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Design, Fabrication, and Testing of a Miniature Impulse Turbine Driven by Compressed Gas

By
Daniel B. Holt

A Thesis Submitted in Partial Fulfillment
of the Requirements for the Degree
Of
MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

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By

Daniel B. Holt

Title of Thesis

Design, Fabrication, and Testing of a Miniature Impulse Turbine Driven by Compressed Gas

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Abstract

A miniature impulse turbine has been developed at the Rochester Institute of Technology. The goal of this project was to design, fabricate, and test a miniature turbine intended to power small vehicles such as micro air vehicles (MAVs). MAVs are vehicles with a maximum dimension of less than 15 cm and are used for surveillance and scouting. Due to the small size of MAVs, weight is a key design parameter. The batteries alone can account for more than 50% of the weight of the entire vehicle. A miniature turbine driven by compressed gas and coupled to a generator has been proposed as a replacement for batteries on these and other small vehicles. By decreasing the weight of the power systems, MAVs will be able to carry more instrumentation, fly longer, and be better able to complete their mission. To investigate the feasibility of this concept a turbine with a 6 mm impeller was designed, fabricated, and tested. The turbine produced 13.8 W of mechanical power. This corresponds to a power density of 230 W/N and a power to weight ratio of 44 W/kg. The generator attached to the turbine produced 208 mW of electrical power. The feasibility of a fuel tank, designed to hold compressed nitrogen at 69 MPa (10,000 psi), has also been investigated. The tank was designed to hold enough nitrogen to power a MAV for fifteen minutes. It was shown that the tank is feasible and could power a miniature turbine onboard a MAV.

Dedication

This thesis is dedicated to my wife, Christine.

Acknowledgement

I would like to thank Dr. Kozak for his contributions to this work. He allowed me the freedom to push this project in directions that interested me as well as providing guidance in a number of aspects of this work. I would also like to thank Dr. Boedo and Dr. Kempinski for their technical help. I would also like to acknowledge the countless hours of support Dave Hathaway has given to me. Without his help this project could not have been done. Finally, I would like to thank Dr. Hensel, Dr. Ogut, Dr. Boedo, and Dr. Kozak for agreeing to be on my defense panel and for taking the time to review my work.

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Nomenclature

A_{surf}	Surface Area of Cylinder
A_t	Area of the Nozzle Throat
C_p	Specific Heat at Constant Pressure
D	Diameter
$\dot{H}$	Heat Energy
K	Minor Loss Coefficient
L	Length
L_e	Equivalent Length
M	Mass
P	Pressure
P_{axial}	Axial Pressure
P_{Elec}	Electric Power
P_i	Initial Pressure
P_{turbine}	Turbine Power
Q	Volumetric Flow Rate
R	Gas Constant of Air
R	Gas Constant of Nitrogen
Re	Reynolds Number
T	Temperature
U	Velocity
W	Work

a	Inside Radius
b	Outside Radius
e	Roughness
f	Friction Factor
g	Local Acceleration due to Gravity
h	Enthalpy
h_l	Head Loss
i	Current Time Step
k	Conductivity of Silicon Carbide
$\dot{m}$	Mass Flow Rate
dt	Time Step
r	Radius
s	Isentropic Point
u	Internal Energy
z	Height
α	Kinetic Energy Coefficient
γ	Ratio of Specific Heats
η_{turbine}	Turbine Efficiency
η_{mech}	Mechanical Efficiency
η_{overall}	Overall Efficiency
μ	Viscosity
v	Specific Volume
ρ	Density

σ_r	Stress in Radial Direction
σ_θ	Stress in Theta Direction
σ_z	Stress in Z Direction
ω	Rotational Speed
V	Volume

1.0 Introduction

Recent advances in miniaturization and aerospace engineering have led to the development of Micro Air Vehicles (MAVs). MAVs are air vehicles that are less than fifteen centimeters in length and travel at speeds between thirty and sixty-five kilometers per hour [Mueller, 1999]. Since these vehicles are quite small, they have a number of applications such as surveillance and data retrieval. Over the past couple of years, both federal and academic institutions have begun developing MAVs that are smaller and more capable with each generation. However, MAVs still face numerous problems such as finding low weight power sources, operating in a relatively unknown flow regime, and miniaturization of components. Designers of MAVs need to devise creative solutions to these problems. A possible solution to the problem of a power source is presented in this thesis.

Current MAVs use batteries, but these place a lower limit on the size of the vehicle and significantly decrease the maximum payload. For example, one of the smallest functional MAVs is the Black Widow, which was designed by Aeroenvironment and currently being used by U.S. troops in Iraq [Grasmeyer, 2001]. The Black Widow has a 15 cm span and uses batteries that comprise 51% of the entire vehicle weight. As MAVs get smaller, current state of the art batteries are too heavy, so they must remain tethered to the ground. One of the options which has been presented is a miniature turbine - generator system to replace conventional batteries.

A miniature turbine is a fluid powered machine that produces either electrical or mechanical power. A miniature turbine, as opposed to a micro turbine, is one manufactured using conventional fabrication techniques. For miniature turbines to be successful on MAVs the turbine and fuel must weigh less than 50 grams, produce 5 Watts of power, and take up less than

50 cm². The purpose of this work was to investigate whether a miniature turbine could serve as a power source on MAVs instead of batteries. This thesis describes the design, fabrication, and testing of a miniature turbine as well as the design of a fuel tank as a power source.

1.1 Turbine Overview

The turbine described in this work was powered by compressed air during testing. However, the goal of the project is to use compressed nitrogen to drive the turbine. Nitrogen was chosen because it is inert and safe to release directly into the environment. Most of the research groups working on microturbines are using some type of combustible fuel such as liquid hydrogen. The big advantage to using a combustible fuel is the higher energy density. Liquid hydrogen has an energy density of about 4500 KJ kg⁻¹ [Peirs, 2003] while compressed nitrogen has an energy density of about 50 KJ kg⁻¹. However, there are a number of advantages to using a compressed gas. Combustible fuels greatly increase the size of the turbine because they require a compressor and a combustor sections, which a compressed gas turbine does not need. Another advantage of the compressed gas turbine is it exhausts cold gas, at or below room temperature, as opposed to hot gas, which is exhausted at a minimum of 300°C [Epstein, 1997]. This is an important consideration when the turbine is in close proximity to electronic equipment or any materials that are sensitive to high temperatures as on MAVs. In addition, a MAV exhausting hot gas will easily be seen by infrared scans, while one driven by compressed gas will not.

Liquid nitrogen will be inserted into silicon carbide fuel tanks, which are then capped and allowed to warm up to room temperature. This could potentially give a pressure of 46,000 psi in the tanks, but for safety and use with a thinner walled tube, 10,000 psi is the current goal. Once the tank is opened, a jet of nitrogen is directed at the impeller. Figure 1.1 shows an overview of

the basic layout of the miniature turbine. The nitrogen tank can be seen on the left side of the Figure 1.1.

Figure 1.1: Miniature Turbine Overview

The impeller was designed to be as simple as possible for its size but also with the possible fabrication difficulties in mind. A future goal of the project is to scale the miniature turbine down to a Micro Electrical Mechanical System (MEMS). MEMS devices are built using semiconductor fabrication techniques that severely limit the design. Axial, radial, and impulse turbines were investigated and because of its simplicity and ease to scale down to MEMS size, the impulse type was chosen. The reason simplicity was a factor in the choice of the turbine type is because the turbine will be competing with batteries, which are inexpensive. The simplicity of the turbine greatly lowered fabrication costs and will make the turbine more marketable.

Another reason an impulse turbine was chosen was based on the specific speed (N_s), a dimensionless coefficient based on the rotational speed, power, and available upstream pressure or head of the turbine shown in Equation 1.1. Different types of turbines are more efficient at

various specific speeds. Specific speed is defined in several ways, but most often given in U.S. customary units as shown in equation 1.1.

$$N_s = \frac{N(rpm) * [P(hp)]^{1/2}}{[H(ft)]^{5/4}} \quad \text{Eqn. 1.1}$$

Initial estimations for these numbers based on other miniature turbines and the fuel type were 50,000 rpm, 0.0067 hp (5 Watts), and 575 ft (5.5 bar). Substituting these numbers into equation 1.1 give a specific speed of about three, which is in the impulse turbine region [Fox, 1998]. As a result, for this application, impulse turbines are the most efficient.

Impulse turbines produce power by directing a fluid jet at the buckets of the impeller as shown in Figure 1.2. The buckets redirect the incoming flow, which causes a portion of the fluid's momentum to be transferred to the turbine, which in turn produces a torque on the impeller shaft. Figure 1.1 displays the impulse turbine and shows how the impeller is oriented to the shaft. Impulse turbines have been used successfully for hydroelectric power generation. Hydraulic turbines have been extensively tested and analyzed but relatively little work has been published on using air with this type of turbine to produce power.

Figure 1.2: Impulse Turbine Bucket

1.2 Previous Work

The word “micro” is used very loosely in the turbine industry. For microturbines, the main difference depends on whether an academic institution or industry is using the word micro. In academia, the word micro usually refers to MEMS where very specialized fabrication techniques have been created to work on the micron scale. A number of these very small micro turbines are being developed and will be outlined later in this section. In industry, where turbines are designed to produce up to a thousand megaWatts, a micro turbine is a turbine that produces less than five hundred kiloWatts. On the smaller end of this very large gap is where the miniature turbine presented in this work lies.

Figure 1.3: Capstone Turbine

The current leader in industry of “microturbines” is the company Capstone, Inc. [Capstone, 2003]. This company builds turbines down to 30 KiloWatt range and several cubic feet in size as shown in Figure 1.3. These turbines can run on a variety of fuels including natural gas, propane, and diesel. Capstone’s turbines are used for a variety of applications such as power generation for manufacturing plants to powering city buses.

In recent years, a number of institutes have been working on MEMS sized turbines. Although there have been many advances in the fabrication of MEMS devices, there are still many difficulties. A technique called deep reactive ion etching allows a Silicon or Silicon Carbide wafer to be cut at 90 degrees to the surface with a very high level of accuracy. This allows two-dimensional shapes to be formed, but the major challenge is connecting the parts together. When scaling a turbine down to the MEMS size, a designer would ideally want to scale the tolerances down the same amount. This is usually not possible which means the tolerances are relatively large in MEMS parts. Large tolerances are one of the main issues facing designers who are attempting to scale turbines down to a MEMS machine.

The first group to work on a microturbine using silicon fabrication techniques was the researchers at AT&T Bell Laboratories [Mehregany, et al., 1987]. They were able to fabricate a turbine that was 900 μm in diameter and turned at approximately 24000 rpm. The smallest turbine they fabricated was 600 μm in diameter and 40 μm thick. The AT&T team was focused on showing that turbines can be built to this size, not in fabricating a turbine for power generation. Therefore, they did not attempt to attach the turbine to a generator or measure the torque being produced by the turbine.

The first successful fabrication of a true microturbine that produced power continuously was done by a joint project between the University of Wisconsin and Motorola [Wiegele, 1996]. Their design produced 17 mW with an impulse turbine similar to the ones presented in this work. The largest difference between their work and this work is they integrated the turbine impeller and the generator rotor into one part. This is shown in Figure 1.4. By combining the impeller and rotor, this group significantly decreased the size and complexity of the design, but also decreased the efficiency of the turbine. The turbine was able to run up to 70,000 rpm with a

pressure ratio of 1.3. The rotor required 4.6×10^{-3} N-m/m to turn. Although they were able to produce power through an innovative design, the amount produced is not sufficient to power a MAV. In addition, since the design was focused on integrating the impeller and generator, they did not try to optimize the shape of the rotor for power production.

Figure 1.4: University of Wisconsin and Motorola's Micro Turbine

Early attempts at microturbines ran on compressed air. However, most of the academic research currently being done on microturbines are investigating the use of a combustible fuel as a power source. Groups at the Massachusetts Institute of Technology [Epstein, 1997], Tohoku University [Tanaka, 2003], and the University of Tokyo [Matsuo, 2003] are currently researching the use of a turbine where gases such as hydrogen or methane are burned to produce power. The MIT project, which is being sponsored by DARPA, was started in the early nineties.

They are currently using liquid hydrogen as a fuel and are predicating the use of hydrocarbons in the future. Significant numbers of papers have been published on their work as well as several patents. MIT's design, shown in Figure 1.5, assembles the generator, compressor, fuel manifold, combustion chamber, and turbine into a one cm by one cm by three mm package.

Figure 1.5: MIT's Micro Turbine Design

The microturbine is predicted to produce 5 Watts of power at a speed of around 1.4 million rpm but has yet to achieve this goal. The use of hydrocarbons for fuel could produce up to a hundred Watts of power. At 5 Watts of power, the turbine has a power density of $4,000 \text{ MW m}^{-3}$ based on the turbomachinery volume.

The group at MIT is simultaneously working on a micro generator, but has so far not tried to couple the generator to the turbine. The groups at Tohoku University and the University of Tokyo are working with MIT but on a larger scale turbine. They are trying to use the larger

scale turbines to show the feasibility of MIT's design as well as validate fabrication methodologies and numerical simulations.

