Statistical model prediction of fatigue life for diffusion bonded Inconel 600

Timothy Nowicki

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Statistical Model Prediction of Fatigue Life for Diffusion Bonded Inconel 600

By

Timothy Nowicki

A Thesis Submitted
In Partial Fulfillment
Of the Requirement for the

MASTER OF SCIENCE
IN
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# Table of Contents

Acknowledgements ........................................................................................................... ii  

Table of Contents ............................................................................................................. iii  

List of Figures ................................................................................................................ vi  

List of Tables .................................................................................................................. viii  

## 1 Literature Review ................................................................................................. 1  

1.1 Diffusion Bonding Overview .............................................................................. 1  

1.2 Defect Based Fatigue Failures ............................................................................ 3  
  
  1.2.1 Fracture Mechanics .................................................................................... 4  
  
  1.2.2 Probabilistic Fatigue Life ........................................................................... 7  
  
  1.2.3 Modeling Defect Populations .................................................................. 10  

1.3 Diffusion Bonding Defects ............................................................................... 11  
  
  1.3.1 Description of Non-Fusion Defects .......................................................... 12  
  
  1.3.2 Grain Diffusion Number ........................................................................... 14  

1.4 Summary ........................................................................................................... 16  

## 2 Objectives ............................................................................................................ 18  

## 3 Analysis of Prior Results ................................................................................... 21 

3.1 Rineferd’s Bonding Setup ................................................................................ 21  

3.2 Prior Tensile and Fatigue Test Details .............................................................. 23  

## 4 Image Analysis and Statistics .......................................................................... 29  

4.1 SEM Results ...................................................................................................... 30  

4.2 Image Analysis .................................................................................................. 33  
  
  4.2.1 Image Analysis Results ............................................................................ 36
4.3 Defect Population .............................................................................................. 39

4.3.1 Defect Density Results .............................................................................. 41

4.4 Statistical Analysis .......................................................................................... 43

4.4.1 Defect Sizes .............................................................................................. 43

4.4.2 Defect Population Density ........................................................................ 47

5 Crack Growth Modeling ..................................................................................... 51

5.1 Priddle Fatigue Model ...................................................................................... 52

5.2 NASGRO Model .............................................................................................. 55

5.3 Fracture Mechanics ........................................................................................ 60

5.3.1 Internal Elliptical Cracks ......................................................................... 61

5.3.2 Surface Cracks .......................................................................................... 64

5.3.3 Crack Geometry and Stress Intensity Solution Transition......................... 66

5.4 Monte Carlo Simulation .................................................................................... 67

5.4.1 Implementation of the Monte Carlo Simulation ........................................ 68

5.4.2 Phase 1 – Single Defect Placed Centrically ............................................. 69

5.4.3 Phase 2 – Single Defect Randomly Located ............................................. 71

5.4.4 Phase 3 – Multiple Defect Model .............................................................. 71

6 Results ..................................................................................................................... 76

6.1 Phase 1 Results ................................................................................................. 77

6.2 Phase 2 Results ................................................................................................. 79

6.3 Phase 3 Results ................................................................................................. 81

6.3.1 Final Crack Size Results for Multiple Defect Model ................................ 85

6.4 Summary of Results ........................................................................................ 87
7 Conclusion .......................................................................................................................... 89

7.1 Recommendations for Future Work......................................................................... 92

7.1.1 Interaction of Multiple Defects ........................................................................... 92

7.1.2 Elastic Plastic Fracture Mechanics ................................................................. 94

7.2 Summary .................................................................................................................... 95

References ......................................................................................................................... 96

Appendices ....................................................................................................................... 99
List of Figures

Figure 1-1 - Mode I Crack Cycling ..................................................................................... 5
Figure 1-2 - Generic Fatigue Crack Growth Rate Plot ....................................................... 6
Figure 1-3 - Simple Monte Carlo Flowchart ....................................................................... 9
Figure 1-4 - Bond line [3] ................................................................................................. 13
Figure 1-5 – Tensile Failure Sample of Non-Fusion Defect Areas [3] ......................... 14
Figure 1-6 - GDN vs. Tensile Strength [3] ....................................................................... 16
Figure 3-1 – Diffusion Bonding Setup [4] ........................................................................ 22
Figure 3-2 – Theoretical and Experimental Strain vs. Life for Block G [4] ................. 27
Figure 4-1 - SEM Mapping of Fracture Surface ............................................................... 30
Figure 4-2 - G-2 SEM Images ......................................................................................... 31
Figure 4-3 - G-3 SEM Images ......................................................................................... 32
Figure 4-4 – ImageJ Analysis ........................................................................................... 34
Figure 4-5 – Repeated Defect Measurements ................................................................ 35
Figure 4-6 – Cumulative Distribution Functions of G-2 and G-3 ................................. 37
Figure 4-7 – Major Diameter vs. Area ............................................................................... 38
Figure 4-8 – Minor Diameter vs. Area ............................................................................... 38
Figure 4-9 – Grid Mapping for Defect Count ................................................................. 40
Figure 4-10 – G2-6 Defect Count ..................................................................................... 41
Figure 4-11 – Lognormal Probability Plot of G2 and G3 .............................................. 44
Figure 4-12 – Normalized CDF of Areas ......................................................................... 45
Figure 4-13 – Combined Lognormal Probability Plot of Areas ................................... 46
Figure 4-14 – Normal Probability plot for G2 and G3 Defect Density ......................... 48
List of Tables

Table 3-1 - Unplated Diffusion Bonding Configurations [4] ........................................... 23
Table 3-2 - Block G LCF Results [4] ................................................................................ 24
Table 3-3 - Comparison of Experimental and Theoretical Strain Ranges for Block G .... 28
Table 4-1 – Results of Repeated Defect Measurements ................................................... 35
Table 4-2 – Basic Statistics for Measured Defect Areas .................................................. 36
Table 4-3 – Defect Count Results ..................................................................................... 42
Table 4-4 – Sample Diameters Before and After Tensile Testing .................................... 47
Table 5-1 – Parameters for FCGR models [9] .................................................................. 53
Table 5-2 - Difference Between NASGRO Parameters at Varying Temperatures .......... 57
Table 5-3 – NASGRO Equation Constants for Sheet/Plate Inconel 600 .......................... 58
Table 5-4 – Geometry Ratio for Valid Surface Defect Solution ...................................... 67
Table 5-5 – Modeling implementation ............................................................................. 68
Table 6-1 – Actual LCF failures of Simulated Samples [4] ............................................. 76
Table 6-2 – Phase 1 Results .............................................................................................. 77
Table 6-3 – Phase 2 Results .............................................................................................. 79
Table 6-4 – Phase 3 Results .............................................................................................. 83
Table 6-5 – Direct Comparison of Priddle and NASGRO models................................. 85
Table 6-6 – General Results for Final Crack Sizes ........................................................... 86
Table 7-1 – Strain Ranges of Experimental Samples [4] .................................................. 91
Table 7-2 – Estimations of Number of Crack Interactions on Bonding Surface with 2500
Defects .......................................................................................................................... 93
Abstract

Diffusion bonding is a solid-state joining process, used in a wide range of fields, allowing the joining of both metallic and ceramic materials to create complicated parts and geometries [1]. Diffusion bonded material has been found to contain inherent defects caused by the bond process that have been loosely associated to the lower tensile and fatigue strengths of the parent material [2]. The process is relatively new, and little research has been done to characterize these defects or their relationship to bond quality, with the exception of a recent study that did relate tensile strength to the bond quality [3].

Previous work resulted in successful diffusion bonding of Inconel 600, a high strength super alloy [4], optimization of bond process parameters, and the development of a method for qualitative analysis of bond quality [3]. The first goal of the present research is to develop a 2-dimensional characterization of the diffusion bonding defects for use in predicting fatigue life. Then, by treating the defects as pre-existing cracks, fracture mechanics can be used to predict life to failure based on initial material quality. A Monte Carlo Simulation will be used to capture the variability in the defects and quality of the bond.

The overall goal of this research will be to enhance the previous qualitative bond quality assessment by using defect measurements taken with the SEM. This will allow for not only a more quantitative relationship between bond quality and tensile strength, but can also provide a relationship bond quality and fatigue strength. The result would lead to less of a need for destructive testing of samples, saving overall costs.

The diffusion bonded defects were successfully modeled and characterized statistically by defining their, sizes, shapes, locations, and populations. Defect areas were characterized as following a log normal distribution. Although very random in shape, the defects were simplified down to elliptical geometries defined by their major and minor axis. Locations were determined to be truly random on the bonding surface. Many defects were present on the bonding surface and the population density followed a normal distribution with a high amount of deviation.

The model was developed in three separate phases, each increased in complexity and accuracy of the bond surface. Each stage improved overall predictions but was still far of from the low cycle fatigue results that were being used as a comparison. It was determined that assumptions of independent multiple crack growth on the bond surface and the validity of linear elastic fracture mechanics were incorrect, causing an over prediction of fatigue life. Defect interaction and possibly analysis into elastic-plastic fracture mechanics must be undertaken to improve the predictions of the diffusion bonding fatigue life.
1 Literature Review

1.1 Diffusion Bonding Overview

Diffusion Bonding is a relatively new manufacturing process that is a simple solid state joining between two materials with the use of high temperature and pressure over a period of time. Controlling the various process factors allows for joining of similar or dissimilar materials with crystalline structures. Diffusion bonding results in only a small amount of deformation and does not melt any of the material. Although an interface layer of material can be used between the two surfaces, the best and most desirable case is for direct bonding without any layer [4].

The diffusion bonding process takes a relatively long period of time, allowing for grains of the crystalline material to slowly grow across the material interface so that the material becomes one solid piece. This process is done in three stages: microasperity deformation, diffusion controlled mass transport, and finally the interface migration [1]. For a theoretically perfect bond, no evidence of any bond line should be present, meaning that any voids that existed on the bond surface were eliminated. With an optimum bond the mechanical properties of the two joined surfaces would remain the same as the original material.

Diffusion bonding can offer several benefits when compared with other joining techniques. The bonding of multiple layers that fit together precisely allows for the ability to create much more accurate and complex parts, all in one step, than any casting or welding process. Diffusion bonding, unlike most joining methods, also results in
similar physical, chemical and mechanical properties as the parent material. A broad range of industries can benefit from the use of diffusion bonding, including aerospace, nuclear, or microelectronic companies. Many engineering designs that are thought to be nearly impossible or infeasible to create by conventional processing can be easily manufactured by diffusion bonding. [5]

Two past studies have investigated and optimized diffusion bonding applications. Robert Rinefierd [4] examined the possibilities of bonding two pieces of Inconel 600, a nickel-based superalloy, both with and without an interface layer. By investigating the effects of five different variables on the diffusion bonding process, a high strength bond was developed. Rinefierd showed that it was possible to bond Inconel 600 together with bond strengths close to parent material strength. Rinefierd also found that fractures occurred at the bond line in 100% of the tensile samples and 94% of the fatigue specimens. Fracture surface analysis revealed small defects across the bond area where crack growth occurred that may have affected the performance during testing.

A follow up project by Sarah Lagoon [3] was conducted to improve and further optimize the bond strength of Inconel 600 while examining the bond surface further in order to develop a method to evaluate the bond quality to reduce the use of costly and time consuming destructive techniques such as tensile and fatigue testing. Lagoon was able to maintain 90.4% of the parent material’s tensile strength through a fine tuning of the process that Rinefierd developed. The bond surface was examined after fracture using a Scanning Electron Microscope (SEM), showing relatively large areas of defects. Most
importantly the research also attempted to qualitatively show a simple relationship between the bond strength and number of defects found in the bond [3]. This will be discussed in more detail in Section 1.3.3.

1.2 Defect Based Fatigue Failures

Most real life applications for fatigue life predictions of metallic components are not based on the simple stress life or strain life curves. While these life curves may give a quick result for fatigue life, these materials, regardless of manufacturing method, contain some form of defects that can lead to a shorter life. In real world applications these defects are what truly drive fatigue failures in metallics. With increases in use of aluminum and other non-ferrous alloys in both the automotive and aerospace industries, a large amount of research has been conducted on qualitatively and quantitatively analyzing the relationships between the various defect types and fatigue life of non-ferrous materials.

Examples of microstructural features that have been examined include porosity, intermetallics and non-metallic particles. In experiments on cast aluminum alloys, it was found that porosity was the main reason for the shorter fatigue life [6]. Experiments have also shown that sizes and density of pores can affect fatigue life. Increases in both size and density of pores in non-ferrous material can cause materials to have shorter fatigue life [7]. With experiments proving that defects can adversely affect fatigue strength, models can be developed based on the experimental data, allowing the prediction of fatigue life given certain defect parameters.
1.2.1 Fracture Mechanics

Crack growth is what drives defect based fatigue failures in non-ferrous materials. The pre-existing defects act as cracks or stress raisers where cracks can form. Empirical data from fatigue life experiments allow for the development of models to accurately predict fatigue crack growth and ultimately failure. Linear Elastic Fracture Mechanics (LEFM) typically is the method implemented for evaluating the behavior of a cracked body with assumed elastic behavior [8]. The driving parameter in the model is the stress intensity factor, $K$ which evaluates the stress magnification factor of a crack tip. The general equation is shown in Equation 1.1, where $a$ is a crack length measurement, $\sigma$ is the applied stress, and $\beta$ is a geometry function that takes the crack and specimen geometries, as well as loading mode, into account.

$$K = \sigma \sqrt{\pi a \beta} \quad \text{(Equation 1.1)}$$

$$\Delta K = \Delta \sigma \sqrt{\pi a \beta} \quad \text{(Equation 1.2)}$$

For the case of cyclic loading, Equation 1.2 is shown, where $\Delta \sigma$ and $\Delta K$ are the stress and stress intensity factor ranges, respectively. As the load is cycled from $\sigma_{\text{max}}$ to $\sigma_{\text{min}}$, the crack tip opens and closes (Figure 1.1). There are three different ways to calculate stress intensity factors, based on the mode of loading on the given specimen or crack location. The most common form, shown in Figure 1.1 is Mode I (opening), however Modes II (sliding) and III (tearing) can also be modeled this way [8]. This research is focused on Mode I loading.
In a LEFM model, fracture or failure of a cracked specimen typically occurs when the stress intensity factor reaches a maximum value [8]. This value, $K_c$, is called the fracture toughness and occurs either when the crack grows to a large enough size or when the stress increases to a critical value. The fracture toughness, found empirically through controlled experiments, is a material constant. Given the value of $K_c$, a crack size or stress can be solved for using Equation 1.1 to determine when a given specimen may fail catastrophically.

The cyclic stress intensity factor was found by P.C. Paris to be the driving parameter for cyclic crack growth [8]. The relationship was developed into the Paris Equation (Equation 1.3).

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (Equation 1.3)
In Equation 1.3 \( \frac{da}{dN} \) is known as the crack growth rate, based on change in crack size \( a \) per cycles \( N \). Parameters \( C \) and \( m \) are material constants based on the fit of empirical fatigue crack growth rate data. In evaluating the relationship between fatigue crack growth rates and cyclic stress intensity factor, three separate stages can be observed in the experimental plots of fatigue crack growth rate curves.

![Figure 1-2 - Generic Fatigue Crack Growth Rate Plot](image)

Stage I is the threshold region, dominated by \( \Delta K_{th} \), defined as the threshold stress intensity factor. Much like the endurance limit of a stress life curve for ferrous materials, which defines a range of cyclic stress that will not cause fatigue failure, \( \Delta K_{th} \) defines the range where crack growth is minimal, and can be largely ignored. Stage III is the region of imminent failure, dominated by \( K_c \). The general Paris Equation fits Stage II, or steady crack growth. However models can be tailored to better fit any or all of the Stages [8].

The Paris law is the first and most basic form of any LEFM model for crack growth, and
can be integrated to solve for the cycles to failure given an initial and critical crack size, or vice versa.

Other models have been developed to relate the stress intensity factors with crack growth rates of materials. These models based on the Paris Law, although more complex, can fit more of the crack growth stages and are determined by curve fitting techniques of empirical crack growth data. Some of these better fitting models include the stress ratio, $R$ in the equation, and also consider include fracture toughness $K_c$, and the threshold stress intensity factor $K_{th}$ [8]. For Inconel 600, the Priddle and NASGRO models have been determined to better suit the crack growth data than the general Paris Law [9, 10]. This will be discussed in more detail in Chapter 5.

1.2.2 Probabilistic Fatigue Life

The parameters and many of the equations of fracture mechanics models are derived from previous experimental fatigue results fitted to the data. These fatigue experiments are done by controlling either the stress or strain and then also creating and monitoring a defect from which a crack grows in the specimen. It is widely known that the results of fatigue testing and general fatigue life have a large degree of scatter, which hinders the accuracy of prediction techniques [11]. Discrepancies in the material as well as the sizes and shapes of defects play a large role in the variability of the resulting fatigue life [12]. This scatter in results leads many to develop probabilistic models of fatigue life fitted to basic statistical distributions such as a Weibull or lognormal. One common tool for
engineers dealing with variability or scatter in parameters and results is the Monte Carlo Model.

Monte Carlo Simulations are used across many disciplines in modeling that involves a large degree of variability. By quantifying the variability of certain constants in the governing equations, a statistical distribution can be associated with the necessary parameters. By iteratively sampling from each distribution a value is chosen for each parameter which is then put through the governing equations, resulting in a single output that is stored. Repeated sampling and insertion into the governing equation numerous amounts of times may lead to the results following a certain statistical distribution which demonstrates the variability. Figure 1.3 shows a very basic form of a Monte Carlo model flowchart illustrating how parameters with a known probability density function (P.D.F.) can be input into a Monte Carlo Model, resulting in an output P.D.F. of results.
Monte Carlo simulations are not always as simple as just giving every parameter in a model a known independent statistical distribution. Annis [11] discusses the process and the complications that arise from using a Monte Carlo Simulation in developing a probabilistic model, specifically with the Paris Law in fatigue life predictions. Adding independent distributions to every single parameter with an assumed amount of variability, most commonly a mean and standard deviation, into an equation could lead to problems with increased and unnecessary variance of results. Annis [11] shows that the Paris Law constants $C$ and $m$ are usually mistaken to be either independent, fixed, or
linearly dependant on one another, each of which can greatly increase unrealistic scatter in the results of models. The parameters $C$ and $m$ are actually jointly distributed to each other, and follow a complex bivariate distribution that links the two together statistically. However Annis also discusses that a higher deviation in some other parameter containing variability, such as a defect size, can overpower the need to jointly distribute the Paris Law constants, which would then insignificantly change the overall results of the Monte Carlo simulation.

### 1.2.3 Modeling Defect Populations

Prior to predicting fatigue life, the defects causing fatigue cracking must be characterized. Proper analysis of a material, whether along a fracture surface or along a predetermined section cut, can yield the statistical characteristics of the defect population. Statistical distributions for defect size and shape can be determined by doing proper analyses of the fracture surface of specimens [13]. These distributions of defect sizes and shapes are then used for input into the desired LEFM model for the proper fatigue life prediction.

Image analysis is necessary to examine the fracture surface and characterize the general size and shape of the defects. In some situations the use of an optical microscope is possible, but many times samples must be polished to achieve the flat and smooth surface needed to make measurements with the microscopes’ limited depth of field. A Scanning Electron Microscope (SEM) allows for a higher depth of field than normal optical microscopes, allowing for samples to be more rough and unpolished [14]. SEMs also allow for a much higher magnification, enabling much more detail of smaller defects to
be observed. By using a SEM to view the material and piece together images of a two
dimensional cross section, a large section of the fracture surface can be mapped out to
measure the size, shape and location of defects. Miyasato, Magnusen, and Hinkle [15]
discuss the importance of a larger sample of defects by imaging with less magnification,
rather than a larger magnification with a more accurate measurement of each single
defect. With a large magnification, measurements become more accurate but much more
time consuming and expensive to observe multiple flaws. Also, it was determined that
larger flaws are much more important to the fatigue life of specimens and accurate
measurement of their size is more important [15]. The time required to collect
measurements is an important factor in Scanning Electron Microscopy as it is much more
expensive to use than optical microscopy. Lower magnifications, while less accurate,
produce larger areas of coverage, allowing for more defects to be measured quickly and
inexpensively as extreme detail is not required.

1.3 Diffusion Bonding Defects

If diffusion bonded components contain defects along the bond surface, the
manufacturing process can possibly lead to reduced tensile and fatigue strength. With an
optimum situation, all voids between the bond surfaces should theoretically disappear
during the diffusion bonding process. However this optimization has been found to be
very difficult to achieve for some materials, including Inconel 600 [4, 3]. Cross-sectional
examination of some Inconel 600 bond surfaces revealed regions of the surface that failed
to bond during the bonding process. These areas have been described previously as non-
fusion defects and preliminary investigation indicates that they do impact the fatigue
strength of a bond [3, 2]. No quantitative work has been published to correlate these defects with the mechanical properties of the bond, or even whether they are in fact the primary reason for a reduction in bond strength.

1.3.1 Description of Non-Fusion Defects

Diffusion bonding defects can be observed in two ways: examining the bond line that forms around the perimeter of the component, or examining the bond area after destructive testing. Figure 1.4 shows the bond line of a specimen from Lagoon’s work, considered to be low quality [3]. It can be seen with the arrow shown in Figure 1.4 where the bond line disappears. These sections are where grains have grown across the surface. However there are still clear sections where the bonding has appeared to not occur and the bond line is visible, which can be defined as voids. The amount of grain growth and number of voids remaining depict the quality of the bond and can be the quickest and easiest way to examine the component and quantify bond quality.
Figure 1.5 shows a cross-sectional area SEM image where a specimen broke on a bond surface in fatigue testing [3]. The rough areas are where bonded material pulled apart during fatigue testing. The smooth areas shown in the figure are where the material did not bond together: the non-fusion defects. Beachmarks can also clearly be observed close to the defects showing possible evidence of crack growth developing from some of the un-bonded areas. This observation provided the motivation for the present research into whether these areas could be related to fatigue life.
1.3.2 Grain Diffusion Number

Previously, determining the quality of a diffusion bonded component with non-destructive testing was done by simply counting the number of defects present along the bond line. With Lagoon’s attempt to develop a more methodical approach for determining the quality of a diffusion bond by minimizing the amount of after fracture analysis (destructive testing), the grain diffusion number (GDN) was created [3]. The GDN allows for a qualitative and possibly non-destructive assessment of the bond strength, and a relationship between the number and size of defects and the tensile strength. To analyze a sample by using the GDN, first the round sample must be ground.

Figure 1-5 – Tensile Failure Sample of Non-Fusion Defect Areas [3]
down and polished so that a portion of a flat bond line can be examined (as in Figure 1.4).
Using an optical microscope images were taken of the bond line and combined to form
one single composite image showing the entire length of the line. Grinding and
polishing the piece down farther can be done continually to reveal different layers of the
bond line for more imaging if desired. By measuring the length where grain growth
appears and where non-fusion defects appear, a calculation [3] was developed to assign
the component a GDN.

\[
GDN = \frac{(G - V)}{T} \quad \text{(Equation 1.4)}
\]

In Equation 1.4, G describes the length where bonding has occurred and grain growth
across the bond line has occurred, V is total length of all defects, and T is the overall
length of the bond line in the image. The one-dimensional measurement allows for a
comparison of how non-fusion defects can affect the strength of the bond. This is shown
in Figure 1.6 where a trend is shown to exist with the GDN and tensile strength of the
specimens. A GDN of 1 would be the most optimum situation, meaning grain growth
fully is visible across the bond line, and -1 would be the least optimal where no
observable grain growth is visible.
Figure 1.6 shows that the range of GDN falls in between those extreme values, with different points representing the different bonding configurations tested in Rinefierd’s work. Some bonds are clearly better quality than others, with a higher strength and GDN. The GDN is a simple technique to describe a relationship between bond quality and tensile strength using just a one-dimensional measurement. However, this number predicts the stress at which the bond will fail with only moderate accuracy and it gives no indication about fatigue life. Instead the GDN is just a good indicator of the overall bond quality in comparison to other bonds.

1.4 Summary

Based on the two prior works of Sarah Lagoon and Robert Rinefierd [4, 3], diffusion bonding of Inconel 600 was developed and minimally characterized. Although tensile and fatigue testing of the bonds showed results close to the parent material, little is still
known about the inherent defects in the bond process and how they impact the tensile strength and fatigue life. The defects have been observed to be multiple penny-like defects on the fracture surface, where no bonding originally occurred. One-dimensional bond line measurements can be used to calculate GDN, giving some measure of bond strength. However, there has been very little other work to quantitatively predict tensile strength and even less work on the fatigue life of diffusion bonded components.

The prediction of fatigue life for diffusion bonded materials can be approached in a manner to that used for porosity [6, 7, 12, 13]. By characterizing the size and shape of the defects first, as well as the general density of the defect population, a statistical model can be created to fit the distribution of flaws. Then, based on the implementation of a linear elastic fracture mechanics model for the material, size, shape, and location of the defects, a Monte Carlo model can be used to predict a distribution of fatigue life.
2 Objectives

The two previous studies undertaken at the Rochester Institute of Technology [4, 3] both demonstrated the diffusion bonding ability of Inconel 600 with comparable and repeatable strengths similar to the parent material. While Lagoon [3] did evaluate the tensile strength in comparison to a one-dimensional measurement of bond quality, it would be practical to have an additional correlation between overall bond quality and fatigue life. Components in aerospace, automotive, and nuclear fields, where diffusion bonding can be beneficial, experience high cycle fatigue loading.

While both Rinefierd and Lagoon explored many different bonding configurations with various results between each, a single configuration will be chosen to study the impact of defects to the fatigue life. Since most of the previously tested samples of both Rinefierd and Lagoon’s work are available, and all raw test data is available, no additional specimens will be bonded and tested. Samples from one of the bond configurations developed by Rinefierd/Lagoon will be used for the current study. In order to predict fatigue life from non-fusion defects, defects must first be found and characterized in a way that can serve as input to a fatigue crack growth model. The two-dimensional fracture surfaces of the previously tested samples will undergo SEM examination to obtain measurements of defect size, location, and density. These variables will be fit to the appropriate statistical distributions for use in the fatigue crack growth models.

Based on the characteristics of the defects and the general shapes and locations found along the fracture surface, a crack growth model as well as the necessary stress intensity
factors can be implemented into a Monte Carlo simulation for prediction of fatigue life. These simulations will attempt to predict the previous fatigue test results of the diffusion bonding configuration chosen. The model will be developed in stages, each with increasing complexity.

The application of a successful model for fatigue life predictions will allow for further examination of Lagoon’s [3] GDN which will look into the possibility of relating the easier one-dimensional measurement of defects along the bond line to a quantitative value of bond quality. The possibility of taking a GDN measurement and converting it to a more practical two-dimensional defect population will be determined. A correlation then may be developed between the two measurements allowing for a quantitative measurement of bond quality that can be used to predict an average fatigue life. This quantitative measurement could allow for accurate results in prediction of fatigue life allowing for more efficient inspections of bond quality without employing a large scale amount of destructive testing.

Work performed previously:
- Diffusion bonding [4, 3]
- Tensile testing [4, 3]
- High Cycle Fatigue testing [4, 3]
- Low Cycle Fatigue testing [4, 3]
- Grain diffusion number analysis [3]

Work to be performed by author in conjunction with outside resources:
- Scanning Electron Microscope measurements (Center for Integrated Manufacturing Studies, Materials Engineering Laboratory at RIT)

Work to be performed by author:

- Microstructural image analysis
- Statistical modeling
- Bond surface fracture mechanics
- Monte Carlo model development
- Possible enhancement of quantitative grain diffusion number
3 Analysis of Prior Results

Since no new bonds, fatigue testing, or tensile testing will be attempted during this investigation, a full understanding of both Rinefierd’s and Lagoon’s work is necessary. Both the diffusion bonding process and the testing of samples were documented in both cases. However, Lagoon focused on the analysis of Rinefierd’s samples, including the application of the GDN [3]. With the availability of all Rinefierd’s specimens, this study will continue to focus on those samples.

3.1 Rinefierd’s Bonding Setup

The bonding setup Rinefierd developed was created to examine multiple samples from each configuration. A total number of 24 bond configurations were created and labeled “A” through “X” [4]. The bond setup is shown by Figure 3.1 and shows two thin sheets of Inconel bonded between two thicker blocks [3]. The setup therefore contained 3 separate bond surfaces specified as upper, middle and lower bonds. The original goal of the tests was to examine bonding of thin, rolled sheets of Inconel together, which allows a manufacturer to stack many machined layers together to form complex geometries. A total number of 30 samples were cut from each block, assigned a number and tested in either tension, high cycle fatigue or low cycle fatigue.
For Rinefierd’s labeling system, unlike Lagoon’s, it should be noted that the number the sample was given from any of the configurations does not define its location on the block. Rinefierd also tested bonding the Inconel with nickel plating as an interface layer between the bond surfaces, as did Lagoon and described these as plated bonds. It was found that plated bonds resulted in lower tensile strengths and more variability of results [4, 3]. Because of the poor results with the interface layer, the present analysis will be only focus on the unplated configurations without the nickel interface layer. Table 3-1 shows the bond process variables for the 12 unplated configurations that Rinefierd created, as well as the mean and standard deviation of the samples’ ultimate strength ($S_{ult}$) [4].
Table 3.1 - Unplated Diffusion Bonding Configurations [4]

<table>
<thead>
<tr>
<th>Block</th>
<th>Grain Size</th>
<th>Surface Finish (μm)</th>
<th>Temperature °F</th>
<th>Desired % strain</th>
<th>% strain (actual)</th>
<th>S_{ut} Mean (ksi)</th>
<th>S_{ut} Standard Dev (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.5</td>
<td>60</td>
<td>2025</td>
<td>2</td>
<td>2.8</td>
<td>70.14</td>
<td>3.83</td>
</tr>
<tr>
<td>B</td>
<td>4.5</td>
<td>60</td>
<td>1950</td>
<td>2</td>
<td>1.88</td>
<td>58.18</td>
<td>4.18</td>
</tr>
<tr>
<td>G</td>
<td>4.5</td>
<td>4</td>
<td>2025</td>
<td>0.5</td>
<td>2.12</td>
<td>81.64</td>
<td>0.35</td>
</tr>
<tr>
<td>H</td>
<td>4.5</td>
<td>4</td>
<td>1950</td>
<td>0.5</td>
<td>1.67</td>
<td>79.26</td>
<td>9.64</td>
</tr>
<tr>
<td>I</td>
<td>8</td>
<td>60</td>
<td>2025</td>
<td>0.5</td>
<td>1.96</td>
<td>62.7</td>
<td>1.37</td>
</tr>
<tr>
<td>J</td>
<td>8</td>
<td>60</td>
<td>1950</td>
<td>0.5</td>
<td>1.2</td>
<td>54.85</td>
<td>3.16</td>
</tr>
<tr>
<td>O</td>
<td>8</td>
<td>4</td>
<td>2025</td>
<td>2</td>
<td>2.1</td>
<td>79.43</td>
<td>0.88</td>
</tr>
<tr>
<td>P</td>
<td>8</td>
<td>4</td>
<td>1950</td>
<td>2</td>
<td>2.27</td>
<td>79.07</td>
<td>4.42</td>
</tr>
<tr>
<td>S</td>
<td>4.5</td>
<td>4</td>
<td>2025</td>
<td>0.5</td>
<td>0.49</td>
<td>73.05</td>
<td>6.65</td>
</tr>
<tr>
<td>T</td>
<td>4.5</td>
<td>4</td>
<td>2100</td>
<td>2</td>
<td>2.2</td>
<td>76</td>
<td>3.09</td>
</tr>
<tr>
<td>U</td>
<td>8</td>
<td>60</td>
<td>2025</td>
<td>0.5</td>
<td>0.43</td>
<td>42.84</td>
<td>2.83</td>
</tr>
<tr>
<td>V</td>
<td>8</td>
<td>60</td>
<td>2100</td>
<td>2</td>
<td>2.1</td>
<td>59.78</td>
<td>3.44</td>
</tr>
</tbody>
</table>

Grain sizes of the blocks were specified prior to the bonding according to ASTM standards. Surface finish was a result of the rolling of plates during manufacturing and Rinefierd chose not to polish the surfaces prior to bonding. Temperature during bonding has a large effect on the overall characteristics of the bond, and Rinefierd chose values over the annealing temperature of Inconel 600 (1850°F) [16]. Although the strain rate was able to be monitored during testing, the applied pressure to achieve the strain was manually controlled, resulting in some variation between the desired and actual strain levels of the blocks.

3.2 Prior Tensile and Fatigue Test Details

For the present study, only one of Rinefierd’s blocks was chosen for further analysis. The choice of bonding configuration to examine was made based on the last two columns of Table 3-1, as well as availability of fatigue data for comparison to models. The best bond configuration, based on both mean and standard deviation of S_{ut} was Block G. Not
only does G have the best strength, but it also carries a low standard deviation in comparison to the other blocks. Due to the low standard deviation, indicating extremely consistent bond quality, further analysis of block G can be completed using a smaller sample size of specimens than would be required if a different block was used. Samples from Block G were tested in low cycle fatigue as well.

The LCF tests (samples G16-G25) were done over a range of strain from 0.30% to 1.20%. Table 3-2 shows a summary of results from the LCF testing. G-20 was the only sample that did not complete the test, due to an abrupt increase in the strain range that should have been constant throughout the test [4]. LCF testing was completed at Mar-Test, Inc in Cincinnati Ohio.

### Table 3.2 - Block G LCF Results [4]

<table>
<thead>
<tr>
<th>L.D.</th>
<th>E $10^6$ PSI</th>
<th>STRAIN RANGE %</th>
<th>STRESS RANGE KSI</th>
<th>MAX. STRESS KSI</th>
<th>MIN. STRESS KSI</th>
<th>SELECTED CYCLE NEAR START</th>
<th>SELECTED CYCLE</th>
<th>MIN. STRESS KSI</th>
<th>MAX. STRESS KSI</th>
<th>MIN. STRESS KSI</th>
<th>NF CYCLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19</td>
<td>31.9</td>
<td>50</td>
<td>1.20</td>
<td>90.2</td>
<td>45.2</td>
<td>45.0</td>
<td>700</td>
<td>105.5</td>
<td>52.6</td>
<td>-52.9</td>
<td>1465</td>
</tr>
<tr>
<td>G24</td>
<td>31.0</td>
<td>10</td>
<td>1.10</td>
<td>79.7</td>
<td>39.4</td>
<td>-40.2</td>
<td>1500</td>
<td>108.2</td>
<td>53.3</td>
<td>-54.8</td>
<td>2059</td>
</tr>
<tr>
<td>G18</td>
<td>31.0</td>
<td>200</td>
<td>1.00</td>
<td>99.7</td>
<td>49.4</td>
<td>-50.3</td>
<td>3000</td>
<td>100.4</td>
<td>49.7</td>
<td>-50.7</td>
<td>5419</td>
</tr>
<tr>
<td>G16</td>
<td>31.2</td>
<td>200</td>
<td>0.60</td>
<td>84.7</td>
<td>42.4</td>
<td>-42.3</td>
<td>11235</td>
<td>91.1</td>
<td>45.2</td>
<td>-45.9</td>
<td>22230</td>
</tr>
<tr>
<td>G17</td>
<td>29.3</td>
<td>42</td>
<td>0.55</td>
<td>68.5</td>
<td>35.0</td>
<td>-33.5</td>
<td>9620</td>
<td>86.7</td>
<td>43.3</td>
<td>-43.4</td>
<td>16097</td>
</tr>
<tr>
<td>G21</td>
<td>30.3</td>
<td>20</td>
<td>0.45</td>
<td>62.4</td>
<td>30.8</td>
<td>-31.6</td>
<td>25000</td>
<td>80.8</td>
<td>39.6</td>
<td>-41.3</td>
<td>49607</td>
</tr>
<tr>
<td>G22</td>
<td>29.0</td>
<td>10</td>
<td>0.38</td>
<td>56.6</td>
<td>28.3</td>
<td>-28.3</td>
<td>32000</td>
<td>76.7</td>
<td>38.3</td>
<td>-38.4</td>
<td>63440</td>
</tr>
<tr>
<td>G23</td>
<td>30.5</td>
<td>10</td>
<td>0.36</td>
<td>56.0</td>
<td>28.5</td>
<td>-27.6</td>
<td>59000</td>
<td>75.9</td>
<td>37.8</td>
<td>-38.1</td>
<td>116455</td>
</tr>
<tr>
<td>G25</td>
<td>29.7</td>
<td>5</td>
<td>0.30</td>
<td>50.3</td>
<td>24.7</td>
<td>-25.5</td>
<td>51000</td>
<td>67.3</td>
<td>33.0</td>
<td>-34.2</td>
<td>100017</td>
</tr>
</tbody>
</table>

Mar-Test’s LCF testing examined the loading of each single cycle, recording the results of each until failure. Table 3-2 shows two separate cycles for each sample, one selected near the start of the test, and one at least half way through the test. The stress range increases during the test for each sample, due to strain hardening of the material.

Hysteresis loops depicting the stress-strain curves as loads were cycled on the samples.
was recorded for 5 of the 10 samples tested by Mart-Test. These loops showed a gradual increase in stress range while the strange range stayed constant. However, since all the samples were not contained in the summary, the basic loading scenarios and stresses will be chosen using the stabilized cycle results. Using ASTM standards the stabilized cycle is chosen to be the half way point to failure, which is listed as the “Selected cycle” section in Table 3-2 [17].

Using the values listed below, a theoretical strain life curve can be created. For a theoretical strain life curve, the equation is a summation of the two separate calculations, the elastic (Basquin’s Rule) and plastic (Mason-Coffin Rule) strain regions. Equation 3.1 [8] shows the basic strain-life curve which can be compared to the experimental LCF results of Table 3.2. This comparison is shown in Figure 3.2.

\[
\frac{\Delta \varepsilon_{\text{elastic}}}{2} = \frac{\sigma^*}{E} (2N_f)^b
\]

\[
\frac{\Delta \varepsilon_{\text{plastic}}}{2} = \varepsilon^* (2N_f)^c
\]  

(Equation 3.1)

\[
\frac{\Delta \varepsilon_{\text{tot}}}{2} = \frac{\Delta \varepsilon_{\text{elastic}}}{2} + \frac{\Delta \varepsilon_{\text{plastic}}}{2}
\]

\[
\therefore \frac{\Delta \varepsilon_{\text{tot}}}{2} = \frac{\sigma^*}{E} (2N_f)^b + \varepsilon^* (2N_f)^c
\]

The total theoretical strain life equation can be determined by sum of the plastic strain and elastic strain applied to the sample. Therefore if you know a cyclic strain applied on a body, you can predict a fatigue life, vice versa, and also determine the amount of plastic and elastic strain on that body. The strain life constants listed below for the theoretical model can be estimated if values are not published for a specific material using average ranges of values or equations based on previous empirical results of materials [8].
Fatigue Strength Coefficient: $\sigma'_f = 139.8 \text{ ksi}$

Modulus of Elasticity: $E = 30500 \text{ ksi}$

Fatigue Strength Exponent: $b = -0.085$

Fatigue Ductility Exponent: $c = -0.6$

Fracture Ductility Coefficient: $\varepsilon_f' = 1.064$

The fatigue strength coefficient, is often approximated as the true fracture strength of a material is estimated by adding 50 ksi to $S_{ut}$. For a heat treatment temperature of $2000^\circ\text{F}$, close to the bonding temperature for Block G, $S_{ut}$ for hot rolled plate is 89.8 ksi [16]. The modulus of elasticity was determined using the empirical results of block G’s tensile test (Table 3.1). Both fatigue strength and ductility exponents are generally estimated as the average value of ranges based on a wide variety of materials testing, with $-0.14 < b < -0.006$ and $-0.7 < c < -0.5$ [8]. Finally the fracture ductility coefficient $\varepsilon_f'$ is estimated by using $\varepsilon_f$, called the true fracture ductility, which is calculated using Equation 3.2 [8].

$$\varepsilon_f = \ln \left( \frac{1}{1 - RA} \right) \quad \text{(Equation 3.2)}$$

The equation utilizes $RA$, the reduction of area, which for a heat treatment temperature of $2000^\circ\text{F}$ is 0.655 for Inconel 600 [16].
Figure 3-2 – Theoretical and Experimental Strain vs. Life for Block G [4]

It can be seen that for the higher strain ranges toward the left of the total strain curve that the plastic strain dominates. The opposite is true on the right, when the plastic region drops below the elastic region; elastic strain dominates the total strain curve. The region where the plastic and elastic curves cross each is described as the transition point [8]. An increased amount of strain causes the stress on the specimen to exceed yield strength, where plastic deformation dominates. This complicates the model but towards the lower stress ranges it behaves much like a stress life curve, due to the lower amount of plastic strain. Comparing experimental results to the theoretical values in Figure 3.2 no tested sample fails above the total strain life curve shown in pink. However, the lower strain samples of the empirical results seem to follow the light blue elastic curve, where the yellow plastic curve follow the much high strain range samples. This is further
demonstrated in Table 3.3, where based on the experimental failures of 3 Block G samples the estimated plastic and elastic strain ranges using Equation 3.1 are compared to the total actual strain range values used during testing [4].

Table 3.3 - Comparison of Experimental and Theoretical Strain Ranges for Block G

<table>
<thead>
<tr>
<th>Sample</th>
<th>N</th>
<th>Actual Total Strain Range %</th>
<th>Theory Plastic Strain Range %</th>
<th>Theory Elastic Strain Range %</th>
<th>Total Theory Strain Range %</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19</td>
<td>1465</td>
<td>1.2</td>
<td>1.76</td>
<td>0.46</td>
<td>2.22</td>
</tr>
<tr>
<td>G17</td>
<td>16097</td>
<td>0.55</td>
<td>0.42</td>
<td>0.38</td>
<td>0.8</td>
</tr>
<tr>
<td>G25</td>
<td>100017</td>
<td>0.3</td>
<td>0.14</td>
<td>0.32</td>
<td>0.46</td>
</tr>
</tbody>
</table>

By comparing the two extreme values, G19 with its low fatigue life and high strain range is dominated by plastic strain, where as G25’s high fatigue life and low strain range is dominated by elastic strain. G17 is the median of the two and occurs close to the transition point, where both plastic strain and elastic strain are of shared importance. Understanding the amount and type of strain on the specimens is important for the assumption to use linear elastic fracture mechanics, which will be explained further in Section 5.
4 Image Analysis and Statistics

In order to characterize the unbonded areas in the tensile samples, a scanning electron microscope was utilized for both its high magnification and depth of field. The Amray SEM from the Materials Engineering Laboratory in the Center for Integrated Manufacturing Studies (CIMS) was used for the analysis of the specimens. Instead of measuring defects on the LCF samples, the tensile samples were chosen. Utilizing the defects measured on the tensile samples allows for essentially more data to be extracted from Rineferd’s previous work, instead of measuring and characterizing defects from the LCF samples to predict the fatigue life of the LCF samples. Also it was found to be easier to see the non fusion defects on tensile samples compared to the fatigue samples. Since the tensile and LCF samples come from the same configuration block, the bonding characteristics, including the defects, should represent both LCF and tensile samples.

Two specimens were chosen for measurement under the SEM: G-2 and G-3. In order to get the proper distributions of sizes for the defects, an unbiased process of measuring the sample surface was used. Biased measurements could possibly allow for a focus on just the large defects, missing out on the actual distribution and characterization of defect sizes. Figure 4.1 demonstrates how the pictures were taken by scanning up and down the fracture surface with a consistent magnification, recording an image whenever any sized defect was spotted. This also ensured that there would be no double counting of any defects during measurement. Magnification was maintained between 500-700x for consistency, but if a larger defect would not fit in the frame of the microscopes display, the magnification was decreased to encompass the entire geometry.
measurements were taken in the SEM the samples were cleaned with isopropyl alcohol and dried with an air hose to remove the majority of dust particles that may have been left from the paper sleeves where the samples were stored for a couple of years.

![SEM Image]

**Figure 4-1 - SEM Mapping of Fracture Surface**

### 4.1 SEM Results

Overall, 111 pictures were recorded from the two samples: 45 from G-2, and 66 from G-3. Each image was numbered according to the sample and the order in which it was taken. Figures 4.2 and 4.3 show some samples of the images taken from G-2 and G-3, respectively, with each image showing a few of the defects highlighted.
Figure 4-2 - G-2 SEM Images
Figure 4-3 - G-3 SEM Images
The varied sizes and shapes of the defects can be seen, not only between the different pictures, but within a single image. In order to characterize the range of these sizes, image analysis was done on each of the 111 images taken.

4.2 Image Analysis

The software application ImageJ [18] was employed to measure all the defects in every SEM image taken. For each image, the scale was specified by using the image’s micron bar. Each defect was measured by drawing with the cursor around the outside edge. The program measured 3 different values: area, major diameter, and minor diameter. The values for major and minor diameter were given to specify a best fit for an ellipse on the defect, so a generic shape could be applied for each defect. Figure 4.4 shows how each image was measured and processed. The values specified in the results window of the figure are given in μm for the diameters and μm² for area.
In order to ensure that measurements using Image J are consistent and repeatable, one defect on a single image was measured and repeated 10 separate times to compare the tabulated areas, and major and minor diameters. Figure 4.5 shows the defect from sample G3 that was measured and Table 4.1 shows the results of the 10 repeated measurements taken with their mean and standard deviation.
Table 4.1 – Results of Repeated Defect Measurement

<table>
<thead>
<tr>
<th>Measure</th>
<th>Area (μm²)</th>
<th>Maj Dia (μm)</th>
<th>Min Dia (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1710.851</td>
<td>67.501</td>
<td>32.271</td>
</tr>
<tr>
<td>2</td>
<td>1745.906</td>
<td>69.635</td>
<td>31.923</td>
</tr>
<tr>
<td>3</td>
<td>1721.52</td>
<td>68.531</td>
<td>31.984</td>
</tr>
<tr>
<td>4</td>
<td>1716.948</td>
<td>69.032</td>
<td>31.668</td>
</tr>
<tr>
<td>5</td>
<td>1700.468</td>
<td>67.998</td>
<td>31.841</td>
</tr>
<tr>
<td>6</td>
<td>1719.805</td>
<td>68.682</td>
<td>31.882</td>
</tr>
<tr>
<td>7</td>
<td>1718.567</td>
<td>69.28</td>
<td>31.584</td>
</tr>
<tr>
<td>8</td>
<td>1703.802</td>
<td>67.945</td>
<td>31.928</td>
</tr>
<tr>
<td>9</td>
<td>1719.138</td>
<td>67.565</td>
<td>32.397</td>
</tr>
<tr>
<td>10</td>
<td>1668.175</td>
<td>67.911</td>
<td>31.276</td>
</tr>
<tr>
<td>Mean</td>
<td>1712.518</td>
<td>68.408</td>
<td>31.8754</td>
</tr>
<tr>
<td>Std Dev</td>
<td>19.830354</td>
<td>0.738487192</td>
<td>0.32211599</td>
</tr>
</tbody>
</table>

Based on the results of Table 4.1, the standard deviation for all 3 values measured demonstrates a consistent measurement when repeated. The difference between the largest and smallest value of area was only 77.7 μm², or 4.4% difference at most. The
small deviations of the 3 values show that measurements are repeatable, and that the process of measuring the defects using ImageJ is an consistent method.

4.2.1 Image Analysis Results

The 111 images taken for size and shape characterization resulted in measurements of 581 defects. 239 defects were recorded on G-2 and 342 on G-3, with an average over 5 defects per image. Measurements were originally kept separately between the two samples in order to confirm that the distribution of defect sizes on each sample was statistically the same. The basic statistics for each sample are shown in Table 4.2. Also the two samples and their distribution of defect areas were placed together in a cumulative distribution function (CDF) shown in Figure 4.6.

<table>
<thead>
<tr>
<th></th>
<th>G2</th>
<th>G3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, µm²</td>
<td>380.91</td>
<td>419.37</td>
</tr>
<tr>
<td>Std Dev, µm²</td>
<td>610.06</td>
<td>1351.26</td>
</tr>
<tr>
<td>Max, µm²</td>
<td>4998.01</td>
<td>18034.33</td>
</tr>
<tr>
<td>Min, µm²</td>
<td>5.88</td>
<td>4.68</td>
</tr>
<tr>
<td>N</td>
<td>239.00</td>
<td>342.00</td>
</tr>
</tbody>
</table>
Additionally, the general elliptical shape was determined by plotting the major and minor diameters as a function of their area. Figure 4.7 shows the plot of major diameter as a function of its area, while Figure 4.8 shows the minor diameters relationship.
Figure 4-7 – Major Diameter vs. Area

Figure 4-8 – Minor Diameter vs. Area
The added trend line in both figures shows a consistent relationship between the two diameter measurements and their areas. Therefore Equations 4.1 and 4.2 from the trend line’s equation describe these relationships.

\[ D_{maj} = 1.79(Area)^{5055} \quad \text{(Equations 4.1)} \]

\[ D_{min} = .7113(Area)^{4945} \quad \text{(Equation 4.2)} \]

Using these equations for major and minor diameter, the general equation for an elliptical area can be derived:

\[ \left(\frac{D_{maj}}{2}\right)\left(\frac{D_{min}}{2}\right) = \frac{1}{4} \left(7113(Area)^{4945}\right)\left(1.79(Area)^{5055}\right) \quad \text{(Equation 4.3)} \]

\[ (r_{maj})(r_{min}) = \frac{1}{\pi} Area \quad \text{(Equation 4.4)} \]

\[ \therefore \pi(r_{maj})(r_{min}) = Area \quad \text{(Equation 4.5)} \]

Since the relationship between Equations 4.1 and 4.2 results in the general ellipse equation, any defect area can be selected and given an assumed major and minor diameter due to the relationships found during image analysis.

### 4.3 Defect Population

In order to characterize the number of defects present in a single specimen, additional measurements were done on samples G-2 and G-3, also using the SEM. Although a large number of defects were found in the earlier analysis, the data does not give an overall representation of the total population of defects on the bond surface. For an unbiased measure of the population density of the defects making sure no surface area was double counted, a grid was drawn on each fracture surface (Figure 4.9). A lower magnification was used to capture as much of the surface in each of the grid sections as possible.
The voided area in G-2 was due to an irregular shaped grid section resulting from the grid drawing process. However an extra grid area was able to be measured in G-3 for the same reason, which makes a total of 24 images recorded. 16 of the images were taken around the edge of the samples and 8 were taken in the center. After the images were taken image analysis was then completed. Defects were counted on each image, and the overall image viewing area was recorded. Figure 4.10 shows an example of how the defects were counted by simply placing a large red dot in the center of each defect and counting them all up. The total area captured by the SEM image was then measured in each image to determine the population density of the defects in each image.
With the much lower magnification it was not as obvious to decide what could be classified as a defect. The earlier work measuring defects in higher magnification gave some insight into what could be considered a defect at the smaller magnifications. Although some areas that were depicted in Figure 4.10 circled in blue could have possibly been considered a defect, they appeared slightly darker and rougher than those that were chosen as defects. In general it was determined to only choose the obvious defects.

4.3.1 Defect Density Results

Table 4.3 shows the complete results for the 24 images used during the defect count, including how many defects were found per image and the total area covered by each
The location of each image was given a number according to its location in Figure 4.9. The relationship between defect density and the location on the surface also was investigated. To do this, each image was defined as being located either in the center or on the edge in the grid.

**Table 4.3 – Defect Count Results**

<table>
<thead>
<tr>
<th>Image</th>
<th># of defects</th>
<th>Area covered (mm²)</th>
<th>Defect/mm²</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>39</td>
<td>0.466</td>
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<td>2</td>
<td>35</td>
<td>0.559</td>
<td>63</td>
<td>Edge</td>
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<td>3</td>
<td>33</td>
<td>0.478</td>
<td>69</td>
<td>Center</td>
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<td>4</td>
<td>22</td>
<td>0.377</td>
<td>58</td>
<td>Center</td>
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<td>30</td>
<td>0.625</td>
<td>48</td>
<td>Edge</td>
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<td>6</td>
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<td>0.516</td>
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<td>56</td>
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<td>10</td>
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<td>0.607</td>
<td>43</td>
<td>Edge</td>
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<tr>
<td>11</td>
<td>26</td>
<td>0.454</td>
<td>57</td>
<td>Edge</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>G-3</th>
<th></th>
<th></th>
<th></th>
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</tr>
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<td>1</td>
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<td>2</td>
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<td>0.59</td>
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<td>3</td>
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<td>0.503</td>
<td>40</td>
<td>Edge</td>
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<td>4</td>
<td>25</td>
<td>0.294</td>
<td>85</td>
<td>Edge</td>
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<td>5</td>
<td>38</td>
<td>0.662</td>
<td>57</td>
<td>Center</td>
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<tr>
<td>6</td>
<td>36</td>
<td>0.503</td>
<td>72</td>
<td>Center</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>0.747</td>
<td>47</td>
<td>Edge</td>
</tr>
<tr>
<td>8</td>
<td>22</td>
<td>0.502</td>
<td>44</td>
<td>Edge</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>0.625</td>
<td>51</td>
<td>Center</td>
</tr>
<tr>
<td>10</td>
<td>31</td>
<td>0.404</td>
<td>77</td>
<td>Center</td>
</tr>
<tr>
<td>11</td>
<td>20</td>
<td>0.59</td>
<td>34</td>
<td>Edge</td>
</tr>
<tr>
<td>12</td>
<td>26</td>
<td>0.823</td>
<td>32</td>
<td>Edge</td>
</tr>
<tr>
<td>13</td>
<td>25</td>
<td>0.59</td>
<td>42</td>
<td>Edge</td>
</tr>
</tbody>
</table>
4.4 Statistical Analysis

Given the variability in defect sizes, concentration, and location observed through the SEM analysis, it was necessary to use statistics to model the initial flaw size population. Separate analyses were performed to describe defect size, density, and location of the defects on the bond surface. It is assumed that these 3 variables are all independent from each other.

Based on the data from the image analysis, further data analysis was done to both the defect size and defect density variables so that they could be characterized using statistical distributions. It must be proven for each of these variables that both samples G-2 and G-3 belong to the same data set, allowing them to be described by one single distribution. By showing that both samples belong to the same data set, a consistency of defect variables was proven between the two samples. This will support the assumption of the same consistency occurring across the entire bonding surface of the G-block configuration. In turn, the data sets for each variable and specimen can be combined into one single set and used for a statistical distribution in future modeling.

4.4.1 Defect Sizes

Figure 4.6, which compares the defect area CDFs of G2 and G3, shows that both the distributions appear to behave lognormally. Using Minitab® Statistical Software, this can be proven by fitting both samples to a probability plot and checking the null hypothesis for the fit. Figure 4.11 shows the output given in Mini-Tab of a probability plot for both samples.
The statement for the null hypothesis in Minitab is that the data set is lognormal. Thus when $P < \alpha$, the null hypothesis is rejected and the alternative hypothesis stating the data set is not lognormal is accepted. However, the P-values for both samples, as shown in the last column of the table, are both are much higher than .05, meaning that the null hypothesis can not be rejected and the distributions are assumed to be lognormal.

By transforming both lognormal distributions into normal distributions, analysis of variance (ANOVA) can be applied to compare and prove or disprove that there is no difference between the means of the two data sets. If the statement is true, and there is no difference between the means, then the two data sets can be combined into a single set. This is to be expected since both samples result from the same bonding process and
The data sets can be transformed by simply taking the natural log of each value, resulting in both G2 and G3 becoming normal distributions with the same means and standard deviations shown in the lognormal probability plot, Figure 4.11. Figure 4.12 shows the normalized distributions of both G2 and G3.

Figure 4-12 – Normalized CDF of Areas

The ANOVA also has a null hypothesis defined by Minitab, which states that the two data sets contain the same mean and can be characterized by the same normal distribution. Alpha was specified as, \( \alpha = .05 \), a 95% confidence interval. Full results of the ANOVA are shown in Appendix A, with a P-value = .270. Since the P-value value is greater than \( \alpha \), the null hypothesis cannot be rejected and it can be assumed that the two data sets both originate from the same normal distribution.
By proving that the two data series come from the same normalized distribution, the areas measured from G2 and G3 can be combined into a single data set described by a lognormal distribution. This combined distribution is shown in Figure 4.13, and with the 95% confidence interval the P-value is again much greater than .05, indicating that the distribution is in fact lognormal. This distribution can be used to characterize the variable of defect size across the entire G-block bonding surface.

![Figure 4-13 – Combined Lognormal Probability Plot of Areas](image)

It should also be noted that since the measurements were taken from a specimen tested to failure in tension, significant increase in length, and most importantly, shrinking of the diameter and bond surface occurred on each sample. This causes the same reduction in area for all of the defects measured. By computing the change in cross-sectional area before and after testing of the samples, a correction factor can be computed to determine
the original sizes of the defects for input into the model later. Table 4.4 shows 9 of the measured diameters and calculated areas prior to and after tensile testing and the % deviation from the original values.

Table 4.4 – Sample Diameters Before and After Tensile Testing

<table>
<thead>
<tr>
<th>Sample</th>
<th>( D_1 ) (m)</th>
<th>( D_0 ) (m)</th>
<th>( A_1 ) (m(^2))</th>
<th>( A_0 ) (m(^2))</th>
<th>% Diff in Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-1</td>
<td>0.00617</td>
<td>0.00750</td>
<td>0.00002992</td>
<td>0.00004422</td>
<td>32.331</td>
</tr>
<tr>
<td>G-2</td>
<td>0.00605</td>
<td>0.00751</td>
<td>0.00002875</td>
<td>0.00004428</td>
<td>35.066</td>
</tr>
<tr>
<td>G-3</td>
<td>0.00599</td>
<td>0.00750</td>
<td>0.00002822</td>
<td>0.00004422</td>
<td>36.173</td>
</tr>
<tr>
<td>G-4</td>
<td>0.00601</td>
<td>0.00751</td>
<td>0.00002834</td>
<td>0.00004431</td>
<td>36.032</td>
</tr>
<tr>
<td>G-5</td>
<td>0.00592</td>
<td>0.00751</td>
<td>0.00002751</td>
<td>0.00004431</td>
<td>37.912</td>
</tr>
<tr>
<td>G-6</td>
<td>0.00601</td>
<td>0.00751</td>
<td>0.00002841</td>
<td>0.00004428</td>
<td>35.827</td>
</tr>
<tr>
<td>G-8</td>
<td>0.00594</td>
<td>0.00751</td>
<td>0.00002775</td>
<td>0.00004425</td>
<td>37.293</td>
</tr>
<tr>
<td>G-14</td>
<td>0.00574</td>
<td>0.00751</td>
<td>0.00002588</td>
<td>0.00004428</td>
<td>41.547</td>
</tr>
<tr>
<td>G-15</td>
<td>0.00551</td>
<td>0.00750</td>
<td>0.00002386</td>
<td>0.00004422</td>
<td>46.037</td>
</tr>
<tr>
<td>Average</td>
<td>0.00593</td>
<td>0.00751</td>
<td>0.00002763</td>
<td>0.00004426</td>
<td>37.580</td>
</tr>
</tbody>
</table>

The average value for the percent difference in area can be converted into a correction factor of \( Y = 1.376 \). This value can be multiplied by any area generated from the log-normal distribution shown in Figure 4.13 to properly characterize the original size of the defects prior to tensile testing.

### 4.4.2 Defect Population Density

The procedure to analyze the defect density was similar to that used for defect size. Much like the data for the defect areas between the two specimens, the defect density data was first fit to 2 separate distributions. Common statistical distributions such as Normal, 2 and 3 parameter Weibull, and Poisson were all fit using Minitab. This time, both sets of data fit a normal distribution with the best accuracy. The probability plots for both samples are shown in Figure 4.14.
Figure 4-14 – Normal Probability plot for G2 and G3 Defect Density

The table to the right of the figure shows each sample’s mean and standard deviation as well as its P-value for how each set of data fits the normal distribution. With a 95% confidence interval, \( \alpha = .05 \). Since both P-values are much greater than \( \alpha \), again the null hypothesis cannot be rejected and both sets of data are assumed to follow their given normal distribution.

With both sets of data proving to belong to a normal distribution, ANOVA can again determine whether the mean of the two data sets are the same. By completing a one way ANOVA using Minitab, the output resulted in a P-value of .485. Since this value is greater than \( \alpha = .05 \), again the null hypothesis is not rejected and both sets of data contain the same mean and are assumed to come from the same distribution. The complete
output of the ANOVA results can be seen in Appendix A. Since both sets of data have been determined to have the same mean, both can be combined into a single array of data as shown in Figure 4.15. Based on the results, the defect density of a specimen can be characterized by a normal distribution using the mean and standard deviation produced by the probability plot below.

![Combined Normal Probability Plot of Defect Density](image)

**Figure 4-15 – Combined Normal Probability Plot of Defect Density**

In addition to analysis of defect density, the defect count can be used to determine whether or not defects are preferentially located on the bonding surface in a specific configuration, or if they are a random occurrence. This is where the center or edge locations specified during image analysis are utilized. By rearranging the defect count data shown in Table 4.3 into two new data sets by their specified location, either edge or center, another ANOVA was performed. The results of the ANOVA from Minitab using a 95% confidence interval can be seen in Appendix A. The resulting P-value from the
analysis is .103. Although close to the value $\alpha$, the null hypothesis can not be rejected and the two sets of data are assumed to belong to the same distribution. This lack of difference between the edge and center sections was to be expected since each specimen was cut from a larger block with an even surface finish and pressure applied for bonding. Since the edge defects belong to the same distribution as the center defects and there is no difference between the densities of defects at the specific locations, it can be assumed that the locations are purely independent and random. This random location can be characterized by utilizing a uniform distribution of the function. The function, $r(x)$, specifies the location of the defect as a radial distance from the center, and the value $R$ is the specimen’s radius.

$$r(x) = \sqrt{x} \quad for \quad 0 \leq x \leq R^2$$  \hspace{1cm} (Equation 4.6)
5 Crack Growth Modeling

To properly predict defect-based fatigue life, the specific fatigue crack growth (FCG) model must be applied based on the material, the loading scenario, and the defect geometry and location on the specimen. All these factors complicate the model necessary to predict the fatigue life of diffusion bonding components. This chapter outlines the necessary fracture mechanics, crack growth model and modeling of defects for implementation into the model. Two models are examined and used, the Priddle and NASGRO Models.

There are several assumptions that have been made initially in order to develop the crack growth model. For the shapes of the unbonded areas, the microstructural image analysis allowed for the shapes of these defects to be simplified as ellipses. Also since the defects all occur on the same single plane of the bond surface that was pressed together it can be assumed that the defects are very thin, penny shaped cracks. Also it is assumed, based on the geometry of the defects and the process of diffusion bonding, that each defects’ edge has a radius = 0, meaning that the defects behave as cracks. Because of the high amount of pressure during the bond process, the defects become very thin. The difference between a defect, cracked defect and inherent defect can be seen in Figure 5.1. Many times during fatigue testing and modeling based on defects a crack initiation stage must be accounted for prior to crack growth. Since these defects already take on the shape of cracks, any crack initiation stage can be neglected. Finally, the non-fusion defects also appear everywhere randomly along the surface following a uniform distribution. This
distribution means that the unbonded areas can occur along the surface of the specimen as well as internally.

![Figure 5-1 – Crack Initiation versus Inherent Cracks](image)

Another major assumption falls on the overall use of LEFM models developed with the data found from previous LCF testing. LEFM models are only valid in the elastic regions of fatigue and once significant plastic deformation occurs there is no guarantee a model can properly predict fatigue life. It is shown clearly on the strain life plot of Figure 3.2 that the left of the curve is dominated by plastic strain. A significant assumption in this analysis is to use LEFM models even though a high amount of inelastic deformation may have occurred on the specimens.

### 5.1 Priddle Fatigue Model

While the Paris law shown previously in Equation 1.3 fits a basic fatigue crack growth rate (FCGR) curve that is applicable to many situations, particularly in Stage II of the FCG curve, another model has been found to better fit data for Inconel 600. Al-Rubaie,
et al. [9] demonstrated through experimental work that the Priddle Model fits the FCGR curves for Inconel 600. Both a Priddle and modified Priddle Model, shown in Equations 5.1 and 5.2 respectively, fit the crack growth data well, with the modified model fitting the best. The objective of the modified model was to obtain a better fit of crack growth data towards the near threshold area of Stage I [9].

\[
\frac{da}{dN} = C \left( \frac{\Delta K (1 - R) - \Delta K_{th} (1 - R)}{K_c (1 - R) - \Delta K} \right)^n \quad \text{(Equation 5.1)}
\]

\[
\frac{da}{dN} = C \left( \frac{\Delta K (1 - R) - \Delta K_{th} (1 - mR)}{K_c (1 - R) - \Delta K} \right)^n \quad \text{(Equation 5.2)}
\]

These models better fit Stages I and III of the FCGR curves for Inconel 600 [9]. Unlike the Paris model, Priddle incorporates both the threshold stress intensity factor, \( \Delta K_{th} \), and the critical stress intensity, \( K_c \). \( C, n, \) and \( m \), are material constants, and \( R \) is the cyclic stress ratio. The inclusion of \( \Delta K_{th} \) and \( K_c \) in the Priddle models allows for better predictions in Stages I and III respectively. \( \Delta K_{th} \) and \( K_c \) were both determined empirically, \( C, n, \) and \( m \), were calculated and fit analytically after empirical testing to complete a FCGR curve, and \( R \) was the stress ratio used for testing in the Al-Rubaie study. Table 5.1 shows all the values for the model [9].

<table>
<thead>
<tr>
<th>Model</th>
<th>( C )</th>
<th>( n )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priddle Model</td>
<td>2.45E-08</td>
<td>1.151</td>
<td></td>
</tr>
<tr>
<td>Modified Priddle</td>
<td>2.43E-08</td>
<td>1.394</td>
<td>2.463</td>
</tr>
</tbody>
</table>

Table 5.1 – Parameters for FCGR models [9]
It should be noted that all testing for the Al-Rubaie, et al. FCGR model parameters was run with a stress ratio of \( R = 0.1 \). Rineferd’s fatigue testing ran with a stress ratio of \( R = -1 \). This dramatically changes the FCGR curves of both the Priddle and modified Priddle model from the published values of Al-Rubaie et al. Figure 5.2 compares the Priddle and Modified Priddle models with a stress ratio of 0.1 and -1.

There are several differences between the curves shown in Figure 5.2. The curve seems to shift to the right and Stage III is shown to occur at a higher stress intensity factor (i.e., larger crack size or higher stress) with an \( R = -1 \). Also the differences between the regular and modified Priddle model are much more obvious in Region I when \( R = -1 \). This could possibly mean that the fit \( C, n, \) and \( m \) parameters were developed just for the single \( R = 0.1 \) loading scenario. However, since little more information is available on
crack growth rates for Inconel 600, the Priddle model with the current data will continue to be used. Also, due to the small sizes of defects found through image analysis many crack growth rates in the present study will fall within Stage I initially. Because of the large difference in Stage I crack growth region of the Modified Priddle model at $R = -1$ and $R = 0.1$, the normal Priddle model will be used instead, as it is more conservative.

5.2 NASGRO Model

While the Priddle model was demonstrated to fit empirical values of $R = 0.1$ in previous research of Inconel 600 crack growth rates [9], little data is available for any other stress ratios. However the NASGRO equation (also known as the Forman, Newman, de Koning, and Henriksen equation) does have additional parameters published for Inconel 600 [10]. The model can incorporate all the same loading scenarios as the Priddle Model but extends to plane strain and stress constraints. In addition to these plane stress and strain constraints, the model also considers an opening stress, which is the applied stress level that causes a crack to open up fully. This opening stress, and associated opening stress intensity factor have been linked to causing fatigue crack retardation [8]. The full NASGRO model is shown in Equation 5.3.

\[
\frac{da}{dN} = C \left[ \left( \frac{1 - f}{1 - R} \right) \Delta K \right]^\alpha \left( 1 - \frac{\Delta K_{ih}}{\Delta K} \right)^\beta \left( 1 - \frac{K_{max}}{K_{crit}} \right)^\gamma 
\]  

(Equation 5.3)

In this model $f$ is the effective stress intensity ratio, which takes into account $K_{ops}$, the lowered effective stress intensity when the crack tip does not open for some portion of the loading cycle. The conditional equation for $f$ is shown in Equation 5.4.
\[ f = \frac{K_{\text{op}}}{K_{\text{max}}} = \begin{cases} \max(R, A_o + A_1R + A_2R^2 + A_3R^3) & R \geq 0 \\ A_o + A_1R & -2 \leq R < 0 \\ A_o - 2A_1 & R < -2 \end{cases} \]  

(Equation 5.4)

With:

\[ A_o = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)^2 \cos\left(\frac{\pi S_{\text{max}}}{2\sigma_0}\right)^\frac{1}{a} \]  

(Equation 5.5)

\[ A_1 = \left(0.415 - 0.071\alpha\right)\frac{S_{\text{max}}}{\sigma_0} \]  

(Equation 5.6)

\[ A_2 = 1 - A_o - A_1 - A_3 \]  

(Equation 5.7)

\[ A_3 = 2A_o + A_1 - 1 \]  

(Equation 5.8)

The threshold stress intensity factor, shown in Equation 5.9, is defined differently here than it typically is in FCG modeling. Typically, \( \Delta K_{\text{th}} \) is a material constant that may be dependent on \( R \), but in the NASGRO model it is also a function of the crack dimension \( a \) and other material and loading parameters.

\[ \Delta K_{\text{th}} = \Delta K_0 \left(\frac{a}{a + a_o}\right)^{1/2} \left(\frac{1 - f}{(1 - A_o)(1 - R)}\right)^{(1+C_aR)} \]  

(Equation 5.9)

It should be noted that \( \Delta K_0 \) is defined as a reference threshold stress intensity factor for the material when \( R = 0 \). The intrinsic crack length, defined by \( a_o \) is a material constant.

Also, the value \( K_{\text{crit}} \), shown in Equation 5.3, is described as the critical stress intensity factor. Similar to \( \Delta K_{\text{th}} \), \( K_{\text{crit}} \) is defined here differently than \( K_c \) is typically defined, and is shown in Equation 5.10.
\[ K_{\text{crit}} = K_{tc} \left( 1 + B_k e^{-\left( \frac{A_k t}{t_o} \right)^2} \right) \]  
(Equation 5.10)

For a round specimen the thickness parameter, \( t \), can be taken as the diameter. The value \( t_o \) is the thickness where transition from plane stress to plane strain occurs:

\[ t_o = 2.5 \left( \frac{K_{tc}}{S_y} \right)^2 \]  
(Equation 5.11)

All other values not defined by the equations are material constants based on published values given in the NASGRO material database. Two separate data sets are available for plate/sheet Inconel 600, based on heat treatment processing prior to testing. The lower set of data is for a temperature range from 75-800°F, and the higher set for a temperature of 1000°F. The closest value to the high temperatures reached during the diffusion bonding is heat treatment of 1000°F. Though the set of data is still only about half of the temperature at which block G was bonded, the data for the higher temperature will be used for analysis. However, the comparison between the two sets of data from the NASGRO tables show a large difference in crack growth parameter \( C \), and \( \Delta K_o \).

Comparing the two parameters, \( C \) is 100 times higher and \( \Delta K_o \) is almost cut in half at 1000°F compared to the lower temperature range. The difference between the values based on the temperatures are shown in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>75-800°F</th>
<th>1000°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>6.58E-12</td>
<td>4.50E-10</td>
</tr>
<tr>
<td>( \Delta K_o )</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

These parameters alone can increase the crack growth greatly, and it could be assumed that the effects would only be magnified with a set of data closer to the bonding
temperature. Thus it is assumed that the parameters used in NASGRO equation will lead to unconservative fatigue life predictions. All constants for Inconel 600 heat treated at 1000°F are shown in Table 5.2 with their respective descriptions.

Table 5.3 – NASGRO Equation Constants for Sheet/Plate Inconel 600

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{1c}$</td>
<td>109.884</td>
<td>Plane Strain Fracture Toughness (MPa√m)</td>
</tr>
<tr>
<td>$K_{1e}$</td>
<td>153.8</td>
<td>Effective fracture toughness surface/elliptically shaped crack (MPa√m)</td>
</tr>
<tr>
<td>$K_c$</td>
<td>164.826</td>
<td>Plane Stress Fracture Toughness (MPa√m)</td>
</tr>
<tr>
<td>$A_k$</td>
<td>1</td>
<td>Fit Parameter</td>
</tr>
<tr>
<td>$B_k$</td>
<td>0.5</td>
<td>Fit Parameter</td>
</tr>
<tr>
<td>C</td>
<td>8.37E-12</td>
<td>Paris Crack Growth Rate Constant</td>
</tr>
<tr>
<td>n</td>
<td>3.3</td>
<td>Paris Exponent for NASGRO Equation</td>
</tr>
<tr>
<td>p</td>
<td>0.5</td>
<td>Exponent in NASGRO Equation</td>
</tr>
<tr>
<td>q</td>
<td>0.5</td>
<td>Exponent in NASGRO Equation</td>
</tr>
<tr>
<td>$\Delta K_0$</td>
<td>5.494</td>
<td>Threshold Stress intensity Range $R=0$ (MPa√m)</td>
</tr>
<tr>
<td>$C_{th}$</td>
<td>1</td>
<td>Threshold Coefficient</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.5</td>
<td>Plane Stress/Strain constraint factor</td>
</tr>
<tr>
<td>$S_{max}/\sigma_o$</td>
<td>0.3</td>
<td>Ratio of max applied stress to flow stress</td>
</tr>
<tr>
<td>$a_0$</td>
<td>3.81E-05</td>
<td>Intrinsic Crack Length (m)</td>
</tr>
<tr>
<td>$S_y$</td>
<td>193.053</td>
<td>Yield Strength (MPa)</td>
</tr>
<tr>
<td>$S_{ut}$</td>
<td>572.27</td>
<td>Ultimate Strengh (MPa)</td>
</tr>
</tbody>
</table>

$K_{1c}$, $K_{1e}$, and $K_c$ are all different fracture toughness values dependent on the material properties and geometry of the specimen. Because $K_{1e}$ is specified as the fracture toughness for surface or internally elliptical cracks, which most accurately represent diffusion bonding defects, $K_{1e}$ will be used as the critical stress intensity factor that defines failure. Figure 5.3 shows a theoretical NASGRO model crack growth curve, employing all values and equations relevant to the present research, and assuming an average threshold stress intensity factor, based on the mean defect size, of $\Delta K_{th} = 2.144$ MPa√m.
In comparison to the theoretical Priddle crack growth curves shown in Figure 5.2, the NASGRO model appears to have a much wider crack propagation range controlled by $\Delta K$, with a much lower $\Delta K_{th}$ and higher $K_c$ than that of the Priddle models. Also the Stage I-II and Stage II-III transitions occur much more rapidly in the NASGRO model. Figure 5.4 shows a direct comparison between the Priddle and NASGRO crack growth curves. The entire FCGR curve of the Priddle model falls within just the mid-section of Stage II in the NASGRO model. Stage I of the NASGRO model also allows for crack growth at much lower stress intensity factors than Priddle. This causes a more conservative model initially for the smaller cracks and defects. However Stage III occurs at much higher stress intensity values than the Priddle model, which causes a less conservative model for the larger defects and overall failure of specimens.
5.3 Fracture Mechanics

Based on the defects recorded during the image analysis a wide range of flaw sizes and locations on the fracture surface are present, all of which must be accounted for in the modeling. Since these defects are generally very thin due to the bonding process causing the material to be pressed together, these flaws are already shaped like cracks and do not need any time to initiate. Although all defects will be modeled as elliptical cracks, the location on the fracture surface also plays an important role. Two separate stress intensity factor solutions are required: an internal elliptical solution when the defect is located away from the edge, and a surface flaw solution when the defect is located on the edge of the specimen.
5.3.1 Internal Elliptical Cracks

The embedded elliptical crack $\Delta K$ solution can be used for cracks that fall anywhere internally in the specimen except on the edge. The stress intensity factor for this geometry was first proposed by Isida and Noguchi [19] and confirmed by Fett and Matheck [20]. The geometry of these equations is for a semi-infinite body. This assumes that the only edge effects for the crack and $\Delta K$ are caused by the nearest edge. Since the defects in this study are relatively small (the average sized defect takes up approximately 0.00033% of the bond surface) compared to the sample size, it can be assumed that they are located on a semi-infinite body. Figure 5.5 shows the representation for the geometry of the crack on a small section of the bond surface.

![Figure 5-5 – Embedded Elliptical Crack Geometry](image)

The basic stress intensity equation for the internal flaw is shown in Equations 5.12, 5.13, and 5.14 [20]. The points A and B represent the two different stress intensity factors at the locations labeled in Figure 5.5.
\[ K_{A,B} = \sigma \sqrt{\frac{\pi a}{Q}} M_{A,B} \]  
(Equation 5.12)

The value \( Q \), given by Equation 5.13, is the approximate value for the complete elliptical integral. The remote applied stress is defined by \( \sigma \), and the value \( a \) is shown in Figure 5.5. Equation 5.14 shows the polynomial fit for the magnification factor \( M \) found by analytical calculations for the location at point A.

\[ Q = 1 + 1.464 \left( \frac{a}{c} \right)^{1.65} \quad (a \leq c) \]  
(Equation 5.13)

\[
M_A = 0.9995 + 0.0005 \mu - 0.0001 \mu^2 - 0.0001 \mu^3 \\
+ \lambda \left( 0.2038 - 0.3856 \mu + 0.5519 \mu^2 - 0.2746 \mu^3 \right) \\
+ \lambda^2 \left( -0.7489 + 0.6965 \mu - 0.985 \mu^2 + 0.5678 \mu^3 \right) \\
+ \lambda^3 \left( 1.3763 - 1.0266 \mu + 0.4242 \mu^2 - 0.1049 \mu^3 \right) 
\]  
(Equation 5.14)

where: \( \mu = a / c \quad \lambda = a / h \)

The stress intensity factor at Point B differs from Point A. This difference is due to point A being closer to the edge of the body, which causes some edge effects and a higher stress intensity solution for Point A. The equation for the magnification factor at point B is given in Equation 5.15.

\[
M_B = 0.9999 + 0.003 \mu - 0.0004 \mu^2 - 0.0002 \mu^3 \\
+ \lambda \left( 0.0282 - 0.2709 \mu + 0.5235 \mu^2 - 0.2845 \mu^3 \right) \\
+ \lambda^2 \left( 0.0978 + 0.2969 \mu - 1.0351 \mu^2 + 0.6648 \mu^3 \right) \\
+ \lambda^3 \left( 0.1206 - 0.4826 \mu + 0.8366 \mu^2 - 0.04603 \mu^3 \right) 
\]  
(Equation 5.15)

When \( a = c \), which is the preferred aspect ratio for crack growth, the geometry of the crack becomes circular, causing \( \Delta K \) to simplify to what is shown in Equation 5.16. Also the magnification factors for both Point A and B are simplified, shown in Equation 5.17 and 5.18 respectively.
\[ K_{A,B} = \sigma \sqrt{\frac{\pi a}{2.464}} M_{A,B} \]  
\hspace{1cm} (Equation 5.16)

\[ M_A = 0.9998 + 0.0955\lambda - 0.4696\lambda^2 + 0.669\lambda^3 \]  
\hspace{1cm} (Equation 5.17)

\[ M_B = 1.0023 - 0.0037\lambda + 0.0244\lambda^2 + 0.42857\lambda^3 \]  
\hspace{1cm} (Equation 5.18)

The two different stress intensity solutions cause the ellipse to grow at different rates on the two sides, which must be included in the crack growth model. Based on the size, shape and location of the elliptical defect either point can have a higher stress intensity factor than the other. The difference in growth rates on each side of the ellipse also means that a shift of the center of the defect occurs after each cycle, moving it either away from or towards the surface which also must be calculated. Figure 5.6 shows the difference in the stress intensity values at Points A and B for a single defect as the location parameter “h” is changed manually. The single defect was kept at a major diameter of 40\(\mu\)m and minor diameter of 20 \(\mu\)m, loaded with the \(\Delta\sigma = 500\) MPa when \(R = -1\).
5.3.2 Surface Cracks

Based on the model and geometry shown in Figure 5.5, the embedded elliptical stress intensity factor is valid as long as $a < h$. It can be seen in Figure 5.6 that when the defect approaches the specimen surface the stress intensity factor increases greatly. For the cases when these cracks appear on, or grow out to, the edge of the specimen, a separate surface crack stress intensity factor must be used. Forman and Shivakumar [21] developed the stress intensity factor for these surface defects based on empirical results. Much like Figure 5.5, Figure 5.7 shows the geometric representation of the surface cracks.
For the stress intensity factor at point A, the basic equation is slightly modified from the embedded elliptical solution, and is shown in Equation 5.19. The magnification factor $F_0$ is a function of $\eta = a/D$ shown in Equation 5.20. The accuracy of this model decreases as $a \to D$ [21]. The application of the limiting condition is further discussed in section 6.3.1.

$$K_A = \sigma \sqrt{\pi a} F_0(\eta)$$  \hspace{1cm} (Equation 5.19)

$$F_0(\eta) = g(\eta) \left[ .752 + 2.02\eta + .37 \left( 1 - \sin \frac{\pi \eta}{2} \right)^3 \right]$$

where $g(\eta) = \frac{.92}{\pi} \left( \frac{\pi \eta}{2} \right)^{1/2} \frac{\tan \frac{\pi \eta}{2}}{\cos \frac{\pi \eta}{2}}$ \hspace{1cm} (Equation 5.20)

for $0.64 \leq \frac{a}{b} \leq 1$
Although the dimension $b$ shown in Figure 5.7 is not directly used in the equation for the stress intensity factor, the equation is only valid in a range for the ratio of $a$ to $b$ shown in Equation 5.20. This dimension $b$ can be associated with the major radius of the elliptical defect $c$. Since these defects are relatively much smaller than the diameter of the specimen (as stated in Section 5.2.1), then it can be assumed that $b \approx c$.

### 5.3.3 Crack Geometry and Stress Intensity Solution Transition

For the case of cracks propagating under cyclic loading, it is possible for an embedded elliptical crack’s edge to eventually reach the sample surface. In this case the stress intensity factor solution for the embedded defect would be invalid and the surface crack solution would need to be applied. However, little is known about how these cracks progress from internal to surface, and usually the stress intensity factor is simply assumed to transition from one model to the other at a predetermined threshold [10]. Because of this, some modifications must be made to the geometries during this transition.

Based on Figure 5.5, the transition occurs when $a \geq h$. When the crack grows to meet this requirement, the geometry must shift to what is shown in Figure 5.7, creating a surface defect. This transition causes a jump in the crack dimension “$a$” which can be found using Equation 5.21.

$$a_{surface} = a_{embedded} + h$$  \hspace{1cm} (Equation 5.21)

Based on this new dimension, the stress intensity solution for a surface crack can then be applied for the remaining life. Also it is reasonable to assume that when the crack transition occurs, that the ratio shown in Equation 5.20 does indeed fall in the desired
range allowing the surface crack solution to be valid. This can be seen in Table 5.4, when a defect falls in the range of transitioning to a surface defect, that a/b is within the range of values for a valid stress intensity solution. The three defect sizes were chosen to represent the variety of defects found on the surface.

Table 5.4 – Geometry Ratio for Valid Surface Defect Solution

<table>
<thead>
<tr>
<th>Area (μm²)</th>
<th>c (μm)</th>
<th>a (μm)</th>
<th>h (μm)</th>
<th>new a (μm)</th>
<th>b (μm)</th>
<th>a/b</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>9.180</td>
<td>3.468</td>
<td>3.468</td>
<td>6.935</td>
<td>9.180</td>
<td>0.755</td>
</tr>
<tr>
<td>1000</td>
<td>29.398</td>
<td>10.827</td>
<td>10.827</td>
<td>21.655</td>
<td>29.398</td>
<td>0.737</td>
</tr>
<tr>
<td>10000</td>
<td>94.151</td>
<td>33.808</td>
<td>33.808</td>
<td>67.617</td>
<td>94.151</td>
<td>0.718</td>
</tr>
</tbody>
</table>

5.4 Monte Carlo Simulation

Due to all the variables examined from image analysis and the statistical modeling, the ranges of defect size, location, and density must all be accounted for in the crack growth model. Based on the distributions that have been fit for each of the variables, and the equations for fatigue modeling, a Monte Carlo Simulation can be developed in order to predict fatigue life and its variability. It is assumed that the major variables for input into the Monte Carlo Simulation (defect size, defect density, defect location) are independent from each other. Although each of these parameters has been described by a statistical distribution, no correlation has yet to be found between any of them that would otherwise suggest a relationship. Therefore, by assuming all variables are independent of each other, random sampling from each of the distributions can in turn be input into the desired crack growth model.
5.4.1 Implementation of the Monte Carlo Simulation

In order to approach the correct solution, the Monte Carlo simulation was broken down into three separate phases. Each Phase increases the complexity of the stress intensity solutions, the crack growth model, and the generic representation of the bond surface.

The first Phase is the most basic model, incorporating a single defect, placed in the center of the specimen. The first Phase also defines a criterion for failure by fracture. The second Phase continues as a single defect model, but incorporates variable defect location and the possibility of the surface defect stress intensity factor. The third and final Phase includes the multiple defects in the model by incorporating the defect population density, and also includes the possibility of failure by exceeding $S_{ut}$. Each phase will be run using the Priddle and NASGRO models to compare results. Table 5.4 shows the implementation of certain aspects of the model for the 3 separate phases, which will also be described in detail in the following sections.

<table>
<thead>
<tr>
<th>Table 5.5 – Modeling implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priddle &amp; NASGRO</td>
</tr>
<tr>
<td>Phase 1</td>
</tr>
<tr>
<td>Phase 2</td>
</tr>
<tr>
<td>Phase 3</td>
</tr>
</tbody>
</table>

The process for analyzing crack growth in the models was developed early on by utilizing the basic Paris Law model with a simple stress intensity solution. It was proven that the cycle by cycle analysis implemented in a Monte Carlo simulations were valid for predicting crack growth, and is further explained and demonstrated in Appendix B.
5.4.2 Phase 1 – Single Defect Placed Centrally

Utilizing the statistical distributions found for the defect sizes, and Equations 4.1 and 4.2, the Monte Carlo simulation is able to generate defect dimensions for input into the crack growth model. For Phase 1, the only random variable is the defect size, chosen from the lognormal distribution discussed previously. The single stress intensity solution used for Phase 1 is the embedded elliptical defect. Also included in the embedded elliptical solution is a conditional for when the dimension “a”, grows to reach or exceed “c”. When this occurs, the geometry of the crack is a basic circle, and the conditional allows for the crack growth to continue as a circle, causing both dimensions grow at the same rate. Since a centrally located defect eventually will grow into a circular shape once \( a \) reaches \( c \), and grows at the same rate in all directions, the crack will never reach the surface before enough load bearing material will be removed to cause \( S_{ul} \) failure. Thus only the embedded elliptical stress intensity solution is used in Phase 1.

The overall model used for each phase is a step by step crack growth model, with the step size specified by \( \Delta N \). For example this is done by manipulating the Priddle model shown in Equation 5.1 into what is shown in Equation 5.22.

\[
\Delta a = C \left( \frac{\Delta K(1-R) - \Delta K_{th}(1-R)}{K_{th}(1-R) - \Delta K} \right)^n \Delta N \\
\text{(Equation 5.22)}
\]

Since evaluating a crack growth model with a fraction of a cycle is physically meaningless, the minimum value of \( \Delta N \) is 1, making it the most accurate step size. The same manipulation can be applied to the NASGRO model. Early tests determined the optimum value of \( \Delta N \) that would maintain accuracy of the Monte Carlo simulation, while speeding up the overall program. \( \Delta N \) values of 1, 5, 10, 20, 50, 100, and 250 were
examined. Predictions using $\Delta N = 100$ cycles were within 0.5% of predictions when $\Delta N = 1$ and still allowed for the program to run 100 times faster. Thus it was decided to keep the constant value of $\Delta N = 100$ throughout each Monte Carlo simulation. Figure 5.8 shows the basic flowchart for the first stage for the Monte Carlo simulation.

![Monte Carlo Flowchart for Singular Centric Defect](image)

Figure 5-8 – Monte Carlo Flowchart for Singular Centric Defect
The basic failure criterion is defined in the first modeling Phase and is carried throughout each subsequent phase. Failure is defined when the calculated stress intensity factor of the defect is greater than the critical stress intensity factor. If failure does not occur in a specified number of cycles, that particular trial is said to have reached infinite life. Infinite life for this model has been specified as $1 \times 10^6$ cycles. While many time infinite life is defined as $1 \times 10^8$ cycles, this would increase the length of each simulation considerably if the sample was to run to infinite life, thus $1 \times 10^6$ cycles was chosen as the parameter.

### 5.4.3 Phase 2 – Single Defect Randomly Located

Image analysis clearly showed defects scattered across the entire area, and it was proven in the statistical analysis that the location does indeed follow a uniform distribution, as shown in Equation 4.1. A defect placed at the center of the model is essentially the least conservative stress intensity factor and prediction for failure. To increase accuracy of the bond surface with the hope to better predict lives, the second Phase of the model differs by incorporating the uniform distribution for the flaw location parameter, “$h$”, defined in Figure 5.5. By defining a location for each defect, the possibility of one being placed on the edge or growing out to the edge becomes a reality. The second Phase model differs from the first Phase by incorporating a conditional to switch between the two stress intensity solutions. Once the dimension $a \geq h$, the surface crack stress intensity solution is used and the geometry of the crack transitions according to Equation 5.21.

### 5.4.4 Phase 3 – Multiple Defect Model
Although the single randomly located defect model increases the accuracy of representing the bond surface, multiple sites with beach marks have been found on the fracture surfaces (Figure 1.4), indicating that more than one defect propagates and contributes to failure. Because of this the model must include multiple defects to further increase the accuracy with which the bond surface is represented. By sampling from the normal distribution of defect population density shown in Figure 4.15, the varying number of defects present on a specimen can be incorporated into the model. Each defect generated on the specimen surface is allowed to propagate independently from all others. This assumes that there is no interaction between the crack growth of the defects near each other. Figure 5.9 shows the final multiple defect model flowchart for a comparison of how it differs from the first Phase of the model.
Figure 5-9 – Multiple Defect Monte Carlo Simulation Flowchart
With a large number of defects on the bond surface, the growing cracks can cause a significant reduction in the amount of actual load bearing area. The stresses reported after LCF testing were calculated based on the loading and original sample geometry and do not take this reduction in area into account. As the cracks continue to grow this reduction in area may cause an increase in actual stress on the element, which by itself can cause a failure. Due to this reduction of load bearing area on the specimen cross-section, an additional failure criterion is added to the multiple defect model outlined in Figure 5.9. The full conditional is shown in Equation 5.23. Failure occurs when:

$$\frac{\text{Load}}{(A_o - \sum A_{\text{def}})} \geq S_{ul}$$  \hspace{1cm} (Equation 5.23)

Where $A_o$ is the original area of the specimen found by using the measured diameter, and $A_{\text{def}}$ is each defect’s area. “Load” is found by using the original max stress given by the LCF testing results and $A_o$. Appendix C shows both the Priddle and NASGRO model example of the full modular Matlab code for the multiple defect simulation.

An effective stress must also be calculated, based on the reduced area, for the calculation of each defect’s stress intensity factor. Although the equation is much like 5.23 the reduction of area does not include the area of the defect that the stress intensity is being calculated for. This is because the stress intensity solution already considers this reduction in area for each calculation [8]. Equation 5.24 shows the effective stress equation that adjusts for the area.

$$\frac{\text{Load}}{(A_o - \sum A_{\text{def}} + A_t)} = \sigma_{i,\text{eff}}$$  \hspace{1cm} (Equation 5.24)
$A_i$ is the area of the particular defect for which the stress intensity factor is being calculated. In comparing this equation to Equation 5.23, there would be a small difference in the stress intensity factors calculated early on when defects are small. However the equation is crucial over time when certain cracks grow larger. The areas of the larger cracks would play a more important role in defining the effective stresses, and more noticeable differences would occur in their stress intensity factors.
6 Results

The results from all three phases for both Priddle and NASGRO model’s Monte Carlo simulations are presented. For each phase and each model, three loading scenarios were run, corresponding to test samples. The maximum, mid-range, and minimum values of applied strain were used: samples G19, G17 and G25. Infinite life was defined to be $1 \times 10^6$ cycles for every simulation. The failure criterion for each simulation was recorded. Also the assumptions made previously about the stress intensity solutions in Chapter 5 were validated using final crack sizes and other parameters recorded from several of the simulations. Table 6.1 shows the experimental LCF results of G19, G17 and G25 [4].

<table>
<thead>
<tr>
<th>Sample</th>
<th>Strain Range</th>
<th>Cycles to Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19</td>
<td>0.012</td>
<td>1465</td>
</tr>
<tr>
<td>G17</td>
<td>0.0055</td>
<td>16097</td>
</tr>
<tr>
<td>G25</td>
<td>0.003</td>
<td>100017</td>
</tr>
</tbody>
</table>

Due to the large degree of variability in sizes of defects, and the large number of very small defects present, the single defect simulation often resulted in infinite life predictions. A large number of simulations were required to generate finite life distributions. So for all results in Phases 1 and 2, 3000 Monte Carlo simulations were performed. For Phase 3, only 100 samples were run for each loading scenario and model. The 100 samples were confirmed to converge to the same resulting CDF if a higher amount of samples were run during the simulation. Fewer simulations were necessary to generate finite life distributions because a large number of defects were placed on each specimen surface, allowing for a much higher probability of larger defects being
generated, growing, and causing failure. In general all simulations greatly overpredicted the fatigue life compared with the results from Rinefierd’s test data.

### 6.1 Phase 1 Results

The most basic placement occurred on this stage, with each sample’s single defect being placed in the center of the specimen. Based on the internal stress intensity factor shown in Equation 5.16 and as illustrated in Figure 5.6, the lowest stress intensity factors occur at this location. These low stress intensity factors resulted in values below both the Priddle and NASGRO values for threshold stress intensity factors and resulted in many predictions of infinite life. Table 6.2 shows the percentage of the 3000 predictions for each loading scenario and model that reached infinite life.

**Table 6.2 – Phase 1 Results**

<table>
<thead>
<tr>
<th>Test</th>
<th>% of samples @ infinite life</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19 Priddle</td>
<td>99.47</td>
</tr>
<tr>
<td>G17 Priddle</td>
<td>99.83</td>
</tr>
<tr>
<td>G25 Priddle</td>
<td>100.00</td>
</tr>
<tr>
<td>G19 NASGRO</td>
<td>96.63</td>
</tr>
<tr>
<td>G17 NASGRO</td>
<td>99.80</td>
</tr>
<tr>
<td>G25 NASGRO</td>
<td>100.00</td>
</tr>
</tbody>
</table>

For G25, the least severe loading scenario of all the tested specimens, every prediction result in both the Priddle and NASGRO models reaching infinite life. It is clear that due to the lower stress levels, the single defect placed towards the center did not create a large enough stress intensity factor to produce crack growth. The cumulative distribution results for the other 4 results are shown in Figure 6.1. Only the results that did not reach infinite life are shown in the distribution.
It can be seen that the Priddle model generally predicts failure much earlier than the NASGRO model. This is due to the discrepancies between the definitions of the two models’ critical stress intensity factors. Also a large space between the predictions of failure and predictions of infinite life occur in the Priddle model results. This same gap does not occur in the NASGRO results; instead the predictions have a smooth transfer into infinite life. The gap in the Priddle model is due to the numerator in Equation 5.1 and how $\Delta K_{th}$ interacts. Any defect that is small enough not to produce a stress intensity factor above $\Delta K_{th}$ will result absolutely no crack growth in the Priddle model, thus allowing for only a finite range of defects to cause crack growth. Although $\Delta K_{th}$ is included in the NASGRO model as well, the value merely retards crack growth. Even if a defect is small and causes a stress intensity factor below $\Delta K_{th}$, crack growth will still
occur, albeit very slowly, using the NASGRO model. Based on Table 6.1, it is clear that this Phase is not an accurate representation of the actual bonding surface.

6.2 Phase 2 Results

Implementing the uniform defect location distribution for a single defect model complicated the model, producing higher stress intensity factors than for defects placed in the center. However, this variation only affected results slightly. Table 6.3 shows the percentage of the 3000 simulations for each test that reached infinite life.

<table>
<thead>
<tr>
<th>Test</th>
<th>% of samples @ infinite life</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19 Priddle</td>
<td>99.37</td>
</tr>
<tr>
<td>G17 Priddle</td>
<td>99.83</td>
</tr>
<tr>
<td>G25 Priddle</td>
<td>100.00</td>
</tr>
<tr>
<td>G19 NASGRO</td>
<td>95.90</td>
</tr>
<tr>
<td>G17 NASGRO</td>
<td>99.83</td>
</tr>
<tr>
<td>G25 NASGRO</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Just like Phase 1, both NASGRO and Priddle models predicted all samples for G25 to reach infinite life. For G19, slightly fewer values reached infinite life compared to results shown in Table 6.1. Figure 6.2 shows the cumulative distribution results for Phase 2 G17 and G19 predictions.
In comparing results from Figure 6.2 and 6.1, values were shifted slightly to the left for Phase 2, essentially predicting earlier fatigue life. A plot of both stages together can be seen in Figure 6.3. Including the location parameter increased the variability in results, widening the range of predicted life slightly for each simulation. This is to be expected as more statistical variables are introduced into the model. The jump between predicted failure and infinite life still exists in the Priddle model. Also, the results of the Priddle and NASGRO models are still far from each other. Overall the slight variations between Phase 2 and Phase 1 do not vastly improve the predictions of Rinefierd’s LCF results.

Figure 6-2 – Cumulative Distributions for Phase 2
6.3 Phase 3 Results

The incorporation of multiple defects growing independently from one another greatly affected the results compared with the previous phases. Since the presence of multiple defects reduced the overall specimen load bearing area, effective stress levels were increased, causing higher stress intensity factors for all defects. An area fraction of defects can be calculated using Equation 6.1 shown below. First, to understand the total defect area on the specimen surface, a distribution of initial area fraction of defects on the
specimen surface before crack growth begins was created using a Monte Carlo simulation and is shown in Figure 6.4.

\[
\text{Area Fraction} = \sum \frac{A_{\text{Def}}}{A_0}
\]  
(Equation 6.1)

![Cumulative Distribution of Multiple Defect Initial Area Fraction](image)

**Figure 6-4 – Cumulative Distribution of Multiple Defect Initial Area Fraction**

The results show that the area fraction follows a normal distribution with a mean of .0279 and a standard distribution of .0075. This distribution is mainly influenced by the normal distribution used for the defect density. As the stress is cycled on the specimen and crack growth occurs, this area fraction will grow, causing the effective stress to increase.

Due to the multiple defects present on the specimen many more simulations resulted in finite life predictions. Table 6.4 shows the percentages of samples that reached infinite
life, those that failed due to stress intensity factors reaching $K_c$, and those that failed due to stress levels reaching $S_{ut}$.

**Table 6.4 – Phase 3 Results**

<table>
<thead>
<tr>
<th>Test</th>
<th>% of samples @ infinite life</th>
<th>% of samples failing by $K_c$</th>
<th>% of samples failing by $S_{ut}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19 Priddle</td>
<td>3.00</td>
<td>34.00</td>
<td>63.00</td>
</tr>
<tr>
<td>G17 Priddle</td>
<td>9.00</td>
<td>60.00</td>
<td>31.00</td>
</tr>
<tr>
<td>G25 Priddle</td>
<td>93.00</td>
<td>7.00</td>
<td>0.00</td>
</tr>
<tr>
<td>G19 NASGRO</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>G17 NASGRO</td>
<td>1.00</td>
<td>0.00</td>
<td>99.00</td>
</tr>
<tr>
<td>G25 NASGRO</td>
<td>89.00</td>
<td>0.00</td>
<td>11.00</td>
</tr>
</tbody>
</table>

In general, fewer specimens reached infinite life compared with the two earlier phases. However, many G25 simulations still reached infinite life. Also, all failures that occurred in the NASGRO simulations were due to stress levels reaching $S_{ut}$. The much higher value of $K_c$ defined in the NASGRO database prevented single defects on the specimen bond surface from reaching critical size before the ultimate strength was reached. For the Priddle model simulations, more severe loading scenarios cause more failures due to ultimate strength than critical stress intensity. The inverse relationship is true for lower loading scenarios, which cause an increase in failure due to individual defects reaching $K_c$. Overall results plotted as a cumulative distribution can be seen in Figure 6.5.
The results compared to the previous two stages show failures much earlier for every model and loading scenario. For G19, the highest loaded scenario, variability in life decreased for both Priddle and NASGRO compared with the previous two phases. This can be attributed to the large numbers of defects occurring (an average of approximately 2500 defects) on every simulated sample. With such a large number of defects being selected, a few extreme values of areas will certainly be included on each specimen. These very large flaws experience faster growth rates and earlier failures. It is difficult to judge whether the same is true for G17 and G25 due to the lack of failure predicted by the earlier models. The same jump in predictions of failure to infinite life shown in the previous stages still occurs in the Priddle model, which can be seen mostly in the G17 and G25 results.
To obtain a more direct comparison between predictions using the Priddle model or NASGRO model, a consistent set of 2500 defects, a number slightly higher then the 55 defect/mm² defect density average, was pre-generated. The loading scenario of G19 (actual N_f = 1465 cycles) was used for both the Priddle and NASGRO models and the same pre-generated defects were employed. The results are shown below in Table 6.5.

Table 6.5 – Direct Comparison of Priddle and NASGRO models

<table>
<thead>
<tr>
<th>Model</th>
<th>Cycles to Failure</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priddle</td>
<td>127600</td>
<td>Kc</td>
</tr>
<tr>
<td>NASGRO</td>
<td>210400</td>
<td>S_{ul}</td>
</tr>
</tbody>
</table>

Just like the results shown in the cumulative distribution of Figure 6.5, the Priddle model predicts failure at a much earlier loading cycle than the NASGRO simulation. Based on the results of the failure mode between the two models, the difference in predicted failure can again be explained by the large difference in published K_c material values used for the two models.

6.3.1 Final Crack Size Results for Multiple Defect Model

In order to justify previous assumptions about the two stress intensity factor solutions used, and confirm that the solutions are valid, final crack sizes were examined for several samples of the multiple defect models. Assumptions under examination include the validity of using the semi-infinite body stress intensity factor solution for the internal elliptical defects, and the accuracy of the surface flaw model as \( a \to D \). Both NASGRO and Priddle models with loading scenario G19 and G17 were examined. Table 6.6 shows the 10 largest final crack sizes from 4 separate simulations with their respective
dimensions, stress intensity value, and location. It should be noted that the “c” dimension for surface defects is not used in the calculations and therefore was converted into a correction factor for the defect area. Thus it will not be presented in the table. For internal defects, all had $a = c$ because the initial elliptical flaws had all grown to be circular.

Table 6.6 – General Results for Final Crack Sizes

<table>
<thead>
<tr>
<th>NASGRO G19 ($K_c = 153.8$ MPa)</th>
<th>NASGRO G17 ($K_c = 153.8$ MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ (mm)</td>
<td>$a/D$</td>
</tr>
<tr>
<td>3.131</td>
<td>0.4175</td>
</tr>
<tr>
<td>0.475</td>
<td>0.0634</td>
</tr>
<tr>
<td>0.222</td>
<td>0.0297</td>
</tr>
<tr>
<td>0.169</td>
<td>0.0225</td>
</tr>
<tr>
<td>0.161</td>
<td>0.0214</td>
</tr>
<tr>
<td>0.146</td>
<td>0.0195</td>
</tr>
<tr>
<td>0.141</td>
<td>0.0188</td>
</tr>
<tr>
<td>0.140</td>
<td>0.0186</td>
</tr>
<tr>
<td>0.131</td>
<td>0.0175</td>
</tr>
<tr>
<td>0.126</td>
<td>0.0168</td>
</tr>
<tr>
<td>0.109</td>
<td>0.0145</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Priddle G19 ($K_c = 40.08$ MPa)</th>
<th>Priddle G17 ($K_c = 40.08$ MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ (mm)</td>
<td>$a/D$</td>
</tr>
<tr>
<td>2.552</td>
<td>0.3402</td>
</tr>
<tr>
<td>0.889</td>
<td>0.1186</td>
</tr>
<tr>
<td>0.771</td>
<td>0.1028</td>
</tr>
<tr>
<td>0.586</td>
<td>0.0782</td>
</tr>
<tr>
<td>0.506</td>
<td>0.0675</td>
</tr>
<tr>
<td>0.455</td>
<td>0.0606</td>
</tr>
<tr>
<td>0.351</td>
<td>0.0468</td>
</tr>
<tr>
<td>0.264</td>
<td>0.0353</td>
</tr>
<tr>
<td>0.174</td>
<td>0.0232</td>
</tr>
<tr>
<td>0.116</td>
<td>0.0155</td>
</tr>
<tr>
<td>0.113</td>
<td>0.0151</td>
</tr>
</tbody>
</table>

For each of the 4 cases, the largest crack was a surface flaw. A surface defect causes a higher stress intensity factor than the internal defects, causing faster crack growth rates and the overall failure. It can be seen that for the both of the Priddle models, the large surface defect reached the defined $K_c$ of 40.08 MPa $\sqrt{m}$. For both NASGRO models,
much like the overall results of Phase 3 shown in Table 6.4, the largest defect did not reach the defined $K_c$. Instead, failure occurred due to stress exceeding ultimate strength.

For the semi-elliptical surface defect solution, it was stated that the model is less accurate as $a \rightarrow D$ [21]. The largest surface defect found between the 4 samples ($a = 3.61\text{mm}$) occurs in the NASGRO G17 results. Even then, the crack dimension is still less than half of the specimen’s diameter of 7.5 mm. Therefore, the stress intensity solution for a semi-elliptical surface crack is valid as the crack grows to critical sizes.

The internal elliptical stress intensity factor used was that of a semi-infinite body [20]. It was assumed early on that the defects would remain small enough for only the closest edge, with its distance defined by “$h$”, would be the only important edge effect. Based on the results from Table 6.6, the largest final internal crack found ($a_f = 1.05\text{mm}$) was in the Priddle model for G17. The overall area of the largest internal defect takes up less than 8% of the entire bonding surface. This shows that the defect remains small enough to where only the closest side provides the only edge effect of value, and that the semi-infinite body assumption is correct.

### 6.4 Summary of Results

In general, all 3 phases of the Monte Carlo Simulation overpredicted fatigue life to a large degree. As the complexity of the phases increased to more accurately represent the bond surface, the predictions began to better predict the empirical failures as well. The best result, Phase 3 can be seen compared to the 3 tested LCF specimens in Figure 6.6.
Figure 6-6 – Phase 3 Results Compared With Tested Specimens

It can be seen from the above figure that although the predictions are far from the actual testing results, that predictions slightly improves with the lower loading scenarios of G17 and G25. Comparing the simulations to the actual test results shows that the Priddle Model appears to be more conservative and a better predictor of the tested samples compared to the NASGRO results.
7 Conclusion

Overall the general sizes, shapes and locations of diffusion bonding defects on a surface have been examined and characterized statistically for a previously optimized bonding configuration of Inconel 600. Defect areas follow a lognormal distribution and sizes on the bonding surface were found ranging from $5 \text{ } \mu m^2$ to well over $10,000 \text{ } \mu m^2$. The defect densities on the bonding surface follow a normal distribution with an average of about 55 defects/mm$^2$. Locations of defects were found to occur uniformly on the bond surface. Finally the shapes of the defects, although random, could be simplified down to elliptical areas, and the resulting major and minor diameters could be used for crack growth modeling.

Two LEFM models were utilized throughout the three stages of the Monte Carlo simulations: Priddle and NASGRO. Inconel 600 material constants for the respective models were very different from each other, resulting in a large difference in predicted failure ranges. The NASGRO material database only contained two sets of Inconel 600 data, with the heat treatment only at half of the amount that the diffusion bonding temperatures reached [10, 4]. The effects of heat treatment on the two parameters were demonstrated in Figure 5.2, which illustrated the large difference in the crack growth rate constant, $C$, of the two published temperature ranges for the NASGRO solution. The crack growth rate constant at 1000°F was much higher than that of the temperature range 75-800°F. Assuming that a higher temperature will in turn produce larger crack growth rate constants, it can be assumed that the proper data for the temperature range used in
the manufacturing of the diffusion bonded samples would result in predictions of shorter fatigue life, allowing for results closer to that of the tested samples.

The Priddle model appears to be more accurate and conservative in predicting the fatigue life of the tested samples. However, NASGRO appeared to behave more realistically as a crack growth model for the diffusion bonded samples than Priddle based on further analysis of the overall Monte Carlo results. Due to inherent problems in the Priddle model equation, failures would suddenly jump to the defined infinite life of $1 \times 10^6$ cycles. NASGRO on the other hand allowed crack growth for flaws of any size and gave a smooth transition to infinite life. Therefore it is determined that the NASGRO model would be the better candidate to use over Priddle if a better set of material parameters were available.

Overall predictions of much higher fatigue life than those yielded by LCF tests can be attributed to the large amount of plastic deformation that occurs in strain life test. Linear elastic fracture mechanics are utilized when assuming that the material behaves elastically and there is theoretically little plastic strain occurring on the specimen [8]. It can be seen in the strain life curve shown in Figure 3.2 that this is clearly not the case, especially when predicting failure of those samples at higher strain levels, where the curve is plastically dominated. Due to this high amount of plastic strain, LEFM may not be able to properly predict fatigue life of all LCF samples. Table 7.1 shows the measured values of total, plastic, and elastic strain for G17, G19 and G25 from the Mar-Test data [4].


Table 7.1 – Strain Ranges of Experimental Samples [4]

<table>
<thead>
<tr>
<th></th>
<th>Total Strain Range %</th>
<th>Elastic Strain Range %</th>
<th>Plastic Strain Range %</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19</td>
<td>1.20</td>
<td>0.33</td>
<td>0.87</td>
</tr>
<tr>
<td>G17</td>
<td>0.55</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>G25</td>
<td>0.30</td>
<td>0.23</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Although it can be seen that the plastic strain range for G25 is much lower than the elastic strain, it still is almost 30% of the total amount of strain and can still contribute to the over-predictions of fatigue life.

The Monte Carlo simulation was developed in 3 Phases, incorporating more of the statistical properties of the bond surface as it progressed. The third and final Phase, a multiple defect model, incorporating the locations of defects and two separate stress intensity factors, was the most accurate representation of the actual bonding surface. The non-conservative predictions of fatigue life can also be attributed to the assumption that each crack grows independently from one another. Due to the large number of defects on the bonding surface interactions may occur between the cracks, either causing a higher stress intensity factor, or a link up of cracks as they grow together.

Due to the inability to accurately predict the fatigue life of the chosen diffusion bonding specimens, the application of the current model providing an enhanced version of the GDN was deemed unnecessary. Although the newly developed two dimensional defect characteristics could be related to the one dimensional GDN, it must first be proven that the defect model does give the correct fatigue life prediction. Additional work is necessary to improve the model’s accuracy.
7.1 Recommendations for Future Work

Due to the inability of the LEFM models in their current state to predict failures of the previously tested LCF samples, there are many directions in which further research can go. One recommendation is to create new diffusion bonding samples using a more widely available, cheaper material, with published diffusion bonding configurations, and a wide range of published crack growth data, such as an aluminum alloy used in the aircraft or automotive industry. Instead of the testing focused on the bondability of the material, additional samples would allow a more thorough fatigue test, where HCF testing can be the main goal. Predicting HCF life using the LEFM model developed in this research would be much more valid than the LCF samples used currently. The limited availability of material and fatigue data for Inconel 600 limited some aspects of the present work.

Analysis into the interactions of multiple defects on the bond surface could also be incorporated into the model, possibly correcting the stress intensity factor solutions to a level that would allow LEFM to properly predict the LCF tested samples. Also, an examination into elastic plastic fracture mechanics (EPFM) could be undertaken to replace the current LEFM model being used.

7.1.1 Interaction of Multiple Defects

Currently, very little research is available for the interactions and coalescence of multiple internal defects. Xiao, Lim, and Liew [22] investigated the interactions of just two internal co-planar elliptical defects. They concluded that the effects of crack interactions
become minimal when the centroidal distance between the two cracks is more than two times the sum of their major axis. Since the diffusion bonding defects in the Inconel 600 specimens in this study are uniformly distributed on the bonding surface, an average interaction effect can be developed based on the statistical analysis to determine whether a crack interaction model may be necessary.

Using the set of 2500 pre-generated defects created for the direct comparison of the Priddle and NASGRO models (shown in Table 6.5), an average defect spacing was calculated. Since the overall surface of a specimen is 4.418E-05 m², and there are 2500 defects on the specimen, that means there is 1 defect per every 1.767E-08 m². By converting this area into a square, the length of each side becomes 0.000133m. This distance can be defined as the average centroidal distance between two defects on the bonding surface, $L_{avg}$. By generically assuming that each defect from the pre-generated list is the neighbor of the next defect listed, and that every defect is spaced apart from one another by $L_{avg}$, a comparison can be made between 2 times the sum of their major axis and $L_{avg}$, as described by Xiao, Lim and Liew [22]. This comparison was made both for the defects as they occur initially on the specimen, and for the defects at the time of failure. Analysis of the pregenerated defects’ dimensions and the potential for crack interactions using both the Priddle and NASGRO model is shown in Table 7.2.

<table>
<thead>
<tr>
<th>Test</th>
<th>Initial Crack Interactions</th>
<th>Final Crack Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>G19 Priddle</td>
<td>22</td>
<td>49</td>
</tr>
<tr>
<td>G19 NASGRO</td>
<td>22</td>
<td>262</td>
</tr>
</tbody>
</table>
Although only a few noticeable interactions occur initially on the specimen surface, the numbers increase as the cracks grow. Although the Priddle model predicts fewer interactions towards fracture, the significance of 49 separate interactions means that as many as 98 cracks have higher stress intensity factors than those found by assuming independent growth. The NASGRO model suggests many more interactions towards failure since all cracks grow regardless of $\Delta K_{th}$. Regardless of the model chosen, early estimations show that there are a significant number of crack interactions that occur on a material specimen that should be accounted for. Analysis into how more than 2 elliptical defects interact with one another would possibly create even more interactions, increase stress intensity factors, and allow for predictions of earlier failures and more accurate fatigue life.

### 7.1.2 Elastic Plastic Fracture Mechanics

If the next step of research proceeds with the prediction of fatigue life using Rineferd’s LCF test specimens, the current research has shown that LEFM models may not be adequate. An investigation into the use of EPFM may be necessary to properly predict LCF fatigue life. EPFM considers the plastic zone, and the energy density generated in front of the crack tip [23]. The basic EPFM model is very similar to that of the Paris law shown in Equation 1.2, and is shown in Equation 7.1.

$$\frac{da}{dN} = C' \Delta J'''$$  \hspace{1cm} (Equation 7.1)

Like the Paris Law, $C'$ and $m'$ are material crack growth constants determined empirically. A much more complicated alternative to the stress intensity factor, $\Delta J$ is the contour integral range which considers energy density in front of the crack tip and both
the plastic and elastic portions of strain on the body [23]. However, model parameters for materials in EPFM are even less common than LEFM parameters, none of which have been found for the particular form of Inconel 600 used in this study.

Although more complicated of a model, there has been recent interest in developing J integral solutions for various crack geometries such as the semi-elliptical surface defect found on the bonding surface [24]. Additionally, other research has shown that traditional LEFM models can greatly underpredict small crack growth, and that EPFM can account for the true behavior of smaller defects [25]. Many of the diffusion bonding defects fall within a range that can be considered small (1μm-1000μm) [25], with many smaller than 100 μm across. Thus the modification from a LEFM to an EPFM model could be the next step in accurately predicting the fatigue life of the previous LCF tested diffusion bonding samples.

### 7.2 Summary

Over the course of this investigation, previous diffusion bonding LCF samples were analyzed for characterization of defects in conjunction with exploration into the prediction of fatigue life using LEFM. Although the efforts to use the more common method of defect based fatigue failures led to highly under-conservative predictions, the next steps have been laid out in two different ways in order to continue the development of a quantitative measurement of quality for a diffusion bonded component.
References


Circumferential Plane of Solid and Hollow Cylinders,” Fracture Mechanics:
Seventeenth Volume, ASTM STP 905, J. H. Underwood, R. Chait, C. W. Smith,

Elliptical Cracks in a Three-Dimensional Solid,” Engineering Fracture Mechanics

Damage During Fatigue Crack Growth,” International Journal of Fatigue, Vol. 20,


Fatigue,” Engineering Fracture Mechanics, Vol. 56, 1997 Pergamon Press Inc,
357-377.
APPENDIX A – Mini-Tab ANOVA Output

One-way ANOVA: G2 G3 DEFECT SIZE

Source  DF  SS   MS  F    P
Factor    1  2.26 2.26 1.22 0.270
Error   579 1074.18 1.86
Total   580 1076.44

S = 1.362  R-Sq = 0.21%  R-Sq(adj) = 0.04%

Individual 95% CIs For Mean Based on Pooled StDev

| Level | N | Mean | StDev | +---------+---------+---------+---------|
|-------|---|------|-------|+---------+---------+---------+---------|
| G2    | 239 | 5.069 | 1.360 |              | (-------------*--------------) |
| G3    | 342 | 4.942 | 1.363 | (-----------*-----------) |

Pooled StDev = 1.362

One-way ANOVA: G2, G3 – Defect Density Between Specimens

Source  DF  SS   MS  F    P
Factor    1  122  122  0.57 0.458
Error   22 4728 215
Total   23 4850

S = 14.66  R-Sq = 2.52%  R-Sq(adj) = 0.00%

Individual 95% CIs For Mean Based on Pooled StDev

| Level | N | Mean | StDev | +---------+---------+---------+---------|
|-------|---|------|-------|+---------+---------+---------+---------|
| G2    | 11 | 57.91 | 11.81 | (---------------*--------------) |
| G3    | 13 | 53.38 | 16.67 | (-------------*-------------) |

Pooled StDev = 14.66
One-way ANOVA: Defect Densities on Edge Vs Center

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>1</td>
<td>563</td>
<td>563</td>
<td>2.89</td>
<td>0.103</td>
</tr>
<tr>
<td>Error</td>
<td>22</td>
<td>4287</td>
<td>195</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>4850</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = 13.96  R-Sq = 11.61%  R-Sq(adj) = 7.59%

Individual 95% CIs For Mean Based on Pooled StDev

| Level  | N  | Mean | StDev | ------+---------+---------+---------+--- |
|--------|----|------|-------|-------+---------+---------+---------+--- |
| Edge   | 16 | 52.03| 15.58 | (---------*----------) |
| Center | 8  | 62.30| 9.61  | (--------------*--------------) |

Pooled StDev = 13.96
APPENDIX B - Validation of methodology for crack growth modeling in the Monte Carlo simulation

Verification is necessary to show that the implementation of a crack growth model in the Monte Carlo simulation is correct. Due to the complex Priddle and NASGRO crack growth models and the Stress intensity factors used, verification cannot easily be done. Therefore it was determined early on to demonstrate the accuracy of the modeling process used in the simulation by utilizing a simpler crack growth model and stress intensity solution. The Paris law was chosen as the model and a basic through-crack on an infinite plate will be the sample crack geometry due to the simple stress intensity solution. Regardless of the model, stress intensity solution or parameters, the fundamental methodology is the same.

For a through crack on an infinite plate, the stress intensity solution is simple, and shown below in Equation A.1 [8]. Due to the infinite height and width of the plate, the lack of edge effects removes the normally complicated shape function of the stress intensity solution.

\[ \Delta K = \Delta \sigma \sqrt{\pi a} \]  
\hspace{2cm} \text{(Equation A.1)}

The model can be applied to a generic material “X” with defined crack growth properties shown below in Table A.1. Because the validity for implementation of crack growth models into the simulation is being proven, any material can be utilized.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.00E-10</td>
</tr>
<tr>
<td>n</td>
<td>2.5</td>
</tr>
<tr>
<td>(K_c) (Mpa(\sqrt{\text{m}}))</td>
<td>100</td>
</tr>
</tbody>
</table>
By defining a stress range on the object of 200 MPa, with a stress ration $R=-1$, a critical crack size $a_c$ can be solved by implementing the critical stress intensity value into Equation A.1. This is shown below:

\[
\Delta K_c = \Delta \sigma \sqrt{\pi a_c} \\
\frac{a_c}{\Delta K_c} = \left( \frac{\Delta K_c}{\Delta \sigma} \right) \frac{1}{\pi} \\
a_c = \left( \frac{200}{200} \right)^2 \frac{1}{\pi} = 0.3183 \text{m}
\]

By manipulating the Paris Law, the number of cycles to failure can be solved for by integrating from the initial crack size to the critical crack size. Assuming an initial crack size of $a_i=0.00001\text{m}$, this is shown below:

\[
\frac{da}{dN} = C(\Delta K)^n = C(\Delta \sigma \sqrt{\pi a})^n \\
0 \int a_i \int dN = \int a_i \left( \frac{\Delta \sigma \sqrt{\pi a}}{C} \right)^n da \\
N_f = \int_{0.00001}^{0.3183} \left( \frac{200}{200} \right)^{2.5} \frac{1}{1.00E-10} da = 278133 \text{cycles}
\]

Therefore material “X” with the given loading scenario and initial crack size will fail at 278,133 cycles. This number of cycles is the true theoretical value for fatigue failure.

A simplified program has been written to focus only on the crack growth model and methodology to predicting fatigue life. This model does not solve by direct integration, instead it evaluates crack growth on a cycle-by-cycle basis. By inputting the same Paris Law model, stress intensity solution and material “X” properties shown above, the program has predicted a fatigue life of 278,141 cycles. Also the final crack size is never directly input in the model, yet the computed final crack size causing failure is found as
a_c=0.3184m. The program is shown attached at the very end of this Appendix. Table A.2 below shows the distinction between the two models and their percent difference.

<table>
<thead>
<tr>
<th>Method</th>
<th>Cycles to Failure</th>
<th>Final Crack Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Integration</td>
<td>278133</td>
<td>0.3183</td>
</tr>
<tr>
<td>Matlab Program</td>
<td>278141</td>
<td>0.3184</td>
</tr>
<tr>
<td>% Error</td>
<td>0.0029</td>
<td>0.0314</td>
</tr>
</tbody>
</table>

It can be seen there is a very small difference between predictions from the Matlab code of crack size and cycles to failure compared to the direct integration. Since the Matlab program predicts failure close to that of the true theoretical value obtained from direct integration, it was determined that the cycle-by-cycle evaluations of crack growth models utilized in the Monte Carlo simulations are valid.

%%%%% Matlab code %%%%%%% 
%%%%% Validation of Model - Paris Law crack growth of Material "X" on an infinite plate 

% Define Properties
C=0.0000000001; %Paris Coefficient
n=2.5; %Paris Exponent
dSig=200; %Stress Range
Kc=100; %Critical Stress intensity
R=-1; %Stress Ratio
a=0.00001; %initial crack size
Fracture=0; %Fracture conditional
dN=1; %Step Size
N=0; %Cycle Count

while Fracture < 1
    dK=dSig*sqrt(pi()*a); %Stress intensity
da=C*dK^n*dN; %Paris Law
    a=da+a; %new crack size
    N=N+dN; %next cycle
    if Kc<=dK/(1-R)
        Fracture=1;
    end
end
N %Display final cycle
a %display final crack size
APPENDIX C – Stage 3 Monte Carlo Matlab Examples

Priddle Model Sample:

%Monte Carlo Simulation
%Multiple Defect Model
%G19 - Low Cycle Fatigue
%Experimentally failed at 1465 cycles

function [Nf]=CrackGrowth()
clear all
cic

%constants and values for model
i=1;               %i-samples
C=2.454*10^-8;      %Priddle Coefficient
dSig=727.397;       %Stress Range MPa
Y=1.376;            %Area Correction Factor
n=1.151;            %priddle exponent
R=-1;               %Stress Ratio
mean1=4.994;        %Area mean
sd1=  1.362;        %Area standard deviation
kc=40.08 ;          %Critical Stress intensity
kth=6.38  ;         %threshold stress intensity range
H=.00375;           %Radius
D=2*H;              %Diam
AREA=H^2*pi;        %Specimen Area
Load=.5*dSig*AREA;  %Constant Force on specimen
Su=619.2;           %Ultimate Strength
dN=100;               %cycle count
infinlife=1000000;   %infinite life parameter
meandensity=55;  %mean of defect density mm^2
sd2=14;          %stdev of defect density mm^2

defects=(randn([i,1])*sd2+meandensity)*3.75^2*pi;  %assumed normal
distribution
defectnumi=round(defects);  %rounded values for # of defects per
specimen

N=zeros(i,1);           %preallocations N=total cycles of failure of
sample
Failure=zeros(i,1);     %array for type of failure

for j=1:i
    defectnum=defectnumi(j);
    area=randn([defectnum,1])*sd1 + mean1;   %generates total number of
areas based on samples defect amount
    Area=exp(area)*Y;
    ci=(1.79/2)*Area.^0.5055*10^-6;  %major rad
    aia=(.7113/2)*Area.^0.4945*10^-6; %minor rad, on side A
    aib=aia;                        %minor rad, on side B
    loc=rand([defectnum,1])*H^2;
ri=sqrt(loc);                   %radial distance
hi=H-ri;                        %surface distance
q=0;                            %cycle count
fracture=0;                     %resets failure condition
Qi=zeros(defectnum,1);          %defines elliptical integral approx
Mib=zeros(defectnum,1);         %defines mag factor for pt B
Mia=zeros(defectnum,1);         %defines mag factor for pt A
Kia=zeros(defectnum,1);         %defines dK for pt A
Kib=zeros(defectnum,1);         %defines dk for pt B
surface=hi;                     %surface defect conditional

while fracture<1

ai=(aia+aib)/2;
hi=hi+(aib-aia)/2;
ri=H-hi;

mu=ai./ci;
lam=ai./hi;

[surface]=SurfaceDefectConditional(defectnum,ai,ri,H,hi);

[Qi,Mib,Mia]=CreateBasicMagFactor(defectnum,ai,ci,Qi,Mib,Mia,lam,mu);
[Qi,Mia,Mib,ci]=CircleDefect(defectnum,ai,ci,Qi,Mib,Mia,lam);

[ai,ci,Qi,Mia,Mib,surface]=SurfaceDefect(defectnum,ai,ri,H,surface,ci,
Qi,Mia,Mib,D);

defectarea=ci.*ai*pi;
SumArea=sum(pi*ai.*ci);
AREAS=SumArea-defectarea;

dSig=2*Load./(abs(AREA-AREAS));

Kia=dSig.*Mia.*sqrt(pi*ai./Qi);   %Stress intensity
Kib=dSig.*Mia.*sqrt(pi*ai./Qi);

numeratora=Kia*(1-R)-kth*(1-R);   %priddle Numerator
numeratorb=Kib*(1-R)-kth*(1-R);

[fracture]=Kfracture(defectnum,Kia,kc,R,Kib,fracture);

[numeratora,numeratorb]=PriddleNumerator(defectnum,numeratora,
numeratorb);

daia=C*((numeratora)./(kc*(1-R)-Kia)).^n*dN;   %priddle model
daib=C*((numeratorb)./(kc*(1-R)-Kib)).^n*dN;

aia=ai+daia;   % adds crack growth
aib=ai+daib;

[fracture]=SuCriteria(Load,AREA,ai,ci,Su,fracture);

105
q=q+dN;  \% new cycle count

[fracture]=Infinitelife(q,infinlife,fracture);

end
N(j)=q;
Failure(j)=fracture;

end
Nf=sort(N,'ascend');
A=[1:1:i];
PNf=A/(1+i);
scatter(Nf,PNf);

return

\% SurfaceDefectConditional - for solution transition
\% SurfaceDefectConditional(defectnum,ai,ri,H,hi,surface)
function
end

\% CreateBasicMagFactor - For elliptical internal defect
\% CreateBasicMagFactor(defectnum,ai,ci,Qi,Mib,Mia,lam,mu)
function
end
end
return

% CircleDefect - simplified elliptical Model
% function [Qi,Mia,Mib,ci]=CircleDefect(defectnum,ai,ci,Qi,Mib,Mia,lambda)
for k=1:defectnum
    if ai(k)>ci(k) % when a=c becomes a circular defect
        ci(k)=ai(k);
        Qi(k)=2.1464;
        Mib(k)=1.0027+(lambda(k))*-0.0037+(lambda(k))^2*.0244+(lambda(k))^3*.42857; % new mag factor
        Mia(k)=.9998+(lambda(k))*.0955+(lambda(k))^2*-.4696+(lambda(k))^3*.669;
    end
end
return

% SurfaceDefect - When defect reaches or formed on surface
% function [ai,ci,Qi,Mia,Mib,surface]=SurfaceDefect(defectnum,ai,ri,H,surface,ci,Qi,Mia,Mib,D)
for k=1:defectnum
    if ai(k)+ri(k)>=H % surface defect conditional
        ai(k)=ai(k)+surface(k);
        ci(k)=ai(k)/2;
        Qi(k)=1;
        l=ai(k)/D;
        Fo=.752+2.02*l+.37*(1-sin(pi*l/2))^3;
        Go=.92*(2/pi)*((tan(pi*l/2))/(pi*l/2))^0.5/cos(pi*l/2);
        Mia(k)=Go*Fo; % new mag factor
        Mib(k)=0;
        surface(k)=0;
    end
end
return

% KFracture - Defines failure by reaching critical stress intensity value
% function [fracture]=KFracture(defectnum,Kia,kc,R,Kib,fracture)
for k=1:defectnum
    if Kia(k)/(1-R)>kc % failure criteria for single defect reaching critical stress intensity
        fracture=1;
    end
    if Kib(k)/(1-R)>kc % failure criteria for single defect reaching critical stress intensity
        fracture=1;
    end
end
return
function [numeratora,numeratorb]=PriddleNumerator(defectnum,numeratora,numeratorb)
for k=1:defectnum
    if numeratora(k)<0        % sets crack growth to 0 if numerator of 
        priddle<0
        numeratora(k)=0;
    end
    if numeratorb(k)<0        % sets crack growth to 0 if numerator of 
        priddle<0
        numeratorb(k)=0;
    end
end
return

function [fracture]=SuCriteria(Load,AREA,ai,ci,Su,fracture)
if Load/(abs(AREA-sum(pi*ai.*ci)))>Su  %Su failure criteria
    fracture=2;
end
return

function [fracture]=InfiniteLife(q,infinlife,fracture)
if q>infinlife           %infinite life failure criteria
    fracture=3;
end
return
NASGRO Model Sample:

%Monte Carlo Simulation
%Multiple Defect Model
%G19 - Low Cycle Fatigue
%Experimentally failed at 1465 cycles

function [Nf]=CrackGrowth()
clear all
cic
%constants and values for model
i=100;        %i-samples
D=727.397;    %Stress range MPA
Y=1.376;      %Area Correction Factor
R=1;          %Stress Ratio
mean1=4.994;  %Experimental Mean of ln(a)
sd1=  1.362;  %Std Deviation of ln(a)
H=.00375;     %Radius
D=2*H;        %Diam
AREA=H^2*pi;  %Specimen Area
Load=.5*dSig*AREA; %Force on specimen
Su=619.2;     %Ultimate Strength
dN=100;        %cycle count
infinlife=100000; %infinite life parameter
kc=40.08;     %critical stress

% NASGRO parameters
alpha=2.5;     %plane stress/strain constraint factor
SmaxSigo=.3;   %Ratio of maximum applied stress to flow stress
ao=.0000381;  %intrinsic crack length
ak=1;         %fit parameter
bk=.5;        %fit parameter
cth=1;        %threshold coefficient
p=.5;         %NASGRO exponent
q=.5;         %NASGRO exponent
K1C=109.884;   %plane strain fracture toughness
K1e=153.838;  %effective facture toughness for surface/elliptical defect
dKo=5.494;     %threshold stress intensity at R=0
YS=193.053;   %yield stress
C=8.3745*10^-12; %Paris Crack growth rate constant
n=3.3;         %paris exponent

calculated constants for NASGRO
A0=(.825-.34*alpha+.05*alpha^2)*(cos((pi/2)*SmaxSigo))^(1/alpha);
A1=(.415-.071*alpha)*SmaxSigo;
f=A0+A1*R;
to=2.5*(K1C/YS)^2;
Kcrit=K1C*(1+bk*exp(-(ak*D/to)^2));
nas1=(1-f)/(1-R); %constant section of NASGRO equation

meandensity=55; %mean of defect density mm^2
sd2=14;        %stdev of defect density mm^2
defects=(randn([i,1])*sd2+meandensity)*3.75^2*pi;  %assumed normal distribution
defectnumi=round(defects);  %rounded values for # of defects per specimen

N=zeros(i,1);           %preallocations N=total cycles of failure of sample
Failure=zeros(i,1);     %array for type of failure

for j=1:i
    defectnum=defectnumi(j);
    area=randn([defectnum,1])*sd1 + mean1;   %generates total number of areas based on samples defect amount
    Area=exp(area)*Y;
    %Area=xlsread('RandDefects.xls',1,'A4:A2503'); %pregenerated rand Areas
    % 25 'Q4:Q28'; 10 'S4:S13'
    ci=(1.79/2)*Area.^0.5055*10^-6;  %major rad
    aia=(.7113/2)*Area.^0.4945*10^-6; %minor rad
    aib=aia;

    loc=rand([defectnum,1])*H^2;
    %loc=xlsread('RandDefects.xls',1,'B4:B2503'); %pregenerated rand Loc
    % 25 'R4:R28'; 10 'S4:S13'
    ri=sqrt(loc);                   %radial distance
    hi=H-ri;                        %surface distance
    cycle=0;                            %cycle count
    fracture=0;
    count=0;
    Qi=zeros(defectnum,1);
    Mib=zeros(defectnum,1);
    Mia=zeros(defectnum,1);
    Kia=zeros(defectnum,1);
    Kib=zeros(defectnum,1);
    kth=zeros(defectnum,1);
    dai=zeros(defectnum,1);
    l=zeros(defectnum,1);
    surface=hi;

    while fracture<1

        [surface]=SurfaceDefectConditional(defectnum,ai,ri,H,hi);
        [Qi,Mib,Mia]=CreateBasicMagFactor(defectnum,ai,ci,Qi,Mib,Mia,lam,mu);
        [Qi,Mia,Mib,ci]=CircleDefect(defectnum,ai,ci,Qi,Mib,Mia,ai);
[ai,ci,Qi,Mia,Mib,surface] = SurfaceDefect(defectnum, ai, ri, H, surface, ci, Qi, Mia, Mib, D);

defectarea = ci.*ai*pi;
SumArea = sum(pi*ai.*ci);
AREAS = SumArea - defectarea;

dSig = 2*Load./(abs(AREA - AREAS));

Kia = dSig.*Mia.*sqrt(pi*ai./Qi);  % Stress intensity
Kib = dSig.*Mia.*sqrt(pi*ai./Qi);

numeratora = Kia*(1-R) - kth*(1-R);  % priddle Numerator
numeratorb = Kib*(1-R) - kth*(1-R);

[fracture] = Kfracture(defectnum, Kia, kc, R, Kib, fracture);
[kth] = Kthreshold(dKo, ai, ao, f, A0, R, cth, R)
[daia, daib] = diabothsides(C, nas1, Kia, kth, p, Kcrit, q, dN, Kib)
aia = ai + daia;  % adds crack growth to defect
aib = ai + daib;

cycle = cycle + dN;  % new cycle count
[fracture] = SuCriteria(Load, AREA, ai, ci, Su, fracture);
[fracture] = InfiniteLife(q, infinlife, fracture);

% count = count + 1;  % confirmation of proper crack growth
cycle = cycle + dN;

end
N(j) = cycle;
Failure(j) = fracture;

end
Nf = sort(N, 'ascend');
A = [1:1:i];
PNf = A/(1+i);
scatter(Nf, PNf);

return

% ------------------------------
% SurfaceDefectConditional - for solution transition
% ------------------------------
function
[surface] = SurfaceDefectConditional(defectnum, ai, ri, H, hi, surface)
for k=1:defectnum
    if ai(k)+ri(k)<H
        surface(k)=hi(k);
    end
end
return

% ##################################################################
% CreateBasicMagFactor - For elliptical internal defect
% ##################################################################
function [Qi,Mib,Mia]=CreateBasicMagFactor(defectnum,ai,ci,Qi,Mib,Mia,lam,mu)
    for k=1:defectnum
        if ai(k)<=ci(k)
            Qi(k)=1+1.1464*(ai(k)/ci(k))^1.65; %Elipse integral
            Mib(k)=-.9999+.003*(mu(k))-.0004*(mu(k))^2+.0002*(mu(k))^3 ...
                    +.0282-.2709*(mu(k))+.5235*(mu(k))^2-.2845*(mu(k))^3 ...
                    +.0978+.2969*(mu(k))-.10351*(mu(k))^2+.6648*(mu(k))^3 ...
                    +.1206-.4826*(mu(k))+.8366*(mu(k))^2-.04603*(mu(k))^3;
            Mia(k)=-.9995+.0005*mu(k)-.0001*(mu(k))^2-.0001*(mu(k))^3 ...
                    +.2038-.3856*(mu(k))+.5519*(mu(k))^2-.2746*(mu(k))^3 ...
                    +.2986-.6965*(mu(k))-.985*(mu(k))^2+.5678*(mu(k))^3 ...
                    +1.3763-1.0266*(mu(k))+.4242*(mu(k))^2-.1049*(mu(k))^3;
        end
    end
return

% ##################################################################
% CircleDefect - simplified elliptical Model
% ##################################################################
function [Qi,Mib,Mia,ci]=CircleDefect(defectnum,ai,ci,Qi,Mib,Mia,lam)
    for k=1:defectnum
        if ai(k)>ci(k)          %when a=c becomes a circular defect
            ci(k)=ai(k);
            Qi(k)=2.1464;
            Mib(k)=1.0027+(lam(k))*-.0037+(lam(k))^2*.0244+(lam(k))^3*.42857;  % new mag factor
            Mia(k)=-.9998+(lam(k))*-.0955+(lam(k))^2*-.4696+(lam(k))^3*.669;
        end
    end
return

% ##################################################################
% SurfaceDefect - When defect reaches or formed on surface
% ##################################################################
function
[ai,ci,Qi,Mia,Mib,surface]=SurfaceDefect(defectnum,ai,ri,H,surface,ci,Qi,Mia,Mib,D)
for k=1:defectnum
    if ai(k)+ri(k)>=H % surface defect conditional
        ai(k)=ai(k)+surface(k);
        ci(k)=ai(k)/2;
        Qi(k)=1;
        l=ai(k)/D;
        Fo=.752+2.02*l+.37*(1-sin(pi*l/2))^3;
        Go=.92*(2/pi)*((tan(pi*l/2))/(pi*l/2))^.5/cos(pi*l/2);
        Mia(k)=Go*Fo; %new mag factor
        Mib(k)=0;
        surface(k)=0;
    end
end
return

% KFracture - Defines failure by reaching critical stress intensity value
% % % % %
function [fracture]=Kfracture(defectnum,Kia,kc,R,Kib,fracture)
for k=1:defectnum
    if Kia(k)/(1-R)>kc % failure criteria for single defect reaching critical stress intensity
        fracture=1;
    end
    if Kib(k)/(1-R)>kc % failure criteria for single defect reaching critical stress intensity
        fracture=1;
    end
end
return

% Kthreshold - Defines threshold Stress intensity factor for each defect
% % % % %
function [kth]=Kthreshold(dKo,ai,ao,f,Area,R,cth,R)
kth=(dKo*(ai./(ai+ao)).^5)./(1-f)/((1-A0)*(1-R))^(1+cth*R);
return

% daibothsides - Defines crack growth for both sides of the defect
% % % % %
function [daia,daib]=diabothsides(C,na1,Kia,kth,p,Kcrit,q,dN,Kib)
daia=(C*(nas1*Kia).^n).*(((1-(kth./Kia)).^p)./(1-(.5*Kia/Kcrit)).^q)*dN;
daib=(C*(nas1*Kib).^n).*(((1-(kth./Kib)).^p)./(1-(.5*Kib/Kcrit)).^q)*dN;
return

% SuCriteria - Failure by reaching ultimate strength
% % % % %
function [fracture]=SuCriteria(Load,AREA,ai,ci,Su,fracture)
if Load/(abs(AREA-sum(pi*ai.*ci)))>Su  %Su failure criteria
    fracture=2;
end
return

% #########################################
% InfiniteLife - Sample reaches infinite life
% #########################################
function [fracture]=InfiniteLife(q,infinlife,fracture)
if q>infinlife           %infinite life failure criteria
    fracture=3;
end
return