Characterization and sensitivity analysis of hyperelastic materials in biaxial tension

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CHARACTERIZATION AND SENSITIVITY ANALYSIS OF HYPERELASTIC MATERIALS IN BIAXIAL TENSION

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Submitted in partial fulfillment of the requirement for the

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In
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Abstract

The focus of this study was to improve characterization of hyperelastic materials in biaxial tension through improved design and validation of an existing test fixture and specimen geometry. Additionally, a sensitivity analysis of the material properties to variations in selected test parameters was conducted to better understand material response.

Misalignment and binding in the original tensile test fixture resulted in non-equibiaxial loading and inaccurate stress-strain data. Analysis and modification were required to improve accuracy and repeatability. Vector analysis of the link system and the Minimum Constraint Design method were used to achieve this aim. Based on a proposed set of criteria, a FE analysis was conducted on several biaxial specimen geometries to determine the best shape and scale for obtaining stress and strain data. A stress decay factor (SDF) is proposed to predict internal stresses from measurable data. A test method has been designed around the use of the SDF and was ultimately applied to a cruciform specimen geometry. In addition to the ideal equibiaxial case, numerical simulations have been perturbed in two ways. The first variation involved a specimen gripped clamps offset by up to half the width of the clamp. The second variation involved non-equibiaxial load ratios ranging from 0.85 to 1.15. The goal was to quantify the change in stress-strain response to slight deviations from ideal loading conditions.

Binding has been eliminated from the test fixture and a 1:1 load ratio has been achieved. The new specimen experiences less stress decay while achieving greater experimental strain. A high sensitivity to non-equibiaxial load ratios and low sensitivity to clamp offset are seen in the test parameter analysis. Finally, results from the SDF correction material characterization method is compared with results from an inflated boiling flask geometry.
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1. Background

1.1 Motivation

The study of how aerosols flow through the airways and within the alveoli is applicable to many research areas such as particle deposition during smoking or inhaled medication dispensing. Current models of the lung do not include accurate mechanical properties of lung tissue. This is because pressure-volume (PV) behavior and elasticity of the lung are well defined but have not been correlated to typical engineering material properties at the tissue level. Current models use linear elastic behavior to represent the lung walls. Individual alveoli are too small to reasonably examine on a true-scale level. Therefore, tissue-level mechanical properties are not easily analyzed.

1.2 Existing Research

Previous research done in this area was completed in 2009 by Joseph Ferrara. He developed the test fixture used and analyzed in this current study as well as a method for characterizing the biaxial response of hyperelastic materials. Physical specimens and finite element models were developed and tested for both planar biaxial and inflatable spherical membrane (i.e., boiling flask) geometries. Labview code was developed and used in conjunction with load cells and a camera to collect and measure load and displacement data. A 2-parameter Mooney-Rivlin constitutive equation was also fit to the experimental biaxial data.

The research resulted in the use of material properties estimated by a planar biaxial tension test in a finite element simulation of an inflatable sphere. The result was an improvement over a prior model approximating the lung as linear elastic, but the inflating sphere deformation was still predicted to be much stiffer than experimental results showed.
1.3 Original Goals & Challenges

The original goal of this research was to identify elastomers to simulate the mechanical behavior of healthy and diseased human lungs. Elastomeric materials were to be selected for this goal by characterizing their response to equibiaxial tension. Upon selection, an inflatable bulb geometry representing the alveolar sac of a human lung would be developed. This model would be designed to inflate and deflate along the same pressure-volume curves of healthy and emphysematous human lungs. Additionally, a simulated FE model of the geometry with representative material properties was to be developed for verification purposes. Success of the simple geometry would have proven the concept and led the way to more complex geometries on a true-scaled level for any degree of emphysema.

Challenges in the testing phase affected the scope of this original plan. Preliminary work focused on obtaining improved estimates of biaxially loaded material response from the FE model and comparative physical test specimen. This was to be achieved by adjusting the specimen size and shape as well as the biaxial tensile test fixture itself through minor mechanical and process modifications. Previous studies (Sacks et al. 2005) have shown that the central region of a clamped cruciform specimen (CCS) was a more accurate representation of biaxial loading than the central region of a clamped square specimen (CSS). Improvement in experimental data acquisition was also achieved through the addition of an automated Labview® program (National Instruments 2009). Mechanical modifications to the biaxial fixture, however, proved more difficult. It was clear that the test fixture’s design and manufacturing issues affected its motion and therefore the test results, warranting the need for a more in depth study to better understand why these issues are caused, how they affect experimental data, and how they may be corrected.
1.4 Revised Goals

Significant improvements to the geometry and process of testing hyperelastic materials in biaxial tension were achieved: specimens with larger regions of uniform biaxial tension as well as more efficient data acquisition programs mean that more accurate results can be obtained. However, this test fixture still exhibited dynamic issues that decreased the accuracy of the data.

As a result, the scope of this thesis has been modified to achieve the following goals:

1. Biaxial specimen design (Chapter 3)
2. Sensitivity analysis of material response to test parameter variation (Chapter 3)
3. Quantification of test fixture misalignment (Chapter 4)
4. Redesign of the test fixture (Chapter 5)
5. Material characterization and validation (Chapter 7)
2. Preliminary Research & Experimentation

2.1 Index of Equation Terms

a, b, c, d, e – Constants

B – Volume difference between A and volume at zero recoil pressure

FRC – Functional residual capacity

$K_1, K_2, K_3$ – Shape constants

P – Recoil pressure

$P_{st}$ or $P_{st}(L)$ – Static lung recoil pressure

K or $dV/dP$ – Index of elasticity, Compliance

$K'$ – Constant

REEP – Resting end expiratory pressure

V – Lung volume

$V_{max}, V_0, V_o'$ or A – Theoretical lung volume at infinite pressure, Max pulmonary lung volume

$V_M$ – Upper asymptote

$V_m$ – Lower asymptote
2.2 Literature Review

2.2.1 Fitting of the Pressure-Volume Curve

The motivation for pressure-volume (PV) curve research is that, in order to accurately understand particle motion and distribution with the lung parenchyma, representative models of the lung must be created. It is necessary that these models exhibit the same dynamic response as healthy and diseased lungs. Understanding the equations used to model PV curves is key to developing relationships between model material properties, equation parameters and physiological properties. These relationships may include the ability to use physiological properties of the lung such as degree of emphysema to define strain in the lung wall and even stress, potentially resulting in a stress-strain curve estimation.

Ting and Lyons (1961) studied primarily the linear portion of the PV curve, resulting in a single number for compliance, \( \frac{dV}{dP} \). There was no mention of fitting an equation to the PV curve. Salazar and Knowles (1964) introduced the potential for using an exponential function to describe the upper section of the curve (above the functional residual capacity, or FRC):

\[
V = V_o(1 - e^{-K^p})
\]  

(2.1)

Equation (2.1) is the solution to the differential equation \( \frac{dV}{dP} = K(V_o - V) \) where \( V = 0 \) at \( P = 0 \). In both equations, \( V_o \) represents the volume at which the slope is zero (infinite pressure). Salazar and Knowles claim that PV data from static breathing (holding of one’s breath) along a breathing cycle yields the same values as snapshots of dynamic breathing. A recognized limitation of equation (2.1) is that \( V_o \) is considered constant, which is not the case due to hysteresis. Challenges surrounding modeling of hysteresis of the lung are the progressive
recruitment of alveoli and that it is mostly due to surface forces generated at the gas-liquid interface (Salazar, 1964).

Paiva et al. (1975) suggest a sigmoid model of the static PV curve over the exponential model of Salazar and Knowles. The primary reason for this is that the sigmoid model may fit the full curve well. It is unclear if the authors are analyzing the full range of the curve or simply the same range as Salazar and Knowles, the equation to which they are comparing theirs. The equation that Paiva suggests is

\[
V = \frac{V_0 e^{FRC}}{FRC + (V_0 - FRC) e^{-K'(P_{REEP})}}
\]  (2.2)

The \( K' \) value for this equation matches the traditionally calculated compliance (Ting and Lyons, 1961) better than \( K \) (Salazar and Knowles, 1964). The Paiva study cannot explain the physiological significance of the sigmoid curve. Possible factors mentioned include alveolar recruitment as well as "intrinsic properties of the lung parenchyma." Likewise, Murphy and Engel (1978) proposed a hyperbolic-sigmoid equation that does not seem to possess any physiological significance.

Murphy et al. chose to represent the PV curve with a hyperbolic-sigmoid function with two shape parameters instead of one. The form of this equation is

\[
P = \frac{K_1}{(V_M - V)} + \frac{K_2}{(V_m - V)} + K_3
\]  (2.3)

The reason for this is the belief that the upper and lower curves of the PV diagram are not caused by the same mechanisms. As a result, the two curvilinear sections can be adjusted separately (\( K_1 \) for the lower and \( K_2 \) for the upper section). The shape constants \( K_1 \), \( K_2 \) and \( K_3 \)
range from 20 to 874, 4.57 to 11.5E+6 and -3.23 to 12.8E+2 respectively. This paper seems only interested in quantifying the curve and not concluding any physiological significance about the parameters. As a result, there is no discussion of recoil or elasticity.

Colebatch, Ng and Nikov (1979) attempted to fit an exponential and polynomial function similar to Salazar and Knowles’ to PV curves of healthy subjects and define its physiological significance.

Colebatch et al. (1979) used the models

\[ V = a + b \times P + c \times P^2 + d \times P^3 + e \times P^4 \]  \hspace{1cm} (2.4)

\[ V = A - B \times e^{-KP} \]  \hspace{1cm} (2.5)

to fit the full range of data using the polynomial and from 50-60% total lung capacity (TLC) to 100% (TLC). Equation 2.4 is used for lower pressures (end expiration) and equation 2.5 is used for higher pressures (end inhalation). They define \( K \) as the index of compliance independent of gender and increasing with age. The magnitude of decrease in elastic recoil is unknown because older subjects are not capable of stressing their lungs at the same level as younger subjects. \( K \) is suggested to be a more accurate index of elasticity because of its independence from lung size and fit over a wider data range. A potential explanation for loss of elasticity is the increased resting length of alveolar tissue. The polynomial, though it fits well, is said to have no physiological significance. The exponential however, is much more helpful. The percent value of \( B/A \) indicates position of the curve with respect to the \( P_{st} \) axis (similar to \( K_3 \) in equation (2.3)). The increase in \( K \) with age is not affected by fitting the exponential over different volumes nor is it affected by the amount of stress on the lungs, which differs in younger and
older subjects. Colebatch describes slightly stiffer lungs as having a lower K value (or compliance), which is reasonable because healthy lungs have more elastic recoil and are therefore stiffer than emphysematous lungs, which have lost the ability to recoil, making them less stiff and exhibit an increased K value. Figure 2.1 shows the range over which the exponential fits PV data.

Gibson et al. (1979) did a similar study to Colebatch et al. (1979) but extended it to include diseased lungs including emphysema. They used the same equation as Colebatch (1979) in the form of \( V = V_{\text{max}} - A \cdot e^{-KP} \) where \( V_{\text{max}} \) is the extrapolated maximum volume (occurring at P = \( \infty \)). Recognizing that this model does not fit the curve well below FRC, they suggest the use of a sigmoid model to achieve that fit. However, they suggest that fitting the complete data is unnecessary and perhaps illusory. Since the upper and lower curves seem to be caused by different mechanisms, they claim it is perhaps of no additional physiological benefit to generate one equation for the entire range. In contrast to the Colebatch findings, however, there was no strong correlation between age and K. Regarding the diseased lungs, the mean value of K was considerably higher for patients with emphysema than for normal subjects.

The results of Silvers et al. (1980) showed that elastic recoil in lungs with minimal emphysema is reduced and TLC is increased compared to normal subjects. This conclusion is limited by the belief that emphysema is not the sole cause of decrease in elastic recoil and
perhaps the two are only coincidentally related as opposed to a cause and effect relationship. The study also argues that in vivo studies should provide more realistic data for predicting early emphysema in patients. Using the exponential equation from Salazar and Knowles, Silvers et al. compared grade of emphysema to static lung pressure at 90% TLC (Pst 90%) and found a correlation of r = -0.696. They then compared Vl/TLC to Pst 90%, a correlation of r = -0.612.

Also in 1980, a study by Greaves and Colebatch (1980) was published regarding the elastic behavior and structure both of normal and emphysematous lungs. This study concluded that K is a valid index of distensibility and increased alveolar distensibility, or ability to expand, is found in all grades of emphysema. Greaves and Colebatch also concluded that exponential analysis is more likely to predict emphysema than other noninvasive diagnostic procedures. Excised human lungs were used and the exponential model was again only fit to the upper portion of the curve. K can help distinguish between emphysema and “other causes of chronic airflow limitations.” Several strong correlations were found. Level of emphysema (%E) and mean interalveolar distance (Lm) had a correlation of r = 0.9. The exponential constant, K, and Lm showed a correlation factor of r = 0.9. K and %E had a correlation of r = 0.73. This lower correlation was explained by the “contribution of more grossly diseased regions in the assessment of the latter [(%E)].” Because of the close correlations, it seems that K can also be viewed as an index of distensibility.

In support of the studies by Greaves and Colebatch, Pare et al. (1982) also indicates that exponential analysis of lung PV curves can be a predictor of emphysema. Pare’s study agrees that K is “the most accurate predictor of pulmonary emphysema.” Also, they seem to believe that exponential analysis is superior due to its independence from lung size and patient
capability. Again, only PV data about FRC was used. The same exponential equation as Colebatch et al. (1979) was used. The lowest r value for this fit was 0.9484, signifying a high goodness-of-fit. Also mentioned again is that in vivo studies are more likely to give more accurate data for noninvasively predicting emphysema in patients. Pare et al. were in agreement with Greaves et al. (1980) that differing levels of emphysema result in different K-values.

In 1986, Colebatch et al. published a similar study to Pare’s, examining the exponential model as a model for elastic recoil. They recognized that the model would only be suitable for higher volumes. Figure 2.2 shows the deviation that occurs at lower lung volumes. Additionally, this illustrates the difference between inhalation and expiration curves.

![Figure 2.2: Exponential curves fit to a full breathing cycle by Colebatch et al. (1986).](image)

It was concluded that $K$, $B/A$ and $P_L$ are sufficient to quantify the PV curve. In contrast to his 1979 study, Colebatch found that $K$ increased as the lower volume limit increased (decreasing the range of data over which the exponential was fitted). Silvers, Petty and Stanford (1987) reexamined their 1980 study and found similar results as before.
The next study reviewed regarding an equation to fit the PV curves was done by Gugger, Wraith and Sudlow (1989). They applied a cubic and exponential function, both similar to if not to the same as those presented in previous studies. The difference between this study and others using the $V = A - Be^{-KP}$ model is that the point through which the exponential model was fit was not FRC but instead the calculated inflection point determined by the cubic function (often higher than FRC). The reasoning for using this method of fitting the exponential comes from a less variable value of $K$ compared to the method of fitting through FRC. Gugger et al. also contradicts Salazar and Knowles by claiming that quasi-static PV curves are more sigmoidal than completely static PV curves, like those used in Colebatch et al. (1979).

