An investigation into the viability of heat sources for thermoelectric power generation systems

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An Investigation into the Viability of Heat Sources for Thermoelectric Power Generation Systems

By

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering

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Abstract:

Thermoelectric materials are a potential means of converting thermal energy into clean and reliable electric power. Although current commercially-available modules are not economically viable, there is hope that in the next few years recent breakthroughs in the laboratories will result in a whole new class of high efficiency modules. To access the viability of the next generation of thermoelectric modules, improved system-level modeling tools are necessary.

To this end, a versatile system model is developed, with the capability of accommodating many configurations, including but not limited to the number of modules, type of modules, geometrical parameters, and heat exchanger parameters. With this wide range of variables, it is possible to gain an understanding of the mechanisms of system performance and how they can be manipulated to optimize a thermoelectric system. Analytical tools, however, are necessary to determine the potential viability of the next generation of Thermoelectric Power Generation Systems.

In this work, a model describing the performance of a thermoelectric system is developed and designed to operate over a large range of system configurations. The theoretical model is compared to the experimental results obtained from a Thermoelectric Power Generation System testing box tested under several configurations and conditions. Discrepancies between model and experiments are described with several model improvements developed and implemented. Finally, the model is incorporated with a heat transfer model and a pricing model to develop a preliminary optimization tool. The optimization tool is then used to analyze the viability of thermoelectric power generation in a hypothetical automotive application when compared with the operating costs of an alternator to develop viability curves based off the price of fuel.
Acknowledgements

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Nomenclature

\[ A \] \quad \text{area [m}^2\text{]} \\
\[ A_b \] \quad \text{area of the base [m}^2\text{]} \\
\[ A_o \] \quad \text{minimum free flow area [m}^2\text{]} \\
\[ A_t \] \quad \text{total area [m}^2\text{]} \\
\[ B \] \quad \text{cost per unit energy [$/W]} \\
\[ C_p \] \quad \text{heat capacity [J/kg-K]} \\
\[ CF \] \quad \text{Capacity Factor} \\
\[ D_h \] \quad \text{hydraulic diameter [m]} \\
\[ e \] \quad \text{energy cost [$/kWh]} \\
\[ f \] \quad \text{fanning friction factor} \\
\[ H \] \quad \text{1 Energy Unit Installed [W]} \\
\[ h \] \quad \text{convective coefficient [W/m}^2\text{-K]} \\
\[ I \] \quad \text{current [A]} \\
\[ J \] \quad \text{Colburn factor} \\
\[ j \] \quad \text{electric current density [A/m}^2\text{]} \\
\[ K_{TE} \] \quad \text{thermal conductance [W/K]} \\
\[ k \] \quad \text{thermal conductivity [W/m-K]} \\
\[ L \] \quad \text{length [m]} \\
\[ l \] \quad \text{price of fuel [$/gal/hr]} \\
\[ M \] \quad \text{module cost per module rated power [$]} \\
\[ m \] \quad \text{load resistance ratio} \\
\[ m \] \quad \text{mass flow rate [kg/s]}
$NR_e$ module electrical resistance [Ω]

$N\alpha$ module Seebeck coefficient [V/K]

$n$ number of items

$Nu$ Nusselt number

$OP$ operation time [yr]

$P$ perimeter [m]

$P_a$ pressure [N/m²]

$Pr$ Prandtl number

$q$ heat rate [W]

$\dot{q}$ volumetric joule heating [W/m³]

$q_o$ heat loss [W]

$R_c$ contact resistance [Ω]

$R_e$ electrical resistance [Ω]

$Re$ Reynolds number

$R_{sk}$ dimensionless parameter relating the ratio of hydraulic diameter and wall thickness to the thermal conductivity of the wall to the fluid

$R_{TE/N}$ module thermal resistance [K/W]

$r$ internal specific electrical resistance [Ω-m]

$T$ temperature [K]

$t$ thickness [m]

$U$ Power generated per unit power installed [kWh]

$V$ voltage [V]

$w$ width [m]
\( w_{TE} \)  power generated [W]

\( X \)  dimensionless distance

\( x \)  distance [m]

\( Z \)  thermoelectric figure of merit [K\(^{-1}\)]

**Greek Letters**

\( \alpha \)  Leg level Seebeck coefficient [V/K]

\( \Delta \)  difference

\( \gamma \)  ratio of minimum free flow area to frontal area

\( \eta \)  efficiency

\( \theta \)  dimensionless temperature

\( \pi \)  Peltier coefficient [W/A]

\( \rho \)  density [kg/m\(^3\)]

\( \sigma \)  electrical conductivity [\( \Omega^{-1}m^{-1} \)]

**Subscripts**

\( c \)  cold

\( cond \)  conduction

\( cs \)  cold side

\( D \)  diameter

\( elec \)  electrical

\( fin \)  fin

\( h \)  hot

*heater*  Temperature read from the inline heater temperature controller
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<td><code>hs</code></td>
<td>hot side</td>
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Chapter 1: Background Information

1.1 Motivation

Currently the world’s energy resources are shifting away from a fossil fuel driven economy and towards a renewable resource economy. The world’s petroleum resources are a prime example of constricting supply coupled with increasing demand. The rising price of oil, which exceeded $145 per barrel of oil during the summer of 2008[1], is spurring research into alternative fuels and energy efficient practices. Hybrids, flex fuel vehicles, and fuel cells are just a few of the possibilities to decrease the environmental impact of automobiles. The energy crunch is also driving large scale research into “clean” energies, including super efficient solar cells, wind turbines, geothermal, even generating power from waves. As of December 2006, Germany was the world’s leading producer of wind energy with over 20 GW of power capacity constituting 7.3% of its total power capacity [2]. This illustrates how countries are increasingly depending on renewable power generation systems. This move towards clean power generation is an attempt to minimize the carbon footprint burning fossil fuels generates and reduces the amount of carbon dioxide emissions in the future. Economic instability is driving this move towards energy independence with the public demanding and political figures pledging a solution to the rising prices.

Thermoelectrics could be a part of the next generation of energy technologies, increasing system efficiencies and reducing total emissions from the transportation and industrial sectors. Thermoelectrics are small solid state devices using the Seebeck effect to produce power when a temperature differential is placed across the device. Also the Peltier effect can be utilized to create a temperature differential when a voltage differential is placed across the device. A typical device is shown below in Figure 1.1. This device includes multiple legs composed of semiconductors of both the p- and n-type. In one leg pair, the overall effect is small. In thermoelectric devices, these legs are set up electrically in series and thermally in parallel with electrically conductive plates connecting these legs on either end. The entire system is then sandwiched between two non-electrically conducting plates to allow current to flow freely through the thermoelectric system. Previous work has looked into the possibility of using
thermoelectrics as a source for cooling in various applications such as refrigerators and seat temperature control in automobiles. They are useful for their high reliability, small size, low weight, safety features, and precise control, unfortunately low coefficient of performances limit its competitiveness, confining its use to niche markets [3]. Research into graded materials has been undertaken to increase the maximum cooling limit, with models developed to accurately predict the lowest possible temperature that can be achieved by their modules [4]. These cooling applications have received more attention than power generation systems because despite low efficiencies in the system large temperature gradients can be created.

Figure 1.1: Typical One-Stage 18-Couples TEM with Two Electrodes and Ceramic Plates [5]

Thermoelectric Power Generation Systems have some significant benefits compared to alternative heat recovery options such as small ammonia steam turbine engines. The high reliability of thermoelectrics (TE) minimizes the chances of failure and allows for more time between failures. The overall construction of TE modules is simpler and the power generation portion is solid state with no moving parts. TE modules are also able to generate electrical power from low grade waste heat (<140 °C) unusable by other power reclamation systems such as a steam turbine [6]. They also perform well versus alternative options in the 150-600 °C range because of the simplicity of the system with no moving parts. Unfortunately current thermoelectric modules have extremely low efficiencies in this low temperature range. Using the best performing materials in these lower temperature ranges, current power generation systems operate at 1-2% system efficiencies, while 5-10% efficiencies are necessary to be economically
competitive as this decreases the cost per watt to a more reasonable level [7]. Research in the area of thermoelectric power generation assists in identifying the best fields for implementation of this technology, and helps in reducing the time between the development of advanced materials and cost-effective thermoelectric power generation.

There are two main fields where these modules are being considered for wide scale application: industrial waste heat and automotive exhaust. Much of the industrial waste heat work has been done in Japan by researchers working with the Japanese New Energy Development Organization (NEDO). They have focused on industrial furnaces and incinerators as these two operations produce large amounts of waste heat, which can be utilized for power generation. Using waste heat helps focus the investigation on power generated as opposed to efficiency, which would be of concern if the energy being delivered to the system had some cost associated with it [8-10]. Similarly, car exhaust was a main focus of investigation by researchers, such as Hendricks and Lustbader, Crane and Jackson, and Karri [11-13], since this waste heat can be turned into power, which will lessen the load placed on the alternator resulting in better fuel efficiencies. Early investigations into this field included Hi-Z Technology Inc.’s attempt to develop a cost-effective power generation system to install on semi-trucks, this ultimately failed as the prohibitive costs of the thermoelectric power generation system [14]. Recent material breakthroughs such as Hi-Z’s quantum well technology, however, bode well for the practical application of thermoelectric technology in the automotive industry [15]. After the conclusion of Hi-Z Technology’s semi-truck study, the topic of automotive waste was generally ignored for the better part of a decade and the next time an investigation was undertaken similar results were encountered with the experimental power generation half of the numerically predicted value [7, 12, 16].
1.2 Theoretical Background

1.2.1 Leg Level Theory

To better understand the potential value of thermoelectrics, the theoretical basis for their performance is explained. The physical phenomena contributing to the thermoelectric performance is explained below and leads into the governing thermoelectric equations.

For a junction between a pair of thermoelectric legs, as shown in Figure 1.2, an ideal voltage is induced across the bottom of the legs when a temperature gradient is applied across the legs. This can be expressed as:

\[ V_{12} = \alpha_{pn} (T_h - T_c) \]  

(1.1)

where \( T_h - T_c \) is the temperature drop across the legs and \( \alpha_{pn} \) is the difference in the Seebeck coefficient for the two leg materials.

![Figure 1.2: Schematic of a Typical Thermoelectric Leg](image)

**Figure 1.2: Schematic of a Typical Thermoelectric Leg**
The other mode in which thermoelectrics are operated is in heating or cooling applications. This is the Peltier effect and describes the rate of heating and cooling that arises on either side of a pair when a current is introduced to the system by Kelvin’s Law.

\[ q = \pi_{pn} I \]  

(1.2)

This performance is given by the Peltier coefficient, \( \pi_{pn} \), and can be related to the Seebeck coefficient by the absolute temperature.

\[ \pi_{pn} = \alpha_{pn} T \]  

(1.3)

where \( T \) is the absolute temperature.

These two effects, Seebeck and Peltier, are the two main thermoelectric effects. Another effect that can have a significant effect on module performance is the Thompson effect, which relates to the generation of reversible heat \( q \) given by:

\[ q = \beta I \Delta T \]  

(1.4)

where \( \beta \) is the Thomson coefficient. This effect is generally neglected in lower temperature gradient problems because of the minimal effect it has on module performance.

These three effects are the basis for thermoelectric theory with the following relations describing the effects defining the performance of a thermoelectric material on a module level basis.

The irreversible heat conduction, \( q \), in a thermoelectric leg is given by:

\[ q = -kA \frac{dT}{dx} \]  

(1.5)

where \( k \) is the thermal conductivity, \( A \) is the cross sectional area of a leg, while \( T_h \) and \( T_c \) are the temperatures at the hot and cold side respectively.

Volumetric Joule heating, \( \dot{q} \), is another effect occurring in thermoelectric material. Joule heating is the heat generated by a resistive element when subjected to an electrical current and is given within each leg by:

\[ \dot{q} = \frac{I^2 \rho}{A^2} \]  

(1.6)
where $I$ is the current flowing through the circuit and $\rho$ is the electrical resistivity of the leg.

Assuming equal cross-sectional areas for each leg in a p-n leg pair, the total electrical resistance per leg pair is defined as:

$$R_e = \frac{2\rho L}{A} \quad (1.7)$$

Another important contributor to heat transfer in a thermoelectric system is contact resistance. This phenomenon arises from micro-roughness in the electrical connection between the legs and contacts, defined as the metal strips in Figure 1.2 joining the n and p-type legs, which is the electrical contact resistance. If contact resistances are not carefully considered, then a system may operate well below its theoretical potential.

**1.2.2 Module Level Theory**

These four mechanisms of heat transfer previously described are the main contributors to energy transfer in a thermoelectric system, shown in Figure 1.3 where the total heat entering the system is given by:

$$q_h = K_{TE}(T_h - T_c) + N\alpha_{pn}T_h I - \frac{1}{2} I^2 NR_e - I^2 R_e \quad (1.8)$$

where $K_{TE}$ is the thermal conductance of the module, which includes considerations for thermal conduction through the legs and the electrically insulating ceramic plates present on most modules and any thermal contact resistances present in the module, $N\alpha_{pn}$ is defined as the module level Seebeck coefficient, $T_h$ and $T_c$ are the hot and cold side absolute temperatures respectively, $I$ is current and $R_e$ is the electrical contact resistance due to the metal connections in the module. The derivation for this expression can be seen in Crane and Jackson [12].

Similarly the heat leaving the system, $q_c$, can be derived to show that:

$$q_c = K_{TE}(T_h - T_c) + N\alpha T_c I + \frac{1}{2} I^2 NR_e + I^2 R_e \quad (1.9)$$
The power generated, $w_{TE}$, by this module can then be determined by subtracting (1.9) from (1.8).

$$w_{TE} = N \alpha (T_h - T_c) I - I^2 (NR_c + 2R_e)$$  \hspace{1cm} (1.10)

$Figure\ 1.3:\ Schematic\ Representation\ of\ Thermoelectric\ Module$

This power generation is one metric of importance in evaluating the performance of a thermoelectric system. The other important metric is the efficiency, $\eta$, of the module. The module efficiency is most often defined as the power generated by the thermoelectric divided by the total heat supplied to the module.

$$\eta = \frac{w_{TE}}{q_h}$$  \hspace{1cm} (1.11)

These equations provide an overview of thermoelectric performance and the factors that contribute to a module’s operation. This basic understanding of how a Thermoelectric Model operates allows for the development of a system model power generation using some type of waste heat as the heat source.

1.3 Focus of Investigation

The background on thermoelectric modules provides the starting point for this investigation. How thermoelectrics can be applied to various applications provide an important insight into the future development of the field. Power generation from modules is in its infancy owing to the fact that currently available modules are only able to convert about 5% of thermal energy that flows through the module into electrical energy. Recent advances in material research have spurred increased interest in this field with the hope that the material breakthroughs are incorporated in commercially-available modules in the near future.
The use of thermoelectrics in a system using waste heat to generate power is of great interest for investigation. The development of a system model to evaluate the performance of a system will help to identify the issues affecting the performance of a real world system as the thermoelectric material technology improves. Several researchers have begun to broach this issue, but have several limitations. These are expanded upon in the next chapter. Most of these models are developed for a specific experimental set-up and the theoretical and experimental data can only be compared for one condition. Two specific examples of this rigidity are heat exchanger geometry and the number of modules operating in the system.

By developing a model accommodating different types of thermoelectric (TE) modules, number of modules, and heat exchanger properties among other variables, a useful tool for understanding the performance of a real world system can be examined. To begin validation of the model, a test rig is developed and investigates the performance of an actual system. The system is operated at a range of flow rates and temperatures to obtain a wide range of data for comparison to the model. From this comparison between the experimental system and theoretical model, refinements to the model are made to better replicate the real world effects that decrease the performance of a system.

Once an acceptable system model is developed, a preliminary optimization routine is developed to investigate the prospects of power generation from a specific waste heat source. The optimization combines parasitic power losses and economic concerns with previously developed results. The intent of the this preliminary model is to begin the development of a tool allowing the end user to determine the feasibility of thermoelectric power generation in a wide range of fields quickly and efficiently.
Chapter 2: Literature Review

2.1 Material Research

The field of thermoelectrics has several main areas of study: material research, module modeling, system modeling, and system application. The focus of the material research is on increasing the thermoelectric figure of merit, $ZT$, which results in more cost-effective and energy-efficient systems. The figure of merit is defined as:

$$ZT = \frac{\alpha^2 \sigma}{k} T$$  \hspace{1cm} (2.1)

where $\alpha$ is the Seebeck coefficient, $\sigma$ is the electrical conductivity, $k$ is the thermal conductivity, and $T$ is the absolute temperature.

To increase the effectiveness of a material, the Seebeck coefficient and electrical conductivity are increased while the thermal conductivity is decreased. By increasing the electrical conductivity, the thermal conductivity also increases because electrons are thermal energy carriers. For metals, where thermal transport is almost entirely by electron transport the relationship between electrical and thermal conductivity are directly related. Semiconductors are the material of choice for thermoelectric materials because of their trade-off between thermal and electrical transport, as opposed to metals, where both values are large, and non-metals that have poor thermal and electrical properties. Common commercially-available thermoelectric materials, such as bismuth telluride and lead telluride have $ZT$’s approaching one [17]. Unfortunately, for thermoelectrics to be economically viable in most applications, figures of merit must have values of two or higher.

To this end, advanced materials are currently being developed by various groups investigating several different solutions. Chen, et. al. provide a thorough review of the current areas of material research, including quantum wells, superlattices, quantum wires, and quantum dots, which are able to improve the figure of merit into ranges useful for wide scale application[17]. These low dimensional materials use quantum size effects to increase the power factor ($\alpha^2 \sigma$) while reducing the thermal conductivity by phonon boundary scattering. Through these advanced material processing techniques,
semiconductors can have their figures of merit greatly increased from their previous structures.

The Defense Advanced Research Projects Agency (DARPA) has an interest in thermoelectric material research; especially in the use of advanced thermoelectric materials for various military applications. The motivation for this is the success thermoelectrics have already had in military applications with their many benefits, such as high reliability, no maintenance, silent operation, and environmental compatibility. Many new materials and structures are being evaluated by DARPA researchers, including filled skudderites and new skudderites, mesoporous materials, thin film/quantum well/quantum wire/quantum dot structures, intercalation compounds, heavy fermion/hybridization gap systems, intermetallic semiconductors, doped polymeric materials, functionally graded materials, and quasi-crystals [17-21]. Using these materials and synthetic techniques, new and highly efficient thermoelectrics can be developed and implemented in systems. The research conducted by DARPA in conjunction with Navy and Army researchers has resulted in figures of merit consistently above two and approaching three in the lab. These new materials thermoelectrics can be effectively implemented in systems [21], with a chart detailing recent advances in thermoelectric materials in the lab shown in Figure 2-1.

![Figure 2.1: History of Thermoelectric Figure of Merit, ZT [15] & [21]](image-url)
2.2 Module Research

A study was undertaken by Min and Rowe [5] into a commercially-available thermoelectric cooler operated in the power generation mode. In the theoretical model used by these researchers, contact resistances were taken into account to give a more accurate model of the module. The effects of altering length and cross-sectional area were investigated, while increases in cross-sectional area increased the power output they also increased the volume of the device, leading to increased cost. On the other hand, leg length was found to increase power output when decreased, which results in a lower cost system. Min and Rowe also found that decreasing the leg length only increased power output to a point at which point contact resistance began to be critical with the actual temperature drop across the legs of the module decreasing. Three nearly identical modules with leg lengths of 2.54, 1.52, and 1.14 mm were obtained from the MELCOR, USA. Min and Rowe found that the shorter leg lengths greatly increased power density while only nominally decreasing conversion efficiency. One of the key limitations of this system is that this study failed to take into account the transfer of energy from a real system (such as hot water delivering heat to the module), and that $\alpha$, $\sigma$ and $k$ were temperature independent.

With the development of new materials, improved performance of modules is expected to be achieved. Researchers are creating models to predict the performance of these advanced modules. Xuan looked into the performance of a module operating between two heat exchangers; one for the cold side and one for the hot side [22]. This idealized model determined the minimum cost of manufacturing for a TE cooler while meeting a set of design requirements. Assumptions made in their analysis, include:

- Seebeck coefficients as well as electrical and thermal conductivities are all temperature independent.
- The TE material is well-insulated from the surroundings, with the exception of the heat flow in the cold and hot sides.
- The contact impedances due to interface effects are ignored.
Throughout the author’s analysis the geometry of the n- and p-type legs are optimized along with the heat exchangers to minimize cost. A non-dimensional model, taking irreversibility effects into account, was developed for the cooling power, coefficient of performance, and voltage. This model is then optimized with respect to total cost including the heat exchanger, thermoelectric and operating costs. The leg length is shown to have a significant effect on cost, with smaller lengths greatly decreasing costs thanks to the smaller amount of material needed in manufacturing. While the smaller leg lengths increase efficiency, under a certain point contact resistances begin to dominate reducing the power density.

Lineykin and Ben-Yaakov worked to develop a model of a thermoelectric device in a PSPICE compatible electronic circuit simulator [23]. The authors assumed Peltier cooling/heating is concentrated at the interfaces; Joule heating is uniformly generated throughout the volume of the thermoelectric module, TEM; and that the module operates at steady-state conditions. Using these assumptions, Lineykin and Ben-Yaakov developed an equivalent electrical circuit representing the non-electrical portions of the thermoelectric. The authors also developed conversion equations to allow for comparison thermoelectric coolers and generators performances using the performance parameters provided by the manufacturer for their respective modes of operation. These conversion factors could be extremely useful when comparing the operation of modules in a power generation mode to determine the accuracy of the manufacturer specifications.

Jovanovic and Ghamaty performed some preliminary module modeling for the development of a quantum well thermoelectric generator, which greatly increases module efficiency over currently available modules [15]. Their model was designed to determine the most efficient thickness of this new quantum well material to meet the Navy’s design requirements. This device is designed to have a footprint of 4.5 square centimeters and operate over a temperature difference of ~5 °C producing 10 mW of power per module. An efficient design was created and found to have theoretical efficiencies of nearly 14% with a temperature differential of 256 °C with a quantum well film thickness of 30 microns greatly exceeding currently available modules. Their manufacturing process was described with the testing procedures showing design requirements were met with an
improved sputtering process developed for the molybdenum metal contact with negligible contact resistance.

### 2.3 System Model Research

The studies in module modeling provide a look into the factors affecting the system model performance. Wu developed a single module system model, which takes into account both internal and external irreversibilities to develop a metric predicting a realistic upper bound on thermoelectric performance [24]. The external irreversibilities include friction and heat leak losses while the internal irreversibilities include Joule heating and thermal conduction heat losses. Important assumptions in this study include temperature independent resistivity; thermal conductivity and Seebeck coefficient for the two leg materials; and insulation of the thermoelectric element from its surrounding, except at the junctions. An expression for the power generated by the generator is derived from the current generated and the temperatures at the hot and cold junctions. The expression is then maximized, which leads to an expression of maximum specific power allowing for easy comparison between modules. To evaluate the accuracy of the irreversible system a numerical example is worked out for four cases with different degrees of reversibility. The Real Cycle Model (both internal and external irreversibilities) shows how inaccurate previous models were in predicting theoretical maximums and how using this new model an improved comparison can be made between experimental and theoretical findings.

Min and Rowe undertook a purely theoretical investigation into the optimum design of a system driven by heat combustion in five common configurations. Each of five configurations was calculated with the overall and system efficiencies plotted against the theoretical maximum module efficiency, which was only a function of the temperatures and material properties [25]. The system was modeled using a Differential Method with each thermoelectric couple viewed as an infinitesimal thermal-to-electric converter. Using this analysis, models for overall and system efficiency were developed with the expressions being similar to one another, with the exception of having slightly different denominators represented the total heat extracted by the module and the total thermal energy supplied. The set-up with heat recirculation via thermoelectric modules
with an external stage provided system efficiencies approximately 10% higher than the maximum module efficiency when operated with a preheat temperature of 1200 K. This study showed that it was possible to achieve efficiencies higher than theoretically possible in a single module with creative system design and provided a metric to investigate various geometry configurations to determine several designs worth investigating further. This study looked at thermoelectric performance using only system and overall efficiencies with many limiting assumptions causing this model to be a good measure at maximum theoretical performance. A figure of merit of one was assumed over a large temperature range, 300-1500K; heat transfer through the module was only one dimensional; there was no heat loss due to imperfect insulation; thermal contact resistances were neglected; and any necessary pump power for the working fluids was also neglected.

Bethancourt, et. al. performed numerical calculations on a system model of a thermoelectric generator operating in a counter-flow heat exchanger [26]. The model parameters were then varied to investigate the effect of leg length; the electrical resistance ratio; a non-dimensional parameter relating the hydraulic diameter and thermoelectric thickness ratio; the ratio of thermal conductivities, $R_{sk}$; and the Reynolds number on power generated and the system efficiency. The assumptions in their physical model: the thermoelectric elements are placed between two insulated walls as a partition wall to form a counter flow heat exchanger, axial conduction is negligible, constant thermophysical properties, and equal mass flow rates in both channels. The system was modeled as two differential equations, one for the fluid flow and one for the thermoelectric partition wall. The system of equations was then non-dimensionalized to allow for a maximization procedure, with it solved numerically because of the highly nonlinear nature of the equations. The effects of the various parameters are then graphed to show how changing one parameter affects the power generated and the system efficiency to determine where the most effective ranges for values exist. One of the main findings was that the max power and efficiency depended on the resistance ratio, $m$, and, $R_{sk}$. It was found that deviations from the optimum range of these two parameters significantly decreasing power and system efficiency. This investigation was intensive
with few limiting assumptions and leads to use data for future investigation and with future work concentrated on the best ranges of important parameters found in this study.

Crane and Jackson performed an optimization study designed to simulate the conditions, which may be encountered in automobile exhaust, operating with a hot liquid flow and cool air flowing over finned surfaces on either side of the system, as seen below in Figure 2.2 [12]. The study was designed to optimize heat exchanger geometry and thermoelectric geometry simultaneously. Crane and Jackson’s main assumptions include all hot fluid flow tubes have the same flow rate; the mid-plane boundary in the cross flow passage between adjacent tubes act as an adiabatic surface; and that a single tube model can capture the performance of the entire thermoelectric heat exchanger. The system models included the main forms of heat transfer in the system including conduction, Seebeck phenomena, Joule heating, and contact resistance only neglecting the Thompson effect. Experimental data was also gathered to compare against the numerical model developed in the previous portion of the paper and predicts the surface temperatures within a few degrees except for the cold side that deviates by approximately 20 °C. The heat exchanger model was compared to available radiator data and tracked well validating that portion of the model. The optimization study then used a cost function bounded by the predetermined values along with additional constraints to minimize heat loss through the minor dimension of the tube along with the net power being above 1kW to ensure that the system produced a significant amount of power. A parametric study was also undertaken to discern how sensitive the system was to variation of parameters. This study was a thorough look into system optimization including initial theoretical modeling, numerical analysis, validation testing and finally optimization of the system. It was found that the effectiveness of the system is greatly dependent on the operating temperature range of the thermoelectric device. Their optimization also found that a max power per cost of 1.1 kW/$10,000 could be achieved. The only shortcoming was in the bound placed on power generation, as the most cost efficient design may be no system at all requiring the study of the desired system outputs before optimization is undertaken.
Hendricks and Lustbader [11] performed a similar study into the application of a Thermoelectric Power Generation System for automotive applications of both light and heavy duty vehicles using a model previously developed by Hendricks [26]. The main assumptions of this model were the cascade operated between hot side temperature and cold side temperature; there were no parasitic temperature differentials at the interfaces of the device; the electrical conductivities of both stages were equal, with the currents in both stages being equal; and the parasitic energy losses were proportional to the two total hot side thermal energies. The study used optimum power system parameters with the added benefit of considering thermal interfaces between the heat exchange systems and thermal losses in the heat exchangers and TE device. The heat losses were expressed parametrically because of the difficulty in comparing values of the temperature and configuration dependency. Similar models were chosen to represent the hot and cold sides of the heat exchanger. Three advanced material segmented-leg configurations were considered in this study as single material legs were unable to meet the power requirements of this study. The model was then used to look at various flow scenarios in light duty applications at the common temperature of exhaust gases at the catalytic converter (700 °C). It was found that maximum power occurred at relatively low conversion efficiencies, but significant increases in efficiencies could occur if small
reductions in power generated were allowed. The second part of the study undertook a parametric study on the effects of varying system parameters; one significant finding was the need to limit contact resistances as they have a large effect on cold side system performance at low flow rates. Heavy duty systems were found to be capable of providing power outputs between 5-6 kW, which could significantly reduce the need for large alternators. The parametric study showed how contact resistances and heat losses can greatly reduce power output when not properly accountable. These design guidelines are useful in any real application where special attention needs to be placed on these important variables.

Karri at Clarkson University developed a model for an automotive thermoelectric power generation system [13]. His investigation looked into the modeling of several different aspects of the system. First the thermoelectric model, the Hi-Z 20, was modeled. This was followed by investigation of exhaust system of an automobile, the coolant system, and finally, the entire power generation system. For the Module Model, the key assumptions included the material in the legs were to be all Bi$_2$Te$_3$ as well as neglecting the electrical and thermal resistances of the electrical connectors for the legs. Constant thermal properties and correction coefficients were the main assumptions used in the exhaust model. Other important assumptions included constant elevation, no external work put into the system, pressure drop has a linear relationship with the length to diameter ratio, constant velocity through heater core, and the resistance of the radiator circuit being infinite during a closed thermostat condition for the exhaust model. For the thermoelectric generator (TEG) system, it assumed that the perfect insulation between exteriors and sides of the heat exchanger and between gaps in thermoelectrics and material of the heat exchangers provided negligible resistance to heat flow normal through the TE modules. This thesis was a thorough investigation into the performance of a complete system model consisting of several main parts. These models were then compared against experimental data and appropriate measures were taken to increase the accuracy of the equations. The models for heat transfer coefficients seem to be a poor fit for this system, and more appropriate models could have been chosen to better reflect the Reynold’s number of the flow. Certain assumptions affected the accuracy of the models and should be investigated to determine their validity in this context. Overall this was a
thorough study, laying out the steps in creating an effective model and in validating data to confirm these models. The main conclusions of this study were that more efficient modules are needed to make this system economically viable. Also, that such a system is very design intensive, the system can actually increase the power load if it is not operated at the correct parameters, such as the test at thirty miles per hour.

These investigations all came to the same conclusion; namely, to be economically feasible, higher thermoelectric figures of merit are needed (preferably, above 1.5, to produce cost-effective power generation) as these new materials lead to a lower cost per watt. Past and current research into applications provides a basis for wide scale implementation of advanced materials. So these studies have merit as they produce models and experimental data in an emerging field, invaluable to future thermoelectric system design. This leads to the purpose of this investigation, which is the development of a model compared to experimental data producing useable criteria for evaluating the viability of a specific stationary heat source for power generation. This is to build upon research already conducted by researchers in a variety of topics relating to the field of thermoelectrics. The work done in previous system-level research demonstrates the importance of careful design and proper assumptions in any model developed. Developing a system utilizing waste heat can help reduce emissions and increase productivity. Development of such a model depends on the theoretical work done by researchers building on their findings and experiences.
Chapter 3: Model Development

3.1 Model Investigation

Three models based on theoretical Thermoelectric Generator (TEG) models from the literature are investigated. These models are used to predict the power generation potential for a hot and cold fluid streams with known mass flow rates and thermal properties, thermoelectric thermophysical properties, and heat exchanger geometries. The models are compared by their assumptions and underlying physics to determine which best models a real thermoelectric system. The best model is then compared to experimental measurements of a TEG system. Modifications to the model are made as necessary based on experimental data. The initial models are described below with their limiting assumptions.

3.1.1 Bethancourt Model [27]

Bethancourt, et. al. developed a dimensionless, counter flow, heat exchanger TEG model. Depicted in Figure 3.1, this model is used to solve the heat equation for both the fluid and the TE [27]. It includes equations representing dimensionless power, heat absorbed, and heat released. The use of dimensionless parameters creates an easy comparison between systems to determine the most efficient processes to investigate. This study deals with a simple case of the heat flow with two fluids passing through either side. Not taking into account the complex flows that can develop in realistic heat exchangers. It is, however, a solid first approximation of system performance. Also the terms arising in the two heat transfer terms cause the system to be highly non-linear; thus, requiring a numerical approach to obtain a solution.
The dimensionless governing equation of this system can be expressed as:

\[
\frac{d\theta}{dX} = \frac{Nu}{Re \cdot Pr} (\theta - \theta_w)
\]  

(3.1)

where \( \theta \) is dimensionless temperature defined as \( \theta = \frac{T - T_{2in}}{\Delta T} \), \( X \) is dimensionless axial coordinate, \( Nu \) is the Nusselt number, \( Re \) is the Reynolds’ number, and \( Pr \) is the Prandtl number. With subscripts 1 for hot side, 2 for cold side, \( in \) for inlet and \( w \) for solid partition wall.

The expression for dimensionless power, \( P_{Dim} \), is expressed as:

\[
P_{Dim} = \frac{ZT_{w1} R_{sk}}{(Re \cdot Pr)_1 (1 + m)^2 \left( \theta_{w1} - \theta_{w2} \right)^2} \left( \theta_{w1} + \frac{T_{2in}}{\Delta T} \right)
\]  

(3.2)

where \( m \) is the electric resistance ratio, \( Re \) is the Reynolds Number, \( Pr \) is the Prandtl Number and the flow a dimensionless parameter \( R_{sk} \) is defined as:

\[
R_{sk} = \frac{D_{H1} \cdot k_w}{t \cdot k_{fi}}
\]  

(3.3)

where \( D_{H1} \) is the hydraulic diameter, \( t \) is the thermoelectric module thickness and \( k \) is the thermal conductivity.
This heat absorbed at the hot junction is expressed as:

\[
q_{1Dim} = \frac{ZT_w R_{th} (\theta_{w1} - \theta_{w2})}{(Re Pr)_1} \left[ 1 + \frac{(1 + m) (\theta_{w1} - \theta_{w2})^2}{ZT_{w1}} - \frac{(\theta_{w1} + T_{2in}/\Delta T)}{2(1 + m)} \right]
\]  
(3.4)

The heat released at the cold side is expressed as:

\[
q_{2Dim} = \frac{ZT_w R_{th} (\theta_{w1} - \theta_{w2})}{(Re Pr)_1} \left[ \frac{\theta_{w2} + T_{2in}/\Delta T}{\theta_{w1} + T_{2in}/\Delta T} + \frac{(1 + m)}{ZT_{w1}} - \frac{(\theta_{w1} - \theta_{w2})^2}{2(1 + m)} \right]
\]  
(3.5)

The boundary conditions are assumed to be:

Hot fluid: \( \theta = \theta_{1in} (X = 0) \)  
(3.6)

Cold fluid: \( \theta = \theta_{2in} (X = L) \)

The main assumptions under which this model was developed are: the thermoelectric elements are placed between two insulated walls as a partition wall to form a counter-flow heat exchanger; axial conduction in both channels and in the partition wall is negligible; thermophysical properties of the partition wall and both fluids are constant; mass flow rates in both channels are equal; and Thomson effect is neglected.

This non-dimensional model is very useful when developing a general approach to thermoelectric modeling. This generic nature lends to easy comparison between various scenarios, but can lose details that would be important when performing viability analyses. This method takes into account the variation in temperature along the x-axis thanks to the differential formulation of this model. This model also neglects any kind of fins to increase heat transfer, which may be included in an actual system. To apply this model, an investigation is needed to determine the changes to be made to allow for considerations for various fluids operating at different flow rates and possibly different states (gas or liquid).
3.1.2 Crane and Jackson Model [12]

Crane and Jackson developed a model for a cross flow heat exchanger and TEG. Shown previously in Figure 2.2, this model uses external air cooling and a convective heat transfer coefficient model taken from literature best fitting their system parameters. The pressure loss is also modeled to determine the amount of power required to drive the cooling fan. The main forms of energy transfer are included in this model, while the less important Thomson effect is neglected; these include heat conduction, Seebeck effect, Joule heating. Contact resistance is also considered in this model for the dissipative effect it has on energy delivery to the thermoelectric module. The combination of these terms in an energy balance gave expressions for the total heat delivered and rejected at the thermoelectric module surfaces. By examining the difference of these two terms, the power generated can be determined. Contact resistances can have a large effect on the system performance and have been neglected in several previous studies. The current obtained by using Ohm’s law with the total voltage in all the couples summed and divided by the total resistance in the circuit along with the load resistance, generalized for modules of varying sizes.

To model the convection at the heat exchanger surfaces, Crane and Jackson used:

\[ h_a = \frac{j m_a C_{p,a}}{A_o Pr^{2/3}} \]  

(3.7)

where \( j \) is the Colburn factor, \( m_a \) is the mass flow rate of air, \( C_{p,a} \) is the specific heat of air at constant pressure, \( A_o \) is the minimum free flow area, and \( Pr \) is the Prandtl number.

The pressure drop across the heat exchanger is given by:

\[ \Delta P_a = \frac{m_a}{2A_o \rho_{a,in}} \left[ 1 + \gamma^2 \left( \frac{\rho_{a,in}}{\rho_{a,out}} - 1 \right) + f_a \frac{A_o}{A_o} \frac{\rho_{a,in}}{\rho_{a,mean}} \right] \]  

(3.8)

which is related to mass flow, areas of interest, density (\( \rho \)), the ratio of minimum free flow area to frontal area (\( \gamma \)), and the Fanning friction factor (\( f_a \)).
The total energy flows in the entire system of thermoelectric modules is described by:

\[ q_h = \left( K_{TE} (T_{sh} - T_{sc}) + N\alpha T_{sh} I - I^2 NR_e / 2 - I^2 R_e \right) n_{TE} \]  \hspace{1cm} (3.9)

\[ q_c = \left( K_{TE} (T_{sh} - T_{sc}) + N\alpha T_{sc} I + I^2 NR_e / 2 + I^2 R_e \right) n_{TE} \]  \hspace{1cm} (3.10)

\[ w_{TE} = \left( N\alpha (T_{sh} - T_{sc}) I - I^2 (NR_e + 2R_e) \right) n_{TE} \]  \hspace{1cm} (3.11)

where \( n_{TE} \) is the number of modules in the system. The heat delivered, \( q_h \), and the heat rejected, \( q_c \), can be balanced to determine the power generated, \( w_{TE} \). Energy flows considered in the above equations include heat conduction, Seebeck effect, and Joule heating.

The current flow through the thermoelectric is determined by the number of couples aligned in series and is expressed as:

\[ I_{TE} = \frac{\sum_{i=1}^{n} (N\alpha (T_{sh} - T_{sc}) n_{TE} d_{maj} \Delta x)_i}{R_{load} + \sum_{i=1}^{n} ((NR_e + 2R_e) n_{TE} d_{maj} \Delta x)_i} \]  \hspace{1cm} (3.12)

where \( d_{maj} \) is the major diameter of the rectangular tube and \( x \) is the axial distance along the hot fluid flow axis.

Important assumptions of this model include all hot fluid flow tubes perform identically; the mid-plane boundary in the cross flow passage between adjacent tubes acts as an adiabatic surface; a single tube can capture the performance of the entire thermoelectric heat exchanger; and thermal contact resistances can be simulated by partial air gaps between two adjoining surfaces.

This system involves a more complex, one-dimensional cross flow heat exchanger model as opposed to the one dimensional counter flow heat exchangers. While a cross-flow heat exchanger may present a better option for certain power generation scenarios, the convective portions of this model may need to be changed to reflect a counter flow set up. This model suffers from the opposite problem of Bethancourt’s model in that it is very narrow in scope limiting it specific system configurations. This model provides a thorough look at a specific instance of power generation, but can be generalized by selecting less detailed models for convective heat transfer. This model was solved using a numeric scheme and provides the best option for calculating values of interest.
### 3.1.3 Karri Model [13]

Karri created a model based specifically on the HZ-20 module as opposed to a generic module. The system modeled was provided heat from an automotive exhaust system with exhaust gas as the working hot fluid and radiator liquid as the cooling fluid with a module sandwiched between these two fluids. It was assumed that the exteriors and sides and the gaps between the TEMs are perfectly insulated. Also, the material of the heat exchanger provides negligible thermal resistance to heat flowing normal to the plane of the thermoelectric modules and constant thermal properties.

Three equations for the heat entering the thermoelectric were generated along with two for the heat rejected and one for the power generated with terms for mass flow rate, specific heat, temperature, area, convective coefficient, Seebeck coefficient, current, electrical resistance, and thermal conductivity. These heat equations balanced into four equations for determining outlet temperatures along with the temperatures at the surfaces, while it was assumed that the inlet temperatures were known.

\[
\dot{m}_h C_{ph} (T_{hi} - T_{ho}) - h_h A_h \left(\frac{T_{hi} - T_{ho}}{2} - T_h\right) = 0 \quad (3.13)
\]

\[
\dot{m}_c C_{pc} (T_{co} - T_{ci}) - h_c A_c \left(T_c - \frac{T_{ci} - T_{co}}{2}\right) = 0 \quad (3.14)
\]

\[
h_h A_h \left(\frac{T_{hi} - T_{ho}}{2} - T_h\right) - h_c A_c \left(T_c - \frac{T_{ci} + T_{co}}{2}\right) - \frac{N\alpha^2 (T_h - T_c)^2}{(NR_e + R_{load})^2} R_{load} = 0 \quad (3.15)
\]

\[
h_h A_h \left(\frac{T_{hi} + T_{ho}}{2} - T_h\right) - k\Delta T - N\alpha T_h I + I^2 R_e / 2 = 0 \quad (3.16)
\]

where \( h \) is the convective coefficient, the load resistance is \( R_{load} \) and the subscripts stand for hot \( (h) \), cold \( (c) \), inlet \( (i) \) and outlet \( (o) \).

This model is similar to the above cross-flow model, but utilizes a counter flow arrangement and accounts for the change in temperature along the temperature stream. This model was developed for a moving automotive platform and was solved using a Newton Rhapson method involving the Jacobian of the above system of equations. This model seems to be readily applicable to a stationary waste heat scenario once the convective portion of the model is sufficiently altered to allow for a wider range of heat inputs.
3.2 Model Comparison:

3.2.1 Model Similarities

The models are compared here to determine the differences in formulation and the underlying assumptions to determine whether any models can be eliminated for redundancy or limiting assumptions.

The first task is to simplify the Bethancourt Model to determine whether the equations presented differ from those of the other two models, and to identify the differences.

The first component of heat transfer that is considered in this investigation is the heat due to conduction presented below.

\[
q_{\text{cond}} = \frac{N\alpha j T_{w1} (1 + m)}{Z T_{w1}}
\]  

(3.17)

which is the expression for non-dimensional heat conduction where \( \alpha \) is the Seebeck coefficient, \( j \) is electric current density, \( T \) is temperature at the wall, \( m \) is the electric resistance ratio, and \( Z \) is the figure of merit. Several of the expressions can be written as function of other values as shown.

\[
j = \frac{N\alpha (T_{w1} - T_{w2})}{r(1 + m)\delta} \quad Z = \frac{N\alpha^2}{r k_w}
\]

(3.18 & 19)

where \( \delta \) is the wall thickness and \( r \) is the specific internal resistance. By substituting the second and third equations into the first expression and simplifying the following expression is obtained.

\[
q_{\text{cond}}^* = \frac{k_w}{\delta} (T_{w1} - T_{w2})
\]

(3.20)

From the expression it is apparent that the thermal conductivity over wall thickness is equivalent to the thermal conductance value present in the other models.

The next component to analyze is the Joule heating shown below in its non-dimensional form.

\[
q_j^* = N\alpha j T_{w1} \frac{T_{w1}}{2(1 + m)}
\]

(3.21)
By substituting the equation for current density and using the following equation for current, the expression can be simplified to the form.

\[ I = \frac{N \alpha (T_{w1} - T_{w2})}{(R_i + R_{load})} \]  

(3.22)

\[ q_j = \frac{1}{2} \frac{r l^2}{A \delta} \]  

(3.23)

This expression has the same form as the joule heating term expression presented in the other two models. From this investigation it can be seen that the models use the same expressions and the best way to compare the models is with their assumptions.

First the main assumptions used by all three models can show how the models are simplified to a level where they can be easily evaluated. The first assumption is the Thomson effect is neglected; this term only affects results in higher temperature ranges and has been shown to have a much smaller effect on performance that the other forms of heat transfer. The effects of the Thomson effect have been extensively studied by Omer and Infield [28] and have shown that the testing range available to study will be largely unaffected by this effect.

The next main assumption shared by the models is neglecting heat leakage in the air gaps existing between the thermoelectric legs in the modules. No true convection occurs in these small gaps, radiation has a minimal effect so conduction is the main mode of heat transfer in the gaps. This term has been shown to be an order of magnitude less than the heat conducted through the legs and is therefore generally neglected in system level modeling. This effect was also discussed by Omer and Infield to show the minimal effect is had on system level modeling [28].

The last main assumption is the use of constant thermophysical properties. Since the temperature drop across the thermoelectric is smaller than the drop across the whole system and the small area in which this power generation takes place this is generally a valid assumption. The properties are generally taken as the mean temperature of the system. This could be addressed by using previously developed expressions for the change in temperature for these properties using the thermoelectric test stand available in
the lab and using temperature dependent expressions for fluid properties when appropriate.

These three main assumptions are generally used by all system-level models and result in minimal error introduced in the final results. Now that the similarities between the models have been addressed, it is essential to see the differences between the models to determine the most effective model for implementation.

3.2.2 Model Differences

Crane and Jackson investigate the mechanisms present in the module to see how important they are to the operation of the system; specifically, to determine the percentage of heat going into Joule, conduction, and Seebeck Effect. Contact resistances are accounted for in both electrical and thermal instances; electrically by a term in the heat entering and leaving the thermoelectric, and thermally by partial air gaps simulated between surfaces. Many previously-developed models are used, refined, and tested in the course of this work adding confidence to the final results obtained.

Bethancourt uses a non-dimensional study that loses some important details in his study. The model assumes mass flow rates are equal, which is relevant to the one premise being investigated, but is a critical issue in most examples such as energy reclamation from industrial power sources. Contact resistances are neglected, which can be important in obtaining accurate predictions. The model also fails to use any type of extended surface to increase heat transfer, which could be an important component of any system.

Karri falls in between these two models. He neglects contact resistances, but takes into account heat transfer from two separate fluids. The model is assumed to be insulated from the exterior to minimize heat loss. This model was designed for an automotive system, which places an emphasis on weight that will be less of an issue in a system utilizing a stationary heat source.

From the above investigation, it becomes clear that the Crane and Jackson model has fewer limiting assumption and provides the best starting point for a comprehensive model development.
3.3 Model Development

3.3.1 Previous Attempts

An Excel model was also developed as a first attempt at modeling a Thermoelectric Power Generator (TEPG). The solution to the system of equations was achieved by using a recursive model, which converges to a solution for each set of design parameters. The model outputs are power generated and system efficiency for various load resistances. The basic schematic of the system modeled is shown in Figure 3.2.

Figure 3.2: Excel Model

The parameters needed to determine the system outputs include:

- Overall heat transfer coefficient between the fluid stream and the edge of the thermoelectric, $U$.
- The thermoelectric area, $A$.
- The heat transfer rate, $q$.
- The module Seebeck coefficient, $N\alpha$.
- The system resistances, $R$.
- The current, $I$.
- The temperatures, $T$. 
Several assumptions were made including minimal heat extracted by the system; minimal contact resistances; perfect insulation around the system and constant thermal properties; and the fluid temperature is relatively isothermal and can be represented by a simple temperature difference.

In the below equation $q_1$ represents the heat transferred from hot fluid stream to the hot side of the module, which equal to the difference between $q_{in}$ and $q_{out}$.

$$q_1 = U_1 A_1 (T_1 - T_2)$$  \(3.24\)

The total heat transfer coefficient, $U_1$, for the heat fins across the area, $A_1$, and the temperature difference $(T_1 - T_2)$ represents the drop in temperature from the hot fluid stream to the edge of the thermoelectric module, which includes convection between the fluid stream and fins, conduction in fins and base plate, and contact resistance between the base plate and the thermoelectric surface.

The heat rejected through the cold side heat sink, with the same assumptions as the hot side is:

$$q_2 = U_2 A_2 (T_3 - T_4)$$  \(3.25\)

Now considering both reversible and irreversible processes, the rate of heat transmitted into the thermoelectric module, as previously shown in Section 1.2, is:

$$q_1 = N \alpha T_2 I + \frac{T_2 - T_3}{R_{TE}/N} - \frac{1}{2} I^2 N R_x$$  \(3.26\)

The first part of eq. (3.26) represents the heat transfer rate due to the Peltier effect, while the last two terms represent the irreversible processes of heat leakage due to conduction and Joule heating.

Rearranging eq. (3.26) for $T_2$ the following expression is obtained.

$$T_2 = \frac{q_1 + T_3/R_{TE} + I^2 R_x/2}{(N \alpha d + 1/R_{TE})}$$  \(3.27\)
Similarly the expression for heat rejected can be expressed as a function of the three different mechanisms of heat transfer or generation in the module. It can also be rearranged to create an expression for the temperature at the cold side of the thermoelectric $T_3$.

$$q_2 = N \alpha T_3 I + \frac{T_2 - T_3}{R_{TE}} + \frac{1}{2} I^2 NR_e$$  \hspace{1cm} (3.28)

$$T_3 = \frac{q_2 - T_2 / R_{TE} - I^2 NR_e / 2}{(N \alpha - 1 / R_{TE})}$$  \hspace{1cm} (3.29)

$$I = \frac{N \alpha (T_2 - T_3)}{NR_e + R_{load}}$$  \hspace{1cm} (3.30)

The above equation is an expression for current as a function of the voltage generated in the module, $N \alpha (T_2 - T_3)$ over the total resistance of the system, both load and internal resistances.

The power generated can be dissipated by some load, $R_{load}$, so that:

$$w_{TE} = I^2 R_{load}$$  \hspace{1cm} (3.31)

The efficiency of the modules is given by:

$$\eta_{system} = \frac{w_{TE}}{q_1}$$  \hspace{1cm} (3.32)

The last expression is system efficiency given by the power generated by the thermoelectric over the heat delivered to the system.

The system performance is shown below in Figure 3.4 for the system parameters: $A=0.005625 \text{ m}^2$, $U_1=10^4$ & $U_2=5 \times 10^3 \text{ W/m}^2\text{K}$, $N\alpha=.025 \text{ V/K}$, $R_{TE}/N=.185 \text{ K/W}$, $NR_e=.3 \Omega$, $T_1=503\text{K}$, $T_2=303\text{K}$. The high heat transfer values are used here to focus on the performance of a single module with negligible thermal resistance between the heat source and sink. The area, internal resistance, Seebeck coefficient, and thermal resistance used were chosen for a Hi-Z 20 module. The Hi-Z 20 has a manufacturer rated maximum power generated value of 19 W (min) and a maximum efficiency of 4.5% (min) for a single module operating between its maximum temperature ratings.
The system reached peak power when the load resistance matched the internal resistance as expected. Efficiency was found to follow the same trend, reaching max efficiency very close to the matched load resistance, which was an unexpected outcome, and may point to a deficiency in the model. The maximum values determined from this model fall below the published values for the Hi-Z 20 module as predicted when the system is operated below the optimum with thermal resistances between the fluid and surface temperatures. When the contact resistances are removed, the calculated power slightly exceeds the manufacturer specification for the minimum amount of maximum power generated by simulating an extremely large heat transfer coefficient. The predicted values being very close to the manufacturer specification occurs because this is an ideal model with real world irregularities neglected.

There were several problems found with the Excel model that precipitated an investigation into other programs for modeling. The heat transfer values needed to make the answer converge were far out of the range of values that would be expected in a real system. These two major shortcomings in the investigation of one module operating between two fluid streams causes concern for this approach and led into the development of a model using a more robust programming environment, MATLAB.
3.3.2 MATLAB Cost Function Model

A general system, depicted in Figure 3.4, is assumed to have a hot fluid with some mass flow rate \((\dot{m}_h)\), found from the volumetric flow rate, and temperature \((T_h)\) flowing in opposite direction to a cooling fluid with a mass flow rate \((\dot{m}_c)\) and temperature \((T_c)\). Thermoelectric modules are placed between the two fluid streams and typically there is some fin system to enhance heat transfer between the fluids and adjacent thermoelectrics. The system is split into thermal zones where it is assumed heat transfer can be approximated as one dimensional, i.e. in the direction from the hot to cold fluids as depicted in Figure 3.5. Each zone is coupled to adjacent zones by the fluid streams. Thermal coupling between zones by conduction in the fins or housing is assumed to be negligible for a first approximation.

![Figure 3.4: Schematic of General Thermoelectric Power Generation Thermal System](image-url)
Each zone can have multiple thermoelectric modules. Assuming constant properties and neglecting the Thomson effect the rate of heat absorption by the hot side, \( q_h \), and heat rejection of the cold side, \( q_c \), of \( n_{TE} \) modules in a zone are:

\[
q_h = (K_{TE} (T_{hi} - T_{co}) + N\alpha_{pn} T_{hi} I - I^2 NR_e / 2) n_{TE} \tag{3.33}
\]

\[
q_c = (K_{TE} (T_{hi} - T_{co}) + N\alpha_{pn} T_{co} I + I^2 NR_e / 2) n_{TE} \tag{3.34}
\]

where \( K_{TE} \), \( N\alpha_{pn} \), and \( NR_e \) are module level performance parameters that can be either theoretically derived or measured [29]. \( K_{TE} \) is the module thermal conductance, which is equivalent to the reciprocal of the module thermal resistance \( (R_{te}/N) \). \( N\alpha_{pn} \) and \( NR_e \) are the module Seebeck parameter and module total internal electrical resistance.

The electrical energy generated from of \( n_{TE} \) modules in a zone is obtained by finding the difference between Eqs. (3.33) and (3.34).

\[
w_{TE} = (N\alpha_{pn} (T_{hi} - T_{co}) I - I^2 NR_e) n_{TE} \tag{3.35}
\]
The heat absorbed and rejected by the hot and cold side of \( n_{TE} \) modules also is equal to the heat transfer from the fluids to the surface of the thermoelectric modules. This can be expressed as

\[
q_h = UA_h \Delta T_{ln,h} - q_o \tag{3.36}
\]

\[
q_c = UA_c \Delta T_{ln,c} - q_o \tag{3.37}
\]

\[
\Delta T_{ln} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \tag{3.38}
\]

where \( UA_h \) and \( UA_c \) are the per zone conductance values for the hot and cold side and are equivalent to the inverse of the thermal resistances between the fluid streams and the surfaces of the thermoelectrics. These values account for convection, conduction in fins and base, and contact resistances. These values can be theoretically determined or experimentally measured. \( q_o \) is the heat loss through the gaps between modules and any thermal bridging caused by mounting hardware between the hot and cold sides.

Assuming negligible thermal losses to the environment in each zone the rate of energy removed or added to the hot and cold fluid flows are:

\[
q_h + q_o = \dot{m}_h c_{p,h} (T_{h,in} - T_{h,out}) \tag{3.39}
\]

\[
q_c + q_o = \dot{m}_c c_{p,c} (T_{c,out} - T_{c,in}) \tag{3.40}
\]

where \( \dot{m} \) is the mass flow rate of the hot and cold fluids, \( c_p \) is the specific heat of the fluids, and \( in \) and \( out \) refer to inlet and outlet temperatures.

Equations (3.33)-(3.40) are solved simultaneously in each zone with MATLAB utilizing a cost function approach to solve the non-linear thermoelectric equations. The model couples zones by making a guess at the cold outlet temperature and solving for \( T_{ho} \) and \( T_{ci} \) for each zone which are then inputs for the next zone. Each zone temperature and heat rates are solved sequentially until a final \( T_{ci} \) for the system is obtained. This temperature is compared to the actual \( T_{ci} \), a new guess for the system \( T_{co} \) is made and calculations are iterated until the temperatures converge.
To validate the model, an experimental test stand was developed to test various size systems under a wide range of conditions. Of interest is having the capability to systematically change $T_{hi}$, $m_{hi}$, $(UA)_h$, and $n_{TE}$ as well as be able to test a range of modules under various configurations.

The model code can be found in Appendix B. In the first section of the code, the design variables are defined such as values for the duct dimensions and thermal properties of fin systems. The number of thermoelectric modules, module dimension, insulation properties, finally the temperatures and flow rates of the hot and cold sides are assigned. The second section of the code contains the thermoelectric performance calculations. The hot side heat capacity and density are defined based off the air temperature before it enters the inline heater. Several improvements, shown in Chapter 5, to the model are calculated in this section also.

The third section of the code is where the system model is evaluated. The model runs in a loop until the cold inlet temperature converges. In this loop, the per zone UA value is then assigned based on the developing flow calculations discussed later. The initial guesses for all the zones are defined for the first pass through of the master loop. The module parameters are then defined based on the testing data of the thermoelectric module test stand.

The tests can be run for many different inlet conditions, number and type of modules in addition to various heat transfer conditions. The module parameters used in the system model were obtained by measurement techniques described by Sandoz-Rosado and Stevens [29]. Two modules used in testing were the Melcor HT8 and the Hi-Z 14. The temperature dependent measured modules parameters are as follows:

**Melcor HT8**

\[
N \alpha_{pn} = -4.38 \cdot 10^{-05} T_{ave} + 0.05 \ [V/K] \quad (3.41)
\]

\[
NRe = 6.38 \cdot 10^{-03} T_{ave} + 2.00 \ [\Omega] \quad (3.42)
\]

\[
Rte/N = 2.84 \cdot 10^{-04} T_{ave} + 1.54 \ [K/W] \quad (3.43)
\]
Hi-Z 14

\[ N \alpha_{pn} = 9.54 \cdot 10^{-06} T_{ave} + 0.0128 \ [V/K] \]  
\[ NRe = 4.6 \cdot 10^{-04} T_{ave} + 0.0984 \ [\Omega] \]  
\[ Rte/N = 4 \cdot 10^{-04} T_{ave} + 0.708 \ [K/W] \]  

where \( T_{ave} \) is the average of the hot and cold side absolute temperature of the module.

3.3.3 Heat Exchanger Model

The overall heat transfer coefficients for each zone found in eqs. (3.36) and (3.37) are functions of temperature, mass flow rate and geometrical parameters, the geometrical parameters are defined in Figure 3.6. The model for the hot side heat transfer coefficient was developed to validate the experimental testing of the hot and cold side of the testing box. The model included considerations for the fins, the wall between the fins and cold plate, and finally the cold plate itself. These parameters were evaluated at the original temperature of the air as read by the flow meter. They were then used to generate a final hot side heat transfer coefficient based on the assumption the heat transfer coefficient was constant between zones since the basic equations would have become impossible to solve without this assumption. This assumption was later addressed with the investigation into the developing flow.

