Image motion compensation for an electronic imaging system

James H. Cain

Follow this and additional works at: http://scholarworks.rit.edu/theses

Recommended Citation

This Thesis is brought to you for free and open access by the Thesis/Dissertation Collections at RIT Scholar Works. It has been accepted for inclusion in Theses by an authorized administrator of RIT Scholar Works. For more information, please contact ritscholarworks@rit.edu.
IMAGE MOTION COMPENSATION
FOR AN ELECTRONIC IMAGING SYSTEM

by

JAMES H. CAIN

A design project submitted in partial fulfillment
of the requirements for the Degree of Master of
Science in Mechanical Engineering.

Reviewed: Dr. Wayne W. Walter
Thesis Advisor

Dr. Edward M. Granger
Center of Imaging Science

Dr. Richard G. Budynas

Dr. Bhalchandra V. Karlekar
Department Head

Department of Mechanical Engineering
Rochester Institute of Technology
Rochester, New York
November 1986
Title: Image Motion Compensation for an Electronic Imaging System.

Author: James H. Cain

I, James H. Cain, hereby deny permission to the Wallace Memorial Library of the Rochester Institute of Technology to reproduce by any means the material presented in this thesis, either in part or in whole.

James H. Cain  November 1986
ABSTRACT

The design of an image compensation device is given which is capable of minimizing the in-track image disturbances caused by velocity variations in an optical scanning device. Guidelines for the development of this mechanism are based on known optical principles. Acceptable response capabilities for this device are based on subjective and quantitative image quality standards.

A first order analysis of the device was conducted to specify the performance required to minimize the distortions mentioned above.

The final system specifications take into account actual hardware that is currently available for the typical scanner system presented.
Acknowledgements

As is the case in many endeavors, the success of this project was due to the outstanding assistance of several persons:

To my family and friends for their continued support and encouragement.

To Wayne Walter, RIT Mechanical Engineering Dept. and Edward Granger, RIT Imaging Science Dept. for technical suggestions and guidance.

To Fred Scipione, Eastman Kodak Company, for his insight and technical advice.
Table of Contents

List of Figures ........................................... i
List of Tables ........................................... ii
Nomenclature ............................................ iii
I Introduction ........................................... 1
II Literature Review ..................................... 3
III Overview of the System .............................. 7
IV The Incident Image .................................... 10
V The Optical System and Optical Distortions ........ 15
   A. Optimization ...................................... 15
   B. MTF of the System ................................ 19
   C. Optical Calculations ............................. 22
VI The Compensating Mechanism .......................... 24
   A. Flexural Mounts ................................... 24
   B. The Glass Element ............................... 28
   C. Feedback and Drive System ..................... 29
   D. Mechanical Calculations ....................... 30
VII Optimization and Practicality ...................... 33
VIII Summary of System Specifications Summary .... 34
IX Conclusions and Recommendations ................... 37
   A. Conclusions ..................................... 37
   B. Recommendations ................................ 37
       1. The Optical System .......................... 38
       2. The Mechanical and Drive System .......... 39
X References ............................................ 40

Appendices
   A 1. Derivation of Image Displacement Equation ... A.1
       2. Derivation of Optical Pathlength Variation .. A.2
       3. Comparison of Image Displacement versus
          Tilt Angle to Accepted Equations .......... A.3
       4. Fortran Computer Program ................... A.4
   B 1. Exposure Requirement Calculations ............ B.1
       2. Depth-of-Focus Calculations ............... B.2
   C 1. Explanation of MTF ............................ C.1
       2. Numerical Ray Tracing Program ............. C.4
       3. MTF Profile for an Abberation-Free System C.10
       4. Calculation of MTF versus Tilt ............ C.12
   D Optimization of Parameters ...................... D.1
   E Glossary of Terms ................................ E.1
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a</td>
<td>The IMC Device at a Small Angle of Deflection</td>
<td>2</td>
</tr>
<tr>
<td>1. b</td>
<td>The IMC Device at a Large Angle of Deflection</td>
<td>2</td>
</tr>
<tr>
<td>1. c</td>
<td>The IMC Device in a Typical Scanning System</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Image Motion Compensation in the &quot;Hycam&quot; camera</td>
<td>4</td>
</tr>
<tr>
<td>3.</td>
<td>Image Motion Compensation in a laser scanning system</td>
<td>4</td>
</tr>
<tr>
<td>4.</td>
<td>Typical Flexure Applications</td>
<td>5</td>
</tr>
<tr>
<td>5.</td>
<td>Image Displacement versus Tilt Angle</td>
<td>7</td>
</tr>
<tr>
<td>6.</td>
<td>Block Diagram of the Feedback Control System</td>
<td>8</td>
</tr>
<tr>
<td>7.</td>
<td>Typical Scanner Mechanism</td>
<td>10</td>
</tr>
<tr>
<td>8.</td>
<td>An Image with Uncorrected Velocity Variations</td>
<td>11</td>
</tr>
<tr>
<td>9.</td>
<td>Velocity Error Thresholds at Various Frequencies</td>
<td>13</td>
</tr>
<tr>
<td>10.</td>
<td>MTF Profile for an Abberation-Free System</td>
<td>20</td>
</tr>
<tr>
<td>11.</td>
<td>MTF versus Tilt for the IMC Device</td>
<td>21</td>
</tr>
<tr>
<td>12.</td>
<td>MTF versus Focal Plane location at 40 lp/mm</td>
<td>22</td>
</tr>
<tr>
<td>13.</td>
<td>Sketch of the IMC Device</td>
<td>25</td>
</tr>
<tr>
<td>14.</td>
<td>The Optical Path</td>
<td>26</td>
</tr>
<tr>
<td>15.</td>
<td>System Response Capabilities</td>
<td>36</td>
</tr>
<tr>
<td>A.1</td>
<td>Derivation of Image Displacement versus Tilt</td>
<td>A.1</td>
</tr>
<tr>
<td>A.2</td>
<td>Derivation of the Change in Optical Pathlength</td>
<td>A.2</td>
</tr>
<tr>
<td>A.3</td>
<td>Comparison of Optical Displacement Equations</td>
<td>A.5</td>
</tr>
<tr>
<td>C.1</td>
<td>The Line Spread Function</td>
<td>C.2</td>
</tr>
<tr>
<td>C.2</td>
<td>A Typical MTF Profile</td>
<td>C.3</td>
</tr>
<tr>
<td>D.1</td>
<td>Commercially Available Glass (N versus V-number)</td>
<td>D.2</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Performance Specifications of the Optical System</td>
<td>15</td>
</tr>
<tr>
<td>2.</td>
<td>Practical Limits for Optimal Optical Parameters</td>
<td>18</td>
</tr>
<tr>
<td>A.1</td>
<td>Comparison of Derived Image Displacement Equation to Accepted Equations</td>
<td>A.4</td>
</tr>
<tr>
<td>A.2</td>
<td>Image Displacement and Pathlength Changes</td>
<td>A.7</td>
</tr>
<tr>
<td>C.1</td>
<td>MTF versus Focal Plane Location at 40 lp/mm.</td>
<td>C.9</td>
</tr>
<tr>
<td>C.2</td>
<td>MTF of an Abberation Free System</td>
<td>C.11</td>
</tr>
<tr>
<td>C.3</td>
<td>MTF versus Frequency for various Tilt Angles</td>
<td>C.12</td>
</tr>
</tbody>
</table>
Nomenclature

D  Optical offset distance produced by the IMC device. (inches)
I  Mass Moment of Inertia (oz-in-s^2)
IMC Image Motion Compensation
J_m Inertia of the Motor (oz-in-s^2)
k_T Torsional Spring Rate (oz-in/rad)
MTF Modulation Transfer Function
N Index of Refraction
OPL Optical Pathlength (inches)
\( \pi \) Pi, 3.1415927
SQF Subjective Quality Factor
t Thickness of the glass element (inches)
\( \theta_{\text{glass}} \) light path angle within the glass element, measured from the normal. (radians)
\( \theta_{\text{air}} \) incident light path angle with respect to the normal of the glass element. (radians)
U The half angle produced by the lens with the optical axis. (radians)
V Abbe V-number
Wt Weight (ounces)
I. Introduction

During the transmittance of an image from the object plane to the image plane in a scanner system, several types of disturbances can occur which tend to disturb the final image. The purpose of the compensation mechanism proposed in this paper is to minimize the contribution of disturbances caused by "in-track" velocity variations within the scanner drive system.

The primary component of this mechanism consists of a flat glass element which is placed in the optical path prior to the image plane. During the scanning process, an electrical feedback system monitors the velocity of the scanner and transmits a signal to the Image Motion Compensation (IMC) mechanism, which controls the orientation of the glass element. Tilting of the glass element shifts the image thereby minimizing the amount of distortion at the image plane. Figures 1a and 1b on the next page show the relationship between the tilting of the glass element and the displacement of the optical path. Figure 1c shows the location of the glass element in a typical scanning system.

The ability of the glass element to adjust to velocity variations in the scanner is directly affected by the inertia of the glass element, couplings and drive system (which includes a specialized flexure mounting).

It is the intention of this paper to investigate and optimize those optical and mechanical properties which will produce the desired image motion compensation.
Figure 1a: The IMC Device at a Small Angle of Tilt

Figure 1b: The IMC Device at a Large Angle of Tilt

Figure 1c: The IMC Device in a Typical Scanning System
II. Literature Review

The IMC device presented in this paper is based on the optical principal that the path of a light ray can be offset by passing through a tilted plane glass element. The basic theory for this offsetting is presented in Appendix A of this paper and may also be found in such sources as Modern Optical Engineering by W. J. Smith (p 82-84), Fundamentals of Optics by F. A. Jenkins and H. E. White (p 28) and Practical Optics by W. P. Ewald, et.al. (p165). These sources also identify the optical distortions produced by the addition of the glass element into the optical path.

A practical application for a similar device is the Wollensak high speed motion picture camera. Here, a multi-sided rotating prism is used to keep the image aligned with a film traveling at thousands of frames per second. The "Hycam" high speed motion picture camera manufactured by Red Lake Labs utilizes the same principal. Figure 2 on the next page details the "Hycam" mechanism. In addition, an early version of the Eastman Kodak high speed camera utilized a parallel surface glass block in combination with a split circular member to function as a shutter. This approach is detailed in Laboratory Instruments by A. Elliott and J. Dickson (p 383).

Other practical means of IMC have been achieved by utilizing a first-surface mirror within the optical path and controlling its position by a feedback response from the
Figure 2: Image Motion Compensation in the "Hycam" camera
(Sketch by E. Granger, Inst. Optics course material)

input device. United States patent 4,453,170 assigned to
Canon Kabushiki Kaisha, Tokyo, Japan utilizes such a device
to compensate for vibrations and pitch variations within a
laser scanning system (see Figure 3 and reference 5). NASA
engineers (K.R.Lorell, et. al.) are also utilizing a steerable
first-surface mirror with a CCD as a means of IMC for the
space shuttle infrared telescope facility (SIRTF). See
reference 6, for more details.

Figure 3: Image Motion Compensation in a laser scanning system
(Sketch taken from U.S. Patent Disclosure 4,453,170)
Based on these optical principles and supported by several workable systems currently being used for similar purposes, the development of the IMC device with a plane glass element seems practical. Possibly the greatest concern in addition to the optical considerations, will be the ability of the mechanism to respond to variations which contribute to image motion induced distortions. An inertially light and responsive (i.e. stiff) system is required with minimal start-up lag and overshoot. Because of this, conventional bearings will not meet performance requirements. Consequently, a mounting structure consisting of flexures will be used (see Figure 4 below for typical flexure applications). Applications for flexure systems can also be found in *Flexure Devices* by P. J. Geary and in the references listed in the second chapter of this book, some of which date back to as early as 1899. Additional information is also available from the Fluid Power Division of Bendix Aerospace, manufacturers of flexural pivots.

