Prediction of Small-Scale Densities Using the Chemical and Optical Spread Functions

David S. Cline

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PREDICTION OF SMALL-SCALE DENSITIES USING
THE CHEMICAL AND OPTICAL SPREAD FUNCTIONS

by

David Scott Cline

A thesis submitted in partial fulfillment
of the requirements for the degree of
Bachelor of Science in the School of
Photographic Arts and Sciences in the
College of Graphic Arts and Photography
of the Rochester Institute of Technology

May 1982

Signature of the Author.................................

Photographic Science
and Instrumentation

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Thesis Advisor

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Accepted by...........................................

Supervisor, Undergraduate Research
Title of Thesis  Prediction of Small-Scale Densities Using the Chemical and Optical Spread Functions.

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PREDICTION OF SMALL-SCALE DENSITIES USING
THE CHEMICAL AND OPTICAL SPREAD FUNCTIONS

by

David Scott Cline

Submitted to the
Photographic Science and Instrumentation Division
in partial fulfillment of the requirements
for the Bachelor of Science degree
at the Rochester Institute of Technology

ABSTRACT

Development adjacency effects and optical spread in photographic
systems cause difficulties in predicting small-scale densities. These
effects can be quantified as spread functions. The optical spread func-
tion describes the light scatter of the film. The chemical spread func-
tion describes the lateral diffusion of development-inhibiting and dev-
elopment-accelerating reaction products. These spread functions can be
used along with the sensitometry and covering power relationships of the
system to predict small-area densities from the object exposure profile.
A study based on C. N. Nelson's work was undertaken to develop a math-
ematical model that uses the optical and chemical spread functions to
predict small-area densities of an image from the original object ex-
posure profile. This model was developed for a fine-grain aerial dup-
licating film processed in KODAK D-76 (1:4).
It was found that the model predicted the image profile of a log-periodic target to within ±0.02 D on the peaks of the bars. However, the model overshot the edges of the bars by about ±0.05 D. This difference, even though significantly greater than the experimental error, does not affect the usefulness of this model to image quality studies.
ACKNOWLEDGEMENTS

The author wishes to thank the following people and organisations for their assistance in making the completion of this thesis possible:

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Dale Ewbank - for supplying FORTRAN programs needed to convert scan data to useful files.
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INTRODUCTION

Many applications of photography exploit the fact that the density of a large-scale image on photographic film has a simple relationship to the original input exposure that was used to form the image. Stated more simply, photographic film can be used as a radiometer on a large-scale. However, when images are analyzed on a small-scale, accurate prediction of the density profile becomes complicated by several micro-effects inherent in the photographic system.

The two most significant of these micro-effects are optical spread of the imaging system and film and development adjacency effects.\textsuperscript{1-5} The optical spread includes the spread of the image due to the optics of the camera and the scattering of the image as it penetrates the film. Development adjacency effects, or edge effects, are density variations that are a function of the adjacent densities. These variations are not predictable by the original illuminance of a given point.\textsuperscript{4-7} Adjacency effects are caused by the inhibitory action of the development by-products released from a higher density upon the development of an adjacent lower density area. The formation of an edge effect is represented in Figure 1.
Fresh developer

Reaction products

Figure 1. The nature of the adjacency effect in development.\textsuperscript{6}

Both of these effects occurring at a given point change the small-scale densities of the points adjacent to that point. Thus, density at any given point is not a function of exposure at that point but rather a function of that point and its surrounding points. It is useful to describe the optical and chemical effects as spread functions.\textsuperscript{6-10} If an image of a point is considered to be made up of an infinite number of points of varying intensity, the point spread function of the system is defined as the relationship between distance and intensity on the image. However, it is more convenient to describe a system in terms of its line spread function. A line spread function is defined as the relationship between distance and intensity of geometric lines made up of rows of an infinite number of points. More simply, it is just the point spread function reduced to one dimension.\textsuperscript{9}
By using the line spread function of the optical spread, it is easy to find a relationship for a total exposure at any given point. This is accomplished by adding up or integrating the contribution of each individual optical line spread function by performing the following convolution:

\[ E(x) = a(x) \ast I(x) \]

where \( E(x) \) is the actual exposure at point \( x \), \( a(x) \) is the optical line spread function, and \( I(x) \) is the input exposure at point \( x \).

