A Study of design parameters of a road racing endurance car

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A STUDY OF DESIGN PARAMETERS
OF A
ROAD RACING ENDURANCE CAR

by
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ABSTRACT

Analytical methods are presented to assess design parameters and aerodynamic effects on a road racing endurance car design. A six degree of freedom model of the vehicle dynamics was used to determine the design equations and objectives. Some of the more important parameters investigated were vehicle mass and inertia, aerodynamic drag and down force, rolling resistance, center of gravity, cornering capacity and braking performance. The effects of these parameters on the vehicle acceleration, velocity, and lap times were investigated analytically and verified by data obtained by on-board instrumentation. Given nearly identical engine performance from car to car, the aerodynamic parameters, drag and down force, were identified as having the greatest influence on racing endurance car performance. Modeling and vehicle test results are presented and correlated, which provided the basis of a relative closed-loop design process. Through this process, significant gains in performance were realized by identifying and optimizing the design parameters during the initial stages of the design.
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PREFACE

Author's Background

The author has been involved in auto racing for twenty two years, beginning in 1968 when he began Kart racing at the age of thirteen. His efforts and talents resulted in several major wins: Canadian-American Champion 1973, New York State and Northeast Division Champion 1973, Canadian National Champion 1973, 1974 (two class titles in 1974), and invitation by the Canadian sanctioning organization to represent Canada at the International Race of Champions held in Spain in 1973. The author is also a 1974 graduate of the world-renowned Jim Russell International Racing Drivers School and holds professional driving licenses. He has since competed on the majority of North American racing circuits in Formula Fords, Super Vees, and various Road Racing classes in the Sports Car Club of America, International Motor Sports Association, Canadian Automotive Sports Club, International Kart Federation, and World Karting Association. Due to his interest and mechanical background (BSME), the author has been extensively involved in race car design, development, and engine modifications.

Background

The vehicle designed and fabricated is intended for Formula 125 road racing governed by the Canadian Automobile Sports Club (CASC), World Karting Association (WKA), and International Karting Federation (IKF). Such road racing events are confined to a strict set of rules concerning vehicle design, safety standards, weight, and engines; of which several combinations make up the various racing class structures. It is pointed out that the class
structures of these different governing bodies have dissimilarities, and thus a compromise in the design must be accommodated to fulfill the different organizational rules and requirements. A brief scenario of actual race conditions is made based on current practice and experience. Within the North American organizations, the road race usually runs for one (1) hour with the winner and subsequent finishing positions determined by the greatest distance covered within the hour. A modified LeMans start is employed and starting position is determined by sign-in. Within each organization, the various racing classes are composed of combinations of vehicle weight and engines which differentiate each class. Typical average lap speeds range from 80 to 125 mph for the lower to higher powered classes, with top speed in the 100 to 150 mph range respectively. Combined vehicle and driver weights (dry) run from 300 to over 500 pounds, depending on class. Power output is from 25 to 90 hp, again depending on class. A weight to horsepower ratio of 12 lb / hp to 5 lb / hp is obtained respectively. In comparison, a modern Formula One Grand Prix Car enjoys a 2 lb / hp ratio. Due to the less favorable weight to horsepower ratio, Formula 125 cars spend a much higher percentage of lap time on straightaways where they try to achieve maximum velocity. Fortunately, due to low weight and size, Formula 125 cars achieve very high corner speeds. However, Grand Prix Cars obtain a much higher terminal speed, and have the ability to accelerate much more quickly, resulting in a comparatively higher average lap speed. Key to higher average lap speed is the ability to maintain speed through corners, achieve maximum acceleration and maximize terminal velocity. This paper explores the parameters governing vehicle performance and their influence on design.
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NOMENCLATURE

A  Cross-sectional area (ft²)

a₁, a₂  Coefficients related to rolling resistance

a  L₁ / L₈

Aᵧ, aᵧ  Lateral acceleration (ft / s²)

Aᵥ  Vehicle acceleration = dv / dt, velocity time rate of change (ft / s²)

ARC  Arc length for each corner (ft)

b  Lᵥ / L₈

Bᵥₘₐₓ  Maximum vehicle deceleration (ft/s²)

c  tanᵧ

C  Chord length (ft)

Cₐ  Center of down force Fₐ

Cᵥₐ  Drag area (ft²)

Cᵥ  Drag coefficient

Cᵥ₉  Center of gravity of vehicle

Cᵥ  Lift coefficient

C₁ - C₅  Constants for V₉₄₉₉

D₁ - D₆  Empirical derived coefficients for each corner

DIST  Total distance required for the vehicle to blend into a straight path (ft)

dₓ  Differential distance (ft)

E₁ - E₃  Constants for Braking motion

E  Kinetic energy of translational and rotating parts (ft lb₉)

Fᵥ  Sum of front and rear axle brake forces = Fᵥᵥ + Fᵥ₉ (lb₉)

Fᵥᵥ  Front axle brake force (lb₉)

Fᵥ₉₉₉₉  Maximum braking force (lb₉)
\( F_{BF_{\text{MAX}}} \) Maximum front braking force (lb\( f \))
\( F_{BR_{\text{MAX}}} \) Maximum rear braking force (lb\( f \))
\( F_{BR} \) Rear axle brake force (lb\( f \))
\( F_{R_{\text{Resistive}}} \) Total resistive force during braking (lb\( f \))
\( F_{\text{DRAG}} \) Drag force (lb\( f \))
\( F_{E} \) Propulsive force supplied by the engine (lb\( f \))
\( F_{EMAX} \) Maximum tractive effort (lb\( f \))
\( F_{L} \) Down force (negative lift) (lb\( f \))
\( F_{R_{\text{Resistive}}} \) Sum of resistive forces acting on an accelerating vehicle (lb\( f \))
\( F_{RT} \) Rolling resistance (lb\( f \))
\( F_{T} \) Sum of the total forces acting on an accelerating vehicle (lb\( f \))
\( F_{TC} \) Cornering tractive force (lb\( f \))
\( F_{TR} \) Transmission/engine braking effect (lb\( f \))
\( F_{ZF}, F_{ZR} \) Vehicle normal forces, front, rear (lb\( f \))
\( F_{XA}, F_{YA} \) Vehicle aerodynamic forces (lb\( f \))
\( F_{YF}, F_{YR} \) Vehicle side forces, front, rear (lb\( f \))
\( F_{XA} \) equal to: \( F_{\text{Drag}} = \frac{\rho C_{DA} V^2}{2} \) (lb\( f \))
\( F_{Y} \) Side force (lb\( f \))
\( g \) Gravity, 32.2 (ft./s\(^2\))
\( H \) Height (ft)
\( H_{D} \) Height of \( F_{\text{DRAG}} \) (ft)
\( H_{RW} \) Height of aerofoil from leading edge to ground (ft)
\( H_{V} \) Height of \( C_{GV} \) associated with lateral load transfer (ft)
\( H_{2} \) Equivalent \( C_{g} \) heights associated with longitudinal load transfer (ft)
\( HP \) Engine power (HP)
Subscript for inside, outside wheels

Rotating inertia being decelerated (lb\(\cdot\)ft \(s^2\))

Mass moment of inertia of the front wheel, each (lb\(\cdot\)ft \(s^2\))

Mass moment of inertia of the rear wheel, each (lb\(\cdot\)ft \(s^2\))

Mass moment of inertia of the front brake, each (lb\(\cdot\)ft \(s^2\))

Mass moment of inertia of the rear brake, each (lb\(\cdot\)ft \(s^2\))

Mass moment of inertia of the engine flywheel (lb\(\cdot\)ft \(s^2\))

Polar moment of inertia (lb\(\cdot\)ft \(s^2\))

Front braking proportion

Rear braking proportion

Distance between the front axle and \(C_G\), \(L_1 = L_B - L_V\) (ft)

Distance between the rear axle and aerodynamic center, \(L_2 = L_B - L_L\) (ft)

Length between rear axle and aerodynamic center (ft)

Wheel base (ft)

Distance rear axle to driver \(C_G\) (ft)

Distance rear axle to fuel \(C_G\) (ft)

Distance front axle to \(F_L\) (ft)

Distance rear axle to vehicle \(C_G\) (ft)

Applied brake torque (ft lb\(\cdot\))

Pitching moment (lb\(\cdot\)ft)

Equivalent mass of the vehicle (lb\(\cdot\)s\(^2\)/ft)

Rolling moment (lb\(\cdot\)ft)

Yawing moment (lb\(\cdot\)ft)

Aerodynamic moment (lb\(\cdot\)ft)

Tire inflation pressure (psia)
\[ R_c \quad \text{Radius of curvature (ft)} \\
R_{FW} \quad \text{Radius of the front wheel (ft)} \\
R_{RW} \quad \text{Radius of the rear wheel (ft)} \\
R_e \quad \text{Reynolds number} \quad (V \cdot \frac{L_B}{v}) \\
R_{CI} \quad \text{Vehicle radius at any given point (ft)} \\
R_{Cmin} \quad \text{Minimum vehicle radius (ft)} \\
R_N \quad \text{Nominal corner radius, at centerline apex (ft)} \\
R \quad \text{(N) Total power train gear ratio} \\
S_f \quad \text{Specific fuel consumption (gal./(HP)s)} \\
S \quad \text{Sum of the side reactions acting on the vehicle (lb)} \\
t \quad \text{Time (s)} \\
T \quad \text{Rear wheel track (ft)} \\
V \quad \text{Vehicle velocity (ft./s)} \\
V_f, V_R \quad \text{Front and rear tire velocity (ft./s)} \\
V_i, V_f \quad \text{Initial and final velocity (ft./s)} \\
W_D \quad \text{Weight, driver (lb)} \\
W_F \quad \text{Weight, fuel (lb)} \\
W_{RS} \quad \text{Static rear weight (lb)} \\
W_{RD} \quad \text{Dynamic rear weight (lb)} \\
W_{FS} \quad \text{Static front weight (lb)} \\
W_{FD} \quad \text{Dynamic front weight (lb)} \\
W_{FW} \quad \text{Total weight of the vehicle on the front wheels, } \quad \text{inclus engine weight) (lb)} \\
W_{FW} = W_{FS} - W_{FD} \quad \text{(lb)} \\
W_{RW} \quad \text{Total weight of the vehicle on the rear wheels, } \quad \text{incl engine weight) (lb)} \\
W_{RW} = W_{RS} + W_{RD} \quad \text{(lb)} \\
W_v \quad \text{Weight, vehicle = } \quad \text{(incl engine weight) (lb)} \\
W_v = W_{FW} + W_{RW} \quad \text{(lb)} \\
W_T \quad \text{Total weight \quad = \quad } \quad \text{incl engine weight) (lb)} \\
W_T = W_D + W_v + W_F \quad \text{(lb)} \]
\(x, y, z\)  Body fixed coordinate system
\(X, Y, Z\)  Inertial reference frame
\(x\)  Vehicle displacement (ft.)
\(x_i\)  Vehicle distance from the apex (ft)
\(\alpha\)  Angle of incidence of aerofoil, from horizontal, degrees
\(\alpha_{FM}, \alpha_{RM}\)  Front and rear tire idealized maximum slip angles, governed by tire cornering stiffness.
\(\alpha_f, \alpha_r\)  Tire slip angle, front and rear, radians
\(\beta\)  Aerodynamic side slip angle, radians
\(\delta_F\)  Angular deflection, radians
\(\rho\)  Density of air = 0.00238 (slug/ft.\(^3\))
\(\mu_A\)  Absolute viscosity of air (lb\(f\) sec./ft.\(^2\))
\(\mu_F\)  Coefficient of road adhesion
\(\mu_M\)  Maximum curvilinear coefficient of road adhesion, for banked corners
\(\mu_R\)  Overall lateral coefficient of road adhesion
\(\mu_{OR}\)  Tire friction coefficient as \(F_z \rightarrow 0\)
\(\mu_{1R}\)  \(\partial \mu / \partial F_z\). (lb\(^{-1}\))
\(\nu\)  Kinematic viscosity of air (ft.\(^2\)/sec.)
\(\eta\)  Drive train efficiency
\(\psi\)  Angle between intended travel and actual travel
\(\theta_S\)  Grade angle
\(\omega\)  Generalized angular velocity for rotating parts (1/\(s\))
\(\omega_{an}\)  Angular deceleration for rotating parts (1/\(s^2\))
\(\psi\)  Angle between \(x\) axis and inertial reference, radians
\(\nu\)  Angular curvilinear vehicle acceleration, radians / \(s^2\)
\(\gamma\)  Banking angle of corner, degrees
INTRODUCTION

Nothing has had as great an impact on race car design in the last decade as computer analysis. Although mathematical analyses appeared and were first published before the beginning of the century, it has only been recently that on-board computers, coupled with advanced modeling techniques, have provided the designer with actual performance data and predictive analysis. This ability to combine predictive analysis with measured performance of many variables gives the designer considerable insight and understanding of vehicle performance, handling, and stability. From the analysis presented here, aerodynamics can readily be shown to have a greater effect on vehicle performance than any other variable available to the designer.

Due to the author's interest and involvement with racing cars since 1968, a design project for a Formula 125 road racing car was undertaken and the results are presented here. The project, which involved the design and construction of the vehicle, relied on analysis to identify the design objectives and numerical modeling to generate input for the design. Particular emphasis was placed on aerodynamics and represents the main portion of the work presented.

In sections 2.1 through 2.4, analyses of vehicle performance are formulated and developed for the numerical models. In section 2.5, the design objectives for the vehicle are identified and section 2.6 discusses aerodynamic theory and the use of the Rochester Institute of Technology's subsonic wind tunnel. The results of extensive aerodynamic testing are also presented for several configurations. The numerical models developed to predict and optimize the vehicle performance are covered in detail in section 2.9. Various possible
configurations were evaluated with the models, the results of which were used as input for the design process.

The actual design is described and illustrated in section 3.1. The author's race car business prevents full disclosure of detailed design information. An on-board computer was used to record actual performance and is discussed in section 3.2.

A discussion of the various analyses and test results provide the reader with greater insight and understanding into the parameters influencing vehicle performance. The measured performance coupled with the predictive analysis result in a closed-loop design process.
2.1 PERFORMANCE CHARACTERISTICS OF RACE VEHICLES

A road racing vehicle's performance characteristics are primarily concerned with the car's capability to accelerate and decelerate, and to negotiate grades and corners. The tractive and braking effort, and the resistive forces determine the performance potential of the vehicle and will be discussed later in detail. The major external forces acting on the vehicle are shown in Figure 1

Figure 1, External Forces on a Two Axled Vehicle

From Newton's second law, the generalized equation of motion for the vehicle is expressed by:

\[ \sum \vec{F}_T = M_{eq} \frac{d\vec{v}}{dt} \quad (\text{lbf}) \]
where:
\[ \Sigma \vec{F}_T = \text{Sum of the forces acting on the vehicle (lb}_f) \]
\[ F_Y = \text{Side force (lb}_f) \]
\[ F_L = \text{Down force (negative lift) (lb}_f) \]
\[ M_{\text{eq}} = \text{Equivalent mass of the vehicle (lb}_f s^2 / \text{ft}) \]
\[ M_P = \text{Pitching moment (lb}_f \text{ ft}) \]
\[ M_R = \text{Rolling moment (lb}_f \text{ ft}) \]
\[ M_Y = \text{Yawing moment (lb}_f \text{ ft}) \]
\[ \frac{d^2 v}{dt^2} = \text{Velocity time rate of change (ft / s}^2) \]
\[ V = \text{Velocity (ft / s)} \]
\[ \theta_S = \text{Grade angle (not shown), degrees} \]
\[ \phi = \text{Rolling angle (not shown), degrees} \]
\[ \psi = \text{Heading angle between intended and actual travel} \]

If a single degree of freedom is used to describe the longitudinal motion, the equation of motion is rewritten as:

\[ M_{\text{eq}} \frac{d^2 x}{dt^2} = A_c M_{\text{eq}} = \Sigma F_T = F_E - F_{\text{Resistive}} \quad \text{(lb}_f) \quad (2) \]

where the linear acceleration of the vehicle \( A_c = \frac{d^2 x}{dt^2} \) (ft./s\(^2\)) and \( M_{\text{eq}} \) is the vehicle equivalent mass. The sum of the forces \( F_T \), acting on the vehicle is equal to the resistive forces \( F_{\text{Resistive}} \), and the force supplied by the engine \( F_E \), to propel the vehicle. Figure 2 on the following page illustrates the forces acting on the vehicle along the longitudinal axis.
Figure 2, Longitudinal Forces on a Two Axled Vehicle

where:

- $C_{GV}$ = Center of gravity of vehicle, includes fuel and driver.
- $C_A, C_{GD}$ = Center of aerodynamic down force $F_L$, location of driver
- $F_{ZF}, F_{ZR}$ = Vehicle normal forces, front and rear balancing static and dynamic weight of the vehicle on the front and rear wheels $W_{FW}, W_{RW}$ (lbf)
- $H_D, H_V$ = Height of $F_{DRAG}, C_{GV}$
- $I_{FW}, I_{RW}$ = Mass moment of inertia of the front and rear wheels (lb-ft s$^2$)
- $I_{FB}, I_{RB}$ = Mass moment of inertia of the front and rear brake (lb-ft s$^2$)
- $I_e$ = Mass moment of inertia of the engine flywheel (lb-ft s$^2$)
- $L_B, L_V, L_L, L_1, L_2$ = Wheel base, lengths to $C_G$’s shown (ft)
- $R_{FW}, R_{RW}$ = Radius of the front and rear wheels (ft)
- $W_D, W_F$ = Weight of driver, fuel (not shown) (lbf)
- $W_V$ = Weight of vehicle = $W_{FW} + W_{RW}$ (inclns engine weight) (lbf)
- $W_T$ = Total weight = $W_D + W_V + W_F$ (lbf)
The equivalent mass $M_{eq}$ as defined in Appendix J, includes the weight of driver, fuel, vehicle, and rotational inertia is expressed as:

$$M_{eq} = \frac{W_T}{g} + l_E \left( \frac{R(N)}{R_{RW}} \right)^2 + 2 \frac{l_{FW}}{R_{FW}^2} + 2 \frac{l_{RW}}{R_{RW}^2} + 2 \frac{l_{FB}}{R_{FW}^2} + \frac{l_{RB}}{R_{RW}^2}$$

$$= \frac{W_T}{g} + \left( \frac{R(N)}{R_{RW}} \right)^2 + 2 \frac{l_{FW}}{R_{FW}^2} + \frac{l_{RB}}{R_{RW}^2}$$

(lbf s²/ft) (3)

Gearing of the vehicle, $R(N)$ is defined and illustrated in Appendix E. The mass moments of inertia were evaluated experimentally by measuring the periods of oscillation on a knife edge as derived in Appendix H.

From equation (2), the sum of the forces acting on the vehicle $\Sigma F_T$, consist of the sum of the forces resisting the motion of the vehicle $F_{Resistive}$, and the propulsive force $F_E$, supplied by the engine:

$$\Sigma F_T = F_E - F_{Resistive} = F_E - F_{Drag} - F_{RT} - F_G$$

(lbf) (4)

where:

- $F_{Drag}$ = Drag force due to aerodynamic resistance (lbf)
- $F_{RT}$ = Rolling resistance of the front and rear tires, $F_{RF} + F_{RR}$ (lbf)
- $F_G$ = Grade resistance (lbf)

For grade resistance $F_G$, where $\theta_S$ is defined as the slope angle:

$$F_G = W_T \sin \theta_S$$

(lbf) (5)

For land vehicles, rolling resistance $F_{RT}$, is the force necessary to overcome the deflection of the tires at the tire patch. This force varies widely with various tire designs and inflation pressures, vehicle weight, and vehicle speed.
The rolling resistance for tire pressure $P_t$, is given experimentally by Hoerner as:

$$F_{RT} = [a_1 + a_2 V^2] W_T \quad (lb_f)$$  \hspace{1cm} (6)

The coefficients for the particular tires used were determined empirically and expressed as:

for velocities $< 150 \text{ ft/s}$

$$a_1 = .0085 + .255/P_t ; \quad a_2 = 2.771 \times 10^{-5}/P_t$$

for velocities $> 150 \text{ ft/s}$

$$a_1 = .225/P_t ; \quad a_2 = 5.1 \times 10^{-5}/P_t$$

Equation (6) provides the greatest amount of uncertainty, although it is widely used in automotive theory. It would be preferable to measure actual rolling resistance from coast down tests for the various tire-pressure combinations under investigation.

The aerodynamic resistance $F_{Drag}$, is given as:

$$F_{Drag} = \rho C_D A V^2/2 \quad (lb_f)$$  \hspace{1cm} (7)

where:

$\rho$ = Density of air = 0.00238 (slug/ft.$^3$)

$C_D$ = Drag coefficient

$A$ = Cross-sectional area (ft.$^2$)

$V$ = Vehicle velocity (ft./s)
For the sake of consistency the value of air density $\rho$, was held constant at 0.00238 slug/ft.$^3$ for comparison purposes. The values of the drag coefficient $C_D$, were determined experimentally in the RIT subsonic wind tunnel for various configurations as discussed in section 2.8. The cross sectional area was measured as the frontal area of the vehicle in ft.$^2$. The product of frontal area and drag coefficient $C_D A$, is referenced in this form throughout this paper.

The propulsive force (Refs. 2) $F_E$, driving the vehicle through an infinitely variable transmission and no wheel slip would yield optimum performance as given by:

$$F_E = \text{HP} \eta \frac{550}{V} \quad \text{(lb_f)} \quad (8)$$

where: $\eta$ = Drive train efficiency

$\text{HP}$ = Engine horse power (1 HP = 550 ft lb/s)

Noting the SAE definition for horse power is given as a function of engine RPM and torque $T$, as:

$$\text{HP} = 2 \pi \text{ RPM } T / 60 \frac{(550)}{\text{lb_f ft/s}} \quad (9)$$

The vehicle velocity is related to the mechanical geometry of the rear wheel radius $R_{RW}$, over all transmission and final drive ratios $R$ (N), and the engine RPM as:

$$V = \text{RPM} \frac{2 \pi R_{RW}}{R \ (N) \ 60} \quad \text{(ft/s)} \quad (10)$$
Substituting equations (9) and (10) into equation (8) yields the propulsive force $F_E$, driving the vehicle through the six speed transmission as a function of the engine torque $T$, over all transmission and final drive ratios $R(\text{ N })$, and rear wheel radius $R_{RW}$:

$$F_E = T \eta \frac{R(\text{ N })}{R_{RW}} \quad (\text{lb}f) \quad (11)$$

The engine's performance characteristics were measured on the author's Stuska water brake dynamometer and are shown in Figure 3 on the following page. From the measured data, the equation for engine horsepower $HP$, was derived from a cubic polynomial regression of the dyno data by the least squares method as:

$$HP = B_0 + B_1 (\text{RPM}) + B_2 (\text{RPM})^2 + B_3 (\text{RPM})^3 \quad (\text{ft lb}f) \quad (12)$$

where:

- $B_0 = 517.40$
- $B_1 = -155.94$
- $B_2 = 15.41$
- $B_3 = -0.48$

$B_0$ through $B_3$ represent the coefficients of the polynomial and are valid for the range of 9,500 to 13,000 RPM. The coefficient of determination $R^2 = 1.00$.

Substituting equations (5), (6),(7), and (8) into the equation of motion (2) yields:

$$A_c M_{eq} = HP \eta \frac{550}{V} - \rho C_D AV^2/2 - [a_1 + a_2 V^2] W_T - W_T \sin \theta_S \quad (\text{lb}f) \quad (13)$$
The solution of this second order nonlinear differential equation for distance, velocity, and acceleration, was performed on a digital computer and explained in detail in section 2.9. Several preliminary analyses can be carried out once the engine horse power, aerodynamic \( C_D A \), and rolling resistance \( F_{RT} \), characteristics are known.

