Experimental and analytical investigation of a passive remote electromechanical dynamic absorber

Todd Nichols

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Experimental and Analytical investigation of a Passive Remote Electromechanical Dynamic Absorber

By
Todd Stuart Nichols

A Thesis Submitted in Partial Fulfillment of the Requirement for the Masters of Science in Mechanical Engineering

Approved by:

Dr. Hany Ghoneim
Department of Mechanical Engineering

Dr. Brett Pokines
Department of Mechanical Engineering

Dr. Josef Török
Department of Mechanical Engineering

Dr. Edward C. Hensel
Department Head of Mechanical Engineering

Department of Mechanical Engineering
Rochester Institute of Technology
October, 2001
Experimental and Analytical investigation of a Passive Remote Electromechanical Dynamic Absorber

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Abstract

For this thesis a new concept, the Passive Remote Electromechanical Dynamic Absorber (RDA) is investigated. The current design utilizes piezoelectric elements to convert the mechanical strain energy of a parent system into electrical energy, which is fed into the RDA. The RDA similarly uses piezoelectric elements to convert the applied electrical energy into mechanical self-excitation. A lumped-system model of the coupled system is developed, accounting for the stiffness and mass of both the parent and RDA systems, along with a coupling stiffness term. Additionally, a coupled-system finite element model, developed in Ansys/multiphysics to include a three dimensional representation of the coupled system, is considered. Experimental work is conducted to validate the concept of the lumped system model and to validate the finite element modeling technique.

A reasonable correlation between the experimental results and the analytical predictions validate the qualitative analysis. FEA such as was developed for this thesis in ANSYS/Multiphysics can reasonably predict the actual performance of an RDA. Furthermore, analytical predictions of the RDA show successful reduction of the parent response by up to ~30 db, in a narrow frequency band around its uncoupled resonant frequency. The overall qualitative agreement between the analytical and the experiment confirm the validity and potential of the proposed Remote Electromechanical Dynamic Absorber for vibration suppression of dynamic systems.
Acknowledgments

First, I would like to express my appreciation to the Gleason Foundation, whose funding made this work possible. This work and much future work at RIT will benefit from the non-contact laser vibrometer purchased with funds from the Gleason Foundation.

To my advisor and great friend, Dr. Hany Ghoneim, you inspired my success at RIT and in this work, I am sincerely grateful. Daily conversation at our tea breaks helped break many religious, social, and cultural barriers for both of us, and for all whom we touch in the future. I can only hope that future work will bring us together again.

I am most appreciative of the support and criticism of both Dr. Brett Pokines and Dr. Josef Török, my committee members. As one of my first and most influential professors, Dr. Török instilled the importance of understanding the fundamentals and not relying on the “black box.” Just as I was departing RIT, Dr. Pokines arrived. As a result, we had little time to work together other than reviewing this work. Dr. Pokines, who is much closer to the field of smart structures than I expected, help me to truly understand the importance of this work in relation to past work and future implementations.

Thank you, to all of the faculty and staff of the Mechanical Engineering Department at RIT. Without your guidance over the years, I would not be where I am today. Special thanks to Dr. Kevin Kochersberger, who prepared me for this work more than any other professor, through lectures and labs in Advanced Vibrations and Experimental and Analytical Modal Analysis. Additional thanks to professor John Wellin, whose proficiency with oscilloscopes came in handy more than once, and to other helpful members of the Electrical Engineering Department at RIT.

Over the twenty-four years of my life, know one has supported me more than my parents, Noel and Josephine Nichols, and my brother Andrew. Both financially and emotionally, their support eased many hard times while I was away at school.

To my best friend Andy P. Meihl, thanks for helping me drive a truck through this one!

To my fiancée Allison, who I met only a short two years ago, you are my reason to succeed. Many a long night has she waited up for me to come home... finally, I am home.
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Introduction

The search for methods to suppress vibrations in dynamic systems is a never-ending task. One fundamental method is the classical dynamic absorber. In essence, it is a device that draws vibration energy away from a parent system. The dynamic absorber is typically a sacrificial and inexpensive item, which can be tuned to draw energy away from any resonance or band of frequencies, through a tuning process. Historically, the dynamic absorber is attached to the parent system in a mechanical sense, where space constraints and aesthetics can limit its feasibility.

Additional versions of the classical dynamic absorber have been introduced, including the shunted piezoelectric [1], fluid surface damping [2], and virtual dynamic absorber [3] technique. An electric network is implemented in the shunted piezoelectric and in the virtual dynamic absorber technique. Limiting the applicability of these two techniques, a high inductance (typically achieved through synthetic inductors with external power) is required in the electrical networks. Fluidic dynamic absorbers are applied in the fluid surface damping technique, whose application has not yet been proven. The virtual dynamic absorber is a promising technique that is still in the research and development phase.

For this thesis a new concept, the Passive Remote Electromechanical Dynamic Absorber (RDA) will be investigated. The RDA draws energy away from a parent system through conducting wires. This allows the RDA to be placed at a distance from the parent system, thus reducing space constraints issues. In addition, the RDA may be fashioned to serve a useful purpose, such as to generate electricity or to provide necessary excitation to a tertiary system.

The current design utilizes piezoelectric elements to convert the mechanical strain energy of a parent system into electrical energy, which is fed into the RDA. The RDA similarly uses piezoelectric elements to convert the applied electrical energy into mechanical self-excitation. Future implementations of this device may allow this energy to be transferred through electromagnetic waves, where no physical connection exists.

The objective of this thesis is to develop the Passive Remote Electromechanical Dynamic Absorber, and demonstrate its feasibility both experimentally and analytically. A lumped-system model of the coupled system is developed, accounting for the stiffness
and mass of both the parent and RDA systems, along with a coupling stiffness term. Additionally, a coupled-system finite element model, developed in Ansys/multiphysics to include a three dimensional representation of the coupled system, is considered. Finally, experimental work is conducted to validate the concept of the lumped system model and to validate the finite element modeling technique.

Both the parent system and the RDA investigated in this project are small cantilevered beams of rectangular cross-section, with piezoelectric elements attached near the root; see Figure 1. Two cases are studied, each with a different beam substrate and piezoelectric arrangement. One case involves 8" aluminum beams and a "31" arrangement. The other case uses 6" glass-epoxy beams and a "33" arrangement.

![Figure 1 PZT Treated Beam](image)

---

1 The "31" and "33" arrangements are explained in Appendix-B.
Lumped System Study

Introduction

Often continuous systems can be modeled as lumped systems, which offer a great reduction in complexity. Analysis of continuous systems can be laborious and often require large numbers of parameters. Lumped systems analysis, on the other hand, typically has only a few parameters. With only a few parameters to relate, generalizations about the fundamental working principles of the more complex continuous system can easily be drawn.

Modeling

Figure 2 provides a graphical representation of the lumped system studied. Both the parent and the RDA system consist of a rectangular beam treated with piezoelectric patches, shown in Figure 1. Each of these systems can be represented by a spring, which includes the stiffness of the beam, glue, and PZT patch, and a mass, which represents the mass of the beam, glue, and the PZT patch. The coupled system can be represented as a two degree of freedom (dof) system as shown in Figure 2. The coupled system also includes a small “soft” spring, which represents the piezoelectric coupling between the systems.

The following is a matrix representation of the equations of motion for the coupled system.
\[
\begin{bmatrix}
m_1 & \dot{x}_1 \\
m_2 & \dot{x}_2
\end{bmatrix} + \begin{bmatrix}
k_1 + k_c & -k_c \\
-k_c & k_2 + k_c
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix}
f_1 \\
0
\end{bmatrix}
\]

where,

\[k_1 = \text{stiffness associated with the parent system.}\]
\[k_2 = \text{stiffness associated with the RDA system.}\]
\[k_c = \text{coupling Stiffness associated with the coupled system.}\]
\[m_1 = \text{mass associated with the parent system.}\]
\[m_2 = \text{mass associated with the RDA system.}\]
\[x_1 = \text{displacement associated with the parent system.}\]
\[x_2 = \text{displacement associated with the RDA system.}\]
\[f_1 = \text{forcing associated with the parent system.}\]

The equations of motion can be used to solve for the driving point frequency response function of the parent motion. This is given as,

\[
\frac{x_1}{f_1} = \frac{(k_2 + k_c) - m_2 \omega^2}{\Delta}
\]

where,

\[
\Delta = (k_1 + k_c - m_1 \omega^2)(k_2 + k_c - m_2 \omega^2) - k_c^2
\]

Note that from Equation 2 an antiresonance exists at, \(\omega^2 = ((k_2 + k_c)/m_2)\), the frequency that causes the numerator to equal zero. The implication of this antiresonance is that with forcing on the parent, at the antiresonant frequency, zero parent response is produced. Also, note that this antiresonant frequency is not a function of any of the parent system parameters. Furthermore, by changing the size of dynamic absorber stiffness “\(k_2\)”, mass “\(m_2\)” or the coupling stiffness “\(k_c\)”, motion of the parent can be stopped at any frequency, including its uncoupled resonance frequency.

The RDA response to parent forcing is given as,

\[
\frac{x_2}{f_1} = \frac{k_c}{\Delta}
\]
Unlike the parent response given by Equation 2, the RDA response does not have an anti-resonance. It does have the same resonant frequencies as the parent response.

**Tuning Principles**

If we assume that the coupling stiffness is fixed, it is evident that many combinations of \( m_2 \) and \( k_2 \) exist that will produce zero parent system response at its natural frequency. For each mass "\( m_2 \)" an appropriate stiffness, "\( k_2 \)" can be chosen.

If antiresonance is to be produced at the parent system natural frequency, then setting the numerator of Equation X equal to zero, at \( \omega = \omega_1 \) results with:

\[
0 = (k_2 + k_c) - m_2(\omega_1)^2
\]

Solving for \( k_2 \),

\[
k_2 = \frac{m_2}{m_1}k_1 - k_c
\]

The value of \( k_2 \) given by Equation 6, will result in a 'tuned' system response.

These equations can be nondimensionalized by the following relationships, \( \mu = \frac{m_2}{m_1}, K_r = \frac{k_c}{k_1}, \) and \( r = \frac{\omega_2}{\omega_1} \). The term "\( \mu \)" is referred to as the mass ratio, "\( K_r \)" is referred to as the coupling stiffness ratio, and "\( r \)" is nondimensional frequency. With these relationships and at tuned value of "\( k_2 \)" (Equation 6), Equation 2, can be cast in the following tuned nondimensional form,

\[
\left( \begin{array}{c} x_1 \\ f_1 \\ k_1 \\ \end{array} \right) = \frac{1 - r^2}{\Delta'}
\]

where,

\[
\Delta' = (1 + K_r - r^2)(1 - r^2) - \frac{1}{\mu} K_r^2
\]

Figure 3 shows the 'tuned' frequency response of the uncoupled parent system and both the coupled parent and RDA response. The curve labeled "Coupled Parent Response" on Figure 3 is a plot of Equation 7, where \( \mu = 1 \) and \( |K_r| = 0.25 \). On this curve, note the presence of two resonant frequencies and one anti-resonant frequency. "Tuning" is evident in that the coupled parent response curve anti-resonance frequency is the same as the uncoupled parent response resonant frequency.
If the uncoupled system resonant frequencies for the parent and RDA are identical, the coupled response will have an 'untuned' response, as shown in Figure 4. The work of Anderson [4], suggested that such a configuration could suppress vibration. This is in fact not the case because the coupled system is not 'tuned'. In this instance, the RDA stiffness $k_2=k_1$, is not the tuned result specified by Equation 6, and there is still a parent system resonance condition at $r=1$ in the coupled parent system response.

Note from Figure 4, that with parent and RDA systems of equal natural frequencies, the first natural frequency of the coupled parent response is lower than that of the uncoupled parent at $r=1$. One should expect the opposite from classical systems. It is lower because in reality the stiffness of the PZT patches decreases when they are coupled. This can be effectively modeled as a negative coupling stiffness. Evidence of this effect is presented in the FEA and Experimental Results section.

![Tuned Frequency Response](image)

**Figure 3** Tuned Dynamic Absorber
**Frequency Separation**

The coupled parent response has two resonant frequencies, where one is high and one is low relative to the uncoupled parent system resonance. These resonant frequencies, determined by solving for the roots of equation 8, are given as,

$$ r^2 = \left(1 + \frac{K_r}{2}\right) \pm \frac{K_r}{2} \sqrt{1 + \frac{4}{\mu}} $$  

*eqn 9*

Figure 5, shows a plot of the two natural frequencies against the mass ratio. It is evident from this plot and Equation 9, that increasing mass ratio results in decreasing separation distance between the natural frequencies. This is opposite to that of the classical dynamic absorber, where a larger mass ratio results in greater frequency separation. For this dynamic absorber implementation, having a light dynamic absorber is advantageous to frequency separation. This is ultimately clear in Figure 6, which is a plot of the relative frequency differential against mass ratio, given as $r_{\text{high}} - r_{\text{low}}$. 

Figure 4  Untuned Dynamic Absorber
The resonant frequencies of the tuned system are functions of the coupling stiffness ratio and to the mass ratio. The first term of Equation 9, \((1+.5*Kr)\), determines the frequency mean, and the separation term is given by the term that follows the “±”. An important property to note is that the mean frequency is not a function of the mass ratio. Separation, on the other hand, is a function of both the coupling stiffness and mass ratio. With reference to Figure 6, at high mass ratios on the order of 10 and above, there is 5% separation, but shifting is such that you have both an anti-resonant and a resonant condition in the coupled parent response at the uncoupled parent system resonance \((r=1)\). This is not an advantageous feature of a dynamic absorber, which requires both separation and deviation of the coupled system resonant frequencies from the uncoupled parent system resonance.

Figure 5  Effect of mass ratio on relative natural frequencies (high and low)
Figure 6  Effect of mass ratio on natural frequency separation

Figure 7 shows the effect of relative coupling stiffness on coupled system resonant frequencies for a unity mass ratio. Increasing the coupling stiffness results in an increase in frequency separation, which is a nearly linear relationship up to values of \(|Kr|=1.4\), see Figure 8. This shows that for high separation a higher coupling stiffness is needed—an important feature that affects the robustness of the RDA design.

Robust design of an RDA requires that three things be understood. First, the RDA must be ‘tuned’ to account for the coupling stiffness, as shown in Equation 6. Secondly, frequency separation can be increased by reducing the mass ratio “\(\mu\)” and/or increasing the coupling stiffness ratio. Finally, an ideal frequency shifting can place the tuned anti-resonance equally (or near equally) spaced between the resonant frequencies.
Figure 7  Effect of coupling stiffness ratio on natural frequencies

Figure 8  Effect of mass ratio on natural frequency separation
Experimental Methodology

Introduction

Analytical and theoretical work can be exciting, but experimental work is an exercise in realism. Many things workout on paper; only experimentation can prove feasibility in the real world.

A complete investigation must include an experimental complement to the thoughts and concepts presented in the theoretical work. Analysis of the lumped representation of the coupled RDA system showed conceptually a promising device, which could provide vibration suppression to a parent system. Further analytical work with models reflecting the physics of the PZT patches and their mechanical coupling to the beams will show a promising result. All that is left is to document the phenomena with experiments.

The methodology of the experimental work is to start with the most basics and predictable experiments, slowly increasing the degree of complexity; comparing the experimental results with analytical predictions along the way. The results of the experiments are frequency response functions (FRFs), which are determined from the ratio of tip motion to base excitation force.

The final experiments should produce the desired FRFs and validate the analytical model and RDA concept. Jumping straight into these experiments is not advantageous, given the large number of parameters that govern the dynamics of the system. These parameters include unknown Young’s modulus, poison ratio, effective bonding area, adhesive stiffness, and other real world nuisances. Gluing the PZT patches onto the beams requires some finesse, and practice. Even with experience, the adhesive layer is the largest unknown and source of error in the experiment. Progressing slowly from basic to complex experiments allows the effect of isolated parameters to be studied.

Untreated beam experiments should show a close correlation to analytical models given that it has the least unknown parameters. With an understanding of the untreated beam, PZT crystals are glued to the beams. Still uncoupled, experimental results of these treated beams are obtained and fit with analytical results. The systems are coupled for the final experimental validation.
Setup

Figure 9 is a schematic of the experimental setup, Figure 10 through 12, show a photographs of the actual experimental setup. All experiments were performed in the vibrations laboratory at the Rochester Institute of Technology. Referring to Figure 9, base excitation was provided by a small LDS® model V203 shaker. A plastic “stinger” connects the shaker to a force transducer, which is in turn attached to the base. A PCB 208B01 force transducer measures the base excitation force. Tip response is measured with a noncontact OMETRON® VH300+ laser vibrometer. An Oros® Dynamic FFT Analyzer processes the data and produces the FRF’s.

A linear ball bearing slide allows motion to result from shaker excitation at the root of the parent beam. Fixed base conditions at the RDA beam root are accomplished with a custom clamping fixture. A chirp type broadband signal generated by Oros® was the shaker excitation.

Some additional materials used for gluing the PZT patches to the beams, etc, are shown in Figure 13.

![Figure 9 Pictorial schematic](image-url)
Figure 10  Experimental Setup

Figure 11  Close up View of Parent System Setup
Figure 12  Over all view of experimental setup

Figure 13  Other Equipment and Products used
Compatibility issues

Three types of frequency response functions can be used to describe the dynamic characteristics of a system, mobility, receptance, and accelerance. Response point acceleration, displacement, and velocity measurements relative to a reference point force measurement produce mobility, receptance, and accelerance, respectively. Conversion between the three forms is relatively simple,

Vector form:
$$\ddot{a} + \omega^2 \ddot{x} = (i\omega)\ddot{v}$$

Magnitude and Phase form:
$$|a| = (\omega^2)|x| = (\omega)|v|$$
$$\phi_a = \phi_x + \pi = \phi_x + \frac{\pi}{2}$$

An incompatibility exists between experimental mobility FRFs and receptance FRFs generated in Ansys. A common form was desired for later comparison. Conversion in Ansys from receptance to mobility was easiest, given that the Oros® software was incapable of converting FRFs natively.

Reference types

It should be known that using a reference accelerometer is not the same as using a reference force gauge. Unlike the different forms of the FRF (mobility, accelerance, and receptance), there is not a simple conversion relationship available to convert between the two functions.

To understand the importance of this discrepancy, consider the physical system shown in Figure 14, and its lumped system model, shown in Figure 15.
The parameters shown in Figure 15 are,

\[ m_f = \text{fixture mass} \]
\[ k = \text{beam stiffness} \]
\[ m = \text{beam mass} \]
\[ f_{\text{ref}} = \text{measured shaker excitation} \]
\[ x_{\text{tip}} = \text{beam tip displacement} \]
\[ x_{\text{ref}} = \text{fixture displacement} \]

For the given model shown in Figure 15, the following frequency response functions can be determined.

\[
\frac{x_{\text{ref}}}{f_{\text{ref}}} = \frac{k - m\omega^2}{\omega^2(k(m_f + m) - m_f m\omega^2)} \tag{eqn 10}
\]

\[
\frac{x_{\text{tip}}}{f_{\text{ref}}} = \frac{k}{\omega^2(k(m_f + m) - m_f m\omega^2)} \tag{eqn 11}
\]
or in terms of measured quantities, \( v_{\text{tip}}, a_{\text{ref}}, f_{\text{ref}} \),

\[
\begin{align*}
\frac{a_{\text{ref}}}{f_{\text{ref}}} &= \frac{k - m\omega^2}{(k(m_f + m) - m_f \omega^2)} \\
\frac{v_{\text{tip}}}{f_{\text{ref}}} &= \frac{k}{\omega(k(m_f + m) - m_f \omega^2)}
\end{align*}
\]  

\text{eqn 12} \quad \text{eqn 13}

If Equation 13 is divided by Equation 12, then finally we have expressions for \( \frac{v_{\text{tip}}}{a_{\text{ref}}} \)

\[
\frac{v_{\text{tip}}}{a_{\text{ref}}} = \frac{k}{\omega(k - m\omega^2)}
\]  

\text{eqn 14}

Figure 16, shows the tip velocity relative to accelerometer and force reference measurements. It is evident that using a reference accelerometer is different than using a reference force gauge. The difference is in the denominator, specifically its roots, of Equations 13 vs. 14. From Equation 13, the natural frequencies are 0 and \( \sqrt{(k(m_f + m)/m_f m)} \), which is different than 0 and \( \sqrt{(k/m)} \), given by Equation 14. The force reference natural frequency is always larger than that given by \( \sqrt{(k/m)} \). As the fixture mass \( m_f \) increases, the difference diminishes between these natural frequencies.

It can be said that using a reference accelerometer will ignore the effects of the fixture mass; it is not a function of \( m_f \). More importantly, note that FRFs from a reference accelerometer do not reflect "true" operating natural frequency of a system. To capture "true" FRFs a reference force gauge must be used. Since "true" natural frequencies are desired, a reference force gauge must be used, and the fixture mass must be included in the model. Addition of the fixture mass presents an additional factor, which must be determined.
Figure 16  Response plots showing the difference between reference signal types
Finite Element Analysis

Introduction

Many FEA codes are capable of linear harmonic analysis as is needed for this type of analysis. One code, which is capable of handling the piezoelectric effect utilized in the PZT treatment is, Ansys/Multiphysics, a strong leader in the piezoelectric analysis. Out of the box the Ansys/Multiphysics code can perform 2d and 3d coupled field piezoelectric effect analysis.

Model Summary

Figure 17 shows the 3d model developed in Ansys/Multiphysics. Included are both the parent and RDA beams with PZT patches, and the adhesive layers. The model allows for coupled, uncoupled-treated and uncoupled-untreated responses. All dimensions are independently parametric. Linear material properties were used for each of the three materials, beam, adhesive, and PZT. Low-order solid brick elements (solid 5) without midside nodes were used throughout the model. Constraint equations were used to couple the voltages on the surfaces of the PZT patches and to represent the coupling wires. With correct alignment of the piezoelectric element axes and the selection of proper harvesting surfaces, both a “31” and “33” PZT arrangement can be simulated. Material specific loss-factor damping was employed for each of the three aforementioned materials. A fixed boundary condition was used at the root of the RDA beam. For the parent beam, a symmetric boundary condition and a lumped mass (fixture mass) were used to represent the linear slide. Block Lanczos mode extraction was used to obtain the first ten mode shapes and natural frequencies. For the harmonic response analysis, mode-superposition was used, with all extracted modes over a wide band around the first natural frequency of the uncoupled parent system. All results were exported to and plotted with Matlab for consistency.
Figure 17  Ansys/Multiphysics 3D FEA Model (Including Piezoelectric Effects)
Results

Two types of arrangements ("31" and "33") were investigated with both experimental and analytical work. A reasonable correlation was sought between the finite element analysis and the experimental results. Some unknown parameters were determined through parameter extraction, others were taken from reference materials.

Curve Fitting and Parameter Extraction

Analytical work requires dimensional parameters to describe the particulars of the simulation. Many of these parameters are easily obtained; from reference material, independent experiments, etc. Some of the parameters are not easily obtained.

Preliminary experiments were performed to determine the values of some of the unknown parameters. Beginning with the most basic (reduced) experiments where all but two parameters are known. In small steps, the complexity of the experiment is increased until the final experiment is reached; a coupled system experiment.

The analysis of the untreated beams is governed by the parameters given in Table 1. The length, width, and height were obtained through caliper measurements. The density was calculated from a measured mass and beam dimensions. For each beam type preliminary experiments were performed to determine the modulus and damping ratio of the untreated beam, and the fixture mass.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Untreated Beam</th>
<th>Treated Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Width</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Height</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Density</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Modulus</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Fixture mass</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Glue thickness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glue modulus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glue ramping ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT thickness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT modulus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT coupling Coeff.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT dielectric</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT damping ratio</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1  Governing Parameters

Figure 18 and 19 show the results of the untreated beam curve fitting and the parameters used for in the finite element analyses for both glass-Epoxy and aluminum. Both the acceleration and the force reference FRF were fit simultaneously. Optimization techniques were not employed; rather, trial and error methods were used with a qualitative objective function. The fit is qualitatively reasonable for both reference types. Noteworthy is the fact that the analytical response was higher than the experimental results, with a force reference, and the opposite is true for an acceleration reference. This trend continues throughout the investigation for both cases.

The governing parameters for treated beams are also given in Table 1. The length, width, and height were kept the same from the untreated analysis. The density was calculated from a measured beam mass and dimensions. For each treated beam type, further experiments were performed to determine the glue modules and damping ratio.

Figure 20 and 21 show the results of the treated beam curve fitting and the parameters used for the analyses. In this set of experiments, the beams were treated with glue and oriented PZT ceramic patches. Curve fitting was used to extract the glue modulus and damping used on both beam types. Extracting glue modulus in this way accounts for the
effective bonding area and quality of bond. Material properties for the PZT ceramic, obtained from reference material provided by the manufacture\(^2\), are shown in Table 2.

![Curvefit to Experimental Data (Untreated 6" Glass-Epoxy Beam#1)](image)

**Figure 18 Naked Glass-Epoxy Beam Results**

\(^2\) Sensor Technology Inc.
Figure 19  Naked Aluminum Beam Results

Figure 20  Uncoupled Treated Glass-Epoxy Beam "33" Arrangement
Figure 21 Uncoupled Treated Alum Beam "31" Arrangement

<table>
<thead>
<tr>
<th>Beam</th>
<th>Case 1-Aluminum &quot;31&quot;</th>
<th>Case 2- glass Epoxy &quot;33&quot;</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>8</td>
<td>6</td>
<td>in</td>
</tr>
<tr>
<td>W</td>
<td>1</td>
<td>0.5</td>
<td>in</td>
</tr>
<tr>
<td>H</td>
<td>0.128</td>
<td>0.128</td>
<td>in</td>
</tr>
<tr>
<td>rho</td>
<td>2700</td>
<td>1944</td>
<td>kg/m³</td>
</tr>
<tr>
<td>E</td>
<td>62.19</td>
<td>19.55</td>
<td>GPA</td>
</tr>
<tr>
<td>eta</td>
<td>0.0015</td>
<td>0.002</td>
<td>---</td>
</tr>
<tr>
<td>thickness</td>
<td>0.008</td>
<td>0.004</td>
<td>in</td>
</tr>
<tr>
<td>E</td>
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<td>1.75</td>
<td>GPA</td>
</tr>
<tr>
<td>eta</td>
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<td>0.0025</td>
<td>---</td>
</tr>
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<td>Arrangement</td>
<td>&quot;31&quot;</td>
<td>&quot;33&quot;</td>
<td>---</td>
</tr>
<tr>
<td>thickness</td>
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<td>0.0216</td>
<td>in</td>
</tr>
<tr>
<td>S11</td>
<td>0.015</td>
<td>0.015</td>
<td>(GPA)⁻¹</td>
</tr>
<tr>
<td>S33</td>
<td>0.019</td>
<td>0.019</td>
<td>(GPA)⁻¹</td>
</tr>
<tr>
<td>eta</td>
<td>0.005</td>
<td>0.020</td>
<td>---</td>
</tr>
<tr>
<td>kp</td>
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<td>0.62</td>
<td>---</td>
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<tr>
<td>k31</td>
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<td>0.37</td>
<td>---</td>
</tr>
<tr>
<td>k33</td>
<td>0.72</td>
<td>0.72</td>
<td>---</td>
</tr>
<tr>
<td>d33</td>
<td>-160</td>
<td>-160</td>
<td>C/N</td>
</tr>
<tr>
<td>d31</td>
<td>365</td>
<td>365</td>
<td>C/N</td>
</tr>
<tr>
<td>K33T</td>
<td>1750</td>
<td>1750</td>
<td>---</td>
</tr>
</tbody>
</table>

Table 2 Analysis Parameters
**Coupled System Response**

After having obtained all the governing parameters, (Table 2) the coupled system response could be simulated analytically. The coupled system response is also obtained for qualitative comparison.

**Analytical Predictions**

Figure 22 and 23 show analytical predictions resulting from the FEA for both of the two arrangements under the untuned identical system case. The characteristic shaped of FRFs resulting from the undamped lumped system analysis, are similar to that resulting from the analytical study. A clear anti-resonance is evident in the parent response. A reduced but nonzero response exists in RDA response, corresponding to the anti-resonance of the parent response. Expected mistuning of the type shown in Figure 5 is evident, where the resonance frequency of the coupled parent response is at the same frequency as the uncoupled parent response resonant frequency.

![Analytical Results Coupled & Uncoupled Response](image)

*Figure 22  Analytical Results – Untuned Coupled & Uncoupled Response*
Figure 23 Analytical Results - Untuned Coupled & Uncoupled Response
A tuned system response is shown in Figures 24 and 25 for the “33” and “31” PZT arrangements, respectively. By changing the natural frequency of the RDA system, the antiresonance in the coupled parent response can be collocated to the uncoupled parent system resonance.

Though, tunning of the type shown in Figure 4 has been achieved analytically, the frequency range of operation (given by frequency separation between the 2 natural frequencies of the coupled system) is narrow. The concept is valid (can successfully lower the amplitude resonance of the parent) only if the excitation is limited to a narrow band (~6 hz). If excitation is only in a very narrow frequency band (+/- 6 hz) around uncoupled resonance, the RDA device successfully reduces the response by up to ~30db.

![Analytical Results - Tuned Coupled & Uncoupled Response](image)

**Figure 24** Analytical Results - Tuned Coupled & Uncoupled Response
Experimental Results

FRFs obtained for untuned coupled system experiments are shown in Figure 26 and 27 along with uncoupled parent response. The characteristic features given by the lumped system analysis are seen in results. A clear anti-resonance is visible as well as two resonant peaks. Expected mistuning of the type shown in Figure 5 is also evident in the experimental results. A resonant condition near the uncoupled parent system response still exists.
Figure 26 Experimental Results – Untuned Coupled & Uncoupled Response

Figure 27 Experimental Results – Untuned Coupled & Uncoupled Response
Comparison

Figure 28 and 29 shows coupled parent response for both the experimental and analytical results, for both the “33” and the “31” PZT arrangement, respectively. A reasonable correlation between the experimental results and analytical predictions exists for both arrangements. The higher natural frequencies match well between the each result set.

For the “33” arrangement, Figure 28, the lower resonance and the anti-resonance given by the analytical results are lower than that measured from the experiments. The relative frequency separation given by the analytical analysis is roughly 3.9%; that from the experiment is only 2.4%.

![Figure 28 Coupled Parent Response (6" Glass-Epoxy Beam "33" Arrangement)](image)

For the “31” arrangement, Figure 29, the relative frequency separation given by the analytical analysis is roughly 1.0%; that given by the experiment is only 0.75%. This under estimation comes as a surprise, considering that actual coupling coefficients are lower than experimental ones due to imperfect adhesive quality.
Figure 29  Coupled Parent Response (8'' Alum Beam "31" Arrangement)

Figure 30 and 31 shows coupled RDA response for both the experimental and analytical results, for both the "33" and the "31" arrangement, respectively. A reasonable correlation between the experimental results and analytical predictions exists for both arrangements.
Figure 30  Coupled RDA Response (6" Glass-Epoxy Beam "33" Arrangement)

Figure 31  Coupled RDA Response (8" Alum Beam "31" Arrangement)
The difference between the analytical predictions to the experimental results can be attributed to the use of idealized values of the coupling stiffness in the FEA. Realistic values may be obtained through curve fitting of experiments where both open-loop and closed-loop treated beam FRF are obtained. An adjusted value of mechanical coupling factors obtained through this process by Hagood [1] showed better correlation between experimental and analytical FRFs for shunted piezoelectric method.
Discussion and Conclusions

In this thesis, the concept of a passive remote dynamic absorber was investigated. The complex continuous system representation was reduced to a two degree of freedom lumped system. Basic system characteristics were determined from the lumped system study. A finite element model of the actual coupled and uncoupled systems was developed in ANSYS/MultiPhysics. The analytical FRFs generated in the FEA were compared and contrasted to FRFs obtained from experiments on actual coupled and uncoupled systems and two PZT arrangements. The characteristic FRF shapes predicted by the lumped system are observed in the experimental results.