Kelly attempted to derive a model to linearize the optical and chemical effects of a photographic system. Kelly approached the problem by assuming that the optical and chemical spread in the system would be gaussian in nature. Using this assumption, he calculated optical and chemical transfer functions and scaled them by using experimental data. However, his model failed to accurately describe the chemical spread effect.

Another attempt at describing the chemical spread was made by Simonds. Simonds used least-squares-regression to empirically find a set of "development effect coefficients" for a given film-chemistry combination processed to a given macro-density gamma. His relationship was as follows:

\[ D'_i - D_i = D_i \sum_{j=i-N_2}^{i+N_2} b_{i-j} (D_i - D_j) \]

where \( D_i \) and \( D_j \) are the large-area densities at points \( i \) and \( j \).
respectively, \( D_i^1 \) is the small-area density at point \( i \), \( b_i \) is the development effect coefficient for the film-chemistry combination at a distance of \( j \) away from point \( i \), and \( N_2 \) is the distance from point \( i \) at which the developing silver no longer affects the density at point \( i \). This model accurately describes a photographic system but it is somewhat limited by the requirement that the development effect coefficients be determined for each film-development condition.

Nelson developed a method of predicting densities in fine detail in photographic images by using the chemical and optical spread functions. Nelson defined the chemical spread function as the relationship that describes the change caused by development of a unit mass of silver per unit area in a line having a width approaching zero. Mass of silver per unit area was chosen because it is directly proportional to development action whereas density is not. The relationship between density and mass of silver per unit area is expressed as follows:

\[
M = PD^n
\]  

(3)

where \( M \) is the unit mass per unit area, \( P \) is the reciprocal of the covering power of the silver at a density of 1.0, \( D \) is density, and \( n \) is the degree of nonlinearity. Nelson experimentally discovered that the magnitude of the increase in density on the edge of a dense image (border effect) at any given point is proportional to the product of the large-area density at the high density side of the edge and the large-area density difference across the edge. He also found that the ratio of the border effect to the fringe
effect, the magnitude of the decrease in density of the edge of the area adjacent to the high density area, to be equal to the ratio of the large-area density at the high side to that of the low density side of the edge. From the above observations, Nelson derived the chemical spread function, $b(x)$, to be equal to:

$$b(x) = \frac{\frac{dM}{dx}}{M_2(M_2-M_1)}$$

(4)

where $\frac{dM}{dx}$ is the slope of the microdensitometer edge trace (converted to mass of silver per unit area) at the distance of $x$ from the center of the edge, $M_1$ is the large-area mass at the low density side of the edge, $M_2$ is the large-area mass at the high density side of the edge.

Because the magnitude of the border and fringe effects were found to be proportional, the chemical spread function can be assumed to be symmetrical about $x=0$. Thus, only the high side of the chemical spread function need be calculated. An example of the derivation of the chemical spread function is shown in Figure 2. Nelson also defined the quantity $B$ as the area under the chemical spread function.
Because of the nature of the chemical spread function, the small-area density to the n-th power, $D_s^n$, of a single point surrounded by a large-area of no density would be equal to the predicted large-area density to the n-th power plus the product of B and $D_s^n$ squared or:

$$D_s^n = D^n + BD_s^{2n}$$

(5)

where all densities are raised to the n-th power to correct for the non-linearity between mass of silver and density.