The vehicle maximum velocity can be found from equation (2); for steady state (ie: no acceleration):

\[
A_c \ M_{eq} = F_E - F_{Drag} - F_{RT} - F_G = 0 \quad \text{(for S.S.)} \quad \text{(lb)}
\]

Substituting \( F_E \), equation (8), and rearranging, the total power to propel the vehicle at a given speed \( (V) \) is:

\[
HP = (F_{Drag} + F_{RT} + F_G) V / 550 \eta \quad \text{(HP)} \quad (14)
\]

If the vehicle is on level ground, \( \theta_s = 0 \). With the equations for drag \( F_{Drag} \) and rolling resistance \( F_{RT} \), substituted in the power equation, we have:

\[
HP = \{a_1 W_T + [a_2 W_T + \rho C_D A/2] V^2]\} V/550 \eta \quad \text{(HP)} \quad (15)
\]

For velocities greater than 30 mph, the aerodynamic term becomes increasingly larger relative to the rolling resistance term. The power required to overcome aerodynamic drag only, at maximum velocity, becomes:

\[
HP = \frac{\rho C_D A V^3}{1100 \eta} \quad \text{(HP)}
\]
In this form, the importance of aerodynamics is clearly stated as the dominating factor in race car design. The power required also increases with the cube of the velocity. The importance of usable engine horse power can not be over-stated. In classes where power between cars are more or less equal, aerodynamics represent one of the few areas where the designer can improve performance. For accuracy, the preceding power equation (15) is used. Equation (15) was used to calculate the power consumed overcoming aerodynamic and rolling resistance. Table 2 in section 2.8 outlines the parameters of the vehicles tested. Figure 12 shows the power consumed to overcome rolling resistance and aerodynamic drag for the final design configuration. Maximum theoretical speeds for the vehicle configurations tested in the RIT wind tunnel are shown in section 2.5. The solutions were obtained by performing a simple iteration (Program: Perform) and are presented as a function of the aerodynamic data (CD\). The calculations were performed for a transmission efficiency of η = 0.90, engine horse power HP = 38, and a tire pressure P_t = 36 psi.

Another aspect of performance, which is as important as top speed, is the acceleration capability of the vehicle. Rearranging equation (13) gives the acceleration of the car:

\[ A_c = \frac{500\eta \text{ HP}}{\text{Meq} V} - \frac{\rho \text{CDAV}^2}{2\text{Meq}} - (a_1 + a_2 V^2) \frac{W_I}{\text{Meq}} - \frac{W_I \text{Sin 0S}}{\text{Meq}} \text{ (ft/s}^2) \]  

To evaluate the vehicle performance potential, the maximum tractive effort that the vehicle can develop must be determined. There are two factors that limit the maximum tractive effort of the vehicle. The smaller of the two
factors determines (limits) the performance potential. The first limiting factor is the coefficient of road adhesion, \( \mu_F \). The other factor is determined by the characteristics of the engine and transmission previously described in equation (12). To calculate the maximum tractive effort for a rear wheel drive vehicle, the normal load on the rear axle must be determined by computing the summation of moments about point B in the free body diagram shown in Figure 2. Taking moments (Refs. 3) about point B, the normal load on the rear axle \( W_{RW} \) is:

\[
W_{RW} = \left( W_T L_1 \cos \theta_S + F_{\text{Drag}} H_D + H_V A_c M_{\text{eq}} + F_L L_L + W_T H_V \sin \theta_S \right) / L_B \quad (\text{Ibf}) \tag{17}
\]

where \( M_{\text{eq}} \) is the total equivalent vehicle mass at the center of gravity \( C_{GV} \). The aerodynamic down force or negative lift \( F_L \) acts at a distance \( L_L \). The height at which the drag force \( F_{\text{Drag}} \) acts is \( H_D \). The height of the vehicle center of gravity is defined as \( H_V \). Normally \( H_D \) and \( H_V \) are nearly equal. Note that for downhill travel, the \( \sin \theta_S \) is negative. For small angles \( \cos \theta_S = 1 \), and \( \sin \theta_S = 0 \). Letting \( H = H_D = H_V \) and simplifying equation (16):

\[
W_{RW} = \left( W_T L_1 + F_{\text{Drag}} H + H A_c M_{\text{eq}} + F_L L_L \right) / L_B \quad (\text{Ibf}) \tag{18}
\]

Substituting equation (2) in for \( A_c \), and eliminating terms yields:

\[
W_{RW} = \left( W_T L_1 + H \left( F_E - F_{RT} \right) + F_L L_L \right) / L_B \quad (\text{Ibf}) \tag{19}
\]
This equation can be expressed in the form of a static and dynamic term:

\[ W_{RW} = W_{RS} + W_{RD} \]  \hspace{1cm} (lb) \hspace{1cm} (20)

where:

\[ W_{RS} = W_T \frac{L_1}{L_B} \] = Static rear weight

\[ W_{RD} = \left( H \left( F_E - F_{RT}\right) + F_{LL} \right) / L_B \] = Dynamic rear weight

Note that the dynamic weight transfer term is a function of wheelbase \( L_B \) and vehicle center of gravity height \( H \), but almost independent of weight (\( F_{RT} \) is small). The term for aerodynamic down force \( F_{LL} \), plays significant importance as speeds increase, providing increased road holding ability and stability.

Similarly, for the front axle the simplified equation can be shown of the form:

\[ W_{FW} = W_{FS} - W_{FD} \]  \hspace{1cm} (lb) \hspace{1cm} (21)

where:

\[ W_{FS} = W_T \frac{L_V}{L_B} \] = Static front weight

\[ W_{FD} = \left( H \left( F_E - F_{RT}\right) - F_{LL} \right) / L_B \] = Dynamic front weight

Note that \( L_2 = L_B - L_L \)

The maximum tractive effort for rear wheel drive is defined as:

\[ F_{E_{MAX}} = \mu_F W_{RW} \]  \hspace{1cm} (lb) \hspace{1cm} (22)
This equation shows the maximum tractive force that can be transmitted by the driving wheels as a function of dynamic axle weight and the coefficient of road adhesion. The type of drive; front, rear, or four wheel exerts considerable influence on the magnitude of transferable tractive force. Dynamic weight shift increases rear axle weight making rear wheel drive more effective than front wheel drive. Four wheel drive theoretically utilizes full vehicle weight, but due to the added complexity and weight, was not considered. For longitudinal motion, neglecting corner banking, equation (22) is substituted into equation (19):

\[ W_{RW} = (W_T L_1 + H (\mu_F W_{RW} - F_{RT}) + F_{LL}) / L_B \] (lb) (23)

Rearranging leads to the form:

\[ W_{RW} = (W_T L_1 - H F_{RT} + F_{LL}) / (L_B - \mu_F H) \] (lb) (24)

Substituting back into equation (22), the maximum tractive force is then:

\[ F_{E,MAX} = \mu_F (W_T L_1 - H F_{RT} + F_{LL}) / (L_B - \mu_F H) \] (lb) (25)

This expression includes the dynamic effect of down force \( F_L \), so important to stability and increased road holding. By moving the location of the down force \( L_L \), towards the rear wheels, the rear wheel dynamic weight is increased. Removing the term \( F_LL \) reduces the equation to that typically derived in references 3 and 4.
2.2 BRAKING DYNAMICS

From Figure 2, the same forces act upon the vehicle during deceleration. The braking forces are applied at all four tire-road interfaces, acting as the primary retarding force. When the brake force is below the limit of road adhesion, the braking force is given as:

\[ F_B = \left( M_B - \sum I \omega_n \right) / R_{RW} = F_{BF} + F_{BR} \quad (\text{lb}_f) \quad (26) \]

where:

- \( M_B \) = Applied brake torque (ft lb_f)
- \( I \) = Rotating inertia being decelerated (lb_f ft s^2)
- \( \omega_n \) = Angular deceleration (1/s^2)
- \( R_{RW} \) = Rolling radius of the tire (ft)

In the calculations that follow, \( F_B \) is considered to be the sum of front and rear axle brake forces, or \( F_B = F_{BF} + F_{BR} \). During braking, the total resistive force \( F_{B_{\text{resistive}}} \), includes the forces normally opposed to forward motion (eqn 4) and is expressed as:

\[ F_{B_{\text{resistive}}} = F_B + F_{\text{Drag}} + F_{RT} + F_G + F_{TR} \quad (\text{lb}_f) \quad (27) \]

where \( F_{TR} \) represents the transmission/engine braking effect. Normally \( F_{TR} \) is small and can be neglected in braking performance calculations. During braking there is load transfer from the rear axle to the front axle. Governed by the same physical relationships that set vehicle tractive-force limits, the maximum braking force \( F_{B_{\text{max}}} \), is expressed as:

\[ F_{B_{\text{max}}} = \mu_F W_T \quad (\text{lb}_f) \quad (28) \]
For front and rear braking, using the previous equations (20, 21), and substituting the brake force for the propulsive force; for the front axle:

\[ W_{FW} = W_{FS} - W_{FD} \quad \text{(lb)} \]

where:

\[ W_{FS} = W_T \frac{L_v}{L_B} = \text{Static front weight} \]

\[ W_{FD} = \left( H \left( F_B - F_{RT} \right) - F_{L2} \right) / L_B = \text{Dynamic front weight} \]

Similarly, for the rear axle:

\[ W_{RW} = W_{RS} + W_{RD} \quad \text{(lb)} \]

where:

\[ W_{RS} = W_T \frac{L_1}{L_B} = \text{Static rear weight} \]

\[ W_{RD} = \left( H \left( F_B - F_{RT} \right) + F_{LL} \right) / L_B = \text{Dynamic rear weight} \]

Substituting equation (28), we have for the maximum braking forces on the front and rear axles:

\[ F_{BF_{\text{MAX}}} = \mu_F \left( W_T L_v - H \left( F_B - F_{RT} \right) - F_{L2} \right) / L_B \quad \text{(lb)} \quad (29) \]

and

\[ F_{BR_{\text{MAX}}} = \mu_F \left( W_T L_1 + H \left( F_B - F_{RT} \right) + F_{LL} \right) / L_B \quad \text{(lb)} \quad (30) \]

When the braking forces reach the values determined in equations (29) and (30), the tires are at the skid point. Beyond this point the tires lock up (no rotation), causing loss of control. Distribution of braking forces between front and rear axles is a function of the brake system design and is primarily
dependent on hydraulic pressures and brake cylinder areas. From equations (29) and (30), we see that only when the braking forces between front and rear axles are the same proportion as that of the dynamic loads, will the maximum braking forces at the front and rear tires be developed at the same time. Thus the proportions $K_{BF}$ and $K_{BR}$, of the total braking force on the front and rear axle can be determined for design purposes:

$$K_{BF} / K_{BR} = F_{BF_{max}} / F_{BR_{max}} = \left\{ \frac{W_T L_V - H (F_B - F_{RT}) - F_{LL2}}{W_T L_1 + H (F_B - F_{RT}) + F_{LL2}} \right\} /$$

$$\left\{ \frac{W_T L_1 + H (F_B - F_{RT}) + F_{LL} L}{W_T L_V - H (F_B - F_{RT}) - F_{LL2}} \right\}$$

(31)

If the braking force distribution is not ideal, either the front or rear tires will lock up first. When the rear wheels lock first, the vehicle will lose directional stability as the capability of the rear tires to resist lateral force is reduced to zero. The dynamics of the situation will cause the rear wheels of the vehicle to rotate 180° about the Z axis passing through the vehicle center of gravity. Lockup of the front tires will cause a loss of directional control, as the driver will no longer have effective steering. However front tire lockup does not cause directional instability, as the vehicle will tend to continue in a straight path. Loss of steering control can be regained by release or partial release of the brakes. Generally front wheel lockup is more easily detected by the driver than rear lockup. In rear wheel lockup, once the angular deviation of the vehicle exceeds a certain level, control cannot be regained, regardless of driver skill. This case presents a more critical situation, especially with low road surface adhesion.
The equations for decelerated motion are derived in a similar manner to those used for accelerated motion. The energy theorem states that the change in kinetic energy of a moving body equals the work produced by the external forces. Applying this theorem, where \( dx \) is the differential distance:

\[
dE/\ dt = ( F_{\text{Resistive}} ) \ dx/\ dt \quad (32)
\]

Kinetic energy of translatory and rotating parts (Appendix J) is given as:

\[
E = MV^2/2 + \sum I \omega^2/2
\]

Differentiating yields:

\[
dE/\ dt = MVdV/dt + \sum I \omega d\omega/\ dt
\]

By use of the techniques developed for longitudinal motion, the effect of the rotating parts is represented as:

\[
dE/\ dt = M_{\text{eq}} VdV/dt \quad (33)
\]

Substituting equation (32) into (33) gives us:

\[
M_{\text{eq}} VdV = ( F_{\text{Resistive}} ) \ dx = ( F_B + F_{\text{Resistive}} ) \ dx \quad (34)
\]

Note that \( V = dx/\ dt \). Using equation (1), the decelerated motion becomes:

\[
F_B = M_{\text{eq}} dV/dt - F_{\text{Resistive}} \quad (\text{lbf}) \quad (35)
\]

From this equation, the state of vehicle motion and forces involved in the braking process can be calculated.

The stopping distance can be found by equating equations (32) and (33).
The solution can be determined easily, when considering level grade. Substituting equation (27) yields:

\[ x = \frac{M_{eq}}{V/(F_{B} + F_{D} + F_{RT})} dV \text{ (ft)} \quad (37) \]

Substituting equations (6) and (7) in for \( F_{RT} \) and \( F_{Drag} \) results in:

\[ x = \frac{M_{eq}}{V/(F_{B} + P_{CDAV^2}/2 + [a^2 + a^2V^2]/WT)} dV \text{ (ft)} \quad (38) \]

Making the following substitutions:

\[ E_1 = \frac{P_{CD}A}{2}, \quad E_2 = \frac{3a}{WT}, \quad E_3 = \frac{a^2}{WT} \]

\[ x = \frac{M_{eq}}{2(E_3 + E_1)} \log_e \left( \frac{F_{B} + E_2 + (E_3 + E_1)V_f}{F_{B} + E_2 + (E_3 + E_1)V_f} \right) dV \text{ (ft)} \quad (39) \]

Letting \( Z = V^2/2 \) and \( dZ = VdV \), integration gives the solution:

\[ x = \frac{M_{eq}}{2(E_3 + E_1)} \log_e \left( \frac{F_{B} + E_2 + (E_3 + E_1)V_f}{F_{B} + E_2 + (E_3 + E_1)V_f} \right) dV \text{ (ft)} \quad (40) \]
The distance to brake to a full stop \((V_f = 0)\) from an initial speed \(V_i\) is calculated as:

\[
x = \frac{M_{eq}}{2} \left( E_3 + E_1 \right) \log_e \left[ 1 + \left( E_3 + E_1 \right) \frac{V_i^2}{(F_B + E_2)} \right] \quad \text{(ft)} \quad (41)
\]

Substituting the maximum braking force \(F_{B_{\text{MAX}}}\), for \(F_B\) allows the minimum stopping distance to be calculated. With the previous constants included gives:

\[
x = \frac{M_{eq}}{2} \left( \frac{a_2}{W_T} + \frac{\rho C_D A/2}{2} \right) \log_e \left[ 1 + \left( \frac{a_2}{W_T} + \frac{\rho C_D A/2}{2} \right) \frac{V_i^2}{(F_{B_{\text{MAX}}} + F_{B_{\text{R_{MAX}}}} + a_1/W_T)} \right] \quad \text{(ft)} \quad (42)
\]

where \(F_{B_{\text{MAX}}}\) and \(F_{B_{\text{R_{MAX}}}}\) are found from equations (29) and (30). The time to reduce the vehicle speed from \(V_i\) to \(V_f\) is calculated from equation (36) with the substitution of \(V = dx/dt\), yielding:

\[
t = \frac{M_{eq}}{(1/(F_B + E_2 + (E_3 + E_1)V_2))} \int_{v_f}^{v_i} dV \quad \text{(s)} \quad (43)
\]

After integration:

\[
t = \frac{M_{eq}}{((E_3 + E_1)(F_B + E_2))^5} \tan^{-1} \left[ \frac{(E_3 + E_1)/(F_B + E_2)}{(V_i - V_f)} \right] \quad \text{(s)} \quad (44)
\]

Substituting the constants and \(F_{B_{\text{MAX}}}\), for \(F_B\), for the minimum stopping distance gives:

\[
t = \frac{M_{eq}}{((a_2/W_T + \rho C_D A/2)((F_{B_{\text{MAX}}} + F_{B_{R_{MAX}}}) + a_1/W_T))^5} \tan^{-1} \left[ \frac{(a_2/W_T + \rho C_D A/2)/((F_{B_{\text{MAX}}} + F_{B_{R_{MAX}}}) + a_1/W_T)}{(V_i - V_f)} \right] \quad \text{(s)} \quad (45)
\]
2.3 CURVILINEAR MOTION

To predict vehicle curvilinear performance, two things must be known:

1) The actual vehicle performance in a corner, which is a function of vehicle weight, height of center of gravity, track width, roll resistance, coefficient of road adhesion, down force $F_L$, and so forth.

2) The curvilinear path the vehicle will follow.

Curvilinear analysis is perhaps the most difficult to model and predict with accuracy as vehicle directional stability and side slipping characteristics depend on a dynamic balance of curvilinear forces. For a vehicle negotiating a corner at steady state, the radial or centrifugal force $F_Y$ is held in equilibrium by the sum of the side force reactions on the tires:

$$\Sigma S = M_{eq} R_c \omega^2 = \frac{M_{eq} V^2}{R_c} = F_Y \text{ (lbf)}$$

where:

- $\Sigma S$ = Sum of the side reactions acting on the vehicle (lbf)
- $F_Y$ = Side force (lbf)
- $M_{eq}$ = Equivalent mass of the vehicle (lb ft s$^2$ / ft)
- $R_c$ = Radius of curvature (ft)
- $V$ = Velocity (ft/s)
- $\omega$ = Angular velocity (1/s)

To assist in the analysis of curvilinear motion, an inertial reference frame of the x - y plane of Figure 1 is shown in Figure 4. Figure 5 illustrates the force system in a steady state turn. From equation (46) we see that a 100% increase in velocity will quadruple the radial force $F_Y$. In contrast, a larger radius for a
Figure 4, Curvilinear Coordinate System on a Two Axled Vehicle

where:

\( C_{GV} \) = Center of gravity of vehicle
\( L_B, L_V, L_1 \) = Wheel base, lengths to \( C_G \)'s as shown (ft)
\( x, y, z \) = Body fixed coordinate system
\( X, Y, Z \) = Inertial reference frame
\( V_F, V_R \) = Front and rear tire velocity
\( \alpha_f, \alpha_r \) = Tire slip angle, front and rear, radians
\( \beta \) = Aerodynamic sideslip angle, radians
\( \psi \) = Angle between x axis and inertial reference, radians
\( \nu \) = Angular acceleration, radians / s\(^2\)
given velocity will decrease $F_Y$. Race drivers, in negotiating a corner, generally drive to produce the maximum possible radius which allows the greatest velocity through a corner. This is typically done by driving at the outside edge or radius of the corner upon entering (as shown in Figure 7), and progressively turning in to the corner apex on the inside corner radius. At this point, the driver expands the radius to the outside edge upon exiting the corner. Prior to entering the corner the vehicle must be braked to the corner entrance velocity.

As the vehicle enters a corner, the motion is translated from rectilinear to curvilinear, and the side forces impose an angular momentum rate change $I_p \cdot \nu$. For a vehicle with a polar moment of inertia $I_p$, wheelbase $L_B$, road adhesion $\mu_F$, and angular acceleration $\nu$; the relationship between the side forces and the rate of change of angular momentum is:

$$\sum S = \frac{I_p \cdot \nu}{L_B} \leq W_T \mu_F \text{ (lbf) } \quad (47)$$

From this equation it is apparent that a small polar moment of inertia will yield quick directional response. In comparison, vehicles with larger polar moments exhibit slower directional response and tend more so to maintain the original direction. Note should also be made that weight distribution, fore and aft, is a major determining factor of polar inertia. Heavy components should be located as close as possible to the vehicle center of gravity to minimize inertial moments.

Referring to Figure 5, for an idealized vehicle in a steady state turn of radius $R_C$ at velocity $V$, the external force system acting on the vehicle consists of tire normal and frictional forces and the aerodynamic force and moments
Figure 5, Force System in a Steady State Turn

where:

\( F_{TC} = \) Cornering tractive force (lbf)
\( F_{ZF}, F_{ZR} = \) Vehicle normal forces, front, rear (lbf)
\( F_{XA}, F_{YA} = \) Vehicle aerodynamic forces, (lbf)
\( F_{YF}, F_{YR} = \) Vehicle side forces, front, rear (lbf)
\( L_A = \) Length between rear axle and aerodynamic center
\( M_{YA}, M_{ZA} = \) Aerodynamic moment, (lbf ft)
\( \delta_F = \) Angular deflection, radians
shown. Assuming all angular deflections are small, the following dynamic equations are formulated (Refs. 5) from equations (2), (4), and (46):

\[
F_{ZF} = W_T b - M_{YA} / L_B + F_L L_A / L_B \quad \text{(lb)} \quad (48)
\]

\[
F_{ZR} = W_T a + M_{YA} / L_B + F_L (L_B - L_A) / L_B \quad \text{(lb)} \quad (49)
\]

\[
F_{VF} = M_{EQ} V^2 b / R_C - F_{ XF} \delta_F / L_B - M_{ZA} / L_B - F_{YA} L_A / L_B \quad \text{(lb)} \quad (50)
\]

\[
F_{VR} = M_{EQ} V^2 a / R_C + F_{ XF} \delta_F / L_B + M_{ZA} / L_B - F_{YA} (L_B - L_A) / L_B \quad \text{(lb)} \quad (51)
\]

\[
F_{TC} = -M_{EQ} V^2 \beta / R_C + F_{ XF} \delta_F + F_{XA} + F_{RT} + F_G \quad \text{(lb)} \quad (52)
\]

where:

\[
a = L_1 / L_B
\]

\[
b = L_V / L_B
\]

\[
F_{XA} = F_{Drag} = \rho C_D A V^2 / 2 \quad \text{(lb)}
\]

From the kinematics of Figure 4, we have:

\[
dv / dt = V / R_C \quad \text{ (1/s)}
\]

\[
\beta = a a_r + b (\delta_F + a_r), \quad \text{radians}
\]

\[
\delta_F = L_B / R_C - (a_t - a_r), \quad \text{radians}
\]

The frictional ellipse concept first proposed by Ellis (Refs. 6 and 7), gives the interaction of the tire cornering force and tractive force in the form:

\[
F_{VF} = \left( \frac{a_r}{a_{FM}} \right) \left( \mu_{OF} - \mu_{1F} F_{ZF} \right) F_{ZF} \quad \text{(lb)} \quad (53)
\]

\[
F_{VR} = \left( \frac{a_r}{a_{RM}} \right) \left[ (\mu_{GR} - \mu_{1R} F_{ZR})^2 F_{ZR}^2 - F_{TC}^2 \right]^5 \quad \text{(lb)} \quad (54)
\]
where

\[ \mu_{OR} = \text{Tire friction coefficient as } F_z \to 0 \]
\[ \mu_{1R} = \frac{\partial \mu}{\partial F_z}, \quad (\text{lb}^{-1}) \]
\[ \alpha_{FM}, \alpha_{RM} = \text{front and rear tire idealized maximum slip angles, governed by tire cornering stiffness.} \]

As a general rule, race car cornering capability is limited by the lateral force developed at the rear tires. At this condition, \( \alpha_R \sim \alpha_{RM} \). Thus the maximum lateral force generated by the rear tires (expressed for inside and outside tire and normalized by dividing by \( W_T \)), is:

\[
\frac{F_{YR}}{W_T} = \left[ (\mu_{OR} - \mu_{1R} F_{ZR})^2 \left( \frac{F_{ZR}}{W_T} \right)^2 - \left( \frac{F_{TC}}{2W_T} \right)^2 \right]^{\frac{1}{5}} + \\
\left[ (\mu_{OR} - \mu_{1R} F_{ZRo})^2 \left( \frac{F_{ZRo}}{W_T} \right)^2 - \left( \frac{F_{TC}}{2W_T} \right)^2 \right]^{\frac{1}{5}} \tag{55}
\]

where:

\[
\frac{F_{ZR}}{W_T} = \left[ a + \frac{M_{YA}}{W_T L_B} + \frac{F_L (L_B - L_A)}{W_T L_B} + H_v A_c / (L_B g) \right] / 2 - \frac{V^2 H_2}{(g R_c T)}
\]
\[
\frac{F_{ZRo}}{W_T} = \left[ a + \frac{M_{YA}}{W_T L_B} + \frac{F_L (L_B - L_A)}{W_T L_B} + H_v A_c / (L_B g) \right] / 2 + \frac{V^2 H_2}{(g R_c T)}
\]
\[ H_v, H_2 = \text{Equivalent Cg heights associated with longitudinal and lateral load transfer, (ft)} \]
\[ i, o = \text{Subscript for inside, outside wheels} \]
\[ T = \text{Rear wheel track, (ft)} \]
Note that lateral load transfer is primarily a function of $C_g$ height $H_v$, and rear tire track width $T$, but is adjustable to a certain extent through front and rear roll stiffness. To consider the aerodynamic down force benefits to high speed cornering, a simplified model where $\mu_{1R} = 0$ (constant coefficient) and $F_{TC} = F_{XA}$ (where: $\beta, \delta_F, F_G = 0$, $F_{RT}$ is small compared to $F_{XA}$). Thus:

$$\frac{F_{yr}}{W_t} = \frac{a V^2}{g R_c} = \left[ \mu_{0R}^2 \left( a + \rho V^2 A F_L (L_B - L_A) / 2 W_T L_B \right)^2 \right] \left[ - (\rho V^2 C_D A / 2 W_T)^2 \right]^{5}$$

(56)

Solving for $V$, the maximum cornering velocity is then:

$$V_{\text{MAX}} = \left\{ (g R_c / a) \left[ \mu_{0R}^2 \left( a + \rho V^2 A F_L (L_B - L_A) / 2 W_T L_B \right)^2 \right] \left[ - (\rho V^2 C_D A / 2 W_T)^2 \right]^{5} \right\}^{5} \quad \text{(ft/s)}$$

(57)

The exact solution to equation (57) can be found by reducing the equation to the quadratic form by making the following substitutions.