Macklem et al. (1990) published a paper specifically examining the elastic properties of emphysematous lungs. Their conclusion was in agreement with other studies that an increase in resting length and decreased distensibility are present in emphysematous lungs, causing the larger lung size and increased specific lung elastance ($E_{sL}$). $E_{sL}$ is defined as the change of lung elastic recoil pressure ($P_{el,L}$) required to produce a given fractional change in lung volume ($\Delta V_L/V_{L,0}$). At this point, it is stated that “patients with emphysema exhibit a lower lung pressure, $P_{el,L}$. Macklem et al. (1990) state that bulk modulus and shear modulus are two parameters required to determine the elastic properties of materials. They also suggest the possibility of being able to characterize the elastic properties of materials if one knows Young’s modulus and Poisson’s ratio because “these parameters determine how easy it is to change the shape of a material compared to how easy it is to change volume.”

Bogaard et al. (1995) attempted to compare an exponential model and linear-exponential model to PV curves. The linear-exponential model fit the data better by using two equations, one
linear and one exponential for differing parts of the curve (above FRC). There was, however, no
determined physiological relevance of $K_{EL}$ to elasticity properties. As seen in other studies
however, $K_{EL}$ from the purely exponential model did show a close correlation to compliance.
This study focused on patients with asthma and COPD, not emphysema directly.

The latest study found was by Venegas et al. (1998), who suggested using a sigmoidal
equation of the form

$$V = a + \frac{b}{1 + e^{-(p-c)/d}}$$  \hspace{1cm} (2.6).

This form is similar to that presented by Colebatch et al. (1979) since $k = d$, $A = a + b$ and
$B = b * e^{c/d}$ for the upper limit of pressures. Similar to the other sigmoidal equation, this one
can span the entire curve better than a mono-exponential model and does have some
physiological relevant parameters. There is no mention, however, of an elasticity parameter nor
is there any mention of emphysema being studied.
2.2.2 Specimen Design & Test Methods

Research into current biaxial specimen designs and test methods was conducted to improve the current design and method used at RIT. Developing an improved biaxial test fixture system that accurately predicts boiling flask PV data is necessary in order to easily test candidate lung model materials. Extensive research has been conducted regarding biaxial tensile specimen design for metals and composites. Only a small number of papers have conducted this type of research for hyperelastic materials.

Sacks (Sacks et al. 2005) studied cruciform and square geometries as well as different clamping styles. Two different styles of clamped cruciform specimens (CCS) were studied, one with the clamps at the ends of the legs and the other closer to the specimen center, which essentially eliminated the effects of the legs. Figure 2.3 shows these two different clamping styles.

The study concludes that while this geometry has been widely used to characterize metals, polymers and even representative in-vivo tissue conditions, boundary conditions affect the stress levels across the specimen and alter the ability to get accurate stress readings in the central region of interest. The clamped square specimen (CSS) is also a victim of the boundary condition stress shielding issue although the range over which the central region is subject to

![Figure 2.3: Clamped cruciform specimen clamping styles from Sacks et al. (2005).](image-url)
uniform loading is greater.

Ultimately, the geometry and clamping style most unaffected by boundary conditions was the suture attachment (SA) of a square geometry. By allowing the edges of the specimen to expand in the transverse direction relative to their respective displacement loads, stress at the corners and edges were more uniform across the specimen than for other clamping methods. Additionally, the stress levels across the specimen were closer to that which was being measured at the edges. Figure 2.4 shows the results of simulations for SA and CSS geometries.

Figure 2.5 shows the stress levels estimated throughout the different specimen geometries and clamping methods. A 1MPa stress was applied at the edges. Using these results to relate edge stress to the central region stress is possible and is part of the focus of this thesis. From this plot, it is evident that Saint-Venant’s Principle cannot be ignored and the SA specimen experiences the least stress shielding of all the geometries. In terms of percentage of...
applied stress, the CSS geometry experiences more stress shielding than the SA specimen.

Brieu (Brieu et al. 2007) also studied cruciform specimens clamped along the full length of the legs. By varying the radius of the fillet at the corners, an optimal size could be determined. Radii of 1 and 5 millimeters were studied. Results indicated that in order to reach a higher stress level in the central region, a larger fillet size should be used. Views of the fillets and displacement fields can be seen in Figure 2.6.

The above studies focused on hyperelastic materials. Several studies that focus on metals and composites involved more complex biaxial specimen geometries. These geometries are more complicated and would therefore be more difficult to create from elastomeric materials, which cannot be formed using traditional machining methods; however their methods and designs may provide some additional insights.
Yu (Yu et al. 2002) published a study that removed material from the center region of a clamped cruciform specimen. The removed section was in the shape of a cruciform with more material removed along the legs of the overall geometry. A ¼ finite element (FE) simulated model can be seen in Figure 2.7. The proposed benefit of this geometry is that deformation of the arms will transfer to this small slot, either partly or wholly (Yu, 2002).

Bhatnagar (Bhatnagar et al. 2007) suggested that the entire central region of the cruciform specimen be thinner than the legs. The reason for this was to allow for mechanical failure within the central region, rather than at the radius where the legs join the center section. With respect to the hyperelastic material characterization, mechanical failure is not desired. As a result, the only reason that this geometry would be transferred for use in elastomeric models would be to raise the stress level in the central region to the level experienced in the legs. As shown in Figure 2.8, the thinner central region is tapered at the edges to avoid severe geometric discontinuities. A study into the size of this taper does not seem to have been conducted. Additionally, the filleted kerfs at the corners are not discussed beyond being the origin of the fracture.

2.2.3 Existing Criteria for Specimen Design

In biaxial testing of hyperelastic materials, several criteria must be satisfied in order to provide accurate data. Several studies have proposed such criteria for both sheet metal and hyperelastic materials (Hannon et al. 2008; Smits et al. 2005; Geiger et al. 2005). Several criteria for sheet metal (Demmerle et al. 1992) are 1) the existence of homogenous stress and
strain distribution in the central test section, 2) stress values in the test region are compatible with the nominal stress values, and 3) the highest stress level can be observed in the test section. The aforementioned criteria represent the requirements of an optimization analysis for shape design of a cruciform specimen. While items 1 and 2 of the criteria exist implicitly as requirements in multiple biaxial studies, the third criteria may not be necessary in order to achieve true central region stress-strain data. The reason for this may lie in the basic understanding of Saint Venant’s principle. Fundamentally, Saint Venant’s principle says that the stress distribution near the boundary conditions of a specimen will be affected by that boundary condition and this effect will decrease with the distance from the boundary condition.

Several papers have discussed or studied the effects of Saint Venant’s principle on the acquisition of accurate biaxial stress-strain data (Stubbs, 1984; Waldman et al. 2002; Waldman et al. 2005; Sacks, 2005). Most papers recognize the Saint Venant’s effect of clamping versus suturing a biaxial specimen, regardless of the tested material: metal or rubber, isotropic or anisotropic, biological or composite, etc. The first study to estimate the end effects of Saint Venant’s principle was an investigation of a fabric material (Stubbs, 1984). The findings seemed to discuss the possibility of a “corrective parameter” that could be calculated by dividing the applied load by the approximate stress determined experimentally. This method attempts to account for the highest stress outside of the central region of focus rather than require it be located within the region. Critiques of this study suggest that mechanical properties of the material a priori are required, voiding the usefulness of the method for uncharacterized materials (Waldman, 2002). A further explanation of this critique is not provided and thus is unable to be further investigated.
2.2.4 Gaps in the Literature

There are areas of research that are important to completely understanding the characterization process that are not completely discussed in the reviewed literature. The test methods used are neither standardized nor are they the same for all equibiaxial specimen geometries. No attempts are made to compare test methods or use existing methods to test new geometries or materials. Biaxial specimen geometries also vary in shape and size. This makes it more difficult to compare results from one study to another. Only one attempt is made to compare stress decay across a limited number of varied dimensions for common CCS and CSS geometries. Finally, no studies attempt to analyze predicted changes in results due to test parameter variations. All studies thus far conduct experiments or numerical analyses with ideal loading conditions. Therefore, it is not yet understood how a slight perturbation in these loading conditions will affect the results.
2.3 Analysis of Existing PV Data

2.3.1 Nominal PV Curves

Work has been done to begin to relate lung pressure-volume and material stress-strain curves. The ultimate goal is the ability to estimate a PV curve based on stress-strain from a candidate material and compare with a human lung PV curve. Several studies focused on gathering data from the healthy patients and those with varying levels of emphysema. The data, while often measured differently from one study to another, were manipulated to generate multiple PV curve projections based on the relevant exponential equation (2.5) by Colebatch (Colebatch et al. 1979). Figure 2.9 shows nine curves generated by data taken from three studies (Colebatch, 1979; Greaves et al. 1980; Gibson et al. 1979). The green and red curves illustrate patients with emphysema while the others illustrate healthy individuals. The red curve has an extrapolated section originally produced by the author for $V_{\text{max}}$ estimation. Variations within both healthy
and diseased curve sets exist because these studies conducted work on several lung types and under different conditions. Both genders were included in living and postmortem lung studies. Additionally, lungs from patients of all ages were both excised and non-excised during testing. These differences, however, do not affect certain attributes of the curve and parameter values. Excised lungs tend to require more pressure to achieve greater volumes than non-excised lungs. This is partially due to testing difficulties including an inability to completely seal the excised lungs and avoid fissures along the lung walls. PV curves from older patients tend to require less pressure than younger patients, which can have an effect on TLC. Male patients also generally have increased TLCs compared with women, and living patients (i.e.: non-excised lungs) used their muscles to generate the negative pressure required to inflate the lung whereas non-living, non-excised and excised lungs required manual inflation.

Some of the clearest differences due to disease are in total lung capacity (TLC), location on the plot and radius of curvature. While TLC is simply defined only by $V_{max}$, the general location of the curve on the plot is affected by the ratio $B/A$ (from equation 2.5) while radius of curvature is a function of the exponential coefficient, $K$. These differences are even more clearly represented in Figure 2.10 which brings together healthy and emphysematous curves into two separate, nominal curves. Two sets of these curves were generated, one for excised lungs and the other for non-excised. These curves

![Figure 2.10: Nominal non-excised curves estimated from experimental data.](image-url)
were generated using equation (2.5) and averaged parameters, A, B and K from tabulated data found in the studies examined. The non-excised curves are of more interest to this study due to the ultimate goal of understanding lung mechanics within the patient. Figure 2.11 illustrates the pressures achievable by excised lungs for normal (N) and emphysematous (E) lungs. It is worth recalling that lung elasticity seems to be represented by the value K (Colebatch et al. 1980). For emphysematous patients, K has been found always to be above 0.3, while for healthy patients to be always under 0.3. Again, while this is not the full PV curve, it is the section that describes lung motion as it relates to the relevant fields of study.

The possibility of correlations between the physiological and mechanical state of human lung tissue is promising, yet it requires more research. A correlation between PV equation parameters and engineering material properties may be possible by conversion from one to the other or by the development of an equation that predicts physiological response with engineering material properties.

2.3.2 Material Candidate Search

The most promising element of PV curves is that, again, they are significantly different between healthy and emphysematous. As a result, the parameter values within equation (2.5) are
also different. This implies that a correlation between their differences exists and may be related to the differences that exist between typical mechanical properties of hyperelastic materials.

The property of the lung that seems to be the most important in determining mechanical response is elasticity. In equation (5), the parameter $K$ has proven to be a good index of elasticity in several papers (Colebatch, 1980). While not an immediately representative value of response such as elastic modulus, the way in which $K$ changes with respect to other properties of the lung implies that it may be related to a modulus typically found on material data sheets (i.e.: Tensile Modulus). Other elastomeric materials that seem to be good candidates have had low hardness and tensile strength values paired with a high percent elongation.

2.3.3 Estimate Strain

Previous work was conducted to estimate the strain experienced in the lung wall (Ferrara et al. 2009). This method, which was to estimate the change in radius of a spherical lung, was applied to both healthy and emphysematous curves. Using equations 2.7 and 2.8, the estimated strain was calculated.

\[
\frac{r}{g1/31/g1/64/g1/67/g1/66/g1/59/g1/53/g1/72/g1/61/g1/67/g1/66/g3404} = \left(\frac{\frac{V}{g3495}}{\sqrt[3]{\frac{3V}{4\pi}}}ight) \quad (2.7)
\]

\[
\text{Elongation} = 100 * \left(\frac{r_n - r_0}{r_0}\right) \quad (2.8)
\]

In equation 2.7, $r$ is radius and $V$ is volume. Elongation is determined by the difference between the radius at $V_{\text{max}}$, $r_n$, and the initial radius value of the exponential curve, $r_0$. The strain estimation shown in Table 2.1 were developed using these equations. Percent expansion of the lung was also calculated using equation 2.8 using volume instead of radius. An estimation of the wall tension is given by the law of LaPlace relating internal pressure and radius to stress in wall.
Equation (2.9) gives the general relationship for a sphere (Beer, *Mechanics of Materials*, 4th ed.).

The symbols $P$, $r_i$ and $h$ are pressure, inner radius and wall thickness, respectively.

$$\sigma = \frac{P\times r_i}{2h} \quad (2.9)$$

While the percent expansion is large, the equation shows that the correlating percent elongation will be much less. It should also be noted that even though lungs with emphysema are associated with larger TLC values, it is the inability to contract normally that results in less strain compared to healthy lungs. Figure 2.12 shows the estimated strain and corresponding volume for healthy and emphysematous lungs. The gross spherical lung assumption will eventually need to be revisited in an effort to better represent inflation within the alveoli relative to the larger bronchial tubes. A study on particle deposition (Harding et al. 2010) models rigid airway walls that resist expansion. Based on this, alveoli account for all volume change; however since alveoli are incomplete spheres, this strain may be underestimated.

