Analysis of translating hydrofoil power generation systems (hydrokites)

Kelsey McConnaghy

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Analysis of Translating Hydrofoil Power Generation Systems (Hydrokites)

by

Kelsey McConnaghy

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering

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Abstract

Analysis of Translating Hydrofoil Power Generation Systems
(Hydrokites)

Kelsey McConnaghy

Supervising Professor: Dr. Mario W. Gomes

The hydrokite is a novel hydro-power system that is based on emerging kite wind-energy systems which are currently being designed for use at high altitudes. The hydrokite system is comprised of a hydrofoil and a support system, and is designed to capture kinetic energy from the flow of a river while reducing negative impacts on the river ecology by minimally interfering with the river's natural flow (i.e., no dams or river diversions are needed). This work presents some initial results which demonstrate the power performance capabilities of the hydrokite. Two different steady-state models for this system were studied to determine the effects of model parameters on power generation. A dynamic model was also developed and preliminary results are presented. These simplified initial models provide an upper bound for the power performance of an actual system as well as providing an understanding of the effects that parameter changes have on the system performance. This initial work shows that such a system could be a feasible, low impact method for generating renewable energy from low-head hydro sources.
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Nomenclature

General

- $A_k$: Hydrofoil (or planform) Area
- $AR$: Aspect Ratio of the hydrofoil
- $\alpha$: Angle of Attack
- $C_l$: 2D Lift coefficient \textit{(i.e. for an infinite wing)}
- $C_d$: 2D Drag coefficient \textit{(i.e. for an infinite wing)}
- $C_m$: 2D Moment coefficient \textit{(i.e. for an infinite wing)}
- $C'_L$: 3D Lift coefficient \textit{(i.e. for a finite wing)}
- $C'_D$: 3D Corrected Drag Coefficient \textit{(i.e. for a finite wing)}
- $C'_M$: 3D Moment Coefficient \textit{(i.e. for a finite wing)}
- $c$: Chord Length
- $F_L$: Magnitude of lift force
- $F_D$: Magnitude of drag force
- $L/D$: Lift to Drag Ratio
- $\hat{\lambda}_d$: Unit Vector in the Direction of the Lift Force
- $\hat{\lambda}_l$: Unit Vector in the Direction of the Drag Force
- $M$: Magnitude of the moment on the hydrofoil from the fluid
• $P$: Instantaneous power

• $P_{cyc}$: Cycle power

• $\tau$: Torque acting on the system

• $\mu$: Viscosity of water

• $\rho$: Density of the water

• $s$: Hydrofoil Span

• $V_a$: Apparent Velocity of the fluid as seen from the hydrofoil

• $V_\infty$: River Velocity relative to the stationary river bank

• $V_k$: Instantaneous velocity of the kite

• $X_d$: Subscript denotes the deploy stroke

• $X_r$: Subscript denote the return stroke

**Steady-State Translating Model**

• $\beta$: Angle defined from the horizontal axis to center axis of hydrofoil

• $\gamma$: Angle from the horizontal axis to the apparent velocity vector

• $F_{rail}$: Normal Force on the Rail

• $l$: Length (along the rail) that the hydrofoil travels from starting position to the first flipping position

• $\hat{\lambda}_r$: Unit Vector in the Direction of the Rail

• $r_{len}$: River width in meters

• $t$: Total time for the stroke
• $\phi$: Angle of the fixed rod across the river relative to the river bank (flow velocity vector)

**Steady-State Rotating Model**

• $\beta$: Angle defined from the boom’s axis to center axis of hydrofoil

• $\gamma$: Angle from the boom’s axis to the apparent velocity vector

• $\lambda_g$: Unit vector in the perpendicular direction to the boom

• $\lambda_b$: Unit vector in the direction of the boom

• $L$: Boom length in meters

• $\theta$: Angle between flow velocity vector and the boom arm

• $k_g$: Generator constant

**Dynamic Rotating Model**

• $m_b$: Mass of the boom

• $m_k$: Mass of the kite (or hydrofoil)

• $I_b$: Mass Moment of Inertia of the boom about the pivot point

**Hydrodynamic Model**

• $a_o$: The slope of the lift curve for an infinite wing

• $a$: The slope of the lift curve for a finite wing

• $e$: Span Efficiency Factor (Oswald Efficiency)

• $e_1$: Span Effectiveness Factor for Lift Calculations (different from Oswald Efficiency)
Chapter 1

Introduction

1.1 Motivation

For thousands of years the world has been harnessing renewable energy resources in order to make improvements in the quality of life. From using the sun to grow and dry food, using windmills to pump water or grind grain and using water to transport materials and people, these three natural resources (sun, wind and water) were the first power sources ever utilized. Even though these alternative energy sources are still used today for many of the same tasks, fossil fuels have become a more common mean of achieving these same goals. The unintended, cumulative environmental impact and the dwindling supply of fossil fuels has created a renewed interest in harnessing renewable energy. By focusing on the creation of new systems that generate power from renewable energy sources, more options other than fossil fuels will become increasingly available for people to utilize.

Hydro-power is a renewable energy source that has been used for thousands of years. Large hydroelectric dams are the most common form of energy extraction and in 2010 the US Energy Information Administration (EIA) reported that hydroelectric power was responsible for 31% of the totally renewable energy usage here in the United States, which only provides 8% of the total energy the country consumes [19]. This means that hydroelectric power accounts for less than 2.5% of the total consumption in America.
From the US Geological Survey, the total hydro-electric storage capacity of the United States has been recorded throughout the past 200 years. There was a very large increase from 1940 to 1980 when urbanization was dramatically increasing and the demand for power grew exponentially. This also was the time period where the majority of the hydroelectric power sources (dams) were constructed, along with their associated storage capacity, in the United States. Since 1980 there has only been an incremental increase in the acres of storage capacity for hydroelectric power. This is not because we do not need more electricity, the population continues to expand and the demand for energy increases along with it, it is due to the limited amount of resources that are available for large scale facilities. According to the U.S. Geological Survey the “trend for the future will probably be to build small-scale hydro plants that can generate electricity for a single community” [18]. This is where micro-hydroelectric power systems will become a more prevalent type of energy source. New and innovative systems will be needed to capture energy form these low head hydro resources, which is where a system such as a hydrokite could be implemented.

1.1.1 Local Application

The city of Rochester was originally designed around the Genesee River and power was extracted to grind grain into flour in the early 1800’s. The city and surrounding towns all originally relied on the river for processing goods and transporting them to surrounding towns. The Genesee River is about 140 miles long, beginning in Pennsylvania and ending in Lake Ontario in Rochester, New York [4]. This river currently does not have a large hydroelectric dam, most likely due to the fact it is not financially feasible, however smaller hydroelectric systems could be utilized in this area. Since the Rochester Institute of Technology (RIT) is also located next to the Genesee River, it was used as a reference for some of the parameters.

Data for the Genesee River is available from the New York State Department
Table 1.1: Genesee River velocity data provided by the NYS DEC. Data taken from the USGS-NY station at Ford Street

<table>
<thead>
<tr>
<th>Month</th>
<th>Minimum Velocity</th>
<th>Average Velocity</th>
<th>Maximum Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td>0.007 m/s</td>
<td>0.249 m/s</td>
<td>1.240 m/s</td>
</tr>
<tr>
<td>March</td>
<td>0.296 m/s</td>
<td>0.618 m/s</td>
<td>0.979 m/s</td>
</tr>
<tr>
<td>April</td>
<td>0.181 m/s</td>
<td>0.617 m/s</td>
<td>1.376 m/s</td>
</tr>
</tbody>
</table>

of Environmental Conservation (NYS DEC). The DEC has recorded discharge and stream velocity for multiple river locations. The river velocity data was requested from the Ford Street location for the months of February, March and April of 2011 and this data is shown in Figure 1.1. These months were chosen because they are the months in Rochester, New York when there is a large amount of snow melt and rain and a corresponding large fluctuation in river velocity. The river velocities ranged between 0.006 to 1.37 m/s [25] After looking at the graph, it is clear that there is a drastic increase in the river velocity towards the end of February so the monthly averages were also calculated, so that the change in velocity could be seen. These values are shown in Table 1.1. For simplicity, the river velocity for the different models studied was assumed to be 1 m/s, even though this is higher than the monthly averages.

It is also assumed that the velocity profile is perfectly uniform throughout the entire cross section even though this is not the case in nature. This assumption was also made in order to simplify the model. As seen in Figure 1.2 the true flow of a river the “velocities in (the) river cross-section reduce to zero values at the bottom and sides, and are maximal at the surface in the center of the channel” [24]. Therefore, in order to properly calculate the river velocity profile near the gage station at Ford Street, the geometry of the river must be known, as well as the velocity at multiple points throughout the river, which is outside the scope of this work.
Figure 1.1: Three months of river velocity data from the DEC for February, March and April of 2011. This data was recorded in 15 minute increments at the USGS-NY station at Ford Street in Rochester, NY [25]
1.2 Background

The design of the hydrokite system was inspired by high altitude wind power generation systems. The hydrodynamic system is comprised of a hydrofoil and a support, and is designed to capture kinetic energy from the flow of a river. In this system, the motion of the hydrofoil is coupled to a mechanical generator so when the hydrofoil moves it creates mechanical power by generating torque. High altitude kite power has theoretically shown that significant amounts of energy can be captured when these are introduced into the upper atmosphere. There have also been successful experimental prototypes created but most have not yet been tested in altitudes that exceed 200 m. Since this idea looks to be very promising but very difficult to implement, adapting it to perform in a smaller scale water environment could be more feasible. By using theoretical models and computer simulations, the power generation characteristics of a hydrokite model can be predicted.

Two technologies which would be similar to the hydrokite are high altitude kites and oscillating wings. Oscillating wings are airfoils or hydrofoils that are connected to a rigid support and have a prescribed pitching and plunging motion which capture energy from an incoming stream velocity. Both of these systems have theoretically shown that large amounts of energy can be captured when these are introduced into
fluids with high velocities. However, both of these systems have had issues being implemented because of different concerns. The high-altitude kites have had successful experimental prototypes created but have not yet been tested in high altitudes. Less work has been done on the oscillating wings, however all of the tested models have cited a disconnect between the theoretical predictions and the experimental results. The reason that is provided for this discrepancy is that the theoretical models and simulations do not include the mechanical components which prescribe the motion of the wings. The energy losses caused by the mechanical system dramatically decrease the efficiency of the oscillating wing system.

1.3 Research Goals

The objective of this work is to answer some of the following research questions:

- What are the power generation characteristics of a hydrokite model and how can a feasible system be achieved?

- What is the sensitivity of the system’s power generation with regards to parameter changes?

- How will different models of power capture affect the limits of power generation?

Originally the possible quantifiable environmental effects of this new technology were going to be explored, this however, was removed from the scope of this thesis.

1.4 Literature Review

There have been many articles, dissertations and conference papers that discuss the use of wings in the air and in the water to generate power. The works that have been investigated examine both applications because of their similarities with the proposed hydrokite system.
1.4.1 Kite Models

For the majority of models in air applications, the wing was analyzed as a system that is connected to a tether that is fixed to the ground. The models were developed and the equations of motion were then derived. The tether and plane of motion are the elements that distinguish the two dimensional system models the most. The simplest tether model is a massless, dragless, straight, inelastic tether. More complicated models could take into account the mass of the tether, its elasticity or flexibility. Also the effects of wind drag on the tether can either be neglected or accounted for. The complexity of the tether model can greatly affect the form and complexity of the governing equations used in the model.

Loyd

Figure 1.3: Figure A is the Simple Kite Model. Figure B is the schematic for the crosswind kite power model and the drag power kite model. Figure C is a 3-Dimensional drag power example. The following parameters are defined as follows: $V_w$, Wind Velocity; $V_L$, Load Velocity; $V_a$, Relative Velocity in Air; $V_c$, Velocity Crosswind; $T$, Tether Tension; $L$, Lift of Kite; $D$, Total Drag.[13]

Loyd [13] wrote one of the first published technical articles discussing the application of kites in order to generate large amounts of power. This model is referenced by many other authors within this field and has become a base for many other analyses and experiments. Loyd develops three simple, steady-state kite power models,
and provides the predicted tensions and average power outputs for each. The three models discussed in this article are the simple kite, the crosswind motion kite, and the drag power kite. The models are shown in the first two images of Figures 1.3A and 1.3B. Figure 1.3C shows the schematic of the 3-dimensional airfoil that was used for the power calculations. All models are air applications with a massless simple kite and have an inelastic massless tether. These models are also assumed to be at steady-state as well as have a lift coefficient that is constant, and not determined by the angle of attack. The model parameters were the wing area, the lift-to-drag ratio, the strength-to-weight ratio, and the coefficient of lift.

The analysis begins for each with the calculation of the power produced for each model.

\[ P = \frac{1}{2} \rho C_L A V_w^3 F \]  \hspace{1cm} (1.1)

In this equation \( A \) is the wing reference area, \( C_L \) is the coefficient of lift, \( V_w \) is the wind speed and \( F \) is a variable that represents a specific model. The function \( F \) was used in order to compare each of the models after the system of equations were derived by determining the value of \( F \) that incorporated all of the characteristics of that model. These equations provide an upper bound for the amount of power that could be collected using kites. The model that most clearly relates to the proposed body of work is the crosswind kite, drag power kite model which is shown in Figure 1.3. For this model the power was represented by Eq. (1.2).

\[ P = L V_L \]  \hspace{1cm} (1.2)

Because the system was analyzed with the assumption of static equilibrium, similar triangles are found between the forces and the velocity vectors. Also, the assumption that \( L/D_K \) is a very large value, means that the relative velocity through the air and the crosswind velocity are determined to be equivalent. The function for both is given
in Eq.(1.3) which is then substituted into the equation for lift resulting in Eq. (1.4).

\[ V_A = V_C = \left( \frac{L}{D_K} \right) (V_W - V_L) \]  

(1.3)

\[ L = (1/2)\rho C_L A(V_W - V_L)^2(L/D_K)^2 \]  

(1.4)

When the equation for the power production of a cross wind kite model is substituted into the equations for all kite models and simplified the function F is determined to be:

\[ F_C = (L/D_K)^2(V_L/V_W)(1 - V_L/V_W)^2 \]  

(1.5)

When the ratio \( V_L/V_W = 1/3 \) the maximum value of \( F_C \) is achieved, and this simplifies to:

\[ F_C = (4/27)(L/D_K)^2 \]  

(1.6)

The equations of motion for the kite are given in three dimensions which are used to calculate a three dimensional drag force at a steady-state. This was done in order to provide the calculation of the tension and the average power output for the different inputs. For the conclusion, the drag power kite model was evaluated using a C-5A model aircraft, as seen in Figure 1.3. A table is provided with example calculations that include the weight of the tether and it states that the average power output would be 6.7 MW for the smallest kite. Also, the peak tether tension would be 3.2 MN for a wing with an area of 576 m\(^2\) and a tether length of 400 m. It is mentioned that the gross takeoff weight of the aircraft would be 3.4 MN which is just slightly more than the peak tether tension. This proves that the wing structure is capable of withstanding the proposed stresses [13]. It is also stated that with the strength and size of the proposed aircraft for these systems, a single machine’s output is about three times that of a modern wind turbine. In today’s standards this power output would be more than two Vestas V90 3.0 MW turbines that have a 45 m blade length
Loyd’s models are very helpful as a starting point for this work, however these models do not account for tether drag. Also the moment coefficients and transient information were not used in these models since they were analyzed at a single point of static equilibrium.

Goela

Goela [6] analyzed a system which used a kite, a pulley, and a water reservoir instead of a power generator. The model is very different than a typical system analysis because the power is not measured electrically but mechanically. The analysis has an ascent stroke where the water is lifted in a container and a descent stroke where the container’s weight pulls the kite downward.

Goela includes the weight of the tether per unit length and the weight of the kite in his analysis. This seems to be more practical since a real world scenario will be affected by these forces. The relative velocity of the kite with respect to the incoming
wind was given by Eq. (1.7). The lift force on the kite and the drag force on the kite were given by Eqs. (1.9) and (1.10) respectively where $A_k$ is the characteristic area of the kite and $\rho$ is the density of the air.

\[ V_R^2 = V^2 + V_K^2 - 2VV_K\cos(\beta) \]  
(1.7)

\[ \tan(\phi) = \frac{V_K\sin(\beta)}{V - V_K\cos(\beta)} \]  
(1.8)

\[ F_{LK} = (1/2)\rho V_R^2 C_L A_K \]  
(1.9)

\[ F_{DK} = (1/2)\rho V_R^2 C_D A_K \]  
(1.10)

The final governing equations were derived from Newton’s Second Law of Motion using the diagram in Figure 1.4 and the lift and drag forces. In the governing equations it is assumed that the tether is straight, and the pulley is frictionless. This system of equations was solved numerically using a Runge-Kutta numerical integration method.

\[ (1/g)(W_K + w_t(L_1 + L_2) + W_L)\frac{dV_1}{dt} = \]

\[ F_{LK}\sin(\theta + \phi) + F_{DK}\cos(\theta + \phi) - (W_K + w_tL_1)\sin(\theta) + F_{Di}\cos(\theta) \]

\[ - F_{Di}\sin(\theta) - (W_K + \frac{w_tL_1}{2})(\frac{V_2^2}{gL_1} - W_La - w_tL_2) \]  
(1.11)
In order to maximize the average power this system could generate, the power generated during the ascent stroke must be maximized and the power needed for the descent stroke must be minimized. To do this the lift to drag ratio were initially assumed constant. During the ascent stroke the lift to drag ratio is equal to 6.5, whereas on the descent stroke it is equal to 2.0. It was also assumed that the kite begins and ends one cycle at the same point. In a later analysis, the descent lift to drag ratio is varied from 0.985 to 2.4 with a corresponding increase in the descent load. The kite’s path using this variation is shown in Figure 1.5. It depicts that the change in the descent lift to drag ratio does not greatly impact the overall path of the kite.
As expected, Goela concludes that the maximum power is produced on the ascent stroke. However, in this configuration periodic motions of the pump may not be feasible. “To operate the kite pump for an extended period of time requires that the tether angle of inclination lie within the range covered by the two equilibrium tether angles which correspond to the maximum and minimum $C_L/C_D$ of the kite during the cycle” [6]. Without this limitation the system would require variable lift to drag ratio for the ascent and descent stroke and would not be assumed to be in equilibrium.

**Argatov**

Lansdorp and Williams from Delft University as well as Argatov and Silvennoinen from Tampere University have analyzed a pumping kite system with the tether attached to a drum and generator. Both groups have published numerous articles relating to the theoretical models and some experimental work. Their models move in three dimensions and include transient behavior. The overall system concept is shown in Figure 1.6. Argatov and Silvennoinen used Newton’s law of motion in spherical coordinates to derive the governing Eqs. (1.13) to (1.16), for the kite where
m is the mass of the kite, and $\theta$ is the angle the tether forms with the vertical [2]. The coefficients of lift and drag are assumed constant and in order to complete the simulation, the motion of the tether was prescribed. The maximum power equation for this system is given by Eq. (1.16).

$$F_r = m(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin(\theta)^2)$$  \hspace{1cm} (1.13)

$$F_\theta = m(r\ddot{\theta} - r\dot{\phi}^2\sin(\theta)\cos(\theta) + 2\dot{r}\dot{\theta})$$  \hspace{1cm} (1.14)

$$F_\phi = m(r\dot{\phi}\sin(\theta) - 2r\dot{\theta}\dot{\phi}\cos(\phi) + 2\dot{r}\dot{\phi}\sin(\theta))$$  \hspace{1cm} (1.15)

$$P_{max} = \frac{1}{2} \rho AC_L V^3 \cos(v^*)^3 (4/27) G_e^2$$  \hspace{1cm} (1.16)

"Here, $v^*$ is the mean angle of inclination of the kiteline with respect to the horizon. Note that $v^* = \pi/2 \theta^*$, where $\theta^*$ is the mean angle that the kiteline forms with the vertical" [2]. Where $G_e$ is the effective glide ratio and given by:

$$G_e = \frac{C_L}{C_D + \frac{C_{L,r}}{4A}}$$  \hspace{1cm} (1.17)

**Lansdorp and Williams**

Lansdorp and Williams have also studied a pumping kite system like the one shown in Figure 1.6. The original concept for their work was a laddermill kite system which is a series of connected kites that consists of an endless tether (or a loop) with many wings attached to it, see Figure 1.7. The second concept was a pumping mill that consists of a single ended tether with multiple wings. However, in order to gain a better understanding of kite systems, Lansdorp and Williams initially modeled a single kite pumping mill. The kite was modeled as a flat plate as seen in Figure 1.8.
The equations of motion for the system were derived using Lagrange’s equations (Eq. (1.18)).

\[
\frac{d}{dt} \left( \frac{d\mathcal{L}}{dq} \right) - \frac{\mathcal{L}}{dq} = Q
\]  

(1.18)

\[
m_o = m_0^0 - \rho c L
\]  

(1.19)

The assumptions which comprise the model are, the tether is assumed to be rigid with uniform density, \( \rho_c \), throughout, the ground is flat (\( z = 0 \)), and the base of the system is acted on by a rolling resistance. The kite is attached to a moving ground vehicle and the instantaneous mass of the vehicle is given by (1.19) where \( L \) is the length of the deployed tether [27] and as seen in Figure 1.9, the kite’s ascending stroke is predicted to spiral upwards and the descending stroke would bring the kite back towards the ground. Two different models of this kite were studied. One that had a fixed tether length and one that had a variable tether length. MATLAB and SNOPT were used to optimize the kite trajectory for the maximum power generation in both models for crosswind motions with average velocities ranging from 10 to 15 m/s, see
Overall, the results showed that for crosswind towing, high wind speeds produce more useful work [10]. "In the case of optimal power generation, highly complex trajectories were generated as a function of wind strength. The results show that the kite makes very efficient use of cross-wind motions to generate significantly higher aerodynamic forces, which results in large tensions, demonstrate that the average power generated by the system increases with the cube of the wind velocity." [10]. One concern for this model was the presence of unsteady wind velocities that are not represented in the figures above. In order to avoid this problem a feedback control system was simulated under variable wind conditions. The feedback system is also used for tracking and stability of crosswind motions. This simulation showed that the designed system was able to track the kite motion extremely well, however substantial changes to wind speed and/or direction become very difficult to track. "The results illustrate that it is necessary to be able to have sufficient control of the kite forces to
obtain tight trajectory following capability” [10].

