini-fan inside the box. The temperature of the enclosure is usually set at about 36°C (approximately the temperature of the human body) so that the temperature doesn’t change when an operator places his hands in the glove box.

The control and readout module is very sensitive to electromagnetic interference and the human body. Its location can significantly affect the repeatability of a
Figure 2.5: The sketch of the sample stage of TC-1000 thermal comparator
voltage reading. To reduce the interference from the fan transformer and the operator's body, the module is placed on the top of the enclosure where it is far enough from the temperature controller and the fan transformer. The operator can not come too close to the module when making measurements.

2.4 Calibration Procedure and Determination of an Unknown Conductivity

To obtain the thermal conductivity of a test sample, standard samples with known thermal conductivities are used to establish a calibration curve. Since a slight change of environmental conditions can affect the thermal EMF of a standard sample, a new calibration curve is established for each test run.

Generally speaking, we can choose several standard samples based on an approximate range of the thermal conductivity of a test specimen. For example, let these standard specimens be sample A and sample B with the known thermal conductivities $K_A$ and $K_B$. We can measure them to obtain their voltage readings $V_A$ and $V_B$. (See Figure 2.6) Then we can use these two groups of data to establish a calibration curve on a semi-logarithmic paper.

For a bulk material of unknown conductivity $K_x$, how can we determine its thermal conductivity? We first measure the voltage reading $V_x$ using the direct-reading thermal comparator TC-1000. Then we locate the $V_x$ on the voltage reading axis on the calibration curve graph and draw a parallel straight line to the thermal conductivity axis (in a logarithmic scale) to intersect the calibration curve. From the intersection point, we draw another straight line that is parallel to the voltage reading axis to reach the thermal conductivity axis and which intersects the point
Figure 2.6: The measurements of standard samples A and B
Table 2.2: The thermal conductivities of standard samples

<table>
<thead>
<tr>
<th>Standard Sample</th>
<th>K(W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ebonite</td>
<td>0.18</td>
</tr>
<tr>
<td>Glass (Corning Code 7740)</td>
<td>1.15</td>
</tr>
<tr>
<td>Quartz</td>
<td>1.40</td>
</tr>
<tr>
<td>ZnO₂</td>
<td>1.77</td>
</tr>
<tr>
<td>YAG</td>
<td>4.86</td>
</tr>
<tr>
<td>Ti (A 110AT)</td>
<td>7.80</td>
</tr>
<tr>
<td>SS (316 Stainless Steel)</td>
<td>14.00</td>
</tr>
<tr>
<td>Fe (Armco Iron)</td>
<td>75.00</td>
</tr>
<tr>
<td>Si</td>
<td>149.00</td>
</tr>
<tr>
<td>Cu (Berilco Copper)</td>
<td>215.00</td>
</tr>
</tbody>
</table>

of $K_x$. So, the thermal conductivity of a bulk material, $K_x$ can be obtained. (See Figure 2.7)

A standard calibration curve has been obtained. (See Figure 2.8) The thermal conductivity is plotted on a logarithmic scale and the voltage reading is plotted on a linear scale in the figure. [Lambropoulos et. al 1989] The thermal conductivities of these standard samples are listed in Table 2.2.

2.5 One Dimensional Model of Direct-Reading Thermal Comparator

To obtain a relationship between the intermediate temperature of the sensing tip and the thermal conductivity of a specimen in a stable state, we simplify the
Figure 2.7: The calibration curve of standard samples A and B.
Figure 2.8: The standard calibration curve. Line is drawn as an aid for the eye.
heat transfer process by considering one-dimensional heat conduction. Figure 2.9 is a schematic diagram of heat flowing from a temperature reservoir (heated block) to a low temperature reservoir (enclosure atmosphere).

Before the sensing tip of the probe touches a specimen, the heated block keeps the temperature of the tip at 56°C. When the sensing tip makes contact with the surface of the specimen, its temperature drops quickly to a stable intermediate value $T_p$. This basic phenomenon can be expressed mathematically by using the Fourier law of heat conduction, which states that the local heat flux in a homogeneous medium is proportional to the negative of the local temperature gradient and can be written as following,

$$\frac{\dot{Q}}{A} = -K \frac{\partial T}{\partial x} \quad (2.1)$$

or

$$\frac{\dot{Q}}{A} = -K \frac{\Delta T}{\Delta x} \quad (2.2)$$

where $K$ is the thermal conductivity which depends upon temperature and other thermophysical properties of a given material. ($\dot{Q}/A$) is the heat flux (rate of heat transfer per unit cross-sectional area) and $T$ is the temperature in Celsius.

When the sensing tip is thought of as our reference object, the following expression will be obtained for the heat flow through the probe tip,

$$\frac{\dot{Q}_{\text{tip}}}{A} = -K_P \frac{T_{\text{high}} - T_P}{L_P} \quad (2.3)$$

On the other hand, we can think of the specimen as our reference object,

$$\frac{\dot{Q}_{\text{spec}}}{A} = -K_S \frac{T_P - T_{\text{low}}}{L_S} \quad (2.4)$$

Since there is no heat source in the sensing tip or the specimen, the heat flux through probe tip is equal to the heat flux through specimen

$$\frac{\dot{Q}_{\text{tip}}}{A} = \frac{\dot{Q}_{\text{spec}}}{A}$$
Figure 2.9: The sketch of one dimensional heat flow modeling. \( A \) is area of contact between sample and probe tip.
that is,

\[-K_p \frac{(T_{high} - T_p)}{L_p} = -K_s \frac{(T_p - T_{low})}{L_s}\]

So the intermediate temperature of the sensing tip for a certain material is

\[T_p = \frac{(K_p/L_p)T_{high} + (K_s/L_s)T_{low}}{(K_p/L_p) + (K_s/L_s)} \tag{2.5}\]

The above equation can also be written

\[K_s = K_p \left(\frac{L_s}{L_p}\right) \frac{(T_{high} - T_p)}{(T_p - T_{low})} \tag{2.6}\]

Thus, if the thermal conductivity of the probe tip \(K_p\), the length of the probe tip, \(L_p\), and the thickness of a specimen, \(L_s\), are known, and the high and low temperatures can be controlled and measured, the thermal conductivity of a specimen \(K_s\) depends uniquely on the intermediate temperature \(T_p\).

### 2.6 The Study of the Calibration Curve

We will use the one-dimensional model to derive the calibration behavior (See Figure 2.8) mathematically.

Let us begin with equation (2.5). We subtract \(T_{high}\) from both sides of that equation, and then multiply by a minus sign. Hence,

\[T_{high} - T_p = \frac{K_s(T_{high} - T_{low})}{K_p(L_s/L_p) + K_s}\]

The probe tip actually is a heated thermocouple with a reference temperature equal to \(T_{high}\), the temperature of the heater block. When the probe tip is in
thermal equilibrium at a temperature $T_p$, with a sample, the voltage reading $V$ is proportional to the difference $(T_{\text{high}} - T_p)$.

$$V = C_0(T_{\text{high}} - T_p)$$

where $C_0$ is a constant of proportionality. That is,

$$V = \frac{C_0 K_S (T_{\text{high}} - T_{\text{low}})}{K_P (L_S/L_P) + K_S}$$

or,

$$V = C_1 \frac{K_S}{(C_2 + K_S)}$$

where $C_1$ is equal to $C_0(T_{\text{high}} - T_{\text{low}})$ and $C_2$ is equal to $K_P (L_S/L_P)$. Powell [Powell, 1969] derived an equation of the same form by using the three dimensional model. The only difference is that the constant $C_2$ is the thermal conductivity of the probe tip, $K_P$. We can solve for $K_S$ from the above equation and write it in the form $K_S = f(V)$:

$$K_S = \frac{C_2 V}{(C_1 - V)}$$

Now, the question is how we can obtain the constants $C_1$ and $C_2$ for the calibration curve. Using two calibration standards, we can obtain the points $(K_{S1}, V_1)$ and $(K_{S2}, V_2)$ on the calibration curve:

$$K_{S1} = \frac{C_2 V_1}{(C_1 - V_1)} \quad (2.9)$$

$$K_{S2} = \frac{C_2 V_2}{(C_1 - V_2)} \quad (2.10)$$

We can obtain the constants of $C_1$ and $C_2$ from the above equations (2.9) and (2.10):

$$C_1 = \frac{V_1 V_2 (K_{S1} - K_{S2})}{(V_2 K_{S1} - V_1 K_{S2})}$$
\[ C_2 = \frac{K_{S1}K_{S2}(V_1 - V_2)}{(K_{S1}V_2 - K_{S2}V_1)} \]

For example, we used the following experimental pairs from the calibration data found in Figure 2.8 [Lambropoulos, et al, 1989]: (182, 1), (600, 100). The constants turn out to be:

\[ C_1 = 614 \]
\[ C_2 = 2.38 \]

The theoretical calibration curve based on \([K_S = C_2V/(C_1 - V)]\) is shown in Figure 2.10 for the conductivities between 1 W/mK and 100 W/mK. It is apparent that the curve approximately describes the standard calibration curve found in Figure 2.8.

In Figure 2.8, the curve seems to be a straight line in the range of 100 mV to 400 mV. Taking a look at our measurement data, we find that all of our data fall in this range. Therefore, the linear regression method will be used to fit the standard samples’ data and establish the calibration curve. To allow for inaccuracies in using a straight line to describe the calibration curve, we have enlarged the estimated systematic uncertainty in \(K_{app}\). (See calibration uncertainty in Table 5.2.) For another approach to describing the calibration curve, see [Lambropoulos, 1989].
Figure 2.10: The theoretical calibration curve for the one dimensional model
3 Determination of Thermal Conductivity of Polymer Film

3.1 Definitions of the Apparent Conductivity and the Effective Conductivity

When a film is chosen as a test specimen, the procedure for determining its conductivity, $K_f$, becomes more complicated using the thermal comparator technique. A bulk material with a known thermal conductivity $K_S$ is used as a substrate to support the film. (See Figure 3.1) An interface with thermal conductivity, $K_i$, obviously exists between the film and the substrate. So there are three individual conductivities in the test specimen as follows:

$K_S$: known thermal conductivity of substrate.

$K_i$: unknown thermal conductivity of interface.

$K_f$: unknown thermal conductivity of film.

When measuring the specimen by bringing the probe tip in contact with the film, we can obtain the voltage reading of a specimen. Then we use the calibration curve to get its thermal conductivity. This conductivity is called the apparent thermal conductivity, $K_{app}$, because it represents the conductivity of the substrate "modified" by the film and the interface.

In order to obtain the conductivity of just the film, some mathematical modeling is necessary to remove the contributions of the substrate and the interface to the apparent conductivity. When the contribution of the substrate is removed from $K_{app}$, the effective thermal conductivity, $K_{eff}$ remains. Hence, $K_{eff}$ represents the effective conductivity of the film and interface together.
Figure 3.1: The sketch of a specimen's composition
3.2 Crude One Dimensional Model for Extraction of $K_{eff}$ and $K_f$

In order to extract the effective thermal conductivity $K_{eff}$ and the film thermal conductivity $K_f$, some mathematical modeling should be established to derive the relationship between $K_{app}$ and $K_{eff}$, also between $K_{eff}$ and $K_f$.

Based on the definition of the (standard) thermal resistance (See Appendix B) $R = \Delta T/\dot{Q}$, we find that the total "series" thermal resistance of the film, interface, and substrate is the combination of the individual resistances.

$$R_{total} = R_S + R_{int} + R_f$$  \hspace{1cm} (3.1)

From the definition of the thermal conductivity (Equation: 2.3), the corresponding relationship between the thermal conductivity $K$ and the thermal resistance $R$ is

$$R = \frac{t}{KA}$$  \hspace{1cm} (3.2)

where $t$ is the thickness of the material.

Therefore, for a film specimen with a substrate (See Figure 3.2), there are following expressions when the heat flow section area is assumed the same within the specimen.

$$\frac{(t_f + t_i + t_s)}{K_{app}} = \frac{(t_i + t_f)}{K_{eff}} + \frac{t_s}{K_S}$$  \hspace{1cm} (3.3)

Similarly, only for the film and the interface,

$$\frac{(t_f + t_i)}{K_{eff}} = \frac{t_f}{K_f} + R_{int}$$  \hspace{1cm} (3.4)
Figure 3.2: The sketch of one dimensional modeling of heat flow through a test specimen
Since the thickness of the interface $t_i$ is much smaller than the thickness of the film $t_f$, the above two expressions can be simplified as follows,

$$\frac{(t_f + t_s)}{K_{app}} = \frac{t_f}{K_{eff}} + \frac{t_s}{K_S}$$ \hspace{1cm} (3.5)

and,

$$\frac{t_f}{K_{eff}} = \frac{t_f}{K_f} + R_{int}$$ \hspace{1cm} (3.6)

The $K_{eff}$ can be obtained by using the expression 3.5. But, the $K_f$ can't be obtained directly from the $K_{eff}$ by using the expression 3.6 because the thermal resistance of the interface, $R_{int}$, is unknown. However, the $K_f$ is the inverse of the slope of a straight line if the $(t_f/K_{eff})$ and $t_f$ are considered as two variables. Hence we are able to measure $(t_f/K_{eff})$ for a series of different thicknesses $t_f$ of the same kind of film; we can then obtain the conductivity of the film $K_f$ by using a linear regression method. The results that are obtained by using the expressions 3.5 and 3.6 are not satisfactory. The inferred value of $K_f$ is much too small, because these two expressions are derived under the assumption that the heat flow is in one direction. Actually, the heat flow goes into the specimen in all directions since the area of the sensing tip is much less than the area of the specimen. A three dimensional model will be discussed in the next section.

### 3.3 Three Dimensional Model for Extraction of $K_{eff}$ from $K_{app}$ and $K_S$

In order to extract $K_{eff}$ from $K_{app}$ and $K_S$, it is necessary to set up a three dimensional model since the heat flow conducted from the probe tip to a film specimen is not only in the longitudinal direction, but in the radial direction as well. (See Figure 3.3) Carslaw and Jeager (See Appendix C.2) derived the thermal resistance as follows:
Figure 3.3: The three dimensional modeling of heat flow through a test specimen
\[ R = \frac{1}{4aK_{app}} \]  
(3.7)

Where \( a \) is effective heat flow radius which will be discussed more detail in Section 3.4.

A constant temperature was assumed across the entire contact spot when the above equation was derived. The heat flux distribution function, \( f(r) \) is selected as follows:

\[ f(r) = \frac{\dot{Q}}{2\pi a \sqrt{a^2 - r^2}} \]  
(3.8)

over \( 0 < r < a \) and vanishing otherwise.

Dryden (See Appendix C.3) established an analytical model for calculation of the thermal resistance of a test specimen with a surface coating and evaluated the thermal resistance as,

\[ R = \frac{1}{4K_{eff}a} + \frac{2}{\pi K_{eff}a} \sum_{j=1}^{\infty} \theta^j I(2jt/a) \]  
(3.9)

where \( I(2jt/a) \) is a Bessel function in trigonometric form,

\[ I(2jt/a) = -\frac{2jt}{a} + (D - \frac{1}{2D}) \sqrt{1 - \frac{1}{D^2}} + \frac{1}{2} \arcsin(\frac{1}{D}) \]

and,

\[ D = \frac{jt}{a} + \sqrt{\left(\frac{j}{a}\right)^2 + 1} \]

and \( \theta \) is a function of \( K_{eff}/K_S \):

\[ \theta = \theta(K_{eff}/K_S) = \frac{(K_{eff}/K_S) - 1}{(K_{eff}/K_S) + 1} \]

Equating Equation 3.7 and Equation 3.9, the relationship between the \( K_{eff} \) and \( K_{app} \) in three dimensions is obtained.

\[ \frac{1}{K_{app}} = \frac{1}{K_{eff}} + \frac{8}{\pi K_{eff}} \sum_{j=1}^{\infty} \theta^j I(2jt/a) \]  
(3.10)
Since $K_{eff}$ is implicitly contained in the infinite sum, it is impossible to directly calculate $K_{eff}$ from $K_{app}$ and $K_S$ using Equation 3.10; an iterative method must be used to extract $K_{eff}$ from $K_{app}$ and $K_S$. A range of $K_{eff}$ is assumed and a computer program is used to establish a table of $K_{app}$ corresponding to each $K_{eff}$. Then, $K_{eff}$ for different film thicknesses is determined by using this table and an interpolation method. The book value of $K_S$ is used in the calculation of $K_{eff}$ (See Table 2.2). All these are done by computer. ( See Appendix D )

3.4 Estimation of the Heat Flow Radius “a”

In Section 3.3, the heat flow radius “a” is introduced. The heat flow radius is the effective radius of the circular “thermal contact” area between the probe tip and the film. It is the main parameter in the calculation of the effective conductivity, $K_{eff}$.

The thermal contact area depends on the followings:

- The physical contact area between the probe tip and the film.
- The extent to which convection around the probe tip occurs.

In order to measure the radius of the physical contact area for our test specimen, we took a picture under an optical microscope of the indentation in the Teflon film which the 5 gram load probe tip left. (See Figure 3.4). The radius determined from the picture about $35\mu m$. Due to the recovery of the elastic deformation of the indentation by the probe tip, the actual radius of physical contact area is certainly larger than the radius that we measured.
Figure 3.4 The optical microscope photograph of the probe tip indentation
Magnification: 500x
In Amsden’s thesis [Amsden, 1988], he indicated that the base diameter of the probe tip is about 670\(\mu m\) as determined with an optical comparator. He took pictures of the bare probe tip and the probe tip contacting a dielectric thin film (See Figure 3.5(a) and (b)). From this information, the radius of physical contact area can be calculated geometrically (See Figure 3.5(c) and (d)) and its value is approximately equal to 168\(\mu m\). Since the films (DuPont Kapton and Teflon) of our test are much softer than the dielectric thin film, the radius of physical contact area for the polymer film should be larger than that for the dielectric thin film.

There is no convection if the measurement is made in a vacuum. But, our measurements were made in the environmental medium of air. The convection, no doubt, exists around the probe tip. In order to get a best estimate of the heat flow radius, we take the average of the physical contact radius 168\(\mu m\) and the radius of the base of the probe tip 335\(\mu m\). Hence, the estimation of the thermal heat flow radius is about 250\(\mu m\) which is used in our calculations.
Figure 3.5: (a) The bare probe tip. (b) The probe tip contacting a sample. (c) The sketch of probe tip contacting a sample. (d) The geometrical calculation.
4 Experimental Procedure and Data Acquisition

4.1 Preparation of the Samples

Polymer films (Kapton and Teflon manufactured by DuPont Company. See Appendix E) ranging in thickness from $12.7 \mu m$ to $127 \mu m$ were selected for measurements with the TC-1000 thermal comparator. Silicon, copper, titanium, quartz and glass substrates were also used. Several different ways were used to combine a film and a substrate (See Figure 4.1): (1) a silicone grease interface was placed between a single layer film and a substrate; (2) a direct contact or air interface was used between a film and a substrate; (3) for a multi-layer sample, the silicone grease was placed only between the first layer and the substrate. The substrates were cylindrical discs whose dimensions are listed in the Table 4.1. The different thicknesses of polymer films are cut into the same size as the substrates. The surface condition of substrates is mirror polished level.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Thickness (mm)</th>
<th>Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>10.0</td>
<td>31.8</td>
</tr>
<tr>
<td>Copper</td>
<td>10.0</td>
<td>25.4</td>
</tr>
<tr>
<td>Titanium</td>
<td>25.4</td>
<td>31.8</td>
</tr>
<tr>
<td>Glass</td>
<td>12.7</td>
<td>50.8</td>
</tr>
<tr>
<td>Quartz</td>
<td>12.7</td>
<td>50.8</td>
</tr>
</tbody>
</table>
Figure 4.1: (a) Single layer film with air interface. (b) Single layer film with silicone grease interface. (c) Multi-layer film with air interface. (d) Multi-layer film with silicone grease interface.
4.2 Operating Procedure of the Measurement

Measurements of thermal conductivity were performed with the TC-1000 thermal comparator at the Laboratory for Laser Energetics of the University of Rochester. The samples were placed into a glove-box (enclosing the comparator) at least two hours in advance of a measurement to insure that they would be in thermal equilibrium with the controlled environment. The temperature of the box is maintained at $36^\circ C$ and the temperature of the probe tip is at $56^\circ C$, $20^\circ C$ greater than the temperature of the box. To insure the same contact force with the probe tip for all samples, a load of $5\, mg$ was chosen.

During the measurements, we found that the initial display value of the thermal comparator varies at random with time after it is adjusted to zero. Before the measurement is made, the variation of the initial display value is from $-5$ to $+15$ millivolts. The initial display value at the instant before a measurement correlates closely with the final display value after a measurement. The data in Table 4.2 show how the raw measurement values correlate with the initial display values.

Table 4.2: The variation of raw measurement value with the initial display value

<table>
<thead>
<tr>
<th>Initial display value (mV)</th>
<th>-10</th>
<th>1</th>
<th>3</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement value (mV)</td>
<td>389</td>
<td>401</td>
<td>402</td>
<td>411</td>
<td>415</td>
</tr>
<tr>
<td>Corrected value (mV)</td>
<td>399</td>
<td>400</td>
<td>399</td>
<td>401</td>
<td>400</td>
</tr>
</tbody>
</table>

To correct the raw measurement value, we do the following:

$$(Corrected\ value) = (Raw\ measurement\ value) - (Initial\ display\ value) \quad (4.1)$$

Since the corrected values in Table 4.2 are nearly all the same, we can conclude that there is a definite relationship between the initial display value and the raw
measurement value. This is a very important relationship which was found during our data-taking. This correction will be applied to all measurements.