<table>
<thead>
<tr>
<th>NOMINAL HEALTHY</th>
<th></th>
<th>% Tot. Expsn.</th>
<th>% Tot. Elong.</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (L)</td>
<td>$r$ (cm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.465</td>
<td>9.39</td>
<td></td>
<td></td>
<td>FRC</td>
</tr>
<tr>
<td>3.758</td>
<td>9.64</td>
<td>8.46</td>
<td>2.74</td>
<td></td>
</tr>
<tr>
<td>6.072</td>
<td>11.32</td>
<td>75.25</td>
<td>20.56</td>
<td></td>
</tr>
<tr>
<td>6.088</td>
<td>11.33</td>
<td>75.70</td>
<td><strong>20.67</strong></td>
<td>TLC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NOMINAL EMPHYSEMATOUS</th>
<th></th>
<th>% Tot. Expsn.</th>
<th>% Tot. Elong.</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (L)</td>
<td>$r$ (cm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.027</td>
<td>10.63</td>
<td></td>
<td></td>
<td>FRC</td>
</tr>
<tr>
<td>5.233</td>
<td>10.77</td>
<td>4.09</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>7.638</td>
<td>12.22</td>
<td>51.94</td>
<td>14.96</td>
<td></td>
</tr>
<tr>
<td>7.640</td>
<td>12.22</td>
<td>51.97</td>
<td><strong>14.97</strong></td>
<td>TLC</td>
</tr>
</tbody>
</table>

Table 2.1: Estimated strain in nominally healthy and emphysematous lung walls.
2.4 Preliminary Experimental Work

2.4.1 Existing Test Equipment

Two senior design teams separately developed test fixtures for the study of hyperelastic materials. The biaxial tensile test fixture (Figure 2.13) was developed to study elastomeric specimens such as those described in section 2.2.2.

The test fixture is used to study the response of materials subject to biaxial loads. While this is not an exact reproduction of the load scenario, previous work demonstrated that biaxial data closely represents the expected response of the same material when tested in the boiling flask test stand (Ferrara et al., 2009). The camera pictured is no longer used but instead a smaller pixellink® camera that is more compatible with the National Instruments Labview program is currently used. Due to a combination of manufacturing and design issues this machine is not able to provide acceptably accurate stress-strain data.

The choice to use drawer slides as the sole means of crossbar alignment proved to be insufficient. While the translational
motion and low friction were easily achieved with these slides, acceptable linear motion was not. The lack of support for the drawer slides meant that they were several degrees off from parallel with the bottom crossbar. To correct this, a set of brackets (Figure 2.14) were installed to align the slides perpendicular to the bottom crossbar. The brackets have been machined for adjustability in all three dimensions. In turn, this defined the top crossbar to properly translate perpendicular to the bottom crossbar. In addition, slots were created in the top crossbar to allow for fine adjustments in the alignment of the y-axis clamps. While these modifications improved the system, a deeper analysis of the misalignment was needed and is discussed in Chapter 4.

Another test fixture was built to study deformation of compliant models in the shape of a boiling flask or asymmetric lung model subjected to pressure (Figure 2.15). The boiling flask shaped...
specimen is shown in Figure 2.16.

The syringe pump extracts a volume of glycerin from the chamber in which the boiling flask (compliant model) sits. This decrease in pressure results in expansion of the boiling flask. Images of the expansion are captured with cameras. The design of the system requires that multiple pressure measurements be taken and manipulated to determine the pressure applied to the compliant model. Figure 2.17 illustrates the pressure sensors with relevant corresponding heights and pressures at points of interest. The height, $h$, is constant between the sensors. Pressure of the glycerin in the graduated cylinder, $P_{gc}$, and in the chamber, $P_{ch}$, is measured as the syringe extracts glycerin from the chamber. The blue bulb represents the boiling flask and $P_{in}$ and $P_{out}$ are pressures at the wall of the boiling flask on the inside and outside, respectively.

The following is a derivation of the applied pressure on the boiling flask wall as it expands. A free body diagram of a section of the boiling flask wall results in equation (2.10).

\[ (2.10) \]

At a time, $t > 0$, a known volume of glycerin, $\Delta V$, is removed from the chamber resulting in boiling flask inflation and a decrease in $h_{gc}$. This change in height can be quantified as

\[ (2.11) \]
where $A_{gc}$ is the cross-sectional area of the graduated cylinder. Equation (2.10) can now be rewritten for some time, $t > 0$

$$P_{\text{in}} - P_{\text{out}} = P_{gc}(0) - \rho g \frac{AV}{A_{gc}}(t) - P_{ch}(t) + \rho gh \quad (2.12).$$

For an initially undeformed boiling flask ($\sigma_{\text{wall}} = 0$ at $t = 0$), $P_{ch}(0) = P_{gc}(0) + \rho gh$.

Substituting this into equation (2.12) gives

$$P_{\text{in}} - P_{\text{out}} = P_{ch}(0) - \rho g \frac{AV}{A_{gc}}(t) - P_{ch}(t) \quad (2.13a)$$

or

$$P_{\text{in}} - P_{\text{out}} = P_{ch}(0) - \rho g \Delta h_{gc}(t) - P_{ch}(t) \quad (2.13b).$$

This pressure difference is measured for each incremental volume change and used with wall displacement data for comparison with numerical results.

### 2.4.2 Stress Decay Factor

It has been proposed in past studies that traditional measurements of stress and strain estimate a stiffer material than actually exists (Sacks, 2005; Yu, 2002; Bhatnagar, 2007). In

![Figure 2.18: von Mises and y-direction stress along the line A-A of CSS specimen.](image)
order to investigate this, preliminary FE analysis was done to compare stress profiles along the center of some representative specimen geometries. Path lines in ANSYS® and resulting stress decay plots can be seen in Figure 2.18 and Figure 2.19. The y-directional stress, SY and von Mises stress, SEQV, are plotted. The geometries presented here are further analyzed and discussed in the next chapter on design of a new specimen. The same stress decay data was exported and graphed together (Figure 2.20).

The von Mises stress is given by

$$\sigma_v = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$  \hspace{1cm} (2.14)

The stress decayed in a similar fashion as the material studied by Sacks in Figure 2.5. This decay can be attributed to Saint Venant’s principle and the effect of the adjacent, perpendicular clamps on load measurements. It is proposed that this stress decay can be quantified and used to estimate the true equibiaxial stress state in the center region.
The stress decay factor (SDF) is a ratio of stresses at different points across the specimen. The SDF is conducted by dividing the stress in the equibiaxial region by the edge stress. Equibiaxial tension is defined here as the region where stress in the principal loading directions are within 1% of each other. Although ANSYS outputs true stress (versus engineering stress), the SDF is independent of stress-type. If stress is uniform across a specimen such as in the case of uniaxial tension, the SDF is 1. Therefore, the goal is to reach unity for the value of the SDF. While this is understandably not possible with a biaxial specimen, a value closest to unity means that stress decay effects are minimized.

The SY and SEQV curves in Figure 2.19 do not intersect at the middle of the specimen as they do in Figure 2.18 is because this particular model was loaded with a 7:8 x:y displacement ratio. To generate these plots, several points must be selected along the symmetry line (keypoints serve as the best option since they are known to lie perfectly along the line of symmetry). In order to achieve a level of resolution that accurately defines the equibiaxial region, there need to be about 100 data sets and 80 divisions. Further FEA test settings are discussed in Chapter 3.

The proposed material characterization process, including identification of a geometry-specific SDF, is outlined here:
1. Experimental data is collected. This data consists of the specimen edge stress (a nominal stress calculated from the load cell measurements) and diamond strain (calculated from optically measured displacement in the center of the specimen). Since stress measurements in the center of the specimen, where the actual equibiaxial stress state exists, are not experimentally measurable, what is collected here represents the best available data with which to start.

2. The stress-strain results from 1 are input into the ANSYS® hyperelastic curve-fitting tool to estimate 2-parameter Mooney-Rivlin coefficients that model a best-fit line.

3. The planar biaxial ANSYS® simulation is run with these coefficients and from these results, a SDF is calculated as described above.

4. The experimental data from 1 are multiplied with the SDF, resulting in an estimated central-diamond (equibiaxial) stress-strain response.

5. Step 2 is repeated with the SDF-modified data. The concept of using a modified set of the original data represents the iterative approach of this process that ensures the coefficients represent true equibiaxial stress and strain.

6. The planar biaxial ANSYS® simulation is run with these new coefficients and central diamond as well as edge stress results are predicted and compared with the SDF-modified experimental data.
Results of using the SDF to predict accurate material properties are best seen in the following graphs. Figure 2.21 illustrates the specimen *Edge* stress (calculated directly from load cell measurements) versus optically measured central diamond strain. The ANSYS curve-fitting tool does not let the user define the location on the specimen from where stress measurements were obtained. Instead, the input data is assumed to be biaxial, and in this case, equibiaxial. It is for this reason that in Figure 2.22, the *ANSYS Central Diamond* (equibiaxial) stress results match well with what is proposed as *Edge* stress results. These preliminary numerical results support the accuracy of the ANSYS model but do not accurately predict material properties. Instead, the SDF-modified results in Figure 2.23 aim to achieve this goal.

![Figure 2.21: Raw experimental stress-strain plot for the R04 CCS no leg specimen.](image)

![Figure 2.22: Raw edge stress data modeled and verified as equibiaxial central diamond stress.](image)
The *SDF-modified Edge* data in Figure 2.23 are the product of the SDF and raw experimental edge stress data (Figure 2.21). The correlating numerical results again match well with the curve-fitted experimental data. The response of the numerical model with SDF-modified (true) material properties should predict stress at the specimen edge comparable to the experimental data in Figure 2.21. Figure 2.24 shows this comparison and supports the use of the SDF to predict diamond region stress levels. The *ANSYS Edge* stress was obtained by summing the reaction forces at the edge nodes and dividing by the edge cross-sectional area.

![Figure 2.23: Predicted central diamond stress with resultine ANSYS central diamond stress.](image1)

![Figure 2.24: Raw experimental data compared with ANSYS edge stress using SDF-modified material properties.](image2)
2.4.3 Data Acquisition Process Improvement

Another significant advancement in the material characterization process was the development of a program that completely automated the collection of loads, displacements and images quickly and more reliably than before. The previous process required the user to mount a specimen and adjust the displacement of the clamps until the specimen was just taut and not loaded yet. The data gathering process then involved manually triggering three separate programs to advance the motor, read load cells & capture images for each data point. It is clear how human error could be introduced. Additionally, some post-processing of the images due to file naming was required. To acquire 50 load steps and images worth of data took about 20 minutes. The automated program has changed most of this.

A Labview VI was developed that brought together all three programs. This VI interfaces with the NI Measurement & Automation Controls program (MAX). The structure of the program involves a loop in which a specific order of events occurs. Once position is changed, a load reading and image are captured and stored. Until these two final operations are completed, the position advancement command is not triggered. The *Configuration* tab in Figure 2.25 provides complete control of motor settings as well as options for file location, name and load readings (simulated loads can be selected for other test purposes). In addition to the settings, the *Acquisition* tab in Figure 2.26 shows images and load as they are acquired. Automatic sequential naming of the image files eliminated the need for image post-processing. The only thing that has not changed with this new process is the need to manually set the specimen to an initial taut but unstressed and unstrained position. Data acquisition time was reduced to about 50 loads and images in about 45 seconds.
Figure 2.25: Configuration tab of Biaxial Membrane Tester with Image Grab.vi.

Figure 2.26: Acquisition tab of Biaxial Membrane Tester with Image Grab.vi.
2.4.4 Machine Modification and Results

With the introduction of a new specimen shape and size, a different operating range of the test fixture was required. This, along with a higher step increment resolution, seems to have magnified certain issues while reducing others. While the SDF was brought closer to unity through the use of a new specimen geometry (which will be discussed in Chapter 3), dynamic issues such as shaft binding and skipping increased. Several tests have been conducted to determine where this binding is occurring and to what extent. Figure 2.27 is a plot of the -x, and +y-clamp positions as a function of the number of motor steps taken. The relatively perfect triangular wave seen for the +y-clamp is desired for both x-axis clamps. Binding is experienced on the -x-axis side and can be seen in Figure 2.27. Uniform extension is not achieved. In this case, no specimen was clamped.

A study of the measured images shows that the -x-axis clamp seems to temporarily stop while the other clamps continue to move with each motor step. The bound clamp then seems to “skip” back into position with a slightly higher slope than before binding occurred. This binding does not seem to occur during unloading, perhaps because of the configuration and resulting

![Figure 2.27: Normalized position of the moving clamps with respect to motor steps (no specimen).](image-url)
Several attempts have been made to resolve this non-uniform motion. The first attempt was to study the ball bushings through which the two x-axis shafts travelled (Figure 2.28). It was found that one of the bushings had been pressed into a hole that was too small, resulting in its ID clamping extremely tightly around the shaft. Boring out the bushing hole seemed to eliminate a small amount of binding. Extreme care was taken not to increase the hole more than 0.001-0.002”. Even a small amount of bore adjustment noticeably changes play between the shaft and bushing. This topic will be discussed later in this section. The next attempt involved using a spring on the angled shaft that would counter misalignment caused by the vertical link’s upward motion. This was unsuccessful, probably due to the fact that the spring was not able to realign the bushing, nor was it designed to. Two more options were then considered. The current bushing was reevaluated to determine if it was the best choice for this application and a major missing part of the original design was identified and added to the test stand.

The currently used ball bushing, a Thomson A-61014, is described as a very good bearing for tight tolerance applications. Its precision steel cage increases the load bearing capacity (5,000 lbs) by about 66% (www.thomsonlinear.com). A load bearing capacity of this level is most likely not relevant to this application; however, binding does cause an increase in radial load. Instead, alignment adjustability is of chief concern due to the misalignment that causes the binding. While precision steel bushings allow for some misalignment, the cage does not rotate within the housing and results in uneven pressure on the rolling balls. A self-aligning bushing
may be more appropriate for the application as the entire bushing is designed to rotate within the bore and distribute the load more evenly. More research into the proposed misalignment must be done before the purchase of self-aligning bushings can be justified.

In the original paper on which the currently used fixture was based (Brieu, 2007), a single shaft connected the links from one x-axis pillowblock to the other. Hidden behind the two clamp-carrying shafts in the study’s images, this shaft was not mentioned nor explicitly pictured. The shaft was eventually called out in a more recent paper by another author (Bhatnagar, 2006). As a result, it was not included in the RIT test stand. To modify the machine, two custom pillow blocks were machined to house bearings through which the shaft could pass during extension and contraction (Figure 2.29). It is important that these pillow blocks are never removed from the links into which they are screwed. The alignment of the linear bushings depends on this interface.

By not allowing this shaft to rotate in the plane of the specimen, the shaft is designed to constrain the clamps to travel at the same height. While the addition of this shaft improved the motion of the system, the problem was not completely resolved, requiring more analysis to be conducted. The resulting design analysis is presented in Chapter 5 on Minimum Constraint Design.
3. Finite Element and Specimen Design Analysis

Introduction
A finite element based analysis of biaxial specimen design and test parameter sensitivity is conducted before attempting to characterize actual material properties. Both analyses are meant to improve the material characterization process discussed in Chapter 6. This specific aim of this chapter is to discuss the methods and results of these analyses independent of actual material characterization.

FE models are developed and tested with a nominal set of material properties using the 2-parameter Mooney-Rivlin constitutive equation

\[
\bar{U} = \frac{\mu_1}{2} (\bar{T}_1 - 3) + \frac{\mu_2}{2} (\bar{T}_2 - 3) + \frac{K_1}{2} (J - 1)^2
\]

\[\mu = \mu_1 + \mu_2 = 2(C_{10} + C_{01}) \quad (3.1)
\]

K is the bulk modulus and \(\mu\) is the shear modulus. The use of a nominal set of properties allows for an analysis of response variation due solely to design and misalignment changes. The results of these analyses will help to better predict and understand both material characterization methods and results.