![Figure 4: Typical Flexure Applications (sketches taken from manufacturer's literature)](image-url)
Finally, the quality of the resulting image must be evaluated. One measure of the image quality is the MTF (Modulation Transfer Function). The importance of evaluating the MTF is outlined in Appendix C. Applications for estimating MTF can be found in *Modern Optical Engineering* by W. J. Smith (p 314). The numerical ray tracing program in Appendix C utilizes the second moment method for calculating the MTF.

Another measure of image quality is the SQF (Subjective Quality Factor), which introduces the ability of the human eye to resolve an image. Development and additional information on the second moment method, MTF and SQF can be found in numerous papers by Edward W. Granger, published by SPIE (the Society of Photo-Optical Instrumentation Engineers). See references 8 and 9 for details.
III. Overview of the System

The device proposed in this paper is based on the physical principle that the path of an optical ray can be altered by placing a section of plane glass into the path. The amount of displacement in the optical path is dependent upon the angle of the glass element with respect to the incoming ray ($\theta_{air}$), the index of refraction ($N$), and thickness of the glass ($t$) (See Figure 5 and Appendix A for development of this concept).

![Diagram of image displacement](image)

Image Displacement = $t \times (\sin \theta_{air} \cdot \tan \theta_{glass} \cdot \cos \theta_{air})$

Figure 5: Image Displacement versus Tilt Angle

By incorporating the glass element into an electro-mechanical servo system, the glass may be tilted proportionally to an error signal from an encoding device mounted to an optical scanning system. As shown above, the tilting of this glass element results in a displacement of the optical path at the image plane.

For the purpose of this paper, the receiving device at the image plane will be a linear charge-coupled device (CCD)
commonly used in electronic imaging applications. The alternative would be to consider a moving film or photoreceptor. In general, the clocking accuracy of the CCD element (typically rated near ±0.05%) versus the velocity uniformity of a moving film (estimated to be ±1% to ±10%) is several orders of magnitude in difference. Consequently, the amount of image distortions introduced by the CCD are imperceptable compared to those of most film systems.

Figure 6 shows a block diagram of the feedback control system required to drive the compensating mechanism. Note that the CCD is considered a perfect receiver and does not require input to the error calculation as the film drive system does. The response time of the system must be optimized to allow correction of the image as it approaches the final image plane. To accomplish this, a low inertia mechanism will be designed with the appropriate electrical drive system.

![Figure 6: Block Diagram of the Feedback Control System](image-url)
The acceptability of the system is based on subjective, as well as quantitative image quality standards. Obviously the accuracy required for any system is a tradeoff between customer expectations, application and cost. An example of this is the difference between the requirements for a commercial billboard display and those of a detailed letterhead for business stationery. While a 1/8" image displacement would scarcely be noticeable on a billboard display, such registration would be rejected for the letterhead (and most materials read at arms length). For the purpose of this paper, the object-to-image registration requirement will be 0.005" max. (approximately). This requirement relates the amount of image displacement tolerated in the final image as compared to the original document. The 0.005" value chosen is based upon limitations of the human visual system for recognizing image displacement.

The optical parameters outlined here will be expanded in the following section in detail.
IV. The Incident Image

The incident image to the Image Motion Compensation (IMC) device is produced by a scanning mechanism (similar to the device shown in Figure 7 below). The velocity uniformity of the scanner is dependent upon each element of the drive system. Such factors as motor control, timing belt tolerances, shaft runout, system inertia, acceleration and frictional effects all contribute to the nonuniformity of the scanner velocity. The content of the image is provided by the original document placed on the platen at the object plane. For the purpose of this paper, discrete black and white pattern documents, such as text and line art will be emphasized, although the IMC device is capable for correction in color applications as well.

Comparison of many scanners in the market today indicates that a majority are driven by "periodic disturbance systems" such as timing belts, cables and pulleys and other rotary means. Because of this, most of the velocity disturbances appear as periodic or repeating disturbances. By assuming the velocity variations are sinusoidal in nature, the amplitude and frequency of velocity disturbances which cause objectional displacements in the image can be identified.

Figure 7: Typical Scanning Mechanism (with IMC device)
Figure 8 below shows the effect of uncorrected scanner velocity variations caused by the "periodic disturbance systems" mentioned on the previous page. The IMC device developed in this paper can substantially reduce the distortion shown in the lower illustration.

Figure 8: Image Distortion due to a Sinusoidal Velocity Variation
The following derivation identifies the minimum velocity error (based on frequency) required to produce a given image displacement.

Let the velocity of the lead mirror of the scanner = \( V_{\text{scan}} \)
where \( V_{\text{scan}} \) equals some steady velocity \( (V_0) \) plus an additional sinusoidal variation of magnitude \( \Delta V \) as a function of the frequency of the disturbance (f) and time (t):

\[
V_{\text{scan}} = V_0 + \Delta V \sin (2\pi ft).
\]  
(1)

By integrating from time zero to an arbitrary time T, an expression for the position of the scanner is obtained:

\[
X_{\text{scan}} = \int_0^T V_0 \, dt + \int_0^T \Delta V \sin(2\pi ft) \, dt.
\]
(2)

which reduces to:

\[
X_{\text{scan}} = V_0 \, T + \frac{\Delta V}{2\pi f} (-\cos(2\pi fT) + 1).
\]

Here \( V_0 \, T = X_0 \), the desired position of the image. By comparing this location to the actual image location \( X_{\text{scan}} \), an expression for the image displacement (\( \Delta X \)) is found:

\[
X_{\text{scan}} - X_0 = \Delta X = \frac{\Delta V}{2\pi f} (1 - \cos(2\pi fT)).
\]
(4)

Finally, by rearranging the previous expression, an equation is obtained for the amplitude of the velocity error as a function of the frequency of the disturbance:

\[
\Delta V = \frac{2\pi f \Delta X}{1 - \cos(2\pi fT)}
\]
(5)

for which the minimal case occurs when \( \cos (2\pi fT) = -1 \), yielding:

\[
\Delta V_{\text{min}} = \pi f \Delta X.
\]

Figure 9 on the following page shows the minimum velocity error threshold at various temporal frequencies for image displacements of 0.005" and 0.015".
By compensating to keep velocity errors below the limits shown in Figure 9, image displacement can be reduced. It is important to note that the maximum image displacement (ΔX) shown above is for the uncorrected image and that the compensation device will reduce this value according to its ability to react at various frequencies. As will be shown later, this allows even greater variations to occur within the scanner drive system while still maintaining the image registration required at the image plane.

The natural frequency of the system must be at least 60 Hz to accommodate those frequencies most likely to produce optical distortions. As shown in Figure 9, variations of nearly 1.885 in/sec can be sustained at 120 Hz without reaching the 0.005" registration requirement. This represents a velocity error in the scanner drive system of nearly 24%. Even the most simple
drive system is unlikely to produce such large fluctuations at high frequencies. By designing a system that is responsive at these higher frequencies, the standard 60 cycle electrical noise and lower frequencies can be controlled.

Because of the magnification of the lens in the system, only a small correction is required after the lens to correct for a large disturbance in the scanning velocity. For example, an optical system at 1/10 magnification would only require an offset of 0.010" for a disturbance which effectively moves the object 0.100". This capability will be used to reduce the total tilt required by the glass element.
V. **The Optical System and Optical Distortions**

A. Optimization

Figure 7 on page 10 shows the type of scanning system for which the IMC mechanism is designed. The object is illuminated and an image is projected via a series of moving mirrors to the lens. The lens focuses onto the CCD array where the optical image is transformed into an electronic signal. Since the purpose of this paper is not to design an optical system, but rather to correct for velocity variations within the optical system, the following assumptions will be made regarding the performance specifications* of the optical system:

<table>
<thead>
<tr>
<th>Performance Specifications of the Optical System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scanner Velocity</strong></td>
</tr>
<tr>
<td><strong>Illumination Power</strong></td>
</tr>
<tr>
<td><strong>Lens parameters</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Glass parameters</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>CCD specifications</strong></td>
</tr>
</tbody>
</table>

*The theory presented here is applicable to optical system with various scanner speeds, lens focal lengths, magnifications, etc. These values have been selected for example only, and are typical of the specifications found in many current scanner designs.*
The addition of the glass element between the lens and the CCD element has several effects. With 0.00° of tilt, the glass optically lengthens the distance between the lens and the CCD array. This distance can be calculated and allowances made accordingly (see Ref 1, page 82 and the Optical Calculations section on page 23). As the glass element is tilted, the optical pathlength (OPL) increases. This produces a shift in the plane of best focus for the image. Because this position changes continuously as the glass element tilts, it is necessary to make a compromise in the fixed location of the CCD element (i.e., the fixed location of the imaging plane). The actual location for the CCD image plane is based upon the maximum MTF profile obtainable at various positions. This evaluation will be discussed fully in the MTF section of this report. The derivation for the optical path length is shown in appendix A. In addition, Appendix A shows the relationship for image displacement and optical pathlength as a function of the incident light path angle and the index of refraction of the glass element.

As shown on the previous page, the depth of focus tolerance for this optical system is ± 0.0069 inches (see Appendix B, page B.2 for calculations). By comparison, the maximum change in OPL shown in Appendix A (page A.7) is well below the depth of focus tolerance. If the CCD image plane is located at the middle of the OPL variation, the maximum focus change would be only ± 0.00081 inches, or less than 12% of the allowable tolerance.
By introducing the glass plate into the optical system, the optical pathlength is increased. If not corrected for, spherical aberrations will degrade the resulting image. By relocating the CCD sensor appropriately, spherical aberration in the non-tilted condition can be eliminated (See ref. 1 and the first calculation in the Optical Calculations section for the new sensor location).

Another optical disturbance produced by the tilted glass element is chromatic aberration. The index of refraction for the glass element is a function of the wavelength of the light ray (see Ref. 3, page 23). This difference in index for various wavelengths causes greater angular deflection at the blue end of the spectrum than at the red end. The result is a spectrum of light rays. By utilizing a glass material with a reasonably flat response in the visible light range and/or the thinnest glass possible, chromatic aberration can be minimized. If necessary, color correction of the lens can be utilized to reduce the dispersion caused by chromatic aberrations. It is important that the lens design incorporates the glass element in the IMC device in order to optimize the optical performance of the system.

The MTF calculations performed for this paper include the effects of chromatic aberration at the limiting, or worst case frequencies (red and blue), although it is realized that color correction of the lens can minimize and in some cases eliminate these aberrations. In addition, most of the information content
collected from a black and white object is within the green frequencies and only a fractional amount exists at the limiting red and blue frequencies.

In addition to correcting various disturbances within the optical system, the addition of the glass plate produces several disturbances. As previously mentioned, some effects can be eliminated by relocation of the CCD image plane, and by color correction of the lens. Others (such as spherical aberration) may be minimized by optimizing the f/# of the lens (although the value chosen for this paper will be remain constant due to exposure requirements).

Finally, the parameters of the glass element must be optimized. This will be accomplished by computing a series of numerical ray traces for the total optical system and calculating the Modulation Transfer Function (MTF) versus the tilt angle of the glass element. Practical limitations for the various glass parameters are shown in the table below:

Table 2: Practical Limitations for Glass Parameters
(See manufacturer’s information reference 2)

<table>
<thead>
<tr>
<th></th>
<th>Thickness t (in)</th>
<th>Index of Refraction N</th>
<th>V-number V</th>
<th>Tilt Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.300</td>
<td>1.435</td>
<td>19.5</td>
<td>0.50°</td>
</tr>
<tr>
<td>Maximum</td>
<td>limited by inertia and optical effects</td>
<td>2.096</td>
<td>90.3</td>
<td>12°</td>
</tr>
<tr>
<td>Optimal</td>
<td>as req’d. (&gt; 0.300 )</td>
<td>as req’d.</td>
<td>highest possible</td>
<td>&lt; 10°</td>
</tr>
</tbody>
</table>
Some of the preceding values are easily selected. A high V-number, for example, helps to eliminate chromatic aberrations with no side effects. Values should be as high as possible for the glass material selected (See Figure D.1).