From the numerical analysis Landsdorp, Remes and Ockels [10] designed an experiment to test a remote controlled surfkite to determine if it could be applied in the laddermill application. Using a Peter Lynn Bomba surf kite [11] that was connected with control lines and drag flaps for steering the kite. This set-up did not include steering lines which makes the kite much simpler to control, however decreases the amount of power that could be generated because the angle of attack can not be adjusted. The kite was tested at a variety of different wind speeds and it was observed that the kite became more stable at higher wind speeds. It was also determined during these experiments, at a velocity of 20 km/h that the time needed to steer the kite $45^\circ$ was approximately 3 secs, giving an angular acceleration of 0.17 rads/sec. It was predicted that the angular acceleration of the kite was 0.25 rads/sec. It is concluded that the kite is stable because of the low lift to drag ratio of the of kite used in the
experiment. This is a helpful when it comes to safety concerns, however a higher lift to drag ratio would improve the tension ratio in the tether and increase the power output. Other strategies (for increasing power output) could be reducing the effective surface of the kite, changing the wing profile of the kite and reducing the angle of attack. [11]

1.4.2 Oscillating Wing Models

There are fewer papers than seen for the high altitude kite that describe the use of oscillating wings for power generation. Most of the work that was reviewed is by Platzer, Jones and Lindsey from the Naval Post Graduate School.

McKinney

McKinney explored the mechanical aspects of the Oscillating-Wing windmill (wingmill) and provided a recommendation that the wingmill could be a viable technology worth pursuing for large scale power generation.
The mean power output for this model can be represented as:

\[ P \cong N\dot{h} + M\dot{\alpha} \]  \hspace{1cm} (1.20)

\[ h = h_o \sin(\omega t) \]  \hspace{1cm} (1.21)

\[ \alpha = \alpha_o \sin(\omega t + \phi) \]  \hspace{1cm} (1.22)

Where \( N \) is the normal force, \( M \) is the moment, \( h \) is the vertical translation over time and \( \alpha \) is the change in pitch angle over time. The functions for \( \alpha \) and \( h \) are shown as well. For these equations \( \omega \) represents the frequency of oscillation and \( \phi \) represents the phase angle between the plunging and pitching motions. The ideal Betz limit

\[ P_{\text{ideal}} = \frac{16}{27}((1/2)\rho AU^3) \]  \hspace{1cm} (1.23)

Where \( A \) is the actuator area, which for the windmill, is defined as the maximum swept projected rectangular area normal to the mean wind vector [15]. \( U \) is the velocity of the air flow away from the airfoil and \( \rho \) is the density of the air. The efficiency of the system was calculated by dividing the mean power output by the
ideal power output.

The experimental results are a very important element because the results determined efficiency of this system was 28.3% [15]. Based on the efficiency of the system it was determined that this technology is an option for energy production. Using a wind tunnel, the oscillating-wing was tested using a test rig that varied the pitch and the phasing between pitch and pivoting of the wing. This allowed for the determination of the maximum lift and the minimum drag on the airfoil which allowed for the calculations above to be completed and the efficiency to be calculated.

**Jones, Platzer and Lindsey**

From McKinney’s work, Jones and Platzer develop a computational model provided for unsteady, two dimensional (2D) flows around both a single and a dual airfoil to examine the thrust performance and the effects of the pitch amplitude and plunge amplitude on the propulsive efficiency [9]. This was done by combining many different approaches created by others to simulate turbulent flow conditions for the airfoil(s). Jones and Platzer’s approach utilizes models from many different people. The flow solutions are computed using a panel code for unsteady flow developed by N. Teng in 1987 through the Naval Postgraduate School [8]. The code also includes a calculation for the wake vorticity and circulation created by the airfoil. This is calculated using the Basu and Hancock procedure as well as the Helmholtz theorem, the boundary layer is also calculated in the simulation by using the Keller-Cebeci box method. This was then combined with the steady panel code from Nowak’s Master’s thesis from the Naval Postgraduate School [8].

The results show the effects of airfoil thickness and plunge amplitude on efficiency. According to the results, the thickness of the airfoil did not have a large effect on the efficiency. The experimental and theoretical results were significantly different. The plunge amplitude results did show that larger plunge amplitude does decrease the
efficiency. Plunge amplitude increases exponentially and matched the estimate given by Garrick’s model [8]. Also a wake model was generated using a NACA 0012 airfoil. This model showed that a larger reduced frequency and smaller plunge amplitude produces the largest wake.

Using this knowledge, Jones, Platzer and Lindsey [8] analyzed the computational model they created as well as an experimental model for a two wing tandem system which oscillates out of phase with one another. The analytical model that was created was developed using Eqs. (1.20) to (1.22) from McKinney’s work and the other works discussed above [15], however there were some major changes made in the overall analysis. The two major differences between K.D. Jones and McKinney is that Jones’ model has two wings that oscillate in-line and 90° out of phase from one another and that Jones’ model is in a water tunnel instead of a wind tunnel.

The airfoil is oscillating in both pitch and plunge with an arbitrary phase angle between the two motions and is defined by the Eqs. (1.24) to (1.26) [8].

\[ \alpha(\tau) = \Delta\alpha \sin(k\tau + \phi) \] (1.24)

\[ z(\tau) = h \sin(k\tau) \] (1.25)

\[ k = \frac{2\pi fc}{U}\] (1.26)

“where \( \tau \) is non-dimensional time, \( h \) is the non-dimensional plunge amplitude, \( \Delta\alpha \) is the pitch amplitude, \( \phi \) is the phase angle between the pitch and plunge, and \( k \) is the reduced frequency” [8]. \( C \) is given as the cord length, \( f \) is the frequency of oscillation in Hz and \( U \) is the stream velocity. “Once the lift and moment have been computed as functions of \( \tau \), the power generated by the oscillating airfoil is obtained from the instantaneous non-dimensional power coefficient” shown in Eq. (1.27) and
the average power output is given in Eq. (1.28) where $q_\infty$ is the dynamic pressure and $S$ is the wing area.

$$C_p = C_l \dot{\theta} + C_m \dot{\alpha}$$  \hspace{1cm} (1.27)$$

$$P = q_\infty U_\infty C_p S$$  \hspace{1cm} (1.28)$$

The efficiency of the system is calculated using the power Eq. (1.28) and the overall power available to the system.

The computer simulations analyzed the flow separation behind the first wing and how it affects the second wing. The mechanical system efficiency was also taken into account in the computer simulation. The simulations that were created used Navier-Stokes equations for the base of the model. Unfortunately, the experimental values do not match the simulated results and are not thoroughly discussed in the paper.
The gap in the power coefficient $C_p$ between “the predicted values are some 200% greater than the measured” [12].

Lindsey’s thesis includes detailed drawings and description of the experimental hydroelectric power generator from Jones’ conference paper [8]. The major schematic is shown in Figure 1.12 and an enlarged view of the oscillating wing is shown in Figure 1.13.

The main reasons given for the discrepancy between the predicted values and the experimental values are that the water flow in the tunnel was not able to be verified and mechanical friction in the system that was neglected in the analysis (mostly mechanical). Also, the models used were 2D and the effects of the wake from the upstream wing were not taken into consideration for the downstream wing. Dye was injected into the water but with the turbulent conditions it did not provide any useful information. There are many more sources of errors listed, however Lindsey states
that “they are beyond the ability of [the] study to consider” [12].

**Ripper**

An initial thesis report from M. Ripper of the University of New South Wales at the Australian Defense Force Academy [20] discusses the plan for his analysis using three different models as well as computational fluid dynamic analysis. The final thesis report has not been published, however Ripper includes the initial results for the equations of motion for the three models he created.

Ripper first discusses a two-dimensional mathematical model, which focuses on the pitching motion and heaving motion of the wing. The airfoil is assumed to be symmetric, and the pivot point is located at half the cord length (or in the center of the airfoil). It is also assumed that the airfoil is at its maximum angle of attack. Shown in Figure 1.14, the mathematical model is an airfoil that is allowed free rotation that is only restricted by ends of the airfoils travel. The heaving motion of the airfoil is calculated using Eq. (1.29) and the pitching moment is calculated using Eq. (1.30).

\[
L = m\ddot{y} + cl\dot{y} + ky \tag{1.29}
\]

\[
M = Ig\ddot{\theta} + c_2\dot{\theta} + K_2\theta \tag{1.30}
\]

The instantaneous power that can be extracted from this system is given in Eq. (1.31). Assuming that all energy in the flow is available to generate power, the power extraction efficiency is the mean power output (the instantaneous power integrated over one cycle) divided by the power in the flow (Eq. (1.32)).

\[
P = La + M\Omega \tag{1.31}
\]
The power in the flow is given as a function of the fluid density ($\rho$), flow velocity ($U_\infty$) and the swept area ($d$). "As the foil approaches its maximum travel a physical barrier is simulated that will cause the foil to rotate" [20]. "The amount of rotation the foil undergoes in this region is controlled by the momentum of the foil. In this way the rotation at these points is not fully defined, if the momentum is insufficient then the rotation will not be complete. This form of restriction will allow the foil to stall if insufficient momentum exists to rotate past a zero angle of attack." [20] This relationship is shown in Figure 1.14b and Eq. (1.33) where $c$ is the chord length and "piv" is the pivot point from the leading edge.

$$\theta = \sin\left(\frac{\text{dist}}{c(1 - \text{piv})}\right)^{-1} \quad (1.33)$$

The Quasi-Steady Model presented "ignores all aerodynamic effects other than lift and moment forces" [20] and these are both calculated as a function of the angle of attack. Ripper cites that these assumptions typically are not consistent with real world results but they were used to validate the equations of motion. The assumptions
also provided an initial estimate to the translational and rotational acceleration and “has proven the concept of simulating a mechanical stop to force a foil rotation at the end of its vertical travel.” [20]

The Wake Interaction Method is the third and final method and does exactly what it states, evaluates the airfoils interaction with its own wake. The initial results of this method provide similar graphs to the quasi-steady method, more work needs to be done.

1.4.3 Conclusions for Literary Review

Although high altitude kites and oscillating wings look and behave very differently, models for both systems are similar because the basic principles remain consistent between the two different types. For example, the available power in the fluid, the efficiency of the system and the aerodynamic properties of lift and drag have similar forms in both applications. However, the equations of motion for any system are based upon the assumptions used to create the model for that application. The variations in the assumptions between high altitude kites and oscillating wings are what create the fundamental differences between these systems and also create the differentiated between these system and the hydrokite.

The idea of the hydrokite system was derived from the concept of high altitude wind power systems and upon further review also shares some similarities with oscillating wing models. The hydrokite is assumed to operate at a steady-state condition like the models for high altitude kites and oscillating wings. The hydrokite system also uses the hydrodynamic properties of lift and drag to generate power. Future hydrokite systems which are more complex than the initial models discussed in this work could better correlate to the complex high altitude kite models.

The hydrokite system is different from the models discussed in the literary review in a few critical ways. In one model the hydrokite system includes the moment
coefficients as well as the lift and drag coefficients, which is not the case in all of the high altitude kite models or oscillating wing models. The lift, drag and moment coefficients are also dependent upon the angle of attack of the hydrokite rather than being defined as constant which makes this system significantly different than Loyd’s work. The prescribed pitching and plunging motion that is seen in the oscillating wing models is not used in the hydrokite system. Both systems use a hydrofoil which is fixed to a rigid rod and the incoming water velocity to generate power. The hydrokite also has a defined load resistance for the generator $k_\theta$, which is not seen in any of the models shown in the literary review. Overall the governing assumptions of the hydrokite define how the models differ from the high altitude kite and oscillating wing models.
Chapter 2

Description of Steady-State Translating Model

2.1 Model Description

Figure 2.1: Translating kite system diagram which shows an isometric view of the rail system with a simplified rectangular river cross-section. Note that the river flow is assumed to be spatially uniform with flow velocity equal to \( V_\infty \).

The translating rail model, shown in Figure 2.1, is one of the simplest possible translating hydrofoil energy-generating systems. The system is comprised of a rigid frictionless rail that spans the river from bank to bank at a fixed angle \( \phi \), a hydrofoil that is in the water, and a collar which can slide freely along the rail. With the flow of the river having uniform velocity away from the hydrofoil, equal to \( V_\infty \), and
the prescribed hydrofoil angle, $\beta$, they create the hydrodynamic forces which act on the system and propel the hydrofoil across the river (Figure 2.2). The hydrofoil angle $\beta$ is defined from the horizontal axis to the chord line of the hydrofoil. A massless, inextensible, string or cable, is connected to the collar and wrapped around two circular drums, one located on each side of the river. One drum acts as an ideal frictionless pulley to hold the tether across the span of the river. The other drum is connected to an electrical generator and when the hydrofoil pulls the tether, the drum spins and reaches equilibrium with the a torque created by the generator. When the hydrofoil reaches the edge of the river, the prescribed angle $\beta$ is instantaneously changed, which creates different hydrodynamic forces on the system. If a particular hydrofoil angle is chosen which generates force in the correct direction, the collar and hydrofoil will return to the original side of the river. This motion across the river and back again describes a cycle for this system. A partial free body diagram for this system is shown in Figure 2.2.

Other main assumptions for this model are that the tether is connected to the hydrofoil by a frictionless collar. The stream velocity has a uniform flow profile, see section 1.1.1, and all components the system are massless. These assumptions were all made in order to simplify the system in order to develop a complete understanding of how the other parameters affect the power production of the system.

\[
\Sigma F = ma = 0 \quad (2.1)
\]

\[
f(v_k, v_\infty) = 0 \quad (2.2)
\]

2.1.1 Steady-State

This model is a steady-state model and is thus in equilibrium at any instant. This was done for two major reasons, one being that the provided lift and drag data for the hydrofoil is comprised of steady-state operating points and another being that it
Figure 2.2: A partial free body diagram which shows a top view of the translating model. The light gray square represents the frictionless collar that attaches the hydrofoil to the rail that spans across the river. The bottom of the figure has the mechanical generator with the torque, $\tau$. The rail can be set at any angle $\phi$ and is positive in the counter-clockwise direction.
describes the simplest possible model for a translating hydrofoil system. Experimental airfoil data was used from [21] and [17]. The data used was collected in wind tunnel tests at specific angles of attack, $\alpha$, at a steady-state condition. Please reference the Appendix A for a complete explanation of the data used. Having the system operate at steady-state conditions ignores the transient behavior that the system would have when initially starting or after the system “flips”. This allows for the other parameters to be examined more closely and determined an upper bound for the power the system can produce. It also provides insight into how the system operates, provided it is at a point that would allow for the transients to be neglected (e.g. long rail lengths).

2.1.2 Methods of Flipping

In order for the system to change from the deploy stroke to the return stroke the hydrofoil must change its orientation or “flip”. It is assumed in the model that the “flip” does not require any time or power. Although this is incorrect, it is plausible that for long lengths of travel, the flipping time and power could be neglected. However, the way in which the hydrofoil flips is defined so that the hydrodynamic forces which act on the system for the deploy and return stroke are consistent for any hydrofoil shape. Two types of flips are illustrated in Figure 2.3.

The first way in which the hydrofoil can flip is about an axis that runs through the quarter chord point. The first method requires a single axis to achieve any hydrofoil angle, $\beta=0^\circ - 360^\circ$. The second method flips the hydrofoil using one axis which is perpendicular to the quarter chord axis with angles of $0^\circ$ or $180^\circ$ and requires a second axis along the hydrofoil to change the hydrofoil angle, $\beta$, along the quarter chord axis.

For this system, the first method of flipping was chosen so that the hydrofoil angle is continuously defined from the same axis. It also was chosen because of the less complicated mechanism that would be required to complete the “flip” so that when a system is constructed the references for the hydrofoil angles will be the same.
Figure 2.3: Two different methods of flipping the hydrofoil to change the angle between the deploy and the return stroke. (A) Is flipping around the quarter chord point of the hydrofoil and (B) is flipping around the midpoint of the span of the hydrofoil so the hydrofoil returns leading edge first.

Figure 2.4: Image of the asymmetric hydrofoil flipping $180^\circ$ for the two different methods of flipping.
However, using this flipping method means that the asymmetric hydrofoil will have a leading edge and a trailing edge range of motion depending on the hydrofoil angle, which can be seen in Figure 2.4.

### 2.1.3 Generator Constant

The kite velocity input parameter is used as a representation of the generator torque which would be required to keep the system operating at an equilibrium condition. The torque on the system is equal to the force acting on the generator from the tether $F_g$ times the radius of the generator drum, $r_d$ (Eq. (2.4)). The torque is also assumed to be linearly proportional to the rotational velocity of the generator drum that the tether is connected to by a constant “$k_\theta$” as seen in Figure 2.5a.

\[
\tau = F_g r_d \quad (2.3)
\]
\[
\tau = k_\theta \dot{\theta} = k_\theta \omega \quad (2.4)
\]
\[
\omega = \frac{V_k}{r_d} \quad (2.5)
\]
\[
F_g = \frac{\tau}{r_d} = k_\theta \frac{V_k}{r_d^2} = k_x V_k \quad (2.6)
\]
\[
k_x = \frac{k_\theta}{r_d^2} \quad (2.7)
\]

The rotational velocity of the drum would be represented by the linear velocity of the tether (or the hydrofoil) and by the radius of the drum. Substituting for the force acting on the generator, the torque can be represented as linearly proportional to the hydrofoil velocity. For the model a kite velocity is prescribed, and the power produced is calculated. Alternatively, one could prescribe a generator stiffness, $k_\theta$, and then determine a resulting steady-state kite velocity and its resulting power production. Either method would be valid, however for simplicity we choose to use the former method of prescribing kite velocities.
Figure 2.5: Theoretical model of an electric generator and load and actual generator data to illustrate the linear correlation between the rotational velocity and torque. It can be seen in the experimental data that when varying load resistances were applied all 11 load resistances had a relationship between torque and rotational speed that is approximately linear [23].

2.1.4 Cycle Power

Although a full cycle involves the hydrofoil and collar translating completely across the river and then returning to its starting position, we do not need to simulate the entire trip across the river and back since the steady-state values for instantaneous power are the same for every position across the river on each stroke. Thus, we only need to calculate the instantaneous power for a single deploy position and a single return position. Having determined the instantaneous power for each of these two states as well as the corresponding kite velocities, we can calculate the average cycle power for that configuration by using Eq. (2.17). For this model, strokes were examined independently and the total cycle power was not calculated. The total cycle power could be calculated using the information provided and Eq. (2.17).
\[ t_d = \frac{l}{v_d} \quad (2.8) \]
\[ t_r = \frac{l}{v_r} \quad (2.9) \]
\[ E_d = \int_0^{t_d} P_d dt \quad (2.10) \]
\[ E_r = \int_{t_d}^{t_d+t_r} P_r dt \quad (2.11) \]
\[ E_d = P_d t_d = P_d \frac{l}{v_d} \quad (2.12) \]
\[ E_r = P_r t_r = P_r \frac{l}{v_r} \quad (2.13) \]
\[ P_{cyc} = \frac{E_d + E_r}{t_d + t_r} \quad (2.14) \]
\[ P_{cyc} = \frac{P_d l/v_d + P_r l/v_r}{l/v_d + l/v_r} \quad (2.15) \]
\[ P_{cyc} = \frac{P_d l v_r + P_r l v_d}{v_d v_r} \frac{v_d v_r}{l v_r + l v_d} \quad (2.16) \]
\[ P_{cyc} = \frac{P_d v_r + P_r v_d}{v_r + v_d} \quad (2.17) \]

**Model Equations**

The equations below comprise part of the model and allow for one to determine the angle of attack for any given set of input parameters. The angle of attack for the system is required in order to determine the lift and drag coefficients which are used to calculate the hydrodynamic forces that act on the system. Equations (2.18) to (2.25) are used to calculate the angle of attack for any set of input parameters. The complete vector diagram which correlates to these equations is shown in Figure 2.6.
\begin{align*}
A_k &= cs \quad (2.18) \\
AR &= \frac{s^2}{A_k} \quad (2.19) \\
\hat{\lambda}_r &= -\sin \phi \hat{i} + \cos \phi \hat{j} \quad (2.20) \\
\vec{V}_a &= V_\infty \hat{i} - V_k \hat{\lambda}_r \quad (2.21) \\
\hat{\lambda}_d &= \frac{\vec{V}_a}{|\vec{V}_a|} \quad (2.22) \\
\hat{\lambda}_l &= \hat{k} \times \hat{\lambda}_d \quad (2.23) \\
\gamma &= \begin{cases} 
-\arccos(\hat{\lambda}_d \cdot \hat{i}) & \text{if } \vec{V}_a \cdot \hat{j} > 0 \\
\arccos(\hat{\lambda}_d \cdot \hat{i}) & \text{if } \vec{V}_a \cdot \hat{j} < 0 
\end{cases} \quad (2.24) \\
\alpha &= \beta - \gamma \quad (2.25) \\
\alpha &= \begin{cases} 
q = \text{floor}( (\alpha + \pi)/(2\pi) ) & \text{if } \alpha > \pi \\
q = \text{ceil}( (\alpha - \pi)/(2\pi) ) & \text{if } \alpha < -\pi 
\end{cases} \quad (2.26)
\end{align*}

Given the equations above, \( \alpha \) could be any angle. In order to interpolate the correct lift and drag coefficients after the angle of attack for the system was calculated the value of \( \alpha \) had to be folded into the available range of lift and drag coefficients (-180° to 180°).
\[ \alpha_{\text{new}} = \alpha - 2q\pi \] (2.27)
\[ F_D = \frac{1}{2} \rho C_D A_k |\vec{V}_a|^2 \] (2.28)
\[ F_L = \frac{1}{2} \rho C_L A_k |\vec{V}_a|^2 \] (2.29)
\[ \vec{F} = F_D \hat{\lambda}_d + F_L \hat{\lambda}_l \] (2.30)
\[ P = \vec{F} \cdot V_k \hat{\lambda}_r \] (2.31)

The lift and drag coefficients used were critical in determine the performance for the overall system. For a complete explanation on how the lift and drag coefficients were translated from the published experimental data for infinite wings to the canonical finite wing used in the simulations, please see Appendix A.
In this model the hydrodynamic moment coefficients were not used and the moment calculation was neglected. This was done because the collar is constrained to only move in the direction of the rod and cannot rotate. The hydrofoil and the boom are not allowed to rotate during the deploy and return stroke which means that the moment would not affect the power generated. The summation of the lift and drag forces which act in the direction of the rail for a prescribed hydrofoil angle and hydrofoil velocity was then used to calculate the power that the system could generate given the specified input parameters.