The groups attempting to build a gas burning microturbine are facing a number of problems. The power produced by a turbine is equal to the torque generated multiplied by the rotational speed, as shown in Equation 1.2:

$$P = T\omega \qquad \text{Eqn. 1.2}$$

At these small scales, the torque produced is small, on the order of 30 $\mu\text{N}\cdot\text{m}$, therefore, the speed must increase significantly. This is why MIT is attempting to run their turbine in excess of one million rpm. Micro air bearings are needed that can handle these speeds and have a reasonable lifespan. Current micro bearings do not meet these requirements due to the tolerances being beyond the levels of current fabrication techniques. This is the first major problem being faced by gas fueled microturbine designers. The second is the time needed for combustion. Since the fuel moves through the turbine at high speeds and the combustion chamber is quite small, the burn time of the fuel has become an issue. A number of papers have addressed this problem, but it is still a serious issue designers must face.

Due to these problems, several universities are trying to develop or use more sophisticated fabrication techniques, which could then be scaled down to MEMS size. Two universities that are working in this area are Stanford [Kang, S, 2000] and the Katholieke Universiteit Leuven [Peirs, 2001]. Stanford has developed a process called Mold Shape Deposition Manufacturing (Mold SDM). Mold SDM is a rapid prototyping process that allows parts to be made out of silicon nitrate, alumina, and stainless steel. Using this process, they have

been able to create the complex shapes needed in an axial compressor and turbine, see Figure 1.6. These rotors are made out of silicon nitride, so they are extremely strong. Testing showed that a 12mm blade was able to spin up to 456,000 rpm using compressed air as a power source, but further work has yet to be presented.

Figure 1.6: Stanford's Microturbine

In 2002, a group from Katholieke Universiteit in Leuven, Belgium produced a miniature turbine one and a half times the size of the one described in this thesis but of the axial type instead of the impulse type. Their impeller was 10 mm in diameter and spun up to 100,000 rpm. This group was able to produce 28 Watts of mechanical power using a Laval type turbine. Figure 1.7 shows an exploded view of their design. A Laval turbine is an axial flow turbine with stationary nozzles that direct the flow at the moving blades. The turbine runs on compressed air, which enters on the right side of the figure through the coupling labeled point 1. The air is turned by the stators, labeled number 3 in Figure 1.7, and then hits the impeller, labeled number 5. The component labeled 6 contains the air exits. The Leuven group was able to create the complex geometry of an axial turbine using a technique called die-sinking electro-discharge machining (EDM). Die-sinking EDM is similar to wire EDM but instead of using a wire, a three

dimensional shaped tool, or die, is used to remove material. The removed material leaves a pattern that matches the die. This allows a complex impeller to be fabricated. This process is quite complex and therefore much more difficult to perform on the micron scale which will hamper efforts to scale the turbine down.

Figure 1.7: Katholieke Universiteit Leuven's Miniature Turbine

The Leuven turbine produced 16 Watts of electrical power when coupled with Faulhaber motor. The turbine achieved a turbine efficiency of about 18 percent and an overall efficiency of 10.5 percent. The mechanical power density of the turbine is 780 W kg^{-1} and the electrical power density is 240 W kg^{-1} .

The many problems faced by designers of combustion and axial turbines have led to the goal of designing a simpler turbine. This will be achieved by producing an impulse turbine run on compressed gas. This project started at the Rochester Institute of Technology with the work done by the 2002-2003 Senior Design Project [Miniature, 2003]. The team built their own casing and inserted the impeller, shaft, and bearings from a dentist drill. Dentist drills or handpieces come in a variety of types of turbines, but the most common is the impulse type. This is primarily due to the simplicity of impulse turbines compared to axial turbines. Dentist turbines are generally about twice the size of the turbine presented here, about ten millimeters in diameter. These turbines are designed to spin between 300,000 and 500,000 rpm and produce very little torque. One of the issues faced when investigating dentist turbines is much of the

research is proprietary, therefore very little of it has been published. More commonly published are papers comparing different competitors.

After researching the different types of dentist impellers, the team decided to use a Pelton wheel type of impeller, which has blade curvature in both the radial and axial direction and a splitter. This is the most efficient style of design for an impulse turbine but is not feasible with the current MEMS technology. The team designed a casing to move air to and from the impeller, support two bearings that the impeller shaft sits on, and be scaled down to the MEMS size. The team connected the turbine to a generator and a compressed air source and measured the electrical power output. The turbine produced a maximum electrical power of 18 Watts at pressure ratio of about 9.5. This gave a power to weight ratio of 26 for just the turbine and a power to weight ratio of 18.3 for the turbine, fuel tank, connection, and fuel.

1.3 Thesis Overview

This thesis investigates the feasibility of a micro impulse turbine run on compressed nitrogen. There are two parts to the project. The first part consists of the designing, fabricating, and testing a miniature turbine. A simple impeller that will be easily scaled down to MEMS size was designed. The impeller is at the lower size limits of conventional fabrication techniques. The casing designed by the Senior Design Team was modified and a new casing was built to house the impeller. Three generators were attached and the turbine was tested at a variety of flow conditions. The purpose of the testing was to determine the amount of power produced by the turbine to determine if it can produce enough power for a MAV. The efficiency of the turbine was also examined in hopes that it can be improved upon in the future. Chapter 2 describes the design, fabrication, test set, and data reduction techniques. The results of the testing are then presented and discussed in Chapter 3.

The second part of the project is to theoretically investigate of feasibility of a storage tank for the compressed nitrogen that will act as a power source for the turbine. The storage tank was designed for a specific run time required for MAVs as well as to maintain the high pressure. A thermodynamic analysis was done to determine how long a specified volume of nitrogen would last. A structural analysis was done to determine the wall thickness of the fuel tank. The results from these analyzes are presented in Chapter 4.

2.0 Experimental Methods

This chapter outlines the turbine design, equipment, testing procedure, and data reduction techniques. Several experiments were done to validate the performance of the turbine. The ultimate goal of the experimentation was to determine the power to weight ratio and power density of the turbine. This will enable the comparison of this turbine with the Senior Design turbine, the turbines being worked on by Leuven University and MIT, and commercial gas turbines. The secondary goal was to determine the efficiency of the turbine. Once the efficiency is found and the losses are accounted, a feasibility study can be done to determine what aspects of the turbine should be improved.

2.1 Turbine Production

This section will provide an overview of the turbine production. The impeller and casing design and fabrication processes will be discussed first. A discussion of the assembly is presented next. Drawings of the designed components can be found in Appendix A.

2.1.1 Impeller Design and Fabrication

Three impellers were originally designed for the turbine. Each impeller was designed with increasingly complex geometry as shown in Figure 2.1. These impellers were designed to be similar to Pelton Wheels. Pelton Wheels are optimized for water but it was thought that this

would be a good starting point for an air impulse turbine. The middle impeller in Figure 3.1 has curved blades in the radial direction, but not in the axial direction. The impeller on the right of Figure 3.1 has curved blades in the both the radial and axial directions as well as a splitter. This impeller mirrors conventional Pelton Wheels. Input was taken from several fabrication companies as well as Impact Technologies [Herzog, 2003] and it was decided that the two complicated impellers would be too expensive to fabricate on the project's budget. The goal of the project is to build a turbine that could replace batteries on small vehicles. Batteries are relatively cheap, in the 20 to 50 dollars range, so keeping the cost of the turbine low was a driving factor throughout the design and fabrication process. Therefore, the straight bladed impeller was chosen for use in the turbine. This impeller was modified to have fewer blades for ease of fabrication, and to have an odd number of blades. The odd number of blades was chosen to even out the fluctuations in the torque caused by the two-nozzle design of the casing discussed in section 2.2. This design was fabricated by Palma Tool & Die Co. using electro-discharge machining. The final design of the impeller is shown in Figure 2.2 next to a dime to provide a scale.

Figure 2.1: Initial Impeller Geometry

Figure 2.2: Impeller

2.1.2 Casing Design and Fabrication

A casing design that was originally done by the 2002-2003 Miniature Turbine Senior Design Team was modified to improve performance and scaled down to fit the new smaller impeller. The casing was designed to accomplish several objectives. The air needs to flow into the casing and be directed at the impeller by a nozzle. The impeller shaft needed to be supported by two bearings that would be housed in the casing. The casing should be as light and small as possible. Finally, the casing also had to be designed to minimize losses but also with fabrication difficulties in mind.

Figure 2.3: Turbine Flow Paths

The final design of the casing uses four plates as opposed to the three plate design used by the Senior Design Team. Figure 2.3 shows the flow paths within the casing. The dotted line and the arrows trace the path of the air through the turbine. The first plate is attached to the fitting for the air to enter the casing. The flow is split at point 1 in Figure 2.3 to flow outward (radial direction) from the impeller shaft through the first plate. Once the flow is outside the radius of the impeller it is turned 90°, points 2, and the leaves the first plate parallel to the shaft. The air then enters the second plate, which supports one of the bearings. The air flows through the second plate towards the third plate. The volute for the impeller is cut into both the second and third plates. This was the major modification from the Senior Design Team casing and should improve the performance by allowing the nozzles to direct the flow at the center of the impeller instead over the entire width of the impeller. There, points 3, the air turns toward the impeller and flows through two separate nozzles, points 4, that directs the flow toward the blade.

This is shown in Figure 2.4. The third plate also supports the other bearing and contains the exits for the volute. After the air leaves the volute, it turns 90° at point 5 and flows into the forth plate. This plate supports the bearing and prevents air from flowing through the bearing. Three bolts are used to hold the entire assembly together as well as to support the motor plate. The entire turbine is shown exploded in Figure 2.5.

Figure 2.4: Third Plate of the Casing

Figure 2.5: Exploded Turbine Assembly

This design was chosen largely due to its ease of fabrication since the flow paths can be easily milled into the plastic plates. Since both flow paths are identical, the air exiting both nozzles should be identical. Two nozzles were used to help balance the turbine. Another reason this design was chosen was future work may scale the turbine down to the MEMS size. Since the casing is composed of plates with channels cut into them, the casing could be fabricated using convention MEMS fabrication techniques.

The casing was fabricated in the RIT Mechanical Engineering Machine Shop. One of the main concerns during fabrication was to keep the alignment between the different plates so the shaft will spin smoothly. In order to do this the three boltholes as well as a center hole were drilled first. These holes were used through out the process to try to maintain alignment. From then on each plate worked on separately. The second and third plates were built first. The flow

paths running through the second plate parallel to the shaft and the exit holes in the third plate were drilled first. The exit holes in the third plate were used to locate the channels for the volute exit in the second plate and the flow paths in the second plate were used to locate the volute inlet in the third plate as well as the end of the channels in the first plate. Once the locations of the volute inlets and exits were marked, the volute was milled into the second and third plates. Next, the bearing holes were reamed to 0.25 inches. Then a ¼ inch NPT hole was drilled and tapped in the first plate. On the opposite side of the first plate a channel was milled to split the flow in opposite directions and end at the marked location found from the second plate. The original design of the turbine casing had three plates, not four as a used in the current model. The fourth plate was added to stop air from flowing through the bearing in the third plate.

A motor mount was made to fit on the bolts extending from the turbine casing. The eleven Watt motor is supported using three allen bolts on the motor face while the servomotor is supported using a rubber o-ring.

Several problems were faced during the fabrication process. The first was caused by the use of a brittle plastic material. During machining, as the walls got thinner they began to shatter and break. A number of casings were built. The final turbine casing chosen has several flaws shown in Figures 2.6 and 2.7. Figure 2.6 shows two flaws, the crack in the bearing support and the very small area change in the nozzle. The crack in the bearing support was filled in with Cyanoacrylate Glue and remachined to the proper size. After being remachined, the crack did not cause any problems. The nozzle originally had a much greater area change, but once testing began, backflow became a problem. This will be discussed further in Chapter 3. Figure 2.7 shows side crack in the second plate. During testing, it was checked to make sure no air was

leaking through the side crack. With the bolts tightened, no air appeared to leak through the crack, therefore it is assumed the crack had no effect on the turbine performance.

Figure 2.6: Fabrication Flaws – Bearing Hole Crack

Figure 2.7: Fabrication Flaws - Side Crack

The bearing support crack and side crack were each due to machinist error while fabricating. Another problem was machining on the small scales. Since a 0.046 mill was being used to cut the channels and form the nozzles, the mill shaft speed should be about 85,000 rpm. This is above the maximum shaft speed of the mills in the RIT Machine Shop. This may have also caused some of the problems noted previously. Since neither of these cracks appeared to affect the performance of the turbine this was the casing used for all of the testing.

2.1.3 Turbine Assembly

The impeller, shaft, bearings, gaskets, and plates were assembled into the turbine shown in Figure 2.8. The shaft was made out of a piece of 5/64-inch drill rod. The bearings were ABEC 7, shielded, and stainless steel bearing and purchased from Stock Drive Products. The gaskets were made out of 1/64-inch vegetable fiber sheet gasketing material. Originally, the turbine was to be pressed fit on the shaft, but since the turbine was made out of aluminum and the shaft out of steel, a press fit would not hold. Therefore, Loctite 660 was used to hold the impeller on the shaft. Once the Loctite dried, the bearings were pressed on the shaft as shown in Figure 2.9. This was then inserted into the bearing holes in the casing. The plates and gaskets were then bolted together. Originally, the boltholes in the third plate were tapped in order to compress the entire casing together to seal the gaps between each of the plates. The third plate boltholes are still tapped in the current prototype, but nuts are also use to hold the fourth plate on. Finally, the motor was mounted on the motor plate and this was slide on the three bolts extending out of the casing as shown in Figure 2.10. This completed the assembly of the turbine.

Figure 2.8: Turbine Casing Assembly

Figure 2.9: Impeller and Bearing Assembly

Figure 2.10: Assembled Turbine

2.2 Measurements

This section will outline the measurements taken as well as discuss the accuracy of the measurements.