Once the optical and chemical spread functions are determined, the image of any object can be predicted. The prediction procedure involves
convolving the object illuminance profile with the optical spread function and transferring the output through several macro-sensitometric curves to obtain $D^N$ as a function of position $(x)$. This relationship is then convolved with the chemical spread function to obtain the amount of development retardation caused by the development of the adjacent areas as a function of position. The retardation, weighted by the large-area $D^N$, at any point $x$ is then subtracted from the maximum small-area density, as determined by equation (5), to determine the final small-scale density, $D_c^N$.

$$D_c^N(x) = D^N(x) + BD^2N(x) - \text{Retardation}(x)$$  \hspace{1cm} (6)

Nelson found that the accuracy of this model, in general, was about ±0.02 D at densities less than 2.0. He tested a broad range of films and found that they all behaved as the model predicted to varying degrees. However, he did not apply the model to continuous-tone, ultra-fine-grain films such as aerial duplicating film. The sponsor of this research frequently has the need to predict the densities of images made with this type of film when investigating image quality. The purpose of this was to apply Nelson's model to an ultra-fine-grain aerial duplicating film.
EXPERIMENTAL

This investigation of the chemical and optical spread functions was performed in five major steps. The first of these steps was the selection of the developers to be used. Second was the determination of the mass of silver per unit area and density relationship of the chosen film-chemistry system. The third step was the derivation of the chemical and optical spread functions. Fourth was the development of the micro-density prediction routines. The last step was the testing of the prediction routine with experimental data.

Selection of Developers

One of the developers selected for this investigation was KODAK D-76 (1:4) because it does exhibit a moderate amount of adjacency effects. The other developer that was used had to minimize adjacency effects. Several references have been made in the literature that KODAK D-19 developer exhibits almost no adjacency effects. In order to verify these claims, X-ray exposed edges were processed in these developers. Edge traces of the resultant images were then made with a microdensitometer. Because X-ray exposed edges have no significant optical spread, the edge traces produced reflected only the chemical effects of the system. Thus, the existence of or lack of development adjacency effects was verified.
Determination of Mass of Silver-Density Relationship

To determine the coefficients \( P \) and \( n \) of equation (3), six uniform density patches processed in D-76 (1:4) for 13 minutes at 68°F were sent to the DuPont-Rochester plant for mass of silver per unit area determination. These measurements are listed below in Table 1. From these measurements, the relationship was calculated by using a geometric regression program to be 0.584 \( D^{1.163} \) grams per square meter.

Table 1. Measurements used to calculate mass of silver per unit area to Density relationship.

<table>
<thead>
<tr>
<th>Patch #</th>
<th>Density</th>
<th>Mass Ag/unit area (grams/meter(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.76</td>
<td>0.42</td>
</tr>
<tr>
<td>2</td>
<td>1.14</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>1.08</td>
</tr>
<tr>
<td>4</td>
<td>1.80</td>
<td>1.21</td>
</tr>
<tr>
<td>5</td>
<td>2.04</td>
<td>1.38</td>
</tr>
<tr>
<td>6</td>
<td>2.11</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Derivation of Chemical and Optical Spread Functions

To derive the chemical spread function of the KODAK D-76 (1:4) developer, X-ray exposed edges were processed in D-76 (1:4) for 13 minutes at 68°F. The resulting images were then scanned by a microdensitometer. To smooth the microdensitometer scan data, two negative exponential functions were fit to the trace data. Negative exponentials were chosen because the edge traces appeared to have a negative exponential shape. By using equation (4), the chemical spread function, \( b(x) \), for each image was calculated by using the derivatives of the negative exponentials used to fit the edge scan. An average chemical spread function was calculated by fitting two negative
exponential functions to the average $b(x)$ value at each point.

To derive the optical spread function, a contact print of a perfect edge was processed in KODAK D-19 (undiluted) for 1.5 minutes at $68^\circ F$. The microdensitometer edge trace of this image was converted to relative exposure by using the D-log $H$ curve of the film. In much the same way that the chemical spread function was calculated, the slope of the exposure (intensity) vs. position relationship was fit to a negative exponential function. The optical spread function is simply the negative exponential function normalized to an area of 1.0.

**Development of Micro-Density Prediction Routine**

This stage of the investigation involved the development of a computer program that will predict micro-densities of an image given the exposure profile of the object, optical spread function of the film, macro-sensitometric characteristics of the film-chemistry system, mass of silver per unit area-density relationship of the system, and the chemical spread function of the film-chemistry system. However, at this point in the investigation, all of the input relationships except for the object exposure profile had been determined. Thus, for the given film-chemistry system, the program could be used to predict the micro-density profile given the object exposure profile. Figure 3 shows the sequential stages of the micro-density calculation. The theory of this calculation is discussed in the introduction section of this report.
Figure 3. Sequential stages in the micro-density profile model. The arrows indicate how one proceeds through the model.³

Several steps of the prediction program were approximated by numerical methods. These include the convolutions, the macro D-log H relationship, and the macro D to micro D relationship. The convolutions (steps 3 and 8 in Figure 3) were performed by point-by-point multiplication of the functions and integrating the result by Simpson's approximation. The D-log H relationship (step 4 in Figure 3) was modeled by fitting two cubic functions to the step tablet data. The first cubic function was fit to the first seven steps above base-plus-fog...
density. The other was fit to the remaining steps. The relationship between macro-density (diffuse) and microdensitometer density (specular) was assumed to be linear.

**Experimental Verification of Prediction Program**

The final stage of this investigation was the testing of the prediction program with experimental data. This was accomplished by contact printing a degraded, log-periodic target and processing it in D-76 (1:4) for 13 minutes at 68°F. The density profile of the original target was then entered into the program to produce the predicted micro-density profile. This computed profile was then compared to the actual micro-density profile of the image. A statistical comparison of the difference in the model densities and the actual densities would be, for the most part, misleading because a slight misalignment of data would create a large density difference along the edge areas. A more useful comparison technique is to compare the actual and predicted density traces visually to subjectively determine if they agree with each other on the whole.
RESULTS

Chemical Spread Function

Of the 16 X-ray exposed edges scanned, 12 were usable. The remaining four had dust spots, scratches, or other irregularities. The mass of silver per unit area vs. position profile was fit to two negative exponential functions with the large-area mass added in as follows:

\[ M(x) = ae^{-bx} + ce^{-dx} + M_2 \]  \hspace{1cm} (7)

where \( M_2 \) is the large-area mass value. From this relationship, one can find the chemical spread function as follows:

\[ b(x) = \frac{dM}{dx} = \frac{M_2(M_2-M_1)}{M_2(M_2-M_1)} = \frac{ab e^{-bx} + cde^{-dx}}{M_2(M_2-M_1)} = \frac{se^{-bx} + te^{-dx}}{M_2(M_2-M_1)} \]  \hspace{1cm} (8)

where \( s \) and \( t \) are constants and \( M_1 \) is the large area mass value at the low density side of the edge. The parameters for the chemical spread function for each edge, along with the correlation coefficient \( r^2 \), is listed in Table 2.
Table 2. Results of chemical spread function calculation.