2.2 Results and Discussion

The effects of many different parameter variations could be studied, however, this work focused on a subset of the parameters that were thought to have the largest impact on power production. For all of the results shown, the system dimensions were kept constant and are given in Table 2.1. Two different airfoil shapes were used, the symmetric NACA 0015 and the asymmetric NACA 4412. These were chosen somewhat arbitrarily but their choice was motivated by a desire to test asymmetric and symmetric hydrofoil shapes and to use existing lift/drag/moment experimental data. Both models used lift and drag coefficients for angles of attack ranging from -180° to 180°. A complete description of this data can be found in Appendix A. Another parameter that was explored is the angle the of the rail across the river, \( \phi \). If the rod is perpendicular to the incoming river velocity then \( \phi=0^\circ \) and \( \phi \) increases value as the rod rotates counter clockwise. The results have been divided into three sections, \( \phi = 0 \), maximum power as a function of \( \phi \) and power surface as a function of \( \phi \).
Table 2.1: Base Set of Translational Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Abbreviation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>River Velocity</td>
<td>$V_\infty$</td>
<td>1 m/s</td>
</tr>
<tr>
<td>span</td>
<td>$s$</td>
<td>0.75 m</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>$AR$</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 2.7: Partial free body diagram of the rail system for the special case where the hydrofoil translates perpendicular to the river flow (i.e. $\phi = 0^\circ$)
2.2.1 Case One: Rail Angle is Perpendicular to the River Flow, $\phi = 0^\circ$

The special case, where the rail angle is perpendicular to the flow velocity ($\phi = 0^\circ$), shown in Figure 2.7, was examined in order to gain an understanding of how only the kite velocity ($V_k$) and the hydrofoil angle ($\beta$) affect the power production. This was studied using both the symmetric and asymmetric hydrofoils and originally completed for a large range of kite velocities and angles. This was then refined after the peak power production area was determined, which for both the symmetric and asymmetric hydrofoils is between $\beta = 80^\circ$ and $100^\circ$ hydrofoil angles and $V_k = 0$ and 25 m/s. The power production of the system, shown in Figure 2.8, was calculated for every 0.1$^\circ$ increment and 0.1 m/s increment for hydrofoil angles and kite velocities respectively.

For both the symmetric and asymmetric hydrofoils there is a given hydrofoil angle $\beta$ and kite velocity $V_k$ that generates the maximum power for the system. For the standard inputs given for the system, the maximum instantaneous power production for the NACA 0015 was 1,379 Watts with $V_k = 12.7$ m/s (12.7 times faster than the river velocity) and with $\beta = 92.5^\circ$. The maximum instantaneous power production
for the NACA 4412 is 844 Watts with $V_k = 10.2 \text{ m/s}$ and $\beta = 89^\circ$. The peak power conditions for both the symmetric and asymmetric hydrofoils are illustrated in Figure 2.9. It is interesting that the symmetric hydrofoil produces more power than the asymmetric hydrofoil. This is difficult to explain but believed to be the result of the lift and drag coefficients. Please see Appendix A for a comparison of the coefficients between these two hydrofoils.

It is apparent from Figure 2.9 that the hydrofoil angle peak power production in both the symmetric and asymmetric cases would not be the optimal hydrofoil angle to start the system from rest from. In order to start the system from rest the hydrofoil angle $\beta$ would need to be different and as the hydrofoil began to move fast, the angle could be adjusted. The goal would be to adjust the hydrofoil angle $\beta$ starting from rest so that it would reach the maximum power production conditions ($\beta$ and $V_k$) as quickly as possible so that the system would have the highest instantaneous power for the majority of the travel across the river.

In order to have a better understanding of how the system operated at the point of
maximum power generation, the angle of attack and lift to drag ratio were calculated as a function of the kite velocity and hydrofoil angle. As can be seen in Figure 2.10 near the region where maximum power is produced, the angle of attack is between 5° and 10° for both the symmetric and the asymmetric models, however the symmetric angle of attack is slightly larger around the peak compared to the asymmetric.

It can also be seen in Figure 2.10 that the angle of attack as a function of hydrofoil angle and kite velocity has a very predictable shape. The contour profile is very similar for both the NACA 0015 and NACA 4412 as expected. It can be seen that as the kite velocity approaches zero, the angle of attack approaches the given β angle. If the kite velocity approaches ∞, the angle of attack for the system will approach 0° along the given range of hydrofoil angles.

Figure 2.11 shows that when the system is at the maximum power production conditions, the lift to drag ratio is not at the maximum. The contour of the lift to drag ratio was laid over the power contours and can be seen in Figures 2.11a and 2.11b. This phenomenon can be seen in both the NACA 0015 and NACA 4412 and was an unexpected result because typically hydrofoil and airfoil designers will focus
Figure 2.11: Lift-to-Drag \((L/D)\) ratio contour as a function of \(\beta\) and \(V_k\) for the NACA 0015 and NACA 4412 hydrofoil overlaid on the peak power contour lines, from 85\% to the peak, which appeared by itself in Figure 2.8. Note that the peak power point does not lie on the peak ridge of maximum L/D.

on the maximum lift to drag ratio. In the design of wind turbine blades the shape is determined by equations from blade element theory and momentum theory. For an ideal rotor an angle of attack is “selected where \(\frac{C_d}{C_l}\) is minimal in order to most closely approximate the assumption that \(C_d = 0\)” which is a common assumption in the analysis using blade element moment theory [14]. However, since the hydrofoil in this scenario is being used in a different manner than typical foils, the fact the peak power is not at the maximum lift to drag ratio is an interesting property of the system but does not affect the overall outcome.

Another interesting point that was discovered was the difference in the shape of the peaks for the symmetric compared to asymmetric hydrofoils. It can be seen when comparing Figures 2.8a and 2.8b that the power production surface near the global maximum power peaks for the asymmetric hydrofoil is not as smooth as the symmetric case. Further investigation into the more abrupt changes that the asymmetric hydrofoils power surface revealed that it is directly correlated to slope discontinuities
in the lift and drag coefficient curves. As can be seen in Figure 2.12a, the lift coefficients after the correction from infinite to finite wings are almost linear for the range of angles of attack that produced maximum power. The drag coefficients for the asymmetric (NACA 4412) hydrofoil do not have as smooth of a curve compared to the symmetric hydrofoil. The two angles where the largest change in the slope occurs are labeled on Figure 2.12b. The two angles of attack where the large slope change occurs are drawn over the power contour plot in Figure 2.13 and one can clearly see that they are the cause of the roughness in the power surface for the asymmetric hydrofoil. In order to eliminate the roughness in the surface profile different experimental data for the lift and drag coefficients would be needed.

### 2.2.2 Case Two: Rail Angle Variation Effect on Maximum Power

The hydrofoil does not have to translate perpendicular to the river flow, one could place the frictionless rod at any angle across the river. An added advantage of studying non-zero rail angles, $\phi$, is that the results of that study could be easily related to the rotational system that will be discussed in Rotational Model Chapter. The
optimization was done two different ways, using brute force and using MATLAB’s optimization toolbox. Each individual method had benefits and weaknesses.

**Brute Force:**

In this method, the hydrofoil angle was allowed to be any angle (0° - 360°) and the kite velocity was limited to a range from 0 to 25 m/s. The rail angle was altered in increments of 5°, and the power production was calculated for every combination for hydrofoil angle and kite velocity for a given rail angle. The maximum calculated power was then taken from that array of values, as well as the hydrofoil angle and kite velocity to achieve the peak condition, this was done for all rail angles studied. This method proved to be effective but also had a few limitations when looking at the resolution of the hydrofoil angle and the kite velocity. The curves of the power production with respect to rail angle were not as smooth as originally expected. This was thought to be because of the resolution that the kite velocity and hydrofoil angle, therefore in order to achieve a curve that appeared to be reasonably continuous the
incremental step had to be small for both the hydrofoil angle and the kite velocity, which became computationally time consuming. Figure 2.14 shows the power curves for both the symmetric and asymmetric hydrofoils. Both hydrofoils have smooth power production curves which form a “W” shape. As expected, the power production for the NACA 0015 is equal when $\phi = 0^\circ$ and $\phi = 180^\circ$.

It can been seen in the asymmetric power curve that the peak seen when $\phi = 180^\circ$ has a lower magnitude than the when $\phi = 0^\circ$. This is most likely due the way in which the hydrofoil flip was defined. As seen in Figure 2.4 using the method of flipping where the hydrofoil simply rotates about the axis passing through the quarter chord point does not allow for the leading edge to be used when rotated $180^\circ$. It is expected that if the flipping operated like the second method defined, that the power produced at $\phi = 180^\circ$ would be equal in magnitude to the power produced when $\phi = 0^\circ$. The hydrofoil angle and kite velocity in order to achieve the peak power curve is shown in Figure 2.15 for the symmetric hydrofoil and in Figure 2.16 for the asymmetric hydrofoil.

For both system, the hydrofoil velocity $V_k$ and hydrofoil angle $\beta$ around $\phi = 270^\circ$ behave differently than the rest of the system. This is thought to be because the system is becoming a drag based system due to the fact that the rail angle is aligning with the incoming river velocity. It is also interesting to note that the kite velocity appears to be a consistent value of 0.3 m/s for both the symmetric and asymmetric hydrofoils at rail angles around $270^\circ (\pm 5^\circ)$.

**Unconstrained Optimization:**

Using the MATLAB function fminunc, the same equations were used to calculate exact values for power production, hydrofoil angle, kite velocity and angle of attack. This allowed the computation of many more increments of rail angle more quickly because the previous point of convergence was used for the initial guess for the next
Figure 2.14: Maximum power production as a function of rail angle, $\phi$, from 0 to 360°, in increments of 2.5° using the Brute Force approach for (A) the NACA 0015 and (B) the NACA 4412
Figure 2.15: Hydrofoil angle and kite velocity as a function of rail angle for peak power production for a symmetric hydrofoil (NACA 0015) using the Brute Force approach.
Figure 2.16: Hydrofoil angle and kite velocity as a function of rail angle for peak power production for an asymmetric hydrofoil (NACA 4412) using the Brute Force approach.
rail angle. Utilizing the fminunc function in MATLAB’s optimization toolbox provided values for convergence compared to stepping through every value for hydrofoil angle and kite velocity for each rail angle that were provided to the simulation. This method also eliminated issues with the resolution of the inputs as well because the convergence criteria was set in the fminunc function to $1 \times 10^{-10}$. The power curve shapes and values are the same for the optimization as using the brute force method. The most noticeable result is that the noise that was seen in the brute force method in the curves (especially the kite velocity for the asymmetric hydrofoil) is still present in the optimization. Using the worst case, the asymmetric hydrofoil between $90^\circ$ and $270^\circ$, the angle of attack in this range was found to be around $173.4^\circ$, which is in a section of the lift and drag coefficient curves that are changing drastically. The curves can be seen in Appendix A and this is most likely the reason for the large fluctuations peak power production. One issue with using the fminunc function was the difficulty that it had converging at rail angles of $90^\circ$, $180^\circ$ and $270^\circ$.

Figures 2.18 and 2.19 include converged values of the kite velocity that can generate maximum power. Near a rail angle of $270^\circ$ the kite velocity curves for maximum power production appears to be more continuous than the results seen in the Brute Force approach. This result is believed to be the outcome of using the previous converged value as the initial guess for the next rail angle and not completing a full mapping to determine if there is global maximum that is farther away from the initial guess. If this is the reason for the discrepancies between the kite velocity curves in the two models, it can be assumed that for any given rail angle there are multiple maximums that can be seen for a given hydrofoil angle $\beta$ and kite velocity $V_k$ but only one global maximum.

The following can be observed in both the NACA 0015 and NACA 4412 hydrofoils:

- At $90^\circ$ there is no power produced. This is because the rail is in-line with the direction of the fluid flow, and power would be required to move it “up” the rail
Figure 2.17: Maximum power production as a function of rail angle, $\phi$, from 0 to 360$^\circ$, in increments of 0.1$^\circ$ using optimization routine “fminunc” in MATLAB with optimset values of $1\times 10^{-10}$ for both function and variable values. (A) the NACA 0015 and (B) the NACA 4412.
Figure 2.18: System specifications to achieve the maximum power production for the system for rail angles from 0 to 360° for the NACA 0015 Hydrofoil.
Figure 2.19: System specifications to achieve the maximum power production for the system for rail angles from 0 to $360^\circ$ for the NACA 4412 Hydrofoil.
(away from the base).

- The instantaneous power peak decreases in magnitude when the rail angle is changed from 360° to 270°. The kite velocity and hydrofoil angle for peak power production also decrease (ϕ = 360° to 270°).

- Between 90° and 270° the hydrofoil angle for peak power production decreases linearly, with a discontinuity in both hydrofoil models around 270°. This is thought to be the result of the system becoming a drag based system when the rail becomes in line with the incoming river velocity.

- The kite velocity increases from 90° to 180° and from 270° to 360° and decreases from 0° to 90° and 180° to 270°. This corresponds with the values of power production along these ranges.

2.2.3 Case Three: Rail Angle Effect on Power Production Surface

In order to have a better idea of how the power production is changing near but not at the maximum power point, the contour plots for different rail angles were compared. Figure 2.20 shows the power production for the NACA 0015 for increments of 30° in rail angle, hydrofoil angles between 0 and 360° and kite velocities ranging between 0 and 25 m/s. Figure 2.21 shows contour plots for the same conditions for the NACA 4412 hydrofoil. These contours illustrate the change in the shape and position of the peak. The first sub-plot in Figure 2.20, where rail angle is zero degrees, is the same contour that is shown in Figure 2.8a only on a slightly larger scale with respect to hydrofoil angle, β, and river velocity, \( V_k \). The same holds true for the first sub-plot in Figure 2.21 and Figure 2.8b.

These contours are another way of illustrating the same information provided in Figures 2.14, 2.17, 2.18 and 2.19 but also provide an idea about the steepness of the power production curve near the peak. The figures also show the power production
curves at a rail angles of 90° did not converge in the optimization because the system is flowing into the flow of the river and there is no positive kite velocity that can allow for positive power to be produced.

The rail angles of 0°, 180° and 270° proved to be interesting conditions of the system. The 0° and 180° cases correspond to the largest power production points for both hydrofoil shapes and the 270° case corresponds to a drag based system. In Figures 2.22 and 2.24 it can be seen that there are two peaks (one much larger than the other) for both the symmetric and asymmetric hydrofoils for rail angles of 0° and 180°. There are also two peaks for a rail angle of 270° that are equal for the symmetric hydrofoil and almost equal for the asymmetric.

For the symmetric hydrofoil the hydrofoil angles, β that produce the maximum power at rail angles of 0° and 180° create the same magnitude for the apparent wind velocity vector, which generates the same power due to the symmetry of the shape. Since the asymmetric hydrofoil shape has a different curvature on top and bottom, the peaks are not identical because the apparent wind velocity vector will not be the same when it is rotated a full 180°.

The following can be observed in both the NACA 0015 and NACA 4412 models for the power surfaces:

- In the large mapping at rail angles of 0° and 180° it can be seen that there are two peaks, one larger than the other.

- It is clear that the path which the peak follows with regard to kite velocity and hydrofoil angle is consistent with the optimization paths from the previous section.

- At a rail angle of 270°, there are two peaks with almost equal magnitude and size (exactly equal for the symmetric hydrofoil). The power produced in this state for both hydrofoils is mainly due to the drag acting on the hydrofoil.
Figure 2.20: Power Contour as a function of $\beta$ and $V_k$ for the NACA 0015 Hydrofoil for $\phi = 0^\circ - 330^\circ$ in increments of $30^\circ$. The $360^\circ$ case is the same contour plot as the $0^\circ$ case.
Figure 2.21: Power Contour as a function of $\beta$ and $V_k$ for the NACA 4412 Hydrofoil for different rail angles from 0 to 330° in increments of 30°. The 360° case is the same contour plot as the 0° case.
Figure 2.22: Complete mapping of power production surface as a function of kite velocity and all possible $\beta$ angles for three different Rail Angles for the NACA 0015.

Physical Position of Hydrofoil and the Rail for All Power Peaks

For All Cases: $V_\infty = 1\text{m/s}$

![Diagram showing power production and physical positions for different rail angles and kite velocities](image)

Figure 2.23: Schematic of the Rail System for all 6 peaks shown in Figure 2.22 for the NACA 0015 hydrofoil. Also shown is the required hydrofoil angle and kite velocity in order to obtain the peak power.
Figure 2.24: Complete mapping of power production surface as a function of kite velocity and all possible $\beta$ angles for three different Rail Angles for the NACA 4412.

**Physical Position of Hydrofoil and the Rail for All Power Peaks**

For All Cases: $V_\infty = 1$ m/s

<table>
<thead>
<tr>
<th>Rail Angle: 0 Degrees</th>
<th>Rail Angle: 180 Degrees</th>
<th>Rail Angle: 270 Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Production: 844.1 Watts</td>
<td>Power Production: 211.1 Watts</td>
<td>Power Production: 40.0 Watts</td>
</tr>
<tr>
<td>$\beta$: 89.0°</td>
<td>$\beta$: 92.5°</td>
<td>$\beta$: 282.6°</td>
</tr>
<tr>
<td>$V_k$: 10.2 m/s</td>
<td>$V_k$: 6.5 m/s</td>
<td>$V_k$: 2.9 m/s</td>
</tr>
<tr>
<td>$\beta$: 99.7°</td>
<td>$\beta$: 260.6°</td>
<td>$\beta$: 267.8°</td>
</tr>
</tbody>
</table>

Figure 2.25: Schematic of the Rail System for all 6 peaks shown in Figure 2.24 for the NACA 4412 hydrofoil. Also shown is the required hydrofoil angle and kite velocity in order to obtain the peak power.
After sketching the schematics for all of the peaks seen in the six surface figures, it was interesting to see that the asymmetric hydrofoils orientation to produce peak power at a rail angle of $180^\circ$ is almost the same as the hydrofoil angle that produces peak power at a rail angle of $0^\circ$. This is believed to be the result because of the method of flipping chosen. At a $180^\circ$ rail angle the hydrofoils leading edge cannot be used (see Figure 2.4) and the trailing edge does not allow for large amounts of power to be produced. It is also worth noting that the peak power production at a rail angle of $180^\circ$ is less than $1/4$ the peak power production at a rail angle of $0^\circ$.

### 2.3 Conclusion

The first case showed that there is a clear maximum power production point when the system is analyzed over a range of hydrofoil angles and kite velocities. It also allowed for the investigation into the angle of attack and lift-to-drag ratio around the peak. The shape of the contour plot for the symmetric and asymmetric hydrofoil were explored, which illustrated how dependent the surface’s shape is on the lift and drag coefficients.

The second case allowed for a complete optimization of the hydrofoil angle and kite velocity for any given rail angle. This was done using two different methods, a brute force approach and an optimization tool from MATLAB. Both of these methods showed that there is much more variability in the asymmetric hydrofoil, especially between $90^\circ$ and $270^\circ$ rail angles. The optimization also showed, as expected, that the symmetric hydrofoil produces the same amount of power at $0^\circ$ and $180^\circ$ rail angles. The angle of attack is equal and opposite for the symmetric hydrofoil at these two points, which provides the coefficients with the same magnitude and generate the same force on the system. Around $270^\circ$ rail angles the brute force method clearly illustrates that the system changes to a drag based system. Using the optimization tool in MATLAB showed some discrepancy from the hydrofoil angle, $\beta$, and the kite
velocity, \( V_k \), curves that were produced using the Brute Force method. This is believed to be the result of using the inputs from the systems previous point of convergence for the start of the next rail angles optimization.

The third and final case provides an understanding of how the shape of the power curvature changes as the rail position changes. It also allowed for further investigation for the areas that were not surrounding the maximum power production point. This showed that there is a smaller peak that is also present when the full range of hydrofoil angles for both the asymmetric and symmetric hydrofoils was examined. A schematic for each peak that was seen at rail angles of 0°, 180° and 270° illustrates the system conditions that would be required to achieve those peaks. The orientation of the asymmetric hydrofoil which produced peak power at a rail angle of 180° was unexpected and similar to the hydrofoil angle at \( \phi = 0^\circ \). This is thought to be the result of the method chosen for flipping but the other method described to flip the hydrofoil needs to be evaluated to confirm.

### 2.4 Future Work for Translating Hydrofoil Model

This model provides insight into the relationships that individual parameters have on a hydrokite system. In the future it would be helpful to look at the design parameters, such as the river velocity and aspect ratio to better understand how those parameters will affect the maximum power generation since they were not studied in depth in this model.

Aspect Ratio also will effect the power production of the system because it will alter the hydrodynamic forces on the system. After becoming familiar with the system, I believe that a large aspect ratio will create more power, however this is yet to be determined and may be infeasible once the system is physically built due to the forces that would be acting on the hydrofoil. Optimizing the Aspect Ratio or the individual hydrofoil dimensions (chord length and span) would be help information
to know for building a prototype.

Another step would be including the dynamics into this simulation to gain an understanding of how the mass of the system affects the power production. Ideally the maximum power producing is with a massless system, however this would not apply to the real world therefore understanding the nature of the relationship between power production and mass of the system would be extremely helpful when designing a system.

Attempting to simulate a non-instantaneous flip that requires some amount of power to complete would also allow for a more accurate depiction of the power that a system could produce. Since there would be two flips necessary for each cycle, this could become a major factor in how much net power the system would be able to produce. This system can not function without flipping and if the power needed to complete such a flip is too large compared to the power production of the system then it could make building a physical system unjustifiable. Fully investigating the other method of flipping would also prove to be useful, especially for the case when an asymmetric hydrofoil is used.

Addressing how a non-uniform flow field throughout the cross section of the river would affect the power generating capabilities of this system. Since the fastest moving water is towards the center of the river, compared to the outside edges and near the bottom, when the system reaches the end of the river the slow moving water could greatly decrease the amount of power that could be produced per cycle.

Finally, addressing and quantifying the environmental impacts that a system like this one could have would be very important if it was going to be implemented. Investigating this before a system was permanently installed would be greatly beneficial in determining if this system would be as eco-friendly as it is currently imagined. Obviously, with the system moving as quickly as it does at the maximum power production states, animals that live in the river could have issues with a system that
operates throughout the cross section of the river.
Chapter 3

Description of Steady-State Rotating Model

3.1 Model Description

Figure 3.1: Boom system schematic which shows an isometric view of the system with a simplified rectangular river cross-section. Note that the river flow is assumed to be spatially uniform with flow velocity equal to $V_\infty$.

A rotational model is examined, which is attached to only one side of the river. The reason for developing another type of power generating system is because we believe this system would be much more practical to build and implement. Since the system is only fixed at one side of the river, the construction involved is much less complicated and would also eliminate issues that could arise with a system stretching across a river. The issues with stretching a system across the river range from the
alignment of the rail to traffic across the river (natural or man made). Another benefit of only mounting a system on one side of the river is the decrease in the environmental impact the system has on the river bank, since it only interacts with one edge instead of the both.

Shown in Figure 3.2 the system is comprised of a hydrofoil, rigid boom arm and a base. The base is mounted at the river bank and the rigid rod, or boom, is used to connect the hydrofoil to the base. Power is captured from the torque which is generated by the rotational motion of the boom due to the hydrodynamic forces acting on the hydrofoil. The speed of the rotation is determined based on the fixed angle that the hydrofoil is set and its position in the river, which results in different angles of attacks. To begin, the hydrofoil “flies” upstream, into the river’s current. The forces will eventually decrease on the system as it moves upstream and after the system slows, the hydrofoil is “flipped” (meaning that the angle the hydrofoil is set at is changed) and different forces act on the hydrofoil which cause the boom to rotate the opposite direction, returning the system to the starting point along the bank of the river. A cycle is defined as a deploy stroke with the hydrofoil moving into the flow of the river, a deploy flip, and then a return stroke which brings the boom back to the starting position and finally another hydrofoil flip is executed after the return to bring the system back to the initial state. This cycle is depicted in Figure 3.3.

The first verification of the rotation model was a comparison to the translational model. To complete this, the boom angle was fixed at $\theta=0^\circ$ so that the hydrofoil angles would correlate directly to the translational model at a rail angle $\phi=0^\circ$ (seen in Figure 3.4). The hydrodynamic forces were then evaluated for three different hydrofoil angles ($\beta$), two different kite (or hydrofoil) velocities, which were made equivalent for the rotational and translational systems, and for both the symmetric and asymmetric hydrofoils. The power that each model predicted for every combination of parameters was documented and compared. The difference between the translational model and
the rotational model can be seen in the Table 3.1. When evaluated to fifteen significant figures the power was exceptionally close or exactly the same for each combination of parameters. The average percent error that was seen, which was on the order of $10^{-15}$, can be described as computing error within MATLAB. This check allowed confirmation that the hydro-dynamic equations that were derived for both systems were operating in the same way.

The equations used in the translating system for calculating the unit vectors and angle of attack for the lift and drag calculations were very similar to those used for the rotational system. The complete set of equations are provided in Eqs. (3.1) to (3.21) and the vector diagram for the rotational system is shown in Figure 3.5. Since the system is rotating, the torque that is generated at the base is a function of the lift
Figure 3.3: System diagram which shows the deployment stroke and return stroke as well as the “flips” that are used for the hydrofoil to change directions.

Figure 3.4: System diagram of rail position used for the comparison between the translational model and the rotational model. The $V_k$ direction is the same in both cases (vertical), thus in each case the conditions for maximum power production should also be the same.
Table 3.1: Comparison of Predicted Power Production for the Translational and Rotational Models

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_k$</td>
<td>15m/s</td>
<td>15m/s</td>
<td>15m/s</td>
<td>12m/s</td>
<td>12m/s</td>
<td>12m/s</td>
</tr>
<tr>
<td>$\beta$</td>
<td>88</td>
<td>90</td>
<td>92</td>
<td>88</td>
<td>90</td>
<td>92</td>
</tr>
<tr>
<td>NACA 0015</td>
<td>$1.0232 \times 10^{-12}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>$1.0004 \times 10^{-11}$</td>
<td>0</td>
</tr>
<tr>
<td>NACA 4412</td>
<td>0.0</td>
<td>$1.0004 \times 10^{-11}$</td>
<td>0.0</td>
<td>$1.0004 \times 10^{-11}$</td>
<td>$1.0004 \times 10^{-11}$</td>
<td>0</td>
</tr>
</tbody>
</table>

and drag forces, hydrodynamic moment, as well as the rotational velocity multiplied by a specified generator constant. In this model the hydrodynamic moment that acts on the system is included in the calculations of power because the forces are no longer acting in only the direction of the rail like the translating model.

Using numerical root finding ("fzero" function in MATLAB) the booms angular velocity $\dot{\theta}$ that would keep the system in equilibrium was determined. This was done for the deploy stroke as well as the return stroke, however the deploy stroke was limited to only positive angular velocities and the return stroke was limited to only negative angular velocities to ensure that the system would be rotating the proper way for any hydrofoil angle that was given. It was discovered, and can be seen in Figure 3.8, that some hydrofoil angles have equilibrium points for both the deploy and return stroke, meaning there are different rotational velocities, some positive and some negative, that allow the system to be in an equilibrium state. This will be examined in Section 3.2.1.
$A_k = cs$ \hspace{1cm} (3.1)

$AR = \frac{s^2}{A_k}$ \hspace{1cm} (3.2)

$\lambda_r = \cos \theta \hat{i} + \sin \theta \hat{j}$ \hspace{1cm} (3.3)

$\lambda_a = -\kappa \times \lambda_r$ \hspace{1cm} (3.4)

$V_k = L \dot{\theta}$ \hspace{1cm} (3.5)

$\vec{V}_a = V_\infty \hat{i} + V_k \lambda_a$ \hspace{1cm} (3.6)

$\lambda_d = \frac{\vec{V}_a}{|\vec{V}_a|}$ \hspace{1cm} (3.7)

$\lambda_l = \kappa \times \lambda_d$ \hspace{1cm} (3.8)

$\gamma = \arccos(\lambda_d \cdot \hat{i})$ \hspace{1cm} (3.9)

$\alpha = \beta - \gamma$ \hspace{1cm} (3.10)
\[
\alpha = \begin{cases} 
q = \text{floor}((\alpha + \pi)/(2\pi)) & \text{if } \alpha > \pi \\
q = \text{ceil}((\alpha - \pi)/(2\pi)) & \text{if } \alpha < -\pi 
\end{cases} 
\]

(3.11)

\[
\alpha_{\text{new}} = \alpha - 2q\pi 
\]

(3.12)

\[
F_D = \frac{1}{2} \rho C_D A_k |\vec{V}_a|^2 
\]

(3.13)

\[
F_L = \frac{1}{2} \rho C_L A_k |\vec{V}_a|^2 
\]

(3.14)

\[
M = \frac{1}{2} \rho C_M A_k c |\vec{V}_a|^2 
\]

(3.15)

\[
\vec{F} = (F_D \hat{\lambda}_d) + (F_L \hat{\lambda}_l) 
\]

(3.16)

\[
\vec{r}_{\text{arm}} = L (\cos(\theta) \hat{i} + \sin(\theta) \hat{j}) 
\]

(3.17)

\[
\tau = -M + (\vec{r}_{\text{arm}} \times \vec{F}) \cdot \hat{k} 
\]

(3.18)

\[
0 = \tau - k_\theta \dot{\theta} 
\]

(3.19)

\[
P = \tau \dot{\theta} 
\]

(3.20)

\[
E = \int P \, dt 
\]

(3.21)

\[
E = \int \tau \dot{\theta} \, dt = \int \tau \frac{d\theta}{dt} \, dt = \int \tau d\theta 
\]

(3.22)

\[
t = \int dt = \int \frac{dt}{d\theta} \, d\theta = \int \frac{1}{\dot{\theta}} \, d\theta 
\]

(3.23)

The cycle time, cycle energy and cycle power were calculated using the trapezoidal approximation in MATLAB (“trapz”). The cycle time was determined using the position of the boom and the corresponding steady-state angular velocities (Eq. (3.23)). The cycle energy is a function of the torque and the position on the boom (Eq. (3.22)) and the cycle power is the cycle energy divided by the cycle time for
Figure 3.6: Precent Different in maximum power which was calculated at a 5° boom position provided different increments of boom position.

each incremental step throughout the system’s deploy and return strokes.

In order to verify that the approximation using MATLAB’s “trapz” function was a reasonable way of representing the integration of the cycle power, cycle time, and cycle energy, the position of the boom (which dictates the step size of the trapezoidal approximation) was examined. The boom position refinement was completed with incremental boom positions ranging from 0.05° to 5° for a cycle ranging from $\theta = 0^\circ$ to $\theta = 5^\circ$. The power production at a boom position of 5° was evaluate with the different incremental boom positions. It can be seen in Figure 3.6 that there is less than a 0.01% increase in power production after the 1° increment.

The maximum power was also evaluated for the peak deploy and return angles with a stroke going from $\theta = 0^\circ$ to $\theta = 0^\circ$, the maximum power was calculated by hand using Eq. (2.17). This point is shown in red in Figure 3.7 and the maximum power for the given incremental boom position is also shown. As seen in Figure 3.7
Figure 3.7: Maximum power was calculated provided different incremental boom positions. The red point is the calculated maximum for the given set of inputs shown in Table 3.2 and hydrofoil angles $\beta_D = 92.5^\circ$ and $\beta_R = -92.5^\circ$

the change in the maximum power of the system is very small over the $5^\circ$ range, and after the $1^\circ$ increment the maximum power increases less than $1/3$ Watt, or $0.02\%$, therefore for this analysis the increment of $1^\circ$ was chosen.

3.2 Results and Discussion

The parameters that were examined for peak power production were deployment angle, return angle, generator constant, flipping angle and boom length. These will all be discussed independently and when investigated were the only parameter that was varied. The initial conditions used for all parameter investigations, unless otherwise noted, are shown in Table 3.2.
Table 3.2: Base Set of Rotational Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Abbreviation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>River Velocity</td>
<td>$V_\infty$</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Boom Length</td>
<td>$L$</td>
<td>5 m</td>
</tr>
<tr>
<td>Span</td>
<td>$s$</td>
<td>0.75 m</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>$AR$</td>
<td>6</td>
</tr>
<tr>
<td>Generator Constant</td>
<td>$k_\theta$</td>
<td>200 kg m$^2$/s</td>
</tr>
</tbody>
</table>

3.2.1 Evaluation of Steady-State Operating Points

We discovered that there are multiple equilibrium states for the system. Equilibrium is reached when the hydrodynamic forces on the hydrofoil create equal and opposing forces to the torque that is acting on the generator when the boom moves at a particular angular velocity. For each instantaneous point this angular velocity would be constant, meaning the system is not accelerating at that instant.

Figure 3.8: Shows the multiple roots for a range of angular velocities between -5 rad/sec and 5 rad/sec. 250 data points were used to initialize for every 1° increment of boom position, $\theta$. The hydrofoil angle ($\beta$) is set to 99° and using the NACA 0015. The points highlighted in red denote negative angular velocities, meaning these would be used for a return stroke. The right figure shows the instantaneous power production for the multiple angular velocities shown in on the left, where the red is the power production for the return stroke angular velocities.
When solving for the equilibrium angular velocity of the system it was discovered that different initial guesses would result in the model producing a different maximum and total cycle power for the same inputs. In Figure 3.8, the different angular velocity paths can be seen when an array of initial guesses is given to the system. These equilibrium paths change in magnitude, in quantity and the boom position for maximum angular velocity translates with respect with boom position. In most cases the angular velocity equilibrium paths are continuous throughout the swept boom angles, however in a few scenarios the path stops. These specific cases were evaluated in more depth to determine why these roots would appear and then disappear.

Using 250 initial angular velocities equally spread between -5 and 5 rad/sec, the equilibrium points were found for four different hydrofoil angles across a full range of boom positions stepping in 5° increments. The equilibrium angular velocities were then used to calculate the instantaneous power production for the system. The points highlighted in red denote negative angular velocities, meaning these would be used for a return stroke and the black points correspond to positive angular velocities which would be used for the deploy stroke. The four different hydrofoil angles were also used to evaluate Eq. (3.21) along the range of initial angular velocities. These plots show the roots of that function which are the steady-state boom angular velocities for the system. It can be seen in Figures 3.9, 3.10, 3.11 and 3.12 that with the changing hydrofoil angles some of the roots will “disappear” meaning that the angular velocity for either a deploy or a return stroke will be eliminated at a certain hydrofoil angle and the system will not be able to operate in that regime. Therefore, for some hydrofoil angles, the system could operate continuously for a complete range of boom positions (ranging from ±180°) as seen Figure 3.8 or could only operate continuously for a partial range of boom positions as seen in Figure 3.9a.
(a) Instantaneous Power and Angular Velocities to achieve equilibrium conditions

(b) Function values for varying angular velocities

Figure 3.9: Hydrofoil Angle of 100° for NACA 0015

(a) Instantaneous Power and Angular Velocities to achieve equilibrium conditions

(b) Function values for varying angular velocities

Figure 3.10: Hydrofoil Angle of 102° for NACA 0015

(a) Instantaneous Power and Angular Velocities to achieve equilibrium conditions

(b) Function values for varying angular velocities

Figure 3.11: Hydrofoil Angle of -100° for NACA 0015
Methodology for Finding Maximum Equilibrium Angular Velocity

In order to determine the maximum power the system is capable of for every deploy stroke and return stroke, there is an array of initial guesses to determine what the maximum equilibrium angular velocity is. That is used as the initial guess in the “fzero” function for the first boom position to calculate power production. The converged value from that boom position is then used as the next initial guess for the next boom position.

In an actual application of this system, choosing the maximum equilibrium angular velocity would translate to the system needing a “kick” to begin its deploy or return stroke. This would need to be done consistently since once the system slows, the next closest operating point could be a slower angular velocity, which would produce less power. In the design phase of the system this would need to be addressed to find a solution that would not consume much power but provide the “kick” that the system needs to obtain the maximum power possible. In the cases where there is no equilibrium angular velocity for the deploy stroke and/or the return stroke, the power production is not calculated since this would be an infeasible operating condition.

For any set of input parameters for this system, the instantaneous power during either the deployment stroke or the return stroke is largest when the hydrofoil is closer
to the bank of the river \((i.e. \text{ when } \theta=0^\circ)\). Even though the maximum angular velocity for any given hydrofoil angle is not always at a 0° boom position (meaning the boom would be along the bank of the river) the peak is normally very close to this point. In all cases the equilibrium velocity of the system decreases as the hydrofoil approaches a perpendicular position in the river (the boom outstretched into the river) which can also be seen in Figure 3.8. Having a low angular velocity allows for the hydrofoil to “flip” while the system is moving relatively slow and may decrease the amount of power consumed to do this. However, if the hydrofoil “flips” after the system has been operating in a low instantaneous power zone, the total cycle power is decreased due to the slow rotation and low torque on the system. Therefore it becomes a trade off between when to flip the hydrofoil to possibly reduce the power needed for flipping but still be generating power during the deploy and return strokes.

### 3.2.2 Evaluation of Power Production

For both the symmetric and asymmetric hydrofoil, the system has four separate peaks. After completing the large mapping and determining which hydrofoil angles generate the peaks and which do not have an equilibrium angular velocity for either the deploy or return stroke, smaller mappings were completed for each peak individually in 0.5° increments. Each peak was separated into its own quadrant, which was labeled in the same order for both the symmetric and asymmetric hydrofoils. The maximum cycle power, deploy angle and return angles to achieve that power production can be seen in Table 3.3. In both the symmetric and asymmetric case there is one peak that is much larger than the other three peaks, meaning this would be the ideal operating condition for power generation. Again, the symmetric hydrofoil produces much higher power compared to the asymmetric hydrofoil like in the translating system. All four peaks were mapped at the higher resolution for the symmetric hydrofoil, however the asymmetric hydrofoil was only mapped around the largest peak at the higher
Figure 3.13: Complete mapping of maximum cycle power as a function of deploy angle and return angle from \(-180^\circ\) to \(180^\circ\) in increments of \(5^\circ\) NACA 0015. This mapping was then divided into 4 quadrants, labeled on the plot so that they could each be investigated independently.

The maximum power production for the asymmetric hydrofoil is still smaller than some of the smaller peaks in the symmetric case.

Table 3.3: Maximum Power Production for Rotational Model

<table>
<thead>
<tr>
<th>Hydrofoil</th>
<th>Quadrant</th>
<th>Power Production (Watts)</th>
<th>Deploy Angle (degrees)</th>
<th>Return Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 0015</td>
<td>1</td>
<td>1340</td>
<td>92.5</td>
<td>-92.5</td>
</tr>
<tr>
<td>NACA 0015</td>
<td>2</td>
<td>568</td>
<td>-95.5</td>
<td>-92.5</td>
</tr>
<tr>
<td>NACA 0015</td>
<td>3</td>
<td>241</td>
<td>-95.5</td>
<td>95.5</td>
</tr>
<tr>
<td>NACA 0015</td>
<td>4</td>
<td>569</td>
<td>92.5</td>
<td>95.5</td>
</tr>
<tr>
<td>NACA 4412</td>
<td>4</td>
<td>342</td>
<td>88</td>
<td>100</td>
</tr>
</tbody>
</table>

3.2.3 Evaluation of Additional Parameters

Generator Constant

Our model of the generator lumps the steady-state generator characteristics along with the electrical load and gear ratio between the boom and the generator into a
Figure 3.14: Higher resolution view of quadrant 1 shown in Figure 3.13. The maximum power produced is 1340 Watts and occurs at a deploy angle 92.5° and a return angle of -92.5°.

Figure 3.15: Higher resolution view of quadrant 2 shown in Figure 3.13. The maximum power produced is 568 Watts and occurs at a deploy angle -95.5° and a return angle of -92.5°.

Figure 3.16: Higher resolution view of quadrant 3 shown in Figure 3.13. The maximum power produced is 241 Watts and occurs at a deploy angle -95.5° and a return angle of 95.5°.
Figure 3.17: Higher resolution view of quadrant 4 shown in Figure 3.13. The maximum power produced is 569 Watts and occurs at a deploy angle 92.5° and a return angle of 95.5°.

Figure 3.18: Complete mapping of maximum cycle power as a function of deploy angle and return angle from -180° to 180° in increments of 5° for NACA 4412. This mapping was then divided into 4 quadrants, labeled on the plot so that they could each be investigated independently.
Figure 3.19: Higher resolution (0.5° increment) view of quadrant 4 shown in Figure 3.18. The maximum power produced is 342 Watts and occurs at a deploy angle 88° and a return angle of 100°

single parameter, the generator constant, \( k_\theta \). Since the optimization for the angular velocity is based on Eq. (3.21) the K factor will determine the equilibrium angular velocity, which directly affects the power production. As seen in Figure 3.20, there is an optimum K factor for both the symmetric and asymmetric hydrofoils. These generator constants are very large, which is difficult to duplicate in an actual system, however it is good to have an understanding that this factor does play a large role in the power production of the system.

For this investigation, the parameters in Table 3.2 were used and the peak production point was determined from a cursory system mapping. The resolution for the hydrofoil angles was then increased to every 1/4° and a smaller search area was used. The peak power was then determined as well as the deploy and return angles for generating this power, shown in Figures 3.20 and 3.21. It can be seen in Figure 3.20 that the symmetric and asymmetric hydrofoils have different optimal generator constants but similar trends.

**Position for Changing Hydrofoil Angles (Flipping)**

To evaluate the boom position for flipping the symmetric hydrofoil was used with the original set of conditions used for the evaluation of power production. The largest
Figure 3.20: Power Production as a function of the generator constant (K factor) for the symmetric and asymmetric hydrofoils.
Figure 3.21: The hydrofoil angles to generate maximum power production for both the symmetric and asymmetric hydrofoil shapes.
peak (found in quadrant 1) was used in order to evaluate the effect that changing the flipping angle has on power production. The entire quadrant was mapped for maximum power and the corresponding flipping angle in order to achieve the maximum power was documented. It can been seen in Figure 3.22 that around the location of maximum power, the flipping angles are very small and for large deploy angles and small return angles the flipping angle approaches $90^\circ$.

In order to gain a better understanding of the flipping angle close to the maximum power point, the system was mapped for deploy angles of $88^\circ$ to $95^\circ$ and return angles of $-88^\circ$ to $-95^\circ$, in increments of $1^\circ$. The maximum power that can be produced for each combination of deploy and return angles was recorded (64 combinations), as well as the angle that the system needs to flip at to achieve that power. For all cases to achieve peak power, the flipping angle was $1^\circ$ due to the fact that the code must integrate between two angles, in this case 0 and $1^\circ$. The original mapping is shown in Figure 3.23. This was an expected result after looking at the instantaneous power curves for the given deploy and return angles because the peaks are at a boom position of approximately $0^\circ$. However, this is obviously not the best case scenario for a real world application.