The fin configuration considered creates a series of rectangular ducts as shown in Figure 3.6. The duct parameters are first geometrical parameter defined in the code. The geometrical parameters for the fins are then defined with the hot side fluid properties determined. Fin parameter equations calculate the necessary geometrical parameters utilized in the later fin efficiency calculations.

\[ A_{fin} = 2L_{Duct}L_{fin} \]  
\[ A_b = \left(2W_{Duct} - n_{fin}L_{fin}\right)L_{Duct} \]  
\[ A_t = n_{fin}A_{fin} + A_b \]
These three equations calculate the area of one fin, the area of the base of the fins, and the total hot side heat transfer area.

The next section in the code calculates the temperature dependent air properties including the Prandtl number, thermal conduction value, heat capacity, density, and viscosity. These values are used to find the hot side mass flow rate and the Reynolds number of the air in the duct channels.

![Figure 3.6: Schematic of Relevant Fin Dimensions](image)

The convective coefficient is calculated next. First, the friction factor is calculated then used to find the Nusselt number, which is directly used to find the average convective coefficient in the duct channels. These values are calculated by the following equations and are valid for the following parameters taken from Incropera and DeWitt [30].

\[
f = (0.790 \log(\text{Re}_D) - 1.64)^2
\]  

(3.50)

\[
\text{Nu}_D = \frac{(f/8)(\text{Re}_D - 1000) \text{Pr}}{\left[1 + 12.7(f/8)^{0.5}(\text{Pr}^{1/3}) - 1\right]}
\]  

(3.51)
\[ h = \frac{Nu_D k}{D_h} \quad (3.52) \]

This model is valid in a turbulent, fully developed flow. With the Prandtl number in the range \( 0.5 < Pr < 2000 \), \( 3000 < Re_D < 5 \times 10^6 \), \( (L/D) \geq 10 \). The experimental numbers fell in these ranges.

Fin resistance was calculated to determine the effect on hot side heat transfer and the following equation were taken from Incropera and DeWitt [30]. These equations are used to calculate the fin efficiency which can be used to easily find the fin resistance as shown in the equations below. The first equation determines the theoretical heat transfer coefficients for validating the experimental values calculated.

\[ UA_{hot} = (R_{fin} + R_{wall} + R_c)^{-1} \quad (3.53) \]

where \( R_{wall} \) is the wall thermal resistance due to conduction, \( R_c \) is the thermal contact resistance between the wall and TE, and \( R_{fin} \) is the thermal resistance from the air to the base of the fins. \( R_{fin} \) is described as

\[ R_{fin} = \frac{1}{\eta_{\text{fin}} h A_i} \quad (3.54) \]

The overall efficiency of the fin system is given by the following, which relates the total amount of heat removed by the fins and the base to the maximum amount of heat removed when the fins were at the same temperature as the base.

\[ \eta_o = 1 - \frac{NA_{\text{fin}} (1 - \eta_{\text{fin}})}{A_i} \quad (3.55) \]

The next equation is given for fin efficiency as the ratio of heat absorbed by the fin to the maximum amount of heat absorbed when the fin was at the same temperature as the base.

\[ \eta_{\text{fin}} = \frac{\tanh(mL_{\text{fin}})}{mL_{\text{fin}}} \quad (3.56) \]
The next equation is given for fin efficiency as the ratio of heat absorbed by the fin to the maximum amount of heat absorbed when the fin was at the same temperature as the base.

\[ m = \frac{2h}{\sqrt{kt_{\text{fin}}}} \]  

(3.57)

This final expression is given for a fin performance parameter taken from the non-dimensionalized heat transfer equation.

The cold plate, an aluminum plate that removes heat from the thermoelectric modules by having a fluid pumped through it, acts as a quasi-cross flow heat exchanger. It is desirable to treat the cold plate as a counter flow for simplicity. The following relation compares the log mean temperature of a cross and counter flow heat exchanger taken from Incropera and DeWitt [30].

\[ \Delta T_{\text{lm,counter}} = F \Delta T_{\text{lm,cross}} \]  

(3.58)

The value of F is found from ratios P and R comparing the inlet and outlet temperatures of the hot and cold fluids.

\[ P = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} \]  

(3.59)

\[ R = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}} \]  

(3.60)

For typical values \( T_{hi} = 105.3 \, ^\circ C \), \( T_{ho} = 78.1 \, ^\circ C \), \( T_{ci} = 32.3 \, ^\circ C \), \( T_{co} = 34.6 \, ^\circ C \) \( P \) was found to be 0.033, and \( R \) was found to be 11.7. Other temperatures also generated similar values with \( P \) and \( F \), with the values of \( P < 0.05 \). For values of \( P < 0.1 \), \( F \) converges to one for all values of \( R \). It is therefore valid to treat the cold plate under typical operating conditions as a counter flow heat exchanger for purposes of mathematical investigation.

Once the model was developed, it was used with the Exhaust Simulation Test Stand, described later, to compare the validity of the model.
4.1 Test Stand Specifications

4.1.1 Thermoelectric Module Test Stand

Two test stands have been developed in the Sustainable Energy Lab by two senior design teams. The first stand was developed to test individual module performance while the second was designed to test module performance on a system level by simulating auto exhaust, which can be channeled into a thermoelectric power unit.

The module test stand, shown in Figure 4.1, is a table mounted test stand equipped with multiple sensors to determine the performance of a single thermoelectric module. The test stand consists of a large copper block with three 400 W cartridge heaters installed. The purpose of the copper block is to ensure a constant heat flux entering the thermoelectric module at the interface. Several thermocouples are also installed into the copper block to monitor the temperature and determine how much heat is being delivered to the system. The hot side temperature is controlled by a temperature controller connected to a power controller controlling how fast the system is allowed to heat up. The power controller is also connected to a power analyzer, which records the amount of power delivered by the cartridge heaters. The copper block is insulated on all non-contact sides by insulation and covered with a metal sheet to minimize user contact with the insulation and reduce radiation losses.
The cold side is controlled by a cold plate connected to a chiller keeping the cold temperature at some predetermined setting. The hot and cold sides are then pressed down on a test module by a crank spring system. The loading pressure on a module is monitored by the use of three load cells. A module is placed into the system with thermal paste applied to both sides to minimize thermal contact resistances and surrounded by a sheet of insulation to limit the heat leakage between the hot and cold sides. A data acquisition system makes power, temperature, and electrical measurements to perform a full characterization of a thermoelectric module. A collection of rheostats vary the resistive load applied to the thermoelectric module. The test stand has been thoroughly characterized by Sandoz-Rosado and Stevens [29].

4.1.2 Exhaust Simulator Test Stand

The second test stand was also originally built by a senior design team to create a platform for testing thermoelectric systems for implementation in real world scenarios; most likely an automotive exhaust system. The original system consisted of a blower accelerating the air. It is controlled by a knife valve and bleed off valve. The flow rate is then measured by a flow meter and then the flow enters the inline heater. The inline heater can heat the air up to 600 °C before it is delivered to the testing box. The heated air is then sent to an exhaust vent. The hot side of the system is powered by 208 V three phase power required by the inline heater and blower. The cold side consisted of a cold
plate taking water from a faucet, running it through the cold plate, and then rejecting the water into a sink. The testing box consisted of a small rectangular box with fins extracting heat from the fluid stream and delivering it to modules sandwiched between the testing box and a cold plate. There was no real electrical control; the power was dissipated by rheostats.

This first design of the system test stand had several shortcomings, including poor controls for the inline heater that led to one of the heating coils failing due to too rapid heating. Also, there were many areas for pressure loss in the system because of poorly constructed connections. When used, the bleed off valve created sounds so loud it was difficult to function without extensive hearing protection. The insulation used around the heater was insufficient and was charred by the high temperatures. The open cold side of the system was wasteful and had limited temperature control. The data acquisition system was marginal. The electrical testing portion of the module also needed to be improved to allow for a larger range of modules testing.

The first task in returning the exhaust simulator to operation was to return the inline heater to the manufacturer and get it repaired, and then determine a way to program the temperature controller so that the inline heater did not burn out again. By consulting with the manufacturer, we found that the system had to ramp up and down at about 5.4 °C/minute to ensure that the heater would not fail. The next task was to raise the blower to be inline with the rest of the system to minimize the pressure loss in the first portion of the test stand. This was accompanied with the purchase of a muffler to place on the bleed off valve so that it could be used without the excessively loud noises of the previous design. More effective ceramic insulation was also ordered and placed on the system.

The next set of improvements involved replacing the preheated air connections that consisted of metal connections with sharp constrictions. PVC tubing was chosen for its low cost and wide selection of connectors and replaced all the previous tubing except the portion connected to the knife valve directly after the blower. The cold side was changed to a recirculating system that pumped water into the cold plate and was then sent through a radiator and fan to remove some of the heat and then delivered back to the system. A schematic of system test stand is shown in Figure 4.2.
A proper data acquisition system was written to process all the input data from thermocouples and record it into a file for later analysis. These improvements brought the test stand up to a level where it could be used for testing after several calibration tests on various equipment to ensure our measurements are accurate. This also required the development of new testing boxes, which could be used to work with a wider array of modules in different combinations.

### 4.2 Power Unit Test Boxes

During 2007 and 2008, two senior design teams were tasked with developing two test power boxes. One was designed to operate as a stand-alone system installed in an automotive exhaust system. The second was designed to test a range of thermoelectric system configurations. Both of these systems were created to be easily installed in the current exhaust simulator and be connected to the current data acquisition system. The second system was designed with three separate zones separated by insulation and could handle up to eight modules in each zone. It also features removable fins, which could be used to investigate several different levels of heat extraction. This system was designed to work with either no fins installed, or extruded fins installed held in place by set screws. The one set of fins had every other fin removed, creating another heat exchanger geometry to test. The three zones were also operated electrically by three rheostats dissipating the power generated by each zone. Over twenty thermocouples were used in this system to record the temperature over the entire surface of the modules and the data
was fed into the data acquisitions system. A schematic and photo of the power unit assembly is shown in Figures 4.3 a & b. The modules are sandwiched between the hot side portions of the box with the cold plates used for removing heat from the thermoelectric modules.

**Figure 4.3 a: Power Unit Assembly**

![Power Unit Assembly](image)

**Figure 4.3 b: Picture of Box Installed in the System**

![Box Installed](image)
4.3 Test Stand Calibration

4.3.1 Thermocouple Calibration

To use this power box effectively, calibration of the flow meter was undertaken to ensure the flow rate recorded was correct. To validate the flow meter, it was necessary to have confidence in the performance of the thermocouples. Three thermocouples were used in measuring temperatures of the pre heated air, inlet, and outlet to the power box. To test the uniformity of the thermocouples, we ran the exhaust simulator at four different temperatures, allowing the stand to reach steady state. Steady state was defined as only allowing a variation of less than +/- 0.2 °C over a 10 minute time span. Each thermocouple was placed in the center of the air stream and the temperatures were recorded as shown in Figure 4.4. Thermocouple T1 and T3 were consistent across the entire temperature range of interest with variance less than 0.8 °C while T2 differed by around 3 °C from T1 and T3 over the temperature range of interest. T2 measured the preheat air temperature while T1 and T3 measured inlet and outlet temperatures. These last two measurements were used as a difference when determining the amount of heat removed the fluid stream. The two water thermocouples were also tested in a water bath and found to have a minimal difference between them, observed to be at most 0.1 °C at three different water temperatures.

Table 4.1: Temperature Calibration Data

<table>
<thead>
<tr>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>Average</th>
<th>T2-T1</th>
<th>T1-T3</th>
<th>T2-T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.42</td>
<td>52.53</td>
<td>48.74</td>
<td>50.23</td>
<td>3.11</td>
<td>0.68</td>
<td>3.79</td>
</tr>
<tr>
<td>102.88</td>
<td>105.91</td>
<td>103.06</td>
<td>103.95</td>
<td>3.03</td>
<td>-0.18</td>
<td>2.85</td>
</tr>
<tr>
<td>158.33</td>
<td>161.68</td>
<td>158.15</td>
<td>159.39</td>
<td>3.35</td>
<td>0.18</td>
<td>3.53</td>
</tr>
<tr>
<td>212.04</td>
<td>215.80</td>
<td>212.82</td>
<td>213.55</td>
<td>3.76</td>
<td>-0.78</td>
<td>2.98</td>
</tr>
</tbody>
</table>
After insulating the pipe, the temperature profile of the air stream across the cross-section of the pipe before entering the power unit was measured. There was a 7 °C difference between the surface of the pipe (222 °C) and the center of the flow (229 °C) when the stand was running at approximately 0.053 kg/s. The distribution of the temperature was be expected in a turbulent flow, where there was a significant drop off near the edges of the pipe and roughly a constant temperature in most of the flow. Based on this data, it was found that it was important to locate the thermocouple in a constant position at the center of the flow to receive consistent measurements. Limited error was introduced as the inlet and outlet temperatures were taken in the same location in the pipe, though mixing could have occurred as the air exited the power box.

The thermocouples for measuring the temperature distribution in the power box were placed in an oil bath and tested at temperatures of 50, 130, 165 and 195 °C. They all were within 1 °C of the average temperature, so the thermocouples were expected to provide temperature measurements with minimal variance between them. These thermocouples were placed in the grooves between the thermoelectric modules and the
hot side portion of the heat exchanger and could be seen in Figure 4.4b highlighted by the yellow connectors.

### 4.3.2 Flow Meter Calibration

To confirm the flow rate measurements, the heater and pipe were fully insulated between the inlet and outlet of the heater and a temperature rise was measured using T1 and T3 thermocouples. Then the amount of energy was measured by recording the root mean squared voltage between each leg and current of each leg of the power controller. The temperature data was also collected. Based on these parameters, the mass flow rate was calculated and compared to the value recorded by the flow meter. Tests were run at maximum temperature at three nominal flow rates taken from the flow meter, 0.053, 0.045 and 0.033 kg/s. The data from these tests were used to calculate the amount of energy added to the air stream. Results were then compared to the 10 kW rating of the heater. The mass flow was calculated using data from the test, found from the following expression.

\[
\dot{m} = \frac{\dot{q}_{in}}{C_p (T_{out} - T_{in})}
\]  

(4.1)

where \(\dot{q}_{in}\) is given by the expression

\[
\dot{q}_{in} = \sqrt{3} \bar{V}_{rms} \bar{I}_{rms}
\]  

(4.2)

Where \(\bar{V}_{rms}\) is the average root mean squared voltage between the heater legs and \(\bar{I}_{rms}\) is the average root mean squared current.

The other mass flow rate is found by multiplying the measured flow rate by air density at 23 °C at sea level, taken to be 1.225 kg/m³ for these measurements. From the calculations, it was found that the difference between the measured and calculated mass flow rates was less than 1.5% for all three cases as shown in the data tables below. From this investigation, it was confirmed that the flow meter was accurate for the needs of our experimentation.
Table 4.2: Flow Rate Calibration Data

<table>
<thead>
<tr>
<th>( \dot{m}_h )</th>
<th>( V_{ab} )</th>
<th>( V_{ac} )</th>
<th>( V_{bc} )</th>
<th>( \bar{V}_{rms} )</th>
<th>( I_a )</th>
<th>( I_b )</th>
<th>( I_c )</th>
<th>( \bar{I}_{rms} )</th>
<th>( q_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg/s</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>W</td>
</tr>
<tr>
<td>0.033</td>
<td>210.7</td>
<td>208</td>
<td>208.1</td>
<td>208.9</td>
<td>27.2</td>
<td>27.4</td>
<td>28.8</td>
<td>27.8</td>
<td>10060</td>
</tr>
<tr>
<td>0.045</td>
<td>208.7</td>
<td>206</td>
<td>206</td>
<td>207.0</td>
<td>27.2</td>
<td>26.9</td>
<td>28.8</td>
<td>27.6</td>
<td>9909</td>
</tr>
<tr>
<td>0.053</td>
<td>210.3</td>
<td>207.6</td>
<td>208</td>
<td>208.6</td>
<td>27.9</td>
<td>27.6</td>
<td>28.3</td>
<td>27.9</td>
<td>10094</td>
</tr>
</tbody>
</table>

The cold side flow rate was confirmed by comparing the manual flow rates measurements made with a stop watch and tank to the measured flow rate using a turbine flow meter. No noticeable differences were found. Since the water flow rate was kept constant at its maximum flow rate during operation this measurement, its measurement was not as critical as the air flow measurement.

4.4 Test Operation

The test stand operates along a curve where the maximum temperature increase is inversely related to the flow rate, with extremely large temperature rises occurring at flow rates not of interest in this investigation. In this investigation, the flow rates are operated between .0236 - .055 kg/s, which correspond to 50 - 95 cfm. The temperatures vary from 100 – 340 °C. The maximum temperature reached when operating the system is 250 °C, with Figure 4.6 illustrating the approximate temperature rise from the pre-heated temperature for various flow rates.
A typical testing procedure for the exhaust system is detailed below; that is to be followed once the power box has been installed in the system and all the measuring equipment and connections are tightly secured. The first system in operation is the cold side, ensuring the heat is removed from the system before the air is heated. Then the hot side is initialized. The hot side is turned off prior to the cold side for the same reason. The directions for operating the testing rig are shown below.

**Cold Side**

1) Make sure all water piping connections are tight. (pipe and flow thermocouples)
2) Open bypass valve and main flow valve.
   - Bypass valve is the blue handle valve; place in the open position.
   - Main flow valve, rotary handle, is hooked to the 5 GPM flow meter.
3) Plug in water circulation pump.
4) Shut off bypass valve and adjust flow valve to desired flow rate.
5) Allow to continuously run during hot side operation.

**Figure 4.5: Flow Rate vs. Temperature Rise for Inline Heater**

A typical testing procedure for the exhaust system is detailed below; that is to be followed once the power box has been installed in the system and all the measuring equipment and connections are tightly secured. The first system in operation is the cold side, ensuring the heat is removed from the system before the air is heated. Then the hot side is initialized. The hot side is turned off prior to the cold side for the same reason. The directions for operating the testing rig are shown below.

**Cold Side**

1) Make sure all water piping connections are tight. (pipe and flow thermocouples)
2) Open bypass valve and main flow valve.
   - Bypass valve is the blue handle valve; place in the open position.
   - Main flow valve, rotary handle, is hooked to the 5 GPM flow meter.
3) Plug in water circulation pump.
4) Shut off bypass valve and adjust flow valve to desired flow rate.
5) Allow to continuously run during hot side operation.
**Hot Side**

1) Place vent ducting over test stand outlet.
2) Plug in hot side control unit to 208V outlet and turn breaker on.
3) Open knife blade flow constrictor by fully turning it clockwise and fully opening the bypass valve.
4) Turn on the blower by pressing the large green button next to the blower sign on the electrical box.
5) After the system has been allowed to warm up, press the green heater button below the blower button and set desired temperature on the temperature controller. Specific temperature controller instructions are located in the appendix of the Test Stand User Manual.
6) Adjust temperature and flow rates as necessary.
7) Begin to ramp down heater upon completion of the test by resetting the set point and running the ramping program.
8) When the temperature controller is set to 40 °C, turn off the system and immediately restart the blower.
9) Once the system has cooled enough turn off the blower, but restart if the temperature rises significantly again.
10) After heater is ramped down, turn off pump and open bypass valve to let excess water flow out.