Let: $(L_B - L_A) / L_B = C_6$

Rearranging equation (56), we have:

$$V^2 \left( \frac{a}{g R_c \mu_{0R}} \right)^2 = a^2 + V^2 \left( a \rho A F_L C_6 / W_T \right) + V^4 \left( \rho A / 2 W_T \mu_{0R} \right)^2 \left( F_L^2 C_6^2 \mu_{0R}^2 - C_D^2 \right)$$

Let: $C_1 = \left( a / g R_c \mu_{0R} \right)^2$, $C_2 = a^2$, $C_3 = \left( a \rho A F_L C_6 / W_T \right)$, $C_4 = \left( \rho A / 2 W_T \mu_{0R} \right)^2 \left( F_L^2 C_6^2 \mu_{0R}^2 - C_D^2 \right)$, $Z = V^2$, $C_5 = C_3 - C_1$
The equation reduces to the form:

\[ C_4 Z^2 + C_5 Z + C_2 = 0 \]

The solution with the constants previously defined:

\[ V_{\text{MAX}} = \left\{ \left[ -C_5 + \left( C_5^2 - 4 C_4 C_2 \right) \right] / 2 C_4 \right\}^{\frac{1}{2}} \text{ (ft/s)} \quad (58) \]

From equation (58), the maximum theoretical cornering capability with aerodynamic down force can be found for known vehicle parameters and lateral load transfer.

As demonstrated earlier, the cornering capacity of the vehicle is primarily determined by the lateral coefficient of friction between the wheels and road surface. The side-thrust reaction of the tires must balance the resultant of all forces as seen in equation (46). From a directional stability standpoint, the distribution of side forces is extremely important. In practice, all four tires do not simultaneously operate at the limits of their performance. Limits of safe maneuverability are determined by the wheel that first loses traction.

For modeling purposes, the overall lateral coefficient \( \mu_R \), represents the sum of the four tire-cornering capabilities on a flat turn for the entire car. Banked corners increase cornering capability and associated speeds and can be represented in the form:

\[ \mu_M = \left( \mu_R + c \right) / \left( 1 - \mu_R c \right) \quad (59) \]

where:

\[ c = \tan \gamma \]

\[ \gamma = \text{Banking angle of corner, degrees} \]
Equating equations (46) and (47) and substituting equation (59) for $\mu_F$, we have:

$$V_{\text{MAX}} \leq \left[ g R_c \mu_M \right]^5 \quad (\text{ft} / \text{s}) \quad (60)$$

Cornering capability is often expressed in terms of lateral acceleration or $A_Y$.

$$A_Y = \frac{V^2}{R_c} \quad (\text{ft} / \text{s}^2) \quad (61)$$

The maximum theoretical lateral acceleration can be found by substituting equation (58) into equation (61), or the actual lateral acceleration found by measuring the vehicle steady state velocity on a skid pad and using equations (60) and (61).

The frictional ellipse concept by Ellis mathematically explains that the balance of acceleration, braking, and cornering forces is dependent on the tire's coefficient of friction. When the vehicle is in a corner, it is not always possible to use maximum acceleration or braking, as a portion of the tire generated side force is required to keep the vehicle in its curvilinear path. Regardless of how the forces are divided, the sum of the component forces cannot exceed the resultant force which is governed by the coefficient of friction. The amount of acceleration the driver can use in a corner is limited and is expressed by:

$$A_{Y,\text{MAX}} = \left[ \mu_M g^2 - a_Y^2 \right]^5 \quad (\text{ft} / \text{s}^2) \quad (62)$$

where:

$$a_Y = \frac{V^2}{R_{Cl}} \quad (\text{ft} / \text{s}^2)$$

$$R_{Cl} = \text{Vehicle radius at any given point (ft)}$$
Equation (62) gives the maximum vehicle acceleration $A_{Y_{\text{MAX}}}$ as a function of maximum cornering capability $\mu_{M}$, and actual cornering required $a_Y$. For modeling, the actual vehicle acceleration is determined by either equation (16) or (62), whichever is smaller. Equation (62) and (63) can be expressed in g’s by dividing through by g. Vehicle deceleration, in the same manner, is given as:

$$B_{Y_{\text{MAX}}} = F_{B_{\text{MAX}}} \left[ 1 - \left( \frac{a_Y}{\mu_{M} g} \right)^2 \right]^{5/\text{Meq}} \text{ (ft/s}^2\text{)} \quad (63)$$

Figure 6 illustrates the performance envelope for a typical road racing endurance car employing the Ellis friction ellipse concept. The figure was developed from the preceding equations, representing the basis for the curvilinear simulation.

**Figure 6, Road Racing Endurance Car Performance Envelope**

![Figure 6, Road Racing Endurance Car Performance Envelope](image-url)
CURVILINEAR PATH

Cornell Aeronautical Laboratory (Refs 8) proposed a method for modeling the path of a race car that showed good correlation to actual performance, and is presented and updated here. The theories presented also follow the friction ellipse theory previously described.

A race car follows a path through a corner such that an optimum balance between braking and cornering and acceleration and cornering is maintained. Each corner has an apex, that point roughly half way through a turn where the path radius is minimum. At the apex, the speed of the vehicle is determined by the maximum theoretical cornering force, road banking, and grade. It is also at this point that all tire forces, under maximum cornering conditions are used to supply cornering forces, leaving no excess for acceleration or braking. The path radius approaching or leaving the apex is always larger than the minimum radius value. As a result, the vehicle can brake into a turn, or accelerate out, as the generated cornering force is less than the maximum encountered at the apex. Figure 7 illustrates the following analysis of a curvilinear race car path through a corner. The rate at which the vehicle path radius increases (acceleration out of the corner), or decreases (braking into the corner) is described by a fifth order spiral function given as:

\[ R_{ci} = R_{cmin} + (10000 - R_{cmin}) \left( \frac{x_i}{DIST} \right)^5 \]  

where:

\( R_{ci} \) = Vehicle radius at any given point (ft)

\( R_{cmin} \) = Minimum vehicle radius (ft)

\( x_i \) = Vehicle distance from the apex (ft)

\( DIST \) = Total length of the vehicle curved path (ft)
Figure 7, Curvilinear Path Through a Corner

Note that 10,000 represents an infinite radius on a straight section. $R_{cmin}$ and $DIST$ are given as empirical functions:

$$R_{cmin} = D_1 + D_2 R_N + D_3 / (1 - \cos(ARC/R_N)) \text{ (ft)}$$  \hspace{1cm} (65)$$

$$DIST = D_4 + D_5 R_N + D_6 ARC \text{ (ft)}$$  \hspace{1cm} (66)$$

where:

$$\begin{align*}
D_1 - D_6 & = \text{Empirically derived coefficients for each corner} \\
R_N & = \text{Nominal corner radius, at centerline apex (ft)} \\
ARC & = \text{Arc length for each corner (ft)}
\end{align*}$$
Alternately, if the path of the vehicle is known explicitly, the radius can be described in this manner, which would yield more accurate results in the curvilinear model. For each corner, the track geometry must be known, including radii, banking and grade. The radius is set equal to 10,000 feet (to approximate infinity) at the beginning and end of each straightaway.

Modeling of the corners is started at the apex of the first or slowest turn, depending on the track. At this point, the vehicle speed is computed from the radius of curvature and $\mu_M$. Here the vehicle is at maximum cornering capability and there is no acceleration or braking. Each succeeding section of track is considered with time, computing the cornering force from the path curvature and banking. For each section, distance, velocity, and acceleration is computed with respect to time. Upon entering the next corner, the vehicle velocity will be higher than possible as determined by $\mu_M$ and $R_{Ci}$. From this point, the model can be used to compute the vehicle speed at the new apex, and then the same algorithm is used in reverse direction using the braking equations. This process of backing up continues until braking speed exceeds acceleration speed (coming into the corner), representing the braking point. The program now continues from the second apex as previously described and continues around the circuit in this manner.

In this manner, vehicle distance, velocity, and acceleration are calculated for the entire track with respect to time.
2.5 DESIGN OBJECTIVES

From the previous section where vehicle performance was presented, the design objectives can be stated relative to longitudinal and curvilinear motion. Ideally, a race car would accelerate at a maximum rate to the highest possible speed on all straights. Upon entering a corner the vehicle would decelerate at a maximum rate into a corner approaching the apex, and corner at the maximum velocity. Leaving the corner apex, the car would accelerate out at maximum rate. To formulate the objectives, it is necessary to examine the factors that influence the vehicle motion. Each motion is examined as follows:

1) Maximize vehicle longitudinal acceleration, equation (2, 16)

   A) Minimize vehicle equivalent mass, equation (3), $M_{eq}$ ↓
      - Minimize vehicle weight, $W_T$ ↓
      - Minimize vehicle rotational moments of inertia, $I$ ↓

   B) Maximize propulsive force, equation (8), $F_E$ ↑
      - Maximize engine horsepower, $HP$ ↑
      - Maximize drive train efficiency, $\eta$ ↑
      - Maximize road holding forces, equation (25), $F_{E\text{MAX}}$ ↑
        - Maximize coefficient of road adhesion, $\mu_F$ ↑
        - Maximize dynamic weight via down force, $F_L$ ↑

   C) Minimize external retarding forces, equation (4), $F_{\text{RESISTIVE}}$ ↓
      - Minimize rolling resistance, $F_{RT}$ ↓
      - Minimize aerodynamic resistance, $F_{\text{DRAG}}$ ↓
2) Maximize vehicle longitudinal deceleration, equation (42)

A) Minimize vehicle equivalent mass, equation (42), $M_{eq} \downarrow$
   - Minimize vehicle weight, $W_T \downarrow$
   - Minimize vehicle rotational moments of inertia, $I \downarrow$

B) Maximize braking forces, equation (28), $F_{B,\text{MAX}} \uparrow$
   - Maximize coefficient of road adhesion, $\mu_p \uparrow$
   - Maximize down force, $F_L \uparrow$

3) Maximize vehicle longitudinal velocity, equation (15)

A) Maximize propulsive force, equation (8), $F_E \uparrow$
   - Maximize engine horsepower, $HP \uparrow$
   - Maximize drive train efficiency, $\eta \uparrow$

B) Minimize external retarding forces, equation (4), $F_{\text{RESISTIVE}} \downarrow$
   - Minimize rolling resistance, $F_{RT} \downarrow$
   - Minimize aerodynamic resistance, $F_{\text{DRAG}} \downarrow$

4) Maximize vehicle curvilinear acceleration, equation (61)
   
   Same per longitudinal acceleration with emphasis on

A) Maximize road holding forces, equation (25), $F_{E,\text{MAX}} \uparrow$
   - Maximize cornering coefficient, equation (59), $\mu_M \uparrow$
   - Maximize down force, equation (57), $F_L \uparrow$
   - Minimize vehicle center of gravity height, $H \downarrow$
   - Minimize vehicle polar moment of inertia, equation (47), $I_p \downarrow$

5) Maximize vehicle curvilinear deceleration, equation (63)
   
   Same per longitudinal deceleration with emphasis on

A) Maximize cornering coefficient, equation (59), $\mu_M \uparrow$
- Maximize down force, equation (57), $F_L \uparrow$
- Minimize vehicle center of gravity height, $H \downarrow$
- Minimize vehicle polar moment of inertia, equation (47), $I_p \downarrow$

6) Maximize vehicle curvilinear velocity, equation (57)

Same per longitudinal velocity with emphasis on

A) Maximize cornering coefficient, equation (59), $\mu_M \uparrow$
- Maximize down force, equation (57), $F_L \uparrow$
- Minimize vehicle center of gravity height, $H \downarrow$
- Minimize vehicle polar moment of inertia, equation (47), $I_p \downarrow$

7) Maximize vehicle stability, balance, response, durability, driver control, comfort and safety.

A) Maximize vehicle acceleration, deceleration, maximum velocity

B) Maximize cornering coefficient, equation (59), $\mu_M \uparrow$
- Maximize down force, equation (57), $F_L \uparrow$
- Minimize vehicle center of gravity height, $H \downarrow$
- Minimize vehicle polar moment of inertia, equation (47), $I_p \downarrow$

C) Minimize vehicle equivalent mass, $M_{eq} \downarrow$

To achieve maximum acceleration, the vehicle weight must be at the minimum allowed by the race organizations. Maximum acceleration occurs at low velocity and is achieved by maximizing the dynamic weight via down force $F_L$, but limited by the coefficient of road adhesion, $\mu_F$. Typically vehicle weight is checked at the end of the race and includes driver weight.

By minimizing rolling and aerodynamic resistance, the vehicle will accelerate more rapidly and achieve a higher top speed with a given engine.
Slipstream disadvantage will also be minimized. Maximizing engine horsepower and drive train / transmission efficiency also allows quicker acceleration and greater top speed.

Maximum vehicle deceleration is governed by the same forces that affect acceleration and occurs when maximum braking force $F_{B\, \text{MAX}}$, is achieved. Because braking is critical at high speeds, down force $F_L$ plays a larger roll in determining $F_{B\, \text{MAX}}$.

Maximum velocity is largely dependent on $F_{\text{DRAG}}$ and available horsepower, as the power required increases with the cube of the velocity.

Maximum lateral acceleration is governed by the same forces that affect longitudinal acceleration but includes vehicle polar moment of inertia $I_p$, and limited by the maximum theoretical cornering coefficient of adhesion, $\mu_M$.

Cornering velocity is governed largely by down force $F_L$, vehicle center of gravity height $H$, cornering coefficient $\mu_M$, vehicle weight and wheel base $L_B$. Maximizing down force explains the exceptionally higher cornering speeds generated by modern race cars.

Maximizing vehicle stability and road-holding allows maximum potential of the machine and driver to be realized. Maximum control and comfort prevents driver fatigue where forces approaching 2g’s are encountered during the hour-long event. Primary and secondary safety are paramount to prevention of driver injury.

To capitalize on these design objectives and to design and fabricate an exceptional vehicle, three numerical models were developed to investigate and optimize various design configurations. The results from the models are presented in section 2.8 for aerodynamic analysis and section 3.2 for vehicle
performance. The models allow evaluation of each possible design and the effect of the various design parameters discussed in section 2. The design objectives are stated as:

**TABLE 1 VEHICLE DESIGN OBJECTIVES**

1) Maximize vehicle acceleration, deceleration, velocity, stability, balance, response, durability, driver control, comfort and safety.

   A) Minimize vehicle equivalent mass, equation (3), \( M_{eq} \downarrow 
   - Minimize vehicle weight, \( W_T \downarrow 
   - Minimize vehicle rotational moments of inertia, \( I \downarrow 

   B) Maximize propulsive force, equation (8), \( F_E \uparrow 
   - Maximize engine horsepower, \( HP \uparrow 
   - Maximize drive train efficiency, \( \eta \uparrow 

   C) Minimize external retarding forces, equation (4), \( F_{RESISTIVE} \downarrow 
   - Minimize rolling resistance, \( F_{RT} \downarrow 
   - Minimize aerodynamic resistance, \( F_{DRAG} \downarrow 

   D) Maximize braking forces, equation (28), \( F_{BMAX} \uparrow 

   E) Maximize cornering coefficient, equation (59), \( \mu_M \uparrow 
   - Maximize down force, equation (57), \( F_L \uparrow 
   - Minimize vehicle center of gravity height, \( H \downarrow 
   - Minimize vehicle polar moment of inertia, equation (47), \( I_p \downarrow 

   F) Maximize road holding forces, equation (25), \( F_{E MAX} \uparrow 
   - Maximize coefficient of road adhesion, \( \mu_F \uparrow 
   - Maximize dynamic weight via down force, \( F_L \uparrow 

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Figure 8 shows the effect of aerodynamic drag and rolling resistance on maximum vehicle velocity. Several computer runs evaluated different vehicle drag - area configurations, with successive improvements in aerodynamic drag. Rolling and air resistance are present under all conditions of motion. Tractive force is primarily used to overcome these two resistive forces. Only when excess tractive force is available will the vehicle accelerate or climb grades. All resistance forces except aerodynamic drag are proportional to vehicle weight, where as rolling and aerodynamic drag resistance are functions of vehicle velocity. Comparison of rolling and aerodynamic drag resistance as a function of speed shows that for modern race cars, aerodynamic drag becomes appreciable at speeds as low as 20 mph. Typically aerodynamic and rolling resistance are equal at speeds of approximately 35 mph.

Increasing available horsepower would yield increased maximum velocity for all configurations, but is limited to the horse power produced from engines currently available. Decreasing rolling resistance $F_{RT}$ through higher tire pressures and decreased vehicle weight has limits imposed by the operating constraints on the tires and minimum weight as regulated by the sanctioning organizations. Aerodynamic drag reduction and increased downforce $F_L$ represent the largest design variables available to improve vehicle performance. Examination of the equations show that increased downforce benefits all curvilinear and braking performance and adds vehicle stability at high velocities at the expense of a small drag increase. Drag reduction yields increased vehicle performance in all motions, with the exception of braking. Race car designers twenty years ago made use of
movable aerodynamic devices that increased drag and down force under braking only. Several structural failures from the sudden tremendous load increase lead race organizers to ban movable aerodynamic devices for safety considerations. Thus, a large part of effective aerodynamic design is involved in obtaining a balance between low drag and drag producing down force. This balance is found with hours of wind tunnel testing and computer simulation as described in the following sections.
**FIGURE 8**

**MAXIMUM VEHICLE VELOCITY vs. DRAG AREA (C_D A)**

**CONSTANTS**
- HP = 38
- \( \eta \) = 0.90
- \( \rho \) = 0.00238
- \( P_T \) = 36 psia
- \( W_T \) = 400 lbs

*Limited by minimum weight (400 lb), by regulations, & maximum tire*

Maximum Theoretical Speed Potential at 152 MPH (38 HP)

DATA SOURCE: Fortran program: Perform For
2.6 AERODYNAMIC THEORY

There are two basic ways to consider aerodynamics and its respective applications. One is aerodynamic applications useful to enhancing handling characteristics such as high speed lateral and straight-line stability, increased tire contact patch pressures through negative lift devices, and center of pressure location. Devices such as wings, spoilers, and ground effects devices are used for these applications. Secondly, aerodynamics can be applied in an effort to refine the air penetration quality of a racing vehicle. In this case streamlining and frontal area reduction are employed, thereby reducing drag and allowing increased vehicle acceleration and terminal speeds.

In the case of a racing vehicle, the vehicle moves through the air, thereby producing air velocity (relative wind). Therefore, when considering the boundary-layer, the vehicle becomes the stationary medium. The boundary layer, caused by air viscosity, creates velocity gradients.

Aerodynamic drag at high speed is the major resistance to the motion of a vehicle and is largely responsible for performance limitations in racing vehicles. Generally, aerodynamic drag on a vehicle can be defined as the sum of three components of the total drag force \( F_{\text{Drag}} \), acting on a road vehicle. They are form drag, friction drag, and induced drag. Form drag results when air flow does not close in completely around the rear of the car, but separates to form a wake due to the shape of the body. This wake, referred to as a "slipstream" by race drivers, intensifies as velocity increases. Race drivers take advantage of form drag to draft their competitors. The wake produces less aerodynamic resistance behind the lead race vehicle, allowing a following car to occasionally catch up and sometimes pass the lead car, if timed correctly.
such as at the end of a straight-away. An interesting note is that this reduced aerodynamic resistance can benefit both the lead car, and a closely following car, if the follower remains behind in the slipstream. The two race vehicles drafting each other generally lap faster than each independently. This is due to a more favorable air flow wake dissipation characterized by prolonging the wake turbulence.

Friction drag is generated by air sticking to the surface of a body and forming a boundary layer due to air viscosity. Friction drag can be reduced by making the body very smooth. Induced drag is primarily a function of wings with their associated angles of attack as used on racing vehicles.

In a racing vehicle, the design emphasis is generally placed on producing as little form drag as possible. This is especially true for lower powered race cars, and for race cars which spend a great amount of track time at high speeds. Streamlining, in the true sense of the word, means a body is carefully shaped to produce as small a wake as possible.

An important consideration with drag is its contribution to vehicle stability. Even as drag opposes forward speed, it also acts as a stabilizing influence, similar to an anchor at the rear of the vehicle. This tends to hold straight-line running characteristics beyond wind gusts or quick directional changes.
Due to the physical size of the RIT subsonic wind tunnel, models are used to simulate actual full-scale conditions. It is important to formulate equations to make the models dynamically similar to ensure accuracy of the test results. Several parameters determine the actual model size. The scale of the model to be tested is the first decision made in any wind tunnel test. Today, it is generally accepted that the model size should be as large as possible to reduce what is known as the "scale effect". The scale effect results when the air flow patterns over the actual model are different than that of the full-size vehicle, strictly due to differences in size. The test results are then not totally reliable, with error ranging as high as 20 percent. Actual size of the model is limited to "blockage ratio", which is the ratio of the frontal cross-section of the model divided by the cross-section area of the wind tunnel (perpendicular to the flow). According to F.N. Beauvais of the Ford Motor Company, the ideal blockage ratio is between 5 and 7 percent, not to exceed 25 percent. For practical purposes, however, the model scale is usually chosen at a convenient workable scale, usually 1/10, 1/4, or 3/8 of the actual size, following the guidelines already mentioned.