The thickness directly effects all optical aberrations and the total angle of tilt required for a given image offset, as well as the inertia of the system. Compared to the inertia of the motor and the flexure connection however, minor variations in thickness will have little effect on the performance of the system. The minimal thickness shown in Table 2 is based on the ability of the glass element to produce a given offset within a given angular displacement. This value will be known as the $\Delta D / \Delta \theta$ ratio.

B. MTF of the System

The MTF reflects the ability of the system to respond to various frequencies by comparing the image produced to the original object. As the MTF profile approaches zero, the system is less capable of producing a true image at the indicated frequency. If no aberrations are present, the MTF profile is governed by diffraction effects (see ref. 1, p318). Figure 10 on the next page shows the MTF profile for a diffraction limited system with no aberrations and a uniformly transmitting circular aperture.

The introduction of the glass plate with no tilting reduces the MTF by a small amount, although, as mentioned
previously, much of this loss can be regained by color correction of the lens. As the glass plate is tilted, additional losses occur which further reduce the MTF of the system. Numerical values for various optical aberrations can be found in the Optical Calculations at the end of this section (p 22).

Figure 11 shows the effects of tilting on the MTF of the system (see Appendix C for MTF calculations). As is evident from the numerical values for the optical aberrations, chromatic and spherical aberrations and astigmatism contribute greatly to a reduction in the system MTF. Further optimizations of the glass element should be directed at reducing these aberrations with the greatest influence.

As mentioned in Reference 1 (p319), an MTF profile derived from raytrace data will produce results higher than that of Figure 10. Such a profile is incorrect however, since ray tracing only partially describes electromagnetic radiation.
behavior. The MTF curves based on ray-tracing that are shown in Figure 11 are nonetheless adequate as an approximation for comparing the effects that various parameters have on the system.

Finally, Figure 12 shows the MTF profiles at various image planes near the calculated CCD image plane. By optimizing the worst case profile (the off-axis ray with a full 6° tilt at the glass element), the best system performance can be realized. It is apparent that even at the worst case angle of tilt, the system requirements for an MTF of 0.30 @ 40 lp/mm is easily accomplished.

Figure 11: Computed MTF vs. Tilt Angle of the Glass Element
(See Appendix C, page C.12 for Calculated MTF values)
Figure 12: Computed MTF vs. Focal Plane @ 40 lp/mm
(See Appendix C, page C.9 for calculated MTF values)

C. Optical Calculations

The following calculations define those aberrations with the largest effect on the overall system performance.

1. Lengthening of the Optical Path by the Glass Element.
(See Reference 1, p 82).

\[ OPL = t \times \frac{(N - 1)}{N} \]

\[ = 0.46 \times \frac{(1.456 - 1)}{1.456} = 0.144 \text{ inches} \] (7)

2. Optical Abberations produced by the Glass Element.
(See reference 1, p 83). See page iii for nomenclature.

Chromatic Abberation = \( t \times (N - 1) / (N^2 \times V) \)

\[ = 0.46 \times 0.456 / (1.456^2 \times 90.3) \]

\[ = 0.0011 \text{ inches} \] (8)

Spherical Abberation = \( \frac{t \times N \times \cos U}{(N^2 - \sin^2 U)^{3/2}} \)

\[ = \frac{0.460 \times (1 - 1.456 \times \cos(0.05675))}{1.456} \]

\[ = 0.00027 \text{ inches} \] (9)
Astigmatism

\[ t/N^3 \theta^2 (N^2 - 1) \]  
(at maximum \( \theta \))
\[ = \frac{0.46/1.456^3 \times 0.104^2 \times (1.456^2 - 1)}{2*1.456^3} = 0.00181 \text{ inches} \]  

Sagittal Coma

\[ t \times \theta^2 \frac{(N^2 - 1)}{2*N^3} \]  
(at maximum \( \theta \))
\[ = \frac{0.46 \times 0.05675^2 \times 0.104 \times (1.456^2 - 1)}{2*1.456^3} = 0.00002 \text{ inches} \]  

Lateral Chromatic

\[ t \times \theta \frac{(N - 1)}{N \times V} \]  
(at maximum \( \theta \))
\[ = \frac{0.46 \times 0.104 \times (1.456 - 1)}{1.456 \times 90.3} = 0.00016 \text{ inches} \]  

3. Exposure and Power Requirements for the System.

Based on a maximum power constraint for the illumination.

See Appendix B  page B.1

4. Depth of Focus Tolerance for the System.

See Appendix B  page B.2

5. MTF Calculations for the System.

Numerical Ray Tracing FORTRAN Program using the second moment method for calculating MTF.

See Appendix C  page C.1

6. Optimal CCD Image Plane location

Based on OPL calculations and Optimal MTF profiles.

OPL shift (from calculation 1)  = 0.144 inches
Additional shift away from the lens to achieve best focus (see Figure 12).  = 0.001 inches
Total Image Plane Shift (lengthening)  = 0.145 inches
VI. The Compensating Mechanism

The Compensating Mechanism consists of flexure mounting members linked with a direct drive DC motor system (see Figure 13). The flexures were chosen over conventional bearing mounts because of the following attributes:

- Elimination of static friction effects at start-up
- Elimination of backlash and minimization of hysteresis
- Elimination of uneven wear patterns caused by continuous cycling at minimal rotation angles
- High rigidity for accurate locating in all but one degree-of-freedom
- Minimal restoring force required at small angles
- Insensitivity to contamination
- No need for lubrication

Combined with the capabilities of a DC motor, the flexures provide the low inertia and torsional spring rate required to complete the compensation mechanism. By matching the performance of the motor drive to the mechanical hardware, a responsive IMC mechanism can be designed. See Figure 14 for a detailed view of the complete optical path. The following paragraphs will discuss the selection of the flexures and describe the DC motor drive and feedback control system.

A. Flexural Mounts

The flexures chosen for this design are manufactured by the Fluid Power Division of Bendix Aerospace under the name of Bendix Free Flex Flexural Pivots. As is the case for many practical designs, some of the compromises made in the optimization of the IMC mechanism will be due to limitations of the components chosen, in this case, the flexural pivots. Appendix D outlines the priority of the optimization performed.
Figure 14: The Optical Path
Three of the most important considerations for the proper selection of the flexural pivots are:

1. **Torsional Spring Rate.** High stiffness is required to obtain a system which is responsive at high frequencies (60 Hz). This value will directly effect the performance of the DC motor and feedback control system.

2. **Maximum operating angle.** Based on manufacturer's life test data the maximum operating angle for infinite life at a load of less than 1% of the rated maximum is ±6.25 degrees. Although most of the flexures available will provide nearly 30° of rotation in either direction, only a finite life is available at these angles. (See reference 1 under technical literature and manufacturer's information).

3. **Center Shift due to deflection.** As rotation occurs, the central axis of the flexural pivot drops. Limitations of the DC motor requires that this deflection be held to a minimum (0.003") according to the manufacturer (See reference 3 under technical literature and manufacturer's information).

Beginning with the Torsional Spring Rate, an appropriate flexure is selected so that the natural frequency of the system will be high. The intent is to produce a system which is capable of responding to the frequencies most likely to produce image distortions (i.e. frequencies of 60 Hz and less). In terms of vibration analysis, this corresponds to a frequency ratio ($\frac{w_{\text{forced}}}{w_{\text{natural}}}$) less than 1 (see ref. 11). Calculations for these and other values can be found on the last pages of this section (starting on p 30).

The flexural pivot chosen (Series-Type # 5020-600) has an infinite life at angular deflections of 6.25° or less. This angular deflection value is needed for the image offset equation found on page 7. The torsional spring rate for two
pivots is 425.6 in-oz/rad which produces a undamped natural frequency of 67 Hz when mounted to the IMC device. Finally, the maximum center shift of the flexural pivot is calculated to be 0.0006" (based on manufacturer's information). According to the manufacturer's specifications this value is acceptable for the DC motor based on the concentricity required between the stator and rotor elements.

Mounting techniques for the flexural pivots are outlined in the technical information from the manufacturer. A simple set-screw mounting has been selected for this design (See figure 13).

B. The Glass Element

Another mechanical concern is the ability of the glass element to transmit torque between the flexural pivots. Obviously, if the glass is too thin, the torque load applied may overstress the element. Using formulas found in reference 10 (p290), the shear stress in the glass element at the maximum calculated torque is 138 psi. Although this value is acceptable for some glasses, actual testing of the material is necessary to determine acceptability. In the event that the glass chosen (based on optical requirements) does not meet the shear stress requirement, a framework will be constructed to enclose the glass element and carry the shear stress produced by torsional loading. The additional inertia of the framework would produce minor variations in the natural
frequency of the system and a slight increase in the torque requirement.

C. The Feedback and Drive System

Finally, a description of the drive system is warranted here. Although a complete profile is beyond the scope of this paper, a basic description will be provided.

As shown previously in Figure 6, the scanner velocity is monitored by an encoder. The encoded signal is then compared to the absolute velocity required for the magnification chosen, and a modified error signal is sent to the IMC device (i.e. the DC motor), proportional to the error.

A direct drive DC motor was selected for several reasons. First, by eliminating a flexible coupling mount, secondary spring effects (such as ringing) are eliminated. This produces a stiff system capable of responding at the higher frequencies required. Secondly, as mentioned in the manufacturer's literature, direct drive DC motors are ideally suited for high acceleration applications with rapid starts and stops. The short electrical time constant coupled with the stiff system produces a responsive drive system.

Based on the total system inertia (including the motor), the angular acceleration required to compensate for velocity variations in the scanner and the torsional spring constant provided by the two flexures, the maximum torque load for the system is 62.0 oz-in (see calculations beginning on page 30).
This value enables correction for ± 0.015" of image displacement at 60 Hz (see Figure 15). Such a high displacement, however, reflects a scanner velocity error of nearly 36%.

Based on the required torque of 62.0 oz-in, an Inland frameless direct drive DC motor with a peak torque rating of 77.4 in-oz has been selected. This torque rating was chosen from system requirements plus an additional 25% safety factor. This selection is based not only on the ability of the motor to perform at the torque required, but also due to the motor inertia (J_m), and the size of the motor. Other manufacturers, including Honeywell and Magnetic Technologies, also produce similarly acceptable motors.

Specifications for the motor can be found in the system specifications section on page 35.

D. Mechanical Calculations

The following calculations provide the information necessary to determine the total system inertia, the torque requirements and the frequency response of the system.