4.3 Establishment of a Calibration Curve

The standard samples should have stable voltage reading in every test run and the data range should closely correspond to the required range of calibration data. The following materials were used as the standard samples: Ebonite \((K = 0.18)\), Quartz \((K = 1.4)\), \(ZrO_2(K = 1.77)\), and YAG \((K = 4.86)\) (All units for thermal conductivity are W/mK.). We plotted the thermal conductivities and thermal EMF data of the standard samples on semi-logarithmic paper and found a linear relationship between the logarithmic values of the thermal conductivities and the millivolt readings of the standard samples. We used the linear regression method and obtain the equation of calibration curve for \(\log_{10} K\) versus \(X\), where \(X\) is a voltage reading:

\[
\log_{10} K = A \times X + B
\]

With this equation, it is easy to determine the apparent thermal conductivity of a test specimen from its voltage reading.

\[
K = 10^{A \times X + B}
\]

4.4 Data and Calculations

The following tables contain measurements for polymer films on the different substrates, with or without the silicone grease interface. The calibration data for the standard samples is given in the first table for each set of runs.
### Table 4.3: Calibration Data (June 22, 1989)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Ebonite</th>
<th>Quartz</th>
<th>YAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>75</td>
<td>210</td>
<td>292</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>73</td>
<td>208</td>
<td>290</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>74</td>
<td>209</td>
<td>291</td>
</tr>
<tr>
<td>$K$ (W/mK)</td>
<td>0.18</td>
<td>1.4</td>
<td>4.86</td>
</tr>
</tbody>
</table>

### Table 4.4: Data for Multi-layer Kapton on Quartz. Interface: No Grease (June 22, 1989)

<table>
<thead>
<tr>
<th>Thickness ($\mu$m)</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>165</td>
<td>149</td>
<td>140</td>
<td>134</td>
<td>130</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>165</td>
<td>149</td>
<td>140</td>
<td>133</td>
<td>130</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>165</td>
<td>149</td>
<td>140</td>
<td>133.5</td>
<td>130</td>
</tr>
<tr>
<td>$K_{app}$ (W/mK)</td>
<td>0.717</td>
<td>0.563</td>
<td>0.491</td>
<td>0.445</td>
<td>0.422</td>
</tr>
<tr>
<td>$K_{eff}$ (W/mK)</td>
<td>0.157</td>
<td>0.181</td>
<td>0.199</td>
<td>0.211</td>
<td>0.226</td>
</tr>
</tbody>
</table>

### Table 4.5: Data for Multi-layer Kapton on Silicon. Interface: Silicone Grease between First Layer and Substrate (June 22, 1989)

<table>
<thead>
<tr>
<th>Thickness ($\mu$m)</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>277</td>
<td>227</td>
<td>188</td>
<td>172</td>
<td>166</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>272</td>
<td>224</td>
<td>186</td>
<td>174</td>
<td>166</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>274.5</td>
<td>226.5</td>
<td>187</td>
<td>173</td>
<td>166</td>
</tr>
<tr>
<td>$K_{app}$ (W/mK)</td>
<td>3.784</td>
<td>1.798</td>
<td>1.002</td>
<td>0.810</td>
<td>0.728</td>
</tr>
<tr>
<td>$K_{eff}$ (W/mK)</td>
<td>0.428</td>
<td>0.371</td>
<td>0.288</td>
<td>0.291</td>
<td>0.306</td>
</tr>
</tbody>
</table>
\[ K = 10^{-1.2328 + 0.0066 \cdot (mV)} \quad R : 1.000 \]

Figure 4.2: Calibration Curve, (June 22, 1989), \( R \) is correlation coefficient.

Standard Samples: Ebonite; Quartz; YAG

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Figure 4.3: Graph of $t/K_{eff}$ Versus $t$, (June 22, 1989)

Multi-layer Kapton Polymer Film on Quartz Substrate

Interface: No Grease
\[ \frac{t}{K_{eff}} = -31.97 + \frac{1}{0.28}t \]

\( R = 0.994 \)

Figure 4.4: Graph of \( \frac{t}{K_{eff}} \) Versus \( t \), (June 22, 1989)

Multi-layer Kapton Polymer Film on Silicon Substrate

Interface: Silicone Grease between First Layer and Substrate

43
Table 4.6: Calibration Data (June 26, 1989)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Ebonite (mV)</th>
<th>Quartz (mV)</th>
<th>Glass (mV)</th>
<th>YAG (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>72</td>
<td>216</td>
<td>196</td>
<td>306</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>74</td>
<td>210</td>
<td>190</td>
<td>308</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>73</td>
<td>213</td>
<td>193</td>
<td>307</td>
</tr>
<tr>
<td>(K \text{ (W/mK)})</td>
<td>0.18</td>
<td>1.4</td>
<td>1.15</td>
<td>4.86</td>
</tr>
</tbody>
</table>

Table 4.7: Data for Multi-layer Kapton on Silicon. Interface: Silicone Grease between First Layer and Substrate (June 26, 1989)

<table>
<thead>
<tr>
<th>Thickness ((\mu\text{m}))</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>260</td>
<td>190</td>
<td>174</td>
<td>164</td>
<td>151</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>254</td>
<td>190</td>
<td>170</td>
<td>162</td>
<td>151</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>257</td>
<td>190</td>
<td>172</td>
<td>163</td>
<td>151</td>
</tr>
<tr>
<td>(K_{app} \text{ (W/mK)})</td>
<td>2.558</td>
<td>0.994</td>
<td>0.771</td>
<td>0.679</td>
<td>0.573</td>
</tr>
<tr>
<td>(K_{eff} \text{ (W/mK)})</td>
<td>0.288</td>
<td>0.204</td>
<td>0.222</td>
<td>0.244</td>
<td>0.241</td>
</tr>
</tbody>
</table>

Table 4.8: Data for Multi-layer Kapton on Silicon. Interface: No Grease (June 26, 1989)

<table>
<thead>
<tr>
<th>Thickness ((\mu\text{m}))</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>218</td>
<td>179</td>
<td>163</td>
<td>154</td>
<td>146</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>222</td>
<td>182</td>
<td>165</td>
<td>156</td>
<td>146</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>220</td>
<td>180.5</td>
<td>164</td>
<td>155</td>
<td>146</td>
</tr>
<tr>
<td>(K_{app} \text{ (W/mK)})</td>
<td>1.518</td>
<td>0.869</td>
<td>0.689</td>
<td>0.607</td>
<td>0.534</td>
</tr>
<tr>
<td>(K_{eff} \text{ (W/mK)})</td>
<td>0.170</td>
<td>0.179</td>
<td>0.198</td>
<td>0.218</td>
<td>0.225</td>
</tr>
</tbody>
</table>
Table 4.9: Data for Multi-layer Kapton on Glass. Interface: No Grease (June 26, 1989)

<table>
<thead>
<tr>
<th>Thickness ($\mu m$)</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>163</td>
<td>148</td>
<td>137</td>
<td>130</td>
<td>120</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>164</td>
<td>150</td>
<td>139</td>
<td>130</td>
<td>121</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>163.5</td>
<td>149</td>
<td>138</td>
<td>130</td>
<td>120.5</td>
</tr>
<tr>
<td>$K_{app} (W/m K)$</td>
<td>0.684</td>
<td>0.557</td>
<td>0.477</td>
<td>0.426</td>
<td>0.373</td>
</tr>
<tr>
<td>$K_{eff} (W/m K)$</td>
<td>0.176</td>
<td>0.202</td>
<td>0.209</td>
<td>0.214</td>
<td>0.204</td>
</tr>
</tbody>
</table>

Table 4.10: Data for Multi-layer Kapton on Titanium. Interface: No Grease (June 26, 1989)

<table>
<thead>
<tr>
<th>Thickness ($\mu m$)</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>202</td>
<td>174</td>
<td>154</td>
<td>145</td>
<td>141</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>204</td>
<td>176</td>
<td>156</td>
<td>145</td>
<td>141</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>203</td>
<td>175</td>
<td>155</td>
<td>145</td>
<td>141</td>
</tr>
<tr>
<td>$K_{app} (W/m K)$</td>
<td>1.194</td>
<td>0.804</td>
<td>0.607</td>
<td>0.527</td>
<td>0.498</td>
</tr>
<tr>
<td>$K_{eff} (W/m K)$</td>
<td>0.155</td>
<td>0.182</td>
<td>0.186</td>
<td>0.199</td>
<td>0.219</td>
</tr>
</tbody>
</table>
Figure 4.5: Calibration curve, (June 26, 1989),
Standard samples: Ebonite; Quartz; Glass; YAG
\[
\frac{t}{K_{eff}} = 11.37 + \left( \frac{1}{0.24} \right) t
\]

\[R = 0.990\]

Figure 4.6: Graph of \(t/K_{eff}\) Versus \(t\), (June 26, 1989)

Multi-layer Kapton Polymer Film on Silicon Substrate

Interface: Silicone Grease between First Layer and Substrate
Figure 4.7: Graph of $t/K_{eff}$ Versus $t$, (June 26, 1989)
Multi-layer Kapton Polymer Film on Silicon Substrate
Interface: No Grease
\[ \frac{t}{K_{ef}} = 17.13 + (1/0.21) \times t \quad R = 0.998 \]

Figure 4.8: Graph of \( t/K_{ef} \) Versus \( t \), (June 26, 1989)

Multi-layer Kapton Polymer Film on Glass Substrate

Interface: No Grease
(t/K_{eff}) = 70.08 + (1/0.24) \times t \quad \text{R}=0.994

Figure 4.9: Graph of t/K_{eff} Versus t, (June 26, 1989)
Multi-layer Kapton Polymer Film on Titanium Substrate
Interface: No Grease
Table 4.11: Calibration Data (June 28, 1989)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Ebonite</th>
<th>Glass</th>
<th>Quartz</th>
<th>ZrO₂</th>
<th>YAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>75</td>
<td>200</td>
<td>212</td>
<td>248</td>
<td>313</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>73</td>
<td>196</td>
<td>212</td>
<td>246</td>
<td>312</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>74</td>
<td>198</td>
<td>212</td>
<td>247</td>
<td>312.5</td>
</tr>
<tr>
<td>(K) (W/mK)</td>
<td>0.18</td>
<td>1.15</td>
<td>1.4</td>
<td>1.77</td>
<td>4.86</td>
</tr>
</tbody>
</table>