The specimen geometry investigation focuses on the ability of different geometries to provide biaxial stress-strain data easily and accurately. The resulting best geometry is then subjected to offset clamps and non-equibiaxial load ratios to study the effects of these common test perturbations.


3.1 Proposed Specimen Design Criteria

Despite criticism, the SDF correction method has been studied by others (Daniel et al. 1995; Geiger et al. 2005) and seems to provide accurate estimates for hyperelastic materials. Critics claim that this approach is flawed because \textit{a priori} material properties are required for this method. By taking an iterative approach with reasonable initial estimations, known material properties are not needed. A new set of criteria is proposed for the design of a planar biaxial specimen:

1) A high ratio of observed central stress to applied edge stress at the boundaries is desired for completely measurable data. A value less than 1 is acceptable, yet requires a correction parameter be estimated.

2) Large deformation of the central region is required just as it is required for uniaxial test of hyperelastic materials. Non-linear properties require that a substantial portion of the curve be experimentally tested for adequate comparison.

3) The range of uniform biaxial stress must be wide (if not completely spanning) over the region of focus. This criterion ensures that a correction parameter is unaffected by a change in the size of the equibiaxial and independent of stress variations within the region of focus.

4) Simple geometries must be employed. Due to the inherent compliant properties of most hyperelastic materials, complicated specimen geometries with attributes such as slits, thinned sections and chamfers cannot easily be manufactured. These attributes also pose a significant problem with convergence in computer numerical simulations due to singularities that arise. Additionally, it has been found that most studies of biological
Based on these criteria several candidate specimen designs were identified. Other proposed designs exist both for hyperelastic materials as well as sheet metal. For a number of reasons, these designs have been ruled out for this study. Table 3.1 describes these specimens and the reason(s) for their exclusion. Candidate geometries to be evaluated are presented in Section 3.3.

<table>
<thead>
<tr>
<th>Description</th>
<th>Illustration</th>
<th>Reasons for Exclusion</th>
</tr>
</thead>
</table>
| Suture attachment            | ![Image](image1.png) | • Difficult mounting & repeatability  
                                • Potential for suture point tearing  
                                • Difficult load transfer into specimen center |
| Bubble inflation             | ![Image](image2.png) | • Difficult mounting & repeatability  
                                • Difficult stress-strain measurements |
| Cruciform with kerf          | ![Image](image3.png) | • Difficult mold creation  
                                • Decreased uniform central biaxial loading region |
| Cruciform with slits         | ![Image](image4.png) | • Difficult mold & specimen creation  
                                • Difficult mounting & repeatability  
                                • High potential for singularities at due to slits in FE simulations |
| Cruciform with thinned center| ![Image](image5.png) | • Difficult mold creation  
                                • Difficult repeatability of specimen creation |

Table 3.1: Excluded specimen.
3.2 Finite Element

Remaining candidate geometries were evaluated in ANSYS®. Previously developed finite element models (Ferrara 2009) were modified to accommodate the new specimen shapes, sizes and equibiaxial ranges. Many of the FEA program settings and data extraction techniques are identical to those used by Ferrara. All finite element models use the Shell281 element type with membrane stiffness only (no bending stiffness is desired since a planar specimen is not self-supporting against bending). A mapped mesh with quad elements is believed to be the more appropriate than triangular elements (ANSYS Verification Manual, 2007). Experimental data is input into the hyperelastic material model curve-fitting tool to obtain estimates for the 2-parameter Mooney-Rivlin coefficients, $C_{10}$ and $C_{01}$ (equation 3.1). The material incompressibility parameter, $d$, which is related to the bulk modulus by $d = \frac{2}{K}$, is always estimated to be 0.

For the specimen design analysis, all FE models were compared using the same values for $C_{10}$ and $C_{01}$ for consistency and because the specimen behavior is expected to be relatively insensitive to slight changes in constitutive model. Figure 3.1 illustrates the applied displacements and their relation to one another for equibiaxial tension. Displacements are applied to whichever lines are defined as a clamp-specimen interface. For example, the clamped square specimen is modeled with three separate interface lines and as a result

![Figure 3.1: Displacement parameters for equibiaxial loading.](image-url)
requires all three lines to be equally displaced as described in Figure 3.1. The same applies for
the offset clamp models discussed in the upcoming sections.

The red diamond in Figure 3.1 is defined as the equibiaxial region, where all x- and y-
direction stresses are within 1% of each other. Within this region, equibiaxial tension is said to
exist. Points A and B are located at the edges of the equibiaxial region and can be used to extract
and calculate stress and strain data. The initial length, $l_o$, is defined as the distance between
points A and B.

$$\varepsilon = \frac{\Delta l}{l_o} = \frac{y_B - y_A - l_o}{l_o}$$

(3.3)

### 3.3 Candidate Geometries

Three categories of Table 3.2 were evaluated: CCS legged, CCS no leg, and CSS. Within
each category, several models with slight variations were added.

| CLAMPED CRUCIFORM SPECIMEN (CCS)                                                                 |
|                                                                                                 |
| Shown: CCS R01 leg (current geometry)                                                          |
| Description: Cruciform specimen with exposed legs (clamped at end of leg)                      |
| Dimensions: 25.4mm end to end, 25.4mm leg width                                               |
| Dimensions varied: Fillet radius (1, 5 & 10mm)                                                 |
|                                                                                                 |
| CLAMPED CRUCIFORM SPECIMEN (CCS)                                                                 |
|                                                                                                 |
| Shown: CCS R04 no leg                                                                          |
| Description: Cruciform specimen with no exposed legs (clamped at fillet foot).                  |
| Dimensions: 33.4mm end to end, 25.4mm leg width                                               |
| Dimensions varied: Fillet radius (1, 4, 5, 6 & 10mm)                                           |
**CLAMPED SQUARE SPECIMEN (CSS)**

*Shown: CSS 2x2*

*Description: Square specimen with clamps inside the boundaries.*

*Dimensions: 50.8mm sides, 25.4mm clamp width, 5mm clamp inset*

*Dimensions varied: Side length (50.8 & 76.2mm), clamp inset (5 & 7.5mm)*

<table>
<thead>
<tr>
<th>Table 3.2: Candidate geometries for the preliminary selection process.</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is important to note that previous studies have used the CSS geometry and care was taken here to vary only the scale of the specimen with respect to those studies. Therefore, dimension ratios are identical with respect to one another. Material properties and applied displacements have also been kept constant across geometries.</td>
</tr>
<tr>
<td>This preliminary analysis focused only on criteria 1, 3 and 4 from section 0. Large deformation was ignored but will eventually need to be fulfilled once a specific specimen has been defined. The geometries in Table 3.2 only include designs that can be easily created. Therefore, obtaining a high SDF value and large range of equibiaxial tension are the primary focus of these numerical simulations.</td>
</tr>
<tr>
<td>The most difficult model to mesh was the CSS shape. The simple square design was modeled to simulate a solid clamp on each of the four sides. This created sharp corners, which resulted in singularities that prevented small scale models (38.1 x 38.1 mm) from converging on a solution. Areas were configured in an effort to eliminate elements with extremely slender shapes, which was previously believed to be the cause of excessive deformation. Figure 3.2 shows the different mesh configurations resulting from different area configurations, line meshes and geometric specimen scale. A global mesh configuration for the CCS geometry was</td>
</tr>
</tbody>
</table>
developed by Ferrara and new mesh refinements were relatively easy to apply. A new area configuration for the selected geometry is discussed in section 3.4.

Figure 3.2: CSS specimen mesh configurations. The best mesh can be seen in the top left.

3.4 Specimen Design Results
The results from all three geometries were reasonably close to one another, especially between the two CCS categories, which were expected. Table 3.3 contains a summary of results:
SDF values, as well as the corresponding equibiaxial range in both millimeters and percentage of the effective specimen length (clamp to clamp).

There are certain artifacts from the method of data post-processing that appear in this data. The method by which data is extracted along the centerline of the model is done so in the same manner for each specimen. Four nodes along the line of symmetry (x = 0) are selected as points through which a fit of the desired results such as SX, SY and SEQV are produced (Figure 2.18). Resolution of this path can be adjusted; however four nodes at the default values of resolution are sufficient for convergence. It seems that the range of uniform data stays fairly constant, perhaps because each model includes the same size central diamond region, -4.5mm to 4.5mm, which are two of the picked nodes through which path data is produced. In order to better understand how this central uniform region relates to the overall specimen geometry and scale, the percentage of that region between the clamps is calculated and now the effects of the overall geometry can be more clearly seen. While achieving a uniform range across the entire specimen is desired, it cannot reasonably be expected under clamped boundary conditions due to the inability of the edge material to deform in the transverse direction. As a result, the SDF value may be the most significant result of the three provided in Table 3.3.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>SDF (%)</th>
<th>Range (mm)</th>
<th>Range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCS R01 leg*</td>
<td>80.74%</td>
<td>3.6</td>
<td>7.0%</td>
</tr>
<tr>
<td>CCS R04 no leg</td>
<td><strong>83.15%</strong></td>
<td><strong>3.6</strong></td>
<td><strong>10.8%</strong></td>
</tr>
<tr>
<td>CCS R05 leg</td>
<td>76.99%</td>
<td>3.6</td>
<td>7.0%</td>
</tr>
<tr>
<td>CCS R05 no leg</td>
<td>82.91%</td>
<td>3.6</td>
<td>10.2%</td>
</tr>
<tr>
<td>CCS R06 no leg</td>
<td>82.30%</td>
<td>3.6</td>
<td>9.6%</td>
</tr>
<tr>
<td>CCS R10 leg</td>
<td>74.48%</td>
<td>3.6</td>
<td>5.8%</td>
</tr>
<tr>
<td>CCS R10 no leg</td>
<td>77.98%</td>
<td>3.6</td>
<td>7.9%</td>
</tr>
<tr>
<td>CSS 2x2</td>
<td>83.10%</td>
<td>3.6</td>
<td>8.8%</td>
</tr>
<tr>
<td>CSS 3x3</td>
<td>69.18%</td>
<td>3.6</td>
<td>5.4%</td>
</tr>
</tbody>
</table>

Table 3.3: Specimen results for preliminary selection process. *Geometry from Ferrara
SDF values have been improved 2-3% relative to the geometry defined originally by Joe Ferrara (Ferrara, 2009). Because the CCS R05 no leg specimen achieved better results than R01 and R10, additional fillet sizes around R05 were examined. These results from R04 and R06 geometries indicate that the optimal scale lies somewhere between a fillet size of 1 and 5mm. While the SDF value of the CSS2x2 model is very close to the CCS R04 no leg model, the CCS central diamond is a larger percentage of total specimen width. It should also be noted that a more of the different CCS geometries and scales converged than the different CSS models.

Once the new specimen analysis was complete and the CCS R04 no leg specimen was chosen, similar efforts to those undertaken for the CSS specimen were conducted to maximize the achievable strain. Eventually, a configuration of the areas shown in Figure 3.3a was chosen. These areas were map meshed with a global element size of 1 and a single level of element refinements to the two rows of elements around the fillets. The resulting mesh can also be seen in Figure 3.3b.

Figure 3.3: Area configuration (a) and resulting mapped mesh (b) for CCS R04 no leg model.
The most significant change is that the size of the diamond region has been reduced in an effort to ensure that the points of the diamond are within the equibiaxial stress region over the entire range of applied displacements. This was done when it was determined that the previous diamond region experienced significantly different directional stresses, SX and SY while under equibiaxial load. This region shrinks as the specimen edge displacement is increased. This can be seen by increasing difference between SX and SY levels in the diamond region in Figure 3.4.

![Figure 3.4: The percent difference between SX and SY dictates the equibiaxial region.](image)

The point A is used to take these measurements. As a result, the smallest reasonable diamond that is experimentally measureable was chosen as 3mm due to marking limitations. This ensures that all 1:1 load displacement tests will yield equibiaxial data.

This smaller diamond region proved important in obtaining more comparable results since the previously defined diamond consisted of non-equibiaxial stress regions. The new CCS specimen also allows for greater experimental strain to be measured since the specimen legs have been removed. The numerical divergence problem in simulation that plagued previous research and early parts of this research was solved by curve-fitting a larger range of experimental data. The issue seemed to be in the coefficients that ANSYS calculated to fit the experimental data. With a limited range of data, ANSYS estimated coefficients that, beyond the experimentally
measured range, predicted an unstable material behavior. Material instability is when the slope of the stress-strain curve becomes negative, such as the case with the Ferrara data shown in Figure 3.5. Some elastomers experience this property so this was not initially thought to be a problem, but a simplified Mooney-Rivlin biaxial equation (Bowers, 2010)

was eventually found that could be used to observe how the computer-estimated coefficients were defining the stress-strain curve. Figure 3.5 compares previously estimated coefficients with current estimations. Table 3.4 details the coefficients used to generate the curves in Figure 3.5. It is clear that previously proposed coefficients create an impossibly unstable material; they were only truly valid in the region of experimental data. These coefficients were estimated with 15% strain data. Current coefficients are estimated with 88% strain data that follows the red line and disproves the legitimacy of previous coefficients. It serves then to advise caution when using any sort of extrapolated data, that is, results beyond the range of experimental data obtained.

<table>
<thead>
<tr>
<th></th>
<th>Ferrara 2009</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{10})</td>
<td>3.46E-02</td>
<td>5.064E-03</td>
</tr>
<tr>
<td>(C_{01})</td>
<td>-1.085E-02</td>
<td>-3.69E-05</td>
</tr>
</tbody>
</table>

Table 3.4: Mooney Rivlin Coefficients.
According to the Mooney-Rivlin 2-parameter model, a material exhibits unstable stress-strain behavior when the $C_{01}$ coefficient is negative. A negative coefficient can, however, be offset with a large enough $C_{10}$ value and the material can exhibit a stable response up to a certain level of strain. These newly estimated coefficients generated from the increased strain data still produce a material instability warning from ANSYS prior to running the simulation yet the models can converge on a solution at strains nearly 10 times greater than previously studied because the coefficients are closer to being stable. This correlates to an increased edge displacement of 50mm. Since higher strains are now achievable, the number of substeps in the simulation has also been increased to 100. This increases the running time to about 5 minutes up from about 1 minute with 12 substeps. It seems that the simulation convergence issue has been identified and partially resolved. Steps can now be taken to develop an improved solution that generates stable material properties.

### 3.5 Misalignment Sensitivity: Offset Clamps

The first perturbation to the biaxial test was a lateral offset of the moving clamps. With the offset clamp, sections of the legs are now exposed. Figure 3.6 illustrates the way in which the clamps were oriented and a deformed simulation. Several magnitudes of clamp offset were compared to the nominal, aligned clamp case. Modeling of the offset required a slight modification to the configuration of

![Figure 3.6: Deformed CCS R04 no leg model with 0.5 inch offset clamps.](image)
modeled areas and the addition of specimen next to the offset clamps. The major difference was the placement of a keypoint at the corner tip of the clamps, resulting in two collinear lines that needed to be concatenated in order to achieve a mapped mesh. Figure 3.7 shows the specimen areas and resulting mesh. The sections of material next to the clamps represent material that is normally contained by the clamps.

Just as the displacement loading is applied to the outer lines of the model in the nominal case, the same is done to all clamp-material interface lines as depicted in Figure 3.6. For these simulations, only the physical clamp offset was changed. The same initial material properties were used and a 1:1 load ratio was used. The initial material properties used were generated from the same raw experimental data as used in the nominal case. Because of the offset however, the iterative material characterization process resulted in slightly different SDF-modified data, which is why only the original material properties are identical between the tests. All simulations were displaced 50mm at the top line and 25mm on each set of side lines with the bottom set of lines fixed for All DOF. This is identical to the magnitude and ratio applied to the nominal specimen.

Figure 3.7: Area configuration and mapped mesh for the 0.375 inch clamp offset model.
Figure 3.8 illustrates the varying diamond region stresses experienced by the different offsets. Since the model geometry is different for each offset clamp, the strains at any given substep are different for each model. Even though all models were subject to the same displacement loading, the final diamond region strain decreases as the offset increases. It is for this reason that an exact percent difference calculation cannot be conducted. Regardless, these results seem to imply that the effect on the stress-strain curve is minimal with respect to the magnitude of offset. For example, the 0.375 and 0.5 inch offsets are 37.5% and 50% offset with respect to the specimen width (1 inch). This amount of offset is clearly visible to the eye and therefore has little potential for going unnoticed, and up to 0.25 in. offset, the results are well within 5% of one another.

A calculable difference between the results of the different clamp offset

![Offset Clamp Graph](image)

**Figure 3.8: Numerical stress-strain results for offset clamp simulations.**

![Stress Decay Factor vs. Clamp Offset](image)

**Figure 3.9: Stress decay as a function of clamp offset magnitude.**
perturbations is their amount of stress decay. Figure 3.9 illustrates the increase in measured
decay (quantified by a lower SDF) for greater clamp offsets. While the SDF is not a means of
characterizing a material in the same way as stress and strain, it provides a context for explaining
the varying stress levels in Figure 3.8.

3.6 Misalignment Sensitivity: Non-equibiaxial Loading

The second perturbation to the nominal test case is a change in the biaxial load ratio. It has
been proposed and shown during the initial development phase of the test fixture and
characterization method that load ratio effects the numerical results of the CCS legged specimen.
For this reason, the CCS no leg specimen has been loaded with several non-equibiaxial x/y
displacement ratios: 0.85, 0.95, 1.05 and 1.15. No new models were required for this
perturbation, but instead different displacements were applied to the nominal model described in
section 3.4.

All initial parameters of the test stayed constant for each load ratio. Just as in the case of the
offset clamp tests, the initial material properties were from the ANSYS curve-fit estimation of
the raw experimental data. The same iterative process used in the two previous sections was
employed to generate a load ratio-specific SDF and resulting final stress-strain curve. All
models were displaced 50mm at the top line in the y-direction. This displacement along with the
load ratio defines the displacements applied to in the x-direction. The load ratio is simply
multiplied to the UX loads in Figure 3.1.

Figure 3.10 shows the results of different non-equibiaxial load ratios on the CCS R04 no leg
specimen. These results are comparable with those obtained by Joe Ferrara (Ferrara, 2009).
Since all simulations were run with identical geometry and UY displacements, it is
understandable that the maximum achieved diamond region strain is equal due to the linear
displacement-strain relationship. For load ratios below 1 (UX < UY) the predicted stress is lower than the nominal case (load ratio = 1). This is expected since a lower UX displacement results in less transverse stress and less measured stress for a given strain. Conversely, load ratios above 1 (UX > UY) result in a higher measured stress for the same amount of strain.

For this analysis, it is much easier to calculate percent difference of stress between load ratios. Table 3.5 outlines a truncated set of this data. While the percent difference decreases at first, it begins to increase at about 25% strain and continues to increase over the observed range of strain. It also seems that for the same amount of load ratio difference (5% for 0.95 and 1.05, 15% for 0.85 and 1.15), the load ratios greater than 1 exhibit a slightly greater percent difference. This can probably be attributed to the higher levels of stress being analyzed. Finally, it is worth noting the change in stress decay via the SDF. As the load ratio increases, the SDF experiences a parablic increase. This is expected because an increase in load ratio is an increase in the UX load, which has a greater effect on the center of the specimen. This is more easily recognized in Figure 3.11.
Finally, it is important to recognize that of the two perturbations, characterization metrics such as the SDF and stress are not only more sensitive to a non-equibiaxial load ratio than a clamp offset but also harder to experimentally detect. While a 50% clamp offset is visually obvious, a load ratio of 7:8 is not. It is therefore important to check that both the test fixture is applying the desired load ratio and the specimen, if assumed to be isotropic and incompressible, is experiencing the same desired load ratio in the central diamond region.

Errors in mounting or test fixture design (such as a difference in initial clamp position) can affect the diamond region load ratio and not the applied load ratio.

<table>
<thead>
<tr>
<th>Load Ratio</th>
<th>1</th>
<th>0.85</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDF</td>
<td>0.658</td>
<td>0.613</td>
<td>0.641</td>
</tr>
<tr>
<td>Strain</td>
<td>Stress</td>
<td>Stress</td>
<td>% Difference</td>
</tr>
<tr>
<td>0.04</td>
<td>0.0015</td>
<td>0.0013</td>
<td>15.7%</td>
</tr>
<tr>
<td>0.08</td>
<td>0.0027</td>
<td>0.0023</td>
<td>15.5%</td>
</tr>
<tr>
<td>0.13</td>
<td>0.0037</td>
<td>0.0032</td>
<td>15.4%</td>
</tr>
<tr>
<td>1.16</td>
<td>0.0137</td>
<td>0.0115</td>
<td>17.5%</td>
</tr>
<tr>
<td>1.20</td>
<td>0.0139</td>
<td>0.0117</td>
<td>17.6%</td>
</tr>
<tr>
<td>1.24</td>
<td>0.0142</td>
<td>0.0119</td>
<td>17.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load Ratio</th>
<th>1</th>
<th>1.05</th>
<th>1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDF</td>
<td>0.658</td>
<td>0.678</td>
<td>0.723</td>
</tr>
<tr>
<td>Strain</td>
<td>Stress</td>
<td>Stress</td>
<td>% Difference</td>
</tr>
<tr>
<td>0.04</td>
<td>0.0015</td>
<td>0.0016</td>
<td>5.6%</td>
</tr>
<tr>
<td>0.08</td>
<td>0.0027</td>
<td>0.0028</td>
<td>5.6%</td>
</tr>
<tr>
<td>0.13</td>
<td>0.0037</td>
<td>0.0039</td>
<td>5.5%</td>
</tr>
<tr>
<td>1.16</td>
<td>0.0137</td>
<td>0.0146</td>
<td>6.3%</td>
</tr>
<tr>
<td>1.20</td>
<td>0.0139</td>
<td>0.0149</td>
<td>6.4%</td>
</tr>
<tr>
<td>1.24</td>
<td>0.0142</td>
<td>0.0151</td>
<td>6.4%</td>
</tr>
</tbody>
</table>

Table 3.5: Percent Difference for non-equibiaxial load ratios.

Figure 3.11: Stress decay across the CCS R04 no leg specimen for different load ratios.
4. Clamp Misalignment Analysis

4.1 Introduction

The current test fixture is subject to misalignment, which causes the specimen to deform in an uneven manner. Measurements, 2-D vector calculations, and images have been gathered to determine expected and actual clamp locations. If clamp location can be predicted and verified, it can be incorporated into FE models in an effort to match simulations more closely with experimental results. The significance of verifying misaligned data is to understand the degree to which specimen are affected by clamp misalignment and how much misalignment still generates acceptable results. A 2-D vector analysis of the test fixture linkages is conducted in plane with the mounted biaxial specimen. Figure 4.1 illustrates the location and orientation of the Cartesian coordinate system used. Each of the four clamps, however, are labeled as +y, +x, -y & -x clockwise from top.

4.2 Assumptions

A number of assumptions have been made in order to reasonably estimate clamp location. The first and foremost of these assumptions is that the source of misalignment is in the plane of Figure 4.1: Clamp and linkage labels with defined origin.
the machine, oriented with the major plane of the specimen. Therefore, this analysis is completely two-dimensional and any z-axis misalignment is assumed negligible. While this may not be the case in the event that biaxial response sensitivity is finer than originally predicted, initial estimates are simplified a great deal with this assumption and therefore serve well as a starting point.

Equations were developed to compare sections of the test fixture to determine the cause of misalignment. Two distinct subsystems have been defined; one which controls the y-axis clamps, and the other, which controls the x-axis clamps. Since these two subsystems are inextricably linked, their equations must be linked in order to predict x-axis clamp location for a particular y-axis clamp location. The simple y-axis subsystem consists of the lead screws and top and bottom crossbars. The more complicated x-axis subsystem consists of the links that control the location and motion of the x-axis clamps. For analysis purposes, a distinction has been made between two sections of the x-axis subsystem. The x-motion subsystem consists of the links that directly control the clamps. The y-motion subsystem consists of the links that indirectly affect the x-axis clamps.

Additionally, certain angles are assumed to be fixed in the calculations. This is discussed in greater detail in section 4.4. It was originally assumed that the points where the y-direction links attached to the top and bottom crossbars were vertical with respect to one another. This proved to be an inaccurate assumption and additionally seems to be the main factor in misalignment of the clamps.

4.3 Component Measurements

The test fixture was disassembled so that each component could be measured. Since drawings for the components were misplaced or lacked tolerance definition, measurements could
not simply be compared to a detail to determine acceptability. It is because of this that acceptable component dimensions must eventually be set based on the sensitivity of the specimen to clamp location.

4.3.1 Measurement Instruments and Resolution

Several machines and devices were employed to measure dimensions of components. Error bars for these calculations were developed based on the resolution of the measurement tools employed. The Coordinate Measurement Machine (CMM) offered a way to quickly measure varying types of dimensions with a probe such as hole diameters, inter-planar distances, hole-plane distances, etc. The resolution of the CMM is on the scale of $1 \times 10^{-5}$ inches, which is more than sufficient for initial component measurements. Measurements between two supposedly perpendicular planes sometimes resulted in extremely shallow angles less than a tenth of a degree, which seems to be an artifact of the high resolution. These planes were instead measured with the Profile Measurement Machine (PMM). The resolution of this machine is $1 \times 10^{-3}$ inches and seems to be slightly more prone to user error due to the required use of one’s eye and discretion regarding the alignment of the crosshairs and work piece. Despite this, the measurements acquired from this machine seem to be sufficiently accurate. Calipers, gage pins and micrometers were also employed to verify the dimensions measured by these machines. The measurement tool with the lowest resolution is the caliper. With a resolution of $1 \times 10^{-3}$ inches, components measured with calipers cannot be measured with accuracy finer than this magnitude.

During the measurement process, it became apparent that certain components, particularly pin joints, were made with very loose tolerances. Because of this issue, certain measurements had to be taken with the machine fully assembled since the components settled into a certain position within the range of the loose tolerances.
4.3.2 Pin Play

The system had pin joints with enlarged hole diameters on the order of 0.01 inches. These introduced play and backlash that affected clamp location. Figure 4.2 illustrates the play caused by an oversized hole or undersized pin. The equation here determines the range of play.

\[ \Delta R_h = R_{h1} + R_{h2} - D_p \]  \hspace{1cm} (4.1)

\( D_p \) is the pin diameter and \( R_{h1} \) & \( R_{h2} \) are hole radii. The extremes of the allowable pin position will be used to determine the extremes of clamp position, and will be discussed in section 4.5.

4.3.3 Measurement Example

Figure 4.3 is a sample measurement detail of the M-101 component and results table for one of them. For this particular component, the CMM, gage pins and calipers were employed to obtain the actual dimensions. The hole diameter, \( A \), was measured with a gage pin and the distance \( B \) was measured with calipers instead of the CMM due to its location along a slanted face. Dimension \( B \) was calculated using the caliper measurement combined with the radius of dimension \( A \). The dimension \( F \) was also calculated in a similar fashion. A complete list can be seen in Table 4.1.
These included measurements, however, that required the positions. This involved adding components of the vectors created by each link. Certain links, such as vector $\overrightarrow{NO}$, have varying vector lengths, and as a result, require their own kinematic equations. Calculating clamp location based separately on the x-motion linkage and y-motion linkage provides two systems against which to compare values in addition to image comparisons. The vectors in Figure 4.5 were used to develop the equations. Most of the equations use point A as the origin. This is done in an effort to make image comparison easier.

<table>
<thead>
<tr>
<th>Dimension Label</th>
<th>Modeled Value (in)</th>
<th>Negative X-clamp Side</th>
<th>Positive X-clamp Side</th>
<th>Measurement Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual</td>
<td>Difference</td>
<td>Actual</td>
</tr>
<tr>
<td>A</td>
<td>0.375</td>
<td>0.388</td>
<td>-0.013</td>
<td>0.377</td>
</tr>
<tr>
<td>B</td>
<td>0.974</td>
<td>0.982</td>
<td>-0.008</td>
<td>0.977</td>
</tr>
<tr>
<td>C</td>
<td>0.689</td>
<td>0.768</td>
<td>-0.079</td>
<td>0.768</td>
</tr>
<tr>
<td>D</td>
<td>0.315</td>
<td>0.3145</td>
<td>0.0005</td>
<td>0.3130</td>
</tr>
<tr>
<td>E</td>
<td>0.630</td>
<td>0.628</td>
<td>0.002</td>
<td>0.628</td>
</tr>
<tr>
<td>F</td>
<td>0.315</td>
<td>0.309</td>
<td>0.006</td>
<td>0.310</td>
</tr>
</tbody>
</table>

Table 4.1: Modeled and actual component dimensions.

This process was repeated for all of the individual components. There were several measurements, however, that required the machine to be assembled in order to obtain. These included $\theta_{NO}$, crossbar offset ($d$) and parallelism of the crossbars shown in Figure 4.4.

4.4 Clamp Location: Geometric Equations of the System
Simple vector analysis of the linkages was employed to develop the equations for clamp positions. This involved adding components of the vectors created by each link. Certain links, such as vector $\overrightarrow{NO}$, have varying vector lengths, and as a result, require their own kinematic equations. Calculating clamp location based separately on the x-motion linkage and y-motion linkage provides two systems against which to compare values in addition to image comparisons.
In order to calculate both X and Y coordinates for the clamps at different extensions of the specimen, it was helpful to develop a flowchart of relevant coordinates and vector lengths (Figure 4.6). This ensured that two or more calculations did not create a circular reference with respect to one another.