Inertia of the Glass Element (See Ref. 12).

\[
\text{Volume} = 0.46" \times 0.46" \times 1.50" = 0.3174 \text{ in}^3 \tag{13}
\]

\[
\text{Density (Ref. 12)} = 1.44 \text{ oz/in}^3
\]

\[
\text{Mass} = \text{Density} \times \text{Volume} / g = 0.0011838 \text{ oz-s}^2/\text{in} \tag{14}
\]

\[
\text{Mass Moment of Inertia} = \frac{1}{12} \times \text{mass} \times (0.46^2 + 0.46^2) = 0.0000417 \text{ oz-in-s}^2 \tag{15}
\]

Inertia of the Motor

From manufacturer's specifications = 0.0023 \text{ oz-in-s}^2
Inertia of the Flexures
Approximated by assuming all the mass is located in a cylinder with the dimensions shown below:

\[
\text{Volume} = \left( \frac{\text{Wt. of flexures}}{\text{Density of steel}} \right) = \left( \frac{0.6064 \text{ oz.}}{4.528 \text{ oz/in}^3} \right) = 0.1339 \text{ in}^3
\]  

(16)

\[
\text{Volume} = \pi \times \text{length} \times \left( \text{O.D.}^2 - \text{I.D.}^2 \right) / 4
\]

\[
= 3.1415927 \times 1.000 \times \left( 0.625^2 - 0.469^2 \right) / 4
\]

\[
\text{I.D.} = 0.469 \text{ inches}
\]

(17)

\[
\text{Mass} = \text{Wt.} / g
\]

\[
= 0.6064 \text{ oz} / 386 \text{ in/sec}^2 = 0.00157 \text{ oz-s}^2/\text{in}
\]

(18)

\[
\text{Mass Moment of Inertia} = 0.125 \times \text{Mass} \times \left( \text{O.D.}^2 - \text{I.D.}^2 \right)
\]

\[
= 0.000335 \text{ oz-in-s}^2 \text{ for both flexures}
\]

(19)

Inertia of the Glass Mounting Couplings
Two Simple Cylindrical Mounts

Motor side coupling = 0.0000295
Free side coupling = 0.0000177
Total = 0.000047 oz-in-s^2

Total Inertia for the System

\[
I_{\text{total}} = I_{\text{glass}} + J_m + I_{\text{flexures}}
\]

\[
= 0.000417 + 0.0023 + 0.0000335 + 0.000047
\]

\[
= 0.00242 \text{ oz-in-s}^2
\]

(20)

Natural Frequency of the System (See Ref. 11).

\[
f_n = \frac{1}{2\pi} \left( \frac{k_T}{I_{\text{total}}} \right)^{1/2}
\]

\[
= \frac{1}{2\pi} \left( \frac{425.6}{0.00242} \right)^{1/2}
\]

\[
= 67 \text{ Hz}
\]

(21)

Frequency Response for the System

See System Specifications, page 36.

Torque Requirement for the System

\[
T_{\text{total}} = I_{\text{total}} \times \text{ang. acceleration} + k_T \times \theta_{\text{max}}
\]

\[
= 0.00242 \times 7200 + 425.6 \times 0.105
\]

\[
= 62 \text{ oz-in}
\]

(22)
Shear Stress in the Glass Element (See Ref. 10, page 290) (where the maximum stress is at the midpoint of each side)

\[ S_{max} = 4.8 \times \text{Torque} / t^3 \]
\[ = 4.8 \times \text{Torque} / .46^3 = 50 \times \text{Torque} \] (23)

Using the peak torque generated by the flexures \( k_T \times \theta_{max} = 44.3 \text{ oz-in from Equation 22 on the previous page}, \) the maximum shear stress in the glass element is calculated to be 138 psi. The range of Young's Modulus for glasses of the quality acceptable for the IMC device is from \( 7-11 \times 10^6 \text{ psi} \). The apparent elastic limit for such glasses is approximately \( 5.3 \times 10^3 \text{ psi} \) (see references 4 and 12). These values are given for reference only and the final determination of the suitability of the glass material must be based on the exact material used and tested loading conditions, including any stress concentrations which may be induced by the couplings.

Determination of the System Response Capabilities

The ability of the complete IMC system to respond to variations in the scanner velocity is directly related to the frequency and amplitude of the disturbance. The response capabilities shown in Figure 15 are developed below:

The peak torque rating for the motor chosen is 77.4 oz-in. After subtracting the maximum torque required to overcome the spring force in the flexures (45.1 oz-in), the remaining torque capability is 32.3 oz-in. By utilizing the relationship between torque \( (T) \), the total system inertia \( (I) \) and the angular acceleration \( (\alpha) \) required to respond to various frequencies of velocity variations, a conservative system response profile can be obtained.

\[ \text{Torque} = 32.3 \text{ oz-in} = I_{\text{total}} \times \alpha_{\text{max}} \]
\[ = 0.00242 \times 13306 \text{ rad/sec}^2. \] (24)

By utilizing equation 6 and taking the derivative to determine the acceleration, a limit can be placed on the maximum correction possible, based upon the frequency of the variation.

\[ \text{Linear Acceleration} = r \times \alpha \] (25)

where \( r \) = the change in image displacement over the change in the tilt of the glass element, or simply \( \Delta D / \Delta \theta \).

For the IMC system designed here \( \Delta D / \Delta \theta \) is equal to 0.145, which leads to the following expression for the maximum angular acceleration to which the IMC system is capable of responding:

\[ \alpha_{\text{max}} = 13306 = \Delta V \times 2 \times \pi \times f / (\Delta D / \Delta \theta) \] (26)

Or,

\[ \Delta V = 13306 \times 0.145 / (2 \times \pi \times f) \] (27)

See Figure 15 for the results of this calculation.
VII. Optimization and Practicality

Foremost in the design of the IMC Device was the desire to specify existing hardware that would produce the necessary results. Although general specifications could have been given for the performance of the motor, the flexures and the glass, there would be no guarantee that such products actually existed. By specifying applicable hardware and allowing substitutions where available, the feasibility of the IMC Device is assured.

In some cases, the performance of an individual component governed the selection for the remaining components. Further optimizations must consider the overall system performance to be valid, due to the interaction of the components. Appendix D details the various parameters to be optimized and provides some fundamental considerations for optimizing the IMC device.

Recommendations for further optimizations can be found starting on page 37.
VIII. **Summary of System Specifications**

Based on the typical scanning system defined on page 15, the following system specifications are presented for the IMC device. The values shown have been assigned according to the specifications shown on page 15 or calculated within the context of this paper based upon those specifications.

**The Optical System**

<table>
<thead>
<tr>
<th>Lens Parameters</th>
<th>Focal length of the lens 2.95 inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Depth of focus tolerance ±0.0069</td>
</tr>
<tr>
<td></td>
<td>Overall System MTF 0.30 @ 40 lp/mm</td>
</tr>
<tr>
<td></td>
<td>Magnification -1/10X</td>
</tr>
<tr>
<td></td>
<td>f number f / 8.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Glass Parameters</th>
<th>Slit width at the object plane 0.30&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optical path size at the IMC plane 1.06&quot; wide x .125&quot; tall</td>
</tr>
<tr>
<td></td>
<td>Thickness 0.460&quot;</td>
</tr>
<tr>
<td></td>
<td>Mass Moment of Inertia (calculated) 0.0000417 oz-in-s²</td>
</tr>
<tr>
<td></td>
<td>Max. Angular Rotation 6⁰</td>
</tr>
<tr>
<td></td>
<td>Abbe V-number 90.3</td>
</tr>
<tr>
<td></td>
<td>Index of refraction:</td>
</tr>
<tr>
<td></td>
<td>Nominal 1.456000</td>
</tr>
<tr>
<td></td>
<td>@ hydrogen C line (red) 1.454453</td>
</tr>
<tr>
<td></td>
<td>@ sodium D line (yellow) 1.455954</td>
</tr>
<tr>
<td></td>
<td>@ hydrogen F line (blue) 1.459502</td>
</tr>
<tr>
<td></td>
<td>Glass Type FK02 Ohara Glass Mfg. Co. Substitute Type 458903 Hoya FCD10N</td>
</tr>
<tr>
<td></td>
<td>Distance from rear surface of glass element to CCD image Plane 1.466&quot;</td>
</tr>
<tr>
<td></td>
<td>Size .46&quot; Sq. x 1.500&quot; long</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CCD Parameters</th>
<th>Type Toshiba TCD105C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resolution 300 pixels per inch</td>
</tr>
<tr>
<td></td>
<td>Sensing Elements (3648) 8/μm x 8/μm</td>
</tr>
<tr>
<td></td>
<td>Sensor Area 8/μm tall x 1.149&quot; long</td>
</tr>
<tr>
<td></td>
<td>Chip size 0.39&quot; tall x 1.638&quot; long</td>
</tr>
<tr>
<td></td>
<td>Location 3.392&quot; from lens aperture</td>
</tr>
</tbody>
</table>
The Flexural Pivots

Manufacturer: Bendix Aerospace Fluid Power Division
Type (Catalog No.): 5020-600
Torsional Spring Rate: 425.6 oz-in/rad
Weight: 0.6064 oz.
Size: 0.625" dia x 1.000" long
Mass Moment of Inertia (estimated): 0.0000335 oz-in-s²
Max. Angular Deflection for Infinite Life: 6.25°
Max. Required Angular Deflection: 6°
Max. Centerline Shift: 0.0006"

The Drive System

Motor Parameters

Type: Inland Frameless Direct Drive DC Motor
Catalog Number: T-1421
Peak Torque Rating: 77.4 in-oz
Electrical Time Constant: 0.550 ms
Rotor Inertia (Jₘ): 0.0023 oz-in-s²
Motor Weight: 15.0 oz
Size: 1.94" dia x 1.20" long
Alternate Suppliers: Honeywell Magnetic Technologies

System Response Capabilities

Figure 15 on the next page shows the results of equation 27, the system response capabilities, as a function of the frequency of the velocity variations in the scanner mechanism.

* NOTE: Response Capabilities may be conservative since the peak torque rating of the motor can be exceeded for short periods of time, although a rise in operating temperature occurs. Values shown in Figure 15 may vary substantially near the resonant frequency (67 Hz).
Figure 15: System Response Capabilities
IX. Conclusions and Recommendations

A. Conclusions

Based upon the MTF profiles obtained for the IMC system, the optical performance clearly exceeds the anticipated goal of an MTF of 0.30 at 40 lp/mm by providing at the worst case (off-axis with full glass tilt) an MTF of 0.74. In addition, the drive system easily compensates for various disturbances, allowing up to 33.2% velocity error at 120 Hz. Although this value is below the amplitude of a 120 Hz disturbance producing 0.015" of distortion, it would be difficult to imagine any respectable drive system with non-uniformities greater than 33% at 120 Hz.

The feasibility of using a plane glass element for IMC versus the more popular first-surface mirror approach has been verified. The lens design must however, include the glass element for optimal performance.

The following recommendations are made in the event that further optimizations are requested.

B. Recommendations

The total system performance is based on the individual contributions of each component. Beginning with the optical requirements and continuing through the flexures and drive motor specifications, the final design reflects the interaction of each of the components. Further optimizations must consider the overall system performance and not just the individual component performance for best results.
1. The Optical System

Several optical properties have shown their ability to easily alter the system performance. Obviously, the original parameters (velocity, exposure requirements, f/#, focal length, etc.) govern a major portion of the obtainable MTF profile. The necessity for incorporating the IMC glass element into the lens design is required to achieve the system performance listed on page 36.

By lengthening the focal length of the lens, the off-axis MTF profiles improve considerably. Additionally, a larger f/# lens will also improve the performance of the system.

In the event that the IMC device is used for laser applications, the entire system could be fine-tuned for the specific monochromatic light frequency of the laser.

Several of the glass parameters directly control the final performance of the system, as well. By utilizing a low index of refraction material, a high V number is possible. This results in less chromatic dispersion and a high depth of focus tolerance. If necessary, most of the optical distortions could be completely eliminated by utilizing the more conventional first-surface mirror approach, with a modified drive and flexure system.
2. The Mechanical and Drive System

By designing a glass element with a square cross-section, it was hoped to improve the assembly and reliability by providing a system which can be oriented in several directions. A square cross-section is recommended for future optimizations due to the limited effect on the mass moment of inertia for the system.

The frequency response capability of the system is a delicate balance between the total inertia of the system, the torsional spring rate of the flexures and the ability of the motor to drive the system. Perhaps a custom motor with sufficient torque and yet a low rotor inertia would be best if higher response capability is sought.

One final word on the IMC device is appropriate here. The feasibility for using a single plane glass element as a means of correcting an errant optical signal has been proven. The final application and optimization is dictated only by the bounds of optical instruments with less than acceptable scanning systems. But the basis for using an IMC device, similar to that designed in this report, is built on a solid foundation.
X. References


Technical Literature and Manufacturer's Information

1. Bendix Flexural Pivots. Manufacturer's Literature from the Fluid Power Division of Bendix Aerospace 211 Seward Ave. P.O.Box 457 Utica, NY 13503.