Table 4.12: Data for Teflon Multi-layer on Silicon. Interface: No Grease (June 28, 1989)

<table>
<thead>
<tr>
<th>Thickness ((\mu m))</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>101.6</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>207</td>
<td>165</td>
<td>142</td>
<td>130</td>
<td>124</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>210</td>
<td>165</td>
<td>140</td>
<td>130</td>
<td>124</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>208.5</td>
<td>165</td>
<td>141</td>
<td>130</td>
<td>124</td>
</tr>
<tr>
<td>(K_{app}) (W/mK)</td>
<td>1.197</td>
<td>0.661</td>
<td>0.476</td>
<td>0.410</td>
<td>0.378</td>
</tr>
<tr>
<td>(K_{eff}) (W/mK)</td>
<td>0.133</td>
<td>0.136</td>
<td>0.137</td>
<td>0.147</td>
<td>0.159</td>
</tr>
</tbody>
</table>

Table 4.13: Data for Kapton on Silicon. Interface: No Grease (June 28, 1989)

<table>
<thead>
<tr>
<th>Thickness ((\mu m))</th>
<th>12.7</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>258</td>
<td>228</td>
<td>184</td>
<td>164</td>
<td>144</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>256</td>
<td>224</td>
<td>182</td>
<td>162</td>
<td>146</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>257</td>
<td>226</td>
<td>183</td>
<td>163</td>
<td>145</td>
</tr>
<tr>
<td>(K_{app}) (W/mK)</td>
<td>2.321</td>
<td>1.520</td>
<td>0.845</td>
<td>0.643</td>
<td>0.503</td>
</tr>
<tr>
<td>(K_{eff}) (W/mK)</td>
<td>0.137</td>
<td>0.170</td>
<td>0.174</td>
<td>0.185</td>
<td>0.212</td>
</tr>
</tbody>
</table>
Figure 4.10: Calibration Curve, (June 28, 1989),
Standard Samples: Ebonite; Quartz; Glass; ZrO$_2$; YAG
\[
\frac{t}{K_{eff}} = 62.06 + (1/0.17)t
\]

R = 0.994

Figure 4.11: Graph of \(\frac{t}{K_{eff}}\) Versus \(t\), (June 28, 1989)

Multi-layer Teflon Polymer Film on Silicon Substrate

Interface: No Grease
\[ \frac{t}{K_{eff}} = 47.15 + (1/0.22)*t \quad \text{R} = 0.996 \]

Figure 4.12: Graph of \( t/K_{eff} \) Versus \( t \), (June 28, 1989)
Kapton Polymer Film on Silicon Substrate
Interface: No Grease
Table 4.14: Calibration Data (July 12, 1989)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Ebonite</th>
<th>Quartz</th>
<th>Glass</th>
<th>ZrO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>70</td>
<td>201</td>
<td>194</td>
<td>246</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>71</td>
<td>203</td>
<td>194</td>
<td>243</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>70.5</td>
<td>202</td>
<td>194</td>
<td>244.5</td>
</tr>
<tr>
<td>$K$ (W/mK)</td>
<td>0.18</td>
<td>1.4</td>
<td>1.15</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Table 4.15: Data for Teflon on Copper. Interface: No Grease (July 12, 1989)

<table>
<thead>
<tr>
<th>Thickness (µm)</th>
<th>12.7</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>240</td>
<td>206</td>
<td>180</td>
<td>156</td>
<td>137</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>240</td>
<td>208</td>
<td>176</td>
<td>156</td>
<td>135</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>240</td>
<td>207</td>
<td>178</td>
<td>156</td>
<td>136</td>
</tr>
<tr>
<td>$K_{app}$ (W/mK)</td>
<td>2.002</td>
<td>1.268</td>
<td>0.849</td>
<td>0.627</td>
<td>0.475</td>
</tr>
<tr>
<td>$K_{eff}$ (W/mK)</td>
<td>0.117</td>
<td>0.141</td>
<td>0.174</td>
<td>0.180</td>
<td>0.200</td>
</tr>
</tbody>
</table>

Table 4.16: Data for Teflon on Silicon. Interface: No Grease (July 12, 1989)

<table>
<thead>
<tr>
<th>Thickness (µm)</th>
<th>12.7</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st reading (mV)</td>
<td>250</td>
<td>213</td>
<td>180</td>
<td>158</td>
<td>135</td>
</tr>
<tr>
<td>2nd reading (mV)</td>
<td>250</td>
<td>213</td>
<td>185</td>
<td>158</td>
<td>135</td>
</tr>
<tr>
<td>Average reading (mV)</td>
<td>250</td>
<td>213</td>
<td>182.5</td>
<td>158</td>
<td>135</td>
</tr>
<tr>
<td>$K_{app}$ (W/mK)</td>
<td>2.298</td>
<td>1.378</td>
<td>0.904</td>
<td>0.644</td>
<td>0.469</td>
</tr>
<tr>
<td>$K_{eff}$ (W/mK)</td>
<td>0.136</td>
<td>0.154</td>
<td>0.186</td>
<td>0.185</td>
<td>0.197</td>
</tr>
</tbody>
</table>
$K = 10^{-1.1400 + 0.0060 \cdot (mV)} \quad R: 0.989$

Figure 4.13: Calibration curve, (July 12, 1989),
Standard samples: Ebonite; Quartz; Glass; ZrO$_2$
\( (t/K_{eff}) = 59.48 + (1/0.22) \times t \)  \( \text{R}=0.999 \)

Figure 4.14: Graph of \( t/K_{eff} \) Versus \( t \), (July 12, 1989)
Teflon Polymer Film on Copper Substrate
Interface: No Grease
\[ \frac{t}{K_{eff}} = 37.02 + (1/0.21) \times t \]

\[ R = 1.000 \]

Figure 4.15: Graph of \( t/K_{eff} \) Versus \( t \), (July 12, 1989)

Teflon Polymer Film on Silicon Substrate

Interface: No Grease
5 Conclusion

5.1 Results of Thermal Conductivities and Interfacial Resistances

According to the basic one dimensional modeling of heat transfer, we can obtain the following equation (See equation 3.6):

\[
\left( \frac{t}{K_{\text{eff}}} \right) = \left( \frac{1}{K_f} \right) * t + R_{\text{int}}
\]

From the above equation, we learn that its slope is equal to the inverse of the thermal conductivity of the film and the intercept equals the interfacial resistance.

Table 5.1: Results of Measurements

<table>
<thead>
<tr>
<th>#</th>
<th>Film</th>
<th>Substrate</th>
<th>Single/Multi Layer</th>
<th>Interface</th>
<th>( K_f ) (W/m K)</th>
<th>( R_{\text{int}} ) (10^{-6}m^2K/W)</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kapton</td>
<td>Quartz</td>
<td>Multi</td>
<td>Air</td>
<td>0.25</td>
<td>73</td>
<td>6/22/89</td>
</tr>
<tr>
<td>2</td>
<td>Kapton</td>
<td>Silicon</td>
<td>Multi</td>
<td>Grease</td>
<td>0.28</td>
<td>-32</td>
<td>6/22/89</td>
</tr>
<tr>
<td>3</td>
<td>Kapton</td>
<td>Silicon</td>
<td>Multi</td>
<td>Grease</td>
<td>0.24</td>
<td>11</td>
<td>6/26/89</td>
</tr>
<tr>
<td>4</td>
<td>Kapton</td>
<td>Silicon</td>
<td>Multi</td>
<td>Air</td>
<td>0.25</td>
<td>66</td>
<td>6/26/89</td>
</tr>
<tr>
<td>5</td>
<td>Kapton</td>
<td>Glass</td>
<td>Multi</td>
<td>Air</td>
<td>0.21</td>
<td>17</td>
<td>6/26/89</td>
</tr>
<tr>
<td>6</td>
<td>Kapton</td>
<td>Titanium</td>
<td>Multi</td>
<td>Air</td>
<td>0.24</td>
<td>70</td>
<td>6/26/89</td>
</tr>
<tr>
<td>7</td>
<td>Teflon</td>
<td>Silicon</td>
<td>Multi</td>
<td>Air</td>
<td>0.17</td>
<td>62</td>
<td>6/28/89</td>
</tr>
<tr>
<td>8</td>
<td>Kapton</td>
<td>Silicon</td>
<td>Single</td>
<td>Air</td>
<td>0.22</td>
<td>47</td>
<td>6/28/89</td>
</tr>
<tr>
<td>9</td>
<td>Teflon</td>
<td>Copper</td>
<td>Single</td>
<td>Air</td>
<td>0.22</td>
<td>59</td>
<td>7/12/89</td>
</tr>
<tr>
<td>10</td>
<td>Teflon</td>
<td>Silicon</td>
<td>Single</td>
<td>Air</td>
<td>0.21</td>
<td>37</td>
<td>7/12/89</td>
</tr>
</tbody>
</table>
if \((t/K_{eff})\) and \(t\) are the variables. Hence, we can get our final results by applying the linear regression method to determine the slope and intercept of the straight line. The resulting thermal conductivities of the polymer films and the interfacial resistances are both listed in the Table 5.1.

From the Table 5.1, the following points can be made:

1. The measured thermal conductivity of Teflon polymer film is about 0.20 ± 0.03 (W/m K) (95% confidence level).

2. The measured thermal conductivity of Kapton polymer film is about 0.24 ± 0.02 (W/m K) (95% confidence level).

3. The interfacial resistance of the grease is significantly different from that of the air (except for the results of run #5-6/26/89).

Comparing the known thermal conductivities of the Teflon and Kapton polymer films with our data, we found that our Teflon result 0.20 ± 0.03 (W/m K) is almost the same as the DuPont’s value of 0.20 (W/m K)[DuPont, E67603]; and the Kapton result 0.24 ± 0.02 (W/m K) is two times larger than the value (0.12 W/m K)[DuPont, 1989] issued by the DuPont Company.

Note: The DuPont values were measured for 25.4\(\mu\)m thick films. Since 25.4\(\mu\)m is considered “thick”, we will be assuming that \(K_f \approx K_{bulk}\) for films equal to or thicker than 25.4\(\mu\)m.

5.2 Estimated Uncertainties

The estimated statistical and systematic uncertainties for each stage of data reduction are listed in the Table 5.2.
Table 5.2: Estimated Uncertainties of Measurement Results

<table>
<thead>
<tr>
<th></th>
<th>Statistical</th>
<th>Principal Origin</th>
<th>Systematic</th>
<th>Principal Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC-1000 Voltage Reading</td>
<td>±2%</td>
<td>Electrical</td>
<td>Not</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fluctuation</td>
<td>Applicable</td>
<td></td>
</tr>
<tr>
<td>Apparent Conductivity</td>
<td>±5%</td>
<td>Calibration</td>
<td>±10%</td>
<td>Calibration</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Technique</td>
<td></td>
<td>Technique</td>
</tr>
<tr>
<td>Effective Conductivity</td>
<td>±10%</td>
<td>Data Reduction</td>
<td>±35%</td>
<td>Uncertainty in</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Technique</td>
<td></td>
<td>Heat Flow Radius</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>a = 250 ± 100 (µm)</td>
</tr>
<tr>
<td>Film Conductivity</td>
<td>±15%</td>
<td>Data Reduction</td>
<td>±45%</td>
<td>Use of One</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Technique</td>
<td></td>
<td>Dimensional Equation</td>
</tr>
</tbody>
</table>

Hence, the estimated statistical and systematic uncertainties in a film conductivity are ±15% and ±45%, respectively.

5.3 Sources of Systematic Uncertainty

The sources of systematic uncertainty are mainly from the load of probe tip, the uncertainty in the heat flow radius “a”, and the mathematical modeling of the extraction of $K_f$ from $K_{eff}$.

Sensitivity to Heat Flow Radius. Since the thermal heat flow radius is invisible and is very difficult to measure accurately, we estimated the heat flow radius “a” as 250µm in our calculation for the extraction of $K_{eff}$ from $K_{app}$ (See
Table 5.3: Variation of the Conductivity and Resistance with the Heat Flow Radius

<table>
<thead>
<tr>
<th>Heat Flow Radius $a$ ($\mu m$)</th>
<th>Conductivity $K_f$ (W/m K)</th>
<th>Resistance $R_{int} (10^{-6} m^2 K/W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.37</td>
<td>-24</td>
</tr>
<tr>
<td>150</td>
<td>0.32</td>
<td>-10</td>
</tr>
<tr>
<td>200</td>
<td>0.28</td>
<td>0.99</td>
</tr>
<tr>
<td>250</td>
<td>0.24</td>
<td>11</td>
</tr>
<tr>
<td>300</td>
<td>0.22</td>
<td>21</td>
</tr>
<tr>
<td>350</td>
<td>0.19</td>
<td>30</td>
</tr>
</tbody>
</table>

Section 3.4). In order to get some ideas about how the heat flow radius affects the calculation of $K_f$, we used 100$\mu m$, 150$\mu m$, 200$\mu m$, 300$\mu m$ and 350$\mu m$ as the heat flow radius to make calculations for a certain run (Table 4.7). The results are listed in Table 5.3. The thermal conductivity versus the heat flow radius and the thermal resistance versus heat flow radius are plotted in Figure 5.1 and Figure 5.2, respectively. From these data, we found that there is a deviation of $0.24^{+0.05}_{-0.08}$ (W/m K) for the film conductivity when the deviation of the heat flow radius is $250 \pm 100\mu m$. Also, we found that the interfacial resistance varies as $11^{+19}_{-22} (10^{-6} m^2 K/W)$ when the heat flow radius varies as $250 \pm 100\mu m$.

Mathematical Modeling. When obtaining the value of $K_f$ from the slope of the regression straight line of $t/K_{eff}$ versus $t$, we assumed that the following relation among $K_{eff}$, $K_f$, and $t$ is available:

$$\frac{t}{K_{eff}} = \frac{t}{K_f} + R_{int}$$

In fact, the above relation is just valid when the thickness of the film, $t$, is much smaller than the heat flow radius “$a$". In our experiment, the thicknesses of the
Figure 5.1: The thermal conductivity of film versus heat flow radius
Figure 5.2: The thermal resistance of film versus heat flow radius
films ranged from 12.7µm to 127µm, hence the “t” was nearly comparable to the “a” (estimated to be 250µm). Thus, the heat transfer can not be thought of as only one dimensional between the probe tip, the interface, and the film. This means that the above equation is not completely correct. Since we don’t have a perfect three dimensional mathematical model yet, we have to apply this one dimensional equation to obtain the thermal conductivity of polymer film, $K_f$. This may be another source of the systematic uncertainty of our results.

When establishing the relationship between the apparent conductivity and the effective conductivity, we applied the uniform temperature model and the resistance is: $R_1 = 1/(4aK)$ (See Appendix C.2.1), which means that the temperature is uniform over the probe-film contact area during the measurement. Hence, other uncertainties may come from the assumptions used in our mathematical model. In Appendix C.2.2, a uniform heat flow model was also discussed and the resistance is: $R_2 = 8/(3\pi^2aK)$. In making a comparison between these two models, we found:

$$\frac{R_2}{R_1} = \frac{32}{3\pi^2} = 1.08$$

that is an 8% difference.

**Elasticity/Hardness of a Sample.** When the probe tip contacts a test specimen, the indentation will be different for specimens with the different elasticity/hardness. The indentation of a softer film (Polymer film) is larger than that of a harder film (Dielectric film) (See Figure 5.3). Hence, the effective thermal heat flow radius varies not only with the load of probe tip but with the properties of specimen as well. Amsden [Amsden, 1988] mentioned in his master’s thesis that the load of probe tip would effect the voltage reading, especially for a softer film specimen. The higher load may reduce the noise in the readings taken. However the higher load will also deform the testing surface of a softer film specimen. For
Figure 5.3: The sketch of indentations of specimens with different elasticity/hardness (a) Undistorted harder film (b) Distorted softer film.
a polymer, this will tend to increase the effective heat flow radius, thereby producing too high of a thermal conductivity. The five-gram load was used in our measurements.

5.4 Suggestion for Future Studies

The thermal comparator technique is a relatively effective method in the measurement of thermal conductivity of materials, especially for the thin film. [Lambropoulos, 1989] But, when the thickness of a film is comparative to the heat flow radius, this technique needs more work to be improved. Future studies should be mainly the following:

(1) In order to extract accurately $K_f$ from $K_{eff}$, the three dimensional analytical modeling must be established for a thick film (A film is “thick” if its thickness is comparable to the value of the heat flow radius). At present, what we are able to use is an one dimensional equation 3.6. Note: At the defense of this thesis, some good ideas on a three dimensional approach were described in detail. [Lambropoulos, 1990].

(2) To reduce the uncertainty in the estimation of the heat flow radius, it is necessary to establish an effective method for determining the heat flow radius for different loads of the probe tip and for different degrees of film hardness or elasticity.

(3) Measure $K_{app}$ and $K_{eff}$ for thick films. As a film becomes thicker, the effect of the interface on $K_{app}$ (and $K_{eff}$) decreases. A preliminary study was done on films which ranged in thickness from 12.7 $\mu$m to 508.0 $\mu$m. The voltage readings are
listed in Table 5.4. Figure 5.4 shows the apparent conductivity versus thickness data. We found that the curve approaches the known bulk thermal conductivity (0.20 W/mK) of the Teflon film when the thickness increases and the interface becomes less important to the apparent thermal conductivity. So

$$\lim_{t \to \infty} K_{app} = K_f$$

But if we plot $K_{eff}$ versus $t$ with different heat flow radii (See Figure 5.5) by using the data produced by computer program in Appendix D. we find that $K_{eff}$

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Standard Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness $t$ ($\mu$m)</td>
<td>Voltage Reading (mV)</td>
</tr>
<tr>
<td>12.7</td>
<td>250.0</td>
</tr>
<tr>
<td>25.4</td>
<td>213.0</td>
</tr>
<tr>
<td>50.8</td>
<td>182.5</td>
</tr>
<tr>
<td>76.2</td>
<td>158.0</td>
</tr>
<tr>
<td>127.0</td>
<td>135.0</td>
</tr>
<tr>
<td>177.8</td>
<td>122.0</td>
</tr>
<tr>
<td>254.0</td>
<td>110.0</td>
</tr>
<tr>
<td>355.6</td>
<td>100.0</td>
</tr>
<tr>
<td>508.0</td>
<td>90.0</td>
</tr>
</tbody>
</table>
Figure 5.4: The apparent thermal conductivity versus thickness for Teflon film on silicon substrate.
Figure 5.5: The effective thermal conductivity versus thickness for Teflon film on silicon substrate for different heat flow radii.
approaches 0.20 W/mK, the known film thermal conductivity, for only a heat flow radius of 250 μm. This seems to imply that there is another way to determine the heat flow radius “a” using \( K_{\text{bulk}} \) as the known thermal conductivity of a “thick” film.

(4) From our initial voltage readings (See Table 5.5) and the final results (See Table 5.1), the thermal conductivity of the Kapton film is almost the same as that of the Teflon film. It seems that the relative thermal conductivities issued by the DuPont Company (\( K_{\text{Teflon}} > K_{\text{Kapton}} \)) are not in agreement with the respective relative values of our data (\( K_{\text{Kapton}} \approx K_{\text{Teflon}} \)). Hence, we are unable to use those data to calibrate the heat flow radius. Our suggestion is that someone find an exact standard sample to use as a calibration method for determining the heat flow radius in later studies.

Table 5.5: The Comparison of Voltage Readings and Film Conductivities of Teflon and Kapton

<table>
<thead>
<tr>
<th>Thickness (μm)</th>
<th>25.4</th>
<th>50.8</th>
<th>76.2</th>
<th>127</th>
<th>( K_f ) (W/mK)</th>
<th>Table #</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teflon (Multi)(mV)</td>
<td>208.5</td>
<td>165</td>
<td>141</td>
<td>124</td>
<td>0.23</td>
<td>4.12</td>
</tr>
<tr>
<td>Kapton (Multi)(mV)</td>
<td>220</td>
<td>180.5</td>
<td>164</td>
<td>146</td>
<td>0.25</td>
<td>4.8</td>
</tr>
<tr>
<td>Teflon (Single)(mV)</td>
<td>213</td>
<td>182.5</td>
<td>158</td>
<td>135</td>
<td>0.21</td>
<td>4.16</td>
</tr>
<tr>
<td>Kapton (Single)(mV)</td>
<td>226</td>
<td>183</td>
<td>163</td>
<td>145</td>
<td>0.22</td>
<td>4.13</td>
</tr>
</tbody>
</table>
REFERENCES


APPENDIX A
A The Two-ball Thermal Comparator
(Powell, 1969)

The two-ball comparator built by R.W.Powell in 1957[R.W.Powell, 1957] is illustrated in Figure (A.1). It consists of two matched phosphor-bronze balls inserted into cylindrical holes in balsa wood which is thermally insulating. One of the balls called the reference ball is flush with the base of the block and the other, the touching ball, protrudes slightly. Between these two balls, a thermocouple is connected. The thermocouple produces a differential voltage when the temperatures between these two balls are different.

In operation, the entire thermal comparator is heated to an initial temperature. Then it is placed on the surface of a specimen of a known temperature that differs from the temperature of the comparator. After contact is made between the “touching” ball and the surface of the specimen, the voltage reading on the potentiometer increases with time because the temperature of the contacting ball changes more rapidly than the temperature of the reference ball. Figure (A.2) shows that the EMF readings of four metals, such as, Al, Fe, Alloy Steel, High-alloy Steel, and Air increase with the increasing of time. A specimen of high thermal conductivity produces a larger EMF reading than a specimen of low thermal conductivity in the same time interval.