For the purpose of measurements during the testing, the physical quantities acting on the model must be the same as those on the actual vehicle. Dynamic similitude is used for this purpose, and it is important to identify these physical quantities.

To make the problem analytically solvable, the important quantities are separated from those which have a negligible influence on the model testing.
For the purposes of testing in the RIT subsonic wind tunnel, two dimensionless groups are used at low velocities (under 300 mph); the Reynolds number and Newton's force coefficients. These are given as:

\[ \text{Reynolds number} \quad R_e = \frac{\rho V L_B}{\mu A} = \frac{V L_B}{v} \]

\[ \text{Lift coefficient} \quad C_L = \frac{F_L}{\rho V^2 A} \]

\[ \text{Drag coefficient} \quad C_D = \frac{F_{\text{drag}}}{\rho V^2 A} \]

The Reynolds number, however, becomes independent of the drag coefficient for all objects with sharp leading edges\(^1\). In the case of a partially streamlined object such as a racing vehicle, this independence does not occur until the Reynolds number is above \(2 \times 10^5\) when based on the wheelbase of the vehicle for the characteristic length\(^7\). Above this number, the drag and lift coefficients will remain constant within the limitations of attainable tunnel velocities. The drag coefficients are then applicable to the full-size vehicle allowing the drag to be calculated, though some error is to be expected when relating scales.

All calculations in this paper were based on a Formula 125 road racing car. This car was selected because its geometric properties are known, the author's familiarity with this class of racing vehicles, and its weight to horsepower ratio, which allows substantial performance gains to be realized in terms of
aerodynamic improvements. The vehicle specifications and RIT's subsonic wind tunnel dimensions are listed in the appendix. Of prime importance in wind tunnel testing of a racing vehicle is the evaluation of the drag and lift coefficients (Refs. 10). These two characteristics can be directly related to performance considerations. For use of the wind tunnel, the velocity equation is needed, which is developed in the appendix.

The drag measured in the wind tunnel is the total aerodynamic drag $F_{\text{DRAG}}$, formed by the sum of the form drag, friction drag, and induced drag. However, rolling resistance must be accounted for, as this is not simulated in the tunnel. The analysis developed in section 2.1 was used in all calculations. The results of the wind tunnel testing are presented in the following section 2.8.
2.8 AERODYNAMIC TEST RESULTS

Several models for use in the wind tunnel were built on a 1/15 scale which allowed use of commercially available parts, such as wheels and tires found in model shops.

The operation of the RIT subsonic wind tunnel is fairly straightforward. Data taken during the tests were lift, drag, manometer readings, air temperature, and barometer reading. In all tests, the model was kept parallel to the tunnel floor where the counter on the balance reads 836 at $a = 0$. From this data, as outlined in the appendix, the free stream velocity is calculated. The air velocity chosen for experimentation was 100 mph. This corresponds to a Reynolds number of $1.9 \times 10^6$, well above the critical number.

During the actual runs, changes were made to the model configurations to reduce the drag and thus lower the drag coefficient. The various models are shown in Figures 9 through 11. Due to the limitations of the RIT subsonic wind tunnel, no ground plane could be simulated during the tests. The order of testing was:

1. Model # 1 tested with front fairing body work, driver in upright driving position (not shown)

2. Model # 2 with full under tray, side fairings and rear wing. (Figure 9).

3. Endurance Model # 3 with full body work, driver in prone driving position (Figure 10)

4. Model # 4 Full body work enclosing simulated monocoque. (Figure 11)
In this order drag was reduced, however the lift force did change as shown in the results. The first or base case (Model # 1) represents the way this class of racing vehicles competed several years ago, without full advantage of streamlining.

**FIGURE 9. Model # 2 with full under tray, side fairings and rear wing**

Tubular space frame chassis are used in this configuration and the vehicle undersides are rather "dirty", aerodynamically speaking. A front fairing allowed cleaner penetration into the air by shrouding the front tires to reduce their high drag and positive lift, known as the "Magnus effect".

A full-length under tray, side fairings and rear wing was added to the Model # 1 in the second test, to produce Model # 2 as shown in Figure 9. This configuration would reduce the interactions between the bottom of the car
and the track. Air tends to stagnate in this area resulting in a high drag area. This has been the major area of race car development over the last 10 years.

**FIGURE 10. Model # 3 with full body work, driver in prone position**

The Endurance Model # 3 was tested with front fairing body work, side moldings and rear wing, with the driver in prone driving position (Figure 10). This configuration represents the majority of cars raced today with lower $C_D$ and reduced cross sectional area. Despite the appearance, most of these aerodynamic body panels are afterthoughts, adding additional weight. Opportunity exists to further refine the body shape to reduce drag.

In test four (Model # 4, Figure 11), full body work was added to enclose the vehicle, simulating the actual monocoque of the final design. All air is now gradually directed over the vehicle. No stagnant air pockets remain with drag.
expectedly lower. The actual body configuration follows most present day practice on larger race cars, with the layout being rather practical for ease of accessibility rather than the ultimate streamlined shape. Further work and refinements can be done in this area for future improvements.

**FIGURE 11. Actual vehicle with full body work enclosing monocoque.**

Further work in this area of refining the body drag qualities are planned for the future. Small vortex generators can be added to the front of the body work to induce turbulent boundary layers. This is expected to lengthen the distance of the point of separation of the flow, thus reducing the drag further.

In Table 2, the wind tunnel data for the scaled models of each test run is given. Table 3 tabulates the calculated data for the full size vehicle configuration and illustrates the differences in performance. In Figure 12, the
power required to propel the test vehicle is given as a function of the drag coefficient and the velocity of the vehicle. The terminal velocity can be seen for each vehicle configuration where the available power line intersects with each drag coefficient line for each different test. In Figure 13, the maximum speed is plotted versus drag area of the vehicle. Thus, one can predict potential speed increases with a further reduction in drag coefficient or frontal area.

<table>
<thead>
<tr>
<th>Test Run</th>
<th>T₀ °F</th>
<th>Z₀ IN</th>
<th>Z₁ IN</th>
<th>P₀ PSIA</th>
<th>P₁ PSIA</th>
<th>Lift (lbf)</th>
<th>Drag (lbf)</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meter Read</td>
<td>Fᴸ/7</td>
<td>Meter Read</td>
<td>Dᵣ x 0.019</td>
<td>ft/s</td>
<td>MPH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>83.0</td>
<td>6.0</td>
<td>1.5</td>
<td>14.913</td>
<td>14.737</td>
<td>0</td>
<td>0</td>
<td>522.5</td>
</tr>
<tr>
<td>2</td>
<td>83.0</td>
<td>6.1</td>
<td>1.55</td>
<td>14.916</td>
<td>14.735</td>
<td>0</td>
<td>0</td>
<td>488.6</td>
</tr>
<tr>
<td>3</td>
<td>83.5</td>
<td>5.95</td>
<td>1.60</td>
<td>14.911</td>
<td>14.734</td>
<td>+1.5</td>
<td>0.21</td>
<td>331.5</td>
</tr>
<tr>
<td>4</td>
<td>83.5</td>
<td>5.95</td>
<td>1.45</td>
<td>14.911</td>
<td>14.740</td>
<td>+1.8</td>
<td>0.26</td>
<td>291.2</td>
</tr>
<tr>
<td>5</td>
<td>83.5</td>
<td>6.0</td>
<td>1.45</td>
<td>14.913</td>
<td>14.740</td>
<td>+1.9</td>
<td>0.27</td>
<td>279.5</td>
</tr>
</tbody>
</table>

Lᵦ = Characteristic length = wheel base (model) = 0.695 ft
PᵦMT = 30.2 Hg = 14.791 psia
A = 0.400 ft² (model)
ρ = 5.1 x 10⁻³ lbf s²/ft⁴
v = 1.5 x 10⁻⁵ ft²/s
TABLE 3. WIND TUNNEL CALCULATED DATA
FOR FULL SIZE VEHICLE

<table>
<thead>
<tr>
<th>Test Run</th>
<th>$C_D A_{\text{Actual}}$ ft$^2$</th>
<th>POWER REQUIRED AT A GIVEN VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>50 MPH</td>
</tr>
<tr>
<td>1</td>
<td>5.46</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>4.98</td>
<td>4.8</td>
</tr>
<tr>
<td>3</td>
<td>3.66</td>
<td>3.85</td>
</tr>
<tr>
<td>4</td>
<td>3.00</td>
<td>3.37</td>
</tr>
<tr>
<td>5</td>
<td>2.88</td>
<td>3.29</td>
</tr>
</tbody>
</table>

$P_t = $ Tire inflation pressure = 36 (psia)

$a_1$, $a_2$ Are the coefficients for the rolling resistance, $F_{RT}$

$\eta = $ Drive train efficiency = 0.90

$W_T = 450$ (lb$^f$) (at $T = 0$)

$F_{\text{DRAG}} = C_D A V^2 / 391$ (lb$^f$)

Down force $F_L$, or negative dynamic lift is created with externally mounted inverted aerofoils. On most racing cars a positive lift force acts more towards the rear of the vehicle, reducing rear wheel traction. The aerofoil is mounted over or as near the rear axle as feasibly possible. The negative lift produced by the inverted aerofoil not only counter acts the positive lift force, but induces additional down force increasing vehicle traction. The reader is referred to equation (25) in section 2.1 for an analytical description. Application of
FIGURE 12
VEHICLE VELOCITY
vs. POWER REQUIRED

DATA SOURCE : TABLE 3  PREPARED BY : D. KORDS
FIGURE 13
MAXIMUM VEHICLE VELOCITY VS. DRAG AREA (CdA)

CONSTANTS
HP = 38
\eta = 0.90
\rho = 0.00238
\text{P}_{f} = 36 \text{ psi}
W_{f} = 400 \text{ lbs}

MAXIMUM SPEED POTENTIAL OF FINAL DESIGN AT 122 MPH LIMITED BY USEFUL CdA AND ROLLING RESISTANCE FrR.

DATA SOURCE: TABLE 3 PREPARED BY: D. KORDS
aerofoils on racing cars, for effective performance and optimum efficiency, requires a great deal of research and experimental work, as was done with the aid of wind tunnel testing. The optimum performance and effectiveness of aerofoils used on race cars depends on the following factors:

1) The type and geometrical configuration of the aerofoil profile.
2) The position and location of the aerofoil on the vehicle.
3) The angle of incidence (or attack) to the oncoming air flow.

The investigation was carried out with model # 3 as previously described and shown in Figure 10. The aerofoil selected was a NACA 6409 mounted over the rear axle, adjustable for height and angle of incidence as shown in Figure 14, enabling a comprehensive range of tests to be carried out. The selection of this aerofoil was based on the following reasons:

1) Similar aerofoils have been used on Formula One and other racing classes quite effectively.
2) The overall performance characteristics were better than other aerofoil profiles.
3) The most important factor was the low drag, high lift producing characteristics which is a desired criteria when selecting aerofoils for use on racing cars.

Initially the model was tested without the aerofoil to determine the magnitude of the upward lift force. Having recorded this, the model was tested with the aerofoil attached over the rear axle at various angles of incidence and at several heights. This determined the optimum angle and height to fix the aerofoil to compensate for the positive lift force.
Flow visualization tests showed that the partially open cockpit and bubbleshield induced turbulent wake interference around the driver's neck and helmet area. The tests also showed that if the aerofoil is placed too close to the ground, the effectiveness is greatly reduced due to the shielding from the driver. Placing the foil too high showed a tendency to produce dangerous pitching, which could lift the front steering wheels off the road surface. The
angle of incidence, when set to a high value, produced large down force and correspondingly high drag which was undesirable.

The measured values of lift and drag coefficients versus angle of incidence is shown in Figure 15. Figure 16 illustrates variation in the lift to drag ratio with angle of incidence. Finally, Figure 17 shows the variation of lift and drag coefficients versus aerofoil height for constant angle of incidence.

From the measured results an optimum design was selected at which the height and angle of incidence of the aerofoil performed efficiently. The height was selected at 24 to 26 inches above ground level and an angle of 10 to 12 degrees. This produced a net down force of approximately 1.6 times that of the lift force, effectively eliminating it and providing additional but not excessive down force.
FIGURE 15
Variation of Lift and Drag Coefficients with Angle of Incidence

Angle of Incidence, $\alpha$

DATA SOURCE: Paragon Industries
PREPARED BY: D. KORDS
FIGURE 16
Variation of Lift to Drag Coefficient Ratio with Angle of Incidence

DATA SOURCE: Paragon Industries
PREPARED BY: D. KORDS
FIGURE 17
Variation of Lift and Drag Coefficients with Height

DATA SOURCE: Paragon Industries
PREPARED BY: D. KORDS
2.9 NUMERICAL MODELS

Three comprehensive computer models are presented, based on the previous sections of vehicle and aerodynamic analysis and testing. The models were used to assess different design configurations and the effects of the various design parameters outlined in section 2. Modeling allowed evaluation of each possible design and resultant performance, allowing optimization of the final design. The programs were written in Fortran. The programs and modes of vehicle performance modeled are described as follows.

<table>
<thead>
<tr>
<th>Program Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Perform</td>
<td>Calculates vehicle theoretical performance from wind tunnel data.</td>
</tr>
<tr>
<td>2) Motion</td>
<td>Predicts straight line - longitudinal vehicle motion.</td>
</tr>
<tr>
<td>3) Curve</td>
<td>Predicts cornering or curvilinear vehicle motion</td>
</tr>
<tr>
<td>4) Race</td>
<td>Control program to link several straight line and cornering sections to form a race track. Passes data to the successive link.</td>
</tr>
</tbody>
</table>

Perform was used to calculate the theoretical maximum velocities directly from the raw wind tunnel data. This program proved useful in quickly assessing potential designs or modifications to existing configurations. Motion predicts straight line - longitudinal vehicle motion. In this form, comparisons can made for changes in vehicle mass, gearing, aerodynamics, rolling resistance, etc. Most automotive references refer to data for a quarter
mile distance, with initial conditions at \( t = 0 \) of \( x = 0, V = 0 \). The Motion program was used frequently for this comparison purpose. The Curve program predicts cornering or curvilinear vehicle motion for a given vehicle configuration. Configurational changes affecting vehicle performance in a corner were investigated with this program. Race is a control program to link several Motion and Curve sections to form a real or theoretical race track. The control program passes data to the successive link.

In all modes, output is printed for vehicle distance, velocity, and acceleration as a function of time for specific design parameters that are input into the program. All major design variables taken into account are listed in the initial comment statements of the program.

From section 2.1, the second order differential equation of motion (13) is evaluated numerically using the classic 4th order Runge Kutta method. This method was selected for the small per-step truncation error, self-starting characteristics, and stability. This convenient method also allows relatively easy programming of the equations. The self-starting feature allowed relative ease of passing data to successive links when using Race. Computational time was not important enough to consider more time efficient methods. The equation of motion is of the form:

\[
\frac{d^2x}{dt^2} = f(t, x, dx/dt)
\]
If we let $V = \frac{dx}{dt}$, the problem is transformed into two sets of first order differential equations. The Runge Kutta method is well suited for this application. The two first-order differential equations are:

\[
\frac{dx}{dt} = V \\
\frac{dV}{dt} = f(t, x, \frac{dx}{dt}) = A_c
\]

The equations as developed in reference 11 are:

\[
V_{i+1} = V_i + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)
\]

where:

\[
K_1 = (h) f(t_i, x_i, V_i) \\
K_2 = (h) f(t_i + h/2, x_i + Q1/2, V_i + K_1/2) \\
K_3 = (h) f(t_i + h/2, x_i + Q2/2, V_i + K_2/2) \\
K_4 = (h) f(t_i + h, x_i + Q3/2, V_i + K_3)
\]

\[
h = \text{Step increment}
\]

and

\[
X_{i+1} = X_i + \frac{1}{6} (Q_1 + 2Q_2 + 2Q_3 + Q_4)
\]

where:

\[
Q_1 = hF(V_i) = (h)(V_i) \\
Q_2 = hF(V_i + K_1/2) = h(V_i + K_1/2) \\
Q_3 = hF(V_i + K_2/2) = h(V_i + K_2/2) \\
Q_4 = hF(V_i + K_3) = h(V_i + K_3)
\]
By substituting the equations for Q's in to both expressions for the K's and reference formulas yield:

\[ X_{i+1} = X_i + hV_i + h/6 (K_1 + K_2 + K_3) \]
\[ V_{i+1} = V_i + 1/6 (K_1 + 2K_2 + 2K_3 + K_4) \]

where:

\[ K_1 = (h) f(t_i, x_i, V_i) \]
\[ K_2 = (h) f(t_i + h/2, x_i + hV_i/2, V_i + K_1/2) \]
\[ K_3 = (h) f(t_i + h/2, x_i + hV_i/2 + hK_1/4, V_i + K_2/2) \]
\[ K_4 = (h) f(t_i + h, x_i + hV_i + hK_2/2, V_i + K_3) \]

For the equation of motion (13)

\[ A_c M_{eq} = HP \frac{\eta}{550} / V - \rho C_D A V^2 / 2 - [a_1 + a_2 V^2] W_T - W_T \ Sin \ \theta_S \quad (lbf) \]

If we let \( C_1 = HP \frac{\eta}{550}, \ C_2 = \rho C_D A/2, \ C_3 = \ Sin \ \theta_S \) and rearrange:

\[ A_c = \left( C_1 / V - C_2 V^2 - [a_1 + a_2 V^2] W_T - W_T C_3 \right) / M_{eq} \quad (lbf) \quad (67) \]
\[ V = \frac{dx}{dt} \quad (ft/s) \quad (68) \]

Note that in this form the variable t does not appear. Although the weight of the vehicle \( W_T \), varies with time due to fuel consumption, this variation is very close to being linear and can be approximated by the equation:

\[ W = W_T - FR \cdot t \]
In this form, \( W \) is the overall vehicle weight term as used in the models, and \( FR \) is the fuel consumption rate, measured to be 0.0156 \((\text{lb}f/\text{s})\), from the use of 7 gals of fuel per hour at 8 lbs per gallon.

The numerical solution begins with the substitution of the initial values of \( x \) and \( V \) into the equations (67) and (68) to determine a value for \( K_1 \). The successive \( K \) values are then determined for use in the recurrence formulas previously shown, to obtain values of \( X_{i+1} \) and \( V_{i+1} \). These values are then used to obtain the new \( K \) values and values of \( X_{i+2} \) and \( V_{i+2} \) and so on. This procedure is shown in the flow chart in the appendix.

For longitudinal motion, the Motion model was run for a given straight line distance, such as a quarter mile. Data in this form proved useful in assessing the effects of aerodynamics, drivetrain gearing and efficiency, vehicle weight and inertia, and various horse power / torque curves. Quarter mile data is also comparable to other references for verification purposes.

Gear shifting was performed numerically by one of two preselected procedures, with shifting occurring at a preset engine redline speed, or by an optimization routine which compared current gear acceleration with the next higher gear. For the optimization routine, the numerical shift would be completed when the acceleration in the higher gear exceeded that in the lower gear. This routine proved most useful in determining gearing for a particular track. Once gear selection was finalized, shifting was performed using the redline method, simulating actual driver habits.
Engine power versus engine speed is numerically obtained from equations (9) and (12).

The effect of road grade was included, although pitch effect due to acceleration and deceleration was simply modeled by setting a percentage of vehicle weight acting on the rear and front wheels, as determined in equations (17) and (18). This technique has proved adequate given the limited amount of suspension travel and anti-dive and squat geometry of the suspension design. A friction coefficient of 1.0 was used for all dry weather simulations.

The models are started by assuming an initial vehicle distance and speed for time t. For quarter mile analyses, initial conditions at $t = 0$ are $V_i$ and $x_i = 0$.

Curvilinear motion analysis involved numerically evaluating the equation of motion in an arc. Lateral cornering acceleration $A_{V,\text{MAX}}$ is determined from equation (62), while cornering speed $V_{\text{MAX}}$ is specified from equation (58) and (60). The curvilinear $R_{vi}$ is described by equation (64).

Braking performance is determined from equation (63) and is employed in an iteration routine to determine the braking point on the straight preceding a corner.

Modeling of the corners is started at the apex of the turn, where the vehicle's lowest speed is computed from the radius of curvature and $\mu_M$. At the apex, the vehicle neither accelerates or brakes, and the vehicle is at maximum cornering capability. From the path curvature and banking; distance, velocity, and acceleration are computed with respect to time in the same manner as previously described. Upon entering the next corner, the vehicle velocity will
be higher than possible as governed by $\mu_m$ and $R_{CI}$. From this point, the model computes the vehicle speed at the new apex and uses the same algorithm in reverse direction using the braking equations. This process of backing up continues until braking speed exceeds acceleration speed (coming into the corner), representing the braking point. The program now continues from the second apex as previously described and continues to calculate distance, velocity, and acceleration with respect to time, around the circuit.

Due to various race track configurations, lengths and terrain, it is impossible to design an optimal vehicle for all conditions. Thus the optimized vehicle is further tuned to the individual track within the constraints of the design. Changes in drive train gearing, wheel size, and aerodynamic down force (and resultant drag) represent significant "tunable parameters" for a given race track.

The modeling mode Race links all race track straights, curves, and grades of the actual track. This is accomplished using a control program to link all straights and curves together in appropriate order, along with the appropriate parameters for each straight or curve. Between each link, calculated vehicle performance data is passed on as a function of time and distance. This linking can be used to simulate a complete lap of a known circuit. As the model theoretically drives the vehicle around the race course, the predicted vehicle displacement, velocity, acceleration, and time for individual segments and complete lap are calculated. In this mode, the overall effect of any vehicle variable change can be evaluated at any point on the race track, as well as the overall effect on average speed and time of one lap of the race course.
3.0 RESULTS
The driver compartment was designed to accommodate a 95th percentile USA male, and allows maximum visibility and comfort. To provide additional impact protection; front, rear, and side bumpers are designed to fit flush with the body work, yielding minimal aerodynamic influence. Instrumentation is located forward of the driver for visibility. A combination of analog and digital instruments display water coolant temperature, engine RPM and exhaust gas temperature, and the vehicle velocity (in MPH). All controls for shifting the transmission are located immediately in front of the steering wheel.

Chassis

In a departure from tradition, the chassis design features monocoque construction using E-glass and Kevlar fiber composite. Major factors behind this decision were the design requirements for a light weight, rigid chassis, and minimal additional body panels. Isophthalic polyester resin was used to control costs, although consideration was given to epoxy resin and carbon fiber. The monocoque itself is a two piece, three millimeter thick molding, hand laid-up in a female mold. The two pieces are bonded together in the mold to produce the finished chassis. This design results in a torsional stiffness exceeding 5500 lb-ft per degree, which is exceedingly high and a major contributing factor to the exceptional handling characteristics. For additional local reinforcements in internal high stress points for suspension and engine mountings, machined steel inserts are molded in during the layup process. The forward part of the cockpit surround has an additional monocoque section for added driver forward protection, acting as an energy
absorbing structure. The entire assembly was designed for ease of manufacturing and service.

Bodywork

Figure 18 shows the central monocoque structure, the fuel cell detachable body panel, and (2) engine bay enclosure panels. The body work is constructed of nominal two millimeter thick fiberglass, hand laid up. The outer panels are nonload bearing.