2.2.1 Pressure

The Pressure in plenum was determined using an Omega General Service Pressure Gage with a range of 0 to 200 psi. This provided the total pressure of the air upstream of the turbine. The accuracy of this measurement was $\pm 1\%$.

The atmospheric pressure was found by contacting the local weather service at the Rochester International Airport, which is updated every 30 minutes. The accuracy of this measurement was ± 2.5 Pa.

2.2.2 Voltage

The voltage of the motor was found in order to calculate the power produced as well as the speed of the turbine. A RadioShack 24-Range Digital Multimeter was used to measure the voltage. The accuracy of this measurement was ± 0.0005 Volts.

2.2.3 Flow Rate

The flow rate was measured using a Gilmont Instruments Rotameter, type GF-1560 with a stainless steel ball. It measured from 4.62 to 151 liters per minute. The accuracy of this measurement was $\pm 2\%$.

2.2.4 Temperature

Three K-type thermocouples were used to measure the temperature of the air in the plenum, upstream of the turbine, and downstream of the turbine. The thermocouples were plugged into a data acquisition unit run by the Measurement and Automation software, which is a component of Labview. The accuracy of this measurement was $\pm 0.1\%$.

2.3 Experimental Setup

The experimental setup is shown in Figure 2.11. Pressurized air comes from either the shop compressor or an air tank. The compressor generates pressures up to about 80 psi. The air tank stores higher-pressure air and is cleaner than the compressor, but since the flow rates were high, the air tank could only be used for short durations. Therefore, the bulk of the testing was done with the compressor.

The air leaves the compressor or tank and travels through 30 ft of hose before entering a plenum. The plenum acts as a dampener on the system by steadying out the fluctuations in pressure and temperature caused by the compressor. The plenum also allows the stagnation pressure, measured with the Omega pressure gage, and stagnation temperature, measured by a thermocouple inserted to the center, to be found.

After leaving the plenum, the air passes over a second thermocouple before making a 90-degree turn vertical to flow through the rotameter. Once leaving the rotameter the air flows into the turbine. The air moves through the casing, powers the turbine, and then leaves the casing in the vertical direction. During testing a thermocouple is inserted into the exit stream to measure the downstream temperature of the turbine.

A motor is connected to the turbine by a coupling connected to the motor and turbine shafts. Due to the small size of the shaft, commercial couplings could not be located. Therefore, a number of different types of couplings were tried including electrical wire housing and various kinds of tape. Since the motor shaft and turbine shaft were not perfectly aligned, a flexible couple worked better. Duct tape turned out to be the best option. It was flexible enough to compensate for the misalignment, but strong enough that it would hold the shafts at the high speeds.

Figure 2.11: Experimental Setup

Three different motors were used for the testing. Multiple motors were used to investigate the affect of the motor on the turbine. Most of the testing was done on a Faulhaber

type 1628 T012B, which is a 3-phase, 10-Watt motor. While testing with this motor, a delta circuit was used as shown in Figure 2.12. Each of the resistors shown in the figure can be modified from 0 to 110 ohms. The voltage between two lines was then found by using the Radio Shack Voltmeter and measuring the RMS Voltage. Two other motors were also used during testing. The first was a motor taken from the WES-Technik LS-2.4 Servo, which is a .4-Watt DC motor. The second was a Maxon # 118396, which is a 1.5-Watt DC motor. These DC motors were tested using the setup shown in Figure 2.13. The voltage was then measured across the resistor using the same voltmeter.

Figure 2.12: 3-Phase Motor Electrical Circuit

Figure 2.13: 2-Phase Motor Electrical Circuit

2.4 Procedure

Testing began by contacting the local weather service and recording the atmospheric pressure. Next, the compressor lines were bled to remove any water that had condensed in the line. Initially the test matrix shown on the left Table 2.1 was used to test the turbine. For these tests, a load was set on the motor and the pressure was varied. The voltage, flow rate, and temperatures were then recorded. This procedure was used since it was less complicated to vary the pressure at constant intervals. However, after reviewing the data taken it was decided that holding the turbine at constant speeds would prove more meaningful. Therefore, the matrix on the right of Table 2.1 was used for the remaining tests. For these tests, a load was applied to the turbine, and then the pressure was modified until a specific voltage was reached. Once that voltage was reached, the pressure, flow rate, and temperatures were recorded.

Motor Load (Ohms)	Plenum Pressure (psi)
20	15
30	20
40	25
50	30
60	35
70	40
80	45
90	50
100	55
	60
	65
	70
	75
	80
	85

Motor Load (Ohms)	Motor Voltage (Volts)
20	0.1
30	0.2
40	0.3
50	0.4
60	0.5
70	0.6
80	0.7
	0.8
	0.9
	1
	1.1

Table 2.1: Test Matrix

2.5 Data Reduction

This section will provide an overview of the data reduction techniques used as well as provide some background information. The first section will discuss how the turbine inlet pressure is calculated. The following section will present the details of calculating the turbine power and efficiency. The last section will discuss the overall system power and efficiency. An example problem is worked out in Appendix B for more details.

Throughout this section, several different powers and efficiencies will be discussed. Turbine power is the mechanical power produced by the turbine, which is calculated from the change in pressure and temperature across the turbine. Electrical power or overall system power is the electrical power generated from the motor. The turbine efficiency is defined as the actual turbine power divided by the isentropic power. The mechanical efficiency is the electrical power divided by the turbine power. The overall efficiency is the electrical power divided by the isentropic power.

2.5.1 Turbine Inlet Pressure

In order to find the efficiencies the turbine upstream pressure needed to be found. Therefore, the pressure loss between the plenum and the turbine inlet was found. This calculation assumed the flow was incompressible because the max speed was at or below the Mach 0.3 limit. The flow was also assumed steady, there was no work being done on or by the fluid, and the pressure and internal energy are uniform across the plenum and turbine inlet cross sections. The general energy equation for pipe flow is shown in Eqn 2.1.

$$\frac{P_1 - P_2}{\rho} + \frac{\alpha_1 U_1^2 - \alpha_2 U_2^2}{2} + g(z_1 - z_2) = h_l \quad \text{Eqn. 2.1}$$

Where location 1 is in the plenum and location 2 is where the tubing meets the turbine. To solve Eqn 2.1 the data in Table 2.2 was used.

Pipe Length:	24.5 inches
1 Bend:	90
Changes in Diameter:	8
Entrance:	1
e/D:	0.001889

Table 2.2: Test Conditions used to Calculate Head Loss

In order to solve Eqn. 2.1, the Reynolds number, which is the ratio of inertia forces to viscous forces, had to be found. Once the Reynolds number and the ratio of the pipe roughness to pipe diameter, e/D, the friction factor could be determined. To find the Reynolds number the density, viscosity, and velocity of the fluid had to be determined. The density was found by assuming the fluid was an ideal gas and therefore using the ideal gas equation, Eqn. 2.2.

$$\rho_1 = \frac{P_1}{RT_1} \quad \text{Eqn. 2.2}$$

To find the viscosity, the Sutherland correlation, Eqn 2.3, was used. This is an empirical correlation based on the kinetic theory of gases relating temperature and viscosity.

$$\mu = \frac{bT^{\frac{1}{2}}}{1 + \frac{S}{T}} \quad \text{Eqn. 2.3}$$

Where the constants are S=110.4 K and b = 1.458 x 10⁻⁶ kg (msK^{1/2})⁻¹ for air

The velocity of the fluid was found using Eqn 2.4. At location 1 in Eqn 2.1, the velocity was assumed approximately zero, therefore Eqn 2.4 was only used at location 2.

$$U = \frac{Q}{A} \quad \text{Eqn. 2.4}$$

Using the density, viscosity, and velocity, the Reynolds number was found using Eqn 2.5.

$$\text{Re} = \frac{\rho \ U D}{\mu} \quad \text{Eqn. 2.5}$$

Once the Reynolds number was found, the friction factor could be found using the formula derived by Miller shown in Eqn 2.6.

$$f = 0.25 \left[\log \left(\frac{e/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^{-2} \quad \text{Eqn. 2.6}$$

From Table 8.2 [Fox, 1998], the entrance loss coefficient is 0.15, and from Figure 8.15, the area change loss coefficient is approximated at 0.2. The head loss can then be calculated using Eqn. 2.7.

$$h_l = f \left(\frac{L}{D} + \frac{L_{e,bend}}{D} \right) \frac{U^2}{2} + (K_{entrance} + K_{area_change}) \frac{U^2}{2} \quad \text{Eqn. 2.7}$$

Substituting in the coefficients and simplifying, the head loss becomes:

$$h_l = (f * 253.5 + 1.75) \frac{U^2}{2} \quad \text{Eqn. 2.8}$$

Substituting Eqn 2.8 into Eqn. 2.1 and solving for the pressure at 1:

$$P_2 = P_1 - \rho \left[\left((f * 253.5 + 1.75) \frac{U^2}{2} \right) - \left(\frac{\alpha_1 U_1^2 - \alpha_2 U_2^2}{2} \right) - (g(z_1 - z_2)) \right] \quad \text{Eqn. 2.9}$$

Equation 2.9 was solved for each test condition to provide the upstream pressure for the turbine.

2.5.2 Turbine Power and Efficiency

The impeller extracts work from the air moving over it. This work can be found using the change in enthalpy and velocity as shown in Eqn 2.10 where state 1 is the upstream condition and state 2 is the downstream condition.

$$W_{turbine} = (h_1 - h_2) + \frac{1}{2}(U_1^2 - U_2^2) \quad \text{Eqn. 2.10}$$

The turbine power can then be found by multiplying the work by the mass flow rate:

$$P_{turbine} = \dot{m} * W = \dot{m} \left[(h_1 - h_2) + \frac{1}{2}(U_1^2 - U_2^2) \right] \quad \text{Eqn. 2.11}$$

The mass flow rate can be calculated by multiplying the density by the volumetric flow rate shown in Eqn. 2.12. Density can be found using Equation 2.2 and knowing the pressure and temperature at the rotameter. The enthalpy can be found using the temperature and the specific heat of air. The upstream velocity can be found using the volumetric flow rate because the changes in pressure and temperature are negligible; this is shown in Eqn. 2.13.

$$\dot{m} = \rho_1 Q \quad \text{Eqn. 2.12}$$

$$U_1 = \frac{Q}{A_1} \quad \text{Eqn. 2.13}$$

The downstream velocity is found using the conservation of mass, which says the mass flow into the turbine equals the mass flow out of the turbine. The downstream pressure and temperature are used to find the density at state 2. This yields Eqn. 2.14.

$$U_2 = \frac{\rho * Q}{\frac{P_2}{RT_2} A_2} = \frac{P_1 Q}{RT_1} * \frac{RT_2}{P_2 A_2} = \frac{P_1 T_2 Q}{P_2 T_1 A_2} \quad \text{Eqn. 2.14}$$

Substituting these equations into Eqn. 2.11, an equation for the turbine power is shown in Eqn. 2.15.

$$P_{turbine} = \frac{P_1 Q}{RT_1} \left[Cp^* (T_1 - T_2) + \frac{1}{2} \left(\left(\frac{Q}{A_1} \right)^2 - \left(\frac{P_1 T_2 Q}{P_2 T_1 A_2} \right)^2 \right) \right] \quad \text{Eqn 2.15}$$

This equation allows the turbine power to be plotted verse the pressure ratio knowing the pressure and temperature at state one and two and the volumetric flow rate.

The turbine efficiency can also be found. The turbine efficiency is defined as the actual turbine power divided by the isentropic turbine power. The isentropic power can be found in the same way the turbine power was found but instead of using the actual conditions at state two, calculating the conditions using isentropic relationships. This is shown in Eqn 2.16 where the subscript 's' denotes an isentropic state.

$$\eta_{turbine} = \frac{P_{turbine}}{m \left[(h_1 - h_{2s}) + \frac{1}{2} (U_1^2 - U_{2s}^2) \right]} \quad \text{Eqn. 2.16}$$

The downstream temperature and velocity can be found using equations 2.17 and 2.18. Equation 2.17 uses the isentropic relationships to find the temperature. Equation 2.18 uses the conservation of mass and assumes the flow is steady and incompressible on the control surfaces.

$$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{Eqn. 2.17}$$

$$U_{2s} = \frac{\rho_1 U_1 A_1}{\rho_{2s} A_2} \quad \text{Eqn. 2.19}$$

The downstream density can be found by combining the ideal gas law shown in equation 2.2 at the turbine inlet and the isentropic relationship shown in equation 2.17 to get equation 2.19.

$$\rho_{2s} = \frac{P_2}{RT_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}}} \quad \text{Eqn. 2.19}$$

Equation 2.19 can be substituted into Equation 2.18 to find an equation for the isentropic downstream velocity shown in Equation 2.20.

$$U_{2s} = \frac{\rho_1 U_1 A_1}{\left(\frac{P_2}{RT_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}}} \right) A_2} = \frac{P_1 U_1 A_1 RT_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}}}{RT_1 P_2 A_2} \quad \text{Eqn. 2.20}$$

$$= \frac{P_1}{P_2} \frac{Q}{A_1 A_2} \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}} = \frac{P_1}{P_2} \frac{Q}{A_2} \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}}$$

Equations 2.12, 2.13, 2.17, and 2.20 can be substituted into 2.16 to derive an expression for the turbine efficiency shown in Equation 2.21.

$$\eta_{turbine} = \frac{P_{turbine}}{\frac{P_1}{RT_1} Q \left[Cp \left(T_1 - T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}} \right) + \frac{1}{2} \left(\left(\frac{Q}{A_1} \right)^2 - \left(\frac{P_1}{P_2} \frac{Q}{A_2} \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}} \right)^2 \right) \right]} \quad \text{Eqn 2.21}$$

This equation allows the turbine efficiency to be plotted versus the pressure ratio in order to determine any trends in the efficiency.

2.5.3 Overall System Power and Efficiency

This section will discuss the overall system, which is defined as the turbine, the coupling and the motor. The power of the motor can be determined from the motor voltage and the load being applied to the motor. Several different motors were used during testing, two of which were two-phase motors, and the other was a three-phase motor. The power calculation for two-phase motors is shown in Eqn. 2.22 and the power calculation for three-phase motors is shown in Eqn. 2.23.

$$P_{Elec} = \frac{V^2}{R} \quad \text{Eqn. 2.22}$$

$$P_{Elec} = \frac{3 * V^2}{R} \quad \text{Eqn. 2.23}$$

Using these equations the power generated by the turbine can be compared to other turbines of the same size.

Two more efficiencies were also calculated, the mechanical efficiency and the overall efficiency. The mechanical efficiency is the electrical power produced divided by the turbine power shown in Equation 2.24. The overall efficiency is the electrical power produced divided by the power that would be produced isentropically as shown in Equation 2.25.

$$\eta_{mech} = \frac{P_{Elec}}{P_{turbine}} \quad \text{Eqn. 2.24}$$

$$\eta_o = \frac{P_{Elec}}{P_{ideal}} = \eta_{turbine} * \eta_{mech} \quad \text{Eqn. 2.25}$$

The mechanical efficiency accounts for losses in the bearings, coupling, and motor. The overall efficiency describes the entire system and gives a quantitative measure for how well the energy in the gas entering the turbine is converted into usable electrical power.

3.0 Results

This section will discuss the results the experiments done on the turbine. The turbine power and efficiency will be discussed first. The overall power and the mechanical and overall efficiencies are presented next. Example plots are presented for each of the different motors used during testing. All of the results are provided in appendix D. Once the results are presented, they are then compared to the results from similar sized turbines. An uncertainty analysis was performed and is shown in Appendix C.

3.1 Turbine Power and Efficiency

This section will discuss the results of the turbine, which will allow comparisons among the tests run with different motors. Three different motors were connected to the turbine for testing. The first motor used was the Technik servomotor. Figure 3.1 plots the turbine power versus pressure ratio. The first type of test matrix discussed in Chapter 2 was used to test this motor. Therefore, each line on Figure 3.1 is a constant load or resistance line and is called R in the figure. The turbine power produced with the Faulhaber motor is shown in Figure 3.2 and with the Maxon motor is shown in Figure 3.3. Both these motors were tested using the second type of test matrix. Therefore, each line on Figures 3.2 and 3.3 are lines of approximately constant speed, which is measured by voltage of the motor and is called V in the figures. The uncertainty of the turbine power in Figures 3.1, 3.2, and 3.3 is +/- 10 %.

The power presented in Figures 3.1, 3.2, and 3.3 are calculated based on the change in temperature and pressure of the air. Therefore, the plot shows just the power produced by the impeller and is not affected by losses in the bearings, alignment, or motor.

Figure 3.1: Calculated Turbine Power vs. Pressure Ratio – Servomotor Test #2

Figure 3.2: Calculated Turbine Power Vs Pressure Ratio - Faulhaber Motor Test # 3

Figure 3.3: Calculated Turbine Power Vs Pressure Ratio - Maxon Motor

The maximum power reached was 13.8 $\pm$ 1.4 Watts during the third test with the Faulhaber motor. The max power reached during the test with the Maxon motor was 13 Watts. This shows that the turbine is producing roughly the same power at the same pressure ratio on different days and with different motors. This shows that the turbine is repeatable. In addition, each of these plots shows the turbine power increases linearly with increasing pressure ratio. The tests with the Technik motor followed a trend of about 0.75 to 1.0 Watts per unit pressure ratio while the Faulhaber and Maxon tests followed a trend of 2 to 2.5 Watts per unit pressure ratio. During tests with the Technik motor, the maximum turbine power was 26 Watts. The lower trend and power is largely due to changes made to the casing after the trials with the Technik motor. After studying the results of the Technik motor tests it was noticed the performance of the turbine decreased at higher-pressure ratios. After inspecting the casing, it was determined to be due to reverse flow inside the casing. The reverse flow was happening during all the test points, but was more apparent at higher-pressure ratios. This flow was happening due to the design of the casing. Since there are two nozzles and two exits, a portion of the air flowing into the turbine volute came in one nozzle and left through the wrong exit. To correct this problem the nozzles in the casing were modified to direct the flow better. Unfortunately, the Technik motor was burned out during testing so further tests with that motor were not completed.

The linear trend shown on the turbine power figures also means sonic velocity was not approached in the nozzle. Therefore, a higher-pressure ratio or an improved nozzle would be able to output a higher momentum stream and the turbine would produce more power.

The efficiency of the turbine is shown in Figures 3.4, 3.5, and 3.6 for each of the different cases. This efficiency is the turbine power divided by the isentropic power. The uncertainty of the turbine efficiency in Figures 3.4, 3.5, and 3.6 is +/- 23 %.

Figure 3.4: Turbine Efficiency vs Pressure Ratio – Servomotor Test #2

Figure 3.5: Turbine Efficiency Vs Pres. Ratio @ Const. Volt.-Faulhaber Motor Test # 1

Figure 3.6: Turbine Efficiency Vs Pressure Ratio @ Constant Voltage - Maxon Motor

These figures all show the similar trends so the turbine is repeatable although the numbers varying. A maximum efficiency of 2.8 ± 0.6 % reached was during the maxon motor test. This is low compared to other turbines of a similar size. The microturbine group from Katholieke University in Leuven, Belgium reported a turbine efficiency of 18.5 %. This is largely due to the complexity of the blade. The Leuven impeller is quite intricate, but this allows the air to flow over it efficiently, while the impeller presented here is very simple so there are more losses. This is a trade off made between the cost of the impeller and the desired efficiency.

3.2 Electrical Power, Mechanical Efficiency, and Overall Efficiency

This section will discuss the electrical power produced and compare it to other small turbines. The mechanical overall efficiencies then be presented and discussed. These efficiencies highlight where major improvements need to be made to the turbine in order to have a viable miniature turbine.

The electrical power output for the Technik motor is shown in Figures 3.7, 3.8, and 3.9. The uncertainty of the electrical power in these Figures is ± 0.2 %. These figures show the

power produced on three successive days of testing got more chaotic during each test. The servomotor was not designed to run at the speeds it was subjected to during testing. It was originally used because of its very low starting torque. Therefore, it is believed that the motor burned out during testing which is why the results jump around and become less consistent with each test.

Figure 3.7: Electrical Power vs Pressure – Servomotor Test #1

Figure 3.8: Electrical Power vs Pressure – Servomotor Test #2

Figure 3.9:Electrical Power vs Pressure – Servomotor Test #3

Figures 3.7, 3.8, and 3.9 all show that the power dropped substantially around a pressure ratio of four. It is unsure what caused this. Later testing did not show this trend. It may have been a vibrational issue caused by the setup using that motor. The Technik motor was the lightest of the three motors used, so this may be an issue future designers should take into account.

The electrical power produced by the Faulhaber and Maxon motors is shown in Figure 3.10 and 3.11. The uncertainty of the electrical power in these Figures is +/- 0.2 %.

Figure 3.10: Electrical Power Vs Pressure Ratio @ Constant Voltage - Faulhaber Motor Test # 2

Figure 3.11: Electrical Power Vs Pressure Ratio @ Constant Voltage - Maxon Motor

The maximum electrical power produced by the system was 208 ± 0.4 millie Watts using the Maxon motor. The maximum electrical power produced by the Faulhaber motor was 90 ± 0.2 mW and by the Technik motor was 88 ± 0.2 mW. These figures show several things. First, the choice of motor has a large impact on the electrical power produced. The Faulhaber and the Maxon motors are similar in quality but slightly different in specifications. The Faulhaber is higher power motor, therefore it requires more torque. The turbine was unable to produce enough torque to spin the Faulhaber motor fast enough to get the motor into its efficient region of operation. The second noticeable thing on the graphs is the total electrical power produced is three orders of magnitude lower than the turbine power. The reason for this will be discussed with the efficiencies.

The mechanical efficiency is shown in Figures 3.12 and 3.13. The uncertainty of the mechanical efficiency in these Figures is $\pm 12\%$. The mechanical efficiency includes the losses in the bearings, alignment, and motor. The figures show how low the mechanical

efficiency is with the maximum value being 1.9 +/- 0.2 %. The Maxon motor produced the best efficiency because it is operating in a more efficient condition then the Faulhaber.

Figure 3.12: Mechanical Efficiency Vs Pres. Ratio @ Const Volt - Faulhaber Motor Test#2

Figure 3.13: Mechanical Efficiency Vs Pressure Ratio @ Constant Voltage - Maxon Motor

The low values of mechanical efficiencies are believed to be due to several reasons. First, the alignment of the motor shaft with the impeller shaft was a problem. A tolerance stack up shows the alignment could be off by as much as 5 degrees. This would severely hamper the torque and power being applied to the motor. This problem could be lessened through several solutions. The casing could be CNC milled as opposed to hand milled as it was for the casing used here. Another option is to build a coupling device that will allow the power to be transferred even with a small miss alignment. A second reason the mechanical efficiency is low may be due to misalignment of the bearings. This would be caused by the same problems mentioned above and could be solved by a new design that would better align the bearings. Another option that should be considered is the use of air bearings. A microturbine would have to use air bearings because ball bearings are not made on that scale. Therefore, incorporating air bearings into early designs of the turbine would help in achieving the final goal of a microturbine. A third reason the mechanical efficiency is so low is due to the motors used. As stated before the Maxon motor worked better than the Faulhaber motor because the turbine produced torque and speed that better matched with that motor. Therefore, a motor should be used that better matches the speed and torque of the turbine. The mechanical efficiency of this turbine must be improved in order to have a feasible miniature turbine.

Examples of the overall efficiency are shown in Figures 3.14 and 3.15. The overall efficiency is the electrical power divided by the isentropic power or equivalently, the turbine efficiency times the mechanical efficiency. Since the mechanical efficiency is low, this pulls the overall efficiency down as well. The uncertainty in the overall efficiency shown in Figures 3.14 and 3.15 is $\pm 27\%$.

Figure 3.14: Overall Efficiency Vs Pres Ratio @ Const Volt - Faulhaber Motor Test # 2

Figure 3.15: Overall Efficiency Vs Pressure Ratio @ Constant Voltage - Maxon Motor

The maximum overall efficiency was 0.043 +/- 0.012 %. The overall efficiency of the Leuven turbine is 10.5 % and the mechanical efficiency is 57 %. The turbine efficiency of the turbine presented here is on the same order of magnitude of the Leuven turbine, but the mechanical efficiency is two orders of magnitude lower. This causes the large difference in the

overall efficiency between the two turbines. Significant changes must be made to the turbine to improve the mechanical efficiency in order to produce a competitive turbine.

3.3 Comparisons

Two methods will be used to compare the turbine presented in this work with other similar sized turbines. The first method is the power to weight ratio, which can be either the electrical power or mechanical power divided by the weight of the turbine. Table 3.1 presents these results. Power to weight ratio is most important for aerial vehicles where the weight is a significant factor.

	Power to Weight Ratio (W/N)
Impulse Turbine - Mechanical	44
Senior Design - Mechanical	87
Leuven - Mechanical	80
Impulse Turbine - Electrical	0.23
Senior Design - Electrical	87
Leuven - Electrical	25

Table 3.1: Power to Weight Ratio

The results in Table 3.1 compare the mechanical and electrical power of the impulse turbine presented here with the 2003 Senior Design turbine and the Leuven turbine. The weight of the generator is included in the electrical power to weight ratio but not in the mechanical power to weight ratio. The mechanical power to weight ratio of the Leuven turbine is about twice that of this turbine. This means if the mechanical efficiency of this turbine was increased to match that of the Leuven turbine, the Leuven turbine would still have about twice the

electrical power to weight ratio. This shows that a more complex blade shape is required to raise the power to weight ratio to match the Leuven turbine.

The Senior Design turbine has a much greater electrical power to weight ratio than the turbine presented here. It is important to note that the Senior Design turbine and this turbine used the same casing design, same connectors, same motor, and same testing conditions. The differences were in the size and complexity of the blade, the size of the housing, and the use of aluminum instead of lexan for the casing material. Since the Senior Design impeller was about twice as large, the casing was about twice as large. This, and the use of aluminum instead of lexan for the casing material, made the fabrication of the casing much simpler and more exact, which would of improved the alignment of the turbine. This made the mechanical efficiency of the Senior Design turbine greater then it was for the turbine presented here. Another reason the mechanical efficiency would be better in the Senior Design turbine was the larger blade produced more torque. The greater torque matched the torque needed by the motor to raise the speed to its efficient range. Aside from these differences, the only other change was the shape of the blade. The Senior Design turbine had an intricate, bucket shaped blade while turbine presented in this work is much simpler and easier to manufacture. This tradeoff is estimated to have lowered the mechanical power by at least 50 percent. A compromise is needed between the to blade shapes. The Senior Design impeller is too complicated to fabricate on the micro scale, while the aerodynamic losses are too great for the simple turbine tested here. Therefore, an impeller with curved blades in the radial direction should be a good middle point.

The Senior Design turbine outperformed the Leuven turbine, although at a much higher-pressure ratio. However, achieving a high-pressure ratio using the proposed fuel take is simple and the Leuven turbine cannot produce any more power with the current design because the

turbine has reached the sonic limit. This shows that an impulse turbine is a viable option, but a complex blade is needed.