In the measurement, the differential EMF represents the difference between the temperature of the reference ball and the temperature of the specimen. However, the EMF can be used as a relative value of thermal conductivity when standard samples of known conductivities are available. The thermal conductivity of a test specimen may be obtained by graphically comparing its EMF reading with the
Asbestos Wool

Thermocouples

To Potentiometer

Balsa Wood

Phosphor-bronze Balls

Figure A.1: The two-ball thermal comparator (Powell, 1969)
Figure A.2: EMF reading versus time in air using two-ball thermal comparator (Powell, 1969)
EMF's of the standard samples.

In his early work, Powell measured several standard samples of known thermal conductivities by using his two-ball thermal comparator. He found that the EMF is approximately linearly proportional to time, especially during the first 10 seconds (See Figure A.2). Powell made a graph of the known thermal conductivities versus the EMF at the 10th second (Sometimes, we will refer to this EMF as the "cooling rate" of the touching ball) for various samples and obtained a parabolic calibration curve (See Figure A.3).

Later, he measured more standard samples of known thermal conductivities ranging from Aluminium (239W/m K) to Plasticized PVC (0.21W/m K). When the square root of the known conductivity was plotted versus cooling rate, the calibration points fall on two intersecting straight lines (See Figure A.4). Powell was never able to explain the origin of the observed square-root relationship.

In general, the two-ball thermal comparator can measure a wide range of thermal conductivities, But, the measurement procedure is rather tedious. Not only the temperature of the comparator but also the temperature of the specimen as well as the timing must be controlled accurately in order to obtain a precise result.
Figure A.3: Thermal conductivity versus differential voltage (Powell, 1969)
Figure A.4: Square root of the conductivity versus cooling rate at 10th second (Powell, 1969)
Total Resistance for a series thermal resistances

In an electrical circuit, a conductor obeys Ohm's law: that is the current $I$ which flows through a conductor is proportional to the voltage difference $\Delta V$ across the two ends of the conductor (See Figure B.1). The reciprocal of the proportionality factor is called electrical resistance of the conductor, designated as $R_{el}$.

$$I = \frac{\Delta V}{R_{el}} \quad (B.1)$$

If the cross sectional area of the conductor is $A$ and its electrical resistivity is $\rho$, then,

$$R_{el} = \rho \frac{L}{A} \quad (B.2)$$

or,

$$R_{el} = \frac{L}{(\sigma A)} \quad (B.3)$$

where $\sigma (= 1/\rho)$ is the electrical conductivity of the material.

When several individual electrical conductors are connected in series (See Figure B.2(a)), the total resistance can be expressed as follows,

$$R_{el\ total} = R_{el1} + R_{el2} + R_{el3} + ... \quad (B.4)$$

For a thermal conduction in one dimension, there is relationship between the heat current $\dot{Q}$ and the temperature difference $\Delta T$ across a bar. This is an analogy to Ohm's law in an electrical circuit (See Figure B.2(b)). The reciprocal of the proportionality factor is called the thermal resistance, $R_{th}$,

$$\dot{Q} = \frac{\Delta T}{R_{th}} \quad (B.5)$$
Figure B.1: Sketch of a thermal conductor and an electrical conductor

\[ \Delta T = (T_{\text{high}} - T_{\text{low}}) \]
Figure B.2: Sketch of thermal conductors and electrical conductors in series
The corresponding thermal resistance is given by,

\[ R_{th} = \frac{L}{(KA)} \]  \hspace{1cm} (B.6)

where \( K \) is the thermal conductivity, \( A \) is the cross sectional area of the bar, and \( L \) is the thickness of the bar. Obviously, for the flow of charge and the flow of heat \( I, \Delta V, \text{and} \sigma \) are exactly analogous to \( \dot{Q}, \Delta T, \text{and} \ K \).

Similarly, if several different kinds of thermal conductors are placed in series, based on the definition of thermal resistance in equation B.5, the expression for the total thermal resistance is in the same form as the electrical resistance.

\[ R_{th\,total} = R_{th1} + R_{th2} + R_{th3} + ... \]  \hspace{1cm} (B.7)

This is a basic equation which is used to discuss the measurements of the thermal conductivity of polymer films. The total resistance (for one dimensional heat flow) consists of the sum of the film, interface, and substrate thermal resistances.
C Derivation of the Relationship between $K_{app}$ and $K_{eff}$

C.1 Establishment of the Differential Equation for Heat Flow

Let us consider the heat flowing in time $\delta t$ into a volume element with side $\delta x$, $\delta y$, and $\delta z$. (See rectangular coordinate system shown in Figure C.1) If $T$ is the temperature at the middle of the box, then the temperature of the section ABCD is $[T - (1/2)(\partial T/\partial x)\delta x]$, while the section EFGH is $[T + (1/2)(\partial T/\partial x)\delta x]$.

The quantity of heat passing across ABCD (inwards) is:

$$Q_{x, in} = -K \frac{\partial}{\partial x} [T - \frac{1}{2}(\frac{\partial T}{\partial x})\delta x \delta y \delta z \delta t]$$

For the section of EFGH, the quantity (outwards) is:

$$Q_{x, out} = -\left\{ -K \frac{\partial}{\partial x} [T + \frac{1}{2}(\frac{\partial T}{\partial x})\delta x \delta y \delta z \delta t] \right\}$$

So, the total quantity flowing inwards over these two sections is:

$$Q_x = Q_{x, in} + Q_{x, out} = K \frac{\partial^2 T}{\partial x^2} \delta x \delta y \delta z \delta t$$

For the $y$ direction and $z$ direction, the familiar equations are:

$$Q_y = K \frac{\partial^2 T}{\partial y^2} \delta x \delta y \delta z \delta t$$

and,

$$Q_z = K \frac{\partial^2 T}{\partial z^2} \delta x \delta y \delta z \delta t$$

Hence, for 3 dimensions, the total heat flow into the infinitesimal volume is:

$$Q_{total} = Q_x + Q_y + Q_z = K \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \delta x \delta y \delta z \delta t$$
Figure C.1: Sketch of a volume element in rectangular coordinate system
If there are no sources of heat and no absorption of heat inside the volume element, the inflow of heat is:

\[ Q = \frac{\partial T}{\partial t} \]

where \( C \) is the specific heat and \( \rho \) is the density. So,

\[ K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = C \rho \frac{\partial T}{\partial t} \]  

(C.1)

For a steady heat flow, there is no variation of temperature with respect to \( t \), so, \( (\partial T/\partial t) = 0 \). Hence, we obtain the Laplace equation for heat conduction.

\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \]  

(C.2)

or

\[ \nabla^2 T = 0 \]  

(C.3)

The cylindrical coordinates \((r, \phi \text{ and } z)\) are defined as follows:

\[
\begin{align*}
    r &= \sqrt{x^2 + y^2} \\
    \phi &= \arctg(y/x) \\
    z &= z
\end{align*}
\]

Then, the Laplace equation will be,

\[ \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \]  

(C.4)

If the heat flow is axisymmetric about the Z-axis, that is \( (\partial T/\partial \phi) = 0 \), the above equation becomes,

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \]  

(C.5)
C.2 Derivation of Thermal Resistance in a Semi-infinite Medium (H. S. Carslaw and J. C. Jeager, 1959)

C.2.1 Case I: Temperature is uniform over circular contact area of radius "a"

Suppose the high temperature at the surface \((z = 0)\) is maintained at temperature \(T_0\) within an area, \(0 \leq r < a\) and the low temperature, \(T = 0\), is maintained at \((r = \infty, z = \infty)\). (See Figure C.2) Then the solution of the Laplace equation is as follows,

\[
T(r, 0) = \frac{2T_0}{\pi} \int_0^\infty e^{-\lambda z} J_0(\lambda r) \sin(\lambda a) \frac{d\lambda}{\lambda}
\]

where \(J_0(\lambda r)\) is a Bessel function of order zero. The following relationships holds true. (G. N. Watson, 1980, §13.42(2))

\[
\int_0^\infty J_0(\lambda r) \sin(\lambda a) \frac{d\lambda}{\lambda} = \begin{cases} \arcsin(a/r) & r > a \\ (1/2)\pi & r \leq a \end{cases}
\]

Assuming cylindrical symmetry, the rate of heat flow \(\dot{Q}\) into a circular spot of radius "a" is,

\[
\dot{Q} = -2\pi K \int_0^a \left[ \frac{\partial T}{\partial z} \right]_{z=0} r dr
\]

where, \(K\) is the thermal conductivity of the medium, and

\[
\frac{\partial T}{\partial z} = \frac{d}{dz} \left[ \frac{2T_0}{\pi} \int_0^\infty e^{-\lambda z} J_0(\lambda r) \sin(\lambda a) \frac{d\lambda}{\lambda} \right]
\]

\[
= -\frac{2T_0}{\pi} \int_0^\infty e^{-\lambda r} J_0(\lambda r) \sin(\lambda a) d\lambda
\]

Then,

\[
\left[ \frac{\partial T}{\partial z} \right]_{z=0} = -\frac{2T_0}{\pi} \int_0^\infty J_0(\lambda r) \sin(\lambda a) d\lambda
\]

So, we can now obtain the heat power.

\[
\dot{Q} = 4T_0Ka
\]

C-4
Figure C.2: Sketch of a model with uniform temperature over circular contact area of radius "a"
where we have used the following identities: (1) \( rJ_0(\lambda r) = (1/\lambda)(d/dr)[rJ_1(\lambda r)]dr \) (Murray R. Spiegel, 1971) and (2) \( \int_0^\infty J_1(\lambda a) \sin(\lambda a)(d\lambda/\lambda) = 1 \) (G. N. Watson, 1980, §13.42(2)). Hence, the thermal resistance of a steady flow from a circle of radius “a” into a semi-infinite medium is,

\[
R = \frac{T_0}{\dot{Q}} = \frac{1}{4Ka} \quad (\text{Standard}) \tag{C.6}
\]

\[
R' = \frac{T_0}{\dot{Q}/A} = \frac{\pi a}{4K} \quad (\text{Normalized to contact area}) \tag{C.7}
\]

### C.2.2 Case II: Heat flow is uniform over contact area of radius “a”.

Suppose the heat flow rate \( \dot{Q} \) per unit time over a circular area of radius “a” at surface \((z = 0)\) is a constant. (See Figure C.3) The following condition will be satisfied for a semi-infinite medium.

\[
-K\frac{\partial T}{\partial z}|_{z=0} = \frac{\dot{Q}}{A} \quad 0 < r < a \tag{C.8}
\]

