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Computer aided design and simulation of an integrated photonic delay line system for phased array antenna and other microwave signal processing applications

Kevin Baldwin

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Computer Aided Design and Simulation of an Integrated Photonic Delay Line System for Phased Array Antenna and Other Microwave Signal Processing Applications

by

Kevin Baldwin

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of MASTER OF SCIENCE in Electrical Engineering

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ROCHESTER, NEW YORK
AUGUST, 1993
Over the past few years, phased array antennas and variable RF/Microwave delay lines have been the subject of much research. This thesis presents a photonic solution to the generation of multiple, compact delay lines. Variable time delays are generated by optically tapping points on an acousto-optic cell by the use of a deformable mirror device. Isolation of a particular time delay is accomplished by the conversion of a time delay point into a corresponding spatial frequency by the use of appropriate optics. The desired time delay is recovered by heterodyning a local oscillator with the desired spatial frequency, selected by a tiltable mirror device. Multiple delay lines are produced by the use of a binary optic device. The design and simulation of the integrated optical system was carried out using a real ray tracing program written by the author. Theoretical signal to noise calculations are also carried out.
ACKNOWLEDGMENTS

I would like to thank Dr. Tseng and Dr. DeLorenzo for sitting on my advising committee and especially Dr. Sumberg, without whose patience and guidance this endeavor could not have come to realization. I would also like to thank Captain Ed Toughlian of the Rome Laboratory Photonics Center for extending to me the opportunity to work on this truly novel project in times when original ideas are scarce, Dr. Henry Zmuda for his advice, RIT's Department of Electrical Engineering for the marvelous educational opportunity given to me, Dr. A. Matthew for allowing me to work in one of the best areas on the RIT campus, Mom, Pop, and the Grandparents.

I would also like to thank some special people I have come to know over the past few years. To fully express the roles they have played, I must draw an analogy to a sonnet:

"The house was old, with tangled wings
   out thrown of which no one could even half keep track.
And in a small room, somewhat near the back, was an odd window
   sealed with ancient stone.
There, in a dream plagued childhood, quite alone I used to go,
   when night reigned vague and black,
      parting the cobwebs with a curious lack of fear
   and with a wonder each time grown.
One later day I brought the masons there
   to find what view my dim forbears had shunned,
      but as they pierced the stone
   a rush of air burst from the alien void that yawned beyond.
They fled,
   but I peered through
      and found unrolled all the wild worlds of which my dreams had told."

- Howard Phillips Lovecraft

I would like to thank Khaleda Najeem for revealing to me corridors yet to be explored, Evelyn Monsay for helping remove the rubble from windows re-sealed by others, Lee Minich who stood by my side at the window and offered his council, and, most importantly, Mr. Eric Rogala who has also forsaken the masons and dared to stride through the void with me; treading upon vistas which few men dare. I am proud to call these people my friends.
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<th>Description</th>
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<tr>
<td>DMD</td>
<td>Deformable Mirror Device</td>
</tr>
<tr>
<td>TTD</td>
<td>True Time Delay</td>
</tr>
<tr>
<td>AO</td>
<td>Acousto-Optic</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
</tbody>
</table>
Table of Symbols

j \quad \text{Square root of -1}

Section II.

\( \theta \) \quad \text{Beam steering angle}
\( \lambda \) \quad \text{Wavelength of antenna driving signal}
\( \Delta s \) \quad \text{Difference in perpendicular distances}
\( k \) \quad \text{Wave number of antenna driving signal}
\( \Phi(\theta) \) \quad \text{Progressive phase delay}
\( \omega_o \) \quad \text{Angular frequency of antenna driving signal}
\( v \) \quad \text{Speed at which emitted signal propagates in air}
\( d \) \quad \text{Antenna element spacing}
\( t_d \) \quad \text{Time delay}

Section III.