The second method of comparison uses the power density of the turbines. Power density is calculated by dividing the power by the mass. These results are shown in Table 3.2. The power density is a more important consideration when the miniaturization of components is a high priority such as ground-based robots.

	Power Density	
Impulse Turbine - Mechanical	431	W/kg
Leuven - Mechanical	780	W/kg
Lithium Ion Battery	230	W/kg
MIT - Predicted	2000	W/kg
Commercial Gas Turbines	4,000 – 10,000	W/kg

Table 3.2: Mechanical Power Density

Table 3.2 compares the mechanical power densities of the turbine presented here with the predicted performance of the MIT turbine, the Leuven turbine, lithium ion battery, and a commercial gas turbine. The table shows that the power density of the turbine presented here is at the same order of magnitude as the Leuven turbine and the lithium ion battery. If the mechanical power is transformed into electrical power, there will be losses but they will still be on the same order of magnitude. This is important to note because while batteries are a mature technology, these turbines are not. Therefore, many improvements can be made to increase their power density. The power density of the MIT turbine and commercial turbines are an order of magnitude greater than this turbine or the Leuven turbine. This is due to the much greater power density of combustible fuels compared to compressed gas. However, there are applications where a hot gas is not desirable. For example, a MAV run on liquid hydrogen will be easily seen during an infrared scan, while one run on compressed gas will not.

4.0 Feasibility of a Compressed Nitrogen Storage Tank

A tank was designed to investigate whether or not it could store the compressed nitrogen that supplies the power for a miniature or micro turbine. The tank had to meet two requirements: it must withstand the high-pressure gas and contain enough gas to power the MAV for 15 minutes. This is the run time specified by the Department of Defense for MAVs. The tank was designed to hold a pressure of 69 MPa (10,000 psi). This pressure was chosen because this is the max pressure of commercially available valves. Silicon carbide was chosen as the material for the tank due to its high yield strength. This means that less material is needed to contain the high pressure, which lightens the entire vehicle. Two analyses were done to design the nitrogen storage tank; a thermodynamic analysis to determine the run time for a specified volume of the tank and a structural analysis to determine the required wall thickness of the tank.

4.1 Thermodynamic Analysis of Storage Tank Gas

To determine the amount of nitrogen needed for a specified run time of the turbine which will specify the size of the storage tank, a thermodynamic analysis was done. As the nitrogen leaves the tank, the remaining gas expands causing the temperature to decrease. Due to the long run times of 15 minutes, this temperature decrease will have an effect on the mass flow leaving the tank. In addition, due to the high pressure of 69 MPa (10,000 psi) in the tank, the nitrogen could not be assumed ideal. This required a real gas model to be used. Several assumptions were made to model the problem. The tank was assumed rigid, therefore no change in the volume was considered. The nitrogen was assumed entirely gas. The state of the nitrogen in the

tank is in equilibrium. An iterative process based on the conservation of mass and energy was used to solve the problem. The initial, boundary, and final conditions, the methodology, and the two methods used to implement the process are outlined below.

4.1.1 Initial, Boundary, and Final Conditions

Several boundary conditions were specified. The shape of a cylinder was used for the storage tank. This was chosen because silicon carbide can be bought in tube stock. Therefore, the length, diameter, and thickness of the tubing had to be specified. The volume and surface area of the cylinder could be found from these. The diameter of the exit port also needed to be specified. From this diameter, the exit or throat area of the tank could be determined.

A number of initial conditions needed to be specified to run the analysis. Table 4.1 shows these values. The temperature and pressure were decided upon, then using a thermodynamic table for nitrogen, the specific volume, enthalpy, and internal energy were found.

T=293	Temperature in degrees Kelvin
Pi=68.948	Pressure in MPa (10,000 psi)
v=0.00201975	Specific Volume in m ³ /kg
h=2.680158	Enthalpy in J/kg
u=139634	Internal Energy in J/kg

Table 4.1: Initial Conditions

From these values the initial mass in the tank could be determined by equation 4.1.

$$M_{initial} = \frac{\forall}{v} \quad \text{Eqn. 4.1}$$

The final condition was reached when the velocity of the nitrogen at the throat was no longer sonic. The pressure outside the tank was assumed atmospheric (0.101325 MPa). From

an isentropic flow chart, the ratio of the total pressure to the static pressure needed to have a sonic velocity at the throat is 1.893. This gives a total pressure in the tank to be 0.1918 MPa. When the pressure in the tank decreased to this value, the calculation was finished.

4.1.2 Methodology

Using the conditions outlined above the analysis can now be done. At each time step the mass, pressure, and temperature was determined. This was done using two different methods in order to check the results. The program was first written using analytical equations that model nitrogen. That program was compared to one using table look-ups to find the property values. The two methods are outlined below.

4.1.2.1 Analytical

The analytical method uses equations of state to model the nitrogen as it is expelled from the tank. Starting with the initial and boundary conditions the mass can be found based on the conservation of mass equation shown in equation 4.2.

$$M_{t+\Delta t} = M_t - \dot{m} * dt \quad \text{Eqn. 4.2}$$

where:

$$\dot{m} = \rho * U * A_t \quad \text{Eqn. 4.3}$$

In order to find the mass flow rate the density and the velocity has to be calculated. The density is the mass at $t=1$ divided by the volume. The velocity is found using equation 4.4.

$$U = \sqrt{\gamma * R_N * T} \quad \text{Eqn. 4.4}$$

where:

$$\gamma = \frac{R_N}{Cp - R_N} + 1 \quad \text{Eqn. 4.5}$$

Since specific heat is a function of temperature, an equation of state derived from the kinetic theory of gases by Span et. al. was used to calculate gamma at each time step [Span, 2000].

Once the mass at t=2 is found the new specific volume can be calculated by dividing the volume by the new mass. Using the specific volume and the temperature the pressure at t=2 can be found using the Benedict-Webb-Rubin equation of state shown in equation 4.6.

$$P = \frac{R_N T}{V_0} + (BRT - A - \frac{C}{T^2}) * \frac{1}{V_0^2} + \frac{bRT - a}{V_0^3} + \frac{a\alpha}{V_0^6} + \frac{C}{V_0^3 T^2} * (1 + \frac{\gamma}{V_0^2}) * \exp(-\frac{\gamma}{V_0^2}) \quad \text{Eqn. 4.6}$$

Where A, B, C, a, b, c, gamma, and alpha are constants shown in Table 4.2 and

$$V_0 = v * M \quad \text{Eqn. 4.7}$$

Constant	Value
a =	2.54 * 10 ⁻²
A =	1.0673
b =	2.328 * 10 ⁻³
B =	4.074 * 10 ⁻²
c =	7.37981 * 10 ²
C =	8.164 * 10 ³
α =	1.272 * 10 ⁻⁴
Δ =	5.3 * 10 ⁻³

Table 4.2: Constants for Benedict-Webb-Rubin Equation of State

Using the pressure the enthalpy can be found using equation 4.8.

$$h = u + P * v \quad \text{Eqn. 4.8}$$

The internal energy at t=2 can now be calculated using the conservation of energy equation shown in Eqn 4.9:

$$u_{t+\Delta t} = \frac{1}{M_{t+\Delta t}} * \left[M_t * u_t - (M_t - M_{t+\Delta t}) * (h + \frac{1}{2} U^2) + \dot{H} \right] \quad \text{Eqn. 4.9}$$

where:

$$\dot{H} = \frac{k * dt * A_{surf}}{t} * (T_i - T_{i+\Delta t}) \quad \text{Eqn. 4.10}$$

Once the internal energy at $t=2$ is known, the new temperature can be found using Eqn 4.11

$$u_{t=2} - u_{t=1} = (Cp - R_N)_{t=1} * T_{t=1} - (Cp - R_N)_{t=2} * T_{t=2} \quad \text{Eqn. 4.11}$$

This equation relates the change in internal energy to the change in temperature. Since Eqn 4.11 is a function of specific heat, which is a function of temperature, the equation cannot be solved explicitly for temperature. Therefore, Newton's Method was used to solve for the temperature. The details of this calculation are shown in code provided in Appendix E.1. Once the temperature is found the time is indexed to the new time and the process is started. This process is continued until the final condition of an internal pressure of 0.1918 MPa is met. This gives the run time based on the initial volume and throat diameter.

4.1.2.2 Numerical

The same algorithm that was used to find the run time analytically was used for the numerical calculation but instead of using equations of state, properties were looked up on a chart. Therefore, the specific heat, the pressure, and the temperature are found using nitrogen tables. The code is shown in Appendix E.2.

4.1.3 Results

Two programs were written to determine the total time it took for the nitrogen to leave the tank based on an initial pressure, throat diameter, and tank volume. Figure 4.1 plots the volume of the tank versus the run time for a 69 MPa tank with a throat diameter of 25 microns.

Figure 4.1: Run Time vs Volume

As shown in Figure 4.1, the analytical model predicts a longer run time than the numerical model by about fourteen percent. This is due to differences between the equations of state used and the actual nitrogen data. Therefore, the numerical model should be more accurate.

For a fifteen minute run time, the volume of the tank should be 27,650 mm³ according to the numerical model. A cylinder 10 mm in diameter and 35 cm long will contain this volume. This is large for a MAV, but the volume needed can be brought down by scaling down the throat diameter. For a 15 micron throat diameter a 15 mm diameter by 13 cm long cylinder is required, so small changes in throat diameter have a large change on the run time.

4.2 Structural Analysis

A structural analysis was done on the storage tank to determine the wall thickness required to prevent the tank from deforming. As mentioned before, silicon carbide (SiC) was chosen as a material for the tank. One issue caused by the use of this material is the large variability of material properties depending on specimen fabrications and size. For example, the yield strength of SiC can vary by several orders of magnitude based on how it was formed and whether it's being used on the micro or macro scale. Therefore, material properties were taken from a vendor, MarkeTech International, Inc, who makes SiC in the size range needed for the fuel tank. They reported a yield strength of 4.6 GPa, which was used to calculate the required wall thickness of the tank.

A thin walled cylinder analysis was conducted first, but this gave a wall thickness was not valid for the thin wall analysis. Therefore, a thick walled analysis was conducted based on the equations derived by Boreasi, Schimidt, and Sidebottom, which are shown in equations 4.11-4.13. Figure 4.2 shows the set up of the problem [Boreasi, 1950].

$$\sigma_{rr} = \frac{P_1 a^2 - P_2 b^2}{b^2 - a^2} - \frac{a^2 b^2 (P_1 - P_2)}{r^2 (b^2 - a^2)} \quad \text{Eqn 4.11}$$

$$\sigma_{\theta\theta} = \frac{P_1 a^2 - P_2 b^2}{b^2 - a^2} + \frac{a^2 b^2 (P_1 - P_2)}{r^2 (b^2 - a^2)} \quad \text{Eqn 4.12}$$

$$\sigma_{zz} = \frac{P_1 a^2 - P_2 b^2}{b^2 - a^2} + \frac{P_{axial}}{\pi (b^2 - a^2)} \quad \text{Eqn 4.13}$$

Figure 4.2: Structural Analysis Setup

Where a is the inside radius, b is the outside radius, P_1 is the internal pressure, and P_2 is the external pressure. Polar coordinates are set up with the origin at the center of the cylinder and the z -axis along the centerline of the turbine. Assuming $P_2=0$, $P_{axial}=0$, and $r=a$ for max stress equations 4.11-4.13 reduce to:

$$\sigma_{rr} = -P_1 \quad \text{Eqn 4.14}$$

$$\sigma_{\theta\theta} = \frac{P_1(a^2 - b^2)}{b^2 - a^2} \quad \text{Eqn 4.15}$$

$$\sigma_{zz} = \frac{P_1 a^2}{b^2 - a^2} \quad \text{Eqn 4.16}$$

Using the inside diameter determined by the thermodynamic analysis, the yield strength of silicon carbide, and the internal pressure, the outside diameter can be found. Table 4.3 shows max stresses in each direction as a function of size.

a (m)	b (m)	t _{wall} (mm)	σ_r (MPa)	σ_θ (MPa)	σ_z (MPa)	σ_{yield} (MPa)	FS
0.0050	0.0055	0.5000	68.95	725.60	328.32	4600	6.3
0.0050	0.0060	1.0000	32.91	346.31	156.70	4600	13.3
0.0050	0.0065	1.5000	20.98	220.83	99.92	4600	20.8
0.0050	0.0070	2.0000	15.08	158.72	71.82	4600	29.0
0.0050	0.0075	2.5000	11.58	121.90	55.16	4600	37.7
0.0050	0.0080	3.0000	9.28	97.68	44.20	4600	47.1

Table 4.3: Max Stresses for Various Sizes

For an inside radius of 5 millimeters and an internal pressure of 69 MPa (10,000 psi), an outside radius of 7 millimeters is required. This results in a factor of safety of almost thirty, which should overcome any stress concentration effects where the cylinder and end cap meet.

4.3 Conclusion

It has been shown that a fuel tank can store nitrogen to power the miniature or micro turbine. A vessel that has an outside diameter of six millimeters, a wall thickness of one millimeter, and a length of 38 centimeters will safely store the high-pressure gas needed to run a miniature turbine. This is longer than the length of an MAV but could be built in a circular shape, which would fit on a MAV. A thermodynamic analysis has been done to ensure that this size tank will fully power the MAV for a minimum of fifteen minutes. As the performance of the miniature turbine improves, the flow rate required will decrease. This will allow smaller tanks to be used and longer run times to be achieved.