The solution of the Laplace equation (C.5) is,

\[
T = \frac{a\dot{Q}}{K} \int_0^\infty e^{-\lambda z} J_0(\lambda r)J_1(\lambda a)\frac{d\lambda}{\lambda} \tag{C.9}
\]

where \( J_0(\lambda r) \) and \( J_1(\lambda a) \) are Bessel functions of order zero and one. The following relationships hold true. (G. N. Watson, 1980, §13.42(9))

\[
\int_0^\infty J_0(\lambda r)J_1(\lambda a)d\lambda = \left\{ \begin{array}{l l} 0 & r > a \\ (1/2a) & r = a \\ (1/a) & r < a \end{array} \right.
\]

The average temperature, \( T_{avg} \), at the surface over \( 0 < r < a \) is,

\[
T_{avg} = \frac{1}{\pi a^2} \int_0^a T(r, 0)2\pi r dr
\]
Figure C.3: Sketch of a model with a constant heat flow over a contact area of radius "a"
where we have used the following identities: (1) \( r J_0(\lambda r) = (1/\lambda)(d/dr)[r J_1(\lambda r)] \) (Murray R, Spiegel, 1971) and (2) \( \int_0^\infty J_1^2(\lambda a)(d\lambda/\lambda^2) = (4a/3\pi) \) (G. N. Watson, 1980, §13.33(1)). When the heat flow rate \( \dot{Q} \) is a constant, the thermal resistance of a steady flow from a circle of a radius \( a \) into a semi-infinite medium is,

\[
R = \frac{T_{\text{avg}}}{\dot{Q}} = \frac{8}{3\pi^2 K a} \quad \text{(Standard)} \tag{C.10}
\]

\[
R' = \frac{T_{\text{avg}}}{\left(\dot{Q}/A\right)} = \frac{8a}{3\pi K} \quad \text{(Normalized to contact area)} \tag{C.11}
\]

(Note: As mentioned before, the high temperature is \( T_{\text{avg}} \) for the contact area of radius "a"; the low temperature is \( T = 0 \) at \( r = \infty, z = \infty \)).

C.3 Derivation of Thermal Resistance in a Semi-infinite Medium with a Coating (J. R. Dryden, 1982)

C.3.1 Solution of Laplace Equation by Using Hankel Transforms

The Laplace equation of heat conduction can be transformed into a linear equation by using the Hankel transform procedure (See section A.4 for more details). Let \( T = T(r, z) \).

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{C.5}
\]

Using Hankel transform of order zero,

\[
\mathcal{H}_0\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}; \rho\right] + \mathcal{H}_0\left[\frac{\partial^2 T}{\partial z^2}; \rho\right] = 0
\]
\[-\rho^2 \mathcal{H}_0[T; \rho] + \frac{\partial^2}{\partial z^2} \mathcal{H}_0[T; \rho] = 0\]

So, the Laplace equation of heat conduction becomes a simple linear second order differential equation.

\[(D^2 - \rho^2)\tilde{T} = 0\]  \hspace{1cm} (C.12)

where \(\tilde{T} = \mathcal{H}_0[T, \rho]\), \(D^2 = (\partial^2/\partial z^2)\), and \(\tilde{T}(\rho, z)\) is the Hankel transform of order zero for \(T(r, z)\). The ordinary solution to the above equation is,

\[\tilde{T} = A(\rho)e^{-\rho z} + B(\rho)e^{\rho z}\]  \hspace{1cm} (C.13)

where \(A(\rho)\) and \(B(\rho)\) are coefficients which can be determined by using boundary conditions. And, the solution of equation is given by the inverse Hankel transform.

\[T = \int_0^\infty \rho J_0(\rho r)[A(\rho)e^{-\rho z} + B(\rho)e^{\rho z}]d\rho\]  \hspace{1cm} (C.14)

Figure C.4 shows a model of a spot of radius "a" on a semi-infinite medium with a surface coating whose thickness is \(t\). The Hankel transforms for \(T_1(r, z)\) and \(T_2(r, z)\) should each satisfy the equation and the ordinary solutions (in items of Hankel transforms) for regions 1 and 2, respectively, are:

\[T_1 = A(\rho)e^{-\rho z} + B(\rho)e^{\rho z}\]  \hspace{1cm} (C.15)

\[T_2 = C(\rho)e^{-\rho z}\]  \hspace{1cm} (C.16)

Since the region 2 is an infinite one, the coefficient of the term, \(e^{\rho z}\), must equal zero.

The temperatures and heat flux functions must be continuous at the boundary between the substrate (medium \#1) and the coating (medium \#2). Hence,
Figure C.4: Contact spot of radius “a” on a semi-infinite medium with surface coating of thickness “t”
\[ T_1(r, z) = T_2(r, z) \quad (r > 0, \ z = t) \quad (C.17) \]

\[ K_1 \frac{\partial T_1(r, z)}{\partial z} = K_2 \frac{\partial T_2(r, z)}{\partial z} \quad (r > 0, \ z = t) \quad (C.18) \]

or, in terms of the transforms,

\[ \tilde{T}_1(\rho, z) = \tilde{T}_2(\rho, z) \quad (\rho > 0, \ z = t) \quad (C.19) \]

\[ K_1 \frac{\partial \tilde{T}_1(\rho, z)}{\partial z} = K_2 \frac{\partial \tilde{T}_2(\rho, z)}{\partial z} \quad (\rho > 0, \ z = t) \quad (C.20) \]

Finally, the boundary condition at the surface \((z = 0)\) is,

\[ -K_1 \frac{\partial T_1(r, z)}{\partial z} \bigg|_{z=0} = f(r) \quad (C.21) \]

where \(f(r)\) represents the heat power distribution per unit area across the contact spot (Note: \(\dot{Q}/A = (1/\pi a^2) \int_0^a f(r) 2\pi r dr\), at \(z = 0\)).

Or,

\[ -K_1 \frac{\partial \tilde{T}_1(\rho, z)}{\partial z} \bigg|_{z=0} = \tilde{f}(\rho) \quad (C.22) \]

where the \(\tilde{f}(\rho)\) is the Hankel transform of the heat power distribution across the contact spot.

Using the above boundary conditions \((C.15), (C.16), (C.19), \) and \((C.20)\) to set up two simultaneous equations, the coefficients \(A(\rho)\) and \(B(\rho)\) can be determined in terms of \(C(\rho)\).

\[ A(\rho) = \lambda_1 e^{-2\rho} C(\rho) \]

\[ B(\rho) = \lambda_2 C(\rho) \]
where \( \lambda_1 \) is equal to \((1 - K_2/K_1)/2\) and \( \lambda_2 \) is equal to \((1 + K_2/K_1)/2\).

Using the above results, equation (C.15) and conditions (C.22), we can obtain:

\[
C(\rho) = -\frac{1}{K_1 \rho} \frac{\tilde{f}(\rho)}{(\lambda_1 e^{-2t\rho} - \lambda_2)}
\]

In order to maintain a constant temperature across the contact spot, we select the flux distribution function as follows, (See Figure C.5)

\[
f(\rho) = \frac{\dot{Q}}{2\pi a \sqrt{a^2 - r^2}}
\]

then,

\[
C(\rho) = \frac{\dot{Q}}{2K_1 \pi a} \frac{\sin(\rho a)}{\rho^2} \frac{1}{(\lambda_2 - \lambda_1 e^{-2t\rho})}
\]

So, the solution of the Laplace equation will be,

\[
\tilde{T}_1(\rho, z) = (\lambda_1 e^{\rho(z-2t)} + \lambda_2 e^{-\rho z}) \frac{\dot{Q}}{2K_1 \pi a} \frac{\sin(\rho a)}{\rho^2} \frac{1}{(\lambda_2 - \lambda_1 e^{-2t\rho})}
\]

and finally,

\[
T_1(r, z) = \int_{0}^{\infty} \rho \tilde{T}_1(\rho, z) J_0(\rho, r) d\rho
\]

C.3.2 Thermal Resistance of a Semi-infinite Medium with a Coating

According to the definition of the (standard) thermal resistance of a medium:

\[
R = \frac{\Delta T}{\dot{Q}}
\]

where \( \Delta T \) is the temperature difference, \((T_{high} - T_{low})\). The temperature function for the coating, \( T_1(r, z) \), at the surface approaches zero at a sufficiently large
Figure C.5: The heat flux distribution on the contact spot with radius of "a"

\[
f(r) = \begin{cases} 
\frac{\dot{Q}}{2\pi a^2} & 0 < r < a \\
0 & r > a 
\end{cases}
\]
distance from the contact spot \( r = \infty \). \( \Delta T \) is equal to the average temperature \( T_{avg} \) of the contact spot. So, the thermal resistance \( R \) becomes,

\[
R = \frac{T_{avg}}{Q} = \frac{\int_0^a T_1(r, 0)2\pi r dr}{\pi a^2 Q}
\]

Using the results (C.24) and (C.25), we can get,

\[
R = \int_0^\infty \frac{\sin(\rho a)}{\pi \rho K_1 a^3} \left(1 + \theta e^{-2\rho a}\right) \int_0^a r J_0(\rho r) dr d\rho
\]

where \( \theta \) is equal to \( (K_1 - K_2)/(K_1 + K_2) \).