**4.5 Testing Results**

Testing was undertaken for three distinct setups with the first used to determine the heat transfer values in the model while the next two setups were used to test the Melcor and Hi-Z modules in various configurations.
4.5.1 Heat Exchanger Testing

For the first test, the system was operated without thermoelectric modules to determine how effective the testing box operated as a heat exchanger and to compare with the model predictions. Given the difficulty on accurately modeling the thermal performance of the heat transfer at the fins, the model confirms the range and general trend of the experimental data. To begin this series of tests, the interface surface of the test box was covered in thermal paste to ensure contact resistance was minimized as much as possible. Then the thermocouples were inserted into slots between the hot and cold sides with the assembled test box placed into the system test stand. The test box was exposed to a range of inlet hot side conditions (mass flow rates of 0.028, 0.042, 0.053 kg/s and inlet temperatures of approximately 100, 150, 200 ºC).

The system was allowed to reach steady state conditions before data was taken. The inlet and outlet temperatures, interface surface temperatures, and hot and cold flow rates were recorded. Using this data and making the assumption that the total heat transfer coefficient was constant in each zone, it is possible to determine the overall heat transfer coefficient for each zone.

To determine the total heat transfer coefficient, the equations for each zone were written for the heat lost by the hot air and heat transferred through the heat exchanger from the system. From this, six equations were developed and rearranged values so heat transfer values can be calculated. The two equations for each zone are as follows:

\[ q_h = \dot{m}_h C_{p,h} (T_{h,in} - T_{h,out}) \]  
\[ q_h = UA\Delta T_{lm} \]

where \( \Delta T_{lm} \) is defined in equation (3.38). An iterative solver was used to determine the total heat transfer values and between zone air temperatures. The UA values from this calculation were compared with the values generated by the model in chapter five.
4.5.2 Preliminary Module Testing

At the completion of the heat exchanger portion of the testing, it was time to test modules in the system. For the first power module unit testing, Melcor HT 8 modules were chosen as these modules were compact and allowed for a wide range of setup configurations. The first test undertaken was testing all 24 modules, with eight arranged in each zone. Thermal paste was thoroughly applied to both sides of the modules. Then they were placed on the surface of the hot side of the testing box with insulation to fill in all the gaps between modules. Then the cold plates were placed on top of the modules. The box was tightened down so all nuts were difficult to tighten further (noting that no consistent method currently exists for applying pressure to the modules). The hot and cold side thermocouples were also placed on the hot and cold sides of the modules.

When the first series of module tests was undertaken, extra power resistors were in transit. It was necessary to place these extra resistors in series with the rheostats since the rheostats in each zone were unable to reach the internal resistance of the module. As a result, a preliminary test was undertaken at values less than the internal resistance to determine whether the system worked properly. It was found that reasonable data was being obtained in each zone with no shorts detected.

In theory, it was necessary to match the internal resistance to obtain the peak power of the module. It was then desirable to obtain a resistance sweep of one of the zones to see how sensitive the peak of the power curve was, and whether the error of a few tenths of an ohm would seriously impact the performance of the zone. From the data, it was found there was a six Ω window where the power was within 1% of the maximum power. A significant plateau can be seen in Figure 4.7 showing power versus resistance, matching what would be expected from a resistance sweep of one, two, and three rheostats hooked together. Finally, an extra decade resistor was attached to obtain large load resistance points. For reference, the internal resistance was calculated to be between 21.6 and 21.7 Ω based on the measurements.
4.5.3 Module Testing

Once the 12 $\Omega$ power resistors were installed, it was possible to test the system with all zones fully loaded. The first task was to determine the maximum amount of power generated by the system. The surface temperature of the first thermocouple was monitored to determine how close the temperature came to 175 $^\circ$C, which was used as the maximum allowable temperature to limit the thermal stresses on the modules. The optimum settings on the test stand to achieve this objective was a flow rate of 0.042 kg/s and a temperature controller temperature of 245 $^\circ$C and the inlet air temperature was observed to be near 270 $^\circ$C for all of the testing at this data point. The internal resistance was matched by tuning the manual rheostats until the calculated load resistance was within a few tenths of an ohm for each zone. The derivation of the expression used to determine the internal resistance is as follows.
When taking open and short circuit measurements, data was taken immediately after the electrical leads were removed from the system, with the system allowed to return to steady state before another measurement was taken.

The following expressions show the development of the expression for determining the internal resistance based on the open circuit voltage, load voltage, and load resistance. The first expression relates the measured voltage to the measured current and the load resistance.

\[ V_{\text{meas}} = I_{\text{meas}} R_{\text{load}} \]  \hspace{1cm} (4.5)

The expression for voltage based on open circuit and short circuit measurements is:

\[ V = V_{oc} + yI \]  \hspace{1cm} (4.6)

where \( y \) is given by:

\[ y = -\frac{V_{oc}}{I_{sc}} \]  \hspace{1cm} (4.7)

By combining equations (4.5) and (4.6), the following expression for measured voltage is found:

\[ V_{\text{meas}} = V_{oc} \left(1 - \frac{I_{\text{meas}}}{I_{sc}}\right) \]  \hspace{1cm} (4.8)

Now substituting expressions for both currents based on Ohm’s law, the following expression was obtained that was a function of values readily available:

\[ V_{\text{meas}} = V_{oc} \left(1 - \frac{V_{\text{meas}} R_{i}}{V_{oc} R_{\text{load}}}\right) \]  \hspace{1cm} (4.9)

Rearranging for internal resistance the expression used to determine when matched load is obtained is as follows:

\[ R_{i} = \left(\frac{V_{oc}}{V_{\text{meas}}} - 1\right) R_{\text{load}} \]  \hspace{1cm} (4.10)
Once the matched load point is set the data acquisition system recorded a 200 second interval of data at a sample rate of two seconds, which resulted in 100 data points of each measurement for further analysis. Testing on the system was done at the previous testing points investigated in the heat exchanger portion of the testing (100, 150, 200 °C at flow rates of 0.028, 0.042, 0.053 kg/s). The fins installed for all of the testing were the extruded fins without any of the fins removed, creating 13 channels for the air to flow through. The largest amount of power generated with 24 Melcor modules, with the set up shown in Figure 4.7a, was 63 Watts at 245 °C and 0.042 kg/s, with an overall module efficiency of 2.7 % and system efficiency of .65% based off of the pre and post heated air temperatures. The next set of testing used 12 modules set up thermally in series as shown in Figure 4.7b. After several attempts of assembling this system, it was discovered that upon assembly, one of the modules was damaged and required replacement. The failure of this testing pointed towards using the protocol for tightening the connecting bolts. Rather than risking the failure of another module, a new configuration of 12 modules (shown in Figure 4.7c) was assembled and placed in the fluid stream. After the successful testing of the second twelve module configuration, six modules were tested and the set up can be seen in Figure 4.7d. The six and twelve module tests did not go above 200 °C to limit the possibility of thermal failure.

The six module test generated 18.9 Watts at 200 °C and 0.053 kg/s while the largest amount of power generated in the 12 module case occurred at the 200 °C and 0.053 kg/s test and was 29.7 Watts. This value can be compared to the 48.4 W generated by the 24 module case at the same conditions. The per module average for six modules was slightly over three watts while the 12 module case was about 2.5 watts. The case with 24 modules generated only about two watts per module, so from this inspection it can be seen that with fewer module removing heat each module performs better. The change in the temperature difference across the modules, when extra heat is removed from the system causes this fall in per module performance.
Table 4.3: Summary of Total Power Generated in Watts for Various Inlet Temperatures and Flow Rates in kg/s

<table>
<thead>
<tr>
<th>Modules</th>
<th>$m_i$(kg/s)</th>
<th>Th = 100 °C</th>
<th>Th = 150 °C</th>
<th>Th = 200 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.028</td>
<td>0.042</td>
<td>0.053</td>
<td>0.028</td>
</tr>
<tr>
<td>6</td>
<td>2.95</td>
<td>3.64</td>
<td>3.89</td>
<td>8.02</td>
</tr>
<tr>
<td>12</td>
<td>3.60</td>
<td>4.87</td>
<td>5.82</td>
<td>10.35</td>
</tr>
<tr>
<td>24</td>
<td>5.24</td>
<td>7.39</td>
<td>8.87</td>
<td>14.10</td>
</tr>
</tbody>
</table>

The second set of testing was for Hi-Z 14 modules. Before the modules were placed in the exhaust simulator, one module was tested in the characterization test stand to obtain module level data implemented in the system model. The Hi-Z 14 modules had significantly lower internal resistance because of the large leg pairs. This required a different set of rheostats with much lower resistances. The new set was installed and used throughout the testing.

Modules were placed between two ceramic wafers needed to electrically isolate the modules from being short circuited by the heat sinks. The ceramic wafers were attached with thermal paste to minimize the thermal contact resistance and the six modules were then placed in the same positions as the previous Melcor test, shown in Figure 4-8 d, and two sheets of insulation were placed around the modules because of the larger thickness of these modules. Testing was run to get the surface temperature of the first module close to 250 °C; the maximum module operating temperature limit.
Figure 4.7 a-d: Schematic of the Four Setups of One Side, the Hot Air Enters from the Left. The Three Blue Lines Indicate the Typical Placement of Thermocouples
The results of this testing were less encouraging than the previous set of testing since the recorded values of performance did not come close to the performance values predicted by the characterization testing. The reason for this was unclear and may have been due to mismatches in modules. To test this hypothesis, each individual module was connected to the rheostats individually and they were switched between rheostats to see if that made any difference. Unfortunately, improved module performance did not occur with the individual testing and the recorded electrical values were about half of what was generated when the modules in each zone were connected. Poor electrical connections could be to blame, as the connections were noticeably degraded when removed from the test stand. The same phenomena was noticed by the senior design team that designed the box in some of their preliminary testing, with unexpectedly low power generated very similar to the data obtained by the current round of testing. The true reason for this discrepancy still remains unknown and warrants further study.

This concluded the testing with the exhaust test stand as there were no more modules available in quantity to test and compare to the developed model. This was not possible because the values generated by the characterization test stand could not be compared to published data for modules and the funds were not readily available to purchase more modules. This lack of different modules was not necessarily a concern since the model could still be compared to a large amount of data gathered from the Melcor testing and still provided a good indicator of avenues that could be investigated to improve the performance of the model.
Chapter 5: Model Refinement

5.1 Heat Exchanger Comparison

To validate the theoretical model for the heat sinks, data collected by the testing procedure described in Section 4.5.1 was analyzed. To calculate the total heat transfer coefficient from the data, the following expression was used.

\[ UA_{\text{total}} = \frac{q_h}{\Delta T_{\text{im}}} \]  

(5.1)

where \( \Delta T_1 = T_{n} - T_{co} \) and \( \Delta T_2 = T_{ho} - T_{ci} \) for a counter-flow configuration, and \( q_h \) is defined in equation (3.37).

The hot side total heat transfer coefficient was calculated by equating the heat removed by the fluid to the heat transferred through the heat sink for each zone, equations are the same form as equations (3.36) through (3.40). The equations were then solved using an iterative solver in Excel for the three unknowns, \( UA_h \) and the two intermediate fluid temperatures with a recursive solver. \( UA_h \) was assumed to be constant in each zone and the total hot side heat transfer value was calculated by multiplying the per zone value by the number of zones; in this case, three. The \( UA_c \) value was then found from the following relation of heat transfer coefficients.

\[ UA_c = \left( \frac{1}{UA_{\text{total}}} - \frac{1}{UA_h} \right)^{-1} \]  

(5.2)

Table 5.1 compares the total hot side heat transfer coefficients based on the model and experiment. The assumption of constant zone heat transfer coefficients is addressed later in the model refinement section.
Table 5.1: Comparing $UA_h$ (W/K) Values from Experiment to Values Calculated by Model

<table>
<thead>
<tr>
<th>Exp $Th$ (°C)</th>
<th>$\dot{m}_h$ (kg/s)</th>
<th>Model $Th$ (°C)</th>
<th>$\dot{m}_h$ (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.028</td>
<td>0.028</td>
<td>0.042</td>
</tr>
<tr>
<td>150</td>
<td>0.028</td>
<td>0.042</td>
<td>0.053</td>
</tr>
<tr>
<td>200</td>
<td>0.028</td>
<td>0.042</td>
<td>0.053</td>
</tr>
</tbody>
</table>

The general trend of the experimental and theoretical data appeared to be the same, but with the experimental numbers showing a lower dependence on flow rate. While there were some significant differences between the two sets of data, the general trend of the data supported the values determined experimentally. The experimentally measured total heat transfer coefficient were averaged for each flow rate and implemented in the thermoelectric system model. No temperature-dependent portion was included because no significant and consistent trend was observed.

### 5.2 Model Comparison

The initial comparison between the model and experiment reveals deviations and required further investigation into the assumptions, which may limit the effectiveness of the model. These discrepancies can be seen in Figures 5.1 through 5.3 and Tables 5.2 through 5.4 showing the comparison of the power predicted by the model and the actual amount generated by the experiment. The error bars represents a 95% confidence interval based on a variation of +/- .02 A in the current measurement, and the voltage variation is calculated from the standard deviation in the temperature measurement. It can be seen that the power is significantly over-predicted in the first two figures for the six and 12 module set up while slightly under predicts in the case of 24 modules.
Figure 5.1: Comparison of Model and Experimental Values for Tests Run at $T_{heater} = 200 \, ^\circ C$ at Three Flow Rates, with Six Modules Installed

Figure 5.2: Comparison of Model and Experimental Values for Tests Run at $T_{heater} = 200 \, ^\circ C$ at Three Flow Rates, with Twelve Modules Installed
Figure 5.3: Comparison of Model and Experimental Values for Tests Run at \( T_{\text{heater}} = 200 \, ^\circ \text{C} \) at Three Flow Rates, with Twenty Four Modules Installed

Table 5.2: Inlet and Outlet Temperatures in °C for the Experimental and Model Data with Six Modules

<table>
<thead>
<tr>
<th>( \dot{m}_h )</th>
<th>( T_{h,\text{inlet}} )</th>
<th>( T_{h,\text{outlet exp}} )</th>
<th>( T_{h,\text{outlet theoretical}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.028</td>
<td>104.0</td>
<td>91.7</td>
<td>94.5</td>
</tr>
<tr>
<td>0.042</td>
<td>106.4</td>
<td>97.4</td>
<td>99.5</td>
</tr>
<tr>
<td>0.053</td>
<td>106.4</td>
<td>99.3</td>
<td>101.0</td>
</tr>
<tr>
<td>0.028</td>
<td>159.4</td>
<td>137.5</td>
<td>143.2</td>
</tr>
<tr>
<td>0.042</td>
<td>164.9</td>
<td>148.8</td>
<td>153.1</td>
</tr>
<tr>
<td>0.053</td>
<td>164.9</td>
<td>152.6</td>
<td>155.6</td>
</tr>
<tr>
<td>0.028</td>
<td>208.1</td>
<td>178.5</td>
<td>185.9</td>
</tr>
<tr>
<td>0.042</td>
<td>216.0</td>
<td>194.1</td>
<td>199.7</td>
</tr>
<tr>
<td>0.053</td>
<td>218.1</td>
<td>200.4</td>
<td>204.7</td>
</tr>
</tbody>
</table>
Table 5.3: Inlet and Outlet Temperatures in °C for the Experimental and Model Data with Twelve Modules

<table>
<thead>
<tr>
<th>$\dot{m}_h$</th>
<th>$T_{h,\text{inlet}}$</th>
<th>$T_{h,\text{outlet exp}}$</th>
<th>$T_{h,\text{outlet theoetical}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.028</td>
<td>103.51</td>
<td>88.01</td>
<td>88.56</td>
</tr>
<tr>
<td>0.042</td>
<td>105.52</td>
<td>93.88</td>
<td>94.16</td>
</tr>
<tr>
<td>0.053</td>
<td>105.40</td>
<td>96.19</td>
<td>96.20</td>
</tr>
<tr>
<td>0.028</td>
<td>159.99</td>
<td>132.98</td>
<td>134.58</td>
</tr>
<tr>
<td>0.042</td>
<td>165.41</td>
<td>145.22</td>
<td>146.10</td>
</tr>
<tr>
<td>0.053</td>
<td>166.33</td>
<td>150.22</td>
<td>150.56</td>
</tr>
<tr>
<td>0.028</td>
<td>213.95</td>
<td>175.91</td>
<td>178.27</td>
</tr>
<tr>
<td>0.042</td>
<td>222.54</td>
<td>193.84</td>
<td>195.19</td>
</tr>
<tr>
<td>0.053</td>
<td>222.36</td>
<td>199.13</td>
<td>199.69</td>
</tr>
</tbody>
</table>

Table 5.4: Inlet and Outlet Temperatures in °C for the Experimental and Model Data with 24 Modules

<table>
<thead>
<tr>
<th>$\dot{m}_h$</th>
<th>$T_{h,\text{inlet}}$</th>
<th>$T_{h,\text{outlet exp}}$</th>
<th>$T_{h,\text{outlet theoetical}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.028</td>
<td>103.9</td>
<td>82.8</td>
<td>83.7</td>
</tr>
<tr>
<td>0.042</td>
<td>106.2</td>
<td>89.9</td>
<td>90.5</td>
</tr>
<tr>
<td>0.053</td>
<td>106.3</td>
<td>93.1</td>
<td>93.4</td>
</tr>
<tr>
<td>0.028</td>
<td>158.7</td>
<td>122.5</td>
<td>125.2</td>
</tr>
<tr>
<td>0.042</td>
<td>165.1</td>
<td>136.7</td>
<td>138.1</td>
</tr>
<tr>
<td>0.053</td>
<td>165.8</td>
<td>142.5</td>
<td>143.0</td>
</tr>
<tr>
<td>0.028</td>
<td>214.3</td>
<td>162.9</td>
<td>166.2</td>
</tr>
<tr>
<td>0.042</td>
<td>223.5</td>
<td>183.2</td>
<td>185.1</td>
</tr>
<tr>
<td>0.053</td>
<td>225.6</td>
<td>191.6</td>
<td>192.4</td>
</tr>
</tbody>
</table>

This data indicates that the initial model is not adequate and that all the physics of the system have not been captured. The first discrepancy noticed is the power is greatly over-predicted in the six and twelve module cases. This is also apparent in the tests run at 100 °C and 150 °C inlet temperatures. The outlet temperatures in the model seem to be off more in the six and twelve module case. This indicates some physics not being currently accounted for affecting the first two cases more than the second. This phenomena is most likely linked to three-dimensional heat spreading effects more prevalent in the six and twelve module systems. The model is then found to under-predict the power in the 24 module case. The trend of the data seems to point towards the model not generating enough power in the first zone while producing excessive or nearly correct amount of power in the last zone. This could be due the flow developing when it enters the rectangular testing box from a circular pipe. These two effects, along with
several others, are investigated to determine the impact on the model and make improvements.

5.2.1 Three-dimensional Heat Conduction Effects

To better understand how critical thermal spreading was to the various module configurations COMSOL, a finite element analysis program, was used to analyze the heat transfer through the fin system and module as seen in Figure 5.4.

![Temperature Plot of the Simulated Portion of the Heat Exchanger](image)

**Figure 5.4: Temperature Plot of the Simulated Portion of the Heat Exchanger**

To determine the added thermal resistance due to three-dimensional effects, a simple model was constructed and was investigated for various hot side conditions and module configurations. The heat sink used in the previous investigations and modules tested were modeled. First simulations for the heat sink without a thermoelectric module
attached were performed. Simulations were run for one Hi-Z module, then one, two and four Melcor modules.

Boundary conditions are set, a representative convection term and hot air temperature are defined at the surface of the fins, and the sides of the plate are insulated. For the heat sink without a module attached, the bottom of the plate boundary condition is a specified temperature and isothermal. When modules are attached, the cold side temperature of the heat sink is coupled to the top of the module(s) and the bottom of the plate that is not in contact with the module is insulated.

Simulations were run at the three flow rates (0.028, 0.042, 0.053 kg/s) at two temperatures (100 and 200 °C). Heat into, $q_h$, temperature at the top, $T_{top,fin}$, and bottom temperature, $T_{bottom,fin}$, of the fins were calculated in Comsol and used along with the surface area to determine the thermal resistance of the fins, $R_{th,fin}$, without any modules attached as follows.

$$R_{th,fin} = \frac{T_{top,fin} - T_{bottom,fin}}{q_h} \quad (5.3)$$

Then the thermal resistance between the fin and module in each of the four module configurations at all the temperatures and flow rate tested are calculated. The following expression is used, replacing $T_{bottom,fin}$ with the temperature at the surface of the thermoelectric module, $T_{top,TE}$.

$$R_{th,fin+TE} = \frac{T_{top,fin} - T_{top,TE}}{q_h} \quad (5.4)$$

The added three-dimensional thermal resistance, $R_{3D}$, is then calculated by:

$$R_{3D} = R_{fin+TE} - R_{fin} \quad (5.5)$$

These values have been calculated for one HI-Z, one, two, and four Melcor modules, then averaged for the different temperatures and flow rates tested. On one side of one zone are shown in the below table for both the hot and cold side. The cold side was calculated with similar expressions. For the two module case, the thermal resistance is recorded for the thermal series case, and was observed to be less than the thermal parallel case.
Table 5.5: Calculated Added Resistances in K/W Due to 3D Conduction for Hot and Cold Side Units

<table>
<thead>
<tr>
<th>Module</th>
<th>$R_{3D, hot}$</th>
<th>$R_{3D, cold}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi - Z</td>
<td>0.02</td>
<td>0.0064</td>
</tr>
<tr>
<td>1 Melcor</td>
<td>0.031</td>
<td>0.0167</td>
</tr>
<tr>
<td>2 Melcor</td>
<td>0.01</td>
<td>0.0071</td>
</tr>
<tr>
<td>4 Melcor</td>
<td>0.001</td>
<td>0.00234</td>
</tr>
</tbody>
</table>

These values are then added to the total heat transfer coefficient accounting for three-dimensional effects, $UA_{w/3D}$, based on the original total heat transfer coefficient without the added three-dimensional resistance, $UA_{w/o3D}$, and the added three-dimensional thermal resistance, $R_{3D}$, for both the hot and cold sides by the following expression.

$$\frac{1}{UA_{w/3D}} = \frac{1}{UA_{w/o3D}} + R_{3D}$$

(5.6)

5.2.2 Developing Flow Consideration

As discovered above, it is necessary to investigate the type of effect the developing flow has on the per zone heat transfer values. From Incropera and DeWitt [30], the following describes the relation of the average Nusselt number for the developing flow regime to the fully developed regime.

$$\frac{\overline{Nu}_D}{\overline{Nu}_{D,fd}} = 1 + \frac{C}{(x/D)^m}$$

(5.7)

$C$ and $m$ are coefficients that depend on the nature of the inlet taken from literature [31]. This expression then had to be manipulated for use with the total heat transfer coefficients previously developed using the following approximation.

$$\overline{(UA)}_x = \eta_o \overline{h} A_x = \eta_o \frac{\overline{Nu}_D k}{D_h} P_x$$

(5.8)
By substituting (5.5) into (5.6) the following expression is found.

\[
\overline{UA}_x = \frac{\eta_o \text{Nu}_{D,fd} kP}{D_h} (x + CD_h^m x^{1-m}) \quad (5.9)
\]

The first portion of the expression can be taken as a constant for all locations assuming minimal variation of Nu_{D,fd}. By defining C* as the following

\[
C^* = \frac{\eta_o \text{Nu}_{D,fd} kP}{D_h} \quad (5.10)
\]

then

\[
\overline{UA}_x = C^* (x + CD_h^m x^{1-m}) \quad (5.11)
\]

From this equation, the average UA value for each zone can be calculated with the length of each zone given by L.

The average UA value for each zone can be determined by finding the UA for the entire box, which is achieved by using 3L for the length of all three zones. The total heat transfer coefficient for each zone can then be found based on the average total heat transfer coefficient and several geometrical parameters as shown below.

\[
\overline{UA}_1 = \frac{L + CD_h^m L^{1-m}}{L + \frac{CD_h^m L^{1-m}}{3m}} \quad (5.12)
\]

\[
\overline{UA}_2 = \frac{L + CD_h^m L^{1-m} (2^{1-m} - 1)}{L + \frac{CD_h^m L^{1-m}}{3m}} \quad (5.13)
\]

\[
\overline{UA}_3 = \frac{L + CD_h^m L^{1-m} (3^{1-m} - 2^{1-m})}{L + \frac{CD_h^m L^{1-m}}{3m}} \quad (5.14)
\]
The ratios were calculated with an \( m \) value of 2/3 and with a \( C \) value of 1.8 and the values found from the previously mentioned literature, and were implemented in the model. The adjustments were successful in raising the predicted temperature in zone one and better modeling the fall in zone temperatures that were recorded in the testing. These values only affected the zone temperatures and had minimal effect on the outlet temperatures and the initial overall heat transfer coefficient is conserved by the above expressions.

5.2.3 Module Variation

In the process of testing individual modules to determine their module level parameters, uncertainties were encountered in measurements and calculated values. The errors present were a result of the differences in calculated values for internal electric resistance, Seebeck coefficient, and thermal resistance. The parameters were calculated by a voltage versus current curve fit and a power and energy fit, while a third value of thermal resistance was calculated through heat monitoring. The larger error values were found at the lower temperatures, which could be a result of the smaller temperature difference creating parameters that could be exposed to outside influence easier. A table of values showing the variances present can be observed in Table 5.6 [29].

**Table 5.6: Parameter Variance for Melcor and Hi-Z Tech. Modules Tested [29]**

<table>
<thead>
<tr>
<th>Module</th>
<th>( T_H ) [°C]</th>
<th>( T_C ) [°C]</th>
<th>( N \alpha_{p,n} ) [mV/K]</th>
<th>( N R_e ) [Ω]</th>
<th>( R_1/N ) [K/W]</th>
<th>( P_{max} ) [W]</th>
<th>( \eta_{max} ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melcor 99.2</td>
<td>99.2</td>
<td>29.0</td>
<td>47.3 +/-4.2</td>
<td>2.19 +/-0.30</td>
<td>1.68 +/-0.07</td>
<td>1.3</td>
<td>2.4%</td>
</tr>
<tr>
<td>HT8-12-40-W6</td>
<td>148.9</td>
<td>30.9</td>
<td>46.0 +/-2.6</td>
<td>2.40 +/-0.21</td>
<td>1.73 +/-0.06</td>
<td>3.1</td>
<td>3.6%</td>
</tr>
<tr>
<td></td>
<td>198.5</td>
<td>33.5</td>
<td>47.0 +/-1.9</td>
<td>2.66 +/-0.17</td>
<td>1.68 +/-0.05</td>
<td>5.6</td>
<td>4.7%</td>
</tr>
<tr>
<td>Hi-Z Technology</td>
<td>99.0</td>
<td>29.1</td>
<td>13.4 +/-0.3</td>
<td>0.130 +/-0.005</td>
<td>0.729 +/-0.016</td>
<td>1.7</td>
<td>1.5%</td>
</tr>
<tr>
<td>14 W</td>
<td>148.7</td>
<td>31.8</td>
<td>13.7 +/-0.2</td>
<td>0.137 +/-0.003</td>
<td>0.752 +/-0.014</td>
<td>4.7</td>
<td>2.5%</td>
</tr>
<tr>
<td></td>
<td>198.2</td>
<td>34.1</td>
<td>14.1 +/-0.1</td>
<td>0.150 +/-0.003</td>
<td>0.750 +/-0.013</td>
<td>8.9</td>
<td>3.4%</td>
</tr>
<tr>
<td></td>
<td>247.4</td>
<td>36.7</td>
<td>14.1 +/-0.1</td>
<td>0.164 +/-0.003</td>
<td>0.765 +/-0.013</td>
<td>13.5</td>
<td>4.1%</td>
</tr>
</tbody>
</table>

To determine the effect these parameters have on the model, the largest value of Seebeck and lowest value of internal resistance were used to determine the maximum amount of power modules could have been expected to produce. The opposite was done with the smallest value of Seebeck and largest value of internal resistance. This resulted in larger than expected variations in the reported power. In the case of 24 modules
generating power at 250 °C and 0.042 kg/s nearly +/- 3 Watts or 12% variation from the base value of power was observed in the first zone, while similar percentage variations were seen in the other zones. This was observed. From this variation almost every data point was captured by the model and showed how small deviations in model parameters can result in large variations in reported power.

In the following graphs the above mentioned additions were made to the system model and compared to experimental data. The error bars present represent the range of performance values that could be expected based on the uncertainty in the module parameters.

In Figure 5.5, the same trends noted are present in the unimproved model, but the slope of the power per zone is better captured. The model values are slightly shifted down because of the added three-dimensional thermal resistances. The second zone is still under predicted, which points to the fall in the total heat transfer coefficient not being quite as significant as predicted, but still provides a better trend of the data than before.
Figure 5.5 a-c: Power Generated Per Zone for System with 24 Modules and $T_{heater} = 200^\circ C$ with flow rates of 0.028, 0.042 and 0.053 kg/s
Figure 5.6 shows a similar trend as the previous graph, but varies surface temperature instead of flow rate. The same trends as seen in Figure 5.5 are apparent; that the model works best when lower flow rates and temperatures are used. This phenomenon is only found at the highest flow rates and temperatures in the 24 module case, and not in the twelve or six module case. The best explanation is that the model has trouble accurately modeling the heat extracted from the flow at its most extreme operating point.

Figure 5.7 accurately captures the trend of temperature at the surface of the modules. While there is still some discrepancy between the model and experimental data, it can be explained by the way thermocouple data is averaged for the surface of the modules, and the variation observed in the data collected. Along with the possibility of an increase in the heat transfer coefficient in the first zone associated with the sudden expansion and contraction of the flow into narrow rectangular channels, no significant data was found to indicate how to handle this sudden change in geometry, but would change the rate of fall of temperature in the model closer to that of the experiment. This current model serves as a good preliminary indicator of the trend of the data, and further research into the possibilities described above should allow the model to better match the experimental data.

Figure 5.8 compares the power generated at a specific operating point with the three different numbers of modules. In the six and twelve module cases, the experimental data fails to follow the expected form, which is most likely due to the reuse of modules between different tests. But the second two graphs still tend to over predict the power generated, but there is much better fit between experimental and modeled data compared to those found in Figure 5.2 and 5.3. This shows that the three-dimensional effect is significant and plays a large part in the prediction of accurate power generation.
Figure 5.6: Power Generated Per Zone for 24 Module System at 0.042 kg/s at Inlet Temperatures of $T_{heater} = 100\degree C, 150\degree C, 200\degree C$ and $250\degree C$ and $T_c = 30\degree C$

Figure 5.7: Temperature Drop Between Zones for Twenty Four Modules at 0.042 kg/s, $T_{heater} = 100\degree C, 150\degree C, 200\degree C, 250\degree C$ and $T_c = 30\degree C$
Figure 5.8 a-c: Power Generated at $T_{\text{heater}} = 200 \, ^\circ\text{C}$ and $T_c = 30 \, ^\circ\text{C}$ and 0.053 kg/s with Twenty Four, Twelve and Six Modules
Figure 5.9 shows a significant improvement from Figure 5.1. Ignoring the first zone data points, the power for second and third zones are within about one watt of experiment, significantly smaller than the more than 2 watts when heat spreading was not accounted for. This shows that the three-dimensional effect are very significant in this set up.

**Figure 5.9: Power Generated with Six Modules with \( T_{\text{heater}} = 200 \, ^{\circ}\text{C} \) and \( T_c = 30 \, ^{\circ}\text{C} \) at Three Flow Rates**

While the results in Figure 5.10 are less accurate than those in Figure 5.9, there is still a marked improvement noticed in the third zone where modeled power is usually within a watt of the experimental values. If more consistent experimental data was obtained, it would be easier to compare the improved model to the experiment, but the shifting of the model’s predicted powers shows that the three-dimensional effects also play a part in the performance. The same problems as noted in the results from Figure 5.9 should be noted here, until a good mounting method and method for determining the effects of module mismatch and degradation it is difficult to obtain consistent data for these experiments.
It was found that a tradeoff between outlet temperature accuracy and power prediction occurred when the three-dimensional heat transfer effects were taken into account. The small outlet temperature increases noticed (~1°C), outlet temperatures were already slightly over predicted by the model, were outweighed by the better power prediction achieved by the model after the implementation of three-dimensional heat transfer effects.

5.2.4 Mismatched Module Parameters

If modules have different voltage-current relationships, then their maximum operating points will differ. When differing modules are coupled in series or parallel, then it will be impossible to operate each module at its peak power point. To investigate the interaction between mismatched modules, a theoretical model will be developed along with an experimental investigation to compare to the theory. The theoretical expression will be developed for two modules connected together in series, developing an expression which can easily be extended to more modules. First, the previously-
discussed expressions for power generated (1.10) and for current (3.30), by substituting (3.30) into (1.10) an expression for power using only module properties and temperatures can be obtained under peak operating conditions.

$$w_{TE} = \frac{(N\alpha)^2(T_h - T_c)^2}{4(NR_e)}$$  \hspace{2cm} (5.15)

Next the expression can be extended to two modules in series with matched load to obtain the following:

$$w_{TE1+2} = \frac{(N\alpha_1 + N\alpha_2)^2(T_h - T_c)^2}{4(NR_{e1} + NR_{e2})}$$  \hspace{2cm} (5.16)

To understand the impact of max power generation due to variation in the module parameters, the relationship between two module parameters is assumed to be:

$$E \cdot N\alpha_1 = N\alpha_2$$  \hspace{2cm} (5.17)

$$G \cdot NR_{e1} = NR_{e2}$$  \hspace{2cm} (5.18)

$$E$$ and $$G$$ are some constants indicating the degree of mismatch between the modules Seebeck and electrical resistance mismatch. Substituting Eqs. (5.17) and (5.18) into the power equation, Eq. (5.16), an expression for power for the two modules in series can be found.

$$w_{TE} = \frac{N\alpha_1^2(1 + E)^2(T_h - T_c)^2}{4NR_{e1}(1 + G)}$$  \hspace{2cm} (5.19)

The ratio between power generated by the modules in series to power that would have been generated if each module was operated at its peak power point is dependent on the ratio between module parameters $$E$$ and $$G$$ as follows.

$$\frac{w_{TE1+2}}{w_{TE1} + w_{TE2}} = \frac{G(1 + E)^2}{(G + E^2)(1 + G)}$$  \hspace{2cm} (5.20)

Another useful ratio is the relation between the power generated by modules with unmatched parameters to those with perfectly matched parameters. This ratio is useful in
comparing the results of the ideal module parameters to the expected experimental results.

\[
\frac{w_{TE1+2}}{2 \cdot w_{TE1}} = \frac{(1 + E)^2}{2(1 + G)} \tag{5.21}
\]

To validate these models performance testing was done on a set of four Melcor modules. These were previously used modules in the system-level testing and had not yet showed any signs of mechanical fatigue. The first step was to test the modules individually in the module characterization test stand to obtain good data for comparing the performance of individual models. Testing was done with the hot side at 200 °C and the cold side held at 25 °C to obtain values for module parameters with as little variation as possible, which was observed at the higher temperature tests. A layer of thermal paste was applied to both surfaces of the module and was placed on the surface of the cold plate above the cold thermocouple, with insulation placed around the module and the hot side assembly lowered onto the module. Once the surface was touched, the pressure was zeroed and the crank was turned until the pressure was 700 kPa. Once the temperature leveled out, resistance sweep was undertaken to obtain good current and voltage data used for calculating the module parameters.
Once the testing for the individual modules was completed, the next step was to test all four modules operating in series. The next tests were two modules connected in several different combinations, as shown in Table 5.7, to obtain a good spread of data that could be compared to the first, eq. (5.20), and second, eq. (5.21), power ratios. The ratios for module power performance can be easily extended to multiple modules so data obtained for the four modules connected can be evaluated. The comparison between the experimental power and results of the power ratios just developed are presented in Table 5.7.

**Table 5.7: Power (W) from Experiment and Models for Mismatched Modules**

<table>
<thead>
<tr>
<th>Module ID</th>
<th>$W_{TE1,exp}$</th>
<th>$W_{TE1+2,exp}$</th>
<th>$W_{TE1}$</th>
<th>$W_{TE1+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6128/M5</td>
<td>7.27</td>
<td>0.79</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>M5/M10</td>
<td>7.75</td>
<td>0.92</td>
<td>0.99</td>
<td>0.86</td>
</tr>
<tr>
<td>M5/M16</td>
<td>7.69</td>
<td>0.90</td>
<td>0.99</td>
<td>0.92</td>
</tr>
<tr>
<td>M10/M16</td>
<td>6.13</td>
<td>0.84</td>
<td>1.00</td>
<td>0.93</td>
</tr>
</tbody>
</table>

The first column is the experimental power generated by two connected modules. The second column the summed experimental power produced by two modules operating separately. The third column is the theoretical combined power described by equation (5.20). The fourth column is the theoretical power calculated by equation (5.21). The reference module is chosen such that the power ratios will be less than one.

It can be seen that there is a large difference between the experimental and theoretical values. This indicates the interaction between modules is more complex than eqs. (5.20) and (5.21), and requires a much more in depth investigation to determine the interaction between modules connected in series. There are several items present that point to areas of further investigation. The calculated parameters for the modules connected in pairs and four were different that a simple sum of the individual parameters, for example the module level Seebeck coefficient calculated for four modules was around .145 V/K as opposed to the individual modules that added together would be .176 V/K. The added internal resistance due to the connecting wires could also have an effect, though the added resistance is on the order 0.1Ω, since this is about 4% of the resistance of a single module it is small enough to neglect in the purposes of this experiment where
the percent difference exceeds 20% between the experimental power and derived power. One more avenue of investigation is the heat flow, and how the added number of modules affects the amount of heat that is drawn through each module. This reduction of heat flow through a single module causes individual performance to suffer while more power is generated as a whole. This is observed in the amount of power generated in the main experimentation and in the mismatched module experiment the same phenomena may be present and is taken into account in future investigations.

5.2.5 Losses

The thermal losses of the system to the surrounding environment are a potential source of error that could reduce the amount of power generated by the system. Preliminary calculations were performed to assess how much of a factor these losses would be. The convective losses only accounted for a loss of a few percent of the whole system and a worst case radiation investigation revealed that the amount of heat lost was less than the convective portion and together would account for less than 5 percent of the heat moving through the system and only produces a variation of about 1 °C in the outlet temperature. The losses were minimized by insulation placed around the system and smooth inlet and outlet pipes, which provided a large thermal resistance between the flow in the pipe and the ambient air. The losses could be further reduced in future investigations by adding additional insulation to portions of the test stand that are the biggest losses, and place foil around the added insulation to minimize further radiation losses.

For the preliminary purposes of this investigation, the model has been refined from its earlier version and now better models the experimental data gathered. This model was the first to attempt to model different configurations of modules, and had the ability to operate under various temperatures, flow rates, and total heat transfer coefficients. An improved preliminary model has been developed and modified to account for additional system physics such a flow developing and heat spreading. There does appear to be issues associated with module mismatch, which could not be modeled by simple parameter mismatch models and still needs to be addressed in future research. Thermal losses appear to have a negligible impact on the difference between
experimental and modeling data. Future work is required to develop a more robust model that captures all the physics of general thermoelectric systems.
Chapter 6: Optimization and Feasibility

A preliminary optimization procedure is developed and demonstrated in this chapter. The approach couples the previously-developed thermoelectric and heat transfer models with new models developed for pressure drop and system cost. The combined models allow several system parameters to be adjusted for identifying the least expensive system subject relative to the given power generation requirement and input conditions. This preliminary approach lays the foundation for further research into the field of thermoelectric system-level optimization.

6.1 Optimization Model Development

The goal is to provide an optimized model with the lowest cost per watt possible, or the lowest cost per watt for a given amount of power generated. To accomplish this, a master file is developed, where input variables are specified, such as flow rates, temperatures, and module parameters. The design variables, duct width, duct height, duct length, number of fins, and the number of modules, are defined in the next section. An iteration program is then run, calling files for evaluating the performance of many systems in determining the most cost-effective solution. The hot-side heat transfer coefficient calculation is discussed in Section 3.3.3 and the thermoelectric performance calculation is described in Section 3.3.2. The cost calculation program calculates the total cost and total cost per watt of the system based on all the components included in a Thermoelectric Power Generation System. Results for total power generated, total cost of the system, and cost per watt are then reported. Contour plots are created to provide a graphical representation of the system cost versus system design variables such as number of modules and heat sink geometry. The program flow is shown in Figure 6.1.
6.1.1 Adjustments to Previously Developed Files

Several changes to the hot side heat transfer coefficient and thermoelectric models were made to accommodate the introduction of design variables to the system. The adjusted hot side heat transfer coefficient model, previously discussed in Section 3.3.3, calculates the $UA_h$ value based on geometry, temperature and flow rate instead of using the experimentally derived values found in the earlier total heat transfer experiment. In the thermoelectric system model, previously described in Section 3.3.2, the temperature dependent parameter equations for both the Melcor and Hi-Z modules have been included and chosen in the optimization routine. The thermoelectric performance calculation process has been generalized so other commercially available and theoretical modules can be investigated. The three-dimensional thermal resistances, discussed in Section 5.2.1, have been generalized into a curve fit based on area ratios for both the hot and cold
sides. The developing flow equations, discussed in Section 5.2.2, have also been expanded for different numbers of zones, from one to four. The values for the three-dimensional thermal resistances and developing flow are only useful in a system similar to the currently developed testing box and requires further study to use these values in other systems.

6.1.2 Pressure Loss

An important consideration in the performance of a thermoelectric system is the pressure loss that directly impacts energy loss in the fluid stream. This pressure loss can have a negative effect on the performance of a turbine when placed in the exhaust stream and often requires additional pumping power to make up for the lost flow energy. The pressure loss through a number of parallel channels that may exist in a Thermoelectric Power Generation System can be expressed as

\[
\Delta P = \frac{fL_{Duct} \bar{v}^2 \rho_{fluid} N_{channels}}{2D_h}
\]

where \( f \) is the fanning friction factor, \( L_{Duct} \) is the duct length, \( \bar{v} \) is the average velocity, \( \rho_{fluid} \) is the density of the air, \( N_{channels} \) is the number of channels in the testing box. The additional pumping is defined as:

\[
w_{pump} = \Delta P \dot{V}
\]

where \( \dot{V} \) is the volume flow rate. These equations are included in the hot side heat transfer coefficient model since important geometric and fluid parameters are already evaluated in that model.

6.1.3 Cost Calculator

The cost calculator subroutine determines the cost per unit energy, \( B \), and first assigns a cost per module based on the input from the user then provides a total module cost, \( F_{mod} \), based on the number of modules used in the evaluated system. Costs are also included for electrical support cost, \( F_{elec} \), on a per module basis, which includes a DC-DC converter and other equipment. A cost for the pump, \( F_{pump} \), needed to replace the pressure loss is included. A per zone cost, \( F_{zone} \), is also included to represent the added cost of including more zones in a power generation system. A cost for the heat removed
from the Thermoelectric Modules is included in eq. 6.3 as \( F_{cs} \). Lastly a cost for the hot side heat exchanger is included, \( F_{hs} \), and is determined on a per unit volume basis. These costs are summed and divided by the total power generated by the Thermoelectric Modules, \( w_{TE} \), less power delivered to the pump as shown below, \( w_{pump} \):

\[
B = \frac{F_{mod} + F_{elec} + F_{pump} + F_{zone} + F_{cs} + F_{hs}}{w_{TE} - w_{pump}} \quad (6.3)
\]

The costs used here are approximations, but do include considerations for all the major components expected to contribute to the cost of an actual system. Many of the cost predictions are taken from the testing box costs, and are easily expected to fall if these systems were mass produced. These costs are educated guesses to obtain an idea of how an actual system is impacted by considerations for its various components. More accurate costs may be a topic of further investigation in future work.

### 6.1.4 Optimization Program

The optimization routine can be seen in Appendix B and is the top-level program alluded to earlier and in the process of running calls the three other routines. First, the input variables, the mass flow rate, and input temperature for the hot and cold fluids are defined for a certain application. The power requirement for the system is then defined. The first three designed variables are the dimensions of the heat exchanger core; the fourth is the number of fins; and the fifth and sixth are the number of zones and modules per zone respectively. The first three variables can be viewed as continuous functions operating between two set points while the second three variables are discrete and must be integer values. From investigation of optimization techniques for discrete variables, the most straightforward and complete method is the exhaustive enumeration where every point is calculated. For three choices for each of the six variables, requires 729 simulations, which takes approximately six minutes of computational time on a computer with an AMD Turion X2 64 but processor and 4 GB of RAM. Contour plots are generated from the simulation data to show the area where the lowest cost per watt can be found and direct investigation in post processing. Then the recorded values for cost per watt, total power, and total cost are inspected to find the system generating the desired
amount of power for the lowest cost. The design variables for the optimal system are then recorded.

6.2 Model Results

To achieve good results, the model was investigated for sensitivity to determine the critical variables requiring modification. This investigation revealed the variables reaching minima, and the values requiring artificial constraints to obtain a realistic solution. Once these variables were investigated, the model was used for generating contour maps showing the range of values resulting in the lowest possible cost per watt and total cost.

This investigation is explained further in the following sections and provides the details of these analyses.

6.2.1 Sensitivity Analysis

A range of values for each parameter were used to calculate system performance while the values of the other five parameters were held steady to determine the affect on several important values (e.g., the total hot-side heat transfer coefficient, total power, cost per watt, and pressure loss) in calculating the performance of a system. This investigation provided a direction for minimization procedures and determined where to place limits on minimum or maximum values from the study. This was by no means a definitive investigation into the physics of the system; rather, it provided a quick look at the future direction of determining the optimum range of variables to use.

The base case, shown in Table 6.1, indicated that there was a tradeoff between the total heat transfer coefficient and the pressure drop for the duct width, \( W_{duct} \). As the total heat transfer value increased by increasing the number of fins, and decreased the hydraulic diameter of the channels, there was an increase in the power generated. This increase in heat transfer and power was at the expense of larger pressure drops. The cost per watt was found to not have reached a minimum in this tradeoff before the minimum width allowable, the width of two modules, was reached. The width of two modules was chosen as the width constraint because the elimination of two modules next to each other would have greatly affected the usefulness of the shape factor calculations with the widths expected to work for one module.
When varying duct length, $L_{duct}$, a minimum was found; specifically, the increase in the three-dimensional thermal resistance became more of a factor than the modest increases in the heat transfer coefficient. The overall effect of changing duct length was small compared to the other two geometrical parameters.

The duct height, $H_{duct}$, and number of fins, $N$, were found to have a tradeoff between an increased heat transfer coefficient and increased pressure drop. The combination of the number of zones, $z$, and number of modules per zone, $p$, was found to have a combined minimum, where the total number of modules was minimized for various combinations of zones and modules as can be seen in Figure 6.2. This valley exists because of the tradeoff between the cost of additional modules and smaller amounts of power generated for more modules added to the system.
This investigation provided a valuable source of information to guide the minimization of a specific system as outlined in the following section.

6.2.2 Optimization of an Automobile Exhaust System

Due to the lack of information for larger systems, input data was used for a 1991 Dodge Caravan travelling at 50 mph [13]. Since our system was designed to simulate an auto exhaust system, the previously-developed model improvements could be used in the system optimization to ensure a more accurate and meaningful solution. This also led to several assumptions for using the program already investigated. The biggest was neglecting the impact on the performance of the automobile and only looking at the system as a stationary power generation source. It was also necessary to assume that the shape factors were the same as those found for the Exhaust Simulator Test Stand, even though the geometrical dimensions changed during the optimization process. This was valid because the dimensions stayed relatively similar throughout, and actually produced conservative values when the optimized system was smaller than the experimental system. Another important assumption was the modules used had the same performance parameters as the previously-tested Melcor modules, but operated over a larger temperature range. The cost per module was assumed to be about twice the current price ($40 per module), to reflect the presumed increased cost of such a segmented module. While pressure loss was not a large factor in this investigation, it was still important to keep pressure loss as an important design variable where additional pump costs were neglected. Therefore, the design was also constrained by keeping the total pressure loss below 10 kPa, which assumed to represent a reasonable amount or an allowable pressure loss, without creating a large back pressure affecting the normal performance of the system. Another constraint was needed to keep the aspect ratio below 4:1 in either direction to ensure the hydraulic diameter relation continued to be used [32].

The optimization process is tedious because every design point is evaluated; unlike a method which actively searches for a minimum. The file is run for constant geometrical properties, including duct height, width and length along with the number of fins. Thirty two combinations of number of zones and number of modules are investigated to produce contour plots of the cost of watt for various combinations of these
two variables. Once a minimum cost per watt is found, one of the four geometrical variables is changed to determine whether the cost per watt can be further minimized. If a variable is found to decrease the cost per watt, then the variable is varied until a minimum is reached. Once it is found by increasing and decreasing all the design variables, no lower cost per watt can be found a minimum is reached. This process is less than ideal, but is sufficient for the proof of concept as it takes less than 30 minutes to reach a minimum for this type of system.

The lowest cost per watt of this system was found to be $3.63 per watt, producing 115 watts of power with a duct width of 0.08 m, duct length of 0.33 m, duct height of 0.0165 m, 24 fins, and 2 zones with 4 modules in each zone. The duct width and duct length were at the minimum constraints as described in the previous section. Contour plots of the cost per watt, total cost, and total power versus total number of modules and number of zones are shown in Figures 6.3 to 6.5. Several other possible configurations existed, providing varying amounts of power for nearly the same cost per watt. This led to the interesting development, where a system could be designed for the lowest cost per watt and several possible configurations exist at nearly the same cost per watt. As a result, the most appropriate configuration could be chosen based on power or cost requirements.
**Figure 6.3: Cost per Watt Versus Number for Modules for the Optimum System**

**Figure 6.4: Total Cost Versus Number of Modules for the Optimum System**
This proof of concept shows that it is possible to determine an optimum system using a global method stemming from the integer values of several of the variables. While the system is not an elegant approach, it provides a good starting point for further refinement and extension to compare to the performance of similar systems.

6.3 Feasibility Procedure

To determine the feasibility of a thermoelectric power generation system, it is useful to develop a relationship between the cost per watt of a system and a cost per kilowatt hour for comparison to other forms of power generation, where $/kWh is the metric for comparison between alternative power generation options. The total energy generated over the lifetime of a system, $U$, is the key to determining the viability of a system and can be found from the following expression:

$$U = H \cdot OP \cdot CF$$  \hspace{1cm} (6.4)
This expression determines how much energy can be generated for some installed capacity, \( H \), in kW over the number of years in operation, \( O P \), for a capacity factor, \( CF \). The capacity factor describes how much a power source is actually operated versus how much it would have been operated if it was run at its rated power all the time. The value, \( U \), then determines the cost per kWh, \( e \), with the following expression:

\[ e = \frac{M}{U} \quad (6.5) \]

where \( M \) is the module cost per module rated power. The cost per kWh can be used to compare the cost effectiveness of various power systems to a thermoelectric system.

Next, it is useful to look at a specific power application to determine at what ranges it could be economically feasible to use a thermoelectric power generation system.

Current thermoelectric systems are not feasible in most applications, but looking into what makes a system feasible in different applications can be useful for determining the circumstance under which such a system is viable. To illustrate this point, a simplified example of a car’s alternator is investigated then compared to the performance of a thermoelectric system.

An automobile alternator is used to convert mechanical work done by the engine into electrical power for charging the battery that in turn runs the electrical components of the automobile. Current automotive alternators are around 60% efficient. This is because the engine efficiency for modern automobiles is approximately 25%; the overall efficiency of converting chemical energy from the fuel to useable electrical energy is 15%. The main driver in the cost efficiency of the alternator is the price of gas. To look at feasibility of a thermoelectric system, it is important to determine the cost per watt of the thermoelectric system versus the price of gas will need to be developed.

An expression to relate the price of fuel in \$/W, \( l \), and the cost per unit power, \( B \), needs to be developed to determine when a thermoelectric power generation system is potentially feasible. The number of years in operation and the capacity factor for an automobile are needed to find the cost per watt of an alternator shown in the following expression:

\[ B = l \bullet O P \bullet CF \quad (6.6) \]
Using this expression the threshold for feasible alternative can be found, where a thermoelectric power generation system will be feasible if it can be made for a cost per watt less than the capacity factor as a given cost of fuel. For this investigation into the feasibility of a power generation system in an automobile, it is assumed the vehicle operated for 12 years and calculated values for capacity factors of 1%, 3%, and 5%. These numbers are selected for the capacity factor to determine how the cost per watt is affected at higher and lower capacity factors than the average capacity factor chosen to be 3%, which corresponds to 250,000 km driven over 12 years averaging 80 km/hr. From this, it is determined that the lower the operating factor, the less economically feasible a thermoelectric system becomes due to the price of fuel in the equation. Figure 6.6 represents the cost per watt of power generation versus the price of gas, where the conversion between gallons of gasoline and megajoules is 131.76 gal/MJ.

![Graph: Cost per Watt of Power Generated Versus Price of Gas for Several Capacity Factors](image)

**Figure 6.6: Cost per Watt of Power Generated Versus Price of Gas for Several Capacity Factors**

A system would be considered feasible when it is below the purple line and to the right of the capacity factor lines. For example for gas at $4/gal and a 3% capacity factor a thermoelectric Power Generation System would have to cost less than $1.72/W, but
only $2.86/W if the capacity factor was 5%, these prices don't include the savings from not installing an alternator. From this it can be seen that Thermoelectric Power Generation Systems are best implemented in vehicles with high capacity factors, such as tractor trailers and busses. Until the cost per watt falls significantly (either from cheaper production methods or more efficient modules being created), thermoelectric power generation is not feasible in smaller scale passenger cars. This hypothetical example shows the basic procedure that may be used in future studies to determine the feasibility of an application for thermoelectric power generation. Thoughts on future studies are highlighted in the next section.
Chapter 7 Conclusions and Future Work

This thesis laid the foundation for future research in the field of thermoelectric power generation systems at RIT. Research into this area will help to find the best applications for thermoelectric power generation, allowing future research to focus on these applications. As a result of this work, the exhaust simulator test stand has been refined and calibrated for use with several testing boxes. A system model has been developed with the capability to predict the performance of Thermoelectric Power Generation Systems ranging from several to hundreds of modules. Several module configurations have been tested using a previously-developed testing box with several refinements to the system model being developed. The most interesting product from this thesis is the optimization model, which lays the ground work for unique research in the TEG system design. This model combines findings from the experimental and theoretical testing; uses them to determine an optimum configuration for a system; and estimates the cost of such a system. This information can then be used to compare the performance of a Thermoelectric Power Generation System to other power generation options, which is helpful in predicting when a thermoelectric system might be competitive against alternative technologies.

The Exhaust Simulator Test Stand has been modified with several improvements, such as eliminating excessive pressure losses in the tubing between the compressor and the heater as well as the addition of a recirculating pump connected to the cold side of the system. The measuring equipment was calibrated to ensure data being obtained was correct. Because of this work, the exhaust simulator was a much more useful tool. During the course of this research several areas of improvement were identified and need to be addressed in the future including better temperature control on the cold side of the testing box and more accurate and repeatable cold side temperature measurements. Pressure measurements are needed to confirm the pressure model for use in applications where pressure loss has an effect on the performance of a system. Finally, a mounting rig needs to be created for the testing box. This would allow for more reliability in the placement of modules and thermocouples and more consistent pressure on the modules.
The system model developed is capable of simulating many different module configurations ranging from a few to hundreds of modules. The Thermoelectric Model has been tested and refined, providing the reasonably accurate prediction of system performance. Several outstanding issues remain and need to be addressed to improve its predictive capabilities. The biggest issue is the effect of mismatched module parameters. This has been studied over the course of several weeks to determine whether there is a relatively simple relation describing the performance of modules connected in series. The experimental data gathered did not validate the two proposed models, leading us to believe there are unknown effects coming into play requiring significant study to develop an improved model.

The largest amount of effort is in the further development of the optimization tool. While the optimization routine in its current form is a good first step in the development of a useful optimization tool, more work is needed to provide more meaningful results. Such a tool is vital in determining the feasibility of thermoelectric systems and in providing guidance in future system design. The pressure model is currently a theoretical model with no experimental validation done yet. Further, it is restricted to one specific type of geometry. Developing an improved pressure model is critical for an enhanced costing function. Current costs are estimated and require thorough research to determine more accurate costs, along with any necessary maintenance costs. For automotive applications, weight can affect the performance of the system and requires examination to determine the effect it has on the viability of a system. The feasibility section also requires further research to determine accurate operating costs of comparable technologies such as oil and coal along with rival green technologies.

This research will continue the work begun in this thesis and will provide a useful tool for investigating the viability of new applications. Being on the cusp of a large availability of new thermoelectric materials will not only make this work imperative to the market penetration of thermoelectric technology, but will ultimately determine the success of this technology, currently limited to a niche markets.
References


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Appendix A
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## Twenty Four Module Testing Data

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### Electrical

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<th>Z2 Volt</th>
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<td>50.13777</td>
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<th>Air in</th>
<th>Water in</th>
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### Notes
- mdot: mass flow rate
- vdot: volume flow rate
## Twenty Four Module Testing Data Cont.

### Electrical

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Appendix B

% Optimization Protocol
clc
clear all
close all

global vfair mdotc mdotc Tcinlet Thinlet Wduct Hduct Lduct N z p
T_preheat CostperWatt module UAh_total DelP Total_power eta_f eta_o ReD
Wpump Total_cost

%Input variables
vfair = .02328; %Hot Flow Rate (m^3/sec)
mdotc = 0.2761; % hot mass flow rate
mdotc = .163; %Cold Flow Rate (kg/sec)
Tcinlet = 78+273; %Cold inlet temperature (K)
Thinlet = 492+273; %Hot inlet temperature (K)
T_preheat = 25 + 273;  %Preheated air temeprature (K)
Wmod = 0.04001;
Lmod = 0.04001;
Power_req = 100;  %Minimum power needed (W)
%set Initial Indices
k = 1; r = 1; q = 1; %Wduct, Hduct, Lduct
c = 1; h = 1; d = 1; %N,Z,p

%Design Variables
module = 1;  %Type of module

Wduct = 0.1228; %Duct width (m) (.1228)
Hduct = 0.005; %Duct height (m) (.0485)
Lduct = 0.3572;%[.5 .75 1.0]; %Duct Length (m) (.3572)

N = 24; %[50 70 90]; %Number of fins (24)
z = 3; %Number of Zones (3)
p = 4; %Number of Modules per zone (2)
s = 1;
% for k = 1:8
%    Wduct = Wduct1(k);
% for r = 1:3
%    Hduct = Hduct1(r);
% for q = 1:3
%    Wduct = Wduct1(q);
% for c = 1:9
%    N = N1(c);
% for h = 1:4
%    z = z1(h);
% for d = 1:8
%    p = p1(d);
%Run Sub Programs

%AR = (z*p*Wmod*Lmod)/(2*Lduct*Wduct);
run ('ua_hot_model')
run ('Kevin_Thermoelectric_Model')
run ('performance_calculator')

%if Total_power >= Power_req && CostperWatt <= 13
CperW_v(s) = CostperWatt;
Total_power_v(s) = Total_power;
%Rsfhs(s) = Rsfh;
%ARs(s) = AR;
qeval(s) = Wduct;
ceval(s) = N;
heval(s) = z;
deval(s) = p;
index(s) = s;
Total_cost_v(s) = Total_cost;
%ReDeval(s) = ReD;
s = s+1;
end

Total_cost = Total_power_v.*CperW_v;

Z =
[CperW_v(1:1:8);CperW_v(9:1:16);CperW_v(17:1:24);CperW_v(25:1:32)];
%CperW_v(22:1:28);CperW_v(29:1:35]);
Z1 =
[Total_power_v(1:1:8);Total_power_v(9:1:16);Total_power_v(17:1:24);Total_power_v(25:1:32)];
%Total_cost(22:1:28);Total_cost(29:1:35]);

Z2 =
[Total_cost_v(1:1:8);Total_cost_v(9:1:16);Total_cost_v(17:1:24);Total_cost_v(25:1:32)];

% Z =
[CperW_v(1:1:6);CperW_v(7:1:12);CperW_v(13:1:18);CperW_v(19:1:24)];
%CperW_v(22:1:28);CperW_v(29:1:35]);
% Z1 =
[Total_cost(1:1:6);Total_cost(7:1:12);Total_cost(13:1:18);Total_cost(19 :1:24)];
%Total_cost(22:1:28);Total_cost(29:1:35]);

%contour_p1 = [qeval; ceval; heval; deval; CostperWatt_v; Total_cost];
%contour_p2 = [heval.*deval, qeval, CostperWatt_v];

figure(1)
[X,Y] = meshgrid(1:1:8,1:1:4);
%X = z1;
%Y = p1;
[C,h] = contourf(X,Y,Z,15);
xlabel('Number of Modules per Zone'),ylabel('Number of Zones'),title('Cost per Watt ($/W')
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2)

figure(2)
plot(index, Total_power_v, index, CperW_v)

figure(2)
[C,h] = contourf(X,Y,Z2,15);
xlabel('Number of Modules per Zone'),ylabel('Number of Zones'),title('Total Cost ($)')
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2)

figure(3)
[C,h] = contourf(X,Y,Z1,15);
xlabel('Number of Modules per Zone'),ylabel('Number of Zones'),title('Total Power (W)')
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2)

Low_cost = min(CperW_v)

CostperWatt
UAh_total
DelP
Total_power

%% UA Theoretical Model
function ua_hot_model

global vfair Wduct Hduct Lduct N Thinlet T_preheat ReD UA cost Vhx UAh_total eta_f eta_o Dh mdoth Wpump DelP

%Pipe parameters
Diameter = 0.08;

%Duct Parameters
%Wduct = .1228;
%Hduct = .0485;
tduct = .01;
kduct = 230;
Conductivity (W/m-K)
%Lduct = .3572;
tbase = .01;

% Fin Geometrical Parameters
%N = 24;
both top and bottom)
L = Hduct/2;

thick = 0.00165;

% Hot Side (Air)
%vfair = 95;

%%% Air Flow Rate (CFM)
volflowair = vfair;  % Air Flow Rate
(m^3/sec)[*4.72e-4]
Thi(1) = Thinlet;  % Inlet temperature
T_preheat = 25 + 273;

rhoair = 5.31818e-12*T_preheat^4 - 1.637e-8*T_preheat^3 + 1.96091e-5*T_preheat^2 - 1.13966e-2*T_preheat + 3.20543;  % Air Density (kg/m^3)

% Fin Calculated Parameters
N_channels = (0.5*N) +1;  % Number of Channels
tchannel = (Wduct - 0.5*N*thick)/N_channels;  % Width of Channel (m)
Achannel = tchannel*Hduct;  % Area of Channel (m^2)
Pchannel = 2*tchannel + 2*Hduct;  % Perimeter of the channel (m)
Dh = 4* Achannel/Pchannel;  % Hydraulic Diameter (m)
A_total_xsect = N_channels*Achannel;  % Total Cross-Sectional Area (m^2)
vbar = mdoth/(A_total_xsect*rhoair);  % Average Velocity (m/s)

Pr = -9e-13*Thi(1)^4 + 2e-9*Thi(1)^3 - 7e-7*Thi(1)^2 - 0.0002*Thi(1) + 0.7798;  % Prandtl Number
kcond = (9e-9*Thi(1)^3 - 5e-5*Thi(1)^2 + 0.1046*Thi(1) - 1.168)*10^-3;  % Air Thermal Conductance Value (W/m-K)
Cph = (9e-12*Thi(1)^3 - 2e-8*Thi(1)^2 + 1e-5*Thi(1) + 1.5371)*10^3;  % Air Heat Capacity (kg/kJ-K)
viscosityair = -2e-11*Thi(1)^2 + 6e-8*Thi(1) + 3e-6;  % Air Viscosity ()

ReD = rhoair*vbar*Dh/viscosityair;  % Average Reynold's number in one channel

% Determine the Convective Coefficient
if ReD <= 2100
    A = 0; B = 16; m = 1;
elseif ReD <= 4000 && ReD > 2100
    A = 0.0054; B = 2.3e-8; m = -0.6667;
else
    A = 0.00128; B = 0.1143; m = 3.215;
end
f = A + B/(ReD^(1/m));  % Fanning Friction Factor (Blasius)

%f = (0.790*log(ReD) - 1.64)^-2;  %Friction factor
NuD = (f/8)*(ReD-1000)*Pr/(1+12.7*(f/8)^0.5*(Pr^(1/3)-1));  %Nusselt Number
h = NuD * kcond / Dh;  %Average Convective Coefficient

%Fin Resistance
Af = 2*Lduct*L;  % Area of Fin (m^2)
Ab = (2*Wduct - N*thick)*Lduct;  % Base area (m^2)
At = N*Af + Ab;  % Total area of Heat Transfer (m^2)
m = sqrt(2*h/(kduct*thick));  % m value
eta_f = tanh(m*L)/(m*L);  % Rectangular Fin Efficiency
\[
\eta_o = 1 - N \cdot \frac{A_f(1-\eta_f)}{At}; \quad \% \text{ Total Efficiency}
\]
\[
R_{\text{hotfin}} = \frac{1}{(\eta_o \cdot h \cdot At)}; \quad \% \text{ Fin Resistance (K/W)}
\]

% Wall Conduction
\[
\text{Abase} = W_{\text{duct}} \cdot L_{\text{duct}}; \quad \% \text{ Area of Wall (m}^2)\]
\[
R_{\text{wall}} = \frac{t_{\text{duct}}}{k_{\text{duct}} \cdot \text{Abase}}; \quad \% \text{ Thermal Resistance of the wall (K/W)}
\]

% Contact Resistance (K/W) -- Include considerations for fin attachment
\[
R_{\text{thermal_contact}} = 0;
\]

%% Total UA Value (W/K) %%
\[
U_{\text{Ah_total}} = \left( R_{\text{hotfin}} + R_{\text{wall}} + R_{\text{thermal_contact}} \right)^{-1};
\]
\[
\text{Axc} = 2 \cdot W_{\text{duct}} \cdot (t_{\text{duct}} + t_{\text{base}}) + (W_{\text{duct}} - N_{\text{channels}} \cdot t_{\text{channel}}) \cdot H_{\text{duct}};
\]
\[
V_{\text{hx}} = \text{Axc} \cdot L_{\text{duct}};
\]

% Basecost = 250;

UAunit = 180;

% if \( N \leq 20 \)
%   % fincost = 100;
%   % UAunit = 190;
% else if \( N \leq 26 \)
%   % fincost = 140;
%   % UAunit = 180;
% else
%   % fincost = 170;
%   % UAunit = 170;
% end

%UAcost = Basecost + fincost;
UAcost = V_{\text{hx}} \cdot UAunit;

% rhoi = 5.31818e-12 \cdot T_i^4 - 1.637e-8 \cdot T_i^3 + 1.96091e-5 \cdot T_i^2 - 1.13966e-2 \cdot T_i + 3.20543;
% rhoo = 5.31818e-12 \cdot T_o^4 - 1.637e-8 \cdot T_o^3 + 1.96091e-5 \cdot T_o^2 - 1.13966e-2 \cdot T_o + 3.20543;
% rhoa = (rhoi + rhoo) / 2;

% DelP = \frac{m_{\text{dth}}^2}{2 \cdot A_o} \cdot \left( 1 - \sigma^2 \right) + f \cdot A_{\text{ht}} \cdot \rho_i \cdot \left( 1 - \sigma^2 \right) \cdot \left( \frac{\rho_i}{\rho_o} - 1 \right) + \frac{f \cdot A_{\text{ht}} \cdot \rho_i}{(A_o \cdot \rho_o) - (1 - \sigma^2) \cdot \rho_i \cdot \rho_o};
% DelP = f \cdot L_{\text{duct}} \cdot \mu_bar^2 \cdot \left( \rho_{\text{air}} \cdot N_{\text{channels}} / (2 \cdot D_h) \right); \% \text{ Pressure Loss (Pa)}

% Vdot = m_{\text{dth}} / \rho_i; \quad \% \text{ Volumetric Flow Rate (m}^3/\text{s})
W_{\text{pump}} = \mu_{\text{fair}} \cdot \text{DelP}; \quad \% \text{ Pump Work}
% Kevin Thermoelectric System Model

function Kevin_Thermoelectric_Model

global ThTE Tho TcTE Tco Thi Tci rhoair Dh mdoth Cph UAh UAh_total
mdotc Cpc UAc Rins z p k Rins Rsfh Rsfc Thinlet Tcinlet Total_power
vfair Wduct AR Lduct module

% clear ThTE Tho TcTE Tco Thi Tci
%%%                                      %%%
% Section 1 - Define Operating Constraints %
%%%                                      %%%

%Insulation parameters
tins = 0.0033;                       % Thickness of
Insulation (m)
kins = 0.045;                       % Thermal Conductivity
of Insulation (W/m-K)

%Pipe parameters
Diameter = 0.08;                     % Pipe Diameter (m)

%Duct Parameters
Wduct = .1228;                       % Duct Width (m)
Hduct = .0485;                       % Duct Height (m)
tduct = .01;                         % Duct Wall Thickness (m)
kduct = 230;                         % Duct Material Thermal
Conductivity (W/m-K)
Lduct = .3572;                       % Duct Length (m)

% Assign Number of TE's in Series and Parallel -- Zones
Lzone = (Lduct-(z-1)*tins)/z;       % length
of Each Zone (m)
%z = 3;                             % Number of Zones
%p = 2;                             % Number of Modules
in Each Zone %%

% Define Thermal Break Parameters
tbreak = 0.0033;                     % Thickness of Break
(m)
kbreak = 0.045;                      % Thermal Conductivity
of the Break (m)

% Hot Side (Air)
% vfair = 50.1;                       % Air Flow Rate
(CFM) %%
% volflowair = vfair*4.72e-4;        % Air Flow Rate
(m^3/sec)

% Cold Side (Water/Cold Plate)
%mdotc = .1563;                       % Cold Side Mass
Flow Rate (kg/sec) %
Cpc = 4179; % Cold Side Heat Capacity (kg/kJ-K)
UAc_total = 225; % Cold Total Heat Transfer Coefficient (W/K) 

%%% 248.1/18.2, 221.95/22.56, 208.9/25.58
% if vfair >= 55
%   %UAh_total = 18.2;
%   UAc_total = 250;
% elseif vfair <= 80
%   %UAh_total = 22.56;
%   UAc_total = 221.95;
% else
%   %UAh_total = 25.58;
%   UAc_total = 208.9;
% end

%%% Section 2 - Thermoelectric Performance Calculations %

%%% Shape Factor Calculations %

if module == 1
    Wmod = .04001; % Width of Module (m) melcor - 0.04001, Hi-Z - 0.06007
    Lmod = .04001; % Length of Module (m)
else
    Wmod = .06007;
    Lmod = .06007;
end

AR = (z*p*Wmod*Lmod)/(2*Lduct*Wduct);

Rsfh = 0.0956*exp(-10.064*AR);
Rsfh = 0.0009*AR^(-1.3418);

% if p <= 2 % This makes shape factor zero in opt model
%   Rsfh = 0.031;
%   Rsfc = 0.0167;
% elseif p == 3 && p == 4 && p == 5
%   Rsfh = 0.0095;
%   Rsfc = 0.0071;
% else
%   Rsfh = 0.001;
%   Rsfc = 0.0026;
% end

%Module Dimensions
%Developing Flow parameters
C = 1.8;
m = 2/3;

ThTE= 0;
Tho = 0; TcTE = 0; Tco=0; Thi=0; Tci=0;
%%% Temperature parameters %%%
Tci_act = Tinlet; %T_preheat = 42.3+273;

%%% Define Initial Gueses
Thi(1) = Thinlet;  Tco(1) = 314; Tci(z) = 313.5;

%Calculate Property Values for each zone
Cph = 1080;       % Air Heat Capacity (kg/kJ-K) based off
of Tph ~50 Celcius
% rhoair = 1.186;       % Air Density (kg/m^3) Standard
Operating Conditions
% mdoth = rhoair*volflowair;       % Mass Flow rate (kg/sec)

%%% % Section 3 - System Model Calculations %
%%% %

while abs(Tci(z) - Tci_act) >= .001
    g = 1;
    for k = 1:z
        UAh_zone = UAh_total/z;       %Make UAh = UAh_zone when
        using more realistic system
            if z == 1
                UAh = UAh_zone;
            elseif z == 2
                if k == 1
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m))/((Lzone+C*Dh^m*Lzone^(1-m)/(2*m)));
                else
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m)*(2^(1-m)-1))/(Lzone+C*Dh^m*Lzone^(1-m)/(2*m));
                end
            elseif z == 3
                if k == 1
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m))/(Lzone+C*Dh^m*Lzone^(1-m)/(3*m));
                elseif k == 2
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m)*(2^(1-m)-1))/(Lzone+C*Dh^m*Lzone^(1-m)/(3*m));
                else
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m)*(3^(1-m)-2^(1-m)))/(Lzone+C*Dh^m*Lzone^(1-m)/(3*m));
                end
            else
                if k == 1
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m))/(Lzone+C*Dh^m*Lzone^(1-m)/(4*m));
                elseif k == 2
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m))/(Lzone+C*Dh^m*Lzone^(1-m)/(4*m));
                else
                    UAh=UAh_zone*(Lzone+C*Dh^m*Lzone^(1-m))/(Lzone+C*Dh^m*Lzone^(1-m)/(4*m));
                end
            end
        end
    end
end
UAh = UA\_h\_zone \times (L\_zone + C \times D_h^m \times L\_zone^\{(1-m) \times (2^{(1-m)} - 1)} / (L\_zone + C \times D_h^m \times L\_zone^\{(1-m) / (4^m)\});

\textbf{elseif} k == 3
UAh = UA\_h\_zone \times (L\_zone + C \times D_h^m \times L\_zone^\{(1-m) \times (3^{(1-m)} - 2^{(1-m)}) / (L\_zone + C \times D_h^m \times L\_zone^\{(1-m) / (4^m)\});

\textbf{else}
UAh = UA\_h\_zone \times (L\_zone + C \times D_h^m \times L\_zone^\{(1-m) \times (4^{(1-m)} - 3^{(1-m)}) / (L\_zone + C \times D_h^m \times L\_zone^\{(1-m) / (4^m)\});
\textbf{end}

% UA values for individual zones
UA\_c = UA\_c\_total / 3;
% Total Cold Side heat Transfer Coefficient in one zone based on experimentation

% Insulation Thermal Resistance
A\_ins = 2 \times (W\_duct \times L\_zone) - (p \times W\_mod \times L\_mod);
R\_ins = t\_ins / (k\_ins \times A\_ins);
% Insulation Resistance (K/W)

\textbf{if} g == 1
\textbf{end}
% Zone Initial Guesses
Th\_TE(k) = Th\_i(k) - 10; Tho(k) = Th\_i(k) - 5;
Tc\_TE(k) = Tc\_o(k) - 10; Tci(k) = Tc\_o(k) - 5;

\textbf{if} module == 1
\textbf{else}
Seebeck = 9.54e-6 \times Tav + 1.28e-2;
K\_TE = 1 / (4e-4 \times Tav + 7.08e-1);
R\_int = p \times (4.6e-4 \times Tav + 9.84e-2);
\textbf{end}
q\_insulation(k) = (Th\_TE(k) - Tc\_TE(k)) / R\_ins;
\textbf{if} k < z
Tc\_o(k+1) = Tci(k);
Th_i(k+1) = Tho(k);
\textbf{end}
k = k+1;
```
end
Tco_old = Tco(1);
Tco(1) = Tco_old + (Tci_act - Tci(z));
g = g+1;
end

% Using final temperature values to generate values of interest
qh = mdoth*Cph*(Thi(1)-Tho(z));
qc = mdotc*Cpc*(Tco(1)-Tci(z));
q_diff = qh - qc;

Current = p*Seebeck.*(ThTE - TcTE)./(2*Rint);
Power_zones = Current.^2.*Rint;
Total_power = sum(Power_zones);
%Ri_one*p

Th_outlet = Tho(z) - 273;
Tc_outlet = Tco(1) - 273;
ThTE_out = ThTE - 273;
TcTE_out = TcTE - 273;

% Seebeck(k) = -4.38e-5*Tav + 5e-2 + (0)*10^-3;
% K_TE(k) = 1/(2.84e-4*Tav + 1.54);
% Rint(k) = p*(6.38e-3*Tav +2-0 );

% Seebeck(k) = 9.54e-6*Tav+1.28e-2;
% K_TE(k) = 1/(4e-4*Tav+7.08e-1);
% Rint(k) = p*(4.6e-4*Tav+9.84e-2);

% Performance calculator
% Values recieved from other programs
% UA_cost, # module (z*p), type of module, Total_power, Wpump
function performance_calculator
    global UA_cost z p Total_power Wpump Wduct Lduct CostperWatt module
    mod_cost elec_cost pump_cost cold_cost Total_cost

    % Calculate the total module cost
    if module == 1
        mod1_cost = 40;
    elseif module ==2
        mod1_cost = 139;
    else
        mod1_cost = 75;
    end

    mod_cost = z*p*mod1_cost; % total module cost ($)

    % Include electrical support cost
```
elec1_cost = 1; %electrical support cost for one module ($)
elec_cost = z*p*elec1_cost; %total electrical cost ($)

% Pump Power Costs
pump1_cost = 1; %pump cost for 1 watt of energy ($)
pump_cost = Wpump*pump1_cost; %total pump cost ($)

%Zone Installation costs
Zone1_cost = 25;
Zone_cost = (z-1)*Zone1_cost; % Additional cost for each extra zone added

% Cold side Heat Removal
cold1_cost = 1250; %cold cost pur unit serface area ($/m^2)
S_area = Wduct*Lduct*2; %duct ht surface area (m^2)
cold_cost = S_area*cold1_cost; %total cold cost ($)

Total_cost = mod_cost + elec_cost + cold_cost + UAcost + Zone_cost;

CostperWatt = (mod_cost + elec_cost + cold_cost + UAcost + Zone_cost)/(Total_power); %Wpump