2. Optical Glass. The Optical Industry and Systems Purchasing Directory. P.O.Box 1146 Pittsfield, MA 01202

3. Inland Direct Drive DC Motors. Manufacturer's Catalog from the Specialty Products Division of the Kollmorgen Corporation 501 First St. Radford, VA 24141
Appendix A

1. Derivation of Image Displacement Equation

The following is a derivation of the Image Displacement equation as a function of Tilt ($\theta$), Thickness ($t$) and the Index of Refraction ($N$) of the glass element.

![Diagram of light path through glass element]

Figure A.1: Derivation of Image Displacement versus Tilt

From Snell's Equation:

$$N_{air} \cdot \sin \theta_{air} = N_{glass} \cdot \sin \theta_{glass}$$  \hspace{1cm} (A.1)

From Figure A.1 above:

$$A = t \cdot \tan \theta_{air} \hspace{1cm} \text{and} \hspace{1cm} B = t \cdot \tan \theta_{glass}$$  \hspace{1cm} (A.2,3)

so that:

$$D = (A - B) \cdot \cos \theta_{air}$$  \hspace{1cm} (A.4)

or:

$$D = t \cdot (\sin \theta_{air} - \tan \theta_{glass} \cdot \cos \theta_{air})$$  \hspace{1cm} (A.5)

where $\theta_{glass} = \sin^{-1} \left( N_{air} \cdot \sin \theta_{air} / N_{glass} \right)$.  \hspace{1cm} (A.6)
Appendix A

2. Derivation of Optical Pathlength Variation

The following is a derivation of the change in optical pathlength as a function of Tilt ($\theta$), Thickness ($t$) and the Index of Refraction ($N$) of the glass element.

![Diagram](image)

Figure A.2: Derivation of the Change in Optical Pathlength

The change in Optical Pathlength is due to the increased distance that the image must travel through the glass element. Dimensions $A$, $B$ and $C$, as shown in Figure A.2 above, represent the optical distances before tilting the glass element. Dimensions $A'$, $B'$ and $C'$ represent the changed optical distances due to the tilting of the glass element.

Dimension $A$ becomes shortened by the amount:

$$t/2 \ast (1 - 1/\cos \theta_{air}). \quad (A.7)$$

Dimension $B$ becomes lengthened by the amount:

$$N \ast (D/\sin(\theta_{air} - \theta_{glass}) - t). \quad (A.8)$$

And finally Dimension $C$ becomes lengthened by the amount:

$$D \ast \tan \theta_{air} - (t/2 \ast (1/\cos \theta_{air} - 1)). \quad (A.9)$$

So that the total change in optical pathlength is the combination of the above equations, or simply:

The Change in Pathlength = ($A' + B' + C'$) - ($A + B + C$) \quad (A.10)
Appendix A

3. Comparison of Image Displacement versus Tilt Angle to Accepted Equations

The following FORTRAN Program verifies the Image Displacement versus Tilt Angle equation (A-5) derived in section 1 of this appendix by comparing the results to those of accepted equations from references 1 and 2. Note that equation A-5 is numerically equal to that of equation 2e found on page 29 of reference 2.

CREATED BY JAMES H. CAIN AUGUST 1986

REAL*8 INDEX, THETA, DA(-1:1000), DB(-1:1000)
REAL*8 DC(-1:1000), THK, THETAG

*** INPUT VARIABLES ***
INDEX* IS THE INDEX OF REFRACTION FOR THE GLASS ELEMENT
INDEX = 1.456

THK* IS THE THICKNESS IN INCHES FOR THE GLASS ELEMENT
THK = 0.460

DO 10 I=0,150,5
THETA = FLOAT(I)/1000.0

*** METHOD 1 IS THE SMALL ANGLE APPROXIMATION FROM SMITH ***
*** SEE REFERENCE 1, PAGE 82 ***
DA(I) = THK * THETA * (INDEX-1)/INDEX

*** METHOD 2 IS DERIVED IN THE DESIGN REPORT ***
*** SEE APPENDIX A ON PAGE A.1 ***

THETAG = ASIN((SIN(THETA))/INDEX)
DB(I) = THK*(SIN(THETA)-TAN(THETAG)*COS(THETA))

*** METHOD 3 IS FROM JENKINS AND WHITE ***
*** SEE REFERENCE 2, PAGE 29 ***

DC(I) = THK*SIN(THETA)*(1.0-COS(THETA)/(INDEX*C0S(THETAG)))

*** WRITING THE OUTPUT ***
DO 300 I=0,150,5
THETA = FLOAT(I)/1000.0
WRITE(6,103)THETA, DA(I), DB(I), DC(I)
103 FORMAT(X,F5.3,3F10.6)
DO 302 I=0,150,5
THETA = FLOAT(I)/1000.0
WRITE(4,200)THETA, DB(I)
200 FORMAT(X,F10.5,',',F15.10)

STOP
END
Table A.1: Comparison of Image Displacement versus Tilt Angle to Accepted Equations.

Optical Displacement Distance (inches)

INDEX (N) OF THE GLASS ELEMENT = 1.456
THICKNESS OF THE GLASS ELEMENT = 0.46 INCHES

<table>
<thead>
<tr>
<th>THETA (RAD)</th>
<th>METHOD 1 (INCHES)</th>
<th>METHOD 2 (INCHES)</th>
<th>METHOD 3 (INCHES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.0000720</td>
<td>0.000720</td>
<td>0.000720</td>
</tr>
<tr>
<td>0.010</td>
<td>0.001441</td>
<td>0.001441</td>
<td>0.001441</td>
</tr>
<tr>
<td>0.015</td>
<td>0.002161</td>
<td>0.002161</td>
<td>0.002161</td>
</tr>
<tr>
<td>0.020</td>
<td>0.002882</td>
<td>0.002882</td>
<td>0.002882</td>
</tr>
<tr>
<td>0.025</td>
<td>0.003603</td>
<td>0.003603</td>
<td>0.003603</td>
</tr>
<tr>
<td>0.030</td>
<td>0.004324</td>
<td>0.004324</td>
<td>0.004324</td>
</tr>
<tr>
<td>0.035</td>
<td>0.005045</td>
<td>0.005045</td>
<td>0.005045</td>
</tr>
<tr>
<td>0.040</td>
<td>0.005766</td>
<td>0.005766</td>
<td>0.005766</td>
</tr>
<tr>
<td>0.045</td>
<td>0.006488</td>
<td>0.006488</td>
<td>0.006488</td>
</tr>
<tr>
<td>0.050</td>
<td>0.007211</td>
<td>0.007211</td>
<td>0.007211</td>
</tr>
<tr>
<td>0.055</td>
<td>0.007934</td>
<td>0.007934</td>
<td>0.007934</td>
</tr>
<tr>
<td>0.060</td>
<td>0.008657</td>
<td>0.008657</td>
<td>0.008657</td>
</tr>
<tr>
<td>0.065</td>
<td>0.009381</td>
<td>0.009381</td>
<td>0.009381</td>
</tr>
<tr>
<td>0.070</td>
<td>0.010105</td>
<td>0.010105</td>
<td>0.010105</td>
</tr>
<tr>
<td>0.075</td>
<td>0.010830</td>
<td>0.010830</td>
<td>0.010830</td>
</tr>
<tr>
<td>0.080</td>
<td>0.011556</td>
<td>0.011556</td>
<td>0.011556</td>
</tr>
<tr>
<td>0.085</td>
<td>0.012282</td>
<td>0.012282</td>
<td>0.012282</td>
</tr>
<tr>
<td>0.090</td>
<td>0.013009</td>
<td>0.013009</td>
<td>0.013009</td>
</tr>
<tr>
<td>0.095</td>
<td>0.013737</td>
<td>0.013737</td>
<td>0.013737</td>
</tr>
<tr>
<td>0.100</td>
<td>0.014466</td>
<td>0.014466</td>
<td>0.014466</td>
</tr>
<tr>
<td>0.105</td>
<td>0.015196</td>
<td>0.015196</td>
<td>0.015196</td>
</tr>
<tr>
<td>0.110</td>
<td>0.015927</td>
<td>0.015927</td>
<td>0.015927</td>
</tr>
<tr>
<td>0.115</td>
<td>0.016658</td>
<td>0.016658</td>
<td>0.016658</td>
</tr>
<tr>
<td>0.120</td>
<td>0.017391</td>
<td>0.017391</td>
<td>0.017391</td>
</tr>
<tr>
<td>0.125</td>
<td>0.018125</td>
<td>0.018125</td>
<td>0.018125</td>
</tr>
<tr>
<td>0.130</td>
<td>0.018860</td>
<td>0.018860</td>
<td>0.018860</td>
</tr>
<tr>
<td>0.135</td>
<td>0.019596</td>
<td>0.019596</td>
<td>0.019596</td>
</tr>
<tr>
<td>0.140</td>
<td>0.020333</td>
<td>0.020333</td>
<td>0.020333</td>
</tr>
<tr>
<td>0.145</td>
<td>0.021071</td>
<td>0.021071</td>
<td>0.021071</td>
</tr>
<tr>
<td>0.150</td>
<td>0.021811</td>
<td>0.021811</td>
<td>0.021811</td>
</tr>
</tbody>
</table>
Figure A.3: Comparison of Image Displacement Equations

Image Displacement (inches) versus the Tilt Angle (radians) of the glass element in the IMC device.
Appendix A

4. Image Displacement and OPL changes - FORTRAN program

Based upon the equations A-5 and A-10 derived in sections 1 and 2 of this appendix, the following FORTRAN Program calculates the Image Displacement and change in the Optical Pathlength for various Tilt Angles of the glass element.

```fortran
CREATED BY JAMES H. CAIN JULY 1986

REAL*8 NG,THETAI,THETAG,D(-1:1000),OPL(-1:1000),THK

*** INPUT VARIABLES ***
INDEX OF REFRACTION OF THE GLASS = NG
NG = 1.456
THICKNESS OF THE GLASS = THK (INCHES)
THK = 0.460

*** IMAGE DISPLACEMENT AND PATHLENGTH CHANGE CALCULATIONS ***
DO 10 I = 0,150,5
THETAI = FLOAT(I)/1000.0
THETAG = ASIN((SIN(THETAI))/NG)
D(I) = THK*(SIN(THETAI)-TAN(THETAG)*COS(THETAI))
AA = D(I)*TAN(THETAI)
BB = THK*(1.0-NG-(1.0/C0S(THETAI)))
THETA = SIN(THETAI - THETAG)
IF(THETA.EQ.0.0) CC = THK*NG
IF(THETA.EQ.0.0) GO TO 9
CC = (D(I)*NG)/(THETA)
9 OPL(I) = AA+BB+CC
10 CONTINUE

*** WRITING THE OUTPUT ***
DO 300 I = 0,150,5
THETAI = FLOAT(I)/1000.0
300 WRITE(6,104)THETAI,D(I),OPL(I)
104 FORMAT(3X,F5.3,F14.6,F14.6)
99 STOP
END
```
Table A.2: Image Displacement and Optical Pathlength versus the Tilt Angle of the glass element.