5.0 Conclusions and Recommendations

5.1 Conclusions

In the future, microturbines will have a large impact on small vehicles such as micro air vehicles and possibly on any electronics that currently use batteries for power. Microturbines offer an alternative to batteries that theoretically have a better power to weight ratio and power density [Epstein, 2000]. To achieve the goal of a working microturbine, an initial study of a larger scale turbine has been completed at the Rochester Institute of Technology. This study investigated a design for the turbine's fuel tank, designed and built a miniature turbine, and set up a method for experimentally verifying a small turbine. This report presented an overview of the work done on the miniature turbine.

A simple miniature turbine was built and tested at a range of conditions. These results showed that the turbine was producing up to 13.8 Watts of mechanical power. Due to losses in the alignment/coupling, bearings, and motor the turbine / generator only produced 208 Milliwatts of electrical power. This corresponds to a turbine isentropic efficiency of up to 2.8 percent, a mechanical efficiency of 1.9 percent, and an overall efficiency of 0.043 percent. It was found that the turbine has a mechanical power density of 431 Watts per kilogram, which compares well with other turbines of the same size.

The turbine assembly described in this paper, including the fuel tanks, has a mechanical power to weight ratio of 30 Watts per Newton. Current Lithium Polymer Batteries have a power to weight ratio of about 52 Watts per Newton depending upon their size. This simple prototype turbine has a power to weight ratio on the same orders of magnitude as the mature technology of

current batteries. Future generation miniature turbines will hopefully improve on this and surpass batteries in both power density and power to weight ratio. One area where miniature turbines will not compete well with batteries is cost. Current high performance batteries used on MAVs cost around 20 dollars for the pack. To manufacture a miniature similar to the one presented in this thesis would cost around 650 dollars. Therefore, miniature turbines will probably not be standard electronics. However, in devices where performance is more important than cost miniature turbines will have a place.

The feasibility of a fuel tank for a miniature or micro turbine was investigated. An analysis was done to verify that a tank of nitrogen stored at 10,000 psi could provide enough fuel to supply a turbine for the fifteen minute run time designated by DARPA. The silicon carbide tank was analyzed to confirm it could contain the high-pressure gas. Therefore, a cylinder with an outside diameter of six millimeters, a wall thickness of one millimeter, and a length of 35 centimeters could be used to store the high-pressure gas needed to run a miniature turbine.

Once improvements have been made to the miniature turbine and a fuel tank is tested experimentally, MAVs will have an option to use a turbine instead of batteries. The mechanical power produced by the turbine could be used to drive the propeller of the MAV while a separate shaft drives a micro motor to produce the electrical power needed by the vehicle. As further improvements are made to the turbine, it will outperform batteries and increase the performance of MAVs.

5.2 Recommendations

A number of things should to be investigated in order to produce a microturbine that is competitive with batteries. The most important issue is the mechanical efficiency. This can be improved several ways. One way is to design a casing that will ensure better alignment. This author believes most of the losses were due to misalignment. A second way to improve the mechanical efficiency is by modifying the bearings. This can be done by finding commercially available, high speed, sealed bearings or by investigating air bearings. If the turbine is scaled down to the MEMS size, air bearings will have to be used because there are no commercially available ball bearings at that scale. Therefore, incorporating air bearings into the design of the miniature turbine may help ease the transition to a microturbine. The mechanical efficiency can also be improved by utilizing a motor that better matches the specifications of the turbine.

Several other things can be done to improve the turbine. It is very difficult to model the turbine analytically. This makes it hard to answer a number of questions that need to be answered to improve the turbine. Therefore, a test bed should be built where the impeller and the volute can be interchanged. A test bed could determine experimentally the answers to the following questions:

- How many nozzles should the casing contain?
- What angle should the nozzles impact the blade?
- How many blades should the impeller have?
- Is the extra cost of machining curvature on the blades justified?
- What should the ratio of the inlet area to the exit area be?
- Should the upstream side of the impeller blades be curved?

Answers to the questions could greatly improve the turbine performance.

Another area of the project that could be improved is the experiments. The equipment used during this study made it difficult to maintain a constant pressure. Therefore, a control system should be used with a solenoid valve to adjust the pressure to the desired condition. This would improve the accuracy of the results. The next step would be to automate the entire test. This would provide a more consistent test of the turbine.

The end goal of this project is to have a working microturbine that produces enough power for a MAV and weighs less than commercial batteries. By decreasing the weight of power systems, MAVs will be able to carry more instrumentation, fly longer, and be better able to complete their mission.

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Appendix A: Turbine Design Drawings

This appendix provides the design drawings for the turbine casing, impeller, and assembly. The design drawings for the casing are shown in Figures A.1-4. The impeller design drawing is shown in Figure A.5. Figure A.6 provides a design drawing of the motor faceplate. Finally, Figure A.6 provides an assembly drawing.

Figure A.1: Design Drawing of Casing Plate One

Figure A.2: Design Drawing of Casing Plate Two

Figure A. 3: Design Drawing of Casing Plate Three

Figure A.4: Design Drawing of Casing Plate Four

Figure A.5: Design Drawing of Impeller

Figure A.6: Design Drawing of Motor Face Plate

Figure A.7: Design Drawing of Turbine Assembly

Appendix B: Example Problem

This appendix provides an example of the calculations done to find the results presented in Chapter 3. The data was taken from test 2 of the Faulhaber motor. The turbine inlet pressure was found using the method outlined in section 3.5.1. The turbine power and efficiency calculations follow the technique discussed in section 3.5.2. The overall system power and efficiency were found using the process outlined in section 3.5.3.

Appendix B.1: Data:

$$P_{plen} = 51 \text{ psi, g}$$

$$P_{atm} = 99966 \text{ Pa}$$

$$T_{plen} = 22.7 \text{ C}$$

$$T_{up} = 22.1 \text{ C}$$

$$T_{down} = 18.6 \text{ C}$$

$$Q = 47$$

$$\text{Voltage} = 0.8 \text{ V}$$

$$R = 50 \text{ } \Omega$$

$$r = .25 \text{ in} = .00635 \text{ m}$$

$$\frac{e}{D} = 3.1496 * 10^{-4} \quad \text{from table}$$

$$z_2 = 0.6225 \text{ m}$$

$$z_1 = 0 \text{ m}$$

$$A_{up} = 6.42 * 10^{-6} \text{ m}^2$$

$$A_{down} = 4.09 * 10^{-5} \text{ m}^2$$

$$Cp = 1004.5 \frac{\text{J}}{\text{kg K}}$$

$$R = 286.9 \frac{\text{J}}{\text{kg K}}$$

Appendix B.2: Analysis:

Appendix B.2.1: Turbine Inlet Pressure:

$$P_{plen} = 51 \text{ psi, g} * 6894.75 \frac{\text{Pa, g}}{\text{psi, g}} = 351632 \text{ Pa, g} + 99966 \text{ Pa} = 451599 \text{ Pa}$$

$$T_{plen} = 22.7 \text{ C} + 273.15 = 295.9 \text{ K}$$

$$T_{up} = 22.1 \text{ C} + 273.15 = 295.3 \text{ K}$$

$$T_{down} = 18.6 \text{ C} + 273.15 = 291.8 \text{ K}$$

$$Q = 68.2 \text{ SLPM} - \text{from calibration chart}$$

$$Q = 68.2 \frac{\text{L}}{\text{min}} * \frac{1 \text{ m}^3}{1000 \text{ L}} * \frac{1 \text{ min}}{60 \text{ sec}} * \frac{99966 \text{ Pa}}{451599 \text{ Pa}} = 0.000255 \frac{\text{m}^3}{\text{s}}$$

$$\rho_1 = \frac{P_{plen}}{RT_{plen}} = \frac{451599 \text{ Pa}}{286.9 \frac{\text{J}}{\text{kg K}} * 295.9 \text{ K}} = 5.32 \frac{\text{kg}}{\text{m}^3}$$

$$\mu = \frac{bT^{\frac{1}{2}}}{1 + \frac{S}{T}} = \frac{1.458 * 10^{-6} \frac{\text{kg}}{\text{ms K}^{\frac{1}{2}}} * (295.9 \text{ K})^{\frac{1}{2}}}{1 + \frac{114.4 \text{ K}}{295.9 \text{ K}}} = 1.83 * 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$$U = \frac{Q}{A} = \frac{0.000255 \frac{m^3}{s}}{\pi * (0.00635m)^2} = 2.01 \frac{m}{s}$$

$$Re = \frac{\rho UD}{\mu} = \frac{5.32 \frac{kg}{m^3} * 2.01 \frac{m}{s} * 2 * 0.00635m}{1.83 * 10^{-5} \frac{Ns}{m^2}} = 7447$$

$$f = 0.25 \left[\log \left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}} \right) \right]^{-2} = 0.25 \left[\log \left(\frac{3.1496 * 10^{-4}}{3.7} + \frac{5.74}{7447^{0.9}} \right) \right]^{-2} = 0.0266$$

$$\begin{aligned} P_2 &= P_{plen} - \rho \left[\left((f * 253.5 + 1.75) \frac{U^2}{2} \right) - \left(\frac{\alpha_1 U_1^2 - \alpha_2 U_2^2}{2} \right) - (g(z_1 - z_2)) \right] \\ &= 451599 \text{ Pa} - 5.32 \frac{kg}{m^3} * \left[\left((0.0266 * 253.5 + 1.75) \frac{\left(2.01 \frac{m}{s} \right)^2}{2} \right) - \left(\frac{0 - 2 * \left(2.01 \frac{m}{s} \right)^2}{2} \right) \right] \\ &\quad - \left(9.81 \frac{m}{s^2} (0 - 0.6225m) \right) \\ &= 451522 \text{ Pa} \end{aligned}$$

Appendix B.2.2: Turbine Power

$$\begin{aligned} P_{turbine} &= \frac{P_1 Q}{RT_1} \left[Cp * (T_1 - T_2) + \frac{1}{2} \left(\left(\frac{Q}{A_1} \right)^2 - \left(\frac{P_1 T_2 Q}{P_2 T_1 A_2} \right)^2 \right) \right] \\ P_{turbine} &= \frac{451522 \text{ Pa} * 0.000255 \frac{m^3}{s}}{286.9 \frac{J}{kg \text{ K}} * 295.3 \text{ K}} * \left[1004.5 \frac{J}{kg \text{ K}} * (295.3 \text{ K} - 291.8 \text{ K}) \right. \\ &\quad \left. + \frac{1}{2} \left(\left(\frac{0.000255 \frac{m^3}{s}}{6.42 * 10^{-6} m^2} \right)^2 - \left(\frac{451522 \text{ Pa} * 291.8 \text{ K} * 0.000255 \frac{m^3}{s}}{99966 \text{ Pa} * 295.3 \text{ K} * 4.09 * 10^{-5} m^2} \right)^2 \right) \right] \end{aligned}$$

$$P_{turbine} = 5.33 \text{ W}$$

Appendix B.2.3: Turbine Efficiency

$$P_{turbine\ isentropic} = \frac{P_1 Q}{RT_1} * \left[Cp \left(T_1 - T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}} \right) + \frac{1}{2} \left(\left(\frac{Q}{A_1} \right)^2 - \left(\frac{P_1}{P_2} \frac{Q}{A_2} \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}} \right)^2 \right) \right]$$

$$P_{turbine\ isentropic} = \frac{451522\ Pa * 0.000255\ m^3/s}{286.9\ \frac{J}{kg\ K} * 295.3\ K}$$

$$* \left[1004.5\ \frac{J}{kg\ K} * \left(295.3\ K - 295.3\ K * \left(\frac{99966\ Pa}{451522\ Pa} \right)^{\frac{1.4}{1.4-1}} \right) \right]$$

$$+ \frac{1}{2} \left(\left(\frac{0.000255\ m^3/s}{6.42 * 10^{-6}\ m^2} \right)^2 - \left(\frac{451522\ Pa * 0.000255\ m^3/s}{99966\ Pa * 4.09 * 10^{-5}\ m^2} \right)^2 \right) - \left(\frac{99966\ Pa}{451522\ Pa} \right)^{\frac{1.4}{1.4-1}} \right]$$

$$P_{turbine\ isentropic} = 402\ W$$

$$\eta_{turbine} = \frac{P_{turbine}}{P_{turbine\ isentropic}} = \frac{5.33\ W}{402\ W} = 1.32\ \%$$

Appendix B.2.4: Overall System Power and Efficiency

$$P_{elec} = \frac{3 * Voltage^2}{R} = \frac{3 * 0.8^2}{50\ \Omega} = 0.0384\ W = 38.4\ mW$$

$$\eta_{mech} = \frac{P_{elec}}{P_{turbine}} = \frac{0.0384}{5.33} = 0.72\ \%$$

$$\eta_{overall} = \eta_{turbine} * \eta_{mech} = 1.32\ \% * 0.72\ \% = .00955\ \%$$

Appendix C: Uncertainty Analysis

This thesis was based largely on experimental data. Therefore, an uncertainty analysis was done in order to validate the data. This Appendix will provide an overview of the analysis as well as present the uncertainty of results.

An uncertainty analysis is done in order to estimate the error in the results. Error is defined as the difference between actual value and the measured or calculated value. There are two types of common errors, bias errors and precision errors. Bias errors, or fixed errors, are the same for each measurement. Bias errors are caused by calibration errors, defective equipment, consistent human errors, or any other error that is consistent over all the measurements. Precision errors, or random errors, are different each time a quantity is measured. Precision can be caused by disturbances in the measurement system such as vibrations, human errors, or changing experimental conditions. Bias errors can be kept small by using proper calibration and experimental techniques, but since precision errors are random, they must be dealt with in other ways. One way is to conduct a statistical analysis on the data. In order to complete a statistical analysis, every data point must be measured a number of times. This was not possible in this work; therefore, a single sample precision uncertainty was estimated.