\( q \) \quad \text{Acoustic wave number}
\( \omega_{RF}, \Omega \) \quad \text{Acoustic signal angular frequency}
\( S_0 \) \quad \text{Amplitude of acoustic wave}
\( v_s \) \quad \text{Speed of sound in transducer}
\( n \) \quad \text{Average index of refraction of transducer}
\( \Delta n_0 \) \quad \text{Maximum deviation of transducer index from average value, n}
\( \Phi \) \quad \text{Phase shift of acoustic wave}
\( \theta \) \quad \text{Angular difference between incident optical ray and acoustic wavefront}
$\Delta r$  Incremental change in reflectance
$\Delta x$  Incremental change in position on the transducer
$dr/dx$  Change in reflectance per change in position
$k$  Wave number of optical wave
$dn/dx$  Change in index of refraction per change in position
$dr/dn$  Change in reflectance per change in index of refraction
$\theta_1$  Angle between incident ray and surface normal
$\theta_2$  Angle between transmitted ray and surface normal
$\theta_3$  Angle between reflected ray and surface normal
$n_1$  Index of refraction seen by incident ray
$n_2$  Index of refraction seen by transmitted ray
$L$  Optical beam width projected on transducer
$\Lambda$  Wavelength of acoustic wave
$\theta_B$  Bragg angle
$f_{\text{signal}}$  Frequency of the acoustic signal
$n_{\text{Bragg}}$  Index of refraction of Bragg cell
$r$  Reflectance
$\omega_o, \omega_{\text{optical}}$  Angular frequency of optical wave
$t$  Time
$E_{\text{in}}$  Electric field amplitude incident on the transducer
$E_{\text{out}}$  Electric field amplitude reflected by acoustic wavefronts
$\Psi_S(t)$  Signal wave used in heterodyning
$\Psi_{\text{LO}}(t)$  Local oscillator wave used in heterodyning
$A_s$  Amplitude of signal wave
$A_{\text{LO}}$  Amplitude of Local Oscillator wave
\( I_{Det} \) Intensity seen by the detector
\( \Phi_s \) Phase shift of signal beam with respect to the reference beam
\( \Delta \theta \) Change in Bragg angle
\( \Delta f_{\text{signal}} \) Difference in RF signal in the Bragg cell from central driving frequency

Section IV.
\( x_0 \) Distance from \( x = 0 \) point along the center of the transducer
\( t \) True time delay
\( \theta_m \) Tilt angle of mirror
\( F \) Focal length
\( q \) Angle between desired spatial frequency and optical axis

Section V.
\( d \) Beam diameter
\( F_1 \) Focal length of \( L_1 \)
\( F_2 \) Focal length of \( L_2 \)
\( F_3 \) Focal length of \( L_3 \)
\( n_{\text{glass}}, n_{\text{BK7}} \) Index of refraction of the glass used in the system construction
\( n_{\text{air}} \) Index of refraction of air
\( \theta_w \) Edge angle of \( \text{Wedge1 and Wedge2} \)
\( f_{\text{min}} \) Minimum focal length needed for \( \text{CL1} \)
\( f_{\text{CL1}} \) Focal length of \( \text{CL1} \)
\( \text{ROC} \) Radius of curvature
\( T_{\text{gap1}} \) Thickness of \( \text{Gap1} \)
$\Delta \theta$  One half the angular spread of a ray fan
$f_{L1}$  Focal length of $L1$
$D_{in}$  Diameter of beam entering telescope
$D_{out}$  Diameter of beam leaving telescope
$F_1, F_2$  Focal length of lenses used in telescope
$d$  Length of glass between $CL2$ and $L1$
$\theta_{OA}$  Angle between optical axis before and after the system front end
$\theta_M$  Angle between reflecting surface of $SP1$ and input optical axis
$h_x$  A dimension of $Spacer2$
$d_{est}$  Estimated length of glass between $CL2$ and $L1$
$d_{act}$  Actual length of glass between $CL2$ and $L1$
$\Delta \%$  Percent difference
$bfl$  Back focal length
$y_{last}$  Height of a ray as it exits the optical system
$\theta'_{last}$  Angle exiting ray makes with the optical axis
$t_{BL1est}$  Estimated length of $BL1$
$ROC_{SP1est}$  Estimated radius of curvature of $SP1$ ($L3$)
$f_{SP1est}$  Estimated focal length of $SP1$
$t_{BL1}$  Actual length of $BL1$
$t_{side}$  Length of a side of a pentaprism
$m_x$  Slope of a line
$b_x$  $y$ intercept of a line
$x_o, y_o$  A point on a line
$t_{BL2}$  Length of glass between $SP1$ and $BS3$
$q_x$  $Q$ factor of a Gaussian beam
A, B, C, D    Elements of the paraxial ray matrix
z_0           Rayleigh range of a Gaussian beam
w_0           Waist radius of Gaussian beam
z             Distance from beam waist to current position on beam axis
\theta_o      Beam divergence angle

Section VI.

x_o, y_o      Center of curvature
R             Radius of curvature
\Phi_m        Angle between refracting planer surface and x axis
\Phi_r        Angle between incident ray and x axis
\Phi_s        Angle between incident ray and surface normal
\Phi_r'       Angle between exiting ray and x axis
\Phi_s'       Angle between surface normal and exiting ray
\Phi_m'       Angle between surface normal and x axis
m             Diffractive order number
\theta_q      Angle between mth diffractive order and optical axis
\theta_i      Angle between incident ray and optical axis
\Lambda       Period of diffraction grating
\chi          Angular difference between adjacent diffractive orders

Section VII.

R             Radius of curvature
n             Index of refraction
f             Focal length
Section VIII.

\( \bar{t}^2 \) Mean-squared value of a random variable

\( \sigma_i^2 \) Variance of random variable

\( \sigma_{i,\text{Shot}}^2 \) Noise power associated with shot noise

\( \sigma_{i,\text{Dark}}^2 \) Noise power associated with dark current

\( \sigma_{i,\text{Johnson}}^2 \) Noise power associated with thermal noise

\( <i_{\text{sig}}> \) Average value of signal current

\( \mathcal{R} \) Responsivity of a detector

\( q \) Fundamental charge of an electron

\( h \nu \) Energy stored in a photon

\( <P_s> \) Average optical power in signal beam

\( <P_L> \) Average optical power in reference beam

\( B \) Bandwidth of the detection system

\( R, R_L \) Total resistance in detection circuit

\( k \) Boltzmann's constant

\( T \) Ambient temperature (in Kelvin)

\( i_d \) Dark current

\( T_s \) fraction of total input power placed in signal beam

\( T_{AO} \) Transmission efficiency of Bragg cell

\( T_p \) Transmission efficiency of Polarizer

\( T_{\text{cube}} \) Transmission efficiency of beam splitter

\( G_{\text{pin}} \) Gain of photodetector

\( G_{\text{amp}} \) Gain of detection circuit

\( BOS \) Splitting ratio of Binary Optic

\( BO_{\text{eff}} \) Transmission efficiency of Binary BO
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(A_S)</td>
<td>Amplitude of signal beam</td>
</tr>
<tr>
<td>(f_0)</td>
<td>Optical carrier frequency</td>
</tr>
<tr>
<td>(f_s)</td>
<td>RF signal frequency</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Optical wavelength</td>
</tr>
<tr>
<td>(x, y, z)</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Mirror tilt angle</td>
</tr>
<tr>
<td>(v)</td>
<td>Velocity of sound in the transducer</td>
</tr>
<tr>
<td>(T)</td>
<td>Desired true time delay</td>
</tr>
<tr>
<td>(F)</td>
<td>Effective focal length of the system</td>
</tr>
<tr>
<td>(f_x)</td>
<td>Spatial frequency</td>
</tr>
<tr>
<td>(\Phi, \phi', \phi'')</td>
<td>Relative phase shifts</td>
</tr>
<tr>
<td>(E_S)</td>
<td>Electric field of signal beam</td>
</tr>
<tr>
<td>(E_{LO})</td>
<td>Electric field of Local Oscillator beam</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Intrinsic impedance of air</td>
</tr>
<tr>
<td>(I)</td>
<td>Intensity pattern</td>
</tr>
<tr>
<td>(P_{LO})</td>
<td>Power density of local oscillator beam</td>
</tr>
<tr>
<td>(P_S)</td>
<td>Power density of signal beam</td>
</tr>
<tr>
<td>(P_{DC})</td>
<td>DC component of time varying intensity pattern</td>
</tr>
<tr>
<td>(P_{SIG})</td>
<td>AC component of time varying intensity pattern</td>
</tr>
<tr>
<td>(A_{DET})</td>
<td>Area of the detector</td>
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I. Introduction

Phased arrays have been the subject of intensive research for several decades, with the primary focus of these efforts centered on microwave components and system. More recently, however, optically based systems have been proposed as possible alternatives to some of the more conventional elements.

Many true time delay (TTD) beam steering systems have been proposed, such as the schemes presented by Toughlian & Zmuda\textsuperscript{1,2} and Sumberg & Toughlian\textsuperscript{3,4}. The system addressed by this thesis combines an acousto-optic deflector with a segmented mirror device to obtain time delay by the following process.

An optical beam of diameter \( D \) passes through an acousto-optic deflector to which an RF frequency is applied. The laser beam that passes through the acousto-optic cell carries a finite time slice of the RF signal. By means of a shift in the optical frequency, that time slice is impressed upon a cross section of the laser beam. The signal carrying optical beam is then transmitted through an integrated (all glass) optical system to a photodetector, where it combines with a second beam at the unshifted laser frequency. The two beams produce a heterodyne signal. Using a mirror to steer one of the laser beams, it is possible to select various delay points of the original time slice and reproduce the instantaneous RF signal occurring at that point. Time delays up to \( D/v_s \) are possible, where \( v_s \) is the velocity of sound propagating in the acousto-optic cell.

\textsuperscript{1} H.Zumda and E. Toughlian, "Variable Photonic Delay Line for Phased Array Antennas and RF/Microwave Signal Processing", Final Technical Report, (Rome Laboratory, Griffiss Air Force Base, New York, June 1991)
The potential advantages of the segmented mirror photonic delay line over existing architectures lie in its ability to provide extremely rapid reconfiguration of the antenna pattern and an infinitely variable time delay. An important advantage lies in steering the beam using a TTD technique as opposed to employing a phase shifting technique. In some existing antenna systems, optical heterodyning schemes have been employed to provide an RF phase shift that is, unfortunately, independent of the RF frequency. Such a system suffers from a phenomena know as squint, which results in different frequency components of the RF carrier pointing in different directions. By utilizing a TTD beam steering technique, the effects of squint are eliminated.\(^5\)

This thesis addresses the concept and design of an integrated photonic delay line system, concentrating on the role of a personal computer in determining system specifications and performance evaluation. The integrated system is capable of providing 25 separate delay lines (although only 20 lines will utilized in the actual system) with a high packing density.

The availability of true time delay lines for RF signals may also be utilized in the realization of microwave signal processing architectures. By using the variable photonic delay lines in conjunction with a photonic amplitude weighting device, a microwave transversal filter can be realized. Such an amplitude weighting effect can be included in the optical system by the use of liquid crystal light valves.

---

II. Phase Array Antenna Fundamentals

Phased Array Antenna Operation

A phased-array antenna is an antenna system which steers the output beam of the antenna by adaptively controlling the time it takes for a common signal emitted from the source to reach a particular radiative element. A simplified block diagram of a one-dimensional antenna array is shown below:

Figure 1. A block diagram of a phased array antenna system. The output direction of the beam, specified by the angle $\theta$ with respect to the array normal, is controlled by varying the time required for a signal emitted from a common source to reach a particular element in the antenna. Thus, the amount of time delay needed at a particular element is a function of both the spatial location of a particular element and the desired steering angle.

The amount of time delay depends on the desired signal direction, specified by the angle $\theta$ in Figure 1, and the spatial location of the radiative element, designated by $n_i$. Thus, in order to achieve a desired steering direction, the signal must be delayed an amount of time that is specific to each element.
The mechanism by which the beam is steered can be seen by first considering the output field resulting from the emissions of an array of synchronized point sources. Each point source represents an idealized radiating element in the antenna array.

![Diagram showing phase fronts and output field](image)

**Figure 2.** The resulting output field from an array of synchronized point sources. Spherical wavefronts of equal phase are shown emerging from each point source in the array. It is evident that the superposition of the spherical wavefronts from each source contribute to the generation of a planar wavefront propagating normal to the array.

By considering the linear array of radiating antenna elements as an array of point sources, the output field of the antenna array may be explored. As in Figure 2, the resulting output field from the antenna generated by a linear array of synchronized oscillators will appear to be a planer wave in the far field.

This statement has been supported using a MathCad simulation to determine the behavior of such a field. In the MathCad simulation, the output field has been generated
by summing the spherical fields generated by 11 point sources, symmetrically distributed about the 0 point of the x axis and with a separation distance between adjacent elements of one half a wavelength. The figure below represents a contour plot of the resulting output field.

Figure 3. Contour plot of the field resulting from the superposition of 11 point sources, located along the lower axis of the plot and centered about the "zero" point. The separation used between adjacent elements was one-half the wavelength of the carrier wave. Note that the resulting field appears to be quite planar, propagating normal array orientation.

From MathCad result shown in Figure 3, it is apparent that the resulting output field distribution displays planer wavefronts characteristics in the "forward looking" region, directly in front of the array.

**Derivation of Required Time Delay for Achieving Steering Angle**

As has been already stated, in order to steer the resulting beam a time delay in the signal between adjacent elements must be introduced. Since, in this example, the signal emitted from the common source is a sinusoid of a single frequency, time delay can also
be realized as a phase delay. Since total phase delay is being considered it can be directly used in the quantification of a time delay.

The expression for the required total phase delay in terms of the steer angle can be derived by considering Figure 4:

![Figure 4](image)

Figure 4. The geometry of the situation leading to the derivation of the amount of required phase delay between adjacent antenna elements in order to achieve desired steering angle of the antenna output beam. In the figure, Δs represents the difference in perpendicular distances from the wavefront to adjacent point sources and θ is steering angle of the beam with respect to the array normal.

By taking a "snapshot" in time of the wavefronts that contribute to a planar wavefront, an expression can be derived for the progressive phase delay in terms of the steering angle. From Figure 4:

\[
\sin \theta = \frac{\Delta s}{\text{Element Spacing}} = \frac{\Delta s}{\frac{\lambda}{2}}
\]

(1)

\[
\Delta s = \frac{\lambda \sin \theta}{2}
\]

(2)
The progressive phase delay, $\Phi(\theta)$, will be given as:

$$\Phi(\theta) = k \Delta s = \frac{2\pi \lambda \sin \theta}{\lambda} = \pi \sin \theta$$

where $k$ is the wave number, $\frac{2\pi}{\lambda}$.

From this derivation it can be seen that in order to steer the beam at angle $\theta$ with respect to the array normal, the signal radiated by a particular element should possess a phase delay of $\pm \Phi(\theta)$ compared to the elements adjacent to it. The $\pm$ factor reflects the not knowing if the adjacent element is on the "right" or "left" of the element in question. In order to illustrate the point, several MathCad beam steering simulations have been carried out for various steer angles. In all cases, the antenna is a linear array of 11 elements arranged along the x axis, centered about the origin.

![Phase Fronts of Output Field](image)

Figure 5. A contour plot of the output field distribution for a steer angle of 15° with respect to the normal to the linear antenna array.
Figure 6. A contour plot of the output field distribution for a steer angle of 45° with respect to the normal to the linear antenna array.

Noting that time and phase are linearly related at a signal frequency, the required time delay, $t_j$, maybe expressed in terms of the phase delay:

$$\phi_0 = \frac{2\pi f_0 t_j}{c}$$

Thus, the proper directional orientation of the output beam of a phased array antenna maybe achieved by generating the proper time delays for each element, based upon the desired steering angle.

Phase Shifting versus True Time Delay

Beam Steering

As stated in the Introduction, some phased array antenna schemes make use of a fixed phase delay system to control the direction of the output beam. However, for broadband signals such a directional controller will result in the pointing error known as squint. Squint results from different frequency components of the transmitted signal pointing in different directions. The origin of this pointing error can be seen by...
generalizing the derivation of the beam steering angle, \( \theta \), presented earlier to multiple wavelengths. Recalling from equation 1 that:

\[
\sin \theta = \frac{\Delta s}{\text{Element Spacing}}
\]  

(5)

For the transmission of a broadband signal, it is no longer possible to specify the spacing of the radiating elements in terms of the transmission wavelength, as was done in the earlier derivation. Therefore, let the spacing between adjacent elements be fixed at a value of \( d \). The phase delay required to steer a frequency component of the output beam will be given by:

\[
\Phi = \frac{2\pi}{\lambda} d \sin \theta
\]  

(6)

For a fixed phase system, \( \Phi \) will remain a constant value for all frequencies (wavelengths). Since all values are constrained in equation except for the beam pointing angle, the beam pointing angle will become a function of wavelength, resulting in the specification of a different beam direction for each frequency in the driving signal. This phenomena is known as squint. This situation is shown in Figure 7.

![Diagram of phased array antenna](image)

**Figure 7.** The effect of using a phase shifting scheme to generate the output field of a phased array antenna. Different frequency components will result in different beam steering angles. This pointing error is termed squint.
On the other hand, it can be seen that by using a TTD scheme to steer the output beam direction, that the effects of squint can be eliminated. This can be seen by deriving the required time delay for a given steering angle from equation 6:

$$t_d = \frac{\Phi}{\omega} = \frac{2\pi}{\lambda \omega} d \sin \theta = \frac{d}{v} \sin \theta$$

(7)

where $d$ is the separation distance between elements in the array and $v$ is the velocity of the output beam radiation. From this relationship it is clear that the steering angle is now independent of the signal frequency. Thus the effects of squint are removed from the antenna output.
III. Special Components Used in the Integrated System Design

Operation of the Acousto-Optic Cell in Bragg Mode

The Bragg, or Acousto-Optic, cell is the heart of the integrated optical system, providing a bridge between electrical and optical signals. By utilizing an Acousto-Optic cell, in the Bragg mode, it is possible to deflect (steer) an incident beam proportionally to the acoustic frequency propagating through the cell as well as place the RF frequencies on an optical carrier.

The mechanism by which the Bragg cell accomplishes these tasks can be derived by adopting a geometrical optics point of view of the cell's operation. In this simplistic method of analysis, the acousto-optic cell may be looked at as a piece of material, the transducer, into which an acoustic wave is introduced by the use of a piezo-electric material, as shown in the figure below:

![Diagram of an acoustic wave propagating in the transducer of the acousto-optic cell.](image)

Figure 8. An acoustic wave propagating in the transducer of the acousto-optic cell.
Derivation of the Expected Acoutso-Optic Cell Output: Amplitude Modulation

The acoustic wave traveling in the transducer is given by the expression

\[ s(x,t) = S_0 \cos(qx - \Omega t) \tag{8} \]

where \( \Omega = 2\pi f \) is the angular frequency of the signal and \( q = \frac{2\pi}{\Lambda} \) is the wave number. In this case, the acoustic wave is propagating down (+x direction) the cell. The acoustic wave traveling in the transducer establishes localized changes in the index of refraction of the transducer proportional to amplitude of the acoustic wave. Under these conditions, the index of refraction in the transducer may be written as a function of spatial coordinates in the transducer and time

\[ n(x,t) = n - \Delta n_0 \cos(qx - \Omega t) \tag{9} \]

In order to simplify the determination of the interaction of light with the time-varying sinusoidal index grating, it is observed that the optical frequency is much greater than the acoustic frequency. Under this condition, the periodic index structure will appear to be almost stationary with respect to the light during the interaction. Thus, the index of refraction of the transducer may be described as:

\[ n(x,t) = n - \Delta n_0 \cos(qx - \Phi) \tag{10} \]

where \( \Phi \) is the phase shift in the sinusoidal structure caused by taking a "snap shot" of the structure at a particular point in time.

Next, consider illuminating the transducer with a plane wave that makes an angle of \( \theta \) with respect to the acoustic wavefronts, as shown in Figure 9.

---

From a prior knowledge, the beam reflected off of the periodic structure is the beam desired for analysis. It is assumed that the incident wave is partially reflected by each period in the transducer, due to the changing refractive index in the transducer, and that the reflectance does not significantly reduce the amplitude of the transmitted light.

If $\Delta r = \frac{dr}{dx} \Delta x$ is the incremental reflectance at a point $x$ on the transducer, then the total reflectance over a distance $L$ is given by integrating the infinitesimal reflectance over the illuminated portion of the transducer:

$$ r = \int_{L/2}^{L/2} e^{j2k \sin \theta} \frac{dr}{dx} dx $$

where a phase factor, $\exp[j2k \sin \theta]$, has been introduced in order to account for the difference in phase across the beam with respect to the $x = 0$ point. An expression for $\frac{dr}{dx}$ may be found by considering $\frac{dr}{dx} = \frac{dr}{dn} \frac{dn}{dx}$.

The expression for $\frac{dr}{dn}$ may be obtained by examining the expressions for the reflectance in the transducer when the light incident is either in the TE or TM mode.

---

**TM Case**

In general, the expression for the Fresnel reflection coefficient, describing the portion of the incident electric field reflected off a boundary interface, is given by:

$$r = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$  \hspace{1cm} (12)

where the definitions of the quantities are shown in the figure below:

![Figure 10. The angular definitions required in the analysis of the Fresnel coefficients for the TE and TM cases.](image)

When applying the formula to determine the reflectance from a period index structure, the following values must be adopted: $n_1 = n + \Delta n$, $n_2 = n$, $\theta_1 = \pi/2$ \hspace{0.5cm} $\theta$, and Snell's Law is required for the determination of $\theta_2$. The origin of these values is obvious by reviewing the situation. Upon substituting into the TM Fresnel reflection coefficient expression, equation 13 results.
An expression for the incremental change in $r$ in terms of a small change in $n$ may be found by writing the Fresnel coefficient as a Taylor series expansion, about 0, in terms of $\Delta n$ and extracting terms on the order of $\Delta n$. Thus, an approximation for $\Delta r$ is given by:

$$\Delta r = \left. \frac{dr}{d\Delta n} \right|_{\Delta n=0} \Delta n$$  \hspace{1cm} (14)

Upon substituting:

$$\Delta r = \left. \frac{dr}{d\Delta n} \right|_{\Delta n=0} \Delta n$$  \hspace{1cm} (15)

Simplifying the expression utilizing trigonometric identities:

$$\Delta r = \left. \frac{dr}{d\Delta n} \right|_{\Delta n=0} \Delta n$$  \hspace{1cm} (16)

Further simplification:

$$\Delta r = \frac{-1}{\sin(\theta)} \cdot \Delta n$$  \hspace{1cm} (17)

Resulting in a final expression for the infinitesimal reflection in the TