Optimization of vehicle suspensions subjected to random excitation

Rifaquat Ali Cheema

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OPTIMIZATION OF VEHICLE SUSPENSIONS
SUBJECTED TO RANDOM EXCITATION

by
Rifaquat Ali Cheema

A Thesis Submitted
in
Partial Fulfillment
of the
Requirements for the Degree of
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OPTIMIZATION OF VEHICLE SUSPENSIONS
SUBJECTED TO RANDOM EXCITATIONS

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ABSTRACT

This study provides basic information on the analysis and optimization of vehicle suspension systems with a damped absorber attached to the sprung mass and the unsprung mass. This study is also concerned with comparing these systems with a optimized conventional system.

A two degree of freedom linear model subjected to guideway irregularity, described as a hyperbolic displacement spectral density, random excitation is chosen for studying the system. Dimensionless space and design parameters are selected to allow for adequate generality.

The objective function incorporates the tire-terrain normal force, as an indication of the vehicle controllability, constrained by the sprung mass acceleration as a comfort criteria. Optimum parameter synthesis of damped absorber suspension with the damped absorber attached to sprung and unsprung mass, as well as a conventional suspension system has been obtained. Performance characteristics for the optimum damped absorber suspension and the conventional suspension are presented.

The comparison among the optimised conventional, and the optimized damped absorber suspension systems show that the optimum damped absorber suspension with the absorber attached to the unsprung mass, based on the
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NOMENCLATURE

\( a_n(s) \)  A polynomial of order \((n)\) for the variable \(s\).

\( a_0, a_1, ..., a_{n-1} \) Coefficient of polynomial \(a(s)\).

\( A \) A constant depends on particular terrain.

\( b(s) \) A polynomial of order \((n)\) for the variable \(s\).

\( b_0, b_1, ..., b_{n-1} \) Coefficient of polynomial \(b(s)\).

\( C_2 \) Damping coefficient of the damper of the main suspension system.

\( C_3 \) Damping coefficient of the damper of the dynamic absorber.

\( c_2, c_2b \) Damping coefficient of both dampers of the four degrees of freedom model.

\( d_0, d_1, ..., d_n \) Coefficients of the polynomial \(d(s)\).

\( d_n(s) \) A polynomial of order \((n)\) for the variable \(s\).

\( D(\emptyset) \) Non dimensional characteristic equation of the two degrees of freedom model.

\( f_g \) Dynamic variation of the ground force.

\( F_g \) Non dimensional variation of the ground force.

\( H(\emptyset), H(\Psi) \) Non dimensional transfer function between an output and an input.

\( I_n \) Value of complex time integral with \((n)\) number of poles

\( K_1 \) Stiffness rate of tire of the two degrees of freedom model.

\( K_2 \) Stiffness of the spring of the main suspension of two degrees of freedom.

\( K_3 \) Stiffness of the spring of the dynamic absorber of two degrees of freedom model.
\( K_{1a}, K_{1b}, K_{2a}, K_{2b} \) Stiffness of the four springs of the four degrees of freedom model.

\( K_2, K_3 \) Non dimensional stiffness rate of the springs of the main suspension and the spring of the absorber, for the two degrees of freedom model.

\( L \) The distance between side (a) and (b) of the four degrees of freedom model.

\( m_1 \) Unsprung mass of the two degrees of freedom model.

\( m_2 \) Sprung mass of the two degrees of freedom model.

\( m_3 \) Mass of the dynamic absorber.

\( m_{1a}, m_{1b} \) Unsprung masses of the four degrees of freedom model.

\( m_{22} \) Sprung mass of the four degrees of freedom model.

\( M_1 \) Non dimensional unsprung mass of the two degrees of freedom model.

\( M_3 \) Non dimensional absorber mass.

\( n \) Order of polynomial.

\( p \) An intermediate coefficient.

\( q \) An intermediate coefficient.

\( R(t) \) Autocorrelation function.

\( S \) Laplace operator.

\( S_x(w) \) Input displacement spectral density in the time domain.

\( S_x(w_{nn}) \) Value of input displacement spectral density at \( w \).

\( S_x(\Omega) \) Spectral density of terrain surface in spatial domain.

\( S_x(\gamma) \) Input displacement spectral density in the non dimensional time domain.
\( \mathbf{S}_{x'x}(w) \)  
Sprung mass acceleration spectral density for an input displacement spectral density.

\( t \)  
Time.

\( v \)  
Forward vehicle velocity.

\( x \)  
Input displacement.

\( x_1 \)  
Displacement of the unsprung mass of the two degrees of freedom model.

\( \dot{x}_1 \)  
Velocity of the unsprung mass of the two degrees of freedom model.

\( \ddot{x}_1 \)  
Acceleration of the unsprung mass of the two degrees of freedom model.

\( x_2 \)  
Displacement of the sprung mass of the two degrees of freedom model.

\( \dot{x}_2 \)  
Velocity of the sprung mass of the two degrees of freedom model.

\( \ddot{x}_2 \)  
Acceleration of the sprung mass of the two degrees of freedom model.

\( x_0 \)  
A significant length which describes the magnitude of the input.

\( X \)  
Non dimensional input displacement.

\( X_1 \)  
Non dimensional displacement of the unsprung mass of the two degrees of freedom model.

\( \dot{X}_1 \)  
Non dimensional velocity of the unsprung mass of the two degrees of freedom model.

\( \ddot{X}_1 \)  
Non dimensional acceleration of the unsprung mass of the two degrees of freedom model.

\( X_2 \)  
Non dimensional displacement of the sprung mass of the two degrees of freedom model.

\( \dot{X}_2 \)  
Non dimensional velocity of the sprung mass of two degrees of freedom model.

\( \ddot{X}_2 \)  
Non dimensional acceleration of the sprung mass of the two degrees of freedom model.
\( \ddot{y}_c \) Comfort limit on acceleration.
\( \ddot{y}_c \) Non dimensional comfort limit on acceleration.
\( z_{la}, z_{lb} \) Input displacement of both sides of the four degrees of freedom model.
\( z_2 \) Displacement of the sprung mass of the four degrees of freedom model.
\( \gamma \) Non dimensional frequency.
\( \theta \) Angular displacement of the sprung mass of the four degrees of freedom model.
\( \mu \) Ratio of mass of absorber to the unsprung mass.
\( \dot{\gamma}_2 \) Non dimensional damping factor of the main suspension of the two degrees of freedom model.
\( \dot{\gamma}_3 \) Non dimensional damping factor of the absorber.
\( \sigma \) A multiple of standard deviation.
\( \tau \) Shift or lag of one function with respect to another.
\( \wp \) Non dimensional laplace operator
\( w \) Input circular frequency.
\( w_{n2} \) Natural frequency of the sprung mass.
\( w_{n3} \) Design natural frequency of the absorber.
\( w_{nn} \) Design natural frequency of the two degrees of freedom model.
\( c \) Constraint.
\( \text{max} \) Maximum
\( \text{min} \) Minimum
\( \text{rms} \) The root mean square.
\[
\int_j \quad \text{Integration sign.}
\]

PSD \quad \text{Power spectral density.}
1. **INTRODUCTION**

Vibration isolation concerns means to bring about a reduction in vibratory effect.

A vibration isolator in its most elementary form may be considered as a resilient member connecting the equipment and foundation. The function of an isolator is to reduce the magnitude of motion transmitted from a vibrating foundation to the equipment, or to reduce the magnitude of force transmitted from the equipment to its foundation.

1.1. **Mechanical Isolation**

The performance criteria in the design of mechanical isolation systems are usually evaluated by any of the following characteristics (4).

i. **Absolute Transmissibility**

It is a measure of the reduction of transmitted force or motion afforded by an isolator. If the source of vibration is an oscillating motion of the foundation, transmissibility is the ratio of the vibration amplitude of the equipment to the vibration amplitude of the foundation.
If the source of vibration is an oscillating force originating within the equipment, transmissibility is the ratio of the force amplitude transmitted to the foundation to the amplitude of the exciting force.

ii. **Relative Transmissibility**

It is the ratio of the relative deflection amplitude of the isolator to the displacement amplitude imposed at the foundation. This relative deflection is a measure of the clearance required in the isolator.

iii. **Motion Response**

It is the ratio of the displacement amplitude of the equipment to the quotient obtained by dividing the excitation force amplitude by the static stiffness of the isolator.

1.2 **Vehicle Suspension System**

Vehicle suspension synthesis can be considered in part as an application of mechanical isolation theory. A considerable amount of work has been performed on general vehicle suspension analysis, design, and optimization (5, 6, 8, 9, 14, 16, 19, 20, 21, 24, 30). Few studies are available in the field of optimum design-parameter synthesis (6, 8, 19, 20, 21, 30). Very little has been done in the application of damped absorber to
vehicle suspension (3, 14, 24).

Most of the work has been on simple model and specified disturbance. Analysis of vehicle suspension has been largely confined to one-dimensional two degrees of freedom vehicle model (5, 6, 8, 14, 16, 20, 21). Some studies have been concerned with one-dimensional three degrees of freedom model (30), and few have included multi-degree of freedom systems (19).

The simple single degree of freedom vehicle model is usually considered in case of handling new developed investigation such as synthesis of active air cushion suspension (34), optimum linear preview control (8, 9), vehicle suspension minimizing perceived acceleration, and optimum linear vehicle suspension subjected to simultaneous guideway and external force disturbances.

System disturbances are usually considered individually and are caused by guideway irregularities. They cover step input (8, 9, 20, 21, 30), different pulses (19, 20), Sinusoids (14, 24) and random inputs (5, 6, 8, 9, 16, 21, 30, 34). Hedrik (35) and Young (36) have considered simultaneous effect of guideway and external force (wind gust) disturbances.

Systems which have been considered are usually linear except for few studies (5, 19, 21, 24, 30). Non
linear exponential damping and non linear exponential elastic restoring elements (19, 21) have been widely used. Some times coulomb friction and non-linearity due to loss of contact between tire and terrain (5, 30) are considered. Unsymmetric damping has been treated by Thompson (30), while actual tire characteristic has been investigated by Omata (24).

Solution of randomly excited non-linear system using analogue computer have been widely exploited (5, 30). If the excitation can be idealized as a Gaussian white noise, exact and approximate solutions can be used based on Markove-vector approach (37). In case of non white stationary excitation approximate solutions can be obtained by perturbation and/or equivalent linearization technique. Wen (32) has presented an approximate method for the non stationary random vibration of non-linear systems.

Most of the previous studies select a behavior variable as a performance criterion. These selected variables include body acceleration as a comfort criteria and ground force (normal force between tire and terrain) as a controllability criteria. However, in some cases, perceived acceleration has been chosen as a ride comfort criterion (5).
The mean square value of the behavior variable is usually employed when the excitation is random (5, 6, 8, 9, 16, 21, 30). Despite the method of mean square integral evaluated by Beshora (10) the method developed by Phillipi (22) is still commonly used. The peak value of the behavior variables is usually considered for different impulse inputs.

Some studies have included constraints on some additional behavior variables, where the physical limits should not be exceeded. The constrained optimization problem, in this case usually consists of a behavior variables as an objective function constrained by another behavior variable (20, 21). An alternate formulation of such problems is to consider some combination of the individual behavior variables as a single overall objective function (6, 8, 9, 16, 34, 35, 36).

Several studies use trade off between two behavior variables usually body acceleration and rattle space (6, 8, 9, 36), ground force and rattle space or ground force and body acceleration (20, 21). Other studies (19, 21) employed the behavior variables as an unconstrained objective function.
1.3 Scope of Study

In this study a one dimensional two degrees of freedom vehicle model, subjected to guideway disturbances (random) is selected. Vehicle controllability as indicated by the tire-terrain normal force is considered to be a primary criterion, constraint by a comfort criteria namely body acceleration.

The effect of adding an optimum damped absorber to the vehicle suspension on the characteristic performances of the vehicle is studied and the pertinent results are compared with an optimized conventional suspension.
2. **PROBLEM FORMULATION AND SOLUTION**

2.1 **Vehicle Suspension Model**

The equivalent diagram of an automobile (Figure 1) shows the individual components which are relevant to vibration investigation. It has ten degrees of freedom, the body has six-three in translational and three in rotation and four degrees of freedom for wheel masses which are shown in individual springs.

Since such a large number of degrees of freedom complicates the solution, the model is simplified to a two dimensional (Figure 2), where only heaves and pitching are considered by assuming that roughness elements are equal in the left and right tracks, or by assuming the other translation and rotational motions are not coupled with heaves and pitching oscillations. The later case is a vehicle that is symmetric about the longitudinal axis.

This two dimensional model reduces to one-dimensional if \( J_2 = m_{22} L_a L_b \). Figure 3a shows the resulting one dimensional model, where the vehicle body and load represents the sprung mass. The suspension is modeled as massless element providing force between the body and the unsprung mass, which is in turn supported
Fig. 1. Equivalent diagram of a vehicle

Fig. 2. Two dimensional four degrees of freedom vehicle model.
above the roadway by a linear spring. Although this one dimensional vehicle model is very simple, it nonetheless, contains some important features found in the variety of real vehicle suspension system, when designed to decouple the different motions of the vehicle or to handle each motion separately.

Simplified tire characteristic is adopted since the amount of damping in most tires are not effective. If this simplification is not used the effect of change in the inflation pressure and the geometry of rolling wheel, which results in filtering the input, would make the exact representation of the tire characteristic very complex.

In this study the suspension system considered is a passive suspension represented by a linear spring and a linear shock absorber. The damped absorber is represented by a linear mass-spring-dashpot system.

2.2 The Mathematical Models

Three models are considered: the conventional (Figure 3a) model without a damped absorber, with the damped absorber attached to sprung mass (Figure 3b), with the damped absorber attached to unsprung mass (Figure 3c).
2.2.1 Conventional System (without absorber)

Equations of motion for the system shown in Figure 3a are:

\[ m_1 \ddot{x}_1 = k_1 (x - x_1) - k_2 (x_1 - x_2) - c_2 (\dot{x}_1 - \dot{x}_2) \quad \text{--------(1)} \]

\[ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) \quad \text{--------(2)} \]

For non-dimensionalization we consider the following non-dimensional design parameters which will help us in design process. Particularly when keeping \( m_2 \) and \( k_1 \) constant, then a change in any of \( M_1, M_3, K_2, K_3 \) would allow a direct observation of the proportional change in \( m_1, m_3, k_2 \) and \( k_3 \).

\[ M_1 = \frac{m_1}{m_2} \quad M_3 = \frac{m_3}{m_2} \]

\[ K_2 = \frac{k_2}{k_1} \quad K_3 = \frac{k_3}{k_1} \]

\[ \xi_2 = \frac{c_2}{2m_2 w_{n2}} \quad \xi_3 = \frac{c_2}{2m_2 w_{n3}} \]

where \( w_{n2} = \sqrt{\frac{k_2}{m_2}} \) and \( w_{n3} = \sqrt{\frac{k_3}{m_2}} \)

we define design natural frequency as \( w_{nn} = \sqrt{\frac{k_1}{m_2}} \)

\[ x_1 = \frac{x_1}{x_0} \quad x_2 = \frac{x_2}{x_0} \]

\[ x_3 = \frac{x_3}{x_0} \quad x = \frac{x}{x_0} \]

where \( x_0 \) is the length related to the input magnitude.
Fig. 3. a. The conventional one-dimensional two degree of freedom vehicle model.

Fig. 3. b. One dimensional three degrees of freedom vehicle model with absorber attached to sprung mass.

Fig. 3. c. One dimensional three degrees of freedom vehicle model with absorber attached to unsprung mass.
now

\[ \dot{x}_1 = \frac{\dot{x}_1}{x_{\text{on}}w_{\text{nn}}} \]
\[ \dot{x}_2 = \frac{\dot{x}_2}{x_{\text{on}}w_{\text{nn}}} \]
\[ \dot{x}_3 = \frac{\dot{x}_3}{x_{\text{on}}w_{\text{nn}}} \]
\[ \ddot{x}_1 = \frac{\ddot{x}_1}{x_{\text{on}}w_{\text{nn}}} \]
\[ \ddot{x}_2 = \frac{\ddot{x}_2}{x_{\text{on}}w_{\text{nn}}} \]
\[ \ddot{x}_3 = \frac{\ddot{x}_3}{x_{\text{on}}w_{\text{nn}}} \]

Substituting these non-dimensional parameters in the equations of motion and with simple mathematical operations.

\[ \ddot{x}_1 = \frac{1}{M_1} (x - x_1) - \frac{K_2}{M_1} (x_1 - x_2) - 2 \sqrt{\frac{K_2}{M_1}} (\dot{x}_1 - \dot{x}_2) \quad --(3) \]
\[ \ddot{x}_2 = K_2 (x_1 - x_2) + 2 \sqrt{K_2} (\dot{x}_1 - \dot{x}_2) \quad --(4) \]

Taking the laplace transform and writing in the matrix form.

\[
\begin{bmatrix}
\varphi^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 \sqrt{\frac{K_2}{M_1}} \varphi & - (\frac{K_2}{M_1} + 2 \sqrt{\frac{K_2}{M_1}} \varphi) \\
-(K_2 + 2 \sqrt{K_2} \varphi) & \varphi^2 + K_2 + 2 \sqrt{K_2} \varphi
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
\frac{X}{M_1} \\
0
\end{bmatrix}
\]

Determinant of the matrix
\[
(\ddot{\phi}^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 \frac{\sqrt{K_2}}{M_1} \phi) (\ddot{\phi}^2 + K_2 + 2 \frac{\sqrt{K_2}}{M_1} \phi)
\]

\[-(K_2 + 2 \frac{\sqrt{K_2}}{M_1} \phi) \left( \frac{K_2}{M_1} + 2 \frac{\sqrt{K_2}}{M_1} \phi \right)\]

\[
D(\phi) = d_4 \phi^4 + d_3 \phi^3 + d_2 \phi^2 + d_1 \phi + d_0 \quad \text{----------(5)}
\]

where \(d_4 = 1.0\)

\[
d_3 = 2 \frac{\sqrt{K_2}}{M_1} (1 + \frac{1}{M_1})
\]

\[
d_2 = K_2 + \frac{1}{M_1} + \frac{K_2}{M_1}
\]

\[
d_1 = 2 \frac{\sqrt{K_2}}{M_1}
\]

\[
d_0 = \frac{K_2}{M_1}
\]

Now transfer function for this system.

\[
\frac{X_2}{X}(\phi) = \frac{1}{M_1} \left( K_2 + 2 \frac{\sqrt{K_2}}{M_1} \phi \right) / D(\phi)
\]

\[
\frac{X_1}{X}(\phi) = \frac{1}{M_1} \left( \phi^2 + K_2 + 2 \frac{\sqrt{K_2}}{M_1} \phi \right) / D(\phi)
\]

2.2.2 System with Absorber Attached to Sprung Mass

Equations of motion for this system as shown in Figure 3b will be:
\[ m_1 \ddot{x}_1 = k_1 (x_1 - x_2) - k_2 (x_1 - x_2) - c_2 (\dot{x}_2 - \dot{x}_3) \quad --(6) \]

\[ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) - k_3 (x_2 - x_3) - c_3 (\dot{x}_2 - \dot{x}_3) \quad --(7) \]

\[ m_3 \ddot{x}_3 = k_3 (x_2 - x_3) + c_3 (\dot{x}_2 - \dot{x}_3) \quad --(8) \]

Applying the same non-dimensional parameters and with some simple mathematical operations

\[ \ddot{x}_1 = \frac{1}{M_1} (x - x_1) - \frac{K_2}{M_1} (x_1 - x_2) - 2 \frac{J_2 \sqrt{K_2}}{M_1} (x_1 - x_2) \quad ---(9) \]

\[ \ddot{x}_2 = K_2 (x_1 - x_2) + 2 \frac{J_2 \sqrt{K_2}}{M_1} (x_1 - x_2) - K_3 (x_2 - x_3) - 2 \frac{J_3 \sqrt{K_3}}{M_3} (\dot{x}_2 - \dot{x}_3) \quad ---(10) \]

\[ \ddot{x}_3 = \frac{K_3}{M_3} (x_2 - x_3) + 2 \frac{J_3 \sqrt{K_3}}{M_3} (\dot{x}_2 - \dot{x}_3) \quad ---(11) \]

After laplace transformation and writing in matrix form

\[
\begin{bmatrix}
\phi^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 \frac{J_2 \sqrt{K_2}}{M_1} \phi - \left( \frac{K_2}{M_1} + 2 \frac{J_2 \sqrt{K_2}}{M_1} \right) \phi \\
-(K_2+2 \frac{J_2 \sqrt{K_2}}{M_1}) \phi^2 + K_2+2 \frac{J_2 \sqrt{K_2}}{M_1} \phi + K_3+2 \frac{J_3 \sqrt{K_3}}{M_3} \phi \\
0 - \left( \frac{K_3}{M_3} + 2 \frac{J_3 \sqrt{K_3}}{M_3} \right) \phi^2 + \frac{K_3}{M_3} + 2 \frac{J_3 \sqrt{K_3}}{M_3} \phi
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]
Determinant of the matrix.

\[ D(\emptyset) = d_6 \varphi^6 + d_5 \varphi^5 + d_6 \varphi^6 + d_3 \varphi^3 + d_2 \varphi^2 + d_1 \varphi + d_0 \quad -(12) \]

where

\[ d_6 = 1.0 \]

\[ d_5 = 2 \int_2 \sqrt{K_2} (1 + \frac{1}{M_1}) + 2 \int_3 \sqrt{K_3} (1 + \frac{1}{M_3}) \]

\[ d_4 = \frac{K_3}{M_3} + K_2 + K_3 + \frac{1}{M_1} + \frac{K_2}{M_1} + 4 \int_2 \sqrt{K_2} \sqrt{K_3} (\frac{1}{M_3} + \frac{1}{M_1} + \frac{1}{M_1 M_3}) \]

\[ d_3 = 2 \int_3 K_2 \sqrt{K_3} (\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}) + 2 \int_2 \sqrt{K_2} K_3 (\frac{1}{M_3} + \frac{1}{M_1 M_3}) \]

\[ + \frac{1}{M_1} \] \[ + 2 \int_3 \sqrt{K_3} (\frac{1}{M_3} + \frac{1}{M_1 M_3}) + 2 \int_2 \sqrt{K_2} \]

\[ d_2 = \frac{K_2 K_3}{M_1} + \frac{K_3 M_1}{M_1 M_3} + \frac{K_2}{M_1} + \frac{K_2 K_3}{M_1 M_1} + \frac{K_2 K_3}{M_1} + \frac{K_3}{M_1} + 4 \int_2 \sqrt{K_2} \sqrt{K_3} \]

\[ d_1 = 2 \int_3 \frac{K_2 \sqrt{K_3}}{M_1} + 2 \int_2 \sqrt{K_2} K_3 \]

\[ d_0 = \frac{K_2 K_3}{M_1 M_3} \]

Transfer function for this system

\[ \frac{X_1}{X} (\emptyset) = [(1 + K_2 +2 \int_2 \sqrt{K_2} \varphi + K_3 + 2 \int_3 \sqrt{K_3} \varphi) (1 + \frac{K_3}{M_3}) + \]

\[ 2 \int_3 \sqrt{K_3} \varphi] / D(\emptyset) \quad -(13) \]
\[
\frac{x_2(\varnothing)}{x} = \left[ (K_2 + 2 \sqrt[2]{K_2} \varnothing) (\varnothing^2 + \frac{K_3}{M_3} + 2 \sqrt[3]{\frac{K_3}{M_3}} \varnothing)/M_1 \right]/D(\varnothing) \quad -- (14)
\]

\[
\frac{x_3(\varnothing)}{x} = \left[ (K_2 + 2 \sqrt[2]{K_2} \varnothing) \left( \frac{K_3}{M_3} + 2 \sqrt[3]{\frac{K_3}{M_3}} \varnothing \right)/M_1 \right]/D(\varnothing) \quad -- (15)
\]
2.2.3 System with Absorber Attached to Unsprung Mass

Equations of motion for the system as shown in Figure 3c are

\[ m_1 \ddot{x}_1 = k_1 (x - x_1) - k_2 (x_1 - x_2) - c_2 (\dot{x}_1 - \dot{x}_2) - c_3 (\dot{x}_1 - \dot{x}_3) - k_3 (x_1 - x_3) \quad \text{----(16)} \]

\[ m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) \quad \text{----(17)} \]

\[ m_3 \ddot{x}_3 = k_3 (x_1 - x_3) + c_3 (\dot{x}_1 - \dot{x}_3) \quad \text{----(18)} \]

Applying the previously mentioned non-dimensional parameters and simple mathematical operations.

\[ \ddot{x}_1 = \frac{1}{M_1} (x - x_1) - \frac{K_2}{M_1} (x_1 - x_2) - 2 \sqrt{\frac{K_2}{M_1}} (\dot{x}_1 - \dot{x}_2) \]

\[ \ddot{x}_2 = K_2 (x_1 - x_2) + 2 \sqrt{\frac{K_2}{M_1}} (\dot{x}_1 - \dot{x}_2) \]

\[ \ddot{x}_3 = \frac{K_3}{M_3} (x_1 - x_3) + 2 \sqrt{\frac{K_3}{M_3}} (\dot{x}_1 - \dot{x}_3) \quad \text{----(19)} \]

After the laplace transformation and writing the equations in the matrix form.
\[
\begin{bmatrix}
\phi^2 + \frac{K_2}{M_1} + 2 \sqrt{\frac{K_2}{M_1}} \phi + \frac{K_3}{M_1} + 2 \sqrt{\frac{K_3}{M_1}} \phi - (\frac{K_2}{M_1} + 2 \sqrt{\frac{K_2}{M_1}}) \phi - (\frac{K_3}{M_1} + 2 \sqrt{\frac{K_3}{M_1}}) \phi \\
- (K_2 + 2 \sqrt{\frac{K_2}{M_1}} \phi) \\
- (\frac{K_3}{M_3} + 2 \sqrt{\frac{K_3}{M_3}} \phi)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
0 \\
x_2 = 0 \\
x_3 = 0
\end{bmatrix}
\]

Determinant of the matrix

\[
D(\phi) = d_6 \phi^6 + d_5 \phi^5 + d_4 \phi^4 + d_3 \phi^3 + d_2 \phi^2 + d_1 \phi + d_0
\]

where

\[
d_6 = 1.0
\]

\[
d_5 = 2 \sqrt{\frac{K_2}{M_1}} (1 + \frac{1}{M_1}) + 2 \sqrt{\frac{K_3}{M_1}} \left( \frac{1}{M_1} + \frac{1}{M_3} \right)
\]

\[
d_4 = \frac{1}{M_1} + K_2 \left( \frac{1}{M_1} + 1 \right) + K_3 \left( \frac{1}{M_1} + \frac{1}{M_3} \right) + 4 \sqrt{\frac{K_2}{M_1}} \sqrt{\frac{K_3}{M_1}} \left( \frac{1}{M_1 M_3} + \frac{1}{M_1} + \frac{1}{M_3} \right)
\]
\[ d_3 = 2 \mathbf{d}_2 \sqrt{K_2} \left( \frac{1}{M_1} + K_3 \left( \frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) \right) + 2 \mathbf{d}_3 \sqrt{K_3} \left( \frac{1}{M_1 M_3} + K_2 \left( \frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) \right) \]

\[ d_2 = 4 \mathbf{d}_2 \mathbf{d}_3 \sqrt{K_2 K_3} + K_2 K_3 \left( \frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) + \frac{K_3}{M_1 M_3} + \frac{K_2}{M_1} \]

\[ d_1 = \left( 2 \mathbf{d}_2 \sqrt{K_2} K_3 + 2 \mathbf{d}_3 \sqrt{K_3} K_2 \right) / M_1 M_3 \]

\[ d_0 = K_2 K_3 / M_1 M_3 \]

Main transfer functions for this system are

\[
\frac{x_1}{x} (\varphi) = \frac{(\varphi^2 + K_2 + 2 \mathbf{d}_2 \sqrt{K_2} \varphi) (\varphi^2 + \frac{K_3}{M_3} + 2 \mathbf{d}_3 \sqrt{K_3} \varphi)}{D(\varphi)} / M_1
\]

\[
\frac{x_2}{x} (\varphi) = \frac{(K_2 + 2 \mathbf{d}_2 \sqrt{K_2} \varphi) (\varphi^2 + \frac{K_3}{M_3} + 2 \mathbf{d}_3 \sqrt{K_3} \varphi)}{D(\varphi)} / M_1
\]

\[
\frac{x_3}{x} (\varphi) = \frac{\varphi^2 + K_2 + 2 \mathbf{d}_2 \sqrt{K_2} \varphi \left( \frac{K_3}{M_3} + 2 \mathbf{d}_3 \sqrt{K_3} \varphi \right)}{D(\varphi)} / M_1
\]

(21)
2.3 *Vehicle Disturbances*

In actual environments vehicle suspension systems are subjected to multiple input disturbances. However, in this study, emphasis is placed on guideway disturbances resulting from terrain irregularity. This guideway disturbance is treated as random and is described by its power spectral density.

Experimental data shows that for wide variety of surfaces the spectra may be well approximated by a hyperbolic displacement density function.

\[ S(\Omega) = \frac{A}{\Omega^2} \]

where "A" is roughness parameter in m. and "\( \Omega \)" is spatial frequency rad./m.

However on applying this form of spectra, it should be kept in mind that there will be some situations in which \( A/\Omega^2 \) would not fit the real roadway spectrum very well. Also for very long or very short wave length \( A/\Omega^2 \) may represent extrapolation of data, which may not very well be justified. Finally any one dimensional spectrum represents only an incomplete description of roadway, even in the statistical sense. However the use of such form has the great advantage that all roadways smooth
or rough are represented by a single parameter $A$, thus rather general preliminary design studies may be made which should have wide applicability.

If the vehicle traverses the surface with a constant forward velocity $V$, and $w$ is the circular frequency in time, the height of the roadway under the vehicle may be described by random process in time, realizing that

$$V \Omega = w \quad \text{(22)}$$

The spectral density in spatial domain may be converted to a spectral density in the time domain as

$$S_x(\Omega) \, d\Omega = S_x(w) \, dw \quad \text{(23)}$$

$$V d\Omega = dw$$

$$\frac{A}{\Omega^2} \, d\Omega = S_x(w) \, dw$$

$$S_x(w) = \frac{A}{V \Omega} = \frac{A}{V(w/V)^2}$$

hence

$$S_x(w) = \frac{AV}{w^2}$$

in non dimensional form

$$S_x(\gamma) = \frac{S_x(w)}{S_x(w_{nn})} = \frac{1}{\gamma^2}$$
where \( \psi = \frac{w}{w_{nn}} \)

and \( S_x (w_{nn}) = \frac{AV}{w_{nn}^2} \)

by using expression similar to eq. (22), this gives

\[ X_0 = \sqrt{AV/w_{nn}} \]

for a hyperbolic displacement spectral density input.
2.4 Behavior Variable Representation

Since the input to the vehicle is considered to be random excitation, the behavior variable of the response are expected to be random as well. Thus the response can only be described in terms of any of the following statistical parameters:

i) RMS value.

ii) Amplitude probability distribution, expressed as probability density.

iii) Vibration spectrum, continuous in frequency, expressed as spectral density.

iv) Autocorrelation function.

RMS magnitude may be considered as the most convenient statistical parameter that can be selected for the behavior variable representation. Since it is the only statistical quantitative parameter and also because, in general, all other statistical parameters can be expressed in terms of RMS.

From the autocorrelation definition

\[ R(\tau) = E [x(t)x(t+\tau)] \]  

\[ R(0) = x^2(t) = \int S_X(w)dw = \sigma^2 \] 

If the time of delay \( \tau \) is brought to zero, then where \( R(\tau) \) is the Autocorrelation function and \( \sigma^2 \) is the standard deviation which can define any function with probability distribution considered to be one of the mathematical known function (Guassian or Rayliegh).
However, when the behavior variable is strongly frequency dependent, we therefore need to know more about the frequency distribution, i.e., it would be advisable to study the power spectral density together with the RMS value as representation of behavior variable.

Since the behavior variable can be expressed in terms of the integral of the square magnitude of the transfer function between the behavior variable and the excitation multiplied by the spectral density of the disturbance input, as will be shown later. Thus behavior variable could be obtained in the form of total square integral. Consequently the magnitude of the RMS value of any of the behavior variables could be evaluated through the direct application of the method developed by Booten and Mathew (38).

In general, this method obtains the value of the complex line integral.

\[ I_n = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{a(s)a(-s)}{d(s)d(-s)} \, ds \]  
\[ \text{---------}(30A) \]

where

\[ a(s) = a_{n-1} s^{n-1} + a_{n2} s^{n2} + \ldots \ldots + a_0 \]  
\[ \text{---------}(30B) \]

\[ d(s) = d_n s^n + d_{n-1} s^{n-1} + \ldots \ldots + d_0 \]
A table which lists the values of these integrals for several values of n can be found in the same ref. (22).

2.5 Vehicle Performance

The vehicle performance and vibration characteristics are obtained based on wheel controllability (ride safety) and ride comfort.

2.5.1 Wheel Controllability and Ride Safety

Since the dynamic variation of normal force between the tire of the wheel and the terrain (ground force), in general determines tire-terrain contact area during normal operation, and since traction characteristics which involve side thrust and self aligning torque are dependent on that contact area; directional control and skid resistance of a wheel, in turn, is governed by the traction characteristics. It follows that vehicle controllability can be indicated by the tire-terrain normal force.

Also, since the increase of the variation of the tire-terrain dynamic force relative to static force would increase the change in wheel load, and consequently the possibility of the wheel to leave the ground; the ground force variation of the dynamic tire-terrain normal force with respect to the static wheel force could be
considered a good indicator of ride safety.

2.5.2 **Ride Comfort**

This is one of the basic goals to be provided by vehicle suspension system, particularly in the case of passenger vehicles. The use of integral square of vehicle body acceleration in case of deterministic input, such as isolated bumps and obstacles, has the advantage of giving greater weight to the large values of acceleration resulting from initial impact than to the smaller acceleration experienced during the subsequent decay of the oscillation. In addition, it has been shown that forces transmission to the human body is, in the case of vertical vibration an appropriate measure of discomfort. At frequency up to 5 HZ the force transmission is about the same as if the human body were replaced by pure mass. In this range, therefore the acceleration of the passenger may be taken as proportional to the force transmission, and hence as a measure of discomfort. Thus RMS sprung mass acceleration may be considered as a good criterion of the ride comfort.

However, human vibration sensitivity depends not only on the magnitude of vibratory acceleration, but also on its frequency. Consequently, the power spectral density of the vehicle acceleration would have the same weight as the RMS value.
2.6 Evaluation of Objective Function

2.6.1 Objective Function

The objective function is formed of RMS ground force (as measure of controllability and ride safety), and is constrained by vehicle acceleration (as a measure of ride comfort). Thus the optimization problem will have the form:

Find minimum $F_{g'}$, $x$, rms

Subjected to constraint

$G = \ddot{x}_2$, $x$, rms $- \ddot{y}_c', x', \text{rms} \leq 0$

where $F_{g'}$, $x$, rms and $x_2$, $x$, rms are the non-dimensional RMS ground force and the RMS unsprung mass acceleration respectively, and $\ddot{y}_c', x', \text{rms}$ is the non-dimension RMS comfort limit defined by

$$\ddot{y}_c', x', \text{rms} = \frac{\ddot{y}_c}{\sigma \sqrt{AV w_{nn}}}$$

where $\ddot{y}_c$ is the maximum allowable acceleration level in dimensional term, and $\sigma$ is the multiple of standard deviation required to ensure a reasonable probability of not exceeding the constraint.

2.6.2 Evaluation

For the application of parameter search optimization techniques the performance variables formulating
the objective function (ground force and body acceleration) should be defined in terms of vehicle design parameters.

The RMS of the non-dimensional ground force can be obtained by

\[ F_{g', x, \text{rms}} = \left[ \int_{-\infty}^{\infty} |H_{f_g, X}(\gamma)|^2 S_X(\gamma) \, d\gamma \right]^{1/2} \]  

--- (31)

where \( H_{f_g, X}(\gamma) \) is the non-dimensional transfer function between the ground force and input excitation, and \( S_X(\gamma) \) is the non-dimensional displacement spectral density.

From eq. (24) and replacing \( \gamma \) by \( \theta / j \) we get

\[ F_{g', x, \text{rms}} = \left[ \frac{1}{j} \int_{-\infty}^{\infty} |H_{f_g, X}(\theta) H_{f_g, X}(-\theta)| \frac{1}{-\theta^2} \, d\theta \right]^{1/2} \]  

--- (32)

Noting that \( f_g = K_1 (X - X_1) \)  

--- (33)

and \( F_g = \frac{f_g}{K_1 X_0} \)  

--- (34)

So \( F_g = X - X_1 \)  

--- (35)

and \( H_{f_g, X}(\theta) = 1 - H_{X_1 X}(\theta) \)  

--- (36)

where \( H_{X_1 X}(\theta) \) is defined already by eq. (13)
Consequently, the RMS value is evaluated by applying Bootons method (22)

\[ F_{g'x'rms} = \sqrt{2nI_n} \] \hspace{1cm} (37)

In the case of conventional system (without absorber):

\[ I_n = I_4 \]

\[ I_4 = a_3^2 (-d_0^2 d_3 + d_0 d_1 d_2) + (a_2^2 - 2a_1 a_3) d_0 d_1 d_4 + (a_1^2 - 2a_0 a_2) d_0 d_3 d_4 \] \hspace{1cm} (38)

\[ + a_0^2 (d_1 d_4^2 + d_2 d_3 d_4) / 2d_0 d_4 (-d_0 d_3^2 - d_1^2 d_4 + d_1 d_2 d_3) \]

where

\[ a_3 = 1.0 \]

\[ a_2 = 2 \frac{K_2}{J_2} \sqrt{\frac{1}{M_1}} \]

\[ a_1 = K_2 (1 + \frac{1}{M_1}) \]

\[ a_0 = 0 \]

and \( d \)'s are defined by equation (5).

Similarly the RMS value of sprung mass acceleration can be obtained as

\[ \ddot{x}_{2'rms} = \left[ \int_{-\infty}^{\infty} [H_{\ddot{x}_2, x} (\gamma)]^2 s_x (\gamma) d\gamma \right]^{1/2} \] \hspace{1cm} (40)
where $H_{x_2'x}(\gamma)$ is the non-dimensional transfer function between the sprung mass acceleration and input excitation. Following the same procedure, we get

$$\ddot{x}_2', x'_{\text{rms}} = \sqrt{2\pi I_n}$$

where $I_n$ is defined by eq. (38)

with

$$a_3 = 0.0$$

$$a_2 = 2 \left( \frac{2}{2} \right)^{\frac{1}{2}} \sqrt{K_2 / M_1}$$

$$a_1 = K_2 / M_1$$

$$a_0 = 0$$

and d's are defined by eq. (5)

Thus selecting a suitable value of $\ddot{y}_{c'x'_{\text{rms}}}$ the objective function will be known and expressed in terms of design parameters.

In the case where absorber is attached to sprung mass:

$$I_n = I_6$$
\[ F_{g'x'\text{rms}} = \sqrt{2\pi I_6} \]

where

\[ I_6 = \frac{1}{2P} \left[ a_5 q_0 + (a_4^2 - 2a_3 a_5) q_1 + (a_3^2 - 2a_2 a_4 + 2a_1 a_5) q_2 \right. \]
\[ \left. + (a_2^2 - 2a_1 a_3 + 2a_0 a_4) q_3 + (a_1^2 - 2a_0 a_2) q_4 + a_0^2 q_5 \right] \quad \text{(42)} \]

where

\[ q_0 = \frac{1}{d_6} (d_4 q_1 - d_2 q_2 + d_0 q_3) \]
\[ q_1 = -d_0 d_1 d_5 + d_0 d_3^2 - d_1 d_2 d_3 + d_1^2 d_4 \]
\[ q_2 = d_0 d_3 d_5 + d_1^2 d_6 - d_1 d_2 d_5 \]
\[ q_3 = d_0 d_5^2 + d_1 d_3 d_6 - d_1 d_4 d_5 \]
\[ q_4 = \frac{1}{d_0} (d_2 q_3 - d_4 q_2 + d_6 q_1) \]
\[ q_5 = \frac{1}{d_0} (d_2 q_4 - d_4 q_3 + d_6 q_2) \]
\[ p_6 = d_0 (d_1 q_5 - d_3 q_4 + d_5 q_3) \]

with
\[ a_5 = 1.0 \]

\[ a_4 = 2 \int_0^1 \sqrt{K_3} (1 + \frac{1}{M_3}) + 2 \int_0^1 \sqrt{K_2} (1 + \frac{1}{M_1}) \]

\[ a_3 = 4 \int_0^1 \sqrt{K_2 K_3} (\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}) + K_3 (1 + \frac{1}{M_3}) + K_2 (1 + \frac{1}{M_1}) \]

\[ a_2 = 2 \int_0^1 \sqrt{K_2} K_3 (\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}) + 2 \int_0^1 \sqrt{K_3} K_2 (\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}) \]

\[ a_1 = K_2 K_3 (\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}) \]

\[ a_0 = 0.0 \]

all d's are defined in eq. (12)

In the similar way as for the case (without absorber) sprung mass acceleration can be obtained

\[ \ddot{x}_{2,\text{rms}} = \sqrt{2\pi} I_n \]

now a's will be as follows

\[ a_5 = 0 \]

\[ a_4 = 2 \int_0^1 \frac{\sqrt{K_2}}{M_1} \]

\[ a_3 = 4 \int_0^1 \int_0^1 \int_0^1 \sqrt{K_2 K_3} / M_1 M_3 + K_2 / M_1 \]
Finally in the case where the absorber is attached to the unprung mass:

\[ I_n = I_6 \]

\[ F_{g'x'rms} = \sqrt{2\pi I_6} \]

\[ I_6 = \frac{1}{2F_6} \left[ a_5^2 q_0 + (a_4^2 - 2a_3 a_5) q_1 + (a_3^2 - 2a_2 a_4 + 2a_1 a_5) q_2 \right. \]
\[ \left. + (a_2^2 - 2a_1 a_3 + 2a_0 a_4) q_3 + (a_1^2 - 2a_0 a_2) q_4 + a_0^2 q_5 \right] \]

where all d's are defined by eq. (20)

now for the case of ground force a's will be as follows.

\[ a_5 = 1.0 \]

\[ a_4 = 2 \sqrt{K_2} (1 + \frac{1}{M_1}) + 2 \sqrt{K_3} \left( \frac{1}{M_3} + \frac{1}{M_1} \right) \]

\[ a_3 = K_2 + \frac{K_3}{M_3} + \frac{1}{M_1} (K_2 + K_3) + 4 \sqrt{K_2 K_3} \left( \frac{1}{M_1 M_3} + \frac{1}{M_1} + \frac{1}{M_3} \right) \]
\[ a_2 = \left( 2 \sqrt{\frac{K_2}{K_3}} K_3 + 2 \sqrt{\frac{K_3}{K_2}} K_2 \right) \left( \frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) \]

\[ a_1 = \frac{K_2 K_3}{M_1} \left( \frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) \]

\[ a_0 = 0.0 \]

and for the case of unsprung mass acceleration

\[ a_5 = 0 \]

\[ a_4 = 2 \sqrt{\frac{K_2}{M_1}} \]

\[ a_3 = 4 \sqrt{\frac{\frac{K_2 K_3}{M_1 M_3}}{\frac{K_2}{M_1}} + \frac{K_2}{M_1}} \]

\[ a_2 = \left( 2 \sqrt{\frac{K_2}{K_3}} K_3 + 2 \sqrt{\frac{K_3}{K_2}} K_2 \right) / M_1 M_3 \]

\[ a_1 = \frac{K_2 K_3}{M_1 M_3} \]

\[ a_0 = 0 \]
3. **OPTIMIZATION TECHNIQUE**

With the general availability of digital computers, iterative searching techniques have been developed in recent years for automatically locating the immediate neighborhood of design optimization problem solution point. Many algorithms have been developed for this purpose P519RE (2) is used in this study.

Program P519RE is derived to solve constrained and unconstrained minimization problems. The objective function is accomplished by the introduction of a penalty function "P" to account for constraint violation in the search process, greater the constraint violation the greater will be the positive value of "P".

Two basic types of techniques in search process are direct search or pattern search, and descent search or gradient based search. Direct search techniques have the advantage of avoiding gradient evaluations and of requiring little storage space in a digital computer. However, compared with gradient based search techniques, direct search techniques are generally relatively inefficient, requiring a large number of function evaluations. Also in direct search techniques, premature termination is more likely to occur.

In order to use P519RE, the specific design
optimization is reformulated to a standard format, which is then embedded in a general program for solution by a digital computer. In this study, the initial formulation of the objective function is

$$F_{g',x',\text{rms}} = \sqrt{2\pi A_{I_6}}$$

where $A_{I_6}$ is defined by (42). This objective function is constrained by the body acceleration;

$$\ddot{x}_2'x',\text{rms} = \sqrt{2\pi B_{I_6}}$$

where $B_{I_6}$ is defined by (42). Since no further simplification is required in this initial formulation, it will also be the final formulation, i.e.

$$Q \downarrow F_{g',x',\text{rms}} = \sqrt{2\pi A_{I_6}}$$

where $A_{I_6}$ is expressed in terms of unitized variables given in Appendix II.

3.1 With Absorber Attached to the System

Given Constants:

- CG(1) = $M_1 = 0.1$
- CG(2) = $M_3 = 0.01$
- CG(3) = $K_2\text{max} = 1.0$
\[
\begin{align*}
CG(4) &= K_2 \text{min} = 0.001 \\
CG(5) &= f_2 \text{max} = 1.0 \\
CG(6) &= f_2 \text{min} = 0.001 \\
CG(7) &= K_3 \text{max} = 1.0 \\
CG(8) &= K_3 \text{min} = 0.001 \\
CG(9) &= f_3 \text{max} = 1.0 \\
CG(10) &= f_3 \text{min} = 0.001 \\
CG(11) &= \ddot{y}c'x'\text{rms} = 0.3
\end{align*}
\]

Constraints:

\[
\begin{align*}
K_2 \text{min} &\leq K_2 \leq K_2 \text{max} \\
K_3 \text{min} &\leq K_3 \leq K_3 \text{max} \\
\ddot{f}_2 \text{min} &\leq \ddot{f}_2 \leq \ddot{f}_2 \text{max} \\
\ddot{f}_3 \text{min} &\leq \ddot{f}_3 \leq \ddot{f}_3 \text{max} \\
\ddot{x}_2'x'\text{rms} &\leq \ddot{y}c'x'\text{rms}
\end{align*}
\]

Variables:

\[
K_2', \ K_3', \ \ddot{f}_2', \ \ddot{f}_3
\]

Unitized Variables:

\[
\begin{align*}
V(1) &= \frac{K_2}{K_2 \text{max}} \\
V(2) &= \frac{K_3}{K_3 \text{max}} \\
V(3) &= \frac{\ddot{f}_2}{\ddot{f}_2 \text{max}} \\
V(4) &= \frac{\ddot{f}_3}{\ddot{f}_3 \text{max}}
\end{align*}
\]
Calculated Constants:

\[ C(1) = \frac{K_{2\text{max}}}{K_{2\text{min}}} = \frac{\text{CG}(3)}{\text{CG}(4)} \]

\[ C(2) = \frac{K_{3\text{max}}}{K_{3\text{min}}} = \frac{\text{CG}(5)}{\text{CG}(6)} \]

\[ C(3) = \frac{f_{2\text{max}}}{f_{2\text{min}}} = \frac{\text{CG}(7)}{\text{CG}(8)} \]

\[ C(4) = \frac{f_{3\text{max}}}{f_{3\text{min}}} = \frac{\text{CG}(9)}{\text{CG}(10)} \]

Regional Constraints:

\[ R(1) = K_{2\text{max}} - K_2 \geq 0 \]
\[ R(2) = K_2 - K_{2\text{min}} \geq 0 \]
\[ R(3) = K_{3\text{max}} - K_3 \geq 0 \]
\[ R(4) = K_3 - K_{3\text{min}} \geq 0 \]
\[ R(5) = f_{2\text{max}} - f_2 \geq 0 \]
\[ R(6) = f_2 - f_{2\text{min}} \geq 0 \]
\[ R(7) = f_{3\text{max}} - f_3 \geq 0 \]
\[ R(8) = f_3 - f_{3\text{min}} \geq 0 \]
\[ R(9) = \ddot{y}_c\cdot x'\cdot \text{rms} - \dddot{x}_2\cdot x'\cdot \text{rms} \geq 0 \]

Unitized Constraints:

\[ 1 - V(1) \geq 0 \]
\[ V(1) \cdot x \cdot C(1) - 1 \geq 0 \]
\[ 1 - V(2) \geq 0 \]
\[ V(2) \cdot x \cdot C(2) - 1 \geq 0 \]
\[ 1 - V(3) \geq 0 \]
\[ V(3) \cdot x \cdot C(3) - 1 \geq 0 \]
\[ 1 - V(4) \geq 0 \]
\[ V(4) \cdot x \cdot C(4) - 1 \geq 0 \]
\[ 1 - \text{ACC}/\text{CG}(11) \geq 0 \]

Final Output:

\[ F(1) = Q = \text{Objective (ground force)} \]
\[ F(2) = K_2 = V(1) \cdot x \cdot \text{CG}(3) \]
\[ F(3) = K_3 = V(2) \cdot x \cdot \text{CG}(5) \]
\[ F(4) = f_2 = V(3) \cdot x \cdot \text{CG}(7) \]
\[ F(5) = f_3 = V(4) \cdot x \cdot \text{CG}(9) \]
\[ F(6) = M_1 = CG(1) \]
\[ F(7) = M_3 = CG(2) \]
\[ F(8) = \ddot{y}_{c'x'}_{rms} = CG(11) \]

3.2 Without Absorber

Given Constants:

\[ CG(1) = M_1 = 0.1 \]
\[ CG(2) = K_2^{\text{min}} = 0.001 \]
\[ CG(3) = K_2^{\text{max}} = 1.0 \]
\[ CG(4) = \ddot{\delta}_2^{\text{min}} = 0.001 \]
\[ CG(5) = \ddot{\delta}_2^{\text{max}} = 1.0 \]
\[ CG(6) = \ddot{y}_{c'x'}_{rms} = 0.3 \]

Constraints:

\[ K_2^{\text{max}} \gg K_2 \gg K_2^{\text{min}} \]
\[ \ddot{\delta}_2^{\text{max}} \gg \ddot{\delta}_2 \gg \ddot{\delta}_2^{\text{min}} \]
\[ \ddot{x}_{2'x'}_{rms} \leq \ddot{y}_{c'x'}_{rms} \]

Variables:

\[ K_2, \ddot{\delta}_2 \]

Unitized Variables:

\[ V(1) = \frac{K_2}{K_2^{\text{max}}} \]
\[ V(2) = \frac{\ddot{\delta}_2}{\ddot{\delta}_2^{\text{max}}} \]
Calculated Constants:

\[ C(1) = \frac{K_2^{\text{max}}}{K_2^{\text{min}}} = \frac{CG(2)}{CG(3)} \]

\[ C(2) = \frac{f_2^{\text{max}}}{f_2^{\text{min}}} = \frac{CG(4)}{CG(5)} \]

Regional Constraints:

\[ R(1) = K_2^{\text{max}} - K_2 \geq 0 \quad 1 - V(1) \geq 0 \]

\[ R(2) = K_2 - K_2^{\text{min}} \geq 0 \quad V(1) \times C(1) - 1 \geq 0 \]

\[ R(3) = f_2^{\text{max}} - f_2 \geq 0 \quad 1 - V(2) \geq 0 \]

\[ R(4) = f_2 - f_2^{\text{min}} \geq 0 \quad V(2) \times C(2) - 1 \geq 0 \]

\[ R(5) = \ddot{y}_c \times \text{x'rms} - \dddot{x}_c \times \text{rms} \geq 0 \quad 1 - \frac{\text{ACC}}{CG(6)} \geq 0 \]

Final Output:

\[ F(1) = Q = \text{Objective (ground force)} \]

\[ F(2) = K_2 = V(1) \times CG(2) \]

\[ F(3) = f_2 = V(2) \times CG(4) \]

\[ F(4) = M_1 = CG(1) \]

\[ F(5) = \check{y}_c \times \text{x'rms} = CG(6) \]
RESULTS & DISCUSSIONS

4.1 Optimum Behavior

4.1.1 Trade-Off Curves

The optimum ground force sprung mass acceleration trade-off curves for both optimum damped absorber suspension systems, for different values of $M_1$ and $M_3$ including $M_3=0$ (the conventional system), are shown in Figures 4-12. Corresponding values of the ground forces and sprung mass accelerations are given in Tables 1-9.

From these figures, it is clear that wheel controllability at a specific ride comfort level improves with decreasing $M_1$ for both damped absorber systems. Also, wheel controllability improves with increasing $M_3$, which imposes a compromise upon choosing $M_3$: improving controllability without adding much weight to the vehicle.

In addition, Figures 4-12 demonstrate that the optimum damped absorber systems behave better than the conventional one, i.e., adding damped absorber improves controllability at a given ride comfort. The figures also demonstrate that the optimum damped absorber with the absorber attached to the unsprung mass is superior to that with the absorber attached to the sprung mass.
$M_1 = 0.25$
$M_3 = 0.01$

<table>
<thead>
<tr>
<th>$\tilde{Y}_{c,x,rms}$</th>
<th>$F_{g,x,rms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without absorber</td>
</tr>
<tr>
<td></td>
<td>Sprung mass</td>
</tr>
<tr>
<td>0.3</td>
<td>5.23749</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>3.95264</td>
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<tr>
<td></td>
<td></td>
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<td>0.5</td>
<td>3.19136</td>
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</tr>
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</tr>
<tr>
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<tr>
<td>0.8</td>
<td>2.15765</td>
</tr>
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<td>0.9</td>
<td>2.01463</td>
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<tr>
<td>1.0</td>
<td>1.92807</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>1.88459</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4  Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

(1) No Absorber
(2) Absorber attached to sprung mass.
(3) Absorber attached to unsprung mass.

\[ M_1 = 0.25 \]
\[ M_3 = 0.01 \]
TABLE – 2 –

\[ M_1 = 0.25 \]
\[ M_3 = 0.025 \]

<table>
<thead>
<tr>
<th>( \dot{Y}_{C,x,rms} )</th>
<th>( F_{g,x,rms} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without absorber</td>
</tr>
<tr>
<td></td>
<td>Sprung mass</td>
</tr>
<tr>
<td>0.3</td>
<td>5.23749</td>
</tr>
<tr>
<td>0.4</td>
<td>3.95264</td>
</tr>
<tr>
<td>0.5</td>
<td>3.19136</td>
</tr>
<tr>
<td>0.6</td>
<td>2.70292</td>
</tr>
<tr>
<td>0.7</td>
<td>2.37668</td>
</tr>
<tr>
<td>0.8</td>
<td>2.15765</td>
</tr>
<tr>
<td>0.9</td>
<td>2.01463</td>
</tr>
<tr>
<td>1.0</td>
<td>1.92807</td>
</tr>
<tr>
<td>1.1</td>
<td>1.88459</td>
</tr>
</tbody>
</table>
(1) No Absorber
(2) Absorber attached to sprung mass.
(3) Absorber attached to unsprung mass.

Fig. 5 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.
\[ M_1 = 0.25 \]
\[ M_3 = 0.05 \]

<table>
<thead>
<tr>
<th>( \bar{\gamma}_{c,x,rms} )</th>
<th>Without absorber</th>
<th>With absorber attached to Sprung mass</th>
<th>Unsprung mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>5.23749</td>
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<td>1.92807</td>
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<td>1.88459</td>
<td>1.87113</td>
<td>1.84752</td>
</tr>
</tbody>
</table>
Fig. 6  Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.
\[ M_1 = 0.1 \]
\[ M_3 = 0.005 \]

<table>
<thead>
<tr>
<th>( \dot{y}_{c,x,rms} )</th>
<th>( F_{g,x,rms} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without absorber</td>
<td>With absorber attached to</td>
</tr>
<tr>
<td>Sprung mass</td>
<td>Unsprung mass</td>
</tr>
<tr>
<td>0.3</td>
<td>3.32713</td>
</tr>
<tr>
<td>0.4</td>
<td>2.51880</td>
</tr>
<tr>
<td>0.5</td>
<td>2.05509</td>
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<td>1.77150</td>
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<td>1.59791</td>
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<td>1.49872</td>
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<td>1.45240</td>
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<tr>
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<td>1.44357</td>
</tr>
<tr>
<td>1.1</td>
<td>1.44357</td>
</tr>
</tbody>
</table>
1. No Absorber
2. Absorber attached to sprung mass
3. Absorber attached to unsprung mass.
<table>
<thead>
<tr>
<th>$\dot{y}_{c,x,rms}$</th>
<th>$F_{g,x,rms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without absorber</td>
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<td>1.44357</td>
</tr>
<tr>
<td>1.1</td>
<td>1.44357</td>
</tr>
</tbody>
</table>

$M_1 = 0.1$

$M_3 = 0.01$
Fig. 8  Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

M₁ = 0.1
M₃ = 0.01

(1) No Absorber
(2) Absorber attached to sprung mass.
(3) Absorber attached to unsprung mass.
TABLE - 6 -

\[
\begin{array}{|c|c|c|}
\hline
\hat{Y}_{C,x,rms} & F_{g,x,rms} \\
\hline
\text{Without absorber} & \text{With absorber attached to} \\
& \text{Sprung mass} & \text{Unsprung mass} \\
\hline
0.3 & 3.32713 & 3.22979 & 1.73289 \\
0.4 & 2.51880 & 2.46118 & 1.63587 \\
0.5 & 2.05509 & 2.01146 & 1.55649 \\
0.6 & 1.77150 & 1.73878 & 1.49624 \\
0.7 & 1.59791 & 1.57546 & 1.45590 \\
0.8 & 1.49872 & 1.48506 & 1.43514 \\
0.9 & 1.45240 & 1.44697 & 1.43149 \\
1.0 & 1.44357 & 1.44277 & 1.43149 \\
1.1 & 1.44357 & 1.44277 & 1.43149 \\
\hline
\end{array}
\]
Figure 9: Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.
\[
\begin{array}{ccc}
\Psi_{c,x,\text{rms}} & F_{g,x,\text{rms}} \\
\hline
\text{Without absorber} & \text{With absorber attached to} & \\
\text{Sprung mass} & \text{Unsprung mass} & \\
0.3 & 2.17312 & 2.14418 & 1.69976 \\
0.4 & 1.80376 & 1.79930 & 1.50672 \\
0.5 & 1.49570 & 1.49212 & 1.36257 \\
0.6 & 1.32252 & 1.31554 & 1.26632 \\
0.7 & 1.23356 & 1.23243 & 1.21354 \\
0.8 & 1.20122 & 1.20124 & 1.19671 \\
0.9 & 1.19985 & 1.19812 & 1.19669 \\
1.0 & 1.19986 & 1.19982 & 1.19669 \\
1.1 & 1.19986 & 1.19982 & 1.19669 \\
\end{array}
\]
Fig. 10 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.
TABLE - 8 -

\[ \begin{array}{|c|c|c|c|}
\hline
\Psi_{c,x,rms} & F_{g,x,rms} & \text{Without absorber} & \text{With absorber attached to} \\
& & \text{Sprung mass} & \text{Unsprung mass} \\
\hline
0.3 & 2.17312 & 2.02343 & 1.56498 \\
0.4 & 1.80376 & 1.79572 & 1.42350 \\
0.5 & 1.49570 & 1.48973 & 1.31621 \\
0.6 & 1.32252 & 1.31866 & 1.24398 \\
0.7 & 1.23356 & 1.23152 & 1.20507 \\
0.8 & 1.20128 & 1.20083 & 1.19514 \\
0.9 & 1.19986 & 1.19775 & 1.19514 \\
1.0 & 1.19986 & 1.19735 & 1.19514 \\
1.1 & 1.19986 & 1.19735 & 1.19514 \\
\hline
\end{array} \]
(1) No Absorber
(2) Absorber attached to sprung mass.
(3) Absorber attached to unsprung mass.

Fig. 11 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.
<table>
<thead>
<tr>
<th>$v_{c,x,rms}$</th>
<th>$F_{g,x,rms}$</th>
<th>Without absorber</th>
<th>With absorber attached to Sprung mass</th>
<th>Unsprung mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>2.17312</td>
<td>2.014164</td>
<td>1.44396</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>1.80376</td>
<td>1.78966</td>
<td>1.34547</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1.49570</td>
<td>1.48408</td>
<td>1.27115</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>1.32252</td>
<td>1.31501</td>
<td>1.22199</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>1.23356</td>
<td>1.22957</td>
<td>1.19745</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>1.20128</td>
<td>1.20044</td>
<td>1.19344</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>1.19986</td>
<td>1.20044</td>
<td>1.19344</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>1.19986</td>
<td>1.20044</td>
<td>1.19344</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>1.19986</td>
<td>1.20044</td>
<td>1.19344</td>
<td></td>
</tr>
</tbody>
</table>
\(M_1 = 0.05\)
\(M_3 = 0.01\)

1. No Absorber
2. Absorber attached to sprung mass.
3. Absorber attached to unsprung mass.

Fig. 12 Road holding/ride comfort trade-off for optimized conventional and optimized damped suspension systems.
4.1.2 Power Spectral Density

Figures 13-18 display the PSD of the sprung mass acceleration at the constrained optimum case, for specified values of $M_1$ and for $\mu = 0.01$ ($\mu = \frac{M_1}{M_3}$) which is the most practical ratio in applications (14, 24), for the three optimum systems: 1) the conventional, 2) the damped absorber with the absorber attached to the sprung mass, 3) with the absorber attached to the unsprung mass.

From these figures it is observed that when the optimum damped absorber is attached to the unsprung mass, the peak at the secondary resonance is suppressed. On the other hand, when the absorber is attached to the sprung mass, the peak is slightly reduced. In other words, when the absorber is attached to the unsprung mass, it is better tuned and consequently gives better ride comfort at the neighborhood of the secondary resonance.

Also, it can be shown that decreasing $M_1$, in general, tends to flatten the PSD and distribute it over a wider range of frequency, i.e., decreasing $M_1$ for a given constrained comfort, reduces the peaks of vibration but introduces a detrimental effect on higher frequency vibration.
Fig. 13. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\dot{y}_c, x, \text{rms} = 0.6$.
Fig. 14. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{Y}_{c,x,\text{rms}} = 0.6$.
Fig. 15. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, \( \bar{\gamma}, c, x, \text{rms} = 0.6 \).
Fig. 16. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{\gamma}_{c,x,rms} = 0.3$
Fig. 17. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,\text{rms}} = 0.3$
Fig. 18. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,rms} = 0.3$.
4.2 Optimum Parameters

The values of the constrained optimum $j_2^*, j_3^*$, $K_2^*$ and $K_3^*$ which optimize the constrained objective function ($f_q$), for different values of $M_1$ and $M_3$, are plotted against $\ddot{y}_{c'x'} \text{rms}$ and shown in Figures 19-21 for the case where the absorber is attached to the unsprung mass, and in Figures 22-24 for the case where the absorber is attached to the sprung mass.

For the case of the optimum damped absorber with the absorber attached to the unsprung mass (Figures 19-21), it is observed that for higher comfort constraints (smaller $\ddot{y}_{c'x'} \text{rms}$), higher values of $K_3^*$ and smaller values of $K_2^*$ are required. On the other hand, for the optimum damped absorber with the absorber attached to the sprung mass (Figures 22-24), higher comfort constraints require smaller $K_2^*$ as well as $K_3^*$. Moreover, for both cases of optimum damped absorber (Figures 19-24), show that $j_2^*$ and $j_3^*$ play less role than $K_2^*$ and $K_3^*$. Except for the case where the absorber is attached to the sprung mass and for $M_1 = 0.05$, $j_2^*$ and $j_3^*$ change slightly with $\ddot{y}_{c'x'} \text{rms}$. 
Fig 49: Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass).
Fig. 20. Constrained optimum parameter values for the constrained optimum damped absorber (damped absorber attached to unsprung mass).
Fig. 21. Constrained optimum parameter values for the constrained damped absorber (absorber attached to unsprung mass).
Fig. 22. Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass).
Fig. 23. Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass)
Fig. 24 Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass).
CONCLUSION

This study concludes that, in general, optimum suspension system with damped absorber achieves better performances (ride comfort and controllability) than the optimum conventional suspension, and that the place of attachment of the damped absorber is important.

Mounting the damped absorber to the unsprung mass yields better controllability/ride comfort performances than in the case where the damped absorber is mounted to the sprung mass. Also, attaching the damped absorber to the unsprung mass helps suppress vibration at the second resonant frequency. Damped absorber attached to the sprung mass does not, it slightly reduces the peak of vibration.

The approach adopted in this study can be extended to more sophisticated models and inputs such as higher order systems, non linear systems, and systems subjected to simultaneous guideway and external force disturbances.
REFERENCES

1. C.M. Harris and C.E. Crede, Shock and Vibration Handbook, Volume I, II and III.


DECLARE DOUBLE B, DOUBLE BA, DOUBLE BB, DOUBLE DB, DOUBLE DC, & DOUBLE DD, DOUBLE DL
DECLARE DOUBLE DS, DOUBLE DT, DOUBLE GM, DOUBLE GN, DOUBLE GU, & DOUBLE GV, DOUBLE GW, DOUBLE GX, DOUBLE GY, DOUBLE GZ, & DOUBLE H, DOUBLE HA
DECLARE DOUBLE HB, DOUBLE OU, DOUBLE OV, DOUBLE OW, DOUBLE P, & DOUBLE PA, DOUBLE PG, DOUBLE Q, DOUBLE QA, DOUBLE QB, & DOUBLE QD, DOUBLE QE
DECLARE DOUBLE QF, DOUBLE QG, DOUBLE RM, DOUBLE TC, DOUBLE TD, & DOUBLE TE, DOUBLE TF, DOUBLE TG, DOUBLE TH, DOUBLE TI, & DOUBLE TJ, DOUBLE TK, DOUBLE TL
DECLARE DOUBLE TM, DOUBLE TN, DOUBLE TP, DOUBLE TO, DOUBLE W, & DOUBLE WA, DOUBLE WB, DOUBLE WD, DOUBLE WE, DOUBLE WF, & DOUBLE WK
DECLARE DOUBLE X, DOUBLE XA, DOUBLE XB, DOUBLE XD, DOUBLE XE, & DOUBLE XF, DOUBLE XG, DOUBLE XI, DOUBLE X3, DOUBLE YJ, & DOUBLE YM
REM FOR DOUBLE PRECISION, VARIABLES B, D, E, G, H, O-Y SHOULD REM BE DBL PREC.
Z$ = "MODSER - APPLICATION OF P519RE FOLLOWS (9-12-83 VERSION)"
DECLARE DOUBLE OD(25), DOUBLE OE(25), DOUBLE OF(25), DOUBLE OG(25), & DOUBLE OI(25), DOUBLE OK(25), DOUBLE OS(25)
PRINT Z$
GOSUB 1000
GOSUB 8500
Z$ = "INITIAL CALCULATIONS:"
PRINT Z$
PRINT
FOR I = 1 TO NV
CD(I) = (CX(I) - CN(I)) / AL
NEXT I
A = 0.
FOR I = 1 TO NV
A = A + (CX(I) - CN(I)) ^ 2
NEXT I
AD = SQR (A) / AB
FD = AB * AD / AW
FS = FD / 5
FX = FS / AT
FH = AD / 2
80 IF NE = 0 AND NR = 0 THEN 81 ELSE 95
81 FM = 0
82 GOTO 110
95 FM = AW / AE / AB
110 Z$ = "AD,FD,FS,FX,FH,FM="
112 PRINT Z$,AD,FD,FS,FX,FH,FM
114 GOSUB 1015
115 GOSUB 1015
116 FS = 100 * FS
117 FX = 100 * FX
118 ME = 0
125 IF MX > 0 THEN GOSUB 2000
140 K = 1
141 KT = 1
142 KE = 0
143 MF = 1
144 MG = 0
145 MR = 0
146 NN = 0
147 KP = 0
148 FP = 1.
149 DC = 0.0E0
150 KA = 0
151 NX = 0
152 DL = AD
153 IT = 0
155 FOR I = 1 TO NV
156 VB(I) = CS(I)
157 V(I) = VB(I)
158 NEXT I
159 IF MS = 0 THEN 160 ELSE 170
160 GOSUB 9000
161 GOTO 200
170 REM FOR DBL PREC, IN LINE 80, VB(I) SHOULD = CDBL(CS(I))
185 FOR IS = 1 TO MS
186 OK(IS) = OS(IS)
187 OI(IS) = OK(IS)
188 NEXT IS
189 GOSUB 9000
200 IF NU > 0 THEN 201 ELSE 215
201 FOR IU = 1 TO NU
202 UB(IU) = U(IU)
203 NEXT IU
215 IF MS = 0 THEN 245
230 FOR IS = 1 TO MS
231 SK(IS) = S(IS)
232 NEXT IS
245 IF KG > 0 THEN 246 ELSE 260
246 FOR JG = 1 TO KG
247 NB(JG) = N(JG)
248 NEXT JG
260 IF NE > 0 THEN 261 ELSE 275
261 FOR IE = 1 TO NE
262 EB(IE) = E(IE)
263 NEXT IE
275 IF NR > 0 THEN 276 ELSE 290
276 FOR IR = 1 TO NR
277 RB(IR) = R(IR)
278 NEXT IR
290 QB = Q
291 BB = B
292 HB = H
293 WB = W
294 PB = P
295 XB = X
296 GOSUB 1000
297 Z$ = "START OF MODSER SEARCH PROCESS:"
298 PRINT Z$
299 GOSUB 1015
300 Z$ = "BASE POINT PRINTOUTS FOLLOW:"
301 PRINT Z$
302 GOSUB 1030
303 GOSUB 1015
304 IF KT = 1 THEN 323 ELSE 335
305 ON NP GOSUB 8195,8240
306 GOSUB 3000
307 GOSUB 5000
308 FE = FR
309 Z$ = "JT,FB,FE=
310 NEXT I
311 X3=SQR(XI)
312 DB = X3
313 PRINT "DB=";DB
314 REM DB SHOULD BE SINGLE PREC.
315 IF KA = 1 THEN 411 ELSE 425
316 DL = DB
317 KA = 0
318 IF DB > FS THEN 500
319 IF ME = 3 THEN 530
320 IF ME = 2 THEN 456 ELSE 470
321 ME = 3
322 GOTO 530
323 IF ME = 0 THEN 471 ELSE 485
324 ME = 1
325 FX = FX / 10
326 FS = FS / 10
327 GOTO 530
328 IF ME = 1 THEN 486 ELSE 500
329 ME = 2
330 FX = FX / 10
331 FS = FS / 10
332 GOTO 530
333 IF KT >= KX THEN 1075
334 GOSUB 1225
335 K = K + 1
336 KT = KT + 1
337 GOTO 320
338 PRINT "ME=";ME
339 IF CP = 0. THEN 560
340 IF FP < FM THEN 590
341 IF K = 1 AND ME = 3 THEN 680
342 K = 0
343 IF CP = 0. THEN 500
344 FP = AM * FP
345 Z$ = "PENALTY INCREASED AT 590,WITH FP=
346 PRINT Z$;FP
347 GOSUB 1030
348 FOR I = 1 TO NV
349 V(I) = VA(I)
607 NEXT I
608 IF MS > 0 THEN 609 ELSE 620
609 FOR IS = 1 TO MS
610 OI(IS) = OA(IS)
611 NEXT IS
620 GOSUB 9000
621 BA = B
622 HA = H
623 WA = W
624 PA = P
625 XA = X
635 IF NE > 0 THEN 636 ELSE 650
636 FOR IE = 1 TO NE
637 EA(IE) = E(IE)
638 NEXT IE
640 GOSUB 9000
645 KA = 1
646 GOTO 500
650 IF NR > 0 THEN 651 ELSE 665
651 FOR IR = 1 TO NR
652 RA(IR) = R(IR)
653 NEXT IR
655 IF NR > 0 THEN 656 ELSE 770
656 FOR I = 1 TO NV
657 XI = XI + CB(I) * CR(I)
660 NEXT I
665 GOTO 500
666 KE = KE + 1
667 Z$ = "TERMINATION TEST COUNT KE="
668 PRINT Z$;KE
669 GOSUB 1030
675 IF AC <= 1 THEN 726 ELSE 740
676 Z$ = "DIFFERENCE ANTICIPATED, WITH AC="
677 PRINT Z$;AC
678 Z$ = "WENT TO 575 FOR NEXT LOOP"
679 PRINT Z$
680 GOSUB 1030
685 IF DB > FT THEN 756 ELSE 770
686 IF DB > FT THEN 756 ELSE 770
687 Z$ = "READY FOR TERMINATION TEST AT 755, AND FT="
688 PRINT Z$;FT
689 GOSUB 1030
690 Z$ = "TERMINATION TEST FAILED AT 755. WENT TO 755 & FOR NEXT LOOP."
691 PRINT Z$
692 GOSUB 1030
693 GOTO 575
694 Z$ = "CONVERGENCE ANTICIPATED, WITH AC="
695 PRINT Z$;AC
696 FT = (AC - 1) * 6 * FS
697 Z$ = "READY FOR TERMINATION TEST AT 755, AND FT="
698 PRINT Z$;FT
699 GOSUB 1030
700 Z$ = "TERMINATION TEST PASSED AT 755"
701 PRINT Z$
702 GOSUB 1000
703 GOSUB 1000
704 Z$ = "SOLUTION FOUND IS AT (K+1). NOW TRANSFERRED TO & (K) STORAGE FOR PRINTOUT"
705 PRINT Z$
706 PRINT
707 KK = 0
708 GOSUB 1225
709 GOSUB 4000
710 XI = 0.0E0
715 FOR I = 1 TO NV
716 XI = XI + CR(I) * CR(I)
806 NEXT I
807 X3=SQR(XI)
808 GM = X3
809 GOSUB 8240
815 PRINT "GM=";GM
816 GOSUB 1015
830 Z$ = "DOUBLE PRECISION VB(I) FOR I=1,...,NV ARE=
831 PRINT Z$
832 FOR I = 1 TO NV
833 PRINT USING "###.##############", VB(I),
834 NEXT I
835 PRINT
836 IF NU = 0 THEN 860
845 Z$ = "DOUBLE PRECISION UB(IU) FOR IU=1,...,NU ARE ="
846 PRINT Z$
847 FOR IU = 1 TO NU
848 PRINT USING "###.##############", UB(IU),
849 NEXT IU
850 PRINT
860 GOSUB 1015
861 FOR I = 1 TO NV
862 V(I) = VB(I)
863 NEXT I
864 IF MS > 0 THEN 865 ELSE 875
865 FOR IS = 1 TO MS
866 OI(IS) = OK(IS)
867 NEXT IS
875 GOSUB 9000
876 GOSUB 10000
877 PRINT
878 GOSUB 1000
890 Z$ = "MF,MG,MR,NX=
891 PRINT Z$;MF,MG,MR,NX
892 CG(11)=CG(11)+.1
893 IF CG(11) < 1.2 GO TO 35
905 GOTO 32000
995 REM
998 REM ***MINOR SUBROUTINES***
999 REM
1000 FOR I=1 TO 72
1001 PRINT "*";
1002 NEXT I
1003 PRINT
1004 RETURN
1015 FOR I = 1 TO 70
1016 PRINT "-";
1017 NEXT I
1018 PRINT
1019 RETURN
1030 FOR I = 1 TO 25
1031 PRINT "-";
1032 NEXT I
1033 PRINT
1034 RETURN
1035 REM
1036 REM
1045 DIM C(KC),CD(NV),CG(NG),CN(NV),CS(NV),CX(NV),F(NF)
1047 DIM N(KG),NA(KG),NB(KG),NC(KG),NS(KG)
1062 RETURN
1063 REM
1064 REM
1075 Z$ = "KT=KX STOP REVIEW;BASE POINT PRINTOUT:"
1076 GOSUB 1015
1077 GOSUB 1015
1078 PRINT Z$
1079 GOSUB 8240
1080 FOR M = 1 TO 3
1081 PRINT
1082 NEXT M
1083 PRINT "ENTER '1' FOR NEXT LOOP, '2' TO END"
1084 INPUT ZZ$
1085 ON ZZ% GOTO 515,32000
1086 REM
1087 REM
1089 REM **NEW BASE POINT ANALYSIS SUBROUTINE 1105:**
1090 REM
1091 FOR I = 1 TO NV
1092 V(I) = VB(I)
1093 NEXT I
1094 IF MS > 0 THEN GOTO 1109 ELSE 1120
1095 FOR IS = 1 TO MS
1096 OI(IS) = OK(IS)
1097 NEXT IS
1100 GOSUB 9000
1101 QB = Q
1102 BB = B
1103 HB = H
1104 WB = W
1105 PB = P
1106 XB = X
1107 IF NU > 0 THEN GOTO 1128 ELSE 1135
1108 FOR IU = 1 TO NU
1109 UB(IU) = U(IU)
1110 NEXT IU
1111 IF MS > 0 THEN GOTO 1136 ELSE 1150
1112 FOR IS = 1 TO MS
1113 SK(IS) = S(IS)
1114 NEXT IS
1115 GOSUB 9000
1116 QB = QA
1117 BB = BA
1118 HB = HA
1119 WB = WA
1120 PB = PA
1121 DD = DC
1122 DC = DB
1123 FOR I = 1 TO NV
1124 GC(I) = GB(I)
1125 SC(I) = SB(I)
1126 NEXT I
1127 QB = QA
1128 BB = BA
1129 HB = HA
1130 WB = WA
1131 PB = PA
1132 DD = DC
1133 DC = DB
1134 FOR I = 1 TO NV
1135 GC(I) = GB(I)
1136 SC(I) = SB(I)
1137 NEXT I
1138 IF MS > 0 THEN GOTO 1128 ELSE 1135
1139 FOR IU = 1 TO NU
1140 UB(IU) = U(IU)
1141 NEXT IU
1142 IF MS > 0 THEN GOTO 1136 ELSE 1150
1143 FOR IS = 1 TO MS
1144 SK(IS) = S(IS)
1145 NEXT IS
1146 IF MS > 0 THEN GOTO 1109 ELSE 1120
1147 FOR IS = 1 TO MS
1148 OI(IS) = OK(IS)
1149 NEXT IS
1150 IF MS > 0 THEN GOTO 1109 ELSE 1120
1151 FOR IS = 1 TO MS
1152 OI(IS) = OK(IS)
1153 NEXT IS
1154 IF MS > 0 THEN GOTO 1109 ELSE 1120
1155 FOR IS = 1 TO MS
1156 OI(IS) = OK(IS)
1157 NEXT IS
1158 IF MS > 0 THEN GOTO 1109 ELSE 1120
1159 FOR IS = 1 TO MS
1160 OI(IS) = OK(IS)
1161 NEXT IS
1162 IF MS > 0 THEN GOTO 1109 ELSE 1120
1163 FOR IS = 1 TO MS
1164 OI(IS) = OK(IS)
1165 NEXT IS
1166 FOR IS = 1 TO MS
1167 SK(IS) = S(IS)
1168 NEXT IS
1169 IF MS > 0 THEN GOTO 1109 ELSE 1120
1170 FOR IS = 1 TO MS
1171 OI(IS) = OK(IS)
1172 NEXT IS
1173 IF MS > 0 THEN GOTO 1109 ELSE 1120
1174 FOR IS = 1 TO MS
1175 OI(IS) = OK(IS)
1176 NEXT IS
1177 IF MS > 0 THEN GOTO 1109 ELSE 1120
1178 FOR IS = 1 TO MS
1179 OI(IS) = OK(IS)
1180 NEXT IS
1181 FOR IS = 1 TO MS
1182 SK(IS) = S(IS)
1183 NEXT IS
1184 IF MS > 0 THEN GOTO 1109 ELSE 1120
1185 FOR IS = 1 TO MS
1186 OI(IS) = OK(IS)
1187 NEXT IS
1188 IF MS > 0 THEN GOTO 1109 ELSE 1120
1189 FOR IS = 1 TO MS
1190 OI(IS) = OK(IS)
1191 NEXT IS
1192 IF MS > 0 THEN GOTO 1109 ELSE 1120
1193 FOR IS = 1 TO MS
1194 OI(IS) = OK(IS)
1195 NEXT IS
1196 IF MS > 0 THEN GOTO 1109 ELSE 1120
1197 FOR IS = 1 TO MS
1198 OI(IS) = OK(IS)
1199 NEXT IS
1200 RETURN
1201 REM
1202 REM **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1203 REM
1204 DD = DC
1205 DC = DB
1206 FOR I = 1 TO NV
1207 GC(I) = GB(I)
1208 SC(I) = SB(I)
1209 NEXT I
1210 REM
1211 REM **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1212 REM
1213 DD = DC
1214 DC = DB
1215 FOR I = 1 TO NV
1216 GC(I) = GB(I)
1217 SC(I) = SB(I)
1218 NEXT I
1219 RETURN
1220 REM
1221 REM **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1222 REM
1223 DD = DC
1224 DC = DB
1225 FOR I = 1 TO NV
1226 GC(I) = GB(I)
1227 SC(I) = SB(I)
1228 NEXT I
1229 RETURN
1230 REM
1231 REM **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1232 REM
1233 DD = DC
1234 DC = DB
1235 FOR I = 1 TO NV
1236 GC(I) = GB(I)
1237 SC(I) = SB(I)
1238 NEXT I
1239 RETURN
1240 REM **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1241 REM
1242 REM **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1243 REM
1244 PB = PA
XB = XA
FOR I = 1 TO NV
VB(I) = VA(I)
NEXT I
IF MS > 0 THEN GOTO 1251 ELSE 1255
FOR IS = 1 TO MS
OK(IS) = OA(IS)
SK(IS) = SA(IS)
NEXT IS
IF NU > 0 THEN 1256 ELSE 1270
FOR IU = 1 TO NU
UB(IU) = UA(IU)
NEXT IU
IF NE > 0 THEN 1271 ELSE 1300
FOR IE = 1 TO NE
EB(IE) = EA(IE)
NEXT IE
IF NR > 0 THEN GOTO 1286 ELSE 1300
FOR IR = 1 TO NR
RB(IR) = RA(IR)
NEXT IR
RETURN
REM ***SUBROUTINE 2000: SG SEARCH***
KS = 1
XG = 1.E19
CP = AG
FP = 1.
PRINT "SG SEARCH FOLLOWS:"
FOR I = 1 TO NV
RM=RND
V(I) = CN(I) + RM * (CX(I) - CN(I))
NEXT I
GOSUB 9000
PRINT "KS,V(I),P,Q=", KS, V(I), P, Q
PRINT
IF X < XG THEN 2031 ELSE 2045
XG = X
QG = Q
PG = P
FOR I = 1 TO NV
VG(I) = V(I)
NEXT I
IF KS < MN THEN 2075
IF PG = 0 THEN 2090
IF KS < MX THEN 2076 ELSE 2090
KS = KS + 1
GOTO 2014
Z$ = "SG SEARCH GAVE FOLLOWING POINT:"
PRINT
PRINT Z$
Z$ = "QG,PG,XG=", QG, PG, XG
PRINT
PRINT
2105 Z$ = "VG(I) FOR I=1,...,NV ARE=
2106 PRINT Z$
2107 FOR I = 1 TO NV
2108 PRINT VG(I),
2109 NEXT I
2110 PRINT
2115 FOR I = 1 TO NV
2116 CS(I) = VG(I)
2117 NEXT I
2118 RETURN
2998 REM ***SUBROUTINE 3000:SEARCH DIRECTION SB(I),TB(I),
3000 Z$ = "SUBROUTINE 3000 STARTED - FOR SEARCH DIRECTION &
AT BASE POINT (K)"
3001 REM PRINT Z$
3002 GOSUB 4000
3015 IF ND = 1 OR DC = 0. THEN 3135
3030 IF KA > 0 THEN 3135
3045 IF ND = 2 OR KG = 0 THEN 3150
3060 JG = 1
3075 IF NB(JG) <> NC(JG) THEN 3120
3090 IF JG = KG THEN 3150
3105 JG = JG + 1
3106 GOTO 3075
3120 NX = NX + 1
3121 Z$ = "NX,JG=
3122 PRINT Z$,NX,JG
3135 FOR I = 1 TO NV
3136 SB(I) = - GB(I)
3137 NEXT I
3138 ID = 1
3139 GOTO 3195
3150 GX = 0.0E0
3151 GY = GX
3165 FOR I = 1 TO NV
3166 GX = GX + (GB(I) - GC(I)) * GB(I)
3167 GY = GY + (GB(I) - GC(I)) * SC(I)
3168 NEXT I
3169 IF GY = 0. THEN 3170 ELSE 3180
3170 Z$ = "GY=0. AT 3165; GOTO 3135"
3171 PRINT Z$
3172 GOTO 3135
3180 WK = GX / GY
3181 FOR I = 1 TO NV
3182 SB(I) = - GB(I) + WK * SC(I)
3183 NEXT I
3184 ID = 2
3195 PRINT "ID=";ID
3210 DT = 0.0E0
3211 FOR I = 1 TO NV
3212 DT = DT + GB(I) * SB(I)
3213 NEXT I
3214 IF DT < 0. THEN 3270
3225 IF DT = 0. AND ID = 1 THEN 3226 ELSE 3240
3226 Z$ = "STATIONARY POINT FOUND AT 3225,WITH ID,DT="
3227 PRINT Z$;ID,DT
3228 RETURN
3240 IF ID = 2 THEN 3241 ELSE 3255
3241 MR = MR + 1
3242 GOTO 3135
3255 Z$ = "FAILED DESCENT DIRECTION TEST AT 3240,WITH GX="
3256 PRINT Z$;GX
3257 GOTO 32000
XI = 0.0E0
FOR I = 1 TO NV
XI = XI + SB(I) * SB(I)
NEXT I
X3 = SQRT(XI)
FOR I = 1 TO NV
XI = XI + SB(I) / X3
GOTO 3315
TB(I) = 0.0E0
GOTO 3315
NEXT I
X3 = SQRT(XI)
FOR I = 1 TO NV
IF X3 > 0.0E0 THEN 3287 ELSE 3300
TB(I) = SB(I) / X3
GOTO 3315
TB(I) = 0.0E0
GOTO 3315
NEXT I
RETURN
REM ***SUBROUTINE 4000: GRADIENT GB(I) AT BASE POINT
REM VB(I); FOR I = 1 TO NV ***
MG = MG + 1
FOR I = 1 TO NV
V(I) = VB(I)
NEXT I
IF MS = 0 THEN 4030
FOR IS = 1 TO MS
OKIS) = SK(IS)
NEXT IS
FOR I = 1 TO NV
VU(I) = V(I)
V(I) = VB(I) + CD(I)
GOSUB 9000
DP(I) = X
V(I) = VB(I) - CD(I)
GOSUB 9000
DN(I) = X
GB(I) = (DP(I) - DN(I)) / 2 / CD(I)
V(I) = VU(I)
NEXT I
RETURN
REM ***SUBROUTINE 5000: LINE SEARCH FOR (K+1)
REM BASE POINT - VA(I), I = 1 TO NV; AND CP TUNING
REM CALCULATIONS ***
Z$ = "SUBROUTINE 5000 STARTED - LINE SEARCH FOR V AT &
(K+1); AND CP TUNING"
PRINT Z$
IF DT = 0. AND ID = 1 AND KT > 1. AND CP <> 0. THEN &
5016 ELSE 5030
FR = 0.
FB = 0.
J = 0
GOTO 5375
IF KT = 1 THEN 5031 ELSE 5045
FR = AD
GOTO 5105
IF KA > 0 THEN 5046 ELSE 5060
FR = CL * DL
GOTO 5105
IF DC = 0. THEN 5061 ELSE 5075
FR = FS / 2
GOTO 5105
IF DC < AD THEN 5076 ELSE 5090
FR = CA * DC
GOTO 5105
FR = AD
FB = FR
FOR I = 1 TO NV
DV(I) = FR * TB(I)
NEXT I
JT = 0
J = 0
QD = QB
WD = WB
XD = XB
FOR I = 1 TO NV
YD(I) = 0.0E0
VD(I) = VB(I)
NEXT I
IF MS = 0 THEN
FOR IS = 1 TO MS
OG(IS) = SK(IS)
OD(IS) = OK(IS)
SD(IS) = SK(IS)
NEXT IS
QF = QE
WF = WE
XF = XE
FOR I = 1 TO NV
YF(I) = YE(I)
VF(I) = VE(I)
NEXT I
IF MS = 0 THEN
FOR IS = 1 TO MS
OE(IS) = OD(IS)
SE(IS) = SD(IS)
NEXT IS
QE = QD
WE = WD
XE = XD
FOR I = 1 TO NV
YE(I) = YD(I)
VE(I) = VD(I)
NEXT I
IF MS = 0 THEN
FOR IS = 1 TO MS
OD(IS) = OG(IS)
OK(IS) = OD(IS)
NEXT IS
GOSUB 9000
IF MS = 0 THEN
FOR IS = 1 TO MS
SD(IS) = S(IS)
OG(IS) = SD(IS)
NEXT IS
MF = MF + 1
QD = Q
WD = W
XD = X
IF CP = 0. AND WD >= WE AND WD > 0 THEN 5945
5330  NN = 0
5345  IF J >= 2 THEN 5420
5360  IF XD <= XE THEN 5361 ELSE 5375
5361  MC = 1
5362  GOTO 5165
5375  GZ = FX * CF
5376  IF FR > GZ THEN 5405
5377  FOR I = 1 TO NV
5378  VA(I) = VD(I)
5379  V(I) = VA(I)
5380  NEXT I
5381  IF MS = 0 THEN 5555
5390  FOR IS = 1 TO MS
5391  OA(IS) = OD(IS)
5392  OI(IS) = OA(IS)
5393  NEXT IS
5394  GOTO 5555
5405  J = 0
5406  FR = CF * FR
5407  FOR I = 1 TO NV
5408  DV(I) = CF * DV(I)
5409  NEXT I
5410  GOTO 5225
5420  IF XD < XE THEN 5855
5435  GX = XD + XF - 2 * XE
5436  IF GX = 0 THEN 5437 ELSE 5450
5437  DS = FR
5438  GOTO 5465
5450  DS = FR * (XD - XE) / GX
5465  XI = 0.0E0
5466  FOR I = 1 TO NV
5467  XI = XI + YD(I) * YD(I)
5468  NEXT I
5469  X3=SQR(XI)
5470  YJ = X3
5471  YM = YJ - FR / 2 - DS
5480  IF YM < 0.0E0 THEN 5481 ELSE 5495
5481  Z$ = "FAILED TEST AT 5480"
5482  PRINT Z$
5483  GOTO 5795
5495  FOR I = 1 TO NV
5496  VS(I) = YM * TB(I)
5497  VA(I) = VB(I) + VS(I)
5498  V(I) = VA(I)
5499  NEXT I
5500  GN = DS + FR / 2
5501  GU = FR * FR
5502  GV = 2 * FR
5510  IF MS = 0 THEN 5555
5525  FOR IS = 1 TO MS
5540  OU = OD(IS)
5541  OV = OE(IS)
5542  OW = OF(IS)
5543  GOSUB 5570
5544  OI(IS) = GZ
5545  NEXT IS
5555  GOSUB 9000
5556  GOTO 5585
5570  GW = OV - OU
5571  GX = OW - OU
5572  GY = (4 * GW - GX) / GV
5573  GX = (GW - GY * FR) / GU
5574  GW = GN * (GX * GN + GY)
IF NU > 0 THEN 5601 ELSE 5615
5601 FOR IU = 1 TO NU
5602 UA(IU) = U(IU)
5603 NEXT IU
5615 IF MS = 0 THEN 5645
5630 FOR IS = 1 TO MS
5631 OA(IS) = OI(IS)
5634 SA(IS) = S(IS)
5637 NEXT IS
5645 IF NE > 0 THEN 5647 ELSE 5660
5647 FOR IE = 1 TO NE
5648 EA(IE) = E(IE)
5651 NEXT IE
5660 IF NR > 0 THEN 5661 ELSE 5675
5661 FOR IR = 1 TO NR
5663 RA(IR) = R(IR)
5666 NEXT IR
5675 IF KG > 0 THEN 5676 ELSE 5690
5676 FOR JG = 1 TO KG
5678 NA(JG) = N(JG)
5681 NEXT JG
5690 IF XA > XB THEN 5720
5705 RETURN
5720 Z$ = "FAILED FUNCTION DECREASE TEST AT 5690"
5723 PRINT Z$
5735 GZ = FX * CF
5738 IF FR > GZ THEN 5739 ELSE 5750
5739 PRINT "WENT TO 5825 AND 5405"
5741 GOTO 5825
5750 RETURN
5765 GZ = FX * CF
5768 IF FR > GZ THEN 5769 ELSE 5780
5769 PRINT "WENT TO 5823 FROM 5765"
5771 GOTO 5825
5780 IF ABS(YM) > GZ THEN 5781 ELSE 5795
5781 PRINT "WENT TO 5825 FROM 5780"
5783 GOTO 5825
5795 FOR I = 1 TO NV
5799 VA(I) = VB(I)
5802 V(I) = VA(I)
5805 NEXT I
5807 IF MS = 0 THEN 5555
5810 FOR IS = 1 TO MS
5813 OA(IS) = OK(IS)
5816 OI(IS) = OA(IS)
5819 NEXT IS
5820 GOTO 5555
5825 QE = QF
5826 WE = WF
5827 XE = XF
5828 FOR I = 1 TO NV
5829 YE(I) = YF(I)
5830 VE(I) = VF(I)
5831 NEXT I
IF MS > 0 THEN 5833 ELSE 5840
5833 FOR IS = 1 TO MS
5834 OE(IS) = OF(IS)
5835 SE(IS) = SF(IS)
5836 NEXT IS
5840 GOTO 5405
5855 IF J > JX THEN 5856 ELSE 5870
5856 Z$ = "J>\text{JX AT } 5855; \text{STOP REVIEW}"
5857 GOSUB 1000
5858 PRINT Z$
5859 GOSUB 8240
5860 GOTO 32000
5870 IF FR > FH THEN 5165
5885 IF MC < 1 THEN 5886 ELSE 5900
5886 MC = 1
5887 GOTO 5165
5890 FR = 2 * FR
5891 QE = QF
5892 WE = WF
5893 XE = XF
5894 FOR I = 1 TO NV
5895 DV(I) = 2 * DV(I)
5896 YE(I) = YF(I)
5897 VE(I) = VF(I)
5898 NEXT I
5899 IF MS = 0 THEN 5910 ELSE 5915
5900 MC = 0
5911 GOTO 5930
5915 FOR IS = 1 TO MS
5916 OE(IS) = OF(IS)
5917 SE(IS) = SF(IS)
5918 NEXT IS
5919 MC = 0
5930 GOTO 5165
5945 IF KP > 0 THEN 5975
5960 IF QD > QE THEN 5961 ELSE 5975
5961 NN = 0
5962 Z$ = "IN PENALTY ZONE AT 5945 WITH WD>WE, BUT NO & TUNE SINCE QD>QE"
5963 PRINT Z$
5964 GOTO 5345
5970 IF NN = 0 THEN 6035 ELSE 5990
5976 REM ***AT FIRST PROBE EDGE OF PENALTY ZONE***
5990 IF KP = 0 THEN 5991 ELSE 6005
5991 KP = 1
5992 GOTO 5165
6000 FR = 2
6006 GOTO 6095
6007 REM ***CP CALCULATION***
6020 REM **SEGMENT 6035: FOR CP SELF-TUNING PROCESS**
6035 Z$ = "START SEGMENT 6035 - AT EDGE OF PENALTY & ZONE WITH WD>WE AND QD<QE"
6036 PRINT Z$
6050 FOR I = 1 TO NV
6051 VB(I) = VE(I)
6052 NEXT I
6053 IF MS = 0 THEN 6080
6060 FOR IS = 1 TO MS
6066 OK(IS) = OE(IS)
6067 NEXT IS
6080 GOSUB 1105
6081 NT = ND
6082 ND = 1
GOSUB 3000
FB = FR
NN = 1
KP = 0
ND = NT
GOTO 5120
Z$ = "READY FOR CP CALCULATION AT 6095"
PRINT Z$
IF WD <> WF THEN CP = .75 *(QF - QD) / (WD - WF)
Z$ = "AT (J-2), AND KP=0: QF,WF=
PRINT Z$;QF,WF
Z$ = "AT (J-1), AND KP=1:QE,WE=
PRINT Z$;QE,WE
Z$ = "AT (J), AND KP=2:QD,WD=
PRINT Z$;QD,WD
Z$ = "REVIEW CALCULATED CP=
PRINT Z$;CP
FOR M = 1 TO 3
PRINT
NEXT M
PRINT "ENTER '1' TO CHANGE CP OR '2' TO RETAIN CP"
ON ZZ% GOTO 6200, 6215
INPUT "ENTER NEW CP="; CP
REM IF CP < 0. THEN CP=(QD-QF)/(WE-WF)
Z$ = "FOR USE IN SEARCH TO FOLLOW, CP=
PRINT Z$; CP
FOR I = 1 TO NV
VB(I) = VE(I)
NEXT I
IF MS = 0 THEN 6249 ELSE 6260
GOSUB 1105
GOTO 6275
FOR IS = 1 TO MS
OK(IS) = OE(IS)
NEXT IS
GOSUB 1105
Z$ = "RETURNING TO LINE SEARCH WITH TUNED CP AND NEW BASE POINT:
PRINT Z$
GOSUB 1015
GOSUB 8240
NT = ND
ND = 1
GOSUB 3000
FR = AD
NN = 2
ND = NT
FB = FR
GOTO 5120
REM *END OF CP SELF-TUNING PROCESS*
REM *** SUBROUTINE 8000: PRINTOUT ***
REM * INPUT DATA PRINTOUT *
Z$ = "AB,AE,AG,AL,AM,AW,AT,CA,CL,CF,JX,KX="
PRINT Z$
PRINT AB,AE,AG,AL,AM,AW,AT,CA,CL,CF,JX,KX
Z$ = "KC,KG,MD,MN,MS,MX,ND,NE,NF,NG,NP,NR,NU,NV=
PRINT Z$
PRINT KC,KG,MD,MN,MS,MX,ND,NE,NF,NG,NP,NR,NU,NV
Z$ = "CN(I) FOR I=1,...,NV ARE=
PRINT Z$
PRINT 8031
8033 PRINT CN(I),
8034 NEXT I
8036 PRINT
8045 Z$ = "CX(I) FOR I=1,....,NV ARE=
8046 PRINT Z$
8047 FOR I = 1 TO NV
8048 PRINT CX(I),
8049 NEXT I
8050 PRINT
8060 IF NG = 0 THEN 8090 ELSE 8075
8075 Z$ = "CG(IG) FOR IG=1,....,NG ARE=
8076 PRINT Z$
8077 FOR IG = 1 TO NG
8078 PRINT CG(IG),
8079 NEXT IG
8080 PRINT
8090 IF MX > 0 THEN 8150 ELSE 8105
8105 Z$ = "SPECIFIED CS(I) FOR I=1,....,NV ARE=
8106 PRINT Z$
8107 FOR I = 1 TO NV
8108 PRINT CS(I),
8109 NEXT I
8110 PRINT
8120 IF MS = 0 THEN 8150 ELSE 8135
8135 Z$ = "SPECIFIED OS(IS) FOR IS=1,....,MS ARE="
8136 PRINT Z$
8137 FOR IS = 1 TO MS
8138 PRINT OS(IS),
8139 NEXT IS
8140 PRINT
8150 IF KC = 0 THEN 8210 ELSE 8165
8165 Z$ = "CALCULATED CONSTANTS C(IC) FOR IC=1,....,KC ARE="
8166 PRINT Z$
8167 FOR IC = 1 TO KC
8168 PRINT C(IC),
8169 NEXT IC
8170 PRINT
8171 GOTO 8210
8180 REM *NP=1, MINIMAL BASE POINT PRINTOUT OPTION*
8195 Z$ = "KT,VB(1),XB,QB=
8196 PRINT
8197 PRINT Z$
8198 PRINT KT,VB(1),XB,QB
8199 PRINT
8210 GOSUB 1015
8211 GOSUB 1030
8212 RETURN
8225 REM *NP=2, FULL BASE POINT PRINTOUT OPTION*
8240 Z$ = "KT,K,VB,QB=
8241 PRINT
8242 PRINT Z$
8243 PRINT KT,K,VB,QB
8244 PRINT
8255 Z$ = "VB(I) FOR I=1,....,NV ARE=
8256 PRINT Z$
8257 FOR I = 1 TO NV
8258 PRINT VB(I),
8259 NEXT I
8260 PRINT
8270 IF MS = 0 THEN 8315
8285 Z$ = "OK(IS) FOR IS=1,....,MS ARE=
8286 PRINT Z$
8287 FOR IS = 1 TO MS

8288 NEXT IS
8288 PRINT OK(IS),
8289 NEXT IS
8290 PRINT
8300 Z$ = "SK(IS) FOR IS=1,...,MS ARE=
8301 PRINT Z$
8302 FOR IS = 1 TO MS
8303 PRINT SK(IS),
8304 NEXT IS
8305 PRINT
8315 IF NU > 0 THEN 8316 ELSE 8330
8316 Z$ = "UB(IU) FOR IU=1,...,NU ARE=
8317 PRINT Z$
8318 FOR IU = 1 TO NU
8319 PRINT UB(IU),
8320 NEXT IU
8321 PRINT
8330 IF NE > 0 THEN 8331 ELSE 8345
8331 Z$ = "EB(IE) FOR IE=1,...,NE ARE=
8332 PRINT Z$
8333 FOR IE = 1 TO NE
8334 PRINT EB(IE),
8335 NEXT IE
8336 PRINT
8345 IF NR > 0 THEN 8346 ELSE 8360
8346 Z$ = "RB(IR) FOR IR=1,...,NR ARE=
8347 PRINT Z$
8348 FOR IR = 1 TO NR
8349 PRINT RB(IR),
8350 NEXT IR
8351 PRINT
8360 IF KG > 0 THEN 8361 ELSE 8390
8361 Z$ = "NB(JG) FOR JG=1,...,KG ARE=
8362 PRINT Z$
8363 FOR JG = 1 TO KG
8364 PRINT NB(JG),
8365 NEXT JG
8366 PRINT
8370 GOSUB 1015
8371 GOSUB 1030
8372 RETURN
8496 REM ***SUBROUTINE 8500:INPUT DATA AND CALCULATED CONSTANTS***
8498 REM **INACTIVATE ANY 'DATA' NOT IN USE, BY DATA**
8500 Z$ = "INPUT DATA:"
8501 PRINT Z$
8502 PRINT
8505 AP = 3.141593
8506 AR = 57.29578
8507 CP = 0.
8508 KP = 0
8510 READ AB,AE,AG,AL,AM,AW,AT,CA,CL,CF,JX,KX
8512 DATA 10.,.5,1.E03,1.E05,8.,2.E04,2.,.85,.25.,.25,100,300
8520 READ KC,KG,MD,MN,MS,MX,ND,NE,NF,NG,NP,NR,NU,NV
8529 GOSUB 1045
8530 REM FOR ARRAY DIMENSIONING
8531 FOR I = 1 TO NV
8532 READ CN(I)
8533 NEXT I
8540 FOR I = 1 TO NV
8541 READ CX(I)
8542 NEXT I
8549 IF NG = 0 THEN 8554
8550 FOR IG = 1 TO NG
8551 READ CG(IG)
8552 NEXT IG
8553 REM DATA
8554 IF MX > 0 THEN 8560
8555 FOR I = 1 TO NV
8556 READ CS(I)
8557 NEXT I
8558 REM **SPECIFIED START POINT DATA (CASE OF NO SG)**
8559 IF MS = 0 THEN 8565
8560 FOR IS = 1 TO MS
8561 READ OS(IS)
8562 NEXT IS
8563 NEXT IS
8564 REM DATA
8565 IF KC = 0 THEN 8985
8566 REM ***HERE USER PROGRAMS EQUATIONS FOR CALCULATED
8567 REM CONSTANTS C(IC), FOR IC=1 TO KC***
8568 REM *** SEE SUBROUTINE 18570 LINES 18571 - 18998 ARE AVAILABLE *
8569 REM **
8570 REM *** USE L COUNTER IF NECESSARY, INSTEAD OF I ***
8571 REM ** SEE SUBROUTINE 19000 LINES 19001 - 19998 ARE AVAILABLE *
8572 REM *
8573 GOSUB 18570
8574 GOSUB 8000
8575 RETURN
8592 REM ***SUBROUTINE 9000:ANALYSIS AT GIVEN V POINT***
8594 REM **HERE USER PROGRAMS EQUATIONS FOR: (1) U(IU), FOR
8595 REM IU=1 TO NU; (2) N(JG), FOR JG=1 TO KG; (3) Q;
8596 REM (4) E(IE), FOR IE=1 TO NE; (5) R(IR), FOR IR=1 TO NR**
8597 REM * USE MODSER REFORMULATION STRATEGY FOR THESE
8598 REM EQUATIONS AND DECISION-MAKING LOGIC PROGRAMMED HERE *
8599 REM ** USE L COUNTER IF NECESSARY, INSTEAD OF I *
8600 REM ** SEE SUBROUTINE 19000 LINES 19001 - 19998 ARE AVAILABLE *
8601 REM *
8602 GOSUB 19000
8603 REM *
8604 REM *** DO NOT ALTER LINES 9950 - 9993
8605 B = 0.
8606 H = 0.
8607 IF NE = 0 THEN 9970
8608 FOR IE = 1 TO NE
8609 B = B + E(IE) * E(IE)
8610 NEXT IE
8611 IF NR = 0 THEN 9990
8612 FOR IR = 1 TO NR
8613 IF R(IR) < 0. THEN H = H + R(IR) * R(IR)
8614 NEXT IR
8615 W = B + H
8616 P = CP * FP * W
8617 X = Q + P
8618 RETURN
8619 REM ***SUBROUTINE 10000:FINAL ITEMS ***
8620 REM ** HERE USER PROGRAMS FINAL ITEMS OF INTEREST,
8621 REM BY F(JF) EQUATIONS, FOR JF=1 TO NF **
8622 REM** SEE SUBROUTINE 20010 LINES 20011 - 29999 ARE AVAILABLE *
8623 IF NF = 0 THEN RETURN
8624 GOSUB 1015
8625 ZF$ = "FINAL ITEMS F(JF) FOR JF=1,...,NF ARE="
8626 PRINT ZF$
8627 REM ***
8628 REM **
8629 REM *
10010 GOSUB 20010
10989 REM *
10990 REM **
10991 REM ***
10992 FOR JF = 1 TO NF
10993 PRINT F(JF),
10994 NEXT JF
10995 RETURN
APPENDIX II
PROGRAM : Subroutine for the car suspension system without damped absorber.

WRITTEN BY : Rifaquat Cheema

DATA 2.,0.,0.,0.,0.,0.,2.,0.,5.,6,1.,5,0.,2
DATA 0.001,0.001
DATA 1.,1.
DATA 0.05,1,1.,1,.,1,.,.001
DATA 0.05,.5
DATA
RETURN
DATA
C(1)=CG(3)/CG(4)
C(2)=CG(5)/CG(6)
RETURN
DATA
A3=1.
A2=2.*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))*(1.+1./CG(1))
A1=CG(3)*V(1)+CG(3)*V(1)/CG(1)
A0=0.
D4=1.0
D3=2.*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))*(1.+1./CG(1))
D2=CG(3)*V(1)+1./CG(1)+CG(3)*V(1)/CG(1)
D1=2.*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))/CG(1)
D0=CG(3)*V(1)/CG(1)
B3=0.
B2=2*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))/CG(1)
B1=CG(3)*V(1)/CG(1)
B0=0.
A16=((A0**2/D0)*(D2*D3-D1*D4)+D3*(A1**2-2*A0*A2)+&
D1*(A2**2-2*A1*A3)+(A3**2/D4)*(D1*D2-D0*D3))/(&
D1*(D2*D3-D1*D4)-D0*D3**2)
B16=((B0**2/D0)*(D2*D3-D1*D4)&
+D3*(B1**2-2*B0*B2)+D1*(B2**2-2*B1*B3)+&
(B3**2/D4)*(D1*D2-D0*D3))/(D1*(D2*D3-D1*D4)-&
D0*D3**2)
ACC=SQR(AP*ABS(B16))
Q=SQR(AP*ABS(A16))
R(1)=1.-V(1)
R(2)=V(1)*C(1)-1.
R(3)=1.-V(2)
R(4)=V(2)*C(2)-1.
R(5)=1.-ACC/CG(2)
RETURN
F(1)=Q
F(2)=CG(3)*V(1)
F(3)=CG(5)*V(2)
F(4)=CG(1)
F(5)=CG(2)
RETURN
END
Subroutine for the car suspension system when the damped absorber is attached to the sprung mass.

Written by: Rifaqut Cheema

```
18522 DATA 4.,0.,0.,0.,2.,8.,11.,2.,9.,0.,4.
18532 DATA 0.001,0.001,0.001
18542 DATA 1.,1.,1.,1.
18552 DATA 0.05,0.01,0.001,1.,0.01,1.,0.01,1.,0.01,1.,0.01,3
18562 DATA 0.05,0.05,0.5,0.05
18564 REM DATA
18565 RETURN
18570 DATA
18571 C(1)=CG(3)/CG(4)
18572 C(2)=CG(5)/CG(6)
18573 C(3)=CG(7)/CG(8)
18574 C(4)=CG(9)/CG(10)
18575 RETURN
19000 DATA
19001 A5=1.
19002 A4=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))/(1.+1./CG(1))&
2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))/(1.+1./CG(2))&
A3=4.*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))/&
SQR(ABS(CG(5)*V(2)))/(1.+1./CG(1))&
A2=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(5)*V(2)&
A1=CG(3)*V(1)+CG(5)*V(2)/(1.+1./CG(1))&
18008 D5=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))/(1.+1./CG(1))&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)&
(1.+1./CG(1))&
18009 D4=1./CG(1)+CG(3)*V(1)/(1.+1./CG(1))&
CG(5)*V(2)*CG(7)*V(3)&
A9=4.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(CG(5)*V(2)))/&
SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)&
18012 D2=CG(3)*V(1)*CG(5)*V(2)/(1.+1./CG(1))&
CG(7)*V(3)&
A7=4.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(CG(5)*V(2)))*&
SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)&
18011 D1=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)&
2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*CG(5)*V(2)&
A6=0.
18007 D6=1.0
18007 A0=0.
18007 P6=D0*(D1*Q5-D3*Q4+D5*Q3)
```

\[ A_0 = 1 - A_1 A_5 + (A_4 A_5 - A_5 A_3) + 2 A_3 A_2 - 2 \cdot A_2 A_4 + \ldots \]

\[ B_5 = 0. \]

\[ B_4 = 2 \cdot C(7) \cdot V(3) \cdot SQR(ABS(C(3) \cdot V(1))) / C(1) \]

\[ B_3 = 4 \cdot C(7) \cdot V(3) \cdot C(9) \cdot V(4) \cdot SQR(ABS(C(3) \cdot V(1))) \cdot SQR(ABS(C(5) \cdot V(2))) / (C(1) \cdot C(2)) + C(3) \cdot V(1) / C(1) \]

\[ B_2 = (2 \cdot C(7) \cdot V(3) \cdot C(5) \cdot V(2) \cdot SQR(ABS(C(3) \cdot V(1)))) + (C(9) \cdot V(4) \cdot C(3) \cdot V(1) \cdot SQR(ABS(C(5) \cdot V(2)))) / (C(1) \cdot C(2)) \]

\[ B_1 = (C(3) \cdot V(1) \cdot C(5) \cdot V(2)) / (C(1) \cdot C(2)) \]

\[ B_0 = 0. \]

\[ B_{16} = 1. / (2 \cdot P6) \cdot (B_5 \cdot 2 \cdot Q0 + Q1 \cdot (B_4 \cdot 2 - 2 \cdot B_3 \cdot B_5) + (B_3 \cdot 2 - 2 \cdot B_2 \cdot B_4 + (B_2 \cdot B_5) \cdot Q2 + Q3 \cdot (B_2 \cdot 2 - 2 \cdot B_1 \cdot B_3 + 2 \cdot B_0 \cdot B_4) + Q4 \cdot (B_1 \cdot 2 - 2 \cdot B_0 \cdot B_2) + B_0 \cdot 2 \cdot Q5) \]

\[ ACC = SQR(2 \cdot A \cdot P \cdot A \cdot ABS(B_{16})) \]

\[ Q = SQR(2 \cdot A \cdot P \cdot A \cdot ABS(A_{16})) \]

\[ R(1) = 1 - V(1) \]

\[ R(2) = V(1) \cdot C(1) - 1. \]

\[ R(3) = 1 - V(2) \]

\[ R(4) = V(2) \cdot C(2) - 1. \]

\[ R(5) = 1 - V(3) \]

\[ R(6) = V(3) \cdot C(3) - 1. \]

\[ R(7) = 1 - V(4) \]

\[ R(8) = V(4) \cdot C(4) - 1. \]

\[ R(9) = 1 - ACC / C(11) \]

\[ R(10) = Q \]

\[ F(1) = C(3) \cdot V(1) \]

\[ F(2) = C(5) \cdot V(2) \]

\[ F(3) = C(7) \cdot V(3) \]

\[ F(4) = C(9) \cdot V(4) \]

\[ F(5) = C(11) \]

\[ F(6) = C(1) \]

\[ F(7) = C(2) \]

\[ F(8) = C(11) \]

\[ F(9) = Q \]

\[ 30000 \]

\[ 32000 \]

\[ END \]
PROGRAM : Subroutine for the car suspension system when the damped absorber is attached to unsprung mass.

WRITTEN BY : Rifaquat Cheema

DATA 4.,0.,0.,0.,0.,0.,2.,0.,8.,11.,1.,9.,0.,4.
DATA 0.001,0.001,0.001,0.001
DATA 1.1.,1.1.
DATA 0.05,0.01,1.,0.001,1.,0.001,1.,0.001,3
DATA 0.05,0.05,0.5,0.05
REM DATA
RETURN
DATA
C(1)=CG(3)/CG(4)
C(2)=CG(5)/CG(6)
C(3)=CG(7)/CG(8)
C(4)=CG(9)/CG(10)
RETURN
DATA
D0=D0*(D1*Q5-D3*Q4+D5*Q3)
Q0=1
Q4=l
D2=CG(3)*V(1)*CG(5)*V(2)*(1/CG(1)+1/CG(2)+1/(CG(1)*CG(2)))
A5=1.
A5**2*Q0+Q1*(A4**2-2.*A3*A5)+(A3**2-2.*A2*A4+&
1/CG(1)+1/CG(2)+1/(CG(1)*CG(2)))&
D3=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(CG(5)*V(2)))*&
(1./CG(1)+1/CG(2)+1/(CG(1)*CG(2)))&
D4=1./CG(1)+CG(3)*V(1)*(1.+1./CG(1))+CG(5)*V(2)*&
(1./CG(1)+1./CG(2))+4.*CG(7)*V(3)*&
D5=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)*&
1./CG(1)+1/CG(2)+1/(CG(1)*CG(2)))&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)*&
(1./CG(1)+1/CG(2)+1/(CG(1)*CG(2)))&
A1=CG(3)*V(1)*CG(5)*V(2)+(1/CG(1)+1/CG(2)+1/(CG(2)*CG(1))&
A0=0.0
D6=1.0
D8=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*1.+1./CG(1))&
D9=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*1./CG(1)+&
D0=D0*(D1*Q5-D3*Q4+D5*Q3)
Q1=-(-D0*D1*D5+D0*D3**2-D1*D2*D3+D1**2*D4
Q2=D0*D3*D5+(D1**2)*D6-D1*D2*D5
Q3=D0*D5**2+D1*D3*D6-D1*D4*D5
Q4=1./D0*(D2*D3-D4*D2+D6*D0)
Q5=1./D0*(D2*D4-D3*D3+D6*D2)
Q6=1./D6*(D4*Q1-D2*D2+D0*Q3)
P6=D0*(D1*Q5-D3*Q4+D5*Q3)
A6=1./.(2.*P6)**(A5**2-2.*Q0+Q1*(A4**2-2.*A3*A5)+(A3**2-2.*A2*A4+&
\[2 \times (A1 \times A5) \times Q2 + Q3 \times (A2 \times 2 - 2 \times A1 \times A3 + 2 \times A0 \times A4) + Q4 \times (A1 \times 2 - 2 \times A0 \times A2) \& + A0 \times 2 \times Q5)\]

\[B5 = 0.\]

\[B4 = 2 \times CG(7) \times V(3) \times SQR(ABS(CG(3) \times V(1))) / CG(1)\]

\[B3 = 5 \times CG(7) \times V(3) \times CG(9) \times V(4) \times SQR(ABS(CG(3) \times V(1))) \& SQR(ABS(CG(5) \times V(2))) / (CG(1) \times CG(2)) + CG(3) \times V(1) / CG(1)\]

\[B2 = (2 \times CG(7) \times V(3) \times CG(5) \times V(2) \times SQR(ABS(CG(3) \times V(1))) + 2 \times (CG(9) \times V(4) \times CG(3) \times V(1) \times SQR(ABS(CG(5) \times V(2)))) / & (CG(1) \times CG(2))\]

\[B1 = (CG(3) \times V(1) \times CG(5) \times V(2)) / (CG(1) \times CG(2))\]

\[B0 = 0.\]

\[B16 = 1 / (2 \times P6) \times (B5 \times 2 \times Q0 + Q1 \times (B4 \times 2 - 2 \times B3 \times B5) + (B3 \times 2 - 2 \times B2 \times B4) + & 2 \times B1 \times B5) \times Q2 + Q3 \times (B2 \times 2 - 2 \times B1 \times B3 + 2 \times B0 \times B4) + Q4 \times (B1 \times 2 - 2 \times B0 \times B2) \& + B0 \times 2 \times Q5)\]

\[ACC = SQR(2 \times AP \times ABS(B16))\]

\[Q = SQR(2 \times AP \times ABS(A16))\]

\[R(1) = 1 - V(1)\]

\[R(2) = V(1) \times C(1) - 1.\]

\[R(3) = 1 - V(2)\]

\[R(4) = V(2) \times C(2) - 1.\]

\[R(5) = 1 - V(3)\]

\[R(6) = V(3) \times C(3) - 1.\]

\[R(7) = 1 - V(4)\]

\[R(8) = V(4) \times C(4) - 1.\]

\[R(9) = 1 - ACC / CG(11)\]

\[RETURN\]

\[F(1) = Q\]

\[F(2) = CG(3) \times V(1)\]

\[F(3) = CG(5) \times V(2)\]

\[F(4) = CG(7) \times V(3)\]

\[F(5) = CG(9) \times V(4)\]

\[F(6) = CG(1)\]

\[F(7) = CG(2)\]

\[F(8) = CG(11)\]

\[RETURN\]

\[END\]