The physical coupled system of the two PZT treated beams, was successfully reduced to a 2 degree of freedom system. The equations of motion were nondimensionalized resulting in two characteristic parameters, $K_c$ and $\mu$, the coupling stiffness and mass ratio. The relationship between the coupling stiffness and the mass ratio to natural frequency separation and shifting was determined.

A finite element model of the actual coupled system was developed, which includes two beams an adhesive layer and piezoelectric patches. Finite element analysis with this model resulted in FRFs similar to that predicted by the lumped system study. A characteristic anti-resonance was evident, along with two resonant frequencies. Analytical predictions of frequency separation were on the order of 3.9% and .75% for the “33” and “31” arrangements respectively. Analytical predictions of the reduction in resonant response of the parent motion were up to ~30db.

Experimental results were obtained for the untreated and treated uncoupled beams and used for system parameter were extraction through curve fitting. Results of the coupled parent beam response show a reasonable correlation to that predicted by lumped system analysis. Characteristic resonant peaks and an anti-resonance frequency were observed. Frequency separation for the actual “33” and “31” arrangements were on the order of 2.4% and 1.0% for the respectively.

Based on both the analytical and experimental work the “33” arrangement produces a larger frequency separation and thus a more appealing design. This result was speculated before hand, but as mentioned in appendix-b application of the “33” arrangement are more delicate than that of the “31”. The result also confirms the lumped system
predictions that increased frequency separation will result from an increased coupling stiffness ratio.

A reasonable correlation between the experimental results and the analytical predictions validate the qualitative analysis. FEA such as was developed for this thesis in ANSYS/Multiphysics can reasonably predict the actual performance of an RDA. Furthermore, analytical predictions of the RDA show successful reduction of the parent response by up to ~30 db, in a narrow frequency band around its uncoupled resonant frequency. The overall qualitative agreement between the analytical and the experiment confirm the validity and potential of the proposed Remote Electromechanical Dynamic Absorber for vibration suppression of dynamic systems.
Further Work

Experimental verification of a tuned system to complement what was presented in the lumped system study and finite element analysis was planned, though not implemented. Additional work could be to execute theses experiments.

Both the adhesive stiffness and adhesion quality are important parameters in this type of analysis, especially for quantitative comparison. Adhesion of the PZT patches to the beam substrate is always a problem. Finding a better adhesive material could enhance the effectiveness of the RDA, through a higher effective coupling stiffness.

Independent experimentation to determine those parameters that were determined through curve fitting would be progress toward a quantitative analysis of the RDA method.

Experiments in this work are somewhat academic; a successful application of the RDA to a real system with a problematic resonance condition would do a great deal to support the validity of such a technique.

The RDA system in this work was a simple cantilevered beam, which could have been a system that serves a useful purpose, such as a fan. Discovery of and experiments with additional electromechanical secondary systems would increase interest in technique.
References


5. Ansys Reference Material, ANSYS Inc.

Appendix

Appendix-A  A Classical Undamped Dynamic Absorber

A classical Dynamic Absorber consists of a single dof spring mass system that is attached to a parent system as shown in Figure 32. Its affect is to draw the dynamic energy away from the parent system into itself at a particular frequency.

![Figure 1 A Classical Dynamic Absorber](image)

The resonant frequency of the parent system occurs at $\omega = \omega_1$, typically this is the frequency at which the dynamic absorber would be tuned. The following is a matrix representation of the equations of motion for the above system

$$
\begin{bmatrix}
  m_1 \\
  m_2
\end{bmatrix}
\ddot{x}(t) +
\begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2
\end{bmatrix}
\ddot{x}(t) = \bar{f}(t)
$$

These equations of motion can be solved for the driving point frequency response function of the parent systems motion. This is given as,

$$
\begin{bmatrix}
  x_1 \\
  f_1
\end{bmatrix} = \frac{k_2 - m_2 \omega^2}{\Delta}
$$

where,

$$
\Delta = (k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2
$$

The Dynamic absorber response is given as,
Note that the numerator of Equation 16 will produce zero response at \( \omega^2= \omega_2^2= (k_2/m_2) \). By tuning the ratio of \( k_2 \) to \( m_2 \) the motion of the parent system can be stopped at any frequency, even its resonance frequency.

For parent anti-resonance at its uncoupled resonant frequency,

\[
k_2=m_2(\omega_1)^2=(m_2/m_1)k_1
\]

These equations can be nondimensionalized by the following relationships,

\[ \mu = \frac{m_2}{m_1}, \quad K_r = \frac{k_c}{k_1}, \quad \text{and} \quad r = \frac{\omega_2}{\omega_1}. \]

The term “\( \mu \)” is referred to as the mass ratio, “\( K_r \)” is referred to as the coupling stiffness ratio, and “\( r \)” is nondimensional frequency. With these relationships and at tuned value of “\( k_2 \)” (Equation 19), Equation 16, can be cast in the following tuned nondimensional form,

\[
\left( \frac{x_1}{f_1/k_1} \right) = \frac{1- r^2}{\Delta'}
\]

where,

\[
\Delta' = (1 + \mu - r^2)(1 - r^2) - \mu
\]

Figure 33 shows the 'tuned' frequency response of the uncoupled parent system and both the coupled parent and dynamic absorber response. The curve labeled “Coupled Parent Response” on Figure 4, is a plot of Equation 7, where \( \mu=1 \) and \( |K_r|=0.25 \). On this curve, note the presence of two resonant frequencies and one anti-resonant frequency. “Tuning” is evident in that the coupled parent response curve anti-resonant frequency is the same as the uncoupled parent response resonant frequency.

Note that the uncoupled resonant frequencies for the parent and RDA systems are identical and that the coupled response is tuned.

The coupled parent response has two resonant frequencies, where one is high and one is low relative to the uncoupled parent system resonance. Theses resonant frequencies, determined by solving for the roots of equation 21, are given as,
Figure 34, shows a plot of the two natural frequencies against the mass ratio. It is evident from this plot and Equation 9, that increasing mass ratio results in increased separation between the high and low natural frequencies. For the classical dynamic absorber implementation, having a heavy dynamic absorber is advantageous to frequency separation. This is ultimately clear in Figure 35, which is a plot of the relative frequency differential against mass ratio, given as $r_{\text{high}} - r_{\text{low}}$.
Relative Natural Frequencies vs. Mass Ratio

Figure 3 Effect of mass ratio on relative natural frequencies (high and low)

Relative Peak Separation vs. Mass Ratio

Figure 4 Effect of mass ratio on relative natural frequencies separation
Appendix-B  Piezoelectric Theory and Ansys Implementation

Piezoelectric Materials

Piezoelectric (From: electric pressure) materials convert mechanical strain energy into electrical potential energy, and vice versa. All materials exhibit this behavior to some degree, a few naturally occurring materials, such as, Quartz, Rochelle Salt and Tourmaline exhibit the effect to a useful degree. Man made materials such as Lead Zirconate Titanates and Barium Titanate can exhibit a strong piezoelectric effect. The magnitude of the piezoelectric effect varies from material to material; some are more efficient than others are. A material’s ability to converts strains to voltage is denoted by “k31”, “k33” or “k_p” for bi-axial piezoelectric materials. These numbers can range from 0% to 75%.

Synthetic materials such as Lead Zirconate Titanates (PZT) are produced by poling the raw material to increase its piezoelectric properties. Once poled, the PZT can be described as an anisotropic material. The poling direction is typically referred to as the “3” direction. The “1” and “2” directions are not typically distinguished and properties of these direction are equal. When applied to a substrate it is convenient to classify the arrangement with respect to primary strain, poling direction, and harvesting direction. Harvesting direction refers to the axes along which the surface potential difference is acquired.

This “arrangement” is denoted by two digits X and Y in the form XY-arrangement. The X is the primary strain direction and the Y refers to harvesting direction, both relative the PZT axes. For instance a “31” arrangement has primary strain along the “3” direction and harvesting across surfaces normal to the “1” direction.

A similar notation is used to describe the mechanical coupling factor k_{XY}, which describes the efficiency of a material to convert strains to electrical displacements. The X and Y are the same as was used to describe the PZT arrangement. For most materials k_{33} is greater than k_{31}, though implementation of a “33” arrangement are more difficult in practice.
Harvesting is typically achieved by attaching a thin conducting metallic foil to paired parallel surfaces. Charge developed on the two surfaces due to strain results in a potential difference, which can be used to drive a secondary device. The opposite is true also, where if a potential difference is applied to the surfaces a strain will result.

**Piezoelectric Materials in ANSYS**

The relationship between the mechanical strains and electrical displacements, to the mechanical stresses and electric fields is given as follows,

\[
\begin{bmatrix}
\dot{\varepsilon} \\
\dot{D}
\end{bmatrix} =
\begin{bmatrix}
[S] & [d] \\
[d^T] & [p]
\end{bmatrix}
\begin{bmatrix}
\dot{\sigma} \\
\dot{E}
\end{bmatrix}
\]

where

- \( \dot{\varepsilon} \) = 6 components of strain
- \( \dot{D} \) = 3 components of electrical flux density
- \( \dot{\sigma} \) = 6 components of stress
- \( \dot{E} \) = 3 components of electrical field
- \([S]\) = compliance matrix
- \([d]\) = dielectric matrix
- \([p]\) = permittivity matrix

The material properties \([S],[d],\) and \([p]\) are what is typically given in manufactures reference data. Ansys is setup to work with a different but representative set of properties, given as

\[
\begin{bmatrix}
\dot{\sigma} \\
\dot{D}
\end{bmatrix} =
\begin{bmatrix}
[Y] & [z] \\
[z^T] & [e]
\end{bmatrix}
\begin{bmatrix}
\dot{\varepsilon} \\
\dot{E}
\end{bmatrix}
\]

where,

- \([Y]\) = Elastic matrix
- \([z]\) = piezoelectric matrix
- \([e]\) = dielectric matrix

Conversion between the two is given by the following relationships.
\[ [Y] = [S]^{-1} \]  
\[ [z] = [d]^T [Y] \]  
\[ [e] = [p] - [d]^T [Y] [d] \]

The material properties \([Y],[z],\) and \([e]\) are required for analytical simulations. Ansys is setup to have input in this form of the equations.