In order to better understand how the power production is affected by the flipping angle the angle that the system would flip at was fixed to different values. The power production was captured at the point where the hydrofoil flips and then compared to the maximum power production that could be achieved for the same 64 combinations of deploy and return angles as used above. The percent difference from the maximum power to the cycle power seen at any flipping point was then calculated. The percent different for the peak power was then taken and plotted as a function of the boom position for flipping, Figure 3.24. This was done for flipping angles ranging from $10^\circ$ to $60^\circ$.

The power produced decreased as the flip angle increased. It was also determined
Figure 3.22: Contours of maximum power production and corresponding flipping angle in order to achieve it for a range of deploy angles from 0 to 102° and range of return angles from 0 to -102° in increments of 2°.
Figure 3.23: Contour of maximum power production around the peak that is shown in Figure 3.22 and corresponding flipping angle in order to achieve it.
throughout this study that the deploy angles and return angles to achieve maximum power were the same for the the range of flipping angles that were investigated. Therefore, the plot of the cycle power at these angles was created as a function of boom position (Figure 3.25) in order to illustrate the decrease in power production.

**Boom Length**

The boom length was also investigated for the symmetric hydrofoil. The flipping angle for this was fixed at 60° and all other parameters were set to the original input conditions used for the evaluation of power production. The boom length was varied from 1 to 90 meters and the hydrofoil angles were mapped multiple times, moving from large searches with coarse increments for hydrofoil angles, to smaller searches in 1° increments. The maximum power, deploy and return angles were then recorded as a function of boom length and can be seen in Figure for three different generator constants. It was expected that an increasing boom length would lead to greater
Figure 3.25: Cycle power as a function of flip angle for $\beta = 92^\circ$ and $\beta = -92^\circ$ which was where the maximum power production occurs on the given range of deploys and return angles.

...power production because it would allow the system to sweep through a larger cross sectional area of the river while still flipping at the same boom position. Instead, what was seen is that there is an optimum boom length for power production given a single generator constant, however, as the generator constant increases, the peak power value that is achieved also increases. Further optimization work would be required to have a complete mapping of power production as a function of deploy and return angles as well as boom position and generator constants.

3.3 Preliminary Result from a Dynamic Rotational Model

In order to better understand an actual system and some of the limitations of the previous steady-state model, we developed a dynamic rotational model which includes the boom and kite mass. The hydrodynamic model remains unchanged, so this is not a full dynamic model. This model allows for the transients in the system to be observed by looking at the system over a designated time period. For this investigation
Figure 3.26: Power production for a flip at 60°, as a function of boom length for various generator constants.

(a) Deploy Angles  
(b) Return Angles

Figure 3.27: Corresponding hydrofoil angles to achieve the peak power as a function of boom length for various generator constants
the time that the system can run for is defined as well as the starting boom position and an initial angular velocity. The angle of attack for the system is calculated in the same fashion as the static model, and used to determine the hydrodynamic coefficients. The resulting forces, moments and boom angular acceleration are calculated. Explicit constant step size fourth order Runge-Kutta integration is used to numerically integrate the equations of motion for the system. Only the symmetric hydrofoil was used to compare the dynamic system to the static system. Hydrofoil flip angles are predetermined unlike the previous steady-state models. The flipping occurs when \( \dot{\theta} < 1 \text{ rad/s or } \theta > 60^\circ \). The system will then change back to the deploy hydrofoil angle when \( \theta < 10^\circ \) boom position.

\[
\tau = -k_\theta \dot{\theta} \tag{3.24}
\]

\[
I_b = \frac{1}{3} m_b L^2 \tag{3.25}
\]

\[
I_k = m_k L^2 \tag{3.26}
\]

\[
\ddot{\theta} = \frac{\tau + M + \vec{r}_{arm} \times \vec{F}}{I_b + I_k} \tag{3.27}
\]

\[
P = k_\theta \dot{\theta}^2 \tag{3.28}
\]

Figure 3.28 shows a single frame of the simulation’s animation.

At the end of the alloted time, the system energy is then approximated using the trapezoidal approximation tool in MATLAB ("trapz") using the instantaneous power over the elapsed time. The average power over the allotted time period is then calculated by dividing the total energy by the total time. This approximation for the average power is not precise due to the fact that the initial transient is taken into account and the calculation does not adjust for the total number of cycles completed in the given time period. The true average power per cycle would need to be calculated differently in order to exclude the transients.
3.3.1 Results

The hydrofoil angles which generated the maximum power in the steady-state model for the symmetric hydrofoil were originally used as well as the parameters shown in Table 3.4. The choice of low masses for the kite and the boom was done to try to replicate the results seen in the previous model as best as possible. The simulation was run with an initial boom position of $0^\circ$ and given an initial angular velocity of $2$ rad/s. The system response was observed for a period of 30 seconds in 0.001 second increments.

Since the flipping position was defined to be $60^\circ$ for the dynamic model, the steady-state model was also run with a fixed flipping position of $60^\circ$. The total cycle power calculated for the steady-state model with the same inputs as shown in Table 3.4 (excluding the mass terms) was 688 Watts. The position of the boom and the power production with respect to time for the dynamic model are shown in Figure 3.29. The initial power production of the system was large, however the power production
Table 3.4: Base Set of Rotational Model Parameters for the Dynamic Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Abbreviation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>River Velocity</td>
<td>$V_\infty$</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Boom Length</td>
<td>$L$</td>
<td>5 m</td>
</tr>
<tr>
<td>Span</td>
<td>$s$</td>
<td>0.75 m</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>$AR$</td>
<td>6</td>
</tr>
<tr>
<td>Generator Constant</td>
<td>$k_\theta$</td>
<td>200 kg m^2/s</td>
</tr>
<tr>
<td>Boom Mass</td>
<td>$m_b$</td>
<td>1 kg</td>
</tr>
<tr>
<td>Kite Mass</td>
<td>$m_k$</td>
<td>0.01 kg</td>
</tr>
<tr>
<td>Deploy Hydrofoil Angle</td>
<td>$\beta_D$</td>
<td>92.5°</td>
</tr>
<tr>
<td>Return Hydrofoil Angle</td>
<td>$\beta_R$</td>
<td>-92.5°</td>
</tr>
<tr>
<td>Flipping Angle</td>
<td>$\theta$</td>
<td>60°</td>
</tr>
</tbody>
</table>

decreased quickly and was minimal for the remaining time. The average power over the designated time period was 14.1 Watts. A complete investigation into where the disconnect between the two models is necessary to be able to make further comparisons, however it is useful to know that the parameters used in the steady-state model do not directly correlate to the dynamic model.

In Figure 3.29 the boom position shows that the system does not instantaneously change the direction which it is rotating after the hydrofoil angle is changed from the deploy angle to the return angle. It can be seen that the system still rotates past 60° before beginning to rotate back towards the starting position. It can also be seen that when the hydrofoil angle flips back to the deploy angle, the system does not rotate forward again, stalling the system. Another observation was made when the power production curve was enlarged around the initial starting point. Shown in Figure 3.30, the power production increase after the initial kick is given to the system, then decreases. This occurs because the system is gaining kinetic energy from the fluid flow at the begin, however as boom position increases the power production begins to decrease.

The hydrofoil angles were changed so that the system could operate for multiple cycles over the given time period. The deploy hydrofoil angle was set to 80° and the
Figure 3.29: Graphical representation of the system outputs for the NACA 0015 for a deploy hydrofoil angle of 92.5° and a return hydrofoil angle of -92.5° over a 30 second time interval which had an incremental step of 0.001 seconds.

Figure 3.30: Enlarged view of the power production curve seen in Figure 3.29
Figure 3.31: Graphical representation of the system outputs for the NACA 0015 for a deploy hydrofoil angle of 80° and a return hydrofoil angle of -70° over a 30 second time interval which had an incremental step of 0.001 seconds.

Figure 3.32: Phase plane for the cycle shown in Figure 3.31
return hydrofoil angle was set to -75°. We are not sure if multiple periodic motions exist for different initial conditions, but we do see a large initial sustained boom angular velocity for the first cycle if given a sufficiently large initial $\dot{\theta}$. It can be seen in Figure 3.31 that the transient drops out after the first stroke and for the set of given system parameters that the system developed a periodic type of motion. The phase plane for this simulation was generated to illustrate the transient more clearly and that the cycles overlap each other, Figure 3.32.

### 3.4 Conclusion

After investigating all of these different parameters; hydrofoil angle for deploy and return strokes, generator constants, flipping position and boom length for the steady-state model, it is apparent that there is an optimum value for power production for each one. In order to fully optimize the system, all parameters would need to be evaluated at the same time. However, this study has shown that power production is very sensitive to the hydrofoil angle, $\beta$, with a narrow range of optimal hydrofoil angles. This correlates to the results shown in the translating system.

Multiple cycle power peaks were found in large mapping of the system even though in both the symmetric and asymmetric case there is one peak that is much larger than the other three peaks. It is also important to see that the peaks are located close to areas where there is zero power production where the system cannot complete a deploy and/or return stroke.

The existence of multiple equilibrium angular velocities at which the system can operate was unexpected but interesting. The multiple steady-state operating conditions would allow the system to produce different amounts of power depending on which equilibrium curve the system is operating on. The maximum angular velocity for any given set of parameters could be difficult for a real world system to achieve, since the real system needs to accelerate from $\dot{\theta} = 0$ at each end of the deploy and
return strokes. If these peak angular velocities cannot physically be met, the predicted power of the system would be significantly larger than the actual power that system will produce. It would be interesting to determine if these multiple roots for equilibrium velocities can be seen in the dynamic model or in a physical prototype of the system.

Both the symmetric and asymmetric foils have distinct generator constants that produce maximum power and have very similar trends of power with respect to $k_\theta$. However, these curves are dependent on all of the system parameters. It is helpful to know that there is an optimal generator constant for peak power and that this input parameter can affect the power output drastically.

The flipping angle would not able to be $0^\circ$ for a real world system because the system would not be able to rotate and the flip would need to be instantaneous. The power decreases as the flip angle increases which means that it would have to be decided where the flipping will occur based on the acceptable reduction in power. Another option would be to redesign the system to rotate in the middle of the river so it could rotate through the peak power zone and therefore increase the overall cycle power. A non-zero flip angle would be required for this system as well.

3.5 Future Work for Rotational Model

Since there were many different parameters investigated for this model, the next step would be to conduct testing in order to determine if the phenomena observed within the model also occur in a physical system. Testing all of these parameters would be extremely helpful for evaluating the validity of the model and determining if the simplifying assumptions reduce the accuracy of this system substantially. One of the most interesting things to test with a real world system would be the hydrofoil angles, near the peak power production, that do not generate any power for the deploy and/or the return stroke. If this is seen in an physical system, it could show that the accuracy
of the hydrofoil position is imperative for successfully producing power.

Another fruitful area for future work would be to optimize the hydrofoil angle, $\beta$, throughout the deploy and return stroke instead of keeping them fixed relative to the boom while the boom changes position. This would hopefully increase power production, however it would also require a more complex control system for a real world application. Therefore, numerically determining how much more power could be achieved by controlling the hydrofoil would be beneficial before trying to design and build the controls and mechanical system to continually adjust the hydrofoil angle.

If the system was optimized for the absolute maximum cycle power, the system would only rotate in an extremely small range or angles near the bank of the river or not at all due to where the peak instantaneous power occurs. Mounting the system in the center of the river so that it could rotate through the maximum power point could increase power production. This was not investigated for this work due to the fact that it would be a more difficult system to build. Evaluating the power production of a system that was mounted in the center of the river could be another aspect worth modeling prior to building an actual system. The current simulation should be capable of determining the power increase with minor modifications, but all parameters would need to be re-evaluated, including examining two different flipping locations instead of only one.

The Dynamic Rotational Model created here needs extensive amounts of exploration in order to fully understand how each parameter affects the power production. We predict that the deploy and return angles for the dynamic system act in a similar way that the hydrofoil angles affected the power production for the static model, however the exact angles may shift due to the fact that the system has to “flip” at a boom position that is not close to $0^\circ$. How the mass of the boom and the mass of the kite affect the power generation of the system should be explored in full.
Chapter 4

Final Conclusions

4.1 Overall Conclusions

The different models examined in this thesis have allowed us to observe the several aspects of the hydrokite system. The original goals achieved in this work were to:

- Determine the power generation characteristics of a hydrokite model and how can a feasible system be achieved
- Determine the sensitivity of the system’s power generation with regards to parameter changes
- Determine how different models of power capture (translating vs. rotational) affect the limits of power generation

The steady-state models discussed in this work provide an upper bound for the power performance of an actual system as well as an understanding of the effects that parameter changes have on the system performance. This initial work shows that such a system could be a feasible, low-impact method for generating renewable energy from low-head hydro sources. This work has provided insight that can inform further studies and guide the design and construction of physical prototypes.

The translational and rotational models have both shown that the hydrofoil angle, $\beta$, has a very large impact on power generation. The power surfaces of both models are relatively steep near the peaks with respect to the hydrofoil angle, therefore even
relatively small angle changes can have a large effect on the amount of power that can be generated by the system. This makes the system very sensitive to changes in the hydrofoil angle $\beta$. The investigation into the rotational model showed that the generator constant, boom length and flipping position also have an effect on the power production of the system. For each of these parameters there was a maximum condition for power production.

Overall, the models are highly sensitive to the hydrofoil lift and drag data that is used. As seen in Figure 2.13, sharp changes in the slope of the drag coefficient made a large impact on the smoothness of the power contour. The difficulty with the hydrofoil data is that it is experimental, which makes it more realistic for real world applications but not as continuous as theoretical data, which then translated to the power surfaces. Table look ups and non-smooth lift and drag coefficient slopes can cause problems for optimization algorithms since they introduce jaggedness into the landscape of the objective function.

The translational and rotational steady-state models investigated showed that the power generating capabilities of the two systems are somewhat different, but this is mainly due to the ways the systems were defined. In the translating model only the instantaneous power was calculated compared to the rotational model which calculated the cycle power. Using the instantaneous power and kite velocity, $V_k$, the cycle power can be calculated for the translating model, however, this would only compare to the rotational model at a boom position of $0^\circ$. When the boom rotates through different positions the power production decreases, which can correlate to the rail angle changing in the translating model. After the initial exploration of the dynamic rotational model it was determined that the optimal power production characteristics of the system are very different.

Outside the scope of mechanical engineering, a complete environmental assessment of a hydrokite system is desirable before permanently installing the system into a river.
With that being said, the Genesee River, which was used as a model for this system is highly polluted already. Though efforts have been made to clean the water, there are still many issues with regards to the current quality of the water. “Major water quality concerns in the watershed are; Urban Stormwater and Industrial Runoff in Rochester area, Agricultural and Other Nonpoint Sources of nutrients and various other pollutants and Protection of Municipal Water Supply in the Hemlock Lake watershed” [4]. Since there are so many other existing pollutants that are currently interfering with the water ecology, it is assumed that the hydrokite system would have a negligible impact on a river like the Genesee.

4.2 Final Recommendations for Future Work

Overall, this analysis has shown that in the future, the hydrokite could be a environmentally friendly micro-hydro power generation system. The majority of the natural resources that are suitable for large hydroelectric dams have been utilized, or are financially infeasible. New hydro-power systems will be needed in order to capture energy from water resources. This analysis has shown that this system could be worth pursuing further. Developing the models to be more comprehensive and robust will allow for more effective prototypes to be designed and built. Even though the current models provide an estimation for how much power could be produced, the amount of over estimation could be quantified in some future work. Refining these assumptions so that they become more realistic to a real-world scenario will be the next step in determining the feasibility of the system. Extensive investigation of the current dynamic model, and possibly others, will also be critical in the design of a prototype hydrokite system. These models have provided a good understanding of how the parameters such as hydrofoil angle, $\beta$, and generator constants, $k_\theta$, will change the amount of power that can be generated.

Discussed individually in each system chapter, the following is a list of different
areas that would be useful in generating more comprehensive and realistic models.

- Experimentally quantifying the amount of power required for flipping the hydrofoil using both flipping methods (Figure 2.3)
- Simplest model for including the power loss due to flipping the hydrofoil in the simulation
- Comparing the power production for the two different flipping methods
- How to best adjust the hydrofoil angle, $\beta$, throughout a stroke (i.e. flying the hydrofoil)
- Developing a more realistic velocity profile for the river flow and determining its effect on power production
- Optimizing the Aspect Ratio, $AR$, of the hydrofoil for increased power production
- Expand the investigation of the dynamic rotational model to determine how changes in the addition parameters (mass of the boom and of the kite) affect the power production
- Optimization of fixed deploy and return hydrofoil angles for the dynamic rotational model
- Identifying the most significant potential environmental impacts and ways in which they would be quantified
- Quantifying the identified environmental impacts

Lastly, an additional model that eliminates the rigid rail or boom and simply attached the hydrofoil to a flexible, inextensible tether we think would be ideal. This would minimize the mass of the boom, however the motion of the tether will be more
challenging to simulate. The addition of the tether would be more similar to the high altitude kite models that were discussed in the literary review of this work. This tethered hydrofoil model would be a very interesting system to analyze and the results of the mathematical model would be instrumental in building and controlling a real world system. This tethered system would hopefully produce more power than any of the other systems constructed here and would be lower cost. This would increase the power production per cost and make the hydrokite a feasible method of energy extraction for micro-hydro applications in the future.
Bibliography


Appendix A

Hydrodynamic Models

The hydrodynamic model used for all of the models is described in this section. This model used 2D steady-state lift, drag and moment coefficients to determine the hydrodynamic forces. The 2D coefficients were then translated to 3D similar to the method used for wings. A Reynolds number of \(1 \times 10^6\) was used, unless otherwise specified, based on preliminary information about the models which makes the model viscous.

A.1 XFOIL Hydrofoil Data

Originally, the hydrofoil lift and drag data was going to be generated using Mark Drela’s software program XFOIL. “XFOIL is an interactive program for the design and analysis of subsonic isolated airfoils” [5]. This software was used to generate the lift, drag and moment coefficients for the NACA 0018 symmetric hydrofoil and the NACA 4412 asymmetric hydrofoil for various Reynolds numbers. The data generated for the lift and drag coefficients are shown in Figure A.1 and Figure A.2.

The difficulty that the XFOIL data presented is that it only is given for a limited range of angle of attacks (-10° to 25° for the asymmetric hydrofoil and -25° to 25° symmetric hydrofoil). It was determined during the simulations that the system operates past the point of stall, which means that after the stall point the forces and moments were unable to be computed in the model. In order to do this, other means of determining hydrodynamic coefficients were investigated.
Figure A.1: 2D Lift and Drag Coefficients for the NACA 0018 symmetric hydrofoil [22]

(a) 2D Lift Coefficients for 3 Different Reynolds Numbers
(b) 2D Drag Coefficients for 3 Different Reynolds Numbers

Figure A.2: 2D Lift and Drag Coefficients for the NACA 4412 asymmetric hydrofoil [22]

(a) 2D Lift Coefficients for 3 Different Reynolds Numbers
(b) 2D Drag Coefficients for 3 Different Reynolds Numbers
A.2 Experimental airfoil data

Both a symmetric (NACA 0015) and an asymmetric (NACA 4412) profile for the hydrofoil cross-section was used in the simulations of the various translating hydrofoil systems. The choice of these two particular cross-sectional shapes was motivated by the availability of experimental lift, drag and moment data and the fact that the shapes are common. Since most applications which use airfoils/hydrofoils operate in a narrow range of angles of attack $\alpha$, usually $0 \leq \alpha \leq \alpha_{\text{stall}}$, the experimental data for many airfoils does not extend to angles of attack past stall. However, we wanted to explore the performance of our system over a wide range of parameters, thus data beyond the stall point was needed. Data for both the lift and drag coefficients which ranged from $-180^\circ$ to $180^\circ$ would be ideal for this system so that for any given inputs, the hydrodynamic forces acting on the system could be calculated. This would eliminate any restrictions on the system due to limited angles of attack in the experimental data.

A.2.1 Symmetric Hydrofoil

For the symmetric hydrofoil, NACA 0015, experimental 2D data for large angles of attack $0 \leq \alpha \leq 180^\circ$ was found in a study which measured section lift/drag/moment coefficients for seven symmetric airfoils which were used in the design of vertical-axis wind turbines [21]. The experimental values for the lift and drag coefficients in this report were measured for a range of Reynolds numbers ($10^4$ to $10^7$). The data for a Reynolds number of $1.0 \times 10^6$ was used since the average Reynolds number seen within the simulation.
Figure A.3: 2D Lift coefficients for an Infinite Wing, NACA 0015, plot of experimental data published in [21]

Figure A.4: 2D Drag coefficients for an Infinite Wing, NACA 0015, plot of experimental data published in [21]
Figure A.5: 2D Moment coefficients for an Infinite Wing, NACA 0015, plot of experimental data published in [21]

A.2.2 Asymmetric Hydrofoil

Experimental data for the desired range ($-180^\circ \leq \alpha \leq 180^\circ$) of angles of attack could not be located for the chosen NACA 4412 asymmetric hydrofoil. However, experimental data for 2D lift/drag/moment coefficients for $-10^\circ \leq \alpha \leq 110^\circ$ was found for infinite wings [17]. Unfortunately, this data was only available for a Reynolds number of $0.25 \times 10^6$, which is slightly lower than the Reynolds number used for the symmetric hydrofoil. The lift and drag coefficients for a Reynolds number of $1.0 \times 10^6$ were provided as well, but only for a much smaller range of angles, (approximately $-10^\circ \leq \alpha \leq 12^\circ$). The coefficient values for the Reynolds number of $1.0 \times 10^6$ appear to almost perfectly overlap with the coefficient values for the Reynolds number of $0.25 \times 10^6$ at the small angles of attack. This, combined with the fact that limited amount of experimental results for high angles of attack were available, was the justification for using the slightly lower Reynolds number for the asymmetric hydrofoil.
Unfortunately, unlike the NACA 0015 data [21], the NACA 4412 data [17] was provided only in graphical format and not in the form of a numeric table. We scanned in the plots provided in the paper for the lift, drag and moment coefficients and digitized them. This code can be found in Appendix C.4. The original plots are shown in Figure A.6. Plots of the digitized data are shown in Figures A.7, A.8 and A.9.