**FIGURE 18 Central Monocoque Structure and Detachable Body Panels**

Engine

The Austrian made Rotax 128 GP single cylinder Road Race engine is used. This 125cc displacement engine currently represents the state of the art in
design and features rotary valve induction, water cooling, a pneumatic
tested exhaust valve, and fully tunable electronic digital ignition. The
stock maximum power output is 38 bhp at 12,800 RPM (see Figure 3). Engine
specifications in detail are located in the appendix.

Transmission
The gearbox features a quick change, side load cartridge gear box. The
internal ratios are fully changeable. The output shaft gear and rear axle gear
are changeable to suit the particular track to be driven. The specifications are
located in the appendix.

Suspension
For optimum cornering and tracking stability, special attention was
directed towards wheel control and accurate suspension movement. Camber
change was carefully constructed over the suspension’s entire vertical
movement. The same values of camber change were applied to both the front
and rear suspension to achieve similar tire cornering characteristics at both
ends of the vehicle at all roll angles.

Stability was further enhanced during turn-in and under braking by
utilizing toe out on bump, and toe in on rebound / bump steer characteristics.
A very low castor angle was employed to minimize steering effort at all
speeds.

Progressive handling behavior with increasing lateral acceleration was
achieved by designing the suspension geometry to give virtually constant roll
center heights relative to the chassis ride level, irrespective of the vehicle’s roll angle.

The short wheel base as imposed by the regulations required some means of controlling pitching during braking. Antidive was built into the front suspension geometry and antilift in the rear geometry to enhance braking stability.

**Steering**

A standard 5/8" shaft with control arm and tubular alloy tie rods were employed, with a turning lock to lock ratio of 0.5. Various values of Ackermann were evaluated, and as anticipated, high values gave good front tire adhesion on small radius corners at the expense of some loss in stability on open sweeping curves. Less Ackermann improved stability at high speeds, but with the inevitable tire scrub and loss of adhesion in tight curves. The best dynamic balance was found to be 60% Ackermann geometry. A Paragon 12” leather steering wheel was used.

**Brake Systems**

Front and rear Paragon hydraulic brakes with twin master cylinders for separate circuits were used. Front to rear brake balance is provided by a mechanical proportioning linkage and can be adjusted by the driver while on the race track. The front system features 6” diameter x 3/8” thick ventilated cast iron discs mounted outboard on alloy wheel hubs. At the rear, an 8” diameter x 3/8” thick ventilated cast iron disc is mounted to a live rear axle.
The calipers at the front feature a single piston full floating design. The rear system features a 2 piston fixed design with spring loaded pads.

**Fuel Cell**

An aluminum fuel cell with internal baffles and fuel pickup sump is used. Capacity is 7 gallons.

**Cooling System**

A side mounted water radiator is located in front of the engine bay. The engine houses an internal water pump. All plumbing is located in the engine bay for minimal weight and complexity.

**Wheels**

Paragon aluminum alloy mono wheels are used. Front wheel dimensions are 5” or 6” diameter x 5.5” width. Rear wheel dimensions are 5” or 6” diameter x 7 or 8” width. Diameter and widths are dictated by weather and race course.

**Manufacturing**

The initial mockup prototype was constructed of wood and fiberglass from full scale drawings. Once the mockup was completed, four molds were constructed for the monocoque and side body work from fiberglass. The monocoque required a two piece mold which bolts together prior to the layup process.
The molds are prepared by cleaning and then waxing with mold release. Gel coat which includes the color coat is first applied, with subsequent cloth and resin layers applied to the mold. During this time, strict control over temperature and humidity are maintained to provide optimum curing conditions. After a 24 hour curing period, the parts are removed from the molds and inspected for flaws. The parts are then trimmed to remove flash, and cut for proper fit. The remaining hardware is then assembled to the chassis to complete it as a "roller".
TABLE 4  
VEHICLE SPECIFICATIONS

The following specifications are current as governed by the Race Sanctioning Organizations.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>125 Road Racing Car</td>
</tr>
<tr>
<td>Chassis Structure</td>
<td>Composite Monocoque</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>48 (in.)</td>
</tr>
<tr>
<td>Caster / Camber</td>
<td>2 degrees / 0 degrees</td>
</tr>
<tr>
<td>Length, Overall</td>
<td>80 (in.) suggested, no maximum</td>
</tr>
<tr>
<td>Width</td>
<td>42 (in.)</td>
</tr>
<tr>
<td>Height</td>
<td>20 Driver's Head (in.)</td>
</tr>
<tr>
<td>Track</td>
<td>38 / 42 Front / Rear (in.)</td>
</tr>
<tr>
<td>Weight, Dry</td>
<td>115 (lb.) minimum</td>
</tr>
<tr>
<td>Weight, Dry, with Driver</td>
<td>400 (lb.) minimum</td>
</tr>
<tr>
<td>Weight Distribution</td>
<td>45 / 55 Front / Rear (%)</td>
</tr>
<tr>
<td>Tire Diameter</td>
<td>11 / 12 Front / Rear (in.)</td>
</tr>
<tr>
<td>Tire Width</td>
<td>5 / 7 Front / Rear (in.)</td>
</tr>
<tr>
<td>Wheels</td>
<td>Paragon Alloy Mono, 5 or 6”</td>
</tr>
<tr>
<td>Engine Type</td>
<td>Rotax, 1 cylinder, Two stroke, watercooled</td>
</tr>
<tr>
<td>Displacement</td>
<td>125 (cubic centimeters)</td>
</tr>
<tr>
<td>Power, Maximum</td>
<td>38 (HP) @ 12,800 RPM</td>
</tr>
<tr>
<td>Ground Clearance</td>
<td>1 (in)</td>
</tr>
<tr>
<td>Rear Axle</td>
<td>1.25” tubular 4130 steel</td>
</tr>
</tbody>
</table>

* * *
3.2 PERFORMANCE VERIFICATION

In this section, simulated vehicle performance is compared to actual performance. The program Motion was used to predict longitudinal performance, where the known input parameters are entered at the start of the simulation. The comparison allows model calibration and determination of error. Actual performance was obtained from measurement of quarter mile times and references listed below. Table 5 tabulates the data for actual and simulated performance and is plotted in Figure 19 for comparison.

**TABLE 5 MEASURED and SIMULATED LONGITUDINAL PERFORMANCE**

<table>
<thead>
<tr>
<th>Time to Accelerate from Zero Velocity</th>
<th>ACCUMULATED TIME (s) VS DISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual Zip 125*</td>
</tr>
<tr>
<td>0 TO 30 MPH</td>
<td>2.7</td>
</tr>
<tr>
<td>0 TO 40 MPH</td>
<td>4.1</td>
</tr>
<tr>
<td>0 TO 50 MPH</td>
<td>5.0</td>
</tr>
<tr>
<td>0 TO 60 MPH</td>
<td>6.7</td>
</tr>
<tr>
<td>0 TO 70 MPH</td>
<td>8.1</td>
</tr>
<tr>
<td>0 TO 80 MPH</td>
<td>10.2</td>
</tr>
<tr>
<td>0 TO 90 MPH</td>
<td>12.5</td>
</tr>
<tr>
<td>0 TO 100 MPH</td>
<td>15.5</td>
</tr>
<tr>
<td>Standing 1/4 mile</td>
<td>15.0 @ 90 MPH</td>
</tr>
<tr>
<td>Top Speed</td>
<td>105</td>
</tr>
<tr>
<td>Braking, 70 to 0 MPH, ft</td>
<td>163</td>
</tr>
<tr>
<td>Lateral Acceleration, g</td>
<td>1.23</td>
</tr>
</tbody>
</table>

The actual data was obtained from a comparison of 1/4 mile acceleration times of a Formula 125 (Refs. 12). The road report included gear ratios, tire sizes, test weight, and engine power. Comparison of plots in Figure 19 show relatively good correlation when one considers the initial conditions, shift points, frontal areas, and driver position were not exactly known. RIT wind tunnel data was used when initial data was missing.

To better determine the improvement in performance of a particular vehicle configuration, it was desirable to simulate performance around a race circuit. As previously discussed in section 2.4, the vehicle was modeled for a complete lap of the well known circuit, Daytona International Raceway, as shown in Figure 20. This track was chosen due to its flat layout and known geometry, allowing ease in modeling the track. The racing line from Refs 13 is shown superimposed on the circuit.

Table 6 lists the actual times of the leader of the 125 class at Daytona speed week, held 12/30/89. The day had excellent conditions with sunny, dry, 70 degree weather. The table lists accumulated time versus distance around the road course, for locations labeled A through I. The subscript i designates the position when the vehicle enters a corner (when the vehicle turns in and begins curvilinear motion), where the subscript o designates the vehicle exiting the corner (beginning longitudinal motion). Values C, F, G, H, and I represent position only. Data was recorded with an on board data collection system and verified against manual timing.
When the actual performance data is compared lap to lap, the overall lap times are found to range within 2 percent. The times for the best actual lap at the listed positions A through I, are compared in Table 7 to the numerical model prediction for the same vehicle. The model predicted a lowest elapsed time of 1 minute, 59.6 seconds over the same distance for an average speed of 107.18 miles per hour.

**TABLE 6 MEASURED PERFORMANCE TIMES**

<table>
<thead>
<tr>
<th>LAP NUMBER</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.73</td>
<td>16.73</td>
<td>23.44</td>
<td>32.97</td>
<td>38.54</td>
<td>44.42</td>
<td>51.51</td>
<td>58.50</td>
<td>1.17.96</td>
</tr>
<tr>
<td>B</td>
<td>16.68</td>
<td>22.96</td>
<td>32.81</td>
<td>37.89</td>
<td>44.11</td>
<td>50.64</td>
<td>57.00</td>
<td>1.17.37</td>
<td>1.34.88</td>
</tr>
<tr>
<td>C</td>
<td>8.14</td>
<td>16.42</td>
<td>22.09</td>
<td>31.70</td>
<td>36.98</td>
<td>43.09</td>
<td>49.92</td>
<td>56.43</td>
<td>1.16.28</td>
</tr>
<tr>
<td>D</td>
<td>17.06</td>
<td>22.90</td>
<td>33.31</td>
<td>37.97</td>
<td>44.10</td>
<td>51.25</td>
<td>57.42</td>
<td>1.17.65</td>
<td>1.34.77</td>
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<tr>
<td>E</td>
<td>16.48</td>
<td>23.58</td>
<td>31.91</td>
<td>37.04</td>
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<tr>
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<td>37.63</td>
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<td>31.88</td>
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<td>50.13</td>
<td>56.11</td>
<td>1.16.26</td>
<td>1.33.07</td>
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<tr>
<td>H</td>
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<td>50.44</td>
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<td>50.62</td>
<td>56.28</td>
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**TABLE 7 SIMULATED PERFORMANCE TIMES**

<table>
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<tr>
<th>LAP NUMBER</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
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<td>32.97</td>
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<td>1.17.96</td>
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</tr>
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<td>C</td>
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<td>31.70</td>
<td>36.98</td>
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</tr>
<tr>
<td>F</td>
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<td>50.62</td>
<td>56.28</td>
<td>1.16.94</td>
<td>1.34.94</td>
</tr>
</tbody>
</table>
The maximum steady state lateral acceleration was taken at 1.6 g's or 51.5 ft/s² as this value was measured earlier in the year at the Watkins Glen skid pad. Braking deceleration was calculated to be 1.1 g. The simulation times are within 5 percent of the actual overall elapsed time. Considering the complexity and interaction of the many variables, this result is quite acceptable as a predictor of actual on-track performance. The difference of 5 percent can be explained, but without extensive investigation, difficult to attribute to any one variable. Differences in air density affect both aerodynamic drag and engine output.

<table>
<thead>
<tr>
<th>LAP NUMBER</th>
<th>ACCUMULATED TIME (s) VS DISTANCE (Total 3.56 miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ai</td>
</tr>
<tr>
<td>ACCUMULATED</td>
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</tr>
<tr>
<td>DISTANCE</td>
<td></td>
</tr>
<tr>
<td>feet</td>
<td></td>
</tr>
<tr>
<td>INCREMENTAL</td>
<td>1166</td>
</tr>
<tr>
<td>DISTANCE</td>
<td></td>
</tr>
<tr>
<td>feet</td>
<td></td>
</tr>
<tr>
<td>ACTUAL</td>
<td></td>
</tr>
<tr>
<td>BEST LAP</td>
<td></td>
</tr>
<tr>
<td></td>
<td>---</td>
</tr>
<tr>
<td>PREDICTED</td>
<td>6.6</td>
</tr>
<tr>
<td>LAP</td>
<td></td>
</tr>
</tbody>
</table>
| Tires diameters increase at high speed and this was accounted for within the model by an algorithm which enlarges the tire diameter with the square of the velocity up to 10 percent at 150 mph.
Down force in excess of 50 percent of the total vehicle weight is estimated to be easily attained, although difficult to measure precisely. The downforce distribution was approximated at 70 percent acting on the rear tires.

Engine output does decline as much as 15 percent within the first 10 to 15 minutes of the race resulting in lower overall lap speed. This is known as engine "wilt" and is a result of heat build up, reducing the engine thermo-efficiency and increasing friction. These effects are difficult to analytically predict and are usually measured empirically.

The value of the model extends beyond theoretical design to areas of optimizing race track setups including transmission gear selection and aerodynamic tradeoffs and are discussed in the following section.
The single most important parameters affecting vehicle performance are drag and down force, in that order. The high drag coefficient measured for a standard Formula 125 is primarily due to the large separated wake resulting from the bluff shape of the driver and secondary cumulative effects of the various small - high drag, exposed components. There are only two ways to reduce this drag, one is to reduce frontal area, the other is to change the shape of the vehicle and its components. From the preceding results, it is clear that a reduction in the drag coefficient of a racing vehicle leads to considerable improvements in performance. From the power analysis, the power to propel the vehicle is a function of the velocity cubed. Therefore, the horsepower to overcome total drag up to fifty miles per hour is approximately five or six horsepower, but by one hundred miles per hour, nearly thirty-five horsepower is required. From this simple analysis, approximately twenty-five percent more power is required to gain a ten percent increase in velocity. It is quite evident that drag reduction pays big dividends in speed as well as acceleration. Finally, a reduced drag coefficient means a decrease in the ease of being drafted, which is another useful return.

Of main interest in aerodynamic testing is obtaining a comparative measure of the drag coefficient of each device tested. It is not necessary to know to the nearest pound what the drag or down force is, rather how much a given aerodynamic change improves or degrades the down force and drag.

It should be noted, in the use of RIT's subsonic wind tunnel, that certain similarities cannot be duplicated in the tests unless elaborate modifications to
the tunnel itself are made. One of the most basic problems is simulating a rotating wheel, as this has been a major source of error associated with the testing of racing vehicles in wind tunnels. Research has shown that a rotating tire can be simulated by putting a flap on top of the tire at its point of flow separation. The absence of a ground plane underneath the car also results in error. In real life, the car is one inch off the ground, restricting air flow and increasing drag. In the wind tunnel, air flows freely under the model, which probably results in an overestimated drag reduction.

Except for the fourth test, in which the full body work was used, all other modifications resulted in fractional improvements by comparison. By far, the full body treatment is the most advantageous in terms of overall performance, as verified by the data.

Aerodynamic down force is substantially increased and is comparable to the vehicle weight. To maintain fairly consistent ride height, which is extremely important to maintaining the ground effects - created down force, and prevent the car from bottoming, the use of progressive rate suspension via springs and geometry is employed. The design must ensure that the entire spring or suspension travel is not used up (Refs. 14, 15). During cornering, if this does occur, the suspension or car bottoms out with disastrous effects. In the case where the springs are completely compressed, the effective spring rate goes to infinity, causing the car to now roll about the tire contact patch, and therefore rolls even further. Ride quality (or lack of) causes the driver and car cornering performance to dramatically suffer. When the car bottoms out, particularly in a corner, the car can momentarily lose traction, causing disastrous directional instabilities which for the most part cannot be
corrected by the driver. Several Indy car crashes have been traced to this phenomena. To deal with these problems, suspension rates have more than doubled on the average current day race car, with a notable loss of ride quality.

There are various philosophies regarding race car design and the best approach to testing and tuning (Refs. 16, 17, 18). Most designers agree that instrumentation of the vehicle and wind tunnel testing, when available, are both extremely valuable and complimentary methods to arrive at a good compromise for performance at any track. The word compromise is used because of the difficult problem facing the designer or racing team in analyzing the performance of the car and driver and deciding how to best improve it. The key to this situation is usually in the ability of the driver to communicate symptoms of handling and performance to his crew chief or engineer. Instrumentation has given race teams considerable insight into vehicle performance, but correcting or improving performance remains primarily one of trial and error, governed by experience.

The model is very effective in predicting performance of a given setup configuration for a particular track and set of conditions, as demonstrated with the Daytona example. The model is a valuable tool to optimize a vehicle’s "tunable parameters", such as drag, downforce, gearing, etc. to obtain the best vehicle performance.

The model is extremely valuable in evaluating different configurations in the design stage, prior to fabrication of any parts. In this way, a potential configuration can be evaluated over several different tracks and conditions, yielding an overall optimized design.
With the modeling analysis and data obtained from on-board instrumentation and driver feedback, it is not difficult to arrive at the best vehicle aerodynamic configuration for any racing class, and then tune the car to the best setup configuration for any track.

The use of ground effects to increase cornering capacity of race cars is still largely a factor of available horsepower. In classes such as Formula 125 which limits engine size, one cannot obviously ignore drag and use all the power to negotiate corners. The balance between drag and down force must be ascertained and this can be accomplished with the model simulations. The only other feasible approach open to most engineers, is to make the necessary changes that are possible, and time the vehicle performance during actual race practice. Although this procedure does work, it is very time consuming at a race and can even be dangerous if the changes are made in the wrong direction.
CONCLUSION

From the preceding results and discussion, it has been shown that improvement of a racing car’s aerodynamics is the most obvious way to improve its performance. The easiest way to reduce aerodynamic drag is through frontal area reduction. However, maintaining a smooth profile may be more practical due to competition regulations.

The models presented and related simulations have shown to be extremely valuable design tools, enabling various vehicle configurations and predicted performance to be studied. The models further reinforce the enormous importance of aerodynamics on racing vehicles. The successful use of “ground effects” to increase vehicle dynamic weight and the associated increase in cornering speeds and braking efficiency have dramatically reduced lap times, as verified by the models.

Measurement of down force in the RIT wind tunnel leaves something to be desired. Stability of a race car is extremely important at high speeds, and it is not enough to know just the down force, but one must know the distribution acting on the vehicle. Once this distribution is known, corrections can be made to the vehicle body to correct unstable aerodynamic conditions. This distribution can be measured by installing pressure taps on the model. Furthermore, a rolling ground plane is needed to ascertain the aerodynamic “ground effects” and related down force which could not be measured. Complete aerodynamic studies of race cars require this data.

The most uncertain portion of the analysis in the development of the longitudinal equation of motion was that of the rolling resistance. A more exact solution could be obtained by the familiar “coast down” test in which
the actual vehicle is allowed to coast to a stop from a known velocity, where its time and distance to stop are recorded.

It is the author's intent to further refine the work presented here, and to produce simulations for the majority of tracks in North America.
6.0 REFERENCES

APPENDIX A

RIT SUBSONIC WIND TUNNEL

The following dimensions of the subsonic tunnel test section are:

Manufacturer: Kenny Engineering Corp.
Pasadena, California
Model 1189, Serial 1189

Width: 2 ft.
Height: 2 ft.
Cross-Sectional Area: 4 ft$^2$ (perpendicular to the flow)
Length: 4 ft.
Attainable Air Flow Velocities: 20-7600 ft./s
Plenum Manometer: Meriam Instruments, 10" CODC
H$_2$O Std. Cleanout Manometer
Test Section Manometer: Dwyer H$_2$O Manometer
Drag and Lift: Measured from Balance
Transducers on Meters
APPENDIX B

SUBSONIC WIND TUNNEL MEASUREMENTS

To determine the wind velocity in the subsonic wind tunnel test section, Figure B.1 is shown below.

**FIGURE B.1 RIT SUBSONIC WIND TUNNEL**

The wind tunnel velocity can be found by the pressure and temperature measured in the tunnel. Two tube manometers and a thermocouple are employed for these measurements. The connections are shown in Figure B.1. The following formula is developed for determining the velocity of the air stream in the test section (Refs. 19, 20, 21).

Utilizing the control volume shown in Figure B.1, we will assume the air flow is one-dimensional adiabatic flow which will give sufficient accuracy. Assuming an ideal gas and steady flow, from the first law of thermodynamics,

$$h_1 + \frac{V_1^2}{2} + gZ_1 = h_0 + \frac{V_0^2}{2} + gZ_0 = \text{constant}$$
where:  
\[ V_i = \text{average flow velocity in the test section} \]
\[ V_0 = \text{average flow velocity in the plenum} \]
\[ h_1 = \text{enthalpy per unit mass in the test section} \]
\[ h_0 = \text{enthalpy per unit mass in the plenum} \]
\[ Z_1 = Z_0 = \text{level of test section} \]
\[ g = \text{gravitational constant} \]

and:

\[ h = U + PV \]

where:  
\[ U = \text{internal} \]
\[ P = \text{pressure} \]
\[ V = \text{specific volume} \]

for an ideal gas:  
\[ PV = RT \]

where:  
\[ T = \text{temperature (absolute)} \]
\[ R = \text{gas constant} = 53.35 \text{ ft} \cdot \text{lb}_f / \text{(lb}_m)^{\circ} \text{R} \]

The velocity in the plenum is sufficiently small compared to the test section velocity, therefore:  
\[ V_i > V_0, \text{ we neglect } V_0. \]
Substituting the values into the first law, we have:

\[ V_i = (2(h_0 - h_1))^5 = \text{velocity in the test section} \]
For reversible adiabatic flow, we have isentropic compression in the test chamber as: \( PV\gamma = \text{constant} \) where \( \gamma \) is the ratio of specific heats, \( C_p \) and \( C_v 
\
For air:

\[
\gamma = \frac{C_p}{C_v} = 1.4
\]

\[
C_p = \frac{dh}{dt} \implies dh = C_p dt
\]

Integrating with the appropriate limits:

\[
\int_{h_1}^{h_0} dh = C_p \int_{T_1}^{T_0} dt = h_0 - h_1 = C_p (T_0 - T_1)
\]

\[
= C_p T_0 (1 - T_1/T_0)
\]

From the specific heats \( C_p - C_v = R \) where \( \gamma = C_p/C_v \n\
Rearranging: \( \frac{C_p}{\gamma} - C_p = R \) or \( (C_p)_\gamma - C_p = \gamma R \n\]

\[
\therefore C_p (\gamma - 1) = \gamma R
\]

Solving for \( C_p \):

\[
C_p = \frac{\gamma R}{\gamma - 1}
\]
From isentropic compression: \( P_1 V_1 \gamma = P_0 V_0 \gamma \) = constant

or rearranging \( P_1 = \frac{V_0 \gamma}{V_1} \) but \( PV = RT \)

or \( V = \frac{RT}{P} \)

Substituting for \( V; \)

\[
\frac{P_1}{P_0} = \frac{RT_0/P_0}{RT_1/P_1} = \frac{T_0 \gamma}{T_1} \frac{P_1 \gamma}{P_0}
\]

Divide by \( \frac{P_1 \gamma}{P_0} \)

\[
\frac{P_1}{P_0} \frac{1}{\gamma} = \frac{T_0}{T_1} \gamma = \frac{P_1}{P_0} (1-\gamma)
\]

or

\[
\frac{P_1}{P_0} \frac{(1-\gamma)/\gamma}{T_1} = \frac{T_0}{T_1} \gamma / \gamma = \frac{T_0}{T_1}
\]

Taking reciprocals to get the form \( T_1/T_0 \)

\[
\frac{T_1}{T_0} = \frac{P_1}{P_0} \frac{(1-\gamma)/\gamma}{(1-\gamma)/\gamma} (1-\gamma)/\gamma = \frac{P_1}{P_0} (\gamma-1)/\gamma
\]

From our equation: \( V_1 = (2(h_0 - h_1))^5 \) we have shown that:

\[
h_0 - h_1 = C_p T_0 (1-T_1/T_0)
\]

\[
C_p = \frac{\gamma R}{(\gamma-1)} \quad \text{and} \quad T_1 = \frac{P_1}{P_0} \frac{(\gamma-1)/\gamma}{(\gamma-1)/\gamma}
\]

Substituting these values in the equation for \( V_1: \)

\[
V_1 = \left( 2R \frac{\gamma}{(\gamma-1)} \right) \frac{T_0}{(1 - (P_1/P_0)^{(\gamma-1)/\gamma})} 5
\]

where \( \gamma = 1.4 \)

\( T_0 = \) temperature measured in the plenum (absolute)

\( P_1 = \) pressure measured at the test section

\( P_0 = \) pressure measured at the plenum
Pressure $P_1$ at the test section and pressure $P_0$ at the plenum are read using the manometers as follows. Shown in Figure B.2 is a model of the manometers used on the wind tunnel. Because each manometer is open to the atmosphere, we can model both manometers as a three-tube manometer$^1$.

**FIGURE B.2**

![Diagram of manometers](image)

\[ P_{\text{atm}} \]

\[ P_0 \]

\[ P_1 \]

\[ P_0 = P_{\text{atm}} + (Z_0) \frac{62.4}{1728} \text{ (psia)} \]

\[ P_1 = P_{\text{atm}} - (Z_1) \frac{62.4}{1728} \text{ (psia)} \]

Temperature in the plenum, $T_0$, is measured by a thermocouple with a digital readout.
SAMPLE CALCULATION

The following data is taken:

Barometric Pressure \( = \) 29.52 in Hg

\( T \) = temperature in plenum = 98°F

Manometer Readings: \( Z_0 = 10.1 \) in., \( Z_1 = 2.5 \) in.

Calculations

\[ \text{Patm} = \frac{29.52 (13.56) (62.4)}{1728} = 14.45 \text{ (psia)} \]

\[ P_0 = \frac{14.45 + (10.1")(62.4)}{1728} = 14.815 \text{ (psia)} \]

\[ P_1 = \frac{14.45 - (2.5") (62.4)}{1728} = 14.360 \text{ (psia)} \]

\[ V_1 = \left( 2(53.35) \text{ ft. lb}_f \frac{1.4(98 + 460)\text{°R} (1 - 14.36/14.815)(1.4-1)/1.4) (32.2 \text{ ft. lb}_m)_{0.4} \text{ lb} \text{ s}^{-2}}{\text{lb}_f \text{ s}^{-2}} \right)^{0.5} \]

\[ V_1 = \frac{243.996 \text{ ft.}}{\text{s}} \frac{1 \text{ mile}}{5280 \text{ ft.}} \frac{3600 \text{ sec.}}{\text{hr.}} \]

\[ V_1 = 166.36 \text{ mph} \]

For many calculations, this equation can be shortened to:

\[ V_1 = \left( 5590.176 \left( T_0 + 460 \right) (1 - (P_1/P_0)^{2857}) \right)^{5} \text{ (mph)} \]

Lift and drag are read directly off the meters on the wind tunnel. From Reference 21, where a free body diagram of the balance was evaluated, the actual drag and lift forces are calculated as shown:

Actual Lift \( = \) (Reading/7) lb\(_f\)

Actual Drag \( = \) (Reading \cdot 0.019045) lb\(_f\)
## APPENDIX C

### TEST VEHICLE DIMENSIONS AND SPECIFICATIONS

The following specifications are current as governed by the Race Sanctioning Organization.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
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</tr>
<tr>
<td>Wheelbase</td>
<td>50 (in.)</td>
</tr>
<tr>
<td>Length, Overall</td>
<td>80 (in.) suggested, no maximum</td>
</tr>
<tr>
<td>Width</td>
<td>44 (in.)</td>
</tr>
<tr>
<td>Height, Top of Track</td>
<td>26 Driver's Head (in.)</td>
</tr>
<tr>
<td>Weight, Dry</td>
<td>115 (lb.) minimum</td>
</tr>
<tr>
<td>Weight, Dry, with Driver</td>
<td>375 (lb.) minimum</td>
</tr>
<tr>
<td>Weight Distribution</td>
<td>45 / 55 Front / Rear (%)</td>
</tr>
<tr>
<td>Tire Diameter</td>
<td>11 / 12 Front / Rear (in.)</td>
</tr>
<tr>
<td>Tire Width</td>
<td>5 / 7 Front / Rear (in.)</td>
</tr>
<tr>
<td>Engine Type</td>
<td>Rotax, 1 cylinder, 2 stroke, water-cooled</td>
</tr>
<tr>
<td>Displacement</td>
<td>125 (cubic centimeters)</td>
</tr>
<tr>
<td>Power, Maximum</td>
<td>38 (HP) @ 12,800 RPM</td>
</tr>
<tr>
<td>Aerodynamics</td>
<td>Body work permitted, no maximum length</td>
</tr>
<tr>
<td>Projected Frontal</td>
<td>6.0 Area (ft.$^2$)</td>
</tr>
</tbody>
</table>
Appendix D

Models

Scale 1/15

Length of Wheelbase Actual = 50" = 4.17 ft.

Scale Wheelbase Length = \( \frac{50''}{(12 \text{ in./ft.})_{15}} \) = .278 ft.

Velocity of Tunnel = 100 mph

\[
\text{Re}^* = \frac{\text{VL}}{v} = \frac{100 \text{ miles}}{hr.} \times \frac{5280 \text{ ft.}}{\text{miles}} \times \frac{1 \text{ hr.}}{3600 \text{ s}} \times \frac{(0.278) \text{ ft.}}{1.5 \times 10^{-5} \text{ ft.}^2/\text{s}}
\]

= \( 1.9 \times 10^6 \)

Projected frontal area (ft.\(^2\)) = \( \frac{6.0}{15} = 0.400 \text{ ft.}^2 \)

Blockage Ratio = 10.0 %

**FIGURE D.1 MODEL MOUNTING POINTS**
APPENDIX E

VEHICLE TRANSMISSION

To determine and clarify the transmission characteristics of the vehicle, Figure E is shown below.

FIGURE E, Vehicle Gearing

The following ratios are defined:

\[ R_p = \text{Primary Drive Ratio} = 3.524 \]

\[ R_T = \text{Transmission Ratio, for six speed gear box, } J = 1 - 6, \text{ where} \]

\[ J (1 - 6) = 2.143, 1.750, 1.529, 1.353, 1.222, 1.150 \]

\[ R_o = \text{Final Drive Ratio} = 0.828 \]

\[ R(N) = \text{Total Drive Train Ratio} = R_p + R_T + R_o \]
The very nature of the wind tunnel and the parameters involved in its use, error can be expected to run high. A "common sense" error analysis is presented. For the plots constructed of the experimentally obtained data where error is present, is given as:

- **Δ Air Density** = ± 2%
- **Δ T₀, Temperature in the Plenum** = ± 1%
- **Δ Z₁, Z₂, Manometer Readings** = ± 2% (each)
- **Δ Patm, Barometer** = ± 1%
- **Δ Lift and Drag Meters** = ± 2% (each)
- **Δ Frontal Area, A** = ± 4%

From these errors, we find the Most Probable Error (M.P.E.) associated with the tunnel velocity, drag, lift, and their coefficients:

- **M.P.E. for Velocity, V** = \((2^2 + 1^2 + 2^2 + 2^2 + 1^2)^{0.5}\) = 3.75%
- **M.P.E. for Drag, \(F_{DRAG}\)** = \((4^2 + 2^2 + 2^2 + 3.75^2)^{0.5}\) = 6.2%
- **M.P.E. for Lift, \(F_L\)** = \((4^2 + 2^2 + 2^2 + 3.75^2)^{0.5}\) = 6.2%
- **M.P.E. for \(C_D\) & \(C_L\) (each)** = \((3.45^2 + 2^2 + 6.2^2 + 4^2)^{0.5}\) = 8.4%

For calculated data, the errors are:

- **Δ \(F_{RT}\), Rolling Resistance** = ± 5%
- **Δ \(S\), Specific Fuel Consumption** = ± 5%
- **Δ \(\eta\), Drive Train Efficiency** = ± 5%
From these errors, we find the M.P.E. for:

\[
\text{M.P.E. for } C_D A = (8.4^2 + 4^2)^{\frac{1}{5}} = 9.3\% \\
\text{M.P.E. for Fuel Consumption} = (3.75^2 + 5^2 + 10.2^2)^{\frac{1}{5}} = 11.9\% \\
\text{M.P.E. for Power, HP} = (6.2^2 + 5^2 + 5^2 + 3.75^2)^{\frac{1}{5}} = 10.2\%
\]

It should be noted that the weight of the vehicle changes with time (fuel load) as does the engine power, both with RPM and with time (the engine power can "wilt" as much as 15% in the first fifteen minutes of a race).
APPENDIX G

ENGINE SPECIFICATIONS

ROTAX 125cc MOTORS FOR 1990

Andover Norton have supplied these details of the 1990 versions of the 125 cc Rotax type 128 kart and 128 Road Racer model engines. The factory report a very successful season in the World and European 125 cc Road-Race Championships 1989 when the type 128 was crowned by the World Championship title of Alex Criville riding a Cobb-rotax motorcycle. Further, the European title was taken by Gabriele Debbia with Alessandro Gramigna as runner-up, and 5th place in the World Championship Series was awarded to Fausto Gresini. All these competitors were riding Aprilia machines using the engine type 128.

As part of the further development of this engine, the following changes will be incorporated in the 1990 series:

1) New crankcase for "Road Racer" version with fins for improved cooling.

2) New rotary valve cover with low friction coating to reduce wear.

3) For better reliability, a new crankshaft is introduced, with the following improvements:
   a) New connecting rod with "L" cross-section, improved surface finish and reinforced small-
      end eye.
   b) Crankpin with low temperature hardening treatment.
   c) New big-end needle bearing with higher load capacity.
   d) Thrust washers with better surface finish.

4) Crankshaft end float to be controlled by thrust washers only no shims available in a wider range of
   thicknesses.
5) New piston, with reinforcement around the boost-port cutaway.
6) Improved stronger clutch drum and clutch hub.
7) Revised gear ratios (the most commonly used ratios will be fitted as standard) and some additional
   ratios are introduced to the option set.
8) New spark plug with longer insulator body and new spark plug cap.

DESIGN NUMBERS: KART 30.128.1413, ROAD RACER 30.128.1313 with balance shaft.

DESCRIPTION: ROTAX two-cycle, single cylinder, rotary valve engine, oil-in-fuel lubrication, water-
cooled, integrated water pump, cartridge gearbox, digital ignition.

MACHINE: 54 mm (2.126 inch).

STROKE: 54.5 mm (2.145 inch).

DISPLACEMENT: 124.8 cm³ (7.616 inch³).

COMPRESSION: theoretical: 13.8:1; effective: 7:1.

IGNITION CHAMBER VOLUME: 9.75 cm³ = 0.4 cm³.

PISTON CENTRE PROTRUDING OVER CYLINDER:
   TOP: 2.05 mm ± 0.1 mm.
   RECOMMENDED OPERATION R.P.M.: 12,500 1/min.

CYLINDER: light alloy cylinder, nikasil plated.

ROAD RACER — with or without pneumatic exhaust valve.

PISTON: aluminium cast piston with coating and one piston ring.

CYLINDER-PISTON CLEARANCE: 0.06 mm
   ± 0.055 mm.

IGNITION UNIT: ROTAX digital ignition unit with 4 different ignition timing curves.

IGNITION TIMING:
   SPARK PLUG: NGK R4600 A-105, thread M14 x 1.25.

ELECTRODE GAP: 0.5 - 0.6 mm.

INTAKE/TIME SYSTEM:
   ROTARY VALVE: 224.400 (symmetrical), cut-off section 162°.

ROTOR PEDALE:
   OPEN: 48.5 mm = 138° T.D.C. / 1.28 mm = 80° after T.D.C. / ROAD RACER closes: 28.3 mm = 85° after T.D.C.

CARBURATOR: Del'Orto flat slide carburettor VHSB 38.

FUEL PUMP: Mikuni DF 44-18.

FUEL: SUPER-gasoline leaded, octane number not below ROZ 95.

LUBRICATION (ENGINE): CASTROL A747, mixing ratio 1:25 (4%).

LUBRICATION (GEARBOX): 0.80 hr ISO VG 100 l/hr operation in KART, 0.10 hr ISO VG 100 (l/hr operation
   on test bench).

COOLING SYSTEM: watercooled, integrated water pump for circulation of coolant, closed water circuit.

COOLANT TEMPERATURE: mm: 45 degrees C (113 F), nominal: 55 degrees C (131 F), max. 65
   degrees C (149 F) measured at water outlet of cylinder.

CLUTCH: Dry multi-plate.

GEAR SHIFTING: Left or right side shifting, neutral between 1st and 2nd speed.

TRANSMISSION: 6-speeds-carrige gearbox, constant mesh, dog engagement, rotary change. By
   means of the cartridge gearbox it is possible to change gear ratios quickly. Choice between 9 x 1 st
   speed, 9 x 2 nd speed, 4 x 3 rd speed, 3 x 4 th speed, 5 x 5 th speed, 5 x 6 th speed.

STANDARD GEAR RATIO: KART 1 st speed (33:15)
   1 st speed (25:14) 1.786, 2nd speed (26:17)
   1.529, 3rd speed (23:17) 1.353, 5th speed (23:16)
   1.25, 6th speed (21:18) 1.167, ROAD RACER 1 st
   speed (32:12) 2.667, 2nd speed (29:15) 1.933, 3rd
   speed (22:14) 1.571, 4th speed (23:17) 1.353, 5th
   speed (21:18) 1.222, 6th speed (21:19) 1.105.

PRIMARY GEAR RATIO: KART 171 241 2.958, ROAD
   RACER 174 211 3.524.

OVERALL REDUCTION RATIO: KART 1 st speed
   6.608, 2nd speed 5.283, 3rd speed 4.525, 4th speed
   4.002, 5th speed 3.698, 6th speed 3.451, ROAD
   RACER 1 st speed 9.397, 2nd speed 8.613, 3rd speed
   7.57, 4th speed 6.768, 5th speed 5.307, 6th speed
   3.895.

STANDARD SPROCKET: 18 teeth, also available
   sprockets with 14. 15. 16. 17. 19 and 20 teeth.

DIMENSION OF CHAIN: 0.6 x 0.6 x 8.51 mm.

EXHAUST SYSTEM: ROTAX exhaust system as per
   drawing KART-VSK 153, ROAD RACER VSX 154.

DRI: WEIGHT: 19 kg, with carburettor, without
   muffler.
APPENDIX G
ENGINE SPECIFICATIONS

ROTAX 250cc MOTORS FOR 1990

Those ever-helpful Rotax specialists, Andover Norton in Hampshire, have rushed us details of the latest 1990 Rotax type 256 "Road Racer" and "Kart" motors.

1) New rotary valve cover with low-friction coating to reduce wear.
2) For better reliability, a new crankshaft is introduced, with the following improvements: a) new connecting rod with "T" cross-section, improved surface finish and reinforced small-end eye; b) crankpin with low temperature hardening treatment; c) new big-end needle bearing with higher load capacity; d) thrust washers with better surface finish.
3) Crankshaft end float to be controlled by thrust washers only (no shims) available in a wider range of thicknesses.
4) New piston, with reinforcement around the boost-port cutaway.
5) Improved stronger clutch drum and clutch hub.
6) New spark plug with longer insulator body and new spark plug cap.

DESIGN NUMBERS: 30.256.1212 ROAD RACER with cartridge gearbox and digital ignition unit: 30.256.1312 ROAD RACER with exhaust valve, cartridge gearbox and digital ignition operation; 30.256.1412 KART with cartridge gearbox and digital ignition unit: 30.256.1512 KART with exhaust valve, cartridge gearbox and digital-ignition unit.

DESCRIPTION: ROTAX twin-cylinder, two-stroke, rotary valve engine, oil-in-fuel lubrication, water-cooled, with integrated water pump, cartridge gearbox, digital ignition.

BORE: 54 mm (2.126 inch).
STROKE: 54.5 mm (2.145 inch).
DISPLACEMENT: 248.6 cm³ (15.233 inch³).
COMPRESSION RATIO: theoretical: 13.8 ± 0.3, effective: 7.1.
COMBUSTION CHAMBER VOLUME: 9.75 ± 0.4 cm³.
PISTON CENTRE PROJECTING OVER CYLINDER TOP: 2.05 mm ± 0.1 mm.
RECOMMENDED OPERATION R.P.M.: 12,500 r.p.m.
CYLINDER: 2 light alloy cylinders, Nikasil plated, with or without pneumatic exhaust valve.
PISTON: aluminium-cast piston with coeung and one piston ring.
PISTON/CYLINDER CLEARANCE: 0.060-0.085 mm.
IGNITION UNIT: ROTAX digital ignition unit with 4 different ignition timing curves.
IGNITION TIMING: 3.75 mm ± 0.1 mm = 27° before T.D.C. between 5,000 and 7,000 r.p.m.
SPARK PLUG: 2 x NGK R 4630 A-10, thread M14 x 1.25.

ELECTRODE GAP: 0.5-0.6 mm.
INTAKE TIMING SYSTEM: rotary valve.
ROTARY VALVE: 224.402 (asymmetric), cut-off section 170° at diameter, 100 mm (3.94 inch), edge via centre = opening edge for ROAD RACER 224.400 (asymmetric), cut-off section 152° for KART.
ROTARY VALVE TIMING: opens: 48.5 mm (1.909 inch) = 136° before T.D.C.; closes: 28.3 mm (1.114 inch) = 85° after T.D.C. for ROAD RACER: opens: 48.5 mm (1.909 inch) = 136° before T.D.C.; closes: 25.8 mm (1.016 inch) = 80° after T.D.C. for KART.
CARBURETTOR: 2 x Dell’Orto flat side carburettor VHSB 36.

LUBRICATION – ENGINE: CASTROL A747 mixing ratio 1:25 (4%).
LUBRICATION – GEARBOX: 0.65 ltr (0.172 gal) ISO VG 100 (at operation in event), 0.64 ltr (0.25 gal) ISO VG 100 (at operation on test bench).
LUBRICATION – PRIMARY DRIVE: 0.20 ltr (0.053 gal) ISO VG 100.
COOLING SYSTEM: watercooled, integrated water pump for circulation of water, closed water circuit.
COOLANT RATE: 55 (mm. 14.5 gals/min).
COOLANT TEMPERATURE: min: 45 degrees C (113 F), nominal: 55 degrees C (131 F), max: 65 degrees C (149 F), measured at water outlet of cylinder.
CLUTCH: dry multi-plate.
GEAR SHIFTING: left side shifting at configuration with cartridge gearbox.

TRANSMISSION: 6-speed cartridge gearbox, constant mesh, dog engagement, rotary change. By means of the cartridge gearbox it is possible to change gear ratios in a short time. Choice between 9 x 1st speed, 9 x 2nd speed, 4 x 3rd speed, 3 x 4th speed, 5 x 5th speed, 5 x 6th speed.


PRIMARY REDUCTION RATIO: (58.22 1.836.


STANDARD SPROCKET: 17 teeth, also available sprockets with 14, 15, 16, 18, 19 and 20 teeth for ROAD RACER, 15 teeth for KART.

DIMENSION OF CHAIN: 9 x 1.4 x 0.16.

EXHAUST SYSTEM: per drawing VSK 153.

DRIY WEIGHT: approx. 29 kg (without mufflers and carburettors).
APPENDIX H
DETERMINATION OF WHEEL INERTIA

The inertia of an object can be found several ways, depending on the equipment available. Presented are two accurate methods for determining inertia of wheels. The first method involves accelerating the tire - wheel combination from rest by a steady torque. From the developed force and torque equations, the inertia is found. In the second method, the period of oscillation is timed by connecting a weight and spring to the wheel and setting the system into oscillation with an initial force.

To employ the first method, the friction in the bearings must first be determined. The procedure is described as follows. The wheel is rotated about its axis of rotation on a stationary stand. The wheel is allowed to decelerate due to friction in the bearings only. During deceleration, the rate is measured. The deceleration rate is determined using a stroboscope to record the rotational velocity at different times. The test is started by rotating the wheel with a steady torque (string wrapped around the wheel and pulled steadily) and the rotational velocity recorded at ten second intervals. The time interval and rpm are recorded. From the data, a plot is drawn and the slope determined. Once the bearing friction is known, a mass of known weight is attached to the wheel diameter by a string and allowed to drop from rest. The distance is measured against time for two different masses and various distances. The freebody diagram is shown in Figure H.1.
FIGURE H.1  DETERMINATION OF INERTIA, METHOD # 1

Wheel, tire combination

\[ R = \text{Radius} \]
\[ \theta = \text{Rotation} \]
\[ R \frac{d\theta}{dt} = \frac{dx}{dt} \]
\[ R \frac{d^2\theta}{dt^2} = \frac{d^2x}{dt^2} \]

Note: string is assumed inextensible, viscous and aerodynamic torques = 0

For the wheel - tire combination shown of known mass, the sum of the unbalanced forces are:

\[ \Sigma F_{un} = M \frac{d^2x}{dt^2} = Mg - F \quad \text{(lb)} \]

\[ F = Mg - M \frac{d^2x}{dt^2} \quad \text{(lb)} \]

The sum of the unbalanced torques is:

\[ \Sigma T_{un} = I_1 \frac{d\theta}{dt} = FR - T_{fc} \quad \text{(ft lb)} \]

Substitute for \( F \):

\[ I_1 \frac{d^2\theta}{dt^2} = (Mg - M \frac{d^2x}{dt^2}) R - T_{fc} \quad \text{(ft lb)} \]
Multiply by $R$:

$$I_1 R \frac{d^2 \theta}{dt^2} = M g R^2 - M R^2 \frac{d^2 x}{dt^2} - R T_{fc} \quad (\text{ft}^2 \text{ lb})$$

or

$$(I_1 + M R^2) \frac{d^2 x}{dt^2} = M g R^2 - R T_{fc} \quad (\text{ft}^2 \text{ lb})$$

Solving for $d^2 x/dt$:

$$\frac{d^2 x}{dt^2} = \frac{(M g R^2 - R T_{fc})}{(I_1 + M R^2)} \quad (\text{ft} s^2)$$

Multiply by $dt$, and let $C = \frac{(M g R^2 - R T_{fc})}{(I_1 + M R^2)}$, and integrate both sides:

$$\int d (dx/dt) = \int C \, dt$$

where initial conditions are: $x(0) = 0$, $dx/dt = 0$; yields

$$\frac{dx}{dt} = Ct + c_1$$

multiply by $dt$ and integrate again:

$$x = \frac{Ct^2}{2} + c_1 t + c_2 \quad (\text{ft})$$

From the initial conditions: $c_2 = 0$. Thus the derivative:

$$\frac{dx}{dt} = Ct$$
Substituting back into the equation for $x$ yields:

$$ x = Ct^2/2 = \left( M g R^2 - R T_{fc} \right) t^2/2 (I_1 + M R^2) \quad \text{(ft)} $$

Solving for $I_1$:

$$ I_1 = \left( M g R^2 - R T_{fc} \right) t^2/2 x - M R^2 \quad \text{(lb} \cdot \text{ft} \cdot \text{s}^2) \quad \text{(1)} $$

where:

- $t$ = Time of the drop (s)
- $x$ = Distance dropped (ft)
- $M$ = Mass attached to spring (lb f s / ft)
- $T_{fc}$ = Coulombic friction torque (ft lbf)
- $R$ = Radius of attached spring (ft)

The data recorded from the coast down test is plotted for angular velocity $d\theta/dt$ (radians / s) versus time, $t$. A lease squares fit to the data determined the slope. The data is relatively linear indicating the viscous and aerodynamic torques are zero. Because there is no applied continuous torque to the tire-wheel assembly, we have:

$$ \sum T_{un} = I_1 d^2\theta/dt = -T_{fc} \quad \text{(ft lbf)} $$
Where \( \frac{d^2\theta}{dt} \) is the slope of the coast down plot in \( (\text{rad}/\text{s}) \). An iteration is now required to find \( T_{fc} \) using the two equations:

\[
I_1 \frac{d^2\theta}{dt} = -T_{fc} \quad \text{(ft lb)} \quad (2)
\]

\[
I_1 = (Mg R^2 - RT_{fc}) t^2/2 x - MR^2 \quad \text{(lb ft/ft)} \quad (3)
\]

The value for \( I_1 \) found from equation (1) is substituted with \( \frac{d^2\theta}{dt} \) into equation (2) to find \( T_{fc} \). This value is substituted into equation (3) to find a new \( I_1 \), which is compared to the old \( I_1 \). The new \( I_1 \) is substituted back into equation (2) and so on until the old and new values are within three decimal places.

The second method refers to the freebody diagram in Figure H.2. In this method, a spring is attached to the tire-wheel combination and to a fixed point at the other end. The spring constant is known or can be found by measuring its deflection with the two different masses. The measurement is started by rotating the system a slight angle \( \theta \) to stretch the spring. The string must be stretched linearly and the mass at the end must not swing, but travel vertically. The system is set into motion and the number of oscillations are counted and timed. This is repeated with the other mass. The moment of inertia is then calculated as follows.

The equation for the unbalanced torques is given as:

\[
\sum T_{un} = I_2 \frac{d^2\theta}{dt} = FR - T_{fc} - K\theta R^2 \quad \text{(ft lb)}
\]
FIGURE H.2  DETERMINATION OF INERTIA, METHOD # 2

Wheel, tire combination

\[ Kx = KR\theta \]

\[ KxR = K\theta R^2 \]

\[ \text{Spring} \]

\[ \text{Mass} \]

Position and forces:

\[ R = \text{Radius} \]
\[ K = \text{spring constant} \]
\[ \theta = \text{Rotation} \]
\[ R\theta = x \]
\[ R \frac{d\theta}{dt} = \frac{dx}{dt} \]
\[ R \frac{d^2\theta}{dt^2} = \frac{d^2x}{dt^2} \]

Multiply by R, substitute for F, and rearranging:

\[ l_2 R \frac{d^2\theta}{dt^2} = MgR^2 - KxR^2 \]

where \( \frac{d^2x}{dt^2} = R \frac{d^2\theta}{dt^2} \). Simplifying and rearranging to a second order linear differential equation:

\[ (l_2 + R^2 M) \frac{d^2\theta}{dt^2} + KR^2 \theta = MgR - T_{fc} \quad \text{(ft lbf)} \]

Let \( \omega_n^2 = \frac{KR^2}{l_2 + R^2 M} \) and \( \beta = \frac{(MgR - T_{fc})}{(l_2 + R^2 M)} \), we have:

\[ \frac{d^2\theta}{dt^2} + \omega_n^2 \theta = \beta \]
The frequency of oscillation is given as:

\[ f = \frac{\omega_n}{2\pi} = \left( \frac{KR^2}{I_2 + R^2 M} \right)\frac{1}{2\pi} \]  

(Hz)

Solving for \( I_2 \):

\[ I_2 = K R^2 / (2\pi f)^2 - M R^2 \]  

(lbf ft s^2)  \hspace{1cm} (4)

From the proceeding analysis, the moments of inertia were calculated for the vehicles front and rear tire-wheel combinations, and front and rear brakes, and engine. Several measurements were taken and the average of the calculated inertia recorded in Table H.1.

**TABLE H.1, MOMENTS OF INERTIA FOR RACE VEHICLE**

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>SYMBOL</th>
<th>MOMENT OF INERTIA (lbf ft s^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRONT WHEELS / TIRES</td>
<td>I_{FW}</td>
<td>0.00540</td>
</tr>
<tr>
<td>REAR WHEELS / TIRES</td>
<td>I_{RW}</td>
<td>0.00652</td>
</tr>
<tr>
<td>FRONT BRAKES</td>
<td>I_{FB}</td>
<td>0.00230</td>
</tr>
<tr>
<td>REAR BRAKES</td>
<td>I_{RB}</td>
<td>0.00410</td>
</tr>
<tr>
<td>ENGINE</td>
<td>I_{E}</td>
<td>0.00550</td>
</tr>
</tbody>
</table>
APPENDIX I

COMPUTER PROGRAMS
Figure I.1, Flow Chart for Program: Perform

START

READ DATA
1) RUN NUMBER
2) VEHICLE WEIGHT
3) DRAG COEFFICIENT x FRONTAL AREA
4) AVAILABLE ENGINE HORSE POWER
5) AIR DENSITY
6) TIRE AIR PRESSURE
7) DRIVE TRAIN EFFICIENCY
8) VELOCITY INCREMENT

Calculate Power Req’d to Propel Vehicle

Write Output Data

Calculate Maximum Velocity

Done?

Y

N

Increment Velocity

Calculate V, A1, A2

Calculate Drag, Rolling Resistance

Print "Max Velocity"

STOP
$ EDIT PERFORM.FOR:41

1 C PERFORMANCE PREDICTION AND OPTIMIZATION PROGRAM 4 / 90
2 C
3 C
4 C
5 C
6 C
7 C
8 C
9 C
10 C
11 C
12 C
13 C
14 C
15 C
16 C
17 C
18 C
19 C
20 C
21 C
22 C
23 C
24 C
25 C
26 C
27 C
28 C
29 C
30 C
31 C
32 C
33 C
34 C
35 C
36 C
37 C
38 C
39 C
40 C
41 C
42 C

* T W

1 C PERFORMANCE PREDICTION AND OPTIMIZATION PROGRAM 4 / 90
2 C
3 C DONALD KORDS
4 C
5 C THIS PROGRAM PREDICTS VEHICLE PERFORMANCE FROM EXPERIMENTAL DATA. OPTIMAL PERFORMANCE IS GENERATED BY VARYING THE INPUT DESIGN PARAMETERS
6 C
7 C
8 C
9 C
10 C
11 C
12 C
13 C
14 C
15 C
16 C
17 C
18 C
19 C
20 C
21 C
22 C
23 C
24 C
25 C
26 C
27 C
28 C
29 C
30 C
31 C
32 C
33 C
34 C
35 C
36 C
37 C
38 C
39 C
40 C
41 C
42 C

INPUT VARIABLES

12 C RUN = PROGRAM RUN NUMBER
13 C W = TOTAL WEIGHT OF VEHICLE = WD+WF+WV (LBS)
14 C WO = DRIVER WEIGHT (LBS)
15 C WF = FUEL WEIGHT (LBS)
16 C WV = VEHICLE WEIGHT. DRY (LBS)
17 C G = GRAVITY (32.17 FT/S**2)
18 C CDA = DRAG COEFFICIENT * FRONTAL AREA(FT**2)
19 C HP = AVAILABLE ENGINE HORSEPOWER
20 C A = FRONTAL AREA (FT**2)
21 C ROW = AIR DENSITY (SLUG/FT**3)
22 C PT = TIRE INFLATION PRESSURE (PSI)
23 C DTE = DRIVE TRAIN EFFICIENCY
24 C FI = ENGINE FLYWHEEL INERTIA (LBS*FT*S**2)
25 C FWI = FRONT WHEEL AND BRAKE INERTIA (LBS*FT*S**2)
26 C RWI = REAR WHEEL AND BRAKE INERTIA (LBS*FT*S**2)
27 C U = COEFFICIENT OF FRICTION ON SURFACE
28 C RN = TOTAL DRIVE TRAIN RATIO = RP+RT+RO
29 C RP = PRIMARY RATIO
30 C RT = TRANSMISSION RATIO
31 C RO = OUTPUT RATIO (BETWEEN ENGINE AND REAR AXLE)
32 C GD = GRADE ANGLE (DEGREES)
33 C WR = REAR WHEEL DYNAMIC WEIGHT (LB)
34 C FW = FRONT WHEEL RADIUS IN (FT)
35 C RW = REAR WHEEL RADIUS IN (FT)
36 C NV = VELOCITY INCREMENT IN MPH
37 C M = MAXIMUM VELOCITY IN MPH
38 C PR = PRINT OUT ALL DATA, YES OR NO!
39 C
40 C
41 C
42 C

OUTPUT VARIABLES

41 C
42 C

I = DO LOOP COUNTER
FACTOR = TIRE INFLATION FACTOR ( 1<150 FT/S, 2>150 FT/S)
A1 = ROLLING RESISTANCE COEFFICIENT
A2 = ROLLING RESISTANCE COEFFICIENT
RN = TOTAL DRIVE TRAIN RATIO = RP+RT+RO
GN = GEAR VEHICLE IS CURRENTLY IN
NV = VELOCITY INCREMENT (MPH)
V1 = VEHICLE VELOCITY (MPH)
V = VEHICLE VELOCITY (FT/S)
W = TOTAL VEHICLE WEIGHT = WD+WF+WV (LBS)
DR = AERODYNAMIC DRAG (LBS)
RR = ROLLING RESISTANCE (LBS)
GR = GRADE RESISTANCE (LBS)
FT = TOTAL EXTERNAL RESISTIVE FORCE = DR+RR+GR (LBS)
MEQ = VEHICLE EQUIVALENT MASS (LBS*S**2/FT)
TE = ENGINE TORQUE ( FT*LBS)
PMAX = MAXIMUM TRACTIVE FORCE AT REAR WHEELS (LBS)
PE = THRUST AT REAR WHEELS (LBS)
P = TOTAL POWER TO PROPEL VEHICLE AT A GIVEN SPEED (HP)

READ IN DATA

WRITE(6,1)
READ(5,*) RUN
WRITE(6,2)
READ(5,*) WD
WRITE(6,3)
READ(5,*) CDA
WRITE(6,4)
READ(5,*) HP
WRITE(6,5)
READ(5,*) ROW
WRITE(6,6)
READ(5,*) PT
WRITE(6,7)
READ(5,*) DTE
WRITE(6,8)
READ(5,*) NV
WRITE(6,9)
89   READ(5,*) PR
90   WRITE(6,10)
91   READ(5,*) WF
92   WRITE(6,11)
93   READ(5,*) WV
94   WRITE(6,12)
95   READ(5,*) FI
96   WRITE(6,13)
97   READ(5,*) FWI
98   WRITE(6,14)
99   READ(5,*) RWI
100  WRITE(6,15)
101  READ(5,*) U
102  WRITE(6,16)
103  READ(5,*) GD
104  WRITE(6,17)
105  READ(5,*) WR
106  WRITE(6,18)
107  READ(5,*) FW
108  WRITE(6,19)
109  READ(5,*) RW
110  WRITE(6,20)
111  READ(5,*) M
112  1 FORMAT(' ENTER RUN NUMBER (RUN #) ',$)
113  2 FORMAT(' ENTER DRIVER WEIGHT IN LBS (WD) ',$)
114  3 FORMAT(' ENTER VEHICLE DRAG COEFF AREA IN FT**2 (CDA) ',$)
115  4 FORMAT(' ENTER MAX. ENGINE HORSEPOWER (HP) ',$)
116  5 FORMAT(' ENTER AIR DENSITY IN SLUG / FT**3 (ROW) ',$)
117  6 FORMAT(' ENTER TIRE INFLATION PRESSURE IN PSI (PT) ',$)
118  7 FORMAT(' ENTER DRIVE TRAIN EFFICIENCY (DTE) ',$)
119  8 FORMAT(' ENTER VELOCITY INCREMENT IN MPH (NV) ',$)
120  9 FORMAT(' ENTER ALL DATA ? (PR): 1.0=YES, 2.0=NO ',$)
121 10 FORMAT(' ENTER FUEL WEIGHT IN LBS (WF) ',$)
122 11 FORMAT(' ENTER VEHICLE WEIGHT IN LBS (WV) ',$)
123 12 FORMAT(' ENTER FLYWHEEL INERTIA IN LBS*FT*S**2 (FI) ',$)
124 13 FORMAT(' ENTER FRONT WHEEL AND BRAKE INERTIA (FWI) ',$)
125 14 FORMAT(' ENTER REAR WHEEL AND BRAKE INERTIA (RWI) ',$)
126 15 FORMAT(' ENTER SURFACE FRICTION COEFFICIENT (U) ',$)
127 16 FORMAT(' ENTER GRADE ANGLE IN DEGREES (GD) ',$)
128 17 FORMAT(' ENTER REAR WHEEL DYNAMIC WEIGHT IN LBS (WR) ',$)
129 18 FORMAT(' ENTER FRONT WHEEL RADIUS IN FT (FW) ',$)
130 19 FORMAT(' ENTER REAR WHEEL RADIUS IN FT (RW) ',$)
131 20 FORMAT(' ENTER MAXIMUM VELOCITY EXPECTED IN MPH (M) ',$)
132  C
133  C WRITE OUT INPUT DATA
134  C

WRITE(6,*) 'INPUT DATA:
WRITE(6,*) 'RUN # =', RUN
WRITE(6,*) 'WD = (LBS)', WD
WRITE(6,*) 'CDA = (FT**2)', CDA
WRITE(6,*) 'HP = ', HP
WRITE(6,*) 'ROW = (SLUGS/FT**3)', ROW
WRITE(6,*) 'PT = (PSI)', PT
WRITE(6,*) 'DTE = ', DTE
WRITE(6,*) 'NV = ', NV
WRITE(6,*) 'M = ', M
WRITE(6,*) 'PRINT ALL DATA, 1 = YES, 2 = NO', PR
WRITE(6,*) 'WF = ', WF
WRITE(6,*) 'WV = ', WV
WRITE(6,*) 'FI = ', FI
WRITE(6,*) 'FWI = ', FWI
WRITE(6,*) 'RWI = ', RWI
WRITE(6,*) 'U = ', U
WRITE(6,*) 'GD = ', GD
WRITE(6,*) 'WR = ', WR
WRITE(6,*) 'FW = ', FW
WRITE(6,*) 'RW = ', RW
WRITE(6,*) '
C
W=WD+WF+WV
WRITE(6,*) 'W = ', W
WRITE(6,*) '
C
CALL A(W,ROW,CDA,DTE,PT,NV,M,FI,FWI,RWI,GD,WR,FW,RW)
END
C
SUBROUTINE A(W,ROW,CDA,DTE,PT,NV,M,FI,FWI,RWI,GD,WR,FW,RW)
REAL RT(6)
DATA RT/2.143,1.750,1.529,1.353,1.222,1.150/
C VELOCITY DO LOOP
M=M+1
DO 100 I=1,M,NV
VMIN=150.
C CALCULATE VELOCITY VI (MPH)
V1=(I-1)*1.

CALCULATE VELOCITY V2 (FT/Min)

V2=(I-1)*88.

CALCULATE VELOCITY V (FT/S)

V=(I-1)*5280./3600.

CALCULATE TIRE INFLATION FACTORS

IF(V.LT.VMIN) THEN
  FACTOR=1
  A1=.0085+.255/PT
  A2=2.771E-5/PT
  ELSE
  FACTOR=2
  A1=.225/PT
  A2=5.1E-5/PT
  END IF

GRAVITY

G=32.17

PIE (22 / 7)

PIE=3.14286

TRANSMISSION RATIO

rp=3.524

TRANSITION RATIO RT

RO=0.828

CALCULATE REAR TIRE CIRCUMFERENCE IN FT

C=2.*PIE*RW

MINIMUM RPM RMIN

RMIN=10500

MAXIMUM RPM RMAX

RMAX=13000

CALCULATE ENGINE RPM

FS=RMAX*C/(RT(1)*RP*RO)

IF(V2.LE.FS) THEN
  J=1
  ELSE
  END IF

RN=RT(J)*RP*RO

RPM=RN*V2/C

IF(RPM.GT.RMAX) THEN
  J=J+1
  IF(J.EQ.7) THEN
    WRITE(6,*)' MAX RPM IN 6TH GEAR '
    GO TO 150
  ELSE
  END IF
RN=RT(J)*RP*RO
RPM=RN*V2/C
ELSE
END IF
C
CALCULATE AERODYNAMIC DRAG (LB)
DR=ROW*CDA*(V**2)/2.
C
CALCULATE ROLLING RESISTANCE (LB)
RR=(A1+A2*(V**2))*W
C
CALCULATE GRADE RESISTANCE (LB)
GR=W*SIN(GD)
C
CALCULATE TOTAL EXTERNAL RESISTIVE FORCE (LB)
FT=DR+RR+GR
C
CALCULATE VEHICLE EQUIVALENT MASS (LBS*S**2/FT)
C
MEQ=W/G+FI*(((RN/RW)**2)+2*FWI/FW**2+2*RWI/RW**2
C
CALCULATE POWER REQUIRED TO PROPEL VEHICLE (HP)
P=(DR+RR+GR)*V/(550*DTE)
C
P.MAX = MAXIMUM TRACTIVE FORCE
C
TRACTION COEFFICIENT U
U=1.00
C
P.MAX=U*WR
C
CALCULATE VEHICLE PROPELLING THRUST AT REAR WHEELS (LB)
B4=517.40
B3=-155.94
B2=15.41
B1=-0.48
EX=RPM
PE=(B1*(EX**3)+B2*(EX**2)+B3*EX+B4)*RN*DTE/RW
IF(PE.GT.PMAX) THEN
WRITE(6,'PE > P.MAX ')
PE=PMAX
END IF
C
MAXIMUM DISTANCE D = 4 MILES
D=4.*5280.
IF(DX.LT.D) THEN
WRITE(6,'*************************** ')
ELSE
WRITE(6,' MAXIMUM DISTANCE OF 4 MILES EXCEEDED ')
GO TO 150
END IF
WRITE OUT CALCULATED FACTORS

WRITE(6,*), ' DO LOOP COUNTER I = ',I
WRITE(6,*), ' VELOCITY V1 (MPH) = ',V1
WRITE(6,*), ' VELOCITY V2 (FT/MIN)= ',V2
WRITE(6,*), ' VELOCITY V (FT/S) = ',V
WRITE(6,*), ' REAR WHEEL CIRCUMFERENCE C (FT) = ',C
WRITE(6,*), ' TRANSMISSION IN GEAR RT = ',J
WRITE(6,*), ' TOTAL DRIVE TRAIN RATIO RN = ',RN
WRITE(6,*), ' ENGINE RPM = ',RPM
WRITE(6,*), ' FACTOR = ',FACTOR
WRITE(6,*), ' TIRE FACTOR A1 = ',A1
WRITE(6,*), ' TIRE FACTOR A2 = ',A2
WRITE(6,*), ' AERODYNAMIC DRAG DR = ',DR
WRITE(6,*), ' ROLLING RESISTANCE RR = ',RR
WRITE(6,*), ' GRADE ANGLE GD = ',GD
WRITE(6,*), ' GRADE RESISTANCE GR = ',GR
WRITE(6,*), ' TOTAL RESISTIVE FORCE FT = ',FT
WRITE(6,*), ' VEHICLE EQUIVALENT MASS EQM = ',EQM
WRITE(6,*), ' POWER (P) TO PROPEL VEHICLE AT VEL V = ',P
WRITE(6,*), ' PMAX (LBS) = ',PMAX
WRITE(6,*), ' VEHICLE THRUST (PE) IN LBS = ',PE
WRITE(6,*), '

100 END DO
150 END

[EOB]
Figure I.2, Flow Chart for Program: Motion

START

Define function $F(x, v)$

READ DATA
1) RUN NUMBER
2) INITIAL CONDITIONS: $t, x, v$
3) VEHICLE, DRIVER, FUEL WEIGHT, COEF FRICTION
4) DRAG COEFFICIENT $\times$ FRONTAL AREA
5) TIRE AIR PRESSURE, DRIVE TRAIN EFFICIENCY
6) FUEL CONSUMPTION RATE, SHIFT METHOD
7) TIME STEP SIZE, PRINT INCREMENT
8) MAXIMUM DISTANCE

Initialize variables

Write Input Data

Calculate Initial Vehicle Conditions

Calculate Engine RPM, Horsepower
Evaluate $F_e \leq F_{\text{MAX}}$

Determine Shift Point, Select Gear Ratio,
Calculate $C_1, C_2, W$

Determine $AK_1 - AK_4$, $X, XD, XDD$ at station under evaluation

Print?

Y

Write Output Data $t, x, v, A_c$

Done?

N

Increment Time, $T = T + H$

Y

Print Final Values

STOP
THE CLASSIC FOURTH ORDER RUNGE KUTTA METHOD APPLIED TO A VEHICLE IN LONGITUDINAL MOTION.

PROGRAM CALCULATES POWER VS RPM, SELECTS THE PROPER GEAR RATIO WITHIN THE POWER BAND SHIFTS AT MIN OR MAX RPM, AND CALCULATES WEIGHT CHANGES, ETC. THE PROGRAM CALCULATES THE FORCES ON THE MOVING VEHICLE INCLUDING DRAG FORCE, ROLLING RESISTANCE, AND GRADE RESISTANCE.

THIS PROGRAM SOLVES SIMULTANEOUS NONLINEAR DIFFERENTIAL EQUATIONS, WHERE THE INITIAL CONDITIONS ARE KNOWN. IN PARTICULAR A SECOND ORDER NONLINEAR EQUATION WITH VARIABLE COEFFICIENTS IS SOLVED.