Using the relationship of \( r J_0(\rho r) dr = (1/\rho) d[r J_1(\rho r)] \), we can obtain \( R \) in the following form.

\[
R = \frac{1}{\pi K_1 a} \int_0^\infty \frac{\sin(\xi)}{\xi^2} \left(1 + \theta e^{-2\xi/a}\right) J_1(\xi) \frac{1}{\left(1 - \theta e^{-2\xi/a}\right)} d\xi
\]

where the \( \xi \) is equal to \( a \rho \). For the term of \( 1/(1 - \theta e^{-2\xi/a}) \), we use the binomial theorem and obtain the follows,

\[
R = \frac{1}{\pi K_1 a} \int_0^\infty \frac{\sin(\xi)}{\xi^2} \left(1 + \theta e^{-2\xi/a}\right) J_1(\xi) \sum_{j=0}^\infty (\theta e^{-2\xi/a})^j d\xi
\]

\[
= \frac{1}{\pi K_1 a} \int_0^\infty \frac{\sin(\xi)}{\xi^2} J_1(\xi) \left(1 + 2 \sum_{j=1}^\infty (\theta e^{-2\xi/a})^j\right) d\xi
\]

Finally, we can get the following result,

\[
R = \frac{1}{4K_1 a} + \frac{2}{\pi K_1 a} \sum_{j=1}^\infty \theta^j I(2jt/a) \quad (Standard) \quad (C.27)
\]

For the thermal resistance normalized to the contact area,

\[
R' = \frac{\pi a}{4K_1} + \frac{2a}{K_1} \sum_{j=1}^\infty \theta^j I(2jt/a) \quad (Normalized) \quad (C.28)
\]

where,

\[
I(2jt/a) = \int_0^\infty e^{-2jt/a} \frac{\sin(\xi)}{\xi^2} J_1(\xi) d\xi
\]
The above equation can be evaluated (I, S, Gradshteyn and I, B, Ryzhik, 1980, p. 763) as follows.

\[ I(2jt/a) = -\frac{2jt}{a} + (D - \frac{1}{2D})\sqrt{1 - \frac{1}{D^2}} + \frac{1}{2}\arcsin\left(\frac{1}{D}\right) \]

where,

\[ D = \frac{jt}{a} + \sqrt{\left(\frac{jt}{a}\right)^2 + 1} \]

### C.4 Hankel Transforms

The definition of the Hankel transform of a function \( f(r) \) is denoted by \( \mathcal{H}_n[f(r); \rho] \),

\[ \mathcal{H}_n[f(r); \rho] = \int_0^\infty rJ_n(\rho r)f(r)dr \]  

where \( J_n(\rho r) \) is a Bessel function of order \( n \). And, the inversion function for the Hankel transform is defined as follows,

\[ f(r) = \int_0^\infty \rho J_n(\rho r)\mathcal{H}_n[f(r); \rho]d\rho \]  

In order to solve the Laplace equation (C.5) of heat conduction by using Hankel transform, the following two formulas should be used:

\[ \mathcal{H}_0[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} ; \rho] = \rho \mathcal{H}_1[\frac{\partial T}{\partial r} ; \rho] \]  

and,

\[ \mathcal{H}_1[\frac{\partial T}{\partial r} ; \rho] = -\rho \mathcal{H}_0[T ; \rho] \]

The differential equation of heat conduction can be transformed into a linear equation by using the Hankel transform procedure.
APPENDIX D
D  The Program of the Calculation of $K_{eff}$

***PROGRAM CALCULATION***

This program will establish a calibration curve by using the linear regression method and calculate the effective thermal conductivity and the thickness over the effective conductivity based on the given values of heat flow radius "a", the voltage reading and the film thickness.

OLDDATA1.DAT: The input file of standard samples to establish the calibration curve.
OLDDATA2.DAT: The input file of measured specimens to yield the effective thermal conductivity.
NEWDATA.DAT: The output file of results.

INTEGER I,J
DOUBLE PRECISION ZJX2(50),ZJX1(50),AC(20),BC(20),C(20),D(20),
+ RADIUS,DPI,KS,THOVRA,FKEFF,RKAPP,K(20),T(20),AA,BB,R
OPEN (UNIT=15,FILE='NEWDATA',STATUS='NEW')
OPEN (UNIT=11,FILE='OLDDATA2',STATUS='OLD')
OPEN (UNIT=10,FILE='OLDDATA1',STATUS='OLD')

1=0
10 I=I+1
READ (10,*,END=20) AC(I),BC(I),D(I)
C(I) = (AC(I)+BC(I))/2
K(I) = LOG10(D(I))
GOTO 10

20 CALL REGRESSION (C,K,AA,BB,R,I)
GOTO 10

30 FORMAT (IX,'THE LINEAR REGRESSION EQUATION: LOG10(K) = ' + 
+ F10.4,'+',F7.4,'*mV')
WRITE(15,40),R
40 FORMAT (IX,'THE CORRELATION COEFFICIENT R IS: ',F7.4)

PRINT*, ('ENTER VALUE OF KS:')
READ*,KS
WRITE(15,50),KS
50 FORMAT(' THE VALUE OF KS IS: ',D12.4)
WRITE(15,60)

60 FORMAT(2x,'thickness t','6x,'mV','8x,'Kapp','8x','Keff','7x','t/Keff')
WRITE(15,70)

70 FORMAT(2x,'..............','6x','..8x','.....','8x','......'/)
I=0
80 J=0
110 READ(11,*,END=110) T(I),MV1(I),MV2(I)
MV(I) = (MV1(I) + MV2(I))/2
KAPP(I) =10.0 ** (AA + BB * MV(I))
RADIUS=250.00
THOVRA=T(I)/RADIUS
DPI=DACOS(-1.0D+00)
I=I+1
READ(11,*,END=110) T(I),MV1(I),MV2(I)
MV(I) = (MV1(I) + MV2(I))/2
KAPP(I) =10.0 ** (AA + BB * MV(I))
RADIUS=250.00
THOVRA=T(I)/RADIUS
DPI=DACOS(-1.0D+00)
J=0
FKEFF=0.00
90 J=J+1
FKEFF=FKEFF+0.025
ZJX2(J)=FKEFF
CALL SUMMATION (THOVRA,FKEFF,SUM,KS)
RKAPP=1.0/(1.0/FKEFF+8.0/DPI/FKEFF*SUM)
ZJX1(J)=RKAPP

D-1
IF (ZJX1(J) .LT. KAPP(I)) GOTO 90
KEFF(I) = ZJX2(J-1) + (KAPP(I) - ZJX1(J-1)) / (ZJX1(J) - ZJX1(J-1)) * 0.025
TOVKE(I) = T(I) / KEFF(I)
WRITE (15, 100) T(I), MV(I), KAPP(I), KEFF(I), TOVKE(I)
FORMAT (5(D12.4))
GOTO 80
110 END

*****************************************************************************

SUBROUTINE SUMMATION (T, RK, S, PK)
INTEGER I
DOUBLE PRECISION T, RK, S, PK, THETA, ALPHA, D, XINT, TERM
THETA = (RK / PK - 1.000) / (RK / PK + 1.000)
I = 0
S = 0.0
120 I = I + 1
ALPHA = 2.0 * I * T
D = 0.500 * ALPHA + DSQRT (1.0 + 0.2500 * ALPHA * ALPHA)
XINT = -ALPHA + (D - 0.5 / D) * DSQRT (1.0 - 1.0 / D / D) + 0.5 * DASIN (1.0 / D)
TERM = (THETA ** I) * XINT
S = S + TERM
IF (DABS(TERM) .GT. 5.0D-05) GOTO 120
RETURN
END

*****************************************************************************

SUBROUTINE REGRESSION (C, K, AA, BB, R, I)
INTEGER I, J
DOUBLE PRECISION SUM1, SUM2, SUM3, SUM4, SUM5, AA, BB, R, C(20), K(20)
SUM1 = 0.0
SUM2 = 0.0
SUM3 = 0.0
SUM4 = 0.0
SUM5 = 0.0
DO 130 J = 1, I - 1
SUM1 = SUM1 + C(J)
SUM2 = SUM2 + K(J)
SUM3 = SUM3 + K(J) * C(J)
SUM4 = SUM4 + C(J) * C(J)
SUM5 = SUM5 + K(J) * K(J)
CONTINUE
JA = J - 1
BB = (JA * SUM3 - SUM1 * SUM2) / (JA * SUM4 - SUM1 * SUM1)
AA = (SUM2 - BB * SUM1) / JA
R = (JA * SUM3 - SUM1 * SUM2) / DSQRT ((JA * SUM4 - SUM1 * SUM1) * (JA * SUM5 - SUM2 * SUM2))
RETURN
END

*****************************************************************************
For example, the data in Table 4.3 and Table 4.5 were used.

$\text{type olddata1.dat}$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>73</td>
<td>.18</td>
</tr>
<tr>
<td>210</td>
<td>208</td>
<td>1.4</td>
</tr>
<tr>
<td>292</td>
<td>290</td>
<td>4.86</td>
</tr>
</tbody>
</table>

This is a data file of standard samples. In the first and second columns are voltage readings of standard samples and in the third column are their thermal conductivities.

$\text{type olddata2.dat}$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25.4</td>
<td>277</td>
<td>272</td>
</tr>
<tr>
<td>50.8</td>
<td>227</td>
<td>224</td>
</tr>
<tr>
<td>76.2</td>
<td>188</td>
<td>186</td>
</tr>
<tr>
<td>101.6</td>
<td>172</td>
<td>174</td>
</tr>
<tr>
<td>127</td>
<td>166</td>
<td>166</td>
</tr>
</tbody>
</table>

This is a data file of test specimens. In the first column are thicknesses of the test specimens. In the second and third columns are their corresponding voltage readings.

$\text{type newdata.dat}$

\[
\begin{align*}
\text{THE LINEAR REGRESSION EQUATION:} & \quad \log_{10}(K) = -1.2328 + 0.0066 \times \text{mV} \\
\text{THE CORRELATION COEFFICIENT R IS:} & \quad 1.0000 \\
\text{THE VALUE OF KS IS:} & \quad 0.1400D+03
\end{align*}
\]

<table>
<thead>
<tr>
<th>thickness $t$</th>
<th>mV</th>
<th>$K_{\text{app}}$</th>
<th>$K_{\text{eff}}$</th>
<th>$t/K_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2540D+02</td>
<td>0.2745D+03</td>
<td>0.3784D+01</td>
<td>0.9559D+00</td>
<td>0.2657D+02</td>
</tr>
<tr>
<td>0.5080D+02</td>
<td>0.2255D+03</td>
<td>0.1798D+01</td>
<td>0.7616D+00</td>
<td>0.6670D+02</td>
</tr>
<tr>
<td>0.7620D+02</td>
<td>0.1870D+03</td>
<td>0.1002D+01</td>
<td>0.5439D+00</td>
<td>0.1401D+03</td>
</tr>
<tr>
<td>0.1016D+03</td>
<td>0.1730D+03</td>
<td>0.8099D+00</td>
<td>0.5078D+00</td>
<td>0.2001D+03</td>
</tr>
<tr>
<td>0.1270D+03</td>
<td>0.1660D+03</td>
<td>0.7282D+00</td>
<td>0.5006D+00</td>
<td>0.2537D+03</td>
</tr>
</tbody>
</table>

D-3
APPENDIX E
# E Mechanical Properties of Teflon and Kapton Films

<table>
<thead>
<tr>
<th></th>
<th>Teflon</th>
<th>Kapton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Polymer</td>
<td>Fluoro-carbon resin</td>
<td>Polyimide</td>
</tr>
<tr>
<td>Yong’s modulus (GPa)</td>
<td>0.48</td>
<td>2.5</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (mm/mm°C)</td>
<td>9.4*10⁻⁵</td>
<td>2.0*10⁻⁵</td>
</tr>
<tr>
<td>Thermal conductivity (25.4μm) (W/mK)</td>
<td>0.195</td>
<td>0.12</td>
</tr>
<tr>
<td>Yield stress (GPa)</td>
<td>0.012</td>
<td>0.069</td>
</tr>
<tr>
<td>Melt Point (°C)</td>
<td>260–280</td>
<td>None</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>2150</td>
<td>1420</td>
</tr>
</tbody>
</table>