<table>
<thead>
<tr>
<th>Edge#</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$s$</th>
<th>$b$</th>
<th>$r^2$</th>
<th>$t$</th>
<th>$d$</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.37</td>
<td>0.06</td>
<td>0.00054</td>
<td>0.017</td>
<td>0.905</td>
<td>0.0044</td>
<td>0.174</td>
<td>0.829</td>
</tr>
<tr>
<td>2</td>
<td>2.37</td>
<td>0.06</td>
<td>0.00093</td>
<td>0.020</td>
<td>0.960</td>
<td>0.0410</td>
<td>0.680</td>
<td>0.932</td>
</tr>
<tr>
<td>3</td>
<td>2.30</td>
<td>0.06</td>
<td>0.00070</td>
<td>0.017</td>
<td>0.949</td>
<td>0.0051</td>
<td>0.200</td>
<td>0.886</td>
</tr>
<tr>
<td>4</td>
<td>2.30</td>
<td>0.06</td>
<td>0.00068</td>
<td>0.017</td>
<td>0.932</td>
<td>0.0048</td>
<td>0.400</td>
<td>0.977</td>
</tr>
<tr>
<td>5</td>
<td>2.35</td>
<td>0.06</td>
<td>0.00064</td>
<td>0.016</td>
<td>0.900</td>
<td>0.0048</td>
<td>0.210</td>
<td>0.884</td>
</tr>
<tr>
<td>6</td>
<td>2.35</td>
<td>0.06</td>
<td>0.00064</td>
<td>0.015</td>
<td>0.940</td>
<td>0.0120</td>
<td>0.320</td>
<td>0.978</td>
</tr>
<tr>
<td>7</td>
<td>2.30</td>
<td>0.06</td>
<td>0.00066</td>
<td>0.016</td>
<td>0.948</td>
<td>0.0070</td>
<td>0.250</td>
<td>0.977</td>
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<tr>
<td>8</td>
<td>2.30</td>
<td>0.06</td>
<td>0.00067</td>
<td>0.016</td>
<td>0.920</td>
<td>0.0140</td>
<td>0.330</td>
<td>0.962</td>
</tr>
<tr>
<td>9</td>
<td>2.45</td>
<td>0.06</td>
<td>0.00059</td>
<td>0.017</td>
<td>0.902</td>
<td>0.0050</td>
<td>0.190</td>
<td>0.945</td>
</tr>
<tr>
<td>10</td>
<td>2.13</td>
<td>0.06</td>
<td>0.00052</td>
<td>0.013</td>
<td>0.910</td>
<td>0.0090</td>
<td>0.250</td>
<td>0.978</td>
</tr>
<tr>
<td>11</td>
<td>2.45</td>
<td>0.06</td>
<td>0.00053</td>
<td>0.015</td>
<td>0.902</td>
<td>0.0060</td>
<td>0.210</td>
<td>0.922</td>
</tr>
<tr>
<td>12</td>
<td>2.13</td>
<td>0.06</td>
<td>0.00051</td>
<td>0.013</td>
<td>0.911</td>
<td>0.0110</td>
<td>0.290</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>average value</td>
<td>.00123</td>
<td>.026</td>
<td>.768</td>
<td>.0080</td>
<td>.330</td>
</tr>
</tbody>
</table>

Note: All $r^2$ values are significantly larger than zero (alpha=0.05).

All $b$ and $d$ values are based on $x$ in 2 micron increments.
Optical Spread Function

The optical spread function was calculated to be:

\[ a(x) = \frac{2}{k} e^{-kx} \]  

(9)

where \( k=0.61 \) and \( x \) is in 2 micron increments.

Prediction Model

By using only the D-log H curve on the input exposure profile of the log-periodic target, the image densities are underestimated. The plot of the density profile as predicted by the D-log H curve and the actual image profile is shown in Figure 4.

When the entire prediction model was used on the exposure profile, the predicted densities were, in general, within \( \pm 0.02 \) D. The corners of the bars were however, overshot by about 0.05D. The plot of the entire model is shown in Figure 5.
Figure 4. Density profile as calculated by the D-log H curve.
Figure 5. Density profile calculated by the prediction model.
DISCUSSION

As one can see from Figure 5, the prediction model accurately predicted the densities of the peaks of the bars but overshot the edges of the bars. There is one possible source of experimental error that can explain this overshoot. The error could be caused by the fact that the original target (input exposure profile) may have been scanned in another portion of the actual target than the actual image was. If there is a slight variation in the target along the bars, error could be introduced to the comparison of the modeled densities to the actual densities. With more scans of the target and image, this possible problem could be resolved.

Considering that in image quality studies, the exact density profile is not always required as long as the general trend of the image effects is known. This model fulfills this requirement and would thus be useful in image quality studies involving the density profile of an image.
REFERENCES


VITA

David Cline was born on 26 September 1960 in Fort Lauderdale, Florida. He studied chemistry for one year at the Broward Community College in Fort Lauderdale before transferring to the Photographic Science and Instrumentation program at the Rochester Institute of Technology. While attending RIT, he received the Raymond Davis Scholarship from the Society of Photographic Scientist and Engineers and a scholarship from the Society of Motion Picture and Television Engineers.

David held a position as a Student Trainee at the Central Intelligence Agency, Office of Development and Engineering during the summer preceding his senior year at RIT. His career plans are to work in the area of image evaluation.