The goal of this analysis was to find the uncertainty in the power and efficiency of the turbine. Since a number of quantities were measured to find the power and efficiency, the uncertainty will propagate through the equations to give a total uncertainty. This propagation on uncertainty is found using the Equation C.1:

$$u_y = \sqrt{\left(\frac{\partial y}{\partial x_1} u_1\right)^2 + \left(\frac{\partial y}{\partial x_2} u_2\right)^2 + \dots + \left(\frac{\partial y}{\partial x_n} u_n\right)^2} \quad \text{Eqn. C.1}$$

Where y is a calculated quantity with uncertainty u_y and x_n is an independent variable in y with uncertainty x_n . The uncertainties in Equation C.1 can be either bias or precision uncertainties. Therefore, the total error in quantity y is found using Equation C.2 where U_B is the bias uncertainty and U_P is the precision uncertainty.

$$U_x = (U_B^2 + U_P^2)^{1/2} \quad \text{Eqn. C.2}$$

Table C.1 presents the uncertainties in each measure value used in this work. The bias uncertainty of these measurements was found from product literature or taken as half the smallest scale. The final two columns show the bias and precision as a percentage, which was used to calculate the uncertainties in the power and efficiencies.

	Bias (+/-)		Estimated Precision	Highest Measured Value		Bias (%)	Precision
Voltage	0.0005	volts	0.003	3	volts	0.0167%	0.1%
Resistance	0.1	ohms	0.1	100	ohms	0.10%	0.1%
Z2	1.59E-03	m	6.23E-05	6.23E-01	m	0.3%	0.01%
Diameter	2.54E-05	m	0.0	1.27E-02	m	0.20%	0.00%
Plenum Pres.	2	Pa	0.2	200	Pa	1.0%	0.1%
Temp.	0.01	C	0.02	23.3	C	0.0429%	0.1%
Flow Rate	2.87E-05	m ³ /s	1.44E-05	1.44E-03	m ³ /s	2.0%	1.0%
Atm. Pres.	2.5	Pa	101	101626	Pa	0.0025%	0.1%

Table C.1: Uncertainties in Measured Values

Using the uncertainties listed in Table C.1, the uncertainties in each of the calculated values were found by using Equation C.1. In this work, Equation C.1 was applied to each part of the equations in Chapter 2 and then the uncertainty of that quantity was used at the next level. For example, instead of apply Equation C.1 to Equation 3.21 (which is the equation to calculate the turbine power); Equation 3.21 was broken down into the mass flow rate, the energy due to change in enthalpy and the change in kinetic energy. Then an uncertainty was calculated for each part. Table C.2 shows the bias and precision uncertainty in each of these values calculated.

Bias Uncertainty		Precision Uncertainty	
Density =	1.0%	Density =	0.1%
Viscosity =	0.04%	Viscosity =	0.1%
Velocity (V) =	2.0%	Velocity (V) =	1.0%
Renold's Number =	2.3%	Renold's Number =	1.0%
Friction Factor =	2.3%	Friction Factor =	1.0%
Upstream Pres. =	3.2%	Upstream Pres. =	1.4%
Downstream V =	3.8%	Downstream V =	1.8%
Isentropic Downstream V =	12%	Isentropic Downstream V =	5%
Upstream V =	2.0%	Upstream V =	1.0%
Isentropic Turbine Power =	24%	Isentropic Turbine Power =	11%
2 Phase Motor Power =	0.1%	2 Phase Motor Power =	0.1%
3 Phase Motor Power =	0.1%	3 Phase Motor Power =	0.2%
Turbine Power =	4.3%	Turbine Power =	2.0%
Turbine Efficiency =	25%	Turbine Efficiency =	11%
Mechanical Efficiency =	4%	Mechanical Efficiency =	2%
Overall Efficiency =	25%	Overall Efficiency =	11%

Table C.2: Bias and Precision Uncertainties in Calculated Values

Using the values in table C.2 and Equation C.2, the total uncertainty was found for the turbine power and efficiencies. These results are shown in Table C.3.

Total Uncertainty	
Turbine Power =	10%
Turbine Efficiency =	23%
Mechanical Efficiency =	12%
Overall Efficiency =	27%

Table C.3: Uncertainty in Turbine Power and Efficiencies

Appendix D: Complete Results

This appendix will provide the complete results of the experimental work. Section D.1 will cover the Faulhaber motor results, Section D.2 will provide the Maxon motor results, and Section D.3 will cover the Servomotor results.

Appendix D.1: Faulhaber Motor

Figure D.1: Calculated Turbine Power Vs Pressure Ratio - Faulhaber Motor Test # 1

Figure D.2: Calculated Turbine Power Vs Pressure Ratio Faulhaber Motor Test # 2

Figure D.3: Calculated Turbine Power Vs Pressure Ratio - Faulhaber Motor Test # 3

Figure D.4: Turbine Efficiency Vs Pressure Ratio @ Constant Voltage - Faulhaber Motor Test # 1

Figure D.5: Turbine Efficiency Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 2

Figure D.6: Turbine Efficiency Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 3

Figure D.7: Electrical Power Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 1

Figure D.8: Electrical Power Vs Pressure Ratio @ Constant Voltage - Faulhaber Motor Test # 2

Figure D.9: Electrical Power Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 3

Figure D.10: Mechanical Efficiency Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 1

Figure D.11: Mechanical Efficiency Vs Pressure Ratio @ Constant Voltage - Faulhaber Motor Test # 2

Figure D.12: Mechanical Efficiency Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 3

Figure D.13: Overall Efficiency Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 1

Figure D.14: Overall Efficiency Vs Pressure Ratio @ Constant Voltage - Faulhaber Motor Test # 2

Figure D.15: Overall Efficiency Vs Pressure Ratio @ Constant Voltage Faulhaber Motor Test # 3

Appendix D.2: Maxon Motor

Figure D.16: Calculated Turbine Power Vs Pressure Ratio @ Constant Voltage - Maxon Motor

Figure D.17: Turbine Efficiency Vs Pressure Ratio @ Constant Voltage - Maxon Motor

Figure D.18: Electrical Power Vs Pressure Ratio @ Constant Voltage - Maxon Motor

Figure D.19: Mechanical Efficiency Vs Pressure Ratio @ Constant Voltage - Maxon Motor

Figure D.20: Overall Efficiency Vs Pressure Ratio @ Constant Voltage - Maxon Motor

Appendix D.3: Servo Motor

Figure D.21: Calculated Turbine Power vs Pressure Ratio at Constant Load Test # 2

Figure D.22: Calculated Turbine Power vs Pressure Ratio at Constant Load Test # 3

Figure D.23: Turbine Efficiency vs Pressure Ratio at Constant Load – Test # 2

Figure D.24: Turbine Efficiency vs Pressure Ratio at Constant Load Test # 3

Figure D.25: Electrical Power vs Pressure at Constant Load Test # 1

Figure D.26:Electrical Power vs Pressure at Constant Load Test # 2

Figure D. 27: Electrical Power vs Pressure at Constant Load Test # 3

Figure D. 28: Mechanical Efficiency vs Pressure Ratio at Constant Load – Test # 2

Figure D.29: Mechanical Efficiency vs Pressure Ratio at Constant Load Test # 3

Figure D.30: Overall Efficiency vs Pressure Ratio at Constant Load – Test # 2

Figure D.31: Overall Efficiency vs Pressure Ratio at Constant Load – Test # 3

Appendix E: Fuel Tank Thermodynamic Analysis Code

This appendix provides the code for the programs written to conduct the thermodynamic analysis outlined in Chapter 4. Section E.1 provides the analytical program and Section E.2 provides the numerical program.

Appendix E.1: Analytical Program

%Run Time Analytical

%Finds the runtime of nitrogen tank based on an initial volume, exit port diameter,

%and specific volume

clc

clear

format short

%CONSTANTS

Mn=28.01348; %kg/kmol

Ru=8.31451; %Universal Gas Constant in KJ/(kmol*K)

R=0.2968; %Nitrogen gas constant in KJ/(kg*K)

%BOUNDARY CONDITIONS

%Cylinder Dimension

L_mm=150; %length in mm

```

D_mm=8; %diameter in mm

Vol=pi*(D_mm/2000)^2*(L_mm/1000);    %Volume in meters

thick=.01; %wall thickness in m

Area_surf=pi*(D_mm/1000)*(L_mm/1000); %Surface area in meters

k=490; %conductivity in W/mK


%Throat Diameter

Dt=25; %in microns

Area_t=pi*(Dt/2000000)^2;            %throat area in meters


%INTIAL CONDITIONS

T=293;                                %Temperature in K

Pi=68.948;                            %Intial Pressure in MPa (10,000 psi)

vf=0.002038344577;                   %Specific Volume in m^3/kg off chart

Min=Vol/vf;                           %Intial mass in kg

h=2.680158;                           %Enthalpy in J/kg off chart

u2=h-Pi*vf;                           %Intial internal energy


dt=.1;                                %delta t in seconds


Pres=Pi*10^6;

M2=Min;T2=T;t=dt;

Cond_const=k*dt*Area_surf/thick*0.001;

```

```

while Pres>.1918*10^5          %Pressure that will cause the throat to no longer be
                                %choked assuming the exit conditions are at STP

M1=M2;T1=T2;u1=u2;

roe=M1/Vol;                    %Calculates the Density in kg/m^3

Vel=V(R,T1);

mdot=roe*Vel*Area_t;  %Finds the mass flow rate in kg/s

M2=-mdot*dt+M1;           %Finds the new mass in kg

vf=Vol/M2;                %Finds the specific Vo

Pres=P(T1,vf,Ru,Mn);

h=u1+Pres*vf;

u2=(M1*u1-(M1-M2)*(h+Vel^2/2)+Cond_const*(T-T1))/(M2);

T2=tsolve(T,u2,R);

t=t+dt

end

t=t/60;

disp('The time is')

disp(t);

disp('The pressure is')

disp(Pres);

disp('The temperature is')

disp(T2);

```

function c=Cp(R,T)

%finds specific heat in KJ/(Kg*K)

%Constants

$h = 6.626176 \times 10^{-34}$; %Js

$k = 1.380662 \times 10^{-23}$; %J/K

$c = 2.997925 \times 10^8$; %m/s

$w1 = 2358.57$; %cm⁻¹

$wx = 14.324$; %cm⁻¹

$v = w1^2 \cdot wx$; %cm⁻¹

$o = (100 \cdot h \cdot c \cdot v) / (k \cdot T)$;

$c = R \cdot ((7/2) + (o^2 \cdot \exp(o) / (\exp(o) - 1)^2))$;

function pressure=P(T,vf,Ru,M)

%finds pressure based on the Benedict-Webb-Rubin Equation of State

$R = Ru \cdot 1000$; % convert to J/(kmol K)

$a = 0.0254$;

$a0 = 1.0673$;

$b = 0.002328$;

$b0 = 0.04074$;

$c = 7.37981 \times 10^2$;

$c0 = 8.164 * 10^3;$

$\alpha = 1.272 * 10^{-4};$

$\delta = 0.0053;$

$v0 = vf * M;$

pressure= $(R*T/v0)+(b0*R*T-a0-(c0/(T^2)))*(1/v0^2)+((b*R*T-a)/(v0^3))$
 $+a*\alpha/(v0^6)+((c/(v0^3*T^2))*(1+\delta/(v0^2))*\exp(-\delta/(v0^2)))$;

function temp=tsolve(Tin,u1,R)

%back solves for the temperature from u and cp

Tol=0.0001;

T1=Tin;

T2=T1-u(T1,Tin,u1,R)/du(T1,R);

while abs(T2-T1)>Tol

 T1=T2;

 T2=T1-((u1+(Cp(R,Tin)*1000*Tin))-(Cp(R,T1)*1000*T1))/(-
(dCp(T1,R)*1000*T1+(Cp(R,T1))*1000));

end

temp=T2;

function vval=V(R,T)

%finds velocity in m/s

```
vval=sqrt(((R/(Cp(R,T)- R))+ 1)*R*T*1000);
```

```
function diff=dCp(x, R)
```

```
%Finds the slope of the Cp function for a given temperature
```

```
h = 6.626176 * 10 ^ -34; %Js
```

```
k = 1.380662 * 10 ^ -23; %J/K
```

```
c = 2.997925 * 10 ^ 8; %m/s
```

```
w1 = 2358.57; %cm-1
```

```
wx = 14.324; %cm-1
```

```
v = w1 ^ 2 * wx; %cm-1
```

```
op = -h * c * v / (k * x ^ 2);
```

```
o1 = h * c * v / (k * x);
```

```
diff      =      R*      (o1*exp(o1)*op/(exp(o1)-1)^2+o1^2*op*exp(o1)/(exp(o1)-1)^2-  
2*o1^2*exp(o1)^2*op/(exp(o1)-1)^3);
```

Appendix E.2: Numerical Program

% Run Time Numerical

%Finds the runtime of nitrogen tank based on an initial volume, exit port diameter,

%and specific volume

clc

clear

format long g

load nitrogen_data.dat

load nitrogen_dataTvf.dat

%CONSTANTS

Mn=28.01348; %kg/kmol

Ru=8.31451; %Universal Gas Constant in KJ/(kmol*K)