Values like NO and PQ are lengths that vary with specimen extension. Subscripts refer to the specific coordinate being determined and the vector path employed, respectively. For example, $R_{y,x}$ is the y-coordinate of the point R as determined through the x-motion subsystem links. Note that $R_y$ can be determined through two different paths, and any
difference in results will cause binding since the system is over-constrained.

Vector analysis of loop 1 is used to determine the y-coordinate of point H and the resulting angles $\theta_{FE}$ and $\theta_{CD}$. While it would be most beneficial to develop equations to determine these angles as a function of $H_y$, it proved easier to determine $H_y$ as a function of these angles. It will be shown that for a particular $H_y$ value, $\theta_{CD}$ is different on both x-axis sides (positive and negative). The determination of $H_y$ therefore is important because it provides a base by which to compare the negative to positive x-clamp calculations. Equations (4.2) and (4.3) are the unmodified negative x-clamp equations that relate $H_y$ and the two unknown angles.

$$H_y = AB \sin \theta_{AB} + BC \sin \theta_{BC} + CD \sin \theta_{CD} + DE \sin \theta_{DE} + EF \sin \theta_{EF} + FG \sin \theta_{FG} + GH \sin \theta_{GH}$$  \hspace{1cm} (4.2)

$$0 = CD \sin \theta_{CD} + DE \sin \theta_{DE} + EF \sin \theta_{EF} + FG \sin \theta_{FG}$$  \hspace{1cm} (4.3)

By setting $\theta_{CD}$ to a range of achievable values and solving for $\theta_{FE} = f(\theta_{CD})$, these two equations can be solved simultaneously. Additionally, since angles $\theta_{AB}$, $\theta_{BC}$, $\theta_{DE}$, $\theta_{FG}$ & $\theta_{GH}$ are always either 0°, 90° or 180°, many sine and cosine terms simplify to 1 or 0. Equations (4.4) and (4.5) show the results of this manipulation.

$$\theta_{EF} = \cos^{-1} \left\{ -\frac{[CD \cos \theta_{AB} + FG]}{EF} \right\}$$  \hspace{1cm} (4.4)

$$H_y = BC + CD \sin \theta_{CD} + DE + EF \sin \cos^{-1} \left\{ -\frac{[CD \cos \theta_{AB} + FG]}{EF} \right\} + GH$$  \hspace{1cm} (4.5)
Since $H_y$ is the dependent variable in this portion of the analysis, it is also necessary to calculate its value as a function of the geometry on the $+x$. Each side’s geometry differs because of the crossbar offset. This can be seen with simple linked lines in SolidWorks® (Figure 4.7). This figure shows the offset and how it affects the y-motion links differently for each side. As a result, $\theta_{CD}$ and $H_y$ values correspond differently on the $+x$ side than the $-x$ side. It should be
noted that the range of $\theta_{CD}$ is limited by clamp-clamp and clamp-pillowblock contact (Figure 4.8). The image here shows the highest and lowest positions of the machine due to these limitations. The resulting range of top crossbar heights is 2.2 inches (4.8 to 7 inches).

Equations (4.6) and (4.7) are the positive x-clamp equations that relate $H_y$ and the two unknown angles. It should be noted that the vectors are slightly different than for the negative x-clamp in order to keep the point $H$ at the most advantageous location on the top crossbar.

$$H_y = AB \cdot \sin \theta_{AB} + BC \cdot \sin \theta_{BC} + CD \cdot \sin \theta_{CD} + DE \cdot \sin \theta_{DE} + EF \cdot \sin \theta_{EF} + GH \cdot \sin \theta_{GH} + HI \cdot \sin \theta_{HI}$$

$$0 = CD \cdot \cos \theta_{CD} + DE \cdot \cos \theta_{DE} + EF \cdot \cos \theta_{EF} + GH \cdot \sin \theta_{GH} + HI \cdot \sin \theta_{HI}$$

Again, by setting $H_y$ to a range of achievable values and solving for $\theta_{FE} = f(\theta_{CD})$, these two equations can be solved simultaneously. Again, since angles $\theta_{AB}$, $\theta_{BC}$, $\theta_{DE}$, $\theta_{GH}$ & $\theta_{HI}$ are either $0^\circ$, $90^\circ$ or $180^\circ$, many sine and cosine terms simplify to 1 or 0. Equations (4.8) and (4.9) show the simplified results of this manipulation.

$$\theta_{FE} = \cos^{-1}\left\{\frac{[HI-CD \cdot \cos \theta_{CD}]}{EF}\right\}$$

$$H_y = BC + CD \cdot \sin \theta_{CD} + DE + EF \cdot \sin \cos^{-1}\left\{\frac{[HI-CD \cdot \cos \theta_{CD}]}{EF}\right\} + GH$$

Now that a base has been determined to compare both sides of the machine, equations for the clamp (x, y) coordinates can be developed. Both sides benefit from the same analysis of loop 2 in the vector diagram. The same method of analysis that was used in the loop 1 calculations can be employed here. Again, there are three unknowns and two equations. Since one of these unknowns is $\theta_{CD}$, the same range can be used from the above analysis to ensure accurate
correlations. The other two unknowns are the lengths NO and PQ (Figure 4.5). The two
trigonometry equations (4.10) and (4.11) are the standard component equations simplified due to
the fact that a section of the loop is fixed. This fixed section is calculated once and represented
by equations (4.12) and (4.13).

\[
0 = -CD \cdot \cos \theta_{CD} - DQ \cdot \cos \theta_{DE} + PQ \cdot \cos \theta_{PR} + OP \cdot \cos \theta_{OP} + NO \cdot \cos \theta_{NO} + CN_x
\]

(4.10)

\[
CN_x = MN \cdot \cos \theta_{MN} + LM \cdot \cos \theta_{LM} + BL \cdot \cos \theta_{BL} - BC \cdot \cos \theta_{BC}
\]

(4.11)

\[
0 = -CD \cdot \sin \theta_{CD} - DQ \cdot \sin \theta_{DE} + PQ \cdot \sin \theta_{PR} + OP \cdot \sin \theta_{OP} + NO \cdot \sin \theta_{NO} + CN_y
\]

(4.12)

\[
CN_y = MN \cdot \sin \theta_{MN} + LM \cdot \sin \theta_{LM} + BL \cdot \sin \theta_{BL} - BC \cdot \sin \theta_{BC}
\]

(4.13)

Solving equation (9) for PQ results in equation (13) below.

\[
PQ = \frac{[CD \cdot \cos \theta_{CD} + DQ \cdot \cos \theta_{DE} - OP \cdot \cos \theta_{OP} - NO \cdot \cos \theta_{NO} - CN_x]}{\cos \theta_{PR}}
\]

(4.14)

Finally, equation (4.14) can be substituted into equation (4.12) to eliminate the variable, PQ. Equations (4.15) and (4.16) show this manipulation along with a solution for NO.

\[
0 =
-CD \cdot \sin \theta_{CD} - DQ \cdot \sin \theta_{DE} +
\left\{ [CD \cdot \cos \theta_{CD} + DQ \cdot \cos \theta_{DE} - OP \cdot \cos \theta_{OP} - NO \cdot \cos \theta_{NO} - CN_x] \right\} / \cos \theta_{PR} \cdot \sin \theta_{PR} +
OP \cdot \sin \theta_{OP} + NO \cdot \sin \theta_{NO} + CN_y
\]

(4.15)

\[
NO = \frac{[CD \cdot \sin \theta_{CD} + DQ \cdot \sin \theta_{DE} - [CD \cdot \cos \theta_{CD} + DQ \cdot \cos \theta_{DE} - OP \cdot \cos \theta_{OP} - CN_x] \cdot \tan \theta_{PR} - OP \cdot \sin \theta_{OP} - CN_y]}{[\sin \theta_{NO} - \cos \theta_{NO} \cdot \tan \theta_{PR}]}
\]

(4.16)
Now that the location of the pillow block has been determined on the angled rod as well as the length of PQ within loop 2, locations of the clamps can be determined. Equations (4.17) through (4.19) represent the clamp coordinates based on either the y or x-motion subsystem of the x-axis subsystem.

\[
R_{x-x} = AB \cdot \cos \theta_{AB} + BL \cdot \cos \theta_{BL} + LM \cdot \cos \theta_{LM} + MN \cdot \cos \theta_{MN} + NO \cdot \cos \theta_{NO} + OP \cdot \cos \theta_{OP} + PR \cdot \cos \theta_{PR}
\]  

(4.17)

\[
R_{y-y} = AB \cdot \sin \theta_{AB} + BC \cdot \sin \theta_{BC} + CD \cdot \sin \theta_{CD} + \frac{1}{2} DE \cdot \sin \theta_{DE} + QR \cdot \sin \theta_{PR} \pm \Delta R
\]  

(4.18)

\[
R_{y-x} = BL \cdot \sin \theta_{BL} + LM \cdot \sin \theta_{LM} + MN \cdot \sin \theta_{MN} + NO \cdot \sin \theta_{NO} + OP \cdot \sin \theta_{OP} + PR \cdot \sin \theta_{PR}
\]  

(4.19)

It should be noted that \( R_{x,y} \) is calculable but irrelevant since the x-coordinate is primarily defined by the angled shaft in the x-motion subsystem. While \( R_{y,y} \) is also primarily defined by the angled shaft, its value is important in understanding possible issues of binding.

4.5 Clamp Location Results

Both the x and y-clamp calculations produce similar results. The magnitude of offset for one x-clamp with respect to the other is similarly seen with the y-clamps. This makes sense because both clamp sets, x and y, are mirrored across an axis of symmetry. Figure 4.9 is a graph illustrating the height of the x-axis clamps as estimated by the calculations. In addition to the nominal clamp location, extremes about nominal that could occur due to play at the joints (as discussed in Section 4.3.2) are represented as dashed lines. Since a new specimen shape and scale has been chosen, a new operating range of the machine is defined; the new and old operating ranges are indicated with vertical lines. From this it is clear why detection of the misalignment was more difficult in the previous study. The previously discussed SolidWorks
vector model was used to estimate clamp height. Finally, in the same way that clamp y-coordinates were calculated, x-coordinates were also determined and compared with the SolidWorks model, which again match very well.

### 4.6 Clamp Location Image Verification and Comparison

Next an image analysis was compared to the calculated clamp locations. Using the same camera and method employed for analyzing the central diamond region of a planar specimen, each clamp was given a unique marking at the central tip of the clamp, as shown in Figure 4.10.

![Figure 4.9: Calculated x-clamp pathlines.](image)

![Figure 4.10: Image analysis picture with reference points circled in red.](image)
A different origin was used than in the calculations outlined earlier in this chapter because it seems that as a point gets further from the center of the camera frame, the effects of perspective seem to grow. It seems this may contribute to the difficulty in consistently identifying the same reference point (i.e.: clamp edge) in a sequence of images. Despite a difference in origin, the image measurements were easily converted to match the calculation origin. Figure 4.11 illustrates how the calculations and image analysis data points compare. Because of backlash created in the joints, extension and contraction paths of both clamps are different.

If the calculations are done correctly, the predicted nominal position will travel between the extension/contraction paths defined by the images. The positive x-clamp calculation seems to do this very well. The negative x-clamp calculation predictions are higher than expected but
still falls almost entirely within the expected range. For each clamp, a maximum and minimum path (Figure 4.12 and Figure 4.13) was also calculated as discussed in section 4.3.2. These lines closely follow the extension and contraction points, which is the expected result.

![Figure 4.12: Calculated -x clamp bounding lines with image data.](image1)

![Figure 4.13: Calculated +x clamp bounding lines with image data.](image2)
The displacement load ratio associated with the misaligned system discussed here is shown graphically in Figure 4.14. The load ratio value of 0.86 compares well with the load ratio previously measured by Joe Ferrara, but based on the analysis presented in Section 3.6 this is not acceptable, since it could result in stress predictions that are off by 15-20%.

Also, it should be noted that while the image analysis system is a proven measurement method, there still exists a measurement resolution of 3-4 pixels. With each pixel accounting for about 0.125 mm, it is possible to have a set of data points off by 0.375 mm simply from where the image analysis program believes the reference point to be. There is also human error when defining these reference points, which could lead to some additional error. Overall, these calculations estimate clamp location well and a tightening of the system tolerances should bring extension and contraction paths closer to the estimated path.
5. Minimum Constraint Design

5.1 Introduction

The goal of the minimum constraint design (MCD) method is to develop a component or device that operates with a particular number of constraints. This method is especially helpful for complex linkage systems like the biaxial test stand. Variations can occur in areas such as component dimensioning, manufacturing or assembly tolerances, or system operation parameters. If designed, manufactured and assembled perfectly, an over-constrained system may in fact operate without flaw. This situation, however, is nearly impossible because of the complexities in achieving perfection in the above variable areas. Therefore it is advantageous that this system be fully defined by the configuration a single degree of freedom (DOF), independent of any potential misalignment.

To have a minimally constrained system, all DOF must either be constrained or explicitly controlled. In three dimensional space an object has six DOF, three translational and three rotational. Each one of these DOF can be constrained, thereby decreasing the number of DOF for that joint. Based on the type of joint between two adjacent components, a certain number of DOF associated with those components are constrained. Table 5.1 details the types of joints used on the original system design and the number of constraints that they introduce.

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>No. of Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>6 (3 Translation, 3 Rotation)</td>
</tr>
<tr>
<td>Pinned</td>
<td>5 (3 Translation, 2 Rotation)</td>
</tr>
<tr>
<td>Sliding Pin</td>
<td>4 (2 Translation, 2 Rotation)</td>
</tr>
<tr>
<td>Ball-Socket</td>
<td>3 (3 Translation, 0 Rotation)</td>
</tr>
</tbody>
</table>

Table 5.1: Joint Types and No. of Constraints

With respect to a system of links comprising a mechanism, the connectivity of neighboring links determines the number of constraints and therefore the number of system DOF. An under-
A constrained system has a greater number of DOF than number of components which are explicitly controlled. In these systems, defining the explicitly controlled components is insufficient. Certain components will still be able to take on various orientations, which is not a desired attribute. Conversely, an over-constrained system has fewer DOF than the number of explicitly controlled components such as the crossbar height.

![Figure 5.1: Link systems with different number of DOF: (a) 0, (b) 1, (c) 2.](image)

Simplified examples of constrained linkage systems can be seen in Figure 5.1. In a 2-dimensional space, all joints in this figure are considered pinned. The 2-link mechanism in (a) is fully defined (0 DOF) simply by the lengths of the legs and location of the two grounded pins. The 3-link mechanism in (b) is defined by the same parameters in (a) but also requires the angular orientation of one of the legs to be defined. Finally, the 4-link mechanism in (c) requires two angular leg orientations to be defined, hence 2 DOF. The table below shows the application of the method that results in the aforementioned final DOF counts.

<table>
<thead>
<tr>
<th>Link System</th>
<th>Introduced Component</th>
<th>Connecting Component</th>
<th>Joint Type</th>
<th>No. Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>GND</td>
<td>Pin</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>GND</td>
<td>Pin</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>c</td>
<td>GND</td>
<td>Pin</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Total DOF = 2<em>3 = 6, Total Const. = 6, Final DOF</em> = 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GND</td>
<td>Pin</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Pin</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Pin</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Total DOF = 3<em>3 = 9, Total Const. = 8, Final DOF</em> = 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GND</td>
<td>Pin</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Pin</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Pin</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Total DOF = 4<em>3 = 12, Total Const. = 10, Final DOF</em> = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Final DOF = TOTAL DOF – TOTAL CONSTRAINTS

Table 5.2: MCD analysis of simplified examples.
5.2 Binding Analysis of Original System

Binding has been discussed as a problem in the initial design of the machine. Figure 5.2 illustrates again the binding discussed in section 2.4.4. While the y-axis clamps have always been directly connected to the crossbars smoothly traveling along the linear screws, the x-axis subsystem of links and ball bushings seemed to bind at intermittent points along the operation path.

The initial design of the system used a fixed joint at point P in Figure 5.3 to connect these two motion subsystems. The schematic shown is not in equilibrium but instead is still allowed to translate within the x-y plane. As a result, two ball bushings were traveling along different shafts with a rigid connection between them. The rigid connection and non-parallel motion of

![Figure 5.3: Proposed torque without x-axis alignment shaft.](image-url)
these two ball bushings seemed to be the cause for a torque load perpendicular to their main axes (Figure 5.3). This kind of torque creates binding between the bushings and the shafts on which they ride. It should be noted that the linear ball bushing at point Q also experiences a torque, which may introduce more binding. While similar systems (Breiu et al. 2007) have introduced an alignment rod spanning from one x-clamp to another, a crossbar offset is not possible since the clamp shafts are no longer collinear with a fixed joint.

5.3 MCD Analysis of Original System Design

The original system design was analyzed in an effort to prove that the system was, in fact, over-constrained. The first step in this process was to define the individual components of the system and the types of joints that connected them. Since each individual component starts with six DOF in three dimensional space, the product of the number of components and six DOF results in the total DOF of the system.

If any number of components are analyzed and found to be over-constrained, the entire system is therefore over-constrained. There is no way to properly constrain an over-constrained section of a system simply by adding additional components with the appropriate number of DOF. Therefore, a single side of x-axis clamp system was investigated, making for a simpler analysis. Figure 5.4 illustrates how the individual components of the system were defined. It should be noted that components fixed to one another were deliberately lumped into a single component since their DOF and constraints cancel out by definition. A 2-dimensional MCD analysis of the y-motion subsystem is identical to the mechanism in Figure 5.1 (b) but a 3-dimensional analysis of the same fully pinned system often introduces more constraints than DOF.
The analysis is begun by grounding a component, usually number 1. Grounding fixes it in space, and therefore fully constrains it in all DOF. From here, the second component can be introduced. Each time a new component is introduced, 6 DOF is added to the total system DOF count. Additionally, constraints are also introduced based on the type of joint that connects this new component to the overall system. Since component 2 is constrained to component 1 by a slide joint, it has four constraints and two remaining DOF. This can be verified conceptually by the recognition that the ball bushing between the components can slide along and rotate around the shaft. Now that the interaction between components 1 and 2 has been defined, components 2 and 3 can now be analyzed. Again, the ball bushing is a slide joint, resulting in two remaining DOF. Table 5.3 outlines this process for all of the investigated components.

<table>
<thead>
<tr>
<th>Introduced Component</th>
<th>Connecting Component</th>
<th>Connecting Joint Type</th>
<th>No. of Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
</tbody>
</table>

Total DOF = 6*6 = 36

Table 5.3: MCD Analysis of original system.

This analysis shows that there are -4 DOF (36 – 40 = -4), which means the system is over-constrained since for this analysis, DOF = 0 is desired. It should be noted that by fixing the top bar (component 6) to ground, we should end up with an equal number of DOF and constraints so
that when we allow the crossbar to translate along the linear screws, one DOF is released, giving
the system its required one significant degree of freedom. Since this portion of the overall
system has been found to be over-constrained, there is no need to further investigate a larger
portion of the system. Instead, modifications must be made to the proper joints to release
constraints that allow the system to operate under near-perfect conditions without compromising
the intended operating goal of the system by leaving it under-constrained.

5.4 System Modifications and Resulting MCD Analysis

Modifying some of the joints of the system will release some constraints on the system. If
chosen carefully, this will make it minimally constrained while avoiding the need to change the
overall machine operation. As stated before, another concern was the fixed joint at point P
(Figure 5.5), where the x-clamp shaft (S-10x) meets the pillowblock (PB-100). Since a release
of constraints was needed, several of the pin joints (1 DOF) were replaced with ball joints (3
DOF). Ball joints were placed at points P, C and F in Figure 5.5. In addition, the x-axis
alignment shaft was added into the analysis with slide joint connections. The proposed system is
shown in Figure 5.6, with corresponding list of DOF, in Table 5.4.

With the introduction of ball and slide joints, it is
particularly important to understand the concept of
significant versus allowable DOF. A system can
quickly appear under-constrained with the
introduction of these types of joints. The result from
Table 5.4 leaves 4 DOF to be defined. These four
are: axial rotation of components 4, 11 & 13 (rear

Figure 5.5: Proposed ball joint locations.
alignment shaft) and axial translation of component 13. These DOF have no effect on the clamp locations, and are therefore not considered significant DOF. Finally, by allowing the top crossbar to translate along the linear screws the system now has one significant DOF, thus satisfying the goals of the design method. With that said, axial rotation of components 4 and 11 is not desired, as this will rotate the clamps out of plane and deform the specimen in an undesired direction. Methods for addressing this issue are discussed in section 5.5. While the system seemed perfect on paper, the next step was to create a CAD model and determine flaws in the design that didn’t present themselves on paper. Upon completion of the modeling, no new issues were apparent.

**Figure 5.6: Modified system component labeling.**
While the theoretical analysis of these modifications was relatively simple, implementation was not so straightforward. The issues of pin play and accurate machine assembly needed to be addressed as well. To do this, four Motion Links (L-102) and Link Mounts (M-102) were remanufactured to new dimensions and both crossbars and X-motion Pillowblocks (PB-100) required modifications to accept the new links and mounts. The modification of inlaying the link mounts into the crossbars made assembly of the system much more accurate and easily repeatable. Precision pins and shoulder bolts also helped to solve backlash problems. Upon complete assembly of the system, preliminary studies showed a greatly reduced amount of backlash and binding. Some backlash still existed in the linear ball bushings and cannot be removed without replacement with tighter bushings.

Upon further inspection of the assembled system, an issue finally presented itself. The modified section of the system (the x & y-motion subsystems) sagged below the crossbars and

<table>
<thead>
<tr>
<th>Introduced Component</th>
<th>Connecting Component</th>
<th>Connecting Joint Type</th>
<th>No. of Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ground</td>
<td>Fixed</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>2 Ground</td>
<td>Fixed</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>3 1 Slide</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>4 3 Ball</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>6 4 Slide</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>7 6 Pin</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>7 1 Ball</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>5 6 Pin</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>5 2 Ball</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>10 1 Ball</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>12 1 Slide</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>11 12 Ball</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>9 11 Slide</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>10 9 Pin</td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>8 9 Pin</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2 8 Ball</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>13 6 Slide</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>13 9 Slide</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td><strong>Total DOF = 6*13 = 78</strong></td>
<td><strong>Total = 74</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: MCD analysis of the modified system.

### 5.5 Implementation of System Redesign

While the theoretical analysis of these modifications was relatively simple, implementation was not so straightforward. The issues of pin play and accurate machine assembly needed to be addressed as well. To do this, four Motion Links (L-102) and Link Mounts (M-102) were remanufactured to new dimensions and both crossbars and X-motion Pillowblocks (PB-100) required modifications to accept the new links and mounts. The modification of inlaying the link mounts into the crossbars made assembly of the system much more accurate and easily repeatable. Precision pins and shoulder bolts also helped to solve backlash problems. Upon complete assembly of the system, preliminary studies showed a greatly reduced amount of backlash and binding. Some backlash still existed in the linear ball bushings and cannot be removed without replacement with tighter bushings.

Upon further inspection of the assembled system, an issue finally presented itself. The modified section of the system (the x & y-motion subsystems) sagged below the crossbars and
the y-axis clamps attached to them. Due to the dimensional tolerances and design of the radial bearings, it seems that the pin joints used in the Motion Links (5, 7, 8 & 10 in Figure 5.5) were acting more like ball joints. The radial ball bearings chosen for this joint were not meant to handle torques about the radial axis. As a result, the inner bearing sleeve seems to have twisted inside the outer sleeve due to the weight of the system. To support this theory, ball joints were placed into the CAD model at these locations and the resulting operation of the system was analyzed. The model showed that the system could now move in and out of the major operating plane in an identical fashion to what was being physically observed.

Several solutions were considered. These included using multiple miniature ball bearings at each pin joint to create radial bearing loads, or using low friction sleeves to achieve the same effect. The other solution, which was chosen mostly for its ease of implementation but also cost, was a platform upon which the Control Pillowblocks could slide freely.

This platform (Figure 5.7) was developed using a machined aluminum plate and precision telescoping legs. To ensure that this new component would not change the final DOF of the system, a new MCD analysis was conducted. The results in Table 5.5 confirm that only 4 DOF remain and have already been accounted for. It should be noted that this analysis required the pin joints to be defined as ball joints since their out-of-plane motion could not be ignored. In an effort to reduce friction between the Control Pillowblocks (component 6 in Figure 5.5) and the
platform, felt material was placed in between. The felt did not completely eliminate friction, which is why differing extension and contraction paths exist. However, the felt smoothed the motion of the system (Figure 5.8). Backlash is the most prominent during a change in loading direction. The remaining backlash is caused by the Thomson linear ball bushings and the small amount of play

<table>
<thead>
<tr>
<th>Introduced Component</th>
<th>Connecting Component</th>
<th>Connecting Joint Type</th>
<th>No. of Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>Ball</td>
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<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
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<td>10</td>
<td>1</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>Ball</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>Ball</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>9</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>14 (Plate)</td>
<td>GND</td>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>14 (Plate)</td>
<td>6</td>
<td>Planar Slide &amp; Rot</td>
<td>3</td>
</tr>
<tr>
<td>14 (Plate)</td>
<td>9</td>
<td>Planar Slide &amp; Rot</td>
<td>3</td>
</tr>
</tbody>
</table>

Total DOF = 6 * 14 = 84, Total Constraints = 80, Final DOF = 4

Table 5.5: MCD analysis of modified system with platform.

Figure 5.8: Improved clamp paths with respect to crossbar height.
still apparent in the pin and ball joints. Manual adjustment will need to be done at the beginning of each test to make sure the specimen is mounted with equal x- and y-clamp gaps. Overall, the backlash (Figure 5.8) has been reduced but more importantly, the x-clamps now move symmetrically about the y-axis and no binding occurs. The load ratio associated with the redesigned system discussed here is shown graphically in Figure 5.9. This redesigned system shows a significant reduction in binding and test parameter variations discussed in Chapter 3.

Figure 5.9: New X:Y displacement load ratio as measured by image analysis.
6. Experimental Methods

6.1 Introduction

The following sections discuss the tools and procedures used to obtain experimental stress-strain data for the thermoplastic elastomer, Ultraflex (medium hardness). Two new molds, CSS and CCS, are introduced in addition to detailed processes for specimen creation, marking and test fixture mounting. The molding, marking and mounting processes can be adapted to most any other open-moldable elastomer. Finally, the data collection procedure is outlined for the newly developed Labview software and camera hardware.

6.2 Molds

Two new aluminum molds have been made, since a new specimen design has been developed. Prior to the numerical specimen design analysis in Chapter 3, A CSS mold was machined similar to geometry by Sacks et al. Dimensions were scaled to work with 1 inch wide clamps, the only dimension not scalable. A detailed drawing of this mold can be seen in Figure 6.2. Since sharp corners were desired for the CSS specimen, two side plates were added after machining, which also allowed for easy removal of the specimen. No longer are sharp blades...
needed to remove the specimen. Instead, the removal of the side plates gives access to an edge from which the specimen can be lifted out. The design of this mold influenced the way that the second mold was designed for the CCS specimen.

The CCS mold was CNC machined after the specimen design analysis was complete and the best geometry was chosen. Again, the clamp width was a constraint that needed to be met so the only difference between this mold and the previous is that a 4 millimeter fillet radius was chosen instead of 1 millimeter. Exact dimensions of the mold can be seen in Figure 6.3. Designed in SolidWorks®, plates similar to those on the CSS mold were used. Again, this made removal of the specimen much safer in terms of avoiding accidental cuts from a sharp blade. Figure 6.4 shows the mold with the removable side plates. In addition to a new mold, a new stencil was created for marking out the central diamond region where equibiaxial stresses are expected. Since a new
diamond region size has been proposed, a stencil (Figure 6.5) has been created to fit into the new mold, creating a more repeatable central diamond region. A SolidWorks® model of the stencil in the mold is shown in Figure 6.6.

6.3 Creation of the New Specimen
These materials and tools needed to create a specimen:

- 5 to 20 grams of Medium Ultraflex
- Scissors
- Weighing scale
- Mold
- Hot plate
- Paperclip or other thin metal pick
- Metal tongs
- Sheet metal shield (large enough to cover mold top)
- Cooling fan
- Stencil
- Marker
- Razor blade
- Plastic sheet (large enough to hold completed specimen)

The process for creating a specimen is:

1. In a well ventilated area, turn on and set hot plate to 200°C.
2. Cut enough ¼ inch cubes of Medium Ultraflex to create the desired thickness (6 grams \(\approx\) 2mm thickness).
3. Ensure plates at leg tips are in place with screws lightly tightened against them.
4. Distribute cubes evenly within the mold.
5. Place mold onto hot plate for approximately 10 minutes.
6. If air bubbles occur in the effective (unclamped) region of the specimen, use the paperclip to move or remove the bubbles.
7. Once Ultraflex has completely melted and is evenly distributed, use the metal tongs to transfer the mold to the cooling fan.
8. Place the sheet metal shield over the mold to protect the melted Ultraflex from being blown out by the fan.
9. Turn the fan to the cool setting (if possible) and direct downward on top of the sheet metal shield and mold for 5 minutes.
10. After 5 minutes, the specimen should be adequately solidified and the sheet metal shield can be removed to speed up the cooling process.
11. Once the specimen is solidified enough for removal, turn off the cooling fan and place the stencil on top of the specimen.
12. Mark all four reference points by lightly touching the marker to the specimen several times at each point.
13. Remove the stencil and loosen the screws to remove the leg tip plates.
14. Run a razor blade between the four plates and specimen to completely separate the two.
15. At the leg tip, lightly rub the top of the specimen toward the center to facilitate separation between the specimen and mold.
16. Separate one leg complete leg from the mold and slide the plastic sheet in between.
17. Repeat step 16 for an adjacent leg.
18. Now that half of the specimen is separated, the remainder can be slowly pulled off of the mold in one motion.
19. Approximately half of the leg length will need to be removed with scissors to allow for clamping of the specimen at the base of the fillets.

6.4 New Issues in Mounting of the Specimen
Similar issues were experienced to those explained in the research by Ferrara (Ferrara 2009) such as difficulty due to tackiness of the specimen. One issue in particular that was not mentioned in the previous study was the deformation of the specimen around the clamps. Upon tightening of the clamps, the specimen, depending on thickness, deforms in the out-of-plane direction (Figure 6.7). This seems to be due to the incompressibility of the specimen. Once compressed within a clamp, Ultraflex seems to expand in the planar direction, thus pushing material out from under the clamps. This seems to create the buckling effect that pushes the specimen out of plane. The current solution is to extend the clamps manually until the specimen returns to a completely planar geometry. Care is taken not to introduce pre-strain.
6.5 Data Collection Procedure
With the introduction of a new Labview VI, the data acquisition time has been greatly reduced compared with the previous method (Ferrara 2009). All test parameters are input prior to the start of the test. The process for data collection is outlined here:

1. Use the program, SMC60WIN (Anaheim Automation, Anaheim CA), and a scale to move the top crossbar to the initial starting height of 5.15 inches (bottom clamp length + top clamp length + specimen length = 1.92 + 1.92 + 1.31 inches).

2. Mount specimen in test fixture. Use SMC60WIN again to carefully expand the specimen to remove out-of-plane deformation discussed in the previous section.

3. Open MAX (National Instruments) and locate the Pixellink® icon in the left menu.

4. Click the Grab button to see real-time camera capture and ensure the specimen is in focus and well-lit. If virtual changes to the image (i.e.: shutter speed, exposure, etc.) are needed, this must be done in the Labview VI block diagram mentioned in step 5. Close MAX upon completion of this step.

5. Open the Labview VI, *Biaxial Membrane Tester with Image Grab.vi*.

6. In the Configuration tab entries for Filename Prefix, Move Parameters and DAQmx Global Virtual Channels should be verified. For maximum strain (maximum machine
extension) the motor controller can move 3500 steps with all micro-stepping switches set to ON. The Move Parameters should be entered as:

a. Move Increment: 50  
b. Number of Moves: 70  
c. Number of Cycles: 1 (for one set of extension and contraction data)  
d. Limits (lbf): 2.0

DAQmx Global Virtual Channels are $X$ Force (lbf) and $Y$ Force (lbf), not Simulated.

7. Release the emergency stop switch to the motor controller (Note: If motor engagement is not heard, check that dashpot on motor controller is set to the 5 volt max).

8. Click the Labview run button and select the folder into which the data will be saved.

A pixellink® camera was required for compatibility with the new Labview VI but the same lens used as in previous studies. Images in the tiff file format are automatically generated, numbered and ready to be processed by the Vision assistant. No new methods for analyzing the images have been introduced since the method was introduced in Joseph Ferrara’s research. As a result, the same level of error (3-4 pixels or 0.375mm) can be expected in the measurements.
7. Material Characterization Results and Discussion

7.1 Nominal Case

The results of the data collection and characterization process (sections 6.5 and 2.4.2, respectively) are discussed here. Figure 7.1 shows the ANSYS-estimated 2-parameter Mooney-Rivlin curve-fit to the raw experimental data ($\sigma_{\text{edge}}$-\$\epsilon_{\text{diamond}}$). The correlating ANSYS-estimated coefficients are shown in Table 7.1.

![Figure 7.1: 2-parameter MR curve-fit for the raw experimental stress-strain data.](image)

<table>
<thead>
<tr>
<th>Mooney-Rivlin 2-parameter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C10</td>
<td>5.06E-03</td>
</tr>
<tr>
<td>C01</td>
<td>-3.69E-05</td>
</tr>
</tbody>
</table>

Table 7.1: 2-parameter MR coefficients for the raw experimental stress-strain data.

As discussed before, since these equibiaxial coefficients were estimated as using what is known to be non-equibiaxial data, the results from this initial simulation can only be used to verify the stress-strain response equibiaxial region. Figure 7.2 illustrates the good match between ANSYS predicted central diamond response and the experimental data that was input into ANSYS as equibiaxial.

![Figure 7.2: Raw edge stress data modeled and verified as equibiaxial central diamond stress.](image)
A SDF of 0.658 was calculated from the initial simulation and the resulting equibiaxial central diamond curve-fit and coefficients are shown in Figure 7.3 and Table 7.2 respectively. The second simulation, now with predicted true biaxial properties can be compared with experimental central diamond predictions. Figure 7.4 illustrates this comparison. The experimental central diamond prediction (red points) matches well with the ANSYS predicted central diamond response (red line) as expected. Additionally, the ANSYS prediction of edge stress (red dash) matches well with the original experimental data (blue dots).

**Table 7.2**: 2-parameter MR coefficients for predicted equibiaxial stress-strain data.

<table>
<thead>
<tr>
<th>Mooney-Rivlin 2-parameter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C10</strong></td>
<td>3.33E-03</td>
</tr>
<tr>
<td><strong>C01</strong></td>
<td>-2.43E-05</td>
</tr>
</tbody>
</table>

**Figure 7.3**: 2-parameter MR curve-fit for the central diamond predicted data.

**Figure 7.4**: True biaxial response prediction of Medium Ultraflex with central diamond and edge stress verification.
7.2 Validation: Boiling Flask

The boiling flask is an inflatable membrane geometry that has been used to represent a simplified human lung alveolar. Experimental pressure and displacement data (Ferrara 2009) has been obtained with the boiling flask geometry in an effort to validate the material properties of Medium Ultraflex estimated through a planar biaxial specimen tests.

Figure 7.5 shows the new results for the boiling flask simulation using the equibiaxial estimates for Mooney-Rivlin 2-parameter coefficients (Table 7.2). The characterization method proposed here estimates that experimental data should show a more compliant model. The method of calculating the applied pressure on the boiling flask presented in section 2.4.1 was developed during this study and therefore not employed for the above experimental results.
Instead, the equation used to generate the plotted pressures was

\[ P_{in} - P_{out} = P_{ch}(0) - P_{ch}(t) \]  

which is an incomplete version of equation (2.13). The change in pressure due to the glycerin in the graduated cylinder, \( \rho g \Delta h_{gc}(t) \), was not considered and therefore over-predicts the applied pressure. Since neither the pressure nor height of the glycerin in the graduated cylinder was recorded for the data in the figure above, it cannot be modified to represent the true applied pressure. Unfortunately, the chamber pressure sensor requires recalibration so a complete set of measurements cannot be obtained at this time. It is reassuring, however, that the determination of actual applied pressures is expected to shift the experimental data to the right (lower pressures required to achieve the same displacement), which will bring it closer to the predictions generated for this study.
8. Conclusions & Recommendations

8.1 Planar Biaxial Testing

This study has focused on improving the characterization process of a hyperelastic material under biaxial tension. This has been achieved through improvements in reliability and accuracy of the tensile test fixture as well as planar biaxial specimen geometry. The MCD method used to improve the operation of the test fixture has proved successful. A displacement load ratio much closer to 1:1 has been achieved as well as smooth and symmetric clamp extension.

The specimen design improvements have increased the experimentally achievable strain with a higher SDF and larger equibiaxial region. Understanding the degree to which characterization results are affected by variations in some test parameters has also been a focus in order to better understand the sensitivity of this material to non-ideal loading.

A process for modifying raw experimental data into accurate biaxial results is also a significant contribution of this work. A deeper understanding of stable hyperelastic material properties has been achieved and will allow future studies to deal more easily with issues that arise when using unstable material properties.

Several recommendations for the biaxial tensile testing are provided here. As stated in the previous study, the Ultraflex material is difficult to manipulate and mark (for strain readings) due to its tackiness and compliance. A material with little to no tackiness would be more manageable. Additionally, choosing a material whose properties have been previously studied to a greater degree than Ultraflex may provide more data with which to compare results. Research into these engineered materials has been started.
In order to achieve greater strains for the currently used *CCS no leg* specimen, pillowblocks on the test fixture would have to be modified to avoid interference with the x-axis clamps as they extend outward.

### 8.2 Boiling Flask

It is believed that the experimentally measured pressures for the boiling flask inflation test data are incorrect. The pressure reading at the base of the chamber is not the only factor in determining the pressure exerted on the walls of the boiling flask specimen. Modification of this data should show a more compliant model and bring the experimental data closer to the newly predicted results.

Simulation convergence issues arise sooner for more compliant material properties. The SDF-modified coefficients proposed in this study can only achieve a 300 Pa pressure load. The raw edge-stress coefficients can only achieve 500 Pa, in comparison to the extremely stiff material properties proposed by Joe Ferrara, which could achieve a simulated pressure of 700 Pa. Improvements to the boiling flask model need to be conducted in order to achieve convergence at greater pressures.

### 8.3 Future Work

The next steps toward reaching the long-term goals of this research include additional analysis of lung PV equations with a focus on stress-strain curve correlations and physiological lung to engineering material property relationships. Continued research into alternative materials designed specifically for engineering applications may help with these correlations. The lung strain estimations discussed in chapter 2 are believed to be overestimates of the strain experienced by the alveoli and therefore require refinement in order to quantify the true desired response of the lung alveolar regions.
Regarding material characterization, the instability of the newly proposed coefficients should be examined, perhaps from the standpoint of the ANSYS hyperelastic curve-fit tool and the optimization methods used to estimate the coefficients. Depending on the optimization methods used, different coefficients may be estimated. Hyperfit® (P. Skacel, 2010) is a program specifically designed for hyperelastic curve-fitting that seems to have the capability of adjusting these optimization methods.

Stress-strain data collected on a 45-degree rotated central diamond should be consistent with the findings here on the loading axes. Both the loading axes and 45-degree axes are symmetric about the specimen so stress-strain results should be identical due to the material’s isotropic nature. In addition, true stress-strain results may be more accurate due to the large deformation.

While convergence issues with the planar biaxial simulations have been improved, the boiling flask convergence issues have increased. Even though they are more stable, the new coefficients limit the boiling flask model to 300 Pa, whereas previous coefficients achieved upwards of 700 Pa. This increased convergence issue is not well understood. An analysis of the ANSYS model and changes to the boundary conditions may improve convergence. Additionally, volume data from the boiling flask simulations would be very helpful in the correlations discuss earlier in this section.

Regarding the validation of these material properties, the chamber pressure sensor must be recalibrated and the graduated cylinder pressure sensor should be upgraded to a more robust and higher resolution model.
References


Appendix

Simulation Setting and Data Extraction Process

Preprocessor

Element Type > Add/Edit/Delete > [Add] > Shell281 > [OK] > [Options] > Element Stiffness: Membrane Only > [OK] > [Close]

Real Constants > Add/Edit/Delete > [Add] > [OK] (selects the only element type available, Shell281) > Real Constant Set No.: 1 (by default) > Shell thickness at node I TK(I): enter specimen thickness > [OK] > [Close]

Material Props > Material Models > Structural > Nonlinear > Elastic > Hyperelastic > Curve Fitting > [Next] (skips Uniaxial test data for Material 1) > [OK] (closes warning)

[Read From File] > (find .txt file) [OK] > [Next] > Curve Fits (left menu in window) > Hyperelastic (middle menu window) > Mooney > 2 Parameter > [Solve] (should solve within a couple seconds) > [Plot] (to view quality of fit) > [Save & Close] > close Material Model window

Solution

Unabridged Menu (not Abridged Menu) > Analysis Type > Solution Controls (see screenshots below)
Notes on Solution Controls:

Large Displacement Static chosen because test is quasi-static

Time at end of loadstep does not affect the simulation and can be changed to suit post-processing needs

Number of substeps affects simulation run time. For current mesh, 10 substeps = about 1 minute.

More substeps means finer results.

Write Item to Results File section (first screenshot): Frequency: write every substep (important for TimeHist Postpro)

Analysis Options (see screenshot below)

Notes on Analysis Options:

Large deform effects ON (top)

Stress stiffness ON (if Large deform effects ON) (bottom)

[OK]

Solve
**General Postproc (POST1)**

To contour plot SX & SY percent difference region:

Element Table > Define Table > [Add] > SX and SY > [Close]

Abs Value Option > Yes > [OK]

Add Items > LabR: numsub (numerator subtraction), FACT1: 1, Lab1: SX, FACT2: -1, Lab2: SY, CONST: blank > [OK]

Add Items > LabR: denadd (denominator addition), FACT1: 1, Lab1: SX, FACT2: 1, Lab2: SY, CONST: blank > [OK]

Multiply > LabR: denmul (denominator multiplication), FACT1: 0.5, Lab1: denadd, FACT2: 1, Lab2: none > [OK]

Exponentiate > LabR: pdiff (percent difference), Lab1: numsub, EXP1: 1, Lab2: denmul, EXP2: -1 > [OK]

Plot Elem Table > Itlab: PDIFF, Avglab: Yes – average > [OK]

*Should produce a contour plot that looks like the top right figure.*

To get stress decay data:

Path Operations > Define Path > By Nodes > Select the following 4 nodes located along the X =0 or Y = 0 axis: Clamp edges (2) and Diamond points (2) > [OK]

For accurate equibiaxial region estimation, 100 data sets are needed.

Path Operations > Map Onto Path > Select Stress SX > Apply > Repeat for SY & SEQV

Path Operations > Plot Path Item > On Graph > Select SX, SY & SEQV (see screenshot) > OK
This image illustrates the true stress along the chosen line (x or y = 0)

Can be extracted into a List:
Plot Path Item > List Path Items

To copy into excel:
Select all
Copy (Ctrl C)

Paste into blank excel cell (if data columns to be pasted are TIME SX SY and SEQV, leave 3 columns after 1st blank column for data separation)

Data tab > Text to Columns (under Data Tools) > Next (spaces default) > Finish

Amount of data in each column is based on number of substeps.

May need to delete duplicate title rows that ANSYS places after every 20 rows of data

**TimeHist Postpro (POST26) (window automatically opens)**

Select the Add Data ( ) button

Choose desired variable (Stress > SY) > [OK]

*Select top diamond node* > [Apply or center click]

Choose next variable (DOF Solution > UY) > [OK]

*Select top diamond node* > [Apply or center click] > [OK] (to select another node for UY)

*Select bottom diamond node* > [OK] (three variables should now be listed in the active window)

Highlight up to 6 variables at a time and select the List Data ( ) button

Copy and paste data into excel (follow instructions above)

Convert true stress in engineering stress