INPUT VARIABLES

RUN = PROGRAM RUN NUMBER
W = TOTAL WEIGHT OF VEHICLE = WD+WF+WV (LBS)
WD = DRIVER WEIGHT (LBS)
WF = FUEL WEIGHT (LBS)
WV = VEHICLE WEIGHT, DRY (LBS)
G = GRAVITY (32.17 FT/S**2)
CDA = DRAG COEFFICIENT * FRONTAL AREA(FT**2)
HP = AVAILABLE ENGINE HORSEPOWER
A = FRONTAL AREA (FT**2)
FR = FUEL CONSUMPTION RATE
ROW = AIR DENSITY (SLUG/FT**3)
PT = TIRE INFLATION PRESSURE (PSI)
DTE = DRIVE TRAIN EFFICIENCY
MR = MOMENT OF INERTIA FOR ROTATIONAL PARTS (LBS*FT*S**2)
FI = ENGINE FLYWHEEL INERTIA (LBS*FT*S**2)
FWI = FRONT WHEEL AND BRAKE INERTIA (LBS*FT*S**2)
RWI = REAR WHEEL AND BRAKE INERTIA (LBS*FT*S**2)
U = COEFFICIENT OF FRICTION ON SURFACE
RN = TOTAL DRIVE TRAIN RATIO = RP+RT+RO
RP = PRIMARY RATIO
RT = TRANSMISSION RATIO
RO = OUTPUT RATIO (BETWEEN ENGINE AND REAR AXLE)
PEI = INITIAL POWER (HP)
GD = GRADE ANGLE (DEGREES)
WR = REAR WHEEL DYNAMIC WEIGHT (LB)
FW = FRONT WHEEL RADIUS IN (FT)
RW = REAR WHEEL RADIUS IN (FT)
X = INITIAL VEHICLE DISTANCE IN (FT)
XD = INITIAL VEHICLE VELOCITY IN (FT/S)

OUTPUT VARIABLES

FACTOR = TIRE INFLATION FACTOR ( 1<150 FT/S, 2>150 FT/S)
A1 = ROLLING RESISTANCE COEFFICIENT
A2 = ROLLING RESISTANCE COEFFICIENT
RN = TOTAL DRIVE TRAIN RATIO = RP+RT+RO
GN = GEAR VEHICLE IS CURRENTLY IN
NV = VELOCITY INCREMENT (MPH)
V1 = VEHICLE VELOCITY (MPH)
V = VEHICLE VELOCITY (FT/S)
W = TOTAL VEHICLE WEIGHT = WD+WF+WV (LBS)
DR = AERODYNAMIC DRAG (LBS)
RR = ROLLING RESISTANCE (LBS)
GR = GRADE RESISTANCE (LBS)
FT = TOTAL EXTERNAL RESISTIVE FORCE = DR+RR+GR (LBS)
MEQ = VEHICLE EQUIVALENT MASS (LBS*S**2/FT)
TE = ENGINE TORQUE ( FT*LBS)
PMA = MAXIMUM TRACTIVE FORCE AT REAR WHEELS (LBS)
PE = THRUST AT REAR WHEELS (LBS)
P = TOTAL POWER TO PROPEL VEHICLE AT A GIVEN SPEED (HP)

REAL RT(6)
DATA RT/2.143,1.750,1.529,1.353,1.222,1.150/

EQUATION

F(X,XD)=(C1/XD-C2*XD*XD-(A1+A2*XD*XD)*W)/MEQ

READ IN INPUT DATA

WRITE(6,2)
FORMAT( ' ENTER X,XD,WV,WD,WF,CDA,FR,H,DTPR,TMAX,DMAX = '.$,)
READ(5,*) X,XD,WV,WD,WF,CDA,FR,H,DTPR,TMAX,DMAX
WRITE(6,3)
FORMAT( ' ENTER PT,FW,RW,ROW,DTE,GD,WR,FI,FWI,RWI,RUN = '.$,)
READ(5,*) PT,FW,RW,ROW,DTE,GD,WR,FI,FWI,RWI,RUN
WT=WD+WV+WF
WRITE(6,*) 'INITIAL WEIGHT = ',WT
WRITE(6,*) ' TIME DISPLACEMENT VELOCITY ACCELERATION W

124
89     T = 0
90     TPR = DTPR
91     C
92     PE = PEI
93     W = WT - FR*T
94     C2 = ROW*CD/2.
95     C1 = 500.*DTE*PE
96     VMIN = 150.
97     C
98     CALCULATE VELOCITY V (FT/S)
99     V = XD
100    C
101    CALCULATE VELOCITY V1 (MPH)
102    V1 = XD*3600./5280.
103    C
104    CALCULATE VELOCITY V2 (FT/MIN)
105    V2 = XD*60.
106    C
107    CALCULATE TIRE INFLATION FACTORS
108    IF (V.LT.VMIN) THEN
109    FACTOR = 1
110    A1 = .0085+.255/PT
111    A2 = 2.771E-5/PT
112    ELSE
113    FACTOR = 2
114    A1 = .225/PT
115    A2 = 5.1E-5/PT
116    END IF
117    C
118    GRAVITY
119    G = 32.17
120    C
121    PIE (22 / 7)
122    PIE = 3.14286
123    C
124    TRANSMISSION RATIO
125    C
126    PRIMARY RATIO RP
127    RP = 3.524
128    C
129    RT = TRANSMISSION GEAR CHANGE RATIOS LISTED IN DATA STATEMENT
130    C
131    OUTPUT RATIO RO
132    RO = 0.828
133    C
134    CALCULATE REAR TIRE CIRCUMFERENCE IN FT
135    C = 2.*PIE*RW
136    C
137    MINIMUM RPM RMIN
138    RMIN = 10500
139    C
140    MAXIMUM RPM RMAX
141    RMAX = 13000
142    C
143    CALCULATE ENGINE RPM
144    FS = RMAX*C/(RT(1)*RP*RO)
145    IF (V2.LE.FS) THEN
146    J = 1
147    ELSE
148    END IF

125
RN=RT(J)*RP*RO
RPM=RN*V2/C
IF(RPM.GT.RMAX) THEN
  J=J+1
  IF(J.EQ.7) THEN
    WRITE(6,*),' MAX RPM IN 6TH GEAR'
    GO TO 150
  END IF
ELSE
  END IF
ENDIF
RN=RT(J)*RP*RO
RPM=RN*V2/C
ELSE
ENDIF
C CALCULATE VEHICLE EQUIVALENT MASS (LBS*S**2/FT)
MEQ=W/G+FI*((RN/RW)**2)+2*FWI/FW**2+2*RWI/RW**2
AK1=H*F(X, XD)
AK2=H*F(X+H/2.*XD, XD+AK1/2.)
AK3=H*F(X+H/2.*(XD+AK1/2.), XD+AK2/2.)
AK4=H*F(X+H*(XD+AK2/2.), XD+AK3)
X=X+H*(XD+(AK1+AK2+AK3)/6.)
XD=XD+(AK1+2.*AK2+2.*AK3+AK4)/6.
XDD=(C1/XD-C2*XD*XD-(A1+A2*XD*XD)*W)/MEQ
C CALCULATE AERODYNAMIC DRAG (LB)
DR=ROW*CDA*(XD**2)/2.
C CALCULATE ROLLING RESISTANCE (LB)
RR=(A1+A2*(XD**2))*W
C CALCULATE GRADE RESISTANCE (LB)
GR=W*SIN(GD)
C CALCULATE TOTAL EXTERNAL RESISTIVE FORCE (LB)
FT=DR+RR+GR
C CALCULATE POWER REQUIRED TO PROPEL VEHICLE (HP)
P=(DR+RR+GR)*XD/(550*DTE)
C PMAX = MAXIMUM TRACTIVE FORCE
C TRACTION COEFFICIENT U
U=1.00
PMAX=U*WR
C CALCULATE VEHICLE PROPELLING THRUST AT REAR WHEELS (LB)
B4=517.40
B3=-155.94
B2=15.41
B1=-0.48
EX=EX
PE=(B1*(EX**3)+B2*(EX**2)+B3*EX+B4)*RN*DTE/RW
IF(PE.GT.PMAX) THEN
  WRITE(6,*),' PE > PMAX'
PE = PMAX
END IF
C MAXIMUM DISTANCE D = 4 MILES
D = 4. * 5280.
IF(X.LT.DMAX) THEN
WRITE(6,*) '*******************************
ELSE
WRITE(6,*) ' MAXIMUM DISTANCE EXCEEDED'
GO TO 150
END IF
T = T + H
IF(ABS(T-TPR) - .005) 5, 5, 4
WRITE(6,6) T, X, XD, XDD, W
F0RMAT(1H , F6.2, 3X, F7.2, 4X, F8.2, 4X, F8.2, 3X, F7.2)
TPR = TPR + DTPR
IF(ABS(T-TMAX) - .005) 7, 7, 4
STOP
150 END
END
Figure I.3, Flow Chart for Program: Curve

START

Define function $F(x, v)$

Define path of curve $R_c$

READ DATA
1) RUN NUMBER
2) INITIAL CONDITIONS: $t, x, v, g_{MAX}$
3) VEHICLE, DRIVER, FUEL WEIGHT, COEF FRICTION
4) DRAG COEFFICIENT x FRONTAL AREA
5) TIRE AIR PRESSURE, DRIVE TRAIN EFFICIENCY
6) FUEL CONSUMPTION RATE, SHIFT METHOD
7) TIME STEP SIZE, PRINT INCREMENT
8) CORNER LENGTH, INSIDE / OUTSIDE RADIUS, $T$

Initialize variables

Write Input Data

Calculate Initial Vehicle Conditions, Braking Distance, Corner Entrance Speed

Calculate Engine RPM, Horsepower Evaluate $F_t \leq F_{t_{MAX}}$

Determine Shift Point, Select Gear Ratio, Calculate $C_1, C_2, W$

Determine $A_{K1} - A_{K4}$, $X, XD, XDD$ at station under evaluation

Print?

Write Output Data $t, x, v, A_c, R, g$

Done?

Increment Time, $T = T + H$

Print Final Values

STOP
THE CLASSIC FOURTH ORDER RUNGE KUTTA METHOD APPLIED TO A VEHICLE IN CURVILINEAR MOTION. PROGRAM CALCULATES POWER VS RPM, SELECTS THE PROPER GEAR RATIO WITHIN THE POWER BAND, SHIFTS AT MIN OR MAX RPM, AND CALCULATES DISTANCE, VELOCITY, ACCELERATION, TIME, WEIGHT CHANGES, ETC. THE PROGRAM CALCULATES THE FORCES ON THE MOVING VEHICLE INCLUDING DRAG FORCE, ROLLING RESISTANCE, AND GRADE RESISTANCE.

This program solves simultaneous nonlinear differential equations, where the initial conditions are known. In particular a second order nonlinear equation with variable coefficients is solved.

Input variables

Run = program run number
W = total weight of vehicle = WD+WF+WV (LBS)
WD = driver weight (LBS)
WF = fuel weight (LBS)
WV = vehicle weight, dry (LBS)
G = gravity (32.17 FT/S**2)
CDA = drag coefficient * frontal area (FT**2)
HP = available engine horsepower
E1-11 = empirical coefficients of corner
FR = fuel consumption rate
ROW = air density (SLUG/FT**3)
PT = tire inflation pressure (PSI)
DTE = drive train efficiency
ARCL = arc length of corner (FT)
FI = engine flywheel inertia (LBS*FT*S**2)
FWI = front wheel and brake inertia (LBS*FT*S**2)
RWI = rear wheel and brake inertia (LBS*FT*S**2)
U = forward coefficient of friction on road surface
RN = total drive train ratio = RP+RT+RO
RP = primary ratio
RT = transmission ratio
RO = output ratio (between engine and rear axle)
PEI = initial power (HP)
GD = grade angle (DEGREES)
44 C WR = REAR WHEEL DYNAMIC WEIGHT (LB)
45 C FW = FRONT WHEEL RADIUS IN (FT)
46 C RW = REAR WHEEL RADIUS IN (FT)
47 C X  = INITIAL VEHICLE DISTANCE IN (FT)
48 C XD = INITIAL VEHICLE VELOCITY IN (FT/S)
49 C UR = LATERAL COEFFICIENT OF FRICTION ON ROAD SURFACE
50 C BA = CORNER BANKING ANGLE (DEGREES)
51 C RAPEX= NOMINAL CORNER RADIUS AT APEX (FT)
52 C OUTPUT VARIABLES
53 C FACTOR = TIRE INFLATION FACTOR ( 1<150 FT/S, 2>150 FT/S)
54 C A1 = ROLLING RESISTANCE COEFFICIENT
55 C A2 = ROLLING RESISTANCE COEFFICIENT
56 C RN = TOTAL DRIVE TRAIN RATIO = RP+RT+RO
57 C GN = GEAR VEHICLE IS CURRENTLY IN
58 C NV = VELOCITY_INCREMENT (MPH)
59 C V1 = VELOCITY_VELOCITY (MPH)
60 C V = VELOCITY_VELOCITY (FT/S)
61 C W = TOTAL VEHICLE WEIGHT = WD+WF+WV (LBS)
62 C DR = AERODYNAMIC DRAG (LBS)
63 C RR = ROLLING RESISTANCE (LBS)
64 C GR = GRADE RESISTANCE (LBS)
65 C FT = TOTAL EXTERNAL RESISTIVE FORCE = DR+RR+GR (LBS)
66 C MEQ = VEHICLE EQUIVALENT MASS (LBS*S**2/FT)
67 C TE = ENGINE TORQUE ( FT*LBS)
68 C PMAX = MAXIMUM TRACTIVE FORCE AT REAR WHEELS (LBS)
69 C PE = THRUST AT REAR WHEELS (LBS)
70 C P  = TOTAL POWER TO PROPEL VEHICLE AT A GIVEN SPEED (HP)
71 C
72 C REAL RT(6)
73 C DATA RT/2.143,1.750,1.529,1.353,1.222,1.150/
74 C
75 C EQUATION
76 C F(X,XD)=ACCEL(I)
77 ACCEL(1)=(C1/XD-C2*XD**2-(A1+A2*XD**2)*W)/MEQ
78 ACCEL(2)=SQRT(C3-XD**4/RX**2)
79 C READ IN INPUT DATA
80 C
81 2 FORMAT(' ENTER X,XD,WV,WD,WF,CDA,FR,H,DTPR,TMAX = ',$) READ(5,* ) X,XD,WV,WD,WF,CDA,FR,H,DTPR,TMAX
82 3 FORMAT(' ENTER PT,FW,RW,ROW,DTE,GD,WR,FI,FWI,RWI,RUN = ',$) WRITE(6,3) READ(5,* ) PT,FW,RW,ROW,DTE,GD,WR,FI,FWI,RWI,RUN
83 4 FORMAT(' ENTER UR,RAPEX,E1,E2,E3,E4,E5,E6,E7,ARCL,BA,I = ',$) WRITE(6,4) UR,RAPEX,E1,E2,E3,E4,E5,E6,E7,ARCL,BA,I
84 WT=WD+WF+WV

130
WRITE(6,*), 'INITIAL WEIGHT = ', WT
WRITE(6,*), ' TIME DISPLACEMENT VELOCITY ACCELERATION W'
T=0
TPR=DTPR
CB=TAN(BA)
UM=(UR+CB)/(1.-UR*CB)
C3=(UM*G)**2
DIST=E4+E5*RAPEX+E6*ARCL
RCM=E1+E2*RAPEX+E3/(1.-COS(ARCL/RAPEX))
RX=RCM+(10000.-RCM)*(X/DIST)**5
PE=PEI
5  W=WT-FR*T
C2=ROW*CDA/2.
C1=500.*DTE*PE
VMIN=150.
C CALCULATE VELOCITY V (FT/S)
V=XD
C CALCULATE VELOCITY V1 (MPH)
V1=XD*3600./5280.
C CALCULATE VELOCITY V2 (FT/MIN)
V2=XD*60.
C CALCULATE TIRE INFLATION FACTORS
IF(V.LT.VMIN) THEN
FACTOR=1
A1=.0085+.255/PT
A2=2.771E-5/PT
ELSE
FACTOR=2
A1=.225/PT
A2=5.1E-5/PT
END IF
C GRAVITY
G=32.17
C PIE (22 / 7)
PIE=3.14286
C TRANSMISSION RATIO
C PRIMARY RATIO RP
RP=3.524
C RT=TRANSMISSION GEAR CHANGE RATIOS LISTED IN DATA STATEMENT
C OUTPUT RATIO RO
RO=0.828
C CALCULATE REAR TIRE CIRCUMFERENCE IN FT
C=2.*PIE*RW
C MINIMUM RPM RMIN
RMIN=10500
C MAXIMUM RPM RMAX
RMAX = 13000

C
CALCULATE ENGINE RPM
FS = RMAX * C / (RT(1) * RP * RO)

IF(V2 .LE. FS) THEN
  J = 1
ELSE
  END IF

RN = RT(J) * RP * RO
RPM = RN * V2 / C

IF(RPM .GT. RMAX) THEN
  J = J + 1
ELSE
  END IF

IF(J .EQ. 7) THEN
  WRITE(6,*) ' MAX RPM IN 6TH GEAR '
  GO TO 150
ELSE
  END IF

RN = RT(J) * RP * RO
RPM = RN * V2 / C

C
CALCULATE VEHICLE EQUIVALENT MASS (LBS*S**2/FT)
MEQ = W/G + FI*((RN/RW)**2) + 2*FWI/FW**2 + 2*RW/RO**2

AK1 = H*F(X, XD)
AK2 = H*F(X + H/2.*XD, XD + AK1/2.)
AK3 = H*F(X + H/2.*(XD + AK1/2.), XD + AK2/2.)
AK4 = H*F(X + H/2.*(XD + AK2/2.), XD + AK3)

X = X + H*(AK1 + AK2 + AK3)/6.
XD = XD + (AK1 + 2.*AK2 + 2.*AK3 + AK4)/6.

XDD1 = (C1/XD - C2*XD*XD - (A1 + A2*XD*XD)*W)/MEQ
XDD2 = SQRT(C3 - XD**4/RX**2)

IF(XDD1.GT.XDD2) THEN
  XDD = XDD2
  I = 2
ELSE
  XDD = XDD1
  I = 1
END IF

C
CALCULATE AERODYNAMIC DRAG (LB)
DR = ROW*CDA*(XD**2)/2.

C
CALCULATE ROLLING RESISTANCE (LB)
RR = (A1 + A2*(XD**2))*W

C
CALCULATE GRADE RESISTANCE (LB)
GR = W*SIN(GD)

C
CALCULATE TOTAL EXTERNAL RESISTIVE FORCE (LB)
FT = DR + RR + GR
CALCULATE POWER REQUIRED TO PROPEL VEHICLE (HP)

\[ P = (D + R + G) \times X / (550 \times D_{TE}) \]

PMAX = MAXIMUM TRACTIVE FORCE

TRACTION COEFFICIENT U

\[ U = 1.00 \]

PMAX = U \times W

CALCULATE VEHICLE PROPELLING THRUST AT REAR WHEELS (LB)

\[ P_E = (B_1 \times E X^3 + B_2 \times E X^2 + B_3 \times E X + B_4) \times R N \times D_{TE} / R W \]

IF \( PE \) > PMAX THEN

WRITE(6,*), ' PE > PMAX '

PE = PMAX

END IF

MAXIMUM DISTANCE DIST

IF \( X \) < DIST THEN

WRITE(6,*), ' MAXIMUM CORNERING DISTANCE EXCEEDED '

ELSE

WRITE(6,*), ' MAXIMUM CORNERING DISTANCE EXCEEDED '

GO TO 150

END IF

T = T + H

IF ABS (T - TPR - 0.005) > 6, 6, 5

WRITE(6,7), T, X, XD, XDD, W

FORMAT(1H, F6.2, 3X, F7.2, 4X, F8.2, 4X, F8.2, 3X, F7.2)

TPR = TPR + DTPR

IF ABS (T - TMAX - 0.005) > 8, 8, 5

STOP

GO TO 150

END

[EOB]

*QUIT

$
APPENDIX J
VEHICLE EQUIVALENT MASS

The translational motion of the vehicle is coupled to the rotational motion of the components connected to the axles and includes brake discs and hubs, wheels and hubs with tires, and the engine and associated drive train. A change in translational speed will therefore be accompanied by a corresponding change of the rotational speed of the rotating components coupled to the wheels. To account for the effect of inertia of the rotating components on the acceleration performance of the vehicle, the equivalent mass $M_{\text{eq}}$, is formulated giving the effective inertia mass of the vehicle, and is introduced as follows.

Every change in speed of a moving body is opposed by an inertia force which is proportional to the product of the mass of the body and the time rate of velocity change. This force, referred to as the inertia resistance $F_I$, is directed against the vector of acceleration, ideally located at the center of gravity of the vehicle mass $M$. From the energy method, the change in kinetic energy of a body equals the work produced by the external forces:

$$dE = (F_E - \sum F_{\text{Resistive}}) \, ds$$

where:

- $E$ = Kinetic energy of the moving vehicle (Ibf)
- $F_E$ = Tractive force to propel the vehicle (Ibf)
- $\sum F_{\text{Resistive}}$ = Summation of all resistive forces except $F_I$ (Ibf)
- $F_I$ = Inertia resistance of the vehicle (Ibf)
\( I \) = Mass moment of inertia about axis of rotation \( \text{(ft-lb s}^2) \)

\( M \) = Vehicle translatory mass \( \text{(lb f s}^2 / \text{ft}) \)

\( \omega \) = Angular velocity, radians / sec.

\( \omega_d \) = Angular velocity of drive axle, radians / sec.

\( R \) = Rolling radius of tire \( \text{(ft)} \)

\( \varsigma \) = Reduction ratio between drive axle and a particular part

From equilibrium of forces, the relation applies:

\[
F_i = F_E - \sum F_{\text{Resistive}} \quad \text{(lb)} \quad (J2)
\]

The kinetic energy of the vehicle is:

\[
E = \frac{MV^2}{2} + \sum l \omega^2 / 2 \quad \text{(ft-lb)} \quad (J3)
\]

Differentiating this equation:

\[
dE = MV(dV) + \sum l \omega (d\omega) \quad (J4)
\]

Separating \( F_i \) and substituting \( ds / dt = V = R \omega_d \) and the relationship

\[
\omega = \omega_d \varsigma, \quad \text{(1/s)} \quad (J5)
\]

yields:

\[
F_i = (dV / dt) (M + \sum l \varsigma^2 / R^2) \quad \text{(lb)} \quad (J6)
\]
Substituting equation (2) from section 2.1, and combining equations (J2) and (J6):

\[
\frac{dV}{dt} M_{eq} = F_i = \frac{dV}{dt} (M + \sum I \zeta^2 / R^2) \quad (lb \cdot ft) \quad (J7)
\]

The equivalent mass is:

\[
M_{eq} = M + \sum I \zeta^2 / R^2 \quad (lb \cdot ft^2 / ft) \quad (J8)
\]

where the first term represents the translational mass, and the second the sum of the rotational mass. For calculation of the rotating parts, the parts can be divided into two groups; those directly attached to the axles and influenced by the tire radius, and those rotating at engine speed through the reduction ratio \( \zeta \). Equation (J8) can be expanded to the following form:

\[
M_{eq} = M + \sum I_{lw} / R^2 + \sum I \zeta^2 / R^2 \quad (lb \cdot ft^2 / ft) \quad (J9)
\]

where

\[
\sum I_{lw} = \text{Mass (polar) moment of inertia of parts attached to axles} \\
\text{(ft-lb} \cdot s^2) \\
\sum I = \text{Mass moment of inertia of any part rotating at engine speed} \\
\text{with speed ratio with respect to the driving axle} \quad \text{(ft-lb} \cdot s^2)
\]
For the vehicle designed, equation (J9) can be divided further into the inertia of the individual components of: vehicle translational mass, inertial mass due to the engine - drivetrain at gear reduction $R (N)$, inertial mass of the front and rear wheels, and front and rear disc brakes. This is given as:

$$M_{eq} = \frac{W_T}{g} + I_e \left( \frac{R (N)}{R_{RW}} \right)^2 + 2 \frac{I_{FW}}{R_{FW}^2} + 2 \frac{I_{RW}}{R_{RW}^2} + 2 \frac{I_{FB}}{R_{FW}^2} + 2 \frac{I_{RB}}{R_{RW}^2} \quad (lbf \text{ s}^2/ft) \quad (3)$$

where:

- $I_{FW, I_{RW}}$ = Mass moment of inertia of the front and rear wheels (lb ft s$^2$)
- $I_{FB, I_{RB}}$ = Mass moment of inertia of the front and rear brake (lb ft s$^2$)
- $I_e$ = Mass moment of inertia of the engine flywheel (lb ft s$^2$)
- $R_{FW, R_{RW}}$ = Radius of the front and rear wheels (ft)
- $W_D, W_f$ = Weight of driver, fuel (lbf)
- $W_v$ = Weight of vehicle = $W_{FW} + W_{RW}$ (incls engine weight) (lbf)
- $W_T$ = Total weight = $W_D + W_v + W_f$ (lbf)
- $R (N)$ = Gearing of the vehicle

Note that there are two front disc brakes and one rear disc brake for the design presented.