R=0.2968*1000; %Nitrogen gas constant in KJ/(kg*K)

%BOUNDARY CONDITIONS

%Cylinder Dimension

L_mm=150; %length in mm

D_mm=7; %diameter in mm

Vol=pi*(D_mm/2000)^2*(L_mm/1000); %Volume in meters

thick=.01; %wall thickness in m

Area_surf=pi*(D_mm/1000)*(L_mm/1000); %Surface area in meters

k=490; %conductivity in W/mK

%Throat Diameter

Dt=25; %in microns

Area_t=pi*(Dt/2000000)^2; %throat area in meters

%INTIAL CONDITIONS

T=293; %Temperature in K

Pi=68.948; %Intial Pressure in MPa (10,000 psi)

vf=0.002038344577; %Specific Volume in m^3/kg off chart

Min=Vol/vf; %Intial mass in kg

u2=139634; %Intial internal energy off char in J/kg

dt=.1; %delta t in seconds

Pres=Pi;

M2=Min;

T2=T;

t=dt;

Cond_const=k*dt*Area_surf/thick*0.001;

while Pres>.1918 %Pressure that will cause the throat to no longer be choked

%assuming the exit conditions are at STP

```

M1=M2;T1=T2;u1=u2;

roe=M1/Vol;

Vel=Vt(R,T1,Pres,nitrogen_data);

mdot=roe*Vel*Area_t;

M2=-mdot*dt+M1;

vf=Vol/M2;

Pres=findP_Tvf(T1,vf,nitrogen_dataTvf);

h=u1+Pres*vf*10^6;

u2=(M1*u1-(M1-M2)*(h+Vel^2/2)+Cond_const*(T-T1))/(M2);

T2=findT_Pu(Pres,u2,nitrogen_data);

t=t+dt;

end

t=t/60;

disp('The time is')

disp(t);

disp('The pressure is')

disp(Pres);

disp('The temperature is')

disp(T2);

```

```

function vval=Vt(R,T,P,nitrogen_data)

```

```

%finds velocity in m/s

```

```
vval=sqrt(((R/(findCp(P,T,nitrogen_data)- R))+ 1)*R*T);
```

```
function Cpval=findCp(P,T,N)
```

```
% Returns Specific Heat value in J/kg*K
```

```
no_of_Plines=32;
```

```
findP=1;
```

```
needinterpP=1;
```

```
n=1;
```

```
while n<=(length(N)) & findP==1 & needinterpP==1
```

```
    if P==N(n,1)
```

```
        findP=0;
```

```
    elseif P<N(n+1,1)
```

```
        needinterpP=0;
```

```
        Plow=N(n,1);
```

```
        Pup=N(n+1,1);
```

```
    else
```

```
        n=n+1;
```

```
    end
```

```
end
```

```
findT=1;
```

```
needinterpT=1;
```

```

if findP==0

    while findT==1 & needinterpT==1

        if T==N(n,2)

            findT=0;

            Cpval=N(n,5);

        elseif T<N(n+1,2)

            needinterpT=0;

            Tlow=N(n,2);

            Cplow=N(n,5);

            Tup=N(n+1,2);

            Cpup=N(n+1,5);

            Cpval=(T-(Tlow-((Tlow-Tup)/(Cplow-Cpup))*Cplow))/((Tlow-Tup)/(Cplow-
Cpup));

        else

            n=n+1;

        end

    end

elseif needinterpP==0

    i=n-no_of_Plines+1;

    while findT==1 & needinterpT==1

        if T==N(i,2)

            findT=0;

            Cplow=N(i,5);

```

```

elseif T<N(i+1,2)

    needinterpT=0;

    PlowTlow=N(i,2);

    Cplow=N(i,5);

    PlowTup=N(i+1,2);

    Cpup=N(i+1,5);

    Cpvalow=(T-(PlowTlow-((PlowTlow-PlowTup)/(Cplow-
Cpup))*Cplow))/((PlowTlow-PlowTup)/(Cplow-Cpup));

    else

        i=i+1;

    end

end

n=n+1;

findT=1;

needinterpT=1;

while findT==1 & needinterpT==1

    if T==N(n,2)

        findT=0;

        Cpup=N(n,5);

        Cpval=(P-(Plow-((Plow-Pup)/(Cplow-Cpup))*Cplow))/((Plow-Pup)/(Cplow-Cpup));

    elseif T<N(n+1,2)

        needinterpT=0;

        PupTlow=N(n,2);

```



```

    Cplow=N(n,5);

    PupTup=N(n+1,2);

    Cpup=N(n+1,5);

    Cpvalup=(T-(PupTlow-((PupTlow-PupTup)/(Cplow-Cpup))*Cplow))/((PupTlow-
PupTup)/(Cplow-Cpup));

    Cpval=(P-(Plow-((Plow-Pup)/(Cpvallow-Cpvalup))*Cpvallow))/((Plow-
Pup)/(Cpvallow-Cpvalup));

    else

        n=n+1;

    end

end

else disp('Error')

end

```

```

function Pval=findP_Tvf(T,vf,u_nitrogen)

```

```

no_of_Tlines=19;

findT=1;

needinterpT=1;

n=1;

while n<=(length(u_nitrogen)) & findT==1 & needinterpT==1

    if T==u_nitrogen(n,2)

        findT=0;
    
```

```

elseif T<u_nitrogen(n+1,2)

    needinterpT=0;

    Tlow=u_nitrogen(n,2);

    Tup=u_nitrogen(n+1,2);

else

    n=n+1;

end

end

end

findvf=1;

needinterpvf=1;

if findT==0

    while findvf==1 & needinterpvf==1

        if vf==u_nitrogen(n,4)

            findvf=0;

            Pval=u_nitrogen(n,1);

        elseif vf<u_nitrogen(n+1,4)

            needinterpvf=0;

            vflow=u_nitrogen(n,4);

            Plow=u_nitrogen(n,1);

            vfup=u_nitrogen(n+1,4);

            Pup=u_nitrogen(n+1,1);

            Pval=(vf-(vflow-((vflow-vfup)/(Plow-Pup))*Plow))/((vflow-vfup)/(Plow-Pup));

```

```

else

    n=n+1;

end

end

elseif needinterpT==0

    i=n-no_of_Tlines+1;

    while findvf==1 & needinterpvf==1

        if vf==u_nitrogen(i,4)

            findvf=0;

            Plow=u_nitrogen(i,1);

        elseif vf<u_nitrogen(i+1,4)

            needinterpvf=0;

            Tlowvflow=u_nitrogen(i,4);

            Plow=u_nitrogen(i,1);

            Tlowvfup=u_nitrogen(i+1,4);

            Pup=u_nitrogen(i+1,1);

            Pvallo=(vf-(Tlowvflow-((Tlowvflow-Tlowvfup)/(Plow-
Pup))*Plow))/((Tlowvflow-Tlowvfup)/(Plow-Pup));

        else

            i=i+1;

        end

    end

end

n=n;

```

```

findvf=1;

needinterpvf=1;

while findvf==1 & needinterpvf==1

    if vf==u_nitrogen(n,4)

        findvf=0;

        Pup=u_nitrogen(n,1);

        Pval=(T-(Tlow-((Tlow-Tup)/(Plow-Pup))*Plow))/((Tlow-Tup)/(Plow-Pup));

    elseif vf<u_nitrogen(n+1,4)

        needinterpvf=0;

        Tupvflow=u_nitrogen(n,4);

        Plow=u_nitrogen(n,1);

        Tupvfup=u_nitrogen(n+1,4);

        Pup=u_nitrogen(n+1,1);

        Pvalup=(vf-(Tupvflow-((Tupvflow-Tupvfup)/(Plow-Pup))*Plow))/((Tupvflow-
Tupvfup)/(Plow-Pup));

        Pval=(T-(Tlow-((Tlow-Tup)/(Pvalup-Pvalup))*Pvalup))/((Tlow-Tup)/(Pvalup-
Pvalup));

    else

        n=n+1;

    end

end

else disp('Error')

end

```

```
function Tval=findT_Pu(P,u,N)
```

```
no_of_Plines=32;
```

```
findP=1;
```

```
needinterpP=1;
```

```
n=1;
```

```
while n<=(length(N)) & findP==1 & needinterpP==1
```

```
    if P==N(n,1)
```

```
        findP=0;
```

```
    elseif P<N(n+1,1)
```

```
        needinterpP=0;
```

```
        Plow=N(n,1);
```

```
        Pup=N(n+1,1);
```

```
    else
```

```
        n=n+1;
```

```
    end
```

```
end
```

```
findu=1;
```

```
needinterpu=1;
```

```
if findP==0
```

```
    while findu==1 & needinterpu==1
```

```

if u==N(n,3)

    findu=0;

    Tval=N(n,2);

elseif u<N(n+1,3)

    needinterpu=0;

    ulow=N(n,3);

    Tlow=N(n,2);

    uup=N(n+1,3);

    Tup=N(n+1,2);

    Tval=(u-(ulow-((ulow-uup)/(Tlow-Tup))*Tlow))/((ulow-uup)/(Tlow-Tup));

else

    n=n+1;

end

end

elseif needinterpP==0

    i=n-no_of_Plines+1;

    while findu==1 & needinterpu==1

        if u==N(i,3)

            findu=0;

            Tlow=N(i,2);

        elseif u<N(i+1,3)

            needinterpu=0;

            Plowulow=N(i,3);

```

```

    Tlow=N(i,2);

    Plowuup=N(i+1,3);

    Tup=N(i+1,2);

    Tval=(u-(Plowulow-((Plowulow-Plowuup)/(Tlow-Tup))*Tlow))/((Plowulow-
Plowuup)/(Tlow-Tup));

    else

        i=i+1;

    end

end

n=n+1;

findu=1;

needinterpu=1;

while findu==1 & needinterpu==1

    if u==N(n,3)

        findu=0;

        Tup=N(n,2);

        Tval=(P-(Plow-((Plow-Pup)/(Tlow-Tup))*Tlow))/((Plow-Pup)/(Tlow-Tup));

    elseif u<N(n+1,3)

        needinterpu=0;

        Pupulow=N(n,3);

        Tlow=N(n,2);

        Pupuup=N(n+1,3);

        Tup=N(n+1,2);

```

```

        Tvalup=(u-(Pupulow-((Pupulow-Pupuup)/(Tlow-Tup))*Tlow))/((Pupulow-
Pupuup)/(Tlow-Tup));

        Tval=(P-(Plow-((Plow-Pup)/(Tallow-Tvalup))*Tallow))/((Plow-Pup)/(Tallow-
Tvalup));

    else

        n=n+1;

    end

end

else disp('Error')

end

```

```

function uval=findu_Pvf(P,vf,u_nitrogen)

```

```

no_of_Plines=30;

findP=1;

needinterpP=1;

n=1;

while n<=(length(u_nitrogen)) & findP==1 & needinterpP==1

    if P==u_nitrogen(n,1)

        findP=0;

    elseif P<u_nitrogen(n+1,1)

        needinterpP=0;

    end
end

```



```

    Plow=u_nitrogen(n,1);

    Pup=u_nitrogen(n+1,1);

else
    n=n+1;

end

end

findvf=1;

needinterpvf=1;

if findP==0

    while findvf==1 & needinterpvf==1

        if vf==u_nitrogen(n,4)

            findvf=0;

            uval=u_nitrogen(n,3);

        elseif vf<u_nitrogen(n+1,4)

            needinterpvf=0;

            vflow=u_nitrogen(n,4);

            ulow=u_nitrogen(n,3);

            vfup=u_nitrogen(n+1,4);

            uup=u_nitrogen(n+1,3);

            uval=(vf-(vflow-((vflow-vfup)/(ulow-uup))*ulow))/((vflow-vfup)/(ulow-uup));

        else

            n=n+1;

        end

    end

end

```

```

        end

    end

elseif needinterpP==0

    i=n-no_of_Plines+1;

    while findvf==1 & needinterpvf==1

        if vf==u_nitrogen(i,4)

            findvf=0;

            ulow=u_nitrogen(i,3);

        elseif vf<u_nitrogen(i+1,4)

            needinterpvf=0;

            Plowvflow=u_nitrogen(i,4);

            ulow=u_nitrogen(i,3);

            Plowvfup=u_nitrogen(i+1,4);

            uup=u_nitrogen(i+1,3);

            uvalow=(vf-(Plowvflow-((Plowvflow-Plowvfup)/(ulow-uup))*ulow))/((Plowvflow-
Plowvfup)/(ulow-uup));

        else

            i=i+1;

        end

    end

end

n=n+1;

findvf=1;

needinterpvf=1;

```

```

while findvf==1 & needinterpvf==1

    if vf==u_nitrogen(n,4)

        findvf=0;

        uup=u_nitrogen(n,3);

        uval=(P-(Plow-((Plow-Pup)/(ulow-uup))*ulow))/((Plow-Pup)/(ulow-uup));

    elseif vf<u_nitrogen(n+1,4)

        needinterpvf=0;

        Pupvflow=u_nitrogen(n,4);

        ulow=u_nitrogen(n,3);

        Pupvfup=u_nitrogen(n+1,4);

        uup=u_nitrogen(n+1,3);

        uvalup=(vf-(Pupvflow-((Pupvflow-Pupvfup)/(ulow-uup))*ulow))/((Pupvflow-
Pupvfup)/(ulow-uup));

        uval=(P-(Plow-((Plow-Pup)/(uvalup-ulow))*uvalup))/((Plow-Pup)/(uvalup-
uvalup));

    else

        n=n+1;

    end

end

else disp('Error')

end

```