A.2.3 Extending experimental results past stall for 0°-360° angles of attack (Viterna Model)

For robustness of the simulation and for a more thorough parameter search, an extension of the data for a complete $-180^\circ \leq \alpha \leq 180^\circ$ was desired for the NACA 4412. Once the flow detaches from the top of an airfoil, and it stalls, the detailed shape of the airfoil is not so critical in determining the fluid forces on it. Thus, modeling post-stall behavior of an airfoil by assuming it is a flat plate can be reasonable step.

The National Renewable Energy Laboratory’s (NREL’s) “User’s Guide to the Wind Turbine Aerodynamics Computer Software” [16] includes a calculation for post-stall airfoil performance characteristics for wind turbines known as the Viterna model. The Viterna model “extrapolates airfoil data from a limited range of angles to the entire range of angles using flat plate characteristics” [16].
Figure A.7: Digitized 2D Lift coefficients for an Infinite Wing, NACA 4412, plot of experimental data published in [17]

Figure A.8: Digitized 2D Drag coefficients for an Infinite Wing, NACA 4412, plot of experimental data published in [17]
Using the experimental data for the NACA 4412, the Viterna model was applied and extensions to the experimental data were made so that the fluid forces could be determined for angles of attack in the range $-180^\circ \leq \alpha \leq 180^\circ$. The Viterna model was applied to all of the different coefficients. At the ends of the range of angle of attack ($-180^\circ$ and $180^\circ$) the coefficients are approximated so that the data is complete. At $\alpha = \pm 180^\circ$ the lift and moment coefficients are approximated as 0 and the drag coefficient is approximated as 0.009 because that is the calculated drag value for $\pm 179^\circ$ angle of attack. The section below outlines the method but for complete instructions of how to apply this model please refer to [7].

Method for applying the Viterna Model to the Lift and Drag Coefficient (See Figure A.10) [7]

- **Section A**: The digitized experimental data

- **Section B**: Data taken from $-25^\circ$ to $93^\circ$ was taken, adjusted to “intercept zero at 90 degrees and reflected around the y axis and x axis. These values were also
scaled down by 30\% for the symmetric hydrofoil. Values are not scaled for the Drag Coefficients.

- **Section C**: Created by mirroring Section B around the -90\(^\circ\) mark.

- **Section D**: Created by mirroring Section C and using the data from 180\(^\circ\) until it intercepted the experimental data.

**Method for applying the Viterna Model to the Moment Coefficient** [7]:

- The center of pressure is assumed to be at the mid chord point at \(\alpha = 90^\circ\) which defines \(C_M\) as \(-C_{D_{max}}/4\).

- “The moment coefficients are reflected to positive values for negative angles of attack” [7].

- Moment coefficients for the following points were defined within the User’s Guide as the following: \(C_M=0.4\) for \(\alpha = -170^\circ\), \(C_M=-0.5\) for \(\alpha = 170^\circ\), \(C_M=0.4\) for \(\alpha = \pm 180^\circ\).
Figure A.11: Experimental 2D Lift coefficients for a NACA 4412 airfoil from [17] along with the extended data from the Viterna model

This approximation for the complete range of angles of attack appears to make reasonable prediction about the lift and drag coefficients that could be expected outside of the typical operating range of the hydrofoil. The extended lift and drag data is shown in Figure A.11 and Figure A.12.

A.2.4 Comparison of XFOIL data to Extended Experimental Data

The extended experimental data was compared to the XFOIL data in order to see if there were any large discrepancies. For the symmetric hydrofoil the data appears to be very similar, however the asymmetric hydrofoil data has substantial differences in the drag coefficients. This difference can also be seen in the lift to drag ration comparison. This discrepancy could not be explained and needs further investigation to determine why there is this large difference in the coefficients.
Figure A.12: Experimental 2D Drag coefficients for a NACA 4412 airfoil from [17] along with the extended data from the Viterna model.

Figure A.13: Experimental 2D Moment coefficients for a NACA 4412 airfoil from [17] along with the extended data from the Viterna model.
Figure A.14: Comparison of lift coefficients from XFOIL to the experimental data for the NACA 0015

Figure A.15: Comparison of drag coefficients from XFOIL to the experimental data for the NACA 0015
Figure A.16: Comparison of lift to drag ratio from XFOIL to the experimental data for the NACA 0015

Figure A.17: Comparison of lift coefficients from XFOIL to the experimental data for the NACA 4412
Figure A.18: Comparison of drag coefficients from XFOIL to the experimental data for the NACA 4412

Figure A.19: Comparison of lift to drag ratio from XFOIL to the experimental data for the NACA 4412
A.3 Infinite to finite wing calculations for 360° Hydrofoil Data

Now that each hydrofoil has lift and drag coefficients which extend for an angle of attack range from -180° to 180° the force on a finite wing needs to be determined since the forces on an infinite wing and finite wing of the same cross-section are significantly different [1]. The 3D coefficients are different compared to the 2D coefficients because they take into account the wingtip vortices [1].

Some of the major assumptions used to translate the experimental data from 2D to 3D were made in the Span Efficiency Factor (Oswald Efficiency) \( e \) and the Span Effectiveness Factor for Lift Calculations \( e_1 \). “For elliptical planforms, \( e = 1 \); for all other planforms \( e < 1 \)” [1]. “For typical subsonic aircraft, \( e \) ranges from 0.85 to 0.95.” [1]. The span effectiveness factor can be defined “where \( e_1 \) and \( e \) are theoretically different but are in practice approximately the same value for a given wing.” [1]. Therefore, for this model the value of 0.9 was chosen for each of the efficiency factors and used throughout this analysis.

A.3.1 Induced angle of attack correction

It is standard practice to correct 2D coefficient data to 3D coefficient data using the following method, which can be found in [1]. Note that, in accordance with standard practice, we use lower case subscripts for 2D lift/drag/moment coefficients for an infinite wing, \( s = \infty \) (i.e. \( C_l, C_d, C_m \)) and upper case subscripts for 3D lift/drag/moment coefficients for a finite wing, \( s \neq \infty \) (i.e. \( C_L, C_D, C_M \)). The correction for lift coefficients for finite wings is defined as a change in the lift curve slope for the linear portion of the lift curve at low angles of attack before stall, (approximately -10° to 15°). The change in the slope is used to reflect the difference in the effective angle of attack to the geometric angle of attack. The change in lift slope is determined using Eq. (A.1a). The 3D coefficient of lift is then calculated using Eq. (A.1b).
Figure A.20: Change in Lift Coefficient from Infinite Wing to Finite Wing for the NACA 0015 where $\alpha_o=0$, $a_o=0.10595$ and $a=0.078027$

\[
\begin{align*}
  a &= \frac{a_o}{1 + \frac{180a_o}{\pi^2e_1AR}} \\
  C_L &= a(\alpha - \alpha_i)
\end{align*}
\]  

(A.1a)  

(A.1b)

In order to calculate the original slopes for both the symmetric and asymmetric hydrofoils, a line fit was done for the linear portion of the angle of attack data. This slope and the x-axis intercept of the line fit were then used to calculate the new slope and applied to this linear portion of the data. The new lift slope was extended until the line would have intersected the original 360° angle of attack data for both the positive and negative angles of attack. At this point, the lift coefficient values are returned to the 360° data. This correction is shown in Figure A.20 for the symmetric hydrofoil and in Figure A.21 for the asymmetric hydrofoil.
Figure A.21: Change in Lift Coefficient from Infinite Wing to Finite Wing for the NACA 4412 where $\alpha_o=-2.94648$, $a_o=0.08856$ and $a=0.068169$

A.3.2 Induced Drag

For each iteration of the simulation, for all models, the lift and drag values are calculated by determining the angle of attack of the hydrofoil and interpolating the corrected experimental data to provide lift and drag coefficients for the exact angle of attack. After the coefficients for lift and the profile drag have been determined, the induced drag is calculated. This correction is taken into account after the values of lift and drag are determined by interpolation. The effect of the induced drag is most noticeable around a zero degree angle of attack and is shown in Figure A.22 and Figure A.23.

\[ C_D = C_{do} + \frac{C_L^2}{(\pi e A R)} \]  

(A.2)

The induced drag is calculated by using Eq. (A.2) which used the interpolated values for lift and drag coefficients.
Figure A.22: Effect of induced drag for the NACA 0015 airfoil we are using with AR = 6.

Figure A.23: Effect of induced drag for the NACA 4412 airfoil we are using with AR = 6.
A.4 Lift to Drag \((C_L/C_D)\) Ratio

The Lift to Drag \((C_L/C_D)\) Ratio was examined after it was determined that the peak power does not occur at the maximum \(C_L/C_D\) ratio. When looking at the difference in \(C_L/C_D\) ratios between the symmetric and asymmetric hydrofoils, Figure A.24, it can be seen that the NACA 0015 has a much larger peak lift-to-drag ratio than the NACA 4412. This was an unexpected result that the asymmetric hydrofoil has a lower \(C_L/C_D\) ratio than the symmetric hydrofoil.

A.5 Conclusions for Hydrodynamic Models

Overall the hydrodynamic coefficients have the largest affect on the power production of the system which is why calculating the angles of attack for a full range of angles was very important. There are many other hydrodynamic phenomena that can affect the power production of the system, however, having hydrofoil data for a complete range of angles of attack is a excellent starting point. Obviously, optimizing the final
hydrofoil have would be ideal, however it is very difficult to find experimental data for a complete range of angle of attacks for any hydrofoil, so optimizing the hydrofoil shape could prove to be very challenging.
Appendix B

Generator Data from ASU

The data shown in Figure 2.5b was reproduced with permission from Dr. Mario Gomes and is listed in Table B.1. The data was from an experimental wind turbine DC generator which different load resistances were applied to at various speeds. The torque on the generator was measured using a load cell. The generator RPM was measured using an optical reflective sensor. This data illustrates the relationship between the rotations per minute and the corresponding torque values and supports our choice of generator model (i.e. $\tau = k_0 \dot{\theta}$).
<table>
<thead>
<tr>
<th>Load Resistance</th>
<th>Rpm</th>
<th>Torque (Nm)</th>
<th>Load Resistance</th>
<th>Rpm</th>
<th>Torque (Nm)</th>
<th>Load Resistance</th>
<th>Rpm</th>
<th>Torque (Nm)</th>
<th>Load Resistance</th>
<th>Rpm</th>
<th>Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30Ω</td>
<td>852</td>
<td>0.399</td>
<td>70Ω</td>
<td>847</td>
<td>0.204</td>
<td>110Ω</td>
<td>849</td>
<td>0.143</td>
<td>150Ω</td>
<td>856</td>
<td>0.114</td>
</tr>
<tr>
<td>40Ω</td>
<td>955</td>
<td>0.450</td>
<td></td>
<td>1019</td>
<td>0.249</td>
<td></td>
<td>1220</td>
<td>0.259</td>
<td></td>
<td>1620</td>
<td>0.184</td>
</tr>
<tr>
<td>50Ω</td>
<td>1013</td>
<td>0.484</td>
<td></td>
<td>1154</td>
<td>0.270</td>
<td></td>
<td>1326</td>
<td>0.305</td>
<td></td>
<td>1826</td>
<td>0.190</td>
</tr>
<tr>
<td>60Ω</td>
<td>1149</td>
<td>0.536</td>
<td></td>
<td>1232</td>
<td>0.289</td>
<td></td>
<td>1442</td>
<td>0.366</td>
<td></td>
<td>2026</td>
<td>0.200</td>
</tr>
<tr>
<td>70Ω</td>
<td>1269</td>
<td>0.599</td>
<td></td>
<td>1275</td>
<td>0.305</td>
<td></td>
<td>1529</td>
<td>0.713</td>
<td></td>
<td>2226</td>
<td>0.223</td>
</tr>
<tr>
<td>80Ω</td>
<td>1387</td>
<td>0.639</td>
<td></td>
<td>1450</td>
<td>0.346</td>
<td></td>
<td>1739</td>
<td>0.459</td>
<td></td>
<td>2526</td>
<td>0.250</td>
</tr>
<tr>
<td>90Ω</td>
<td>1442</td>
<td>0.676</td>
<td></td>
<td>1537</td>
<td>0.351</td>
<td></td>
<td>1860</td>
<td>0.421</td>
<td></td>
<td>3026</td>
<td>0.249</td>
</tr>
<tr>
<td>100Ω</td>
<td>1529</td>
<td>0.713</td>
<td></td>
<td>1689</td>
<td>0.432</td>
<td></td>
<td>1968</td>
<td>0.470</td>
<td></td>
<td>3526</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Figure B.1: Data provided for the load resistances, RPM and Torque
Appendix C

MATLAB Code

C.1 Static Translating Model

C.1.1 Brute Force Code

The brute force approach takes every hydrofoil angle and kite velocity for any rail angle and finds the maximum instantaneous power that can be produced.

Translating Model Run

This is the code that allows for the input parameters to be changed and the plotting of all of the surfaces and contours for the outputs of power production, angle of attack, lift and drag forces and coefficients, and Reynolds number.

```matlab
% by: Kelsey McConnaghy: 2011
% Kite on Rails
% Including Induced Drag

tic
clc
clear all
close all

global data_unsym_cl;
global data_unsym_cd;
global data_sym_cl;
global data_sym_cd;
```
global r_len;

r_len=10; % river width meters check in poweralc

vk=[0:0.1:25]; %Hydrofoil velocity in m/s
beta=[80:0.1:120]; %Hydrofoil angle in degrees
phid=[0:1:360]; % this is the angle of the rail deg.
f=length(phid)
d=1;
e=ceil(f/d);

ind=length(beta)+length(vk)+length(phid)

symmetric = 1;
%This flag will toggle the simulation from a symm to unsymm hydrofoils

if symmetric == 1;
    NACA=0015;
elseif symmetric == 0;
    NACA=4412;
end

%NOTE that these are the only 2 valid choices at this time,
%the graphical output will change but the actual lift and
%drag data is only for 0015 and 4412.

data_sym = xlsread('180_degree_naca0015.xlsx','a3:e119');
data_sym_cl = [data_sym(:,1),data_sym(:,5)]; % [aoa in deg, 2D lift coeff]
data_sym_cd = [data_sym(:,1),data_sym(:,4)]; % [aoa in deg, 2D drag coeff]

%Drag NACA4412
data_unsym_cd_read = xlsread('180_degree_naca4412.xlsx','a2:b79');
data_unsym_cd = [data_unsym_cd_read(:,1),data_unsym_cd_read(:,2)]; % [aoa in deg, 2D lift coeff]

%Lift NACA4412
data_unsym_cl_read = xlsread('180_degree_naca4412.xlsx','d2:f63');
data_unsym_cl = [data_unsym_cl_read(:,1),data_unsym_cl_read(:,3)]; % [aoa in deg, 2D lift coeff]

for i=1:length(phid);

    for j=1:length(beta);

    end

end
j;
beta(:,j);
for n=1:length(vk)
    [eff(n), D(n), L(n), Fy(n), Reyn(n), P(n), ld(n), gamma(n), ... 
        alpha(n)] = powercalc(beta(j), vk(n), phid(i), symmetric);

    if P(n) > 0
        Power(n,j)=P(:,n);
    else
        Power(n,j)=0;
    end

    a(n,j)=alpha(:,n)*180/pi;
    Re(n,j)=Reyn(:,n);
    kia(n,j)=eff(:,n);
    F2(n,j) =Fy(:,n);
    Drag(n,j)=D(:,n);
    Lift(n,j)=L(:,n);
    liftdrag(n,j)=ld(:,n);
end

%Figures for the contours of power production, angle of attack and lift and drag for each rail angle that is provided.
figure(1)
subplot(d,e,i)
contour(beta,vk,Power,...
    [0.5*max(max(Power)) 0.55*max(max(Power)) 0.6*max(max(Power)) ...  
      0.65*max(max(Power)) 0.7*max(max(Power)) 0.75*max(max(Power)) ...  
      0.8*max(max(Power)) 0.85*max(max(Power)) 0.9*max(max(Power)) ...  
      0.95*max(max(Power)) max(max(Power))], 'LineWidth', 2);
title('Power Production for a Asymmetric Hydrofoil', ...  
    'FontWeight','b','Color','k','VerticalAlignment','bottom')
xlabel('Beta Angle (degrees)' , 'FontWeight','b','Color','b',...  
    'VerticalAlignment', 'bottom')
ylabel('Kite Velocity (m/s)' , 'FontWeight','b','Color','b',...  
    'VerticalAlignment', 'bottom')
colorbar('East')
caxis([0 max(max(Power))])
grid on
figure(2)
subplot(d,e,i)
contour(beta,vk,Power,[0.85*max(max(Power)) 0.9*max(max(Power))... 0.95*max(max(Power)) max(max(Power))], 'LineWidth', 2);%
hold on
contour(beta,vk,a, 'LineWidth', 2);
title('Angle of Attack for a Asymmetric Hydrofoil','FontWeight','b',... 'Color','k','VerticalAlignment','bottom','fontsize', 24)
xlabel('Beta Angle (degrees)' ,'FontWeight','b','Color','b',... 'VerticalAlignment','bottom','fontsize', 24)
ylabel('Kite Velocity (m/s)' ,'FontWeight','b','Color','b',... 'VerticalAlignment','bottom','fontsize', 24)
colorbar('EastOutside')
caxis([0 20])
grid on

figure(3)
subplot(d,e,i)
contour(beta,vk,Power,[0.85*max(max(Power)) 0.9*max(max(Power))... 0.95*max(max(Power)) max(max(Power))], 'LineWidth', 2);%
hold on
contour(beta,vk,liftdrag, [0.5*max(max(liftdrag))... 0.6*max(max(liftdrag))... 0.7*max(max(liftdrag))... 0.8*max(max(liftdrag))... 0.9*max(max(liftdrag))... 0.999*max(max(liftdrag))], 'LineWidth', 2);
title('Lift-to-Drag Ratio for a Asymmetric Hydrofoil','FontWeight','b',... 'Color','k','VerticalAlignment','bottom','fontsize', 24)
xlabel('Beta Angle (degrees)' ,'FontWeight','b','Color','b',... 'VerticalAlignment','bottom','fontsize', 24)
ylabel('Kite Velocity (m/s)' ,'FontWeight','b','Color','b',... 'VerticalAlignment','bottom','fontsize', 24)
colorbar('EastOutside')
caxis([0 max(max(liftdrag))])
grid on

%Calculations for maximum power and for any rail angle, also the %plots for the hydrofoil angle and hydrofoil velocity to achieve %the peak power. Also the lift and drag coefficients for peak power IP_max(i)=max(max(Power));
[y,vkind1] = max(Power);
[z,betaind1] = max(max(Power));
vk_max(i)=vk(vkind1(betaind1));
beta_max(i)=beta(betaind1);
liftdrag_max(i)=liftdrag(vkind1(betaind1));

%Outputs to the command windows so that you can see the progression of
%rail angles
phid(i)
end

%Plots for maximum power and for any rail angle, also the plots for the
%hydrofoil angle and hydrofoil velocity to achieve the peak power
figure(3)
plot(phid,IP_max,'o-')
title('Power as a function of Rail Angle for an Hydrofoil','FontSize',24)
xlabel('Phi Angle (degrees)' ,'FontSize',24)
ylabel('Power (Watts)' ,'FontSize',24)
grid on

figure(4)
plot(phid,vk_max,'o-')
title('Kite Velocity as a function of Rail Angle for an Hydrofoil','FontSize',24)
xlabel('Phi Angle (degrees)' ,'FontSize',24)
ylabel('vk (m/s)' ,'FontSize',24)
grid on

figure(5)
plot(phid,beta_max,'o-')
title('Beta as a function of Rail Angle for an Hydrofoil','FontSize',24)
xlabel('Phi Angle (degrees)' ,'FontSize',24)
ylabel('Beta (degrees)' ,'FontSize',24)
grid on
**Power Calculation**

This is the function that calculates the power for the calculations described above. The equations used are provided in the Steady-State Translating Model Chapter.

```matlab
function [eff, D, L, Fy, Reyn, P, ld, gamma, alpha] = powercalc...
    ( beta, vk, phid, symmetric)
%Will calculate the value of k (generator profile) for each force

global data_unsym_cl;
global data_unsym_cd;
global data_sym_cl;
global data_sym_cd;

%OUTPUT variable descriptions:
%eff:
i = [1 0 0];
j = [0 1 0];
k = [0 0 1];

%Default Inputs:
vinf = 1; % river velocity in m/s

%Dimensions
rlen=10; % river width meters
rdep=0.75; % depth in meters
chord=0.75/6; % chord length in meters

%Water properties
rho=1000;
mew=0.001002; %kg/(m s)

%Airfoil Information
e=0.9;
%span efficiency factor (oswald efficiency) – for elliptical planforms e=1;
%typical value e=0.85 to e=0.95
```
el=0.9;
%span effectiveness factor – theoretically different but approx same
%value as e

%Calculations
r_area=r_len*r_dep;
k_area=chord*r_dep;
Power=(0.5*rho*r_area*vinf^3);
AR=(r_dep^2)/(k_area);
%can be reduced to AR2=(r_dep)/(chord) %for rectangular wings

%Alpha Calculation
phi=phid*pi()/180; %converts phi to radians

lambda_rod = -sin(phi)*i + cos(phi)*j;

%the apparent wind direction;
va = vinf*i - vk*lambda_rod;
mag_va = norm(va);

lambda_D = va/mag_va;
lambda_L = cross(k,lambda_D);

if va(2) >= 0
    gamma=-1*acos(1/mag_va*dot(va,i));
else
    gamma=acos(1/mag_va*dot(va,i));
end

alpha=(pi()*beta/180)-gamma; %rads
alpha*180/pi;

%Reynolds Number Calculation
Reyn=rho*mag_va*chord/mew;

if alpha > pi
    %To calculate remainder
    q = floor((alpha+pi)/(2*pi));
    alpha = alpha - q*2*pi;
elseif alpha <= -pi
    q = ceil((alpha-pi)/(2*pi));
alpha = alpha - q*2*pi;
end

%Lift and Drag Coeff.
if symmetric == 1;
    [cl] = interp1q(data_sym_cl(:,1),data_sym_cl(:,2),alpha*180/pi);
    [cd] = interp1q(data_sym_cd(:,1),data_sym_cd(:,2),alpha*180/pi);
elseif symmetric == 0;
    [cl] = interp1q(data_unsym_cl(:,1),data_unsym_cl(:,2),alpha*180/pi);
    [cd] = interp1q(data_unsym_cd(:,1),data_unsym_cd(:,2),alpha*180/pi);
else
    error('there is a problem: symm not defined')
eend

%Induced Drag
cd_id=cd+((clˆ2)/(pi()*e*AR));

%Power Calculations (two ways based on two cd)
D=0.5*rho*k_area*mag_vaˆ2*cd_id;
L=0.5*rho*k_area*mag_vaˆ2*cl;

F = D*lambda_D + L*lambda_L;
Fy = dot(F,lambda_rod);

if D>0
    ld=cl/cd_id;
else
    ld=0;
end

P=dot(F,vk*lambda_rod);

eff=P/Power;
end
C.1.2 Optimization Code

The optimization allows for the hydrofoil angle and kite velocity to be optimized for power production for any given rail angle. This code does not have as many outputs currently as the Brute Force code.