INDEX OF REFRACTION: 1.456  THK: 0.460"

<table>
<thead>
<tr>
<th>THETA (RADIANS)</th>
<th>OFFSET (INCHES)</th>
<th>DELTA OPL (INCHES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.000720</td>
<td>0.000002</td>
</tr>
<tr>
<td>0.010</td>
<td>0.001441</td>
<td>0.000007</td>
</tr>
<tr>
<td>0.015</td>
<td>0.002161</td>
<td>0.000016</td>
</tr>
<tr>
<td>0.020</td>
<td>0.002882</td>
<td>0.000029</td>
</tr>
<tr>
<td>0.025</td>
<td>0.003605</td>
<td>0.000045</td>
</tr>
<tr>
<td>0.030</td>
<td>0.004324</td>
<td>0.000065</td>
</tr>
<tr>
<td>0.035</td>
<td>0.005045</td>
<td>0.000088</td>
</tr>
<tr>
<td>0.040</td>
<td>0.005766</td>
<td>0.000115</td>
</tr>
<tr>
<td>0.045</td>
<td>0.006488</td>
<td>0.000146</td>
</tr>
<tr>
<td>0.050</td>
<td>0.007211</td>
<td>0.000180</td>
</tr>
<tr>
<td>0.055</td>
<td>0.007934</td>
<td>0.000218</td>
</tr>
<tr>
<td>0.060</td>
<td>0.008657</td>
<td>0.000260</td>
</tr>
<tr>
<td>0.065</td>
<td>0.009381</td>
<td>0.000305</td>
</tr>
<tr>
<td>0.070</td>
<td>0.010105</td>
<td>0.000353</td>
</tr>
<tr>
<td>0.075</td>
<td>0.010830</td>
<td>0.000406</td>
</tr>
<tr>
<td>0.080</td>
<td>0.011556</td>
<td>0.000462</td>
</tr>
<tr>
<td>0.085</td>
<td>0.012282</td>
<td>0.000521</td>
</tr>
<tr>
<td>0.090</td>
<td>0.013009</td>
<td>0.000584</td>
</tr>
<tr>
<td>0.095</td>
<td>0.013737</td>
<td>0.000651</td>
</tr>
<tr>
<td>0.100</td>
<td>0.014466</td>
<td>0.000722</td>
</tr>
<tr>
<td>0.105</td>
<td>0.015196</td>
<td>0.000796</td>
</tr>
<tr>
<td>0.110</td>
<td>0.015927</td>
<td>0.000874</td>
</tr>
<tr>
<td>0.115</td>
<td>0.016658</td>
<td>0.000955</td>
</tr>
<tr>
<td>0.120</td>
<td>0.017391</td>
<td>0.001040</td>
</tr>
<tr>
<td>0.125</td>
<td>0.018125</td>
<td>0.001129</td>
</tr>
<tr>
<td>0.130</td>
<td>0.018860</td>
<td>0.001222</td>
</tr>
<tr>
<td>0.135</td>
<td>0.019596</td>
<td>0.001318</td>
</tr>
<tr>
<td>0.140</td>
<td>0.020333</td>
<td>0.001418</td>
</tr>
<tr>
<td>0.145</td>
<td>0.021071</td>
<td>0.001521</td>
</tr>
<tr>
<td>0.150</td>
<td>0.021811</td>
<td>0.001628</td>
</tr>
</tbody>
</table>
Appendix B

1. Exposure Requirements

The following information has been given:

Saturation Voltage of the Sensor = 800 mV
White reference voltage for the sensor = $V_w = 100$ mV
Resolution of the Sensor (dots/inch) = $P = 300$ dpi
Minimum Responsivity = $R = 0.5V/lux\cdot s$
Scanner Speed = $V = 8.0 \text{ in/s}$
Illumination Area at the object plane = $A = 0.0022 \text{ m}^2$
f / # of the lens = $f/# = f/8.0$
Desired Power Limit = 200 W
Magnification = $m = 1/10$
Lens Axial Transmittance (estimate) = $T = 85\%$
Illumination Efficiency (estimate) = $E = 10\%$

Beginning with the white exposure = $X_w = V_w / R = 0.2 \text{ lux} \cdot s$
Exposure time = resolution / scanner speed = $1/\rho V = 0.0004167 \text{ s}$
White Illumination at the image = $X_w / \text{exp. time} = 480 \text{ lux}$
Ratio = Lens axial transmit. / $1+(m+1)^2 f/# = 0.002735$
White Illumination of the object = $I_w \text{ image/Ratio} = 175.503 \text{ lux}$
Flux = $I_w \text{ object} \times \text{Area} = 390.67 \text{ lumens}$
Lamp Flux = Flux / Illumination efficiency = $3906.7 \text{ lumens}$
Conversion Factor = $C = 20 \text{ lumens/watt (estimate)} = 20 \text{ lumens/W}$
Power = Lamp Flux / Conversion Factor = $195 \text{ Watts}$

A simplified version of this procedure is given below:

$$\text{Power (watts)} = \frac{A*V_w*P*V}{C*R*T*E} \left( 1 + ((m+1) \times 2 \times f/#)^2 \right) \quad (B.1)$$

For the system designed in this report the calculated power requirement is $195 \text{ Watts}$. 
Appendix B

2. Depth of Focus Calculations

Given the following information, the depth of focus can be calculated:

<table>
<thead>
<tr>
<th>f / #</th>
<th>f / 8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTF</td>
<td>.30 @ 4 lp/mm</td>
</tr>
<tr>
<td>Magnification</td>
<td>1/10 X</td>
</tr>
</tbody>
</table>

Assuming that the limits of the depth of focus are set by the blur circle diameter (which is based on the f / #), then:

\[
\text{MTF}(f) = \cos^2 \left( 1.4142 \times \pi \times f \times .28 \times \Delta X / (f' / #) \right) = .30
\]  

(B.2)

Here the \( \cos^2 \) function displays the type of MTF falloff associated with a defocused system, and:

\[
f = 4 \text{ lp/mm} \times 1 \text{/magnification} = 40 \text{ lp/mm at the image}
\]

\[
f' / # = f / # \times (1 + \text{magnification}) = (8 \times (1+1)) = f / 8.8
\]

Now using equation B.2 above to solve for the depth of focus (\( \Delta X \)):

\[
\cos^2 \left( 1.4142 \times \pi \times 40 \times .28 \times \Delta X / 8.8 \right) = .30
\]

\[
\Delta X = \pm 0.1753 \text{ mm}
\]

Depth of Focus Tolerance = \( \Delta X = \pm 0.0069 \text{ in} \)
Appendix C

1. Explanation of the Modulation Transfer Function

In order to evaluate the performance of the IMC device, it is necessary to trace the path of the light rays that will be transmitted through the system. By understanding the behavior of each light ray, the system can be optimized for maximum response.

The following FORTRAN program numerically traces the path of a series of optical rays through the system. The theory and application for this program is developed in Reference 1.

As mentioned in the body of the report, the MTF profile obtained by a pure ray trace does not completely describe the performance of the optical system, although the data generated here is adequate for evaluation of the IMC device. The effects of diffraction also effect the final MTF profile. The true system MTF is then a combination of the optical ray tracing effects and that of diffraction limiting effects. For the worst-case MTF profile (an off-axis ray with 6° of tilt at the glass element), the combination of the MTF profile calculated from the ray trace cascaded with the perfect diffraction limited MTF profile produces a worst case MTF value of 0.53 at 40 lp/mm. This value is acceptable according to the system specifications outlined on page 34.
The Modulation Transfer Function (MTF) mentioned previously can be simply defined as a measure of the ability of an optical system to transmit various frequencies. In the electronic discipline this type of analysis would yield the frequency response of the system in question. By comparing the final image produced to the original object, the MTF for the system can be found.

In a more general sense, the MTF reflects the amount of spread or loss of contrast in an image as a function of increasing spatial frequency. Figure C.1 shows the relationship between the physical edge of the object to be imaged and an important expression known as the line spread function. This function is simply the mathematical expression for the derivative of the imaged edge \( e(x) \) which has been degraded due to the performance of the optical system.

$$\text{EDGE}$$

The Physical Object

\[ \text{Edge with Degradation due to the Optical System} \]

\[ \text{The Line Spread Function} \]

\[ \text{The Line Spread Function} \]

Figure C.1: The Line Spread Function
By transforming the line spread function \[ l(x) \] into the frequency domain (by utilizing the Fourier Transform, shown as \[ \mathcal{F} \] in Equation C.1 below), an expression known as the Optical Transfer Function (OTF) is found. The MTF is the magnitude of the OTF.

\[
\frac{d\left[e(x)\right]}{dx} = l(x) \quad \Rightarrow \quad L(f) = OTF(f) = MTF \times e^{i2\pi f} \quad (C.1)
\]

Figure C.2 shows a typical MTF profile. Note that at a zero frequency (the steady-state DC portion of the image), the MTF is exactly 1. As the spatial frequency increases, the ability of the system to reproduce the pattern decreases due to the edge effect shown in Figure C.1. Eventually the frequency becomes so high that the spread of the edge forms into a continuous haze and the edge is no longer discernible.

The final result of this form of evaluation is a rating of the optical performance of an optical element or system, based upon the ability to accurately transmit patterns of various spatial frequencies. This is especially important for the range from 1/2 to 2 lp/mm (the range most sensitive to the human eye).

For additional information consult references 1 and 8.

---

**Figure C.2:** A Typical MTF Profile
Appendix C

2. Numerical Ray Tracing Program for MTF Analysis

This FORTRAN Computer Program is based upon the path that a light ray would travel as it passes from the lens, through the glass element of the IMC device and to the CCD image plane. The amount of deviation or spreading of the light from the ideal target is computed and the MTF is then determined statistically from the standard deviation of the ray traces by utilizing the second moment method.

Created by James H. Cain and Fred J. Scipione August 1986

```
C DECLARATION OF VARIABLES
PARAMETER(PI=3.1415927, JPL0T=4, NPHI=20, NR=5, NLAMBDA=2)
PARAMETER(NRAY=NPHI*NR*NLAMBDA, NPLANE=20)
REAL ENS(NLAMBDA),H(NLAMBDA),EN1(NRAY),EN2(NRAY),SCZ(NRAY)
REAL X(NRAY),SX(NRAY),Y(NRAY),SY(NRAY),Z(NRAY)
REAL CX(NRAY),SCX(NRAY),CY(NRAY),SCY(NRAY),CZ(NRAY)

C LINE FUNCTION TO COMPUTE MTF AT TEST FREQUENCY FROM SECOND MOMENT
F(S) = COS(PI * MIN(0.5, ABS(2.0**0.5*FTEST*S)**2)

C SET UP CONFIGURATION WITH AXIS IN CENTER OF GLASS BLOCK
(UNITS IN MM AND DEGREES) -
F TEST = 40.0
T GLASS = 0.46 * 25.4
THETA MAX = 6.0
ENS(1) = 1.454453
ENS(2) = 1.459502
ENAVG = (ENS(1) + ENS(2)) / 2.0
DT = TGLASS * ((ENAVG - 1.0) / ENAVG)
DP = 0.025
AMAG = 1.0 / 10.0
F NO = B.0 * (1.0 + AMAG)
EFL = 75.0
BFI = 0.517 * EFL
DX = BFI + EFL*AMAG
XA = -(DX + DT)/2.0
XP = XA + DX
XA = XA - .483 * EFL
YP = 0.0

C OPEN FILE FOR PLOTTER OUTPUT
OPEN(JPLOT,CARRIAGECONTROL='LIST',STATUS='NEW',NAME='PLOT2D.DAT')

C TRACE RAYS ON AND OFF AXIS FOR PLATE TILT OF 0, 3 AND 6 DEGREES
DO NAXIS = 0, 1
ZP = NAXIS * 35.0 / 2.0
DO NTHTHA = 0, 2
THETA = NTHTHA * THETAMAX / 360.0

C INITIALIZE RAYS
CALL INIT RAYS(XA, XF, YP, ZP, FNO, TGLASS, DT, TAIR, H.
* NPHI, NR, NLAMBDA, X, Y, Z, CX, CY, CZ, EN1, EN2, ENS)
C UN-ROTATE GLASS PLATE AND ROTATE RAYS
CALL ROTATE BUNDLE(NRAY, X, Y, Z, CX, CY, CZ, -THETA)
C TRACE TO BACK OF PLATE -
CALL XFER BUNDLE(NRAY, X, Y, Z, CX, CY, CZ, -TGLASS / 2.0)
```
CALL DEFLT BUNDLE(NRAY, CX, CY, CZ, EN1, EN2)
CALL XFER BUNDLE (NRAY, X, Y, Z, CX, CY, CZ, TGLASS)
CALL DEFLT BUNDLE(NRAY, CX, CY, CZ, EN2, EN1)
C
RE-ROTATE PLATE AND UN-ROTATE RAYS
CALL XLATE BUNDLE(NRAY, X, TGLASS / 2.0)
CALL ROTATE BUNDLE (NRAY, X, Y, CX, CY, THETA)
C
SAVE RAYS FOR MULTIPLE TRACES TO VARIOUS FOCAL PLANES
DO I = 1, NRAY
SX(K) = X(K)
SY(K) = Y(K)
SZ(K) = Z(K)
SCX(K) = CX(K)
SCY(K) = CY(K)
SCZ(K) = CZ(K)
END DO
C
FIND AND PLOT MTF AT TEST FREQUENCY FOR VARIOUS IMAGE PLANES
DO J = -(NPLANE/2), NPLANE-(NPLANE/2)
COPY SAVED RAYS
DO K = 1, NRAY
X(K) = SX(K)
Y(K) = SY(K)
Z(K) = SZ(K)
CX(K) = SCX(K)
CY(K) = SCY(K)
CZ(K) = SCZ(K)
END DO
C
TRACE TO PLANE
DPLANE = J*DP
T = XP + DT + DPLANE
CALL XFER BUNDLE (NRAY, X, Y, Z, CX, CY, CZ, T)
C
COMPUTE AVERAGE MTF
SMY = SMOMENT(Y, NRAY)
SMZ = SMOMENT(Z, NRAY)
EMTF = (F(SMY) + F(SMZ)) / 2.0
C
PLOT MTF VS FOCAL PLANE DISPLACEMENT
WRITE(JPLOT, 500) DPLANE, EMTF
500 FORMAT (F10.4, ', ', F10.4)
END DO
WRITE (JPLOT, 600)
600 FORMAT ('END')
END DO
! NTHETA
END DO
! NAXIS
999 STOP
C
SUBROUTINE INITRAYS (XA, XP, YP, ZP,FNO, T, DT, THETA, H,
* NPHI, NR, NLAMDBA, XX, YY, ZZ, CXX, CYY, CZZ, EN1, EN2, EMS)
PARAMETER (PI = 3.1415927)
REAL XX(NPHI, NR, NLAMDBA), YY(NPHI, NR, NLAMDBA),
* ZZ(NPHI, NR, NLAMDBA), CXX(NPHI, NR, NLAMDBA),
* CYY(NPHI, NR, NLAMDBA), CZZ(NPHI, NR, NLAMDBA),
* EN1(NPHI, NR, NLAMDBA), EN2(NPHI, NR, NLAMDBA),
* EMS(NLAMDBA), H(NLAMDBA)
C
FIND HEIGHT OF AXIAL RAY AT IMAGE PLANE
DO J = 1, NLAMDBA
XT = XA
YT = 0.0
ZT = 0.0
D = SQRT((XP-XT)**2 + (YP-YT)**2 + (ZP-ZT)**2)
CX = (XP - XT) / D
CY = (YP - YT) / D
CZ = (ZP - ZT) / D
CALL ROTATE BUNDLE(1, XT, YT, CX, CY, -THETA)
CALL XFER RAY(XT, YT, ZT, CX, CY, CZ, -T/2.0)
CALL DEFLT RAY(CX, CY, CZ, 1.0 / EMS(J))
CALL XFER RAY(XT, YT, ZT, CX, CY, CZ, T)
CALL DEFLT RAY(CX, CY, CZ, EMS(J))
XT = XT + T/2.0
CALL ROTATE BUNDLE(1, XT, YT, CX, CY, THETA)
CALL XFER RAY(XT, YT, ZT, CX, CY, CZ, XP + DT)
H(J) = YT
END DO
C
C5
RMAX = \left| \frac{X_P - X_A}{2*FN0} \right|

DX = X_P - X_A

DX2 = DX^2

DO J=1,NPHI

\phi_i = \frac{(2*\pi*(2*J-1))/(2*NPHI)}

SP = \sin(\phi_i)

CP = \cos(\phi_i)

DO K=1, NR

R = RMAX*\sqrt{\left(2.0*K-1\right)/(2*NR)}

Y = R*SP

Z = R*CP

DO L=1, NLAMBD

YPT = Y

ZPT = Z

D = \sqrt{DX^2 + (YPT-Y)^2 + (ZPT-Z)^2}

CX = DX/D

cy = (YPT-Y)/D

CZ = (ZPT-Z)/D

DO M = 1, 4

XT = XA

YT = Y

ZT = Z

CALL ROTATE BUNDLE(1, XT, YT, CX, CY, -\theta)

CALL XFER RAY(XT, YT, ZT, CX, CY, CZ, -T/2.0)

CALL DEFLT RAY(CX, CY, CZ, 1.0 / ENS(L))

CALL XFER RAY(XT, YT, ZT, CX, CY, CZ, T)

CALL DEFLT RAY(CX, CY, CZ, ENS(L))

XT = XT + T/2.0

CALL ROTATE BUNDLE(1, XT, YT, CX, CY, \theta)

CALL XFER RAY(XT, YT, ZT, CX, CY, CZ, XP + DT)

EV = VT - H(L)

EZ = ZT - ZP

YPT = YPT - EY

ZPT = ZPT - EZ

D = \sqrt{DX^2 + (YPT-Y)^2 + (ZPT-Z)^2}

CX = DX/D

cy = (YPT-Y)/D

CZ = (ZPT-Z)/D

END DO

XX(J,K,L) = XA

YY(J,K,L) = Y

ZZ(J,K,L) = Z

CXX(J,K,L) = CX

CYY(J,K,L) = CY

CZZ(J,K,L) = CZ

EN1(J,K,L) = 1.0

EN2(J,K,L) = ENS(L)

END DO

END DO

END DO

RETURN

END
Subroutines for the Numerical Ray Tracing Program

SUBROUTINE XFER RAY(X, Y, Z, CX, CY, CZ, T)

C

E = T*CX - (X*CX + Y*CY + Z*CZ)
EM1X = X + E*CX - T
E1 = ABS(CX)
EL = E + (-2.0*EM1X)/(CX + E1)
X = X + EL*CX - T
Y = Y + EL*CY
Z = Z + EL*CZ

C

RETURN
END

SUBROUTINE DEFLT RAY(CX, CY, CZ, R)

C

G1 = SORT (1.0 - R**2*(1-CX**2)) - R*CX
CX = R*CX + G1
CY = R*CY
CZ = R*CZ

C

RETURN
END

SUBROUTINE XFER BUNDLE(NRAY, X, Y, Z, CX, CY, CZ, T)
REAL X(NRAY), Y(NRAY), Z(NRAY), CX(NRAY), CY(NRAY), CZ(NRAY)
DO J = 1, NRAY
   CALL XFER RAY(X(J), Y(J), Z(J), CX(J), CY(J), CZ(J), T)
END DO
RETURN
END

SUBROUTINE DEFLT BUNDLE(NRAY, CX, CY, CZ, EN1, EN2)
REAL CX(NRAY), CY(NRAY), CZ(NRAY), EN1(NRAY), EN2(NRAY)
DO J = 1, NRAY
   CALL DEFLT RAY(CX(J), CY(J), CZ(J), EN1(J) / EN2(J))
END DO
RETURN
END

SUBROUTINE XLATE BUNDLE(NRAY, X, DX)
REAL X(NRAY)
DO J = 1, NRAY
   X(J) = X(J) + DX
END DO
RETURN
END
SUBROUTINE ROTATE BUNDLE (NRAY, X, Y, CX, CY, THETA)
REAL X(NRAY), Y(NRAY), CX(NRAY), CY(NRAY)

C ROTATE ALL RAYS BY THETA ABOUT THE Z AXIS
CT = COS(THETA)
ST = SIN(THETA)
DO J = 1, NRAY
  XX = X(J) + CX(J)
  YY = Y(J) + CY(J)
  T  = XX*CT - YY*ST
  YY = YY*CT + XX*ST
  XX = T
  T  = X(J)*CT - Y(J)*ST
  Y(J) = Y(J)*CT + X(J)*ST
  X(J) = T
  CX(J) = XX - X(J)
  CY(J) = YY - Y(J)
END DO
RETURN
END

FUNCTION SMOMENT(X, N)
REAL X(N)

C SX = 0.0
DO J = 1, N
  SX = SX + X(J)
END DO
XM = SX / N
SX2 = 0.0
DO J = 1, N
  SX2 = SX2 + (X(J) - XM)**2
END DO
SMOMENT = SQRT(SX2 / N)
RETURN
END
Table C.1: MTF versus Focal Plane Location at 40 lp/mm

<table>
<thead>
<tr>
<th>Location (mm)</th>
<th>MTF</th>
<th>Location (mm)</th>
<th>MTF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ON AXIS WITH NO TILT</strong></td>
<td></td>
<td><strong>OFF AXIS WITH NO TILT</strong></td>
<td></td>
</tr>
<tr>
<td>-0.1000,</td>
<td>0.7660</td>
<td>-0.1000,</td>
<td>0.7651</td>
</tr>
<tr>
<td>-0.0750,</td>
<td>0.8633</td>
<td>-0.0750,</td>
<td>0.8628</td>
</tr>
<tr>
<td>-0.0500,</td>
<td>0.9376</td>
<td>-0.0500,</td>
<td>0.9374</td>
</tr>
<tr>
<td>-0.0250,</td>
<td>0.9842</td>
<td>-0.0250,</td>
<td>0.9841</td>
</tr>
<tr>
<td>0.0000,</td>
<td>1.0000</td>
<td>0.0000,</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.0250,</td>
<td>0.9841</td>
<td>0.0250,</td>
<td>0.9840</td>
</tr>
<tr>
<td>0.0500,</td>
<td>0.9376</td>
<td>0.0500,</td>
<td>0.9373</td>
</tr>
<tr>
<td>0.0750,</td>
<td>0.8633</td>
<td>0.0750,</td>
<td>0.8626</td>
</tr>
<tr>
<td>0.1000,</td>
<td>0.7660</td>
<td>0.1000,</td>
<td>0.7649</td>
</tr>
<tr>
<td><strong>ON AXIS WITH 3 DEGREE TILT</strong></td>
<td></td>
<td><strong>OFF AXIS WITH 3 DEGREE TILT</strong></td>
<td></td>
</tr>
<tr>
<td>-0.1000,</td>
<td>0.7291</td>
<td>-0.1000,</td>
<td>0.6838</td>
</tr>
<tr>
<td>-0.0750,</td>
<td>0.8306</td>
<td>-0.0750,</td>
<td>0.7817</td>
</tr>
<tr>
<td>-0.0500,</td>
<td>0.9108</td>
<td>-0.0500,</td>
<td>0.8591</td>
</tr>
<tr>
<td>-0.0250,</td>
<td>0.9645</td>
<td>-0.0250,</td>
<td>0.9110</td>
</tr>
<tr>
<td>0.0000,</td>
<td>0.9884</td>
<td>0.0000,</td>
<td>0.9341</td>
</tr>
<tr>
<td>0.0250,</td>
<td>0.9810</td>
<td>0.0250,</td>
<td>0.9268</td>
</tr>
<tr>
<td>0.0500,</td>
<td>0.9426</td>
<td>0.0500,</td>
<td>0.8896</td>
</tr>
<tr>
<td>0.0750,</td>
<td>0.8758</td>
<td>0.0750,</td>
<td>0.8250</td>
</tr>
<tr>
<td>0.1000,</td>
<td>0.7848</td>
<td>0.1000,</td>
<td>0.7372</td>
</tr>
<tr>
<td><strong>ON AXIS WITH 6 DEGREE TILT</strong></td>
<td></td>
<td><strong>OFF AXIS WITH 6 DEGREE TILT</strong></td>
<td></td>
</tr>
<tr>
<td>-0.1000,</td>
<td>0.6156</td>
<td>-0.1000,</td>
<td>0.4681</td>
</tr>
<tr>
<td>-0.0750,</td>
<td>0.7237</td>
<td>-0.0750,</td>
<td>0.5589</td>
</tr>
<tr>
<td>-0.0500,</td>
<td>0.8162</td>
<td>-0.0500,</td>
<td>0.6372</td>
</tr>
<tr>
<td>-0.0250,</td>
<td>0.8873</td>
<td>-0.0250,</td>
<td>0.6974</td>
</tr>
<tr>
<td>0.0000,</td>
<td>0.9323</td>
<td>0.0000,</td>
<td>0.7355</td>
</tr>
<tr>
<td>0.0250,</td>
<td>0.9484</td>
<td>0.0250,</td>
<td>0.7487</td>
</tr>
<tr>
<td>0.0500,</td>
<td>0.9344</td>
<td>0.0500,</td>
<td>0.7362</td>
</tr>
<tr>
<td>0.0750,</td>
<td>0.8913</td>
<td>0.0750,</td>
<td>0.6988</td>
</tr>
<tr>
<td>0.1000,</td>
<td>0.8218</td>
<td>0.1000,</td>
<td>0.6391</td>
</tr>
</tbody>
</table>
Appendix C

3. MTF Profile for an Abberation-Free System

If no aberrations are present in an optical system, the MTF profile is governed by diffraction effects (see ref. 1).

Equations C.2 through C.4 are taken from reference 1, page 318.

The following FORTRAN Program computes the MTF profile for an aberration-free system based upon an f/8.0 lens and the worst case wavelength of light (700 nm).

Limiting Resolution \( (v_o) = \frac{1}{\text{wavelength} \times (f/#)} \) \hspace{1cm} (C.2)

\[ = \frac{1}{(0.000000700 \times 8.0)} \]

\[ = 178571.5 \text{ line pairs per meter} \]

\[ = 178.6 \text{ lp/mm} \]

\[
\text{MTF}(v) = \frac{2}{\pi} \times (\psi - \cos(\psi) \times \sin(\psi)) \]

\hspace{1cm} (C.3)

Where \( v \) represents a fraction of the limiting resolution, and

\[
\psi = \cos^{-1} \left( \frac{v}{v_o} \right) \]

\hspace{1cm} (C.4)

```
REAL*B MTF,PSI,V,VO,VDVO,THETA,A
OPEN(4,CARRIAGECONTROL='LIST',STATUS='NEW',NAME='PLOT2D.DAT')

WRITE(6,100)
100 FORMAT(X,'MTF OF AN ABBERATION FREE SYSTEM','/,' LIMITING RESOLUTION = 178 lp/mm','/)
WRITE(6,101)
101 FORMAT(X,'FREQUENCY MTF ','/)

DO 800 V = 0.00,178571.5,8928.57
A = 0.00000056*V
PSI = DACOS(A)
MTF = 0.636620 * (PSI - COS(PSI)*SIN(PSI))
VDVO = 178.5715*A / VO

300 WRITE(6,103)VDVO,MTF
103 FORMAT(X,F7.1,F18.6)
303 WRITE(4,200)VDVO,MTF
800 CONTINUE
WRITE(4,301)
200 FORMAT(X,F10.5,','F15.10)
301 FORMAT('END')
```

99 STOP
END
Table C.2: MTF Profile for an Abberation-Free System

<table>
<thead>
<tr>
<th>FREQUENCY (1p/mm)</th>
<th>MTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.000000</td>
</tr>
<tr>
<td>8.9</td>
<td>0.936365</td>
</tr>
<tr>
<td>17.9</td>
<td>0.872869</td>
</tr>
<tr>
<td>26.8</td>
<td>0.809733</td>
</tr>
<tr>
<td>35.7</td>
<td>0.747060</td>
</tr>
<tr>
<td>44.6</td>
<td>0.685038</td>
</tr>
<tr>
<td>53.6</td>
<td>0.623838</td>
</tr>
<tr>
<td>62.5</td>
<td>0.563640</td>
</tr>
<tr>
<td>71.4</td>
<td>0.504632</td>
</tr>
<tr>
<td>80.4</td>
<td>0.447014</td>
</tr>
<tr>
<td>89.3</td>
<td>0.391002</td>
</tr>
<tr>
<td>98.2</td>
<td>0.336830</td>
</tr>
<tr>
<td>107.1</td>
<td>0.284757</td>
</tr>
<tr>
<td>116.1</td>
<td>0.235075</td>
</tr>
<tr>
<td>125.0</td>
<td>0.188121</td>
</tr>
<tr>
<td>133.9</td>
<td>0.144294</td>
</tr>
<tr>
<td>142.9</td>
<td>0.104088</td>
</tr>
<tr>
<td>151.8</td>
<td>0.068148</td>
</tr>
<tr>
<td>160.7</td>
<td>0.037386</td>
</tr>
<tr>
<td>169.6</td>
<td>0.013320</td>
</tr>
<tr>
<td>178.6</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
Appendix C

4. Calculation of MTF versus Tilt values

The following values were used to plot the MTF profiles of Figure 11. A form of Equation B-2, based on the Second Moment (shown as $S$ in Equation C.5) was used to calculate the values shown below. The Second Moment ($S$) is simply the statistical standard deviation between the resulting location of the image data points and the ideal or perfect location as calculated by the FORTRAN computer program in Appendix C starting on page C.4.

Since the MTF is known at 40 lp/mm for each of the positions listed in Table C.3 below (See Table C.1, page C.9), the Second Moment ($S$) can be calculated. This quantity remains constant for each position. Once the Second Moment ($S$) is found, various frequencies can be substituted into Equation C.5, to obtain a profile of MTF versus Frequency.

From Appendix B, page B.2:

$$MTF(f) = \cos^2 (1.4142 \cdot \pi \cdot f \cdot 0.28 \cdot \Delta X / (f' / #))$$

(B.2)

Here, the Second Moment ($S$) is utilized:

$$MTF(f) = \cos^2 (1.4142 \cdot \pi \cdot f \cdot S)$$

(C.5)

Table C.3: Calculation of MTF versus Tilt values

<table>
<thead>
<tr>
<th>Position</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>On-Axis 0° Tilt</td>
<td>1.00</td>
</tr>
<tr>
<td>On-Axis 3° Tilt</td>
<td>1.00</td>
</tr>
<tr>
<td>On-Axis 6° Tilt</td>
<td>1.00</td>
</tr>
<tr>
<td>Off-Axis 0° Tilt</td>
<td>1.00</td>
</tr>
<tr>
<td>Off-Axis 3° Tilt</td>
<td>1.00</td>
</tr>
<tr>
<td>Off-Axis 6° Tilt</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Appendix D: Optimization of Parameters

Although all of the parameters listed below individually have a range of acceptable values, the combination of each parameter into an acceptable and workable system provided little room for variation. The possibility of custom making and tuning each component still exists, but not without the substantial cost penalty associated with customized equipment. In the interest of developing a system from currently available stock components, the values shown below have been selected. Recommendations for further optimization of the IMC device can be found on page 37.

The Glass Element

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (t)</td>
<td>Directly affects the inertia, offset vs. tilt, torque requirements, depth of focus and optical aberrations. Desire to make the cross-section a square for potentially extending the life in the event of a scratched surface. Values between 0.30&quot; and 0.60&quot; are required to create the necessary $\Delta D / \Delta \theta$ ratio.</td>
</tr>
<tr>
<td>Index of Refraction (N)</td>
<td>Directly related to the V-number of commercially available glass (see Fig. D.1). Also affects the depth of focus, optical aberrations and the $\Delta D / \Delta \theta$ ratio. Chosen in conjunction with the highest V-number possible.</td>
</tr>
<tr>
<td>V-number (V)</td>
<td>Directly related to N. Directly affects optical aberrations including chromatic aberration. Should be maximum for the glass chosen.</td>
</tr>
<tr>
<td>Tilt Angle (θ)</td>
<td>Directly related to image offset, torque, $\Delta D / \Delta \theta$ ratio and life of the flexures. Should be between 0.5 and 10 degrees, and as small as possible.</td>
</tr>
</tbody>
</table>
The Flexures

**Torsional Spring Rate** 
Directly affects the frequency response of the IMC system. Also affects the torque requirements. $k_T$ must be chosen so that the natural frequency of the system is above 60 Hz and within the torque capabilities for the motor selected.

**Mass Moment of Inertia** 
Some minor affects on the torque requirements of the system, although only minor changes in inertia are produced with the various size flexural pivots.

**Life** 
Related to the maximum angle $\theta$. Less than 6.25° is considered acceptable for infinite life.

**Deflection** 
The deflection of the center-line of the pivot is directly related to the maximum angle of rotation and the flexure length. Information from the manufacturer indicates that deflections of $0.1\% \times$ O.D. of the flexure are present near 6°. Motor requirements specify a maximum center shift of 0.003" ($\pm 0.0015$").
Optimization of Parameters, continued

The Motor

Torque \( (T) \)

The torque demand on the motor is a function of the total inertia of the system, the \( \Delta D / \Delta \theta \) ratio and the \( k_T \) of the flexures. Although a larger motor could have been specified to build a workable system with considerably larger inertia, the factor of cost would have also increased.

Inertia \( (J_m) \)

The internal inertia of the motor directly affects the torque needed in the system. As progressively larger motors are selected to meet torque requirements, the inertia also increases, and not always proportionally.

The criteria for optimizing the above parameters is based on the following considerations:

1. Optical Offset must be 0.015" minimum.
2. Total system inertia should be minimized.
3. Flexure spring rate is chosen so that the undamped natural frequency of the system is above 60 Hz, and so that the cycle life rating of the flexure is infinite for the angular rotation required.
4. Peak Torque Rating for the motor cannot be exceeded. See Figure 15 for System Response Capabilities.
Appendix E: Glossary of Terms

The following definitions have been compiled to clarify terms used within the main body of the report. For further information consult the references listed on page 40.

Abbe V-number

Also known as the reciprocal relative dispersion. A measure of difference in the index of refraction for a material at specified frequencies of the visible light spectrum.

\[
V = \frac{N_D - 1}{N_F - N_C}
\]

\(N_D\) = index at 0.5893 microns
\(N_F\) = index at 0.4861 microns
\(N_C\) = index at 0.6563 microns

CCD

Charge-Coupled Device. A photocell arrangement which resolves an image into a series of picture elements (pixels). A voltage proportional to the density of the image is formed at each element.

First-Surface Mirror

Also known as a front-surface mirror. A mirror produced by a reflective coating on the exterior surface of a substrate material (most commonly glass). Used extensively in optical systems to avoid additional diffraction in the image path.

MTF(f)

Modulation Transfer Function (as a function of frequency). A ratio of the resultant image to the original object according to the frequency content of the object. MTF is a useful measure for comparing the performance of many types of optical systems and films.

N

Index of Refraction. A ratio of the velocity of light in a vacuum to the velocity in a specified material.

Second Moment

Also known statistically as the standard deviation. A numerical measure of the square of the distance between a series of points and a reference position. Mechanically analogous to the method for calculating the Moment of Inertia for an area.
Slit Width

The width of the image path being formed by the optical system at the object or image plane. Directly contributes to the overall correction range of the IMC device and the illumination power required. See Figure 14.

SQF

Subjective Quality Factor. An optical merit function which relates the optical capability of a system to the performance of the human eye.