Translating Model Optimization

This code allows for the input parameters to be changed and the plotting of the optimized power production, hydrofoil angle, kite velocity and angle of attack. After the hydrofoil angle and kite velocity are optimized the angle of attack for those given inputs is calculated.

%by: Kelsey McConnaghy: 2011
%Kite on Rails
%Including Induced Drag

clc
close all
clear all
tic

global data_unsym_cl;
global data_unsym_cd;
global data_sym_cl;
global data_sym_cd;
global r_len;
global phi_deg;
global symmetric;

r_len=10; % river width meters check in poweralc
phid = [0:1:360];
% this is the angle of the rail in deg.

symmetric = 1;
%This flag will toggle the simulation from a symmetric to unsymmetric
%hydrofoils
if symmetric == 1;
    NACA=0015;
elseif symmetric == 0;
    NACA=4412;
end

%NOTE that these are the only 2 valid choices at this time,
%the graphical output will change but the actual lift and
%drag data is only for 0015 and 4412.

data_sym = xlsread('180_degree_naca0015.xlsx','a3:e119');
data_sym_cl = [data_sym(:,1),data_sym(:,5)]; % [aoa in deg, 2D lift coeff]
data_sym_cd = [data_sym(:,1),data_sym(:,4)]; % [aoa in deg, 2D drag coeff]

%Drag NACA4412
data_unsym_cd_read = xlsread('180_degree_naca4412.xlsx','a2:b79');
data_unsym_cd = [data_unsym_cd_read(:,1),data_unsym_cd_read(:,2)]; % [aoa in deg, 2D lift coeff]

%Lift NACA4412
data_unsym_cl_read = xlsread('180_degree_naca4412.xlsx','d2:f63');
data_unsym_cl = [data_unsym_cl_read(:,1),data_unsym_cl_read(:,3)]; % [aoa in deg, 2D lift coeff]

%initial guesses for the hysrofoil angle (b1) and the kite velocity (b2)
b1=92;
b2=6;

%Set tolerances for fminunc
options=optimset('TolFun',1e-10,'TolX',1e-10);

for i=1:length(phid)
    phi_deg=phid(i)
    [a,z]=fminunc(@powercalc_optim,[b1,b2],options);
    IP_max(i)=-z;
vk_max(i)=a(2);
    beta_max(i)=a(1);

    %sets the outputs for positive kite velocity values and using these
%outputs in power calc to get the angle of attack, otherwise (if kite
%velocity is negative the power is set to NaN and a new initial guess is
%used
if vk_max(i)>0
  b1=beta_max(i);
  b2=vk_max(i);
  [eff, D, L, Fy, Reyn, P, ld, gamma, a] = powercalc( beta_max(i),...
  vk_max(i), phi_deg, symmetric);
  alpha_max(i)=a*180/pi;
else
  IP_max(i)=NaN;
  vk_max(i)=NaN;
  beta_max(i)=NaN;
  alpha_max(i)=NaN;
  b1=40;
  b2=10;
end
end

%Plots the values that were optimized and the calculate angle attack at
%these points.
figure(1)
subplot(1,4,1)
plot(phid,IP_max,'o-')
title('Power v Phi','FontWeight','b','Color','k','VerticalAlignment')
xlabel('Phi Angle (degs)','FontWeight','b','Color','b', 'VerticalAlignment')
ylabel('Power (Watts)','FontWeight','b','Color','b', 'VerticalAlignment')
grid on

subplot(1,4,2)
plot(phid,vk_max,'o-')
title('Kite Vel v Phi','FontWeight','b','Color','k','VerticalAlignment')
xlabel('Phi Angle (degs)','FontWeight','b','Color','b', 'VerticalAlignment')
ylabel('vk (m/s) ','FontWeight','b','Color','b', 'VerticalAlignment')
grid on

subplot(1,4,3)
plot(phid,beta_max,'o-')
title('Beta v Phi','FontWeight','b','Color','k','VerticalAlignment')
xlabel('Phi Angle (degs)','FontWeight','b','Color','b', 'VerticalAlignment')
ylabel('Beta (degrees) ','FontWeight','b','Color','b', 'VerticalAlignment')
grid on

subplot(1,4,4)
plot(phid,alpha_max,'o-')
title('Alpha v Phi','FontWeight','b','Color','k','VerticalAlignment')
xlabel('Phi Angle (degs)','FontWeight','b','Color','b', 'VerticalAlignment')
ylabel('Alpha (degrees)','FontWeight','b','Color','b', 'VerticalAlignment')
grid on
toc

Power Calculation Optimization

This is the function that calculates the forces and the power for the optimization described above. This code is very similar to the Power Calculation code shown, however it only allows for the hydrofoil angle and kite velocity to the optimized.

function [P] = powercalc_optim(a)
%Will calculate the value of k (generator profile) for each force

beta = a(1);
vk = a(2);

global data_unsym.cl;
global data_unsym.cd;
global data_sym.cl;
global data_sym.cd;
global r_len;
global phi_deg;
global symmetric;

%OUTPUT variable descriptions:
%eff:
i = [1 0 0];
j = [0 1 0];
k = [0 0 1];

%Default Inputs:
vinf = 1; % river velocity in m/s
%Dimensions
r_{dep}=0.75; \quad \% \text{depth in meters}
chord=0.75/6; \quad \% \text{chord length in meters}

%Water properties
rho=1000;
mew=0.001002; \quad \% \text{kg/(m s)}

%Airfoil Information
e=0.9;
%span efficiency factor (oswald efficiency) – for elliptical planforms e=1;
%typical value e=0.85 to e=0.95

e1=0.9;
%span effectiveness factor – theoretically different but approx same
%value as e

%Calculations
r_{area}=r_{len}\times r_{dep};
k_{area}=chord\times r_{dep};
\text{Power}=(0.5\times \rho \times r_{area}\times v_{inf}^3);
\text{AR}=(r_{dep}^2)/(k_{area});
%can be reduced to \text{AR2}=(r_{dep})/(chord) \% for rectangular wings

%Alpha Calculation
phi=\phi_{deg}\times \pi()/180; \quad \% \text{converts \phi to radians}

\text{lambda}_rod = -\sin(\phi)\hat{i} + \cos(\phi)\hat{j};

%the apparent wind direction;
va = v_{inf}\hat{i} - v_k\times \text{lambda}_rod;
\text{mag}_{va} = \text{norm}(va);

\text{lambda}_D = va/\text{mag}_{va};
\text{lambda}_L = \text{cross}(k, \text{lambda}_D);

\textbf{if} \; va(2) \geq 0
\quad \text{gamma}=-1\times \cos(1/\text{mag}_{va}\times \text{dot}(va,i));
\textbf{else}
\quad \text{gamma}=\cos(1/\text{mag}_{va}\times \text{dot}(va,i));
\textbf{end}
180*gamma/pi();

alpha=(pi()*beta/180)−gamma; % (rads)

%Reynolds Number Calculation
Reyn=rho*mag_va*chord/mew;

if alpha > pi
% To calculate remainder
    q = floor((alpha+pi)/(2*pi));
    alpha = alpha − q*2*pi;
elseif alpha < −pi
    q = ceil((alpha−pi)/(2*pi));
    alpha = alpha − q*2*pi;
end

% Lift and Drag Coeff.
if symmetric == 1;
    [cl] = interp1q(data_sym_cl(:,1),data_sym_cl(:,2),alpha*180/pi);
    [cd] = interp1q(data_sym_cd(:,1),data_sym_cd(:,2),alpha*180/pi);
elseif symmetric == 0;
    [cl] = interp1q(data_unsym_cl(:,1),data_unsym_cl(:,2),alpha*180/pi);
    [cd] = interp1q(data_unsym_cd(:,1),data_unsym_cd(:,2),alpha*180/pi);
else
    error('there is a problem: symm not defined')
end

% Induced Drag
cd_id=cd+((cl^2)/(pi()*e*AR));

% Power Calculations (two ways based on two cd)

D=0.5*rho*k_area*mag_va^2*cd_id;
L=0.5*rho*k_area*mag_va^2*cl;

F = D*lambda_D + L*lambda_L;
Fy = dot(F,lambda_rod);

if D>0
    ld=cl/cd;
else
    ld=0;
end

P=-1*(dot(F,vk*lambda.rod));

eff=P/Power;

end

C.2 Static Rotational Model

Cycle Calculations

This code allows for the input parameters to be changed and altered and outputs the surface plots for the peak power production for every deploy and return angle calculations. It seeds each initial boom position to find the maximum angular velocity for that given deploy hydrofoil angle. This is then used for the initial guess for the determining the instantaneous power of the system along the peak power curve. This is then also done for the return stroke. These two instantaneous powers are then combined to determine the cycle power for the system.

clc
clear all
close all
format long
tic

% Dimensions and Constants
vinf = 1;  % river flow velocity [m/s]
r_arm = 5;  % boom length meters
r_dep=.75;  % depth of hydrofoil in meters

global vinf r_arm r_dep chord K data_sym_cl data_sym_cd...
    data_sym_cm data_unsym_cl data_unsym_cd data_unsym_cm state
chord = 0.75/6; % chord length of hydrofoil in meters
K = 200; % torque constant

%% Parameters

%beta = [-93:0.5:-110]*(pi/180); % angle of the airfoil to the rod
beta = [80]*(pi/180); % angle of the airfoil to the rod
beta_r = [-70]*(pi/180);
D_theta = 1; % Incremental Theta [degrees]
theta_m = [0:D_theta:45]; % Swept area of the Tether [degrees]

numb = 500;
f = 1;
state = 'deploy';

symmetric = 1;
% This flag will toggle the simulation from a symm to unsymm hydrofoils
if symmetric == 1;
    NACA = 0015;
elseif symmetric == 0;
    NACA = 4412;
end

% NOTE that these are the only 2 valid choices at this time,
% the graphical output will change but the actual lift and
% drag data is only for 0015 and 4412.

data_sym = xlsread('180_degree_naca0015.xlsx','a3:f119');
data_sym_cl = [data_sym(:,1),data_sym(:,5)]; % [aoa in deg, 2D lift coeff with correction]
data_sym_cd = [data_sym(:,1),data_sym(:,4)]; % [aoa in deg, 2D drag coeff]
data_sym_cm = [data_sym(:,1),data_sym(:,6)]; % [aoa in deg, 2D moment coeff]

% Drag NACA4412
data_unsym_cd_read = xlsread('180_degree_naca4412.xlsx','a2:b79');
data_unsym_cd = [data_unsym_cd_read(:,1),data_unsym_cd_read(:,2)]; % [aoa in deg, 2D lift coeff]

% Lift NACA4412
data_unsym_cl_read = xlsread('180_degree_naca4412.xlsx','d2:e63');
data_unsym_cl = [data_unsym_cl_read(:,1),data_unsym_cl_read(:,2)];
% [aoa in deg, 2D lift coeff]

%moment NACA4412

data_unsym_cm_read = xlsread('180_degree_naca4412.xlsx','g2:h59');
data_unsym_cm = [data_unsym_cm_read(:,1),data_unsym_cm_read(:,2)];
% [aoa in deg, 2D moment coeff]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
length(beta)
length(beta_r)
length(beta_r)*length(beta)

for j=1:length(beta);
    for i=1:length(beta_r)
        m = 1;
        state = 'deploy';
        theta=theta_m(1);

        %Calculating peak theta-dot for starting value
        theta_dot_a = zeros(1,numb);
        theta_dot_matrix = zeros(1,numb);
        for h = 1:numb
            theta_dot_matrix(h) =10*(h-1)/numb;
            [theta_dot_a(h), f_conv, flag]= fzero(@(theta_dot)...
                Torquesd3(theta_dot, beta(j), theta, symmetric),...
                theta_dot_matrix(h));
            if (flag < 0)
                error('fzero: did not converge during scan of thetadots');
            end
        end
        theta_dot_init=max(theta_dot_a);

        mem_size = length(theta_m);
        t = zeros(1,mem_size);
        td = zeros(1,mem_size);
        IP = zeros(1,mem_size);
        theta_dot_recip = zeros(1,mem_size);
        Torque = zeros(1,mem_size);

        if theta_dot_init > 0
for kk = 1:length(theta_m)
    theta=theta_m(kk);
    [theta_dot_conv, f_conv, flag] = fzero(@(theta_dot)...
        Torquesd3(theta_dot, beta(j), theta, symmetric),...
        theta_dot_init);
    if (flag < 0)
        error('fzero: did not converge during theta deploy sweep');
    end
    %[alpha, Reyn]= aero_func(theta_dot_conv, beta(j), theta);
    mag_Torque=K*theta_dot_conv;
    t(kk)=theta;
    td(kk)=theta_dot_conv;
    IP(kk)=mag_Torque*theta_dot_conv;
    theta_dot_recip(kk)=(1/theta_dot_conv);
    Torque(kk)=mag_Torque;
    theta_dot_init = theta_dot_conv;
    %reset initial theta_dot guess to solution
end
else
    fprintf('there is no DEPLOY stroke for: ')
    beta(j)*180/pi
    return_counter=0;
    for kk=1:length(theta_m);
        theta_m(kk)=return_counter;
        t(kk)=theta_m(kk);
        td(kk)=0;
        IP(kk)=0;
        theta_dot_recip(kk)=1000;
        Torque(kk)=0;
        return_counter=return_counter+1;
    end
end

%Subscript (.r) is for "Return"
theta_return=[0:D_theta:theta_m(kk)];
%REVERSESwept area of the Tether on downstroke [degrees]
n=1;
state = 'return';
theta_r=theta_return(1);
%Calculating peak theta-dot for starting value
theta_dot_matrix_r = zeros(1,numb);
theta_dot_a_r = zeros(1,numb);
for g = 1:numb
  g;
  theta_dot_matrix_r(g) = -10*(g-1)/numb;
  try
    [theta_dot_a_r(g), f.conv, flag, output] = fzero(@(theta_dot)... Torquesd3(theta_dot, beta_r(i), theta_r, symmetric),... theta_dot_matrix_r(g));
  catch ME
    theta_dot_a_r(g) = NaN;
  end
end

theta_dot_init_r = min(theta_dot_a_r);

mem_siz = length(theta_return);
t_r = zeros(1,mem_siz);
td_r = zeros(1,mem_siz);
IP_r = zeros(1,mem_siz);
theta_dot_recip_r = zeros(1,mem_siz);
Torque_r = zeros(1,mem_siz);

if theta_dot_init_r < 0
  for n = 1:length(theta_return);
    theta_r = theta_return(n);
    [theta_dot_conv_r, f.conv, flag] = fzero(@(theta_dot)... Torquesd3(theta_dot, beta_r(i), theta_r, symmetric),... theta_dot_init_r);
    if (flag < 0)
      theta_r
      error('fzero: did not converge during sweep on return');
    end
    [alpha, Reyn] = aero_func(theta_dot_conv, beta(j), theta);
    mag_Torque = K*theta_dot_conv_r;
    t_r(n) = theta_r;
    td_r(n) = theta_dot_conv_r;
    IP_r(n) = mag_Torque*theta_dot_conv_r;
    theta_dot_recip_r(n) = (1/theta_dot_conv_r);
    Torque_r(n) = mag_Torque;
    theta_dot_init_r = theta_dot_conv_r;
  end
  %reset initial theta_dot guess to solution
else
    fprintf('there is no RETURN stroke for: ')
    beta_r(i)*180/pi
    for n=1:length(theta_return);
        t_r(n)=theta_return(n);
        td_r(n)=0;
        IP_r(n)=0;
        theta_dot_recip_r(n)=1000;
        Torque_r(n)=0;
    end
end

e=2;
theta_flip=[Dtheta:Dtheta:max(theta_m)];
mem_si = length(theta_flip);

for f=1:length(theta_flip)
    theta_dot_recip_cp=theta_dot_recip(1:e);
    theta_dot_recip_r_cp=theta_dot_recip_r(1:e);
    Torque_cp=Torque(1:e);
    Torque_r_cp=Torque_r(1:e);
    theta_rad_cp=t(1:e)*pi()/180; %Conversion to Radians
    theta_rad_r_cp=t_r(1:e)*pi()/180; %Conversion to Radians
    cycle_time=trapz(theta_rad_cp, abs(theta_dot_recip_cp));
    Cycle_Energy=trapz(theta_rad_cp, abs(Torque_cp));
    Cycle_Power(f)= Cycle_Energy/cycle_time;
    cycle_time_return=trapz(theta_rad_r_cp, abs(theta_dot_recip_r_cp));
    Cycle_Energy_return=trapz(theta_rad_r_cp, abs(Torque_r_cp));
    Cycle_Power_return(f)= Cycle_Energy_return/cycle_time_return;
    Total_Cycle_Time(f)=(cycle_time+cycle_time_return);
    Total_Cycle_Energy(f)=(Cycle_Energy+Cycle_Energy_return);
    Total_Cycle_Power(f)= Total_Cycle_Energy(f)/Total_Cycle_Time(f);
    e=e+1;
end

% Maximum Cycle Power and Flipping Angle
[value, inx]=max(Total_Cycle_Power);
flip_angle(j,i)=theta_flip(inx);
TCP(j,i)=value;

% Power for prescribed flipping angles
TCP(j,i)=Total_Cycle_Power(f);

f=f+1;

% Plots for the instantaneous power for every deploy and return angle
% and the combined cycle power
(j-1)*length(beta_r) + i
figure((j-1)*length(beta_r) + i)
subplot(3,1,1)
plot(t, IP, 'ok')
title('IP Multi roots')
xlabel('theta')
ylabel('Insta. Power (Watts)')

dep=num2str(beta(j)*180/pi);
text(max(theta),max(max(IP)),dep)

subplot(3,1,2)
plot(t_r, IP_r, 'ok')
title('IP Multi roots Return')
xlabel('theta return')
ylabel('Insta. Power (Watts)')

ret=num2str(beta_r(i)*180/pi);
text(max(theta_r),max(max(IP_r)),ret)

subplot(3,1,3)
plot(theta_flip, Total_Cycle_Power, 'ok')
title('Cycle Power vs Theta Flip')
xlabel('theta flip')
ylabel('Cycle Power (Watts)')

end
end
toc

[y, deploy_ind] = max(TCP);
[z, return_ind] = max(max(TCP));
MAX_Deploy=beta(deploy_ind)*180/pi()
MAX_Return=beta_r(return_ind)*180/pi()
MAX_TCP=z

%Plots for the total cycle power as a function of hydrofoil angles
figure(length(beta_r)*length(beta)+2)
surf(beta_r*(180/pi), beta*(180/pi),TCP, 'EdgeColor','none')
title('Cycle Power (Watts)'
ylabel('Deploy Angle (deg)'
xlabel('Return Angle (deg)'
xlabel('Return Angle (deg)'

figure(length(beta_r)*length(beta)+3)
contour(beta_r*(180/pi), beta*(180/pi),TCP)
title('Cycle Power (Watts)'
ylabel('Deploy Angle (deg)'
xlabel('Return Angle (deg)'

figure(length(beta_r)*length(beta)+4)
surf(beta_r*(180/pi), beta*(180/pi),flip_angle)
title('Flipping Angle for Maximum Power (degrees)'
ylabel('Deploy Angle (deg)'
xlabel('Return Angle (deg)'
colorbar

Torque Calculation

This is the function that calculates the torque values for the cycle calculations described above. The equations used are provided in the Steady-State Rotating Model Chapter.

function [f]= Torquesd3(theta_dot, beta, theta, symmetric)

%INPUT:
% theta_dot: angular velocity of the boom [rad/sec]
% beta: angle between the boom and the hydrofoil [rads]
% theta: angle of the boom relative to the fluid flow [deg]
% symmetric: toggle to use symmetric or unsymmetric data (values 0 or 1)
% [unitless]

%OUTPUT:
% f: zero when fluid torque equals the generator torque

global vinf r_arm r_dep chord K data_sym_cl data_sym_cd...
data_sym_cm data_unsym_cl data_unsym_cd data_unsym_cm

i = [1 0 0];
j = [0 1 0];
k = [0 0 1];

theta_rad = theta * pi() / 180; % Conversion to Radians

% Water properties
rho = 1000; % density
mew = 0.001002; % kg/(m s)

% Airfoil Information
e = 0.9;
% span efficiency factor (oswald efficiency) – for elliptical planforms e=1;
% typical value e=0.85 to e=0.95

e1 = 0.9;
% span effectiveness factor – theoretically different but approx same
% value as e

%%% ______Calculations________%%

% Lambda Vectors for Tether
lambda_rod = cos(theta_rad)*i + sin(theta_rad)*j;
mag_lr = norm(lambda_rod);

% cross product changed for speeding up code
% lambda_arm = -cross(k, lambda_rod);
lambda_arm = -[-lambda_rod(2) lambda_rod(1) 0];
%Kite Velocity
vk=theta_dot*r_arm--lambda_arm; % (meters per second)

%the apparent wind direction;
va = vinf*i - vk;
mag_va = norm(va);

%Lambda Vectors for Aerodynamic Forces
lambda_D = va/mag_va;
% cross product changed for speeding up code
%lambda_L = cross(k,lambda_D);
lambda_L = [-lambda_D(2) lambda_D(1) 0];

%for correct angle calculation of Gamma
% cross product changed for speeding up code
%cross_product = cross(va, lambda_rod);
cross_product = [0 0 va(1)*lambda_rod(2)-va(2)*lambda_rod(1)];

% trying to speed up code
% gamma=acos(dot(lambda_rod, va)/(mag.va*mag_lr));
gamma = acos((lambda_rod(1)*va(1)+lambda_rod(2)*va(2))/(mag.va*mag_lr));

%Alpha Calculation
%if state == 'deploy'

if cross_product >= 0
  gamma=gamma;
else
  gamma=-gamma;
end

alpha= beta - gamma; % (rads)

%Code for translating alpha

%this next piece of code should fold the alphas into the range of pi
if alpha > pi
  %To calculate remainder
  q = floor((alpha+pi)/(2*pi));
  alpha = alpha - q*2*pi;
elseif alpha < -pi
    q = ceil((alpha-pi)/(2*pi));
    alpha = alpha - q*2*pi;
end

%Reynolds Number Calculation
Reyn=rho*mag_va*chord/mew;

%Kite Calculations
k_area=chord*r_dep;

AR=(r_dep^2)/(k_area);
%can be reduced to AR2=(r_dep)/(chord) %for rectangular wings

%Lift and Drag Coeff.
if symmetric == 1;
    [cl] = interp1q(data_sym_cl(:,1),data_sym_cl(:,2),alpha*180/pi);
    [cd] = interp1q(data_sym_cd(:,1),data_sym_cd(:,2),alpha*180/pi);
    [cm] = interp1q(data_sym_cm(:,1),data_sym_cm(:,2),alpha*180/pi);
elseif symmetric == 0;
    [cl] = interp1q(data_unsym_cl(:,1),data_unsym_cl(:,2),alpha*180/pi);
    [cd] = interp1q(data_unsym_cd(:,1),data_unsym_cd(:,2),alpha*180/pi);
    [cm] = interp1q(data_unsym_cm(:,1),data_unsym_cm(:,2),alpha*180/pi);
else
    error('there is a problem: symm not defined')
end

%Induced Drag
cd_id=cd+((cl^2)/(pi*e*AR));

%Power Calculations
D=0.5*rho*k_area*mag_va^2*cd_id;
L=0.5*rho*k_area*mag_va^2*cl;
M=0.5*rho*k_area*chord*mag_va^2*cm;
ld=cl/cd_id;

Drag= D*lambda_D;
Lift= L*lambda_L;
Fsum= Lift+Drag;

%F=norm(Fsum);
%arm position calculation
x = r_arm*cos(theta_rad);
y = r_arm*sin(theta_rad);
arm = [x, y, 0];

% cross product changed for speeding up code
% Torque = cross(arm, Fsum);
Torque = -M + ...
    [arm(2)*Fsum(3)-arm(3)*Fsum(2) -arm(1)*Fsum(3)+arm(3)*Fsum(1)...
     arm(1)*Fsum(2)-arm(2)*Fsum(1)];

% trying to speed up code
% mag_Torque = dot(k, Torque);
mag_Torque = Torque(3);

f = mag_Torque - K*theta_dot;
end

Function Value Calculation for Torque Calculation

This code calculates the function value for:

\[ f = \tau - K\dot{\theta} \]  \hspace{1cm} (C.1)

along a given range of angular velocities. This is then plotted to see the intercepts of
the function and the overall function shape. This allows for the visualization of the
disappearing roots.

% Code for F root find
clc
clear

global vinf r_arm r_dep chord K data_sym_cl data_sym_cd

%%% Dimensions and Constants %%%
vinf = 1; % river flow velocity [m/s]
r_arm = 5; % boom length meters
r_dep = .75; % depth of hydrofoil in meters
chord= 0.75/6; % chord length of hydrofoil in meters

K=200; %torque constant

%%% _______Parameters___________________________%%%

beta = [-100]*(pi/180); %angle of the airfoil to the rod

theta=20; %Boom angle
theta_dot=[-2.5:0.001:1]; %Tether Rotation Speed [rads/sec]

data_sym = xlsread('180_degree_naca0015.xlsx','a3:d119');
data_sym_cl = [data_sym(:,1),data_sym(:,3)]; % [aoa in deg, 2D lift coeff]
data_sym_cd = [data_sym(:,1),data_sym(:,4)]; % [aoa in deg, 2D drag coeff]

for j=1:length(theta_dot);
    [out alpha] = Torque1(theta_dot(j), beta, theta);
    A(j) = out;
    a(j) = alpha*180/pi();
end

figure(1)
plot(theta_dot, A,'.')
grid on
xlabel('Angular Velocity');
ylabel('Function Value');

%figure(2)
%plot(theta_dot,a)
%grid on

%%%%%%%%%%%%%%%%
%Torque1 fnction is the same as the other torque calculation except for
%the last line of code that reads:
%f= mag_Torque − K*theta_dot;
%This is so the function value can be calulated

Multi-root Plots Generation

Allows for the generation of the multi-root plots for a given hydrofoil angle and a range of angular velocities. This will seed each increment of boom angle with the
entire range of angular velocities. The fzero function is used to find all the converged values for the given inputs. The output is all of the converged angular velocities and the corresponding instantaneous power curve that is associated with the angular velocities. The points denoted in red are negative velocities, meaning they would be used for the return stroke of the system.

clc
clear all
close all
format long

global vinf r_arm r_dep chord K data_sym_cl data_sym_cd data_sym_cm...
data_unsym_cl data_unsym_cd state

%%%......Dimensions and Constants.................%%%
vinf = 1; % river flow velocity [m/s]
r_arm = 5; %boom length meters
r_dep=.75; % depth of hydrofoil in meters
chord= 0.75/6; % chord length of hydrofoil in meters
K=200; %torque constant

%%%......Parameters...................................%%%
beta = [-102]*(pi/180); %angle of the airfoil to the rod
D_theta =5; %Incrimental Theta [degrees]
theta_m = [-90:D_theta:90]; %Swept area of the Tether [degrees]
numb=250;
td_start = -5; %Smallest Value
td_end = 5; %Largest Value

td_span = abs(td_start-td_end);

symmetric = 1;

%This flag will toggle the simulation from a symm to unsymm hydrofoils

if symmetric == 1;
    NACA=0015;
elseif symmetric == 0;
    NACA=4412;
end

%NOTE that these are the only 2 valid choices at this time, 
%the graphical output will change but the actual lift and 
%drag data is only for 0015 and 4412.

data_sym = xlsread('180_degree_naca0015.xlsx','a3:f119');
data_sym_cl = [data_sym(:,1),data_sym(:,5)];  
% [aoa in deg, 2D lift coeff with correction] 
data_sym_cd = [data_sym(:,1),data_sym(:,4)];  
% [aoa in deg, 2D drag coeff] 
data_sym_cm = [data_sym(:,1),data_sym(:,6)];  
% [aoa in deg, 2D moment coeff]

%Drag NACA4412
data_unsym_cd_read = xlsread('180_degree_naca4412.xlsx','a2:b79');
data_unsym_cd = [data_unsym_cd_read(:,1),data_unsym_cd_read(:,2)];  
% [aoa in deg, 2D lift coeff]

%Lift NACA4412
data_unsym_cl_read = xlsread('180_degree_naca4412.xlsx','d2:e63');
data_unsym_cl = [data_unsym_cl_read(:,1),data_unsym_cl_read(:,2)];  
% [aoa in deg, 2D lift coeff]

%moment NACA4412
data_unsym_cm_read = xlsread('180_degree_naca4412.xlsx','g2:h59');
data_unsym_cm = [data_unsym_cm_read(:,1),data_unsym_cm_read(:,2)];  
% [aoa in deg, 2D moment coeff]

for j=1:length(beta);
    m=1;
    for k = 1:length(theta_m)
        theta=theta_m(m);
        k;
        for h = 1: numb
            theta_dot_init(h) = td_start + td_span*(h-1)/numb;
            [theta_dot_conv, f_conv, flag]= fzero(@(theta_dot) Torquesd3...
                (theta_dot, beta(j), theta, symmetric), theta_dot_init(h));
            if (flag < 0)
                error('fzero: did not converge');
            end
end
mag_Torque=K*theta_dot_conv;
t(m)=theta;
TT(h,m) = theta;
td(h,m)=theta_dot_conv;
IP(h,m)=mag_Torque*theta_dot_conv;

end
m = m+1
end

I = find(td<0);

figure(j)
subplot(1,2,1)
plot(t, td, 'ok')
title('Equilibrium Angular Velocities (rad/sec) for NACA0015',...
     'FontWeight','b','Color','k','VerticalAlignment','bottom',...
     'FontSize', 24)
xlabel('Boom Position [theta] (degrees)', 'FontWeight','b','Color','k', 'VerticalAlignment','bottom', 'FontSize', 24)
ylabel('Angular Velocity (rad/s)', 'FontWeight','b','Color','k', 'FontSize', 24)
hold on
plot(TT(I),td(I),'r+')
grid on
subplot(1,2,2)
plot(t, IP, 'ok')
title('Instanteous Power for Multiple Roots', 'FontWeight','b','Color','k', 'VerticalAlignment','bottom', 'FontSize', 24)
xlabel('Boom Position [theta] (degrees)', 'FontWeight','b','Color','k', 'VerticalAlignment','bottom', 'FontSize', 24)
ylabel('Instanteous Power (Watts)', 'FontWeight','b','Color','k', 'FontSize', 24)
hold on
plot(TT(I),IP(I),'r+')
grid on
end
C.3 Dynamic Rotational Model

Cycle Calculations

This code allows for the initial conditions to be set, including the deploy and return hydrofoil angles, the initial start position of the boom and the initial rotational velocity of the system. It also allows for changing conditions that allow for the hydrofoil to flip at different positions. The outputs is the system schematic that is drawn at every time step that illustrates the resulting force and apparent wind direction. It also outputs the boom position, angular velocity, power production and angle of attack of the system as a function of time.

% Hydro-kite with boom dynamics model

clf
clc
clear all
close all

global first mk mb l beta K r_dep chord vinf ...
   data_sym_cl data_sym_cd data_sym_cm

first = 1;
%used for animation so the lines are created only once and then positions
%updated

mk = 0.01; % mass of the kite in [kg]
mb = 0.1; % mass of the boom in [kg]
l = 5; % length of the boom in [m]
beta = 92.5*pi/180;
% angle of hydrofoil chord relative to the boom in radians;

K = 200;
% value of the braking torque damping (similar to elec. generator) [N*m*sec]

r_dep=.75; % depth of hydrofoil in meters
chord= 0.75/6; % chord length of hydrofoil in meters
vinf = 1;  % river velocity [m/s]

data_sym = xlsread('180_degree_naca0015.xlsx','a3:f119');
data_sym_cl = [data_sym(:,1),data_sym(:,5)];
  % [aoa in deg, 2D lift coeff with correction]
data_sym_cd = [data_sym(:,1),data_sym(:,4)];
  % [aoa in deg, 2D drag coeff]
data_sym_cm = [data_sym(:,1),data_sym(:,6)];
  % [aoa in deg, 2D moment coeff]

tstart = 0;  % start time [sec]
tfinal = 50;  % stop time [sec]

Y0(1) = 0*pi/180;  % initial angle of boom in radians
Y0(2) = 2;  % initial angular velocity of boom in radians/sec

tstep = 0.003;  % constant step size integration time step
numb = (tfinal-tstart)/tstep;
yout(1,:) = [Y0(1), Y0(2)];
tout(1) = tstart;
state = 'deploy';

for ind = 1:numb
  [yout(ind+1,:)] = my_rk4(yout(ind,:)', tstep);
  tout(ind+1) = tout(ind) + tstep;
  theta = yout(ind+1,1);
  thetadot = yout(ind+1,2);

  % this section is just used to calculate the angle of attack
  i = [1 0 0];
  j = [0 1 0];
  k = [0 0 1];
  
  lambda_rod = cos(theta)*i + sin(theta)*j;
  mag_lr = 1;
  lambda_arm = cross(k, lambda_rod);
  vk = thetadot*1*lambda_arm;  % (meters per second)
  va = vinf*i - vk;
  mag_va = norm(va);
  gamma = acos(dot(lambda_rod, va)/(mag_va*mag_lr));
% for correct angle calculation of Gamma

cross_product = cross(va, lambda_rod);
if cross_product >= 0
    gamma = gamma;
else
    gamma = -gamma;
end
alpha(ind+1) = (beta - gamma); % (deg)

if alpha(ind+1) > pi
    % To calculate remainder
    q = floor((alpha(ind+1)+pi)/(2*pi));
    alpha(ind+1) = alpha(ind+1) - q*2*pi;
elseif alpha(ind+1) < -pi
    q = ceil((alpha(ind+1)-pi)/(2*pi));
    alpha(ind+1) = alpha(ind+1) - q*2*pi;
end

alpha(ind+1) = alpha(ind+1)*180/pi;

[junk, f_drag] = kite_f(yout(ind+1,:));
fdrag(ind+1,:) = f_drag;
fdrag_norm(ind+1) = norm(f_drag);

% The following lines consist of the controller parameters used for
% flipping the hydrofoil
if state == 'deploy'
    if ((theta > 10*pi/180) & (thetadot < 1*pi/180)) || ...
        (theta > 90*pi/180)
        beta = -92.5*pi/180;
        state = 'return';
    end
elseif state == 'return'
    if (theta < 0*pi/180)
        beta = 92.5*pi/180;
        state = 'deploy';
    end
end

draw_kite(yout(ind+1,:),va,f_drag); % animate the system
drawnow;
end

figure(2);
subplot(1,5,1)
plot(tout,yout(:,1)*180/pi,'o-');
xlabel('time [s]');
ylabel('angular position of boom [deg]');
grid on;

subplot(1,5,2)
plot(tout,yout(:,2)*180/pi,'o-');
xlabel('time [s]');
ylabel('angular velocity [deg/sec]');

%power calculation, integrate the instantaneous power as a function of time
%over the cycle to get the total energy produced during the half cycle

P = K.*(yout(:,2)).^2;
Eng = trapz(tout,P);
Pavg = Eng/tout(end)

subplot(1,5,3)
plot(tout,alpha,'r');
xlabel('time [s]');
ylabel('alpha [deg]');

subplot(1,5,4)
plot(tout,P,'r');
xlabel('time [s]');
ylabel('Power [W]');

subplot(1,5,5)
plot(tout,fdrag_norm,'b');
xlabel('time [s]');
ylabel('hydro force magnitude [N]');

**Torque and Angular Velocity Calculations**

This code is run at every incremental time step and updates the forces and corresponding torque on the system as well as the angular velocity and the angular acceleration of the system. These outputs are then used to update the system schematic.
function [dydt,Fsum] = kite_f(Y)

global mk mb l beta K r_dep chord vinf data_sym_cl data_sym_cd data_sym_cm

theta = Y(1); % in radians
thetadot = Y(2); % in radians

i = [1,0,0];
j = [0,1,0];
k = [0,0,1];

% Water properties
rho=1000; % density
mew=0.001002; % kg/(m s)
e = 0.9; % wing shape factor for induced drag calc
AR = r_dep/chord; % wind aspect ratio for induced drag calc

T = -K*thetadot; % torque from the generator

rp_o = l*cos(theta)*i + l*sin(theta)*j;

% Lambda Vectors for Tether
lambda_rod = cos(theta)*i + sin(theta)*j;
mag_lr=1;

lambda_arm = cross(k, lambda_rod);

% Kite Velocity
vk=thetadot*l*lambda_arm; % (meters per second)

% the apparent wind direction;
va = vinf*i - vk;
mag_va = norm(va);

% Lambda Vectors for Aerodynamic Forces
lambda_D = va/mag_va;
lambda_L = cross(k,lambda_D);

gamma=acos(dot(lambda_rod, va)/(mag_va*mag_lr));

% for correct angle calculation of Gamma
cross_product = cross(va, lambda_rod);
if cross_product >= 0
    gamma=gamma;
else
    gamma=-gamma;
end

%Alpha Calculation
alpha=beta-gamma; %(rads)

%this next piece of code should fold the alphas into the range of
if alpha > pi
    %To calculate remainder
    q = floor((alpha+pi)/(2*pi));
    alpha = alpha - q*2*pi;
elseif alpha < -pi
    q = ceil((alpha-pi)/(2*pi));
    alpha = alpha - q*2*pi;
end

%Reynolds Number Calculation
Reyn=rho*mag_va*chord/mew;

[cl] = interp1q(data_sym_cl(:,1),data_sym_cl(:,2),alpha*180/pi);
[cd] = interp1q(data_sym_cd(:,1),data_sym_cd(:,2),alpha*180/pi);
[cm] = interp1q(data_sym_cm(:,1),data_sym_cm(:,2),alpha*180/pi);

cd_id=cd+((cl^2)/(pi*e*AR)); %induced drag

k_area = r_dep*chord;

D=0.5*rho*k_area*mag_va^2*cd_id;
L=0.5*rho*k_area*mag_va^2*cl;
M=0.5*rho*k_area*chord*mag_va^2*cm;

Drag= D*lambda_D;
Lift= L*lambda_L;

Fsum = Lift+Drag;

Ibar_o = 1/3*mb*l^2;

thetadotdot = (T - M + dot(cross(rp_o,Fsum),k))/(Ibar_o + mk*l^2);
dydt = [thetadot; thetadotdot];

**Draw Kite, Written By Dr. Mario Gomes**

This code is used to draw the system at any given time step provided the dimensions of the system, the apparent velocity and the force vector.

```matlab
function [] = draw_kite(X,va,f_drag)

% written by Mario W. Gomes
% 16July2010
global first l beta

persistent myline %the handle for the kite line handle graphic
persistent airline % the handles for the airfoil line segments
persistent kite_wind % the handle for the incomming wind at the kite;
persistent force_line % the handle for the fluid forces on the kite

theta = X(1);
thetadot = X(2);

vel_scale = 0.1; %used to scale the relative velocity on the animation
force_scale = 0.02; % used to scale the forces on the animation

figure(1)

u_r = [cos(theta); sin(theta); 0];

u_th = [-sin(theta); cos(theta); 0];

r_vec = l*u_r; %position vector from ground to kite
v_vec = l*thetadot*u_th; %velocity vector of the kite
wind_vec = va*vel_scale; %apparent wind velocity vector

f_drag = force_scale*f_drag;

%draw the kite string and the airfoil to represent the kite
if (first == 1)
    myline = line('xdata', [0, r_vec(1)],...
    'ydata', [0, r_vec(2)],'color','k','linwidth', 3);
    [px,py] = importfoil(r_vec(1),r_vec(2),0.2,theta-beta);

```
for i = 1:length(px)-1
    airline(i)=line('xdata',[px(i),px(i+1)],'ydata',[py(i), py(i+1)]);
end
kite_wind = line('xdata',[r_vect(1),r_vect(1)+wind_vect(1)],...
    'ydata',[r_vect(2),r_vect(2)+wind_vect(2)],'color','g');
force_line = line('xdata',[r_vect(1),r_vect(1)+f_drag(1)],...
    'ydata',[r_vect(2),r_vect(2)+f_drag(2)],'linewidth',2,'color','r');
axis([-0.25 8 -0.25 8]);
axis square;
first = 0;
else
    % just move the already existing lines to the new locations
    set(myline,'xdata',[0, r_vect(1)], 'ydata', [0, r_vect(2)]);
    [px,py] = importfoil(r_vect(1),r_vect(2),0.2,theta-beta);
    for i = 1:length(px)-1
        set(airline(i),'xdata',[px(i), px(i+1)],'ydata', [py(i), py(i+1)]);
    end
    set(kite_wind,'xdata',[r_vect(1),r_vect(1)+wind_vect(1)],...
        'ydata',[r_vect(2),r_vect(2)+wind_vect(2)]);
    set(force_line,'xdata',[r_vect(1),r_vect(1)+f_drag(1)],...
        'ydata',[r_vect(2),r_vect(2)+f_drag(2)]);
end

C.4 Plot Digitization Code

This code allows for digital images of data to be imported into MATLAB and then using designated reference points and a set number of data points (both defined by the user) acquire digital data.

%This m-file will load the image file specified and then ask the user to %click on the (0,0) point and another grid point to provide the scaling and %then will ask the user for how many data points they want to acquire by %clicking on them using the mouse.

clear;
clf;
x = imread('NACA4412Moment.bmp','bmp');
y = imread('NACA4412Moment.bmp','bmp');

figure(1);
image(x);

'please click on the (0,0) point'
[x0,y0] = ginput(1);
X0 = 0;
Y0 = 0;

'please click on another point and enter its coords'
[xf,yf] = ginput(1);
Xf = input('the xcoord of the point you just clicked on');
Yf = input('the ycoord of the point you just clicked on');

n = input('How many points do you want to digitize? ');

[x,y] = ginput(n);

xscale = (Xf-X0)/(xf-x0);
yscale = (Yf-Y0)/(yf-y0);

X = (x-x0).*xscale + X0;
Y = (y-y0).*yscale + Y0;

data = [X, Y];

save '4412_exptdatamoment.txt' data -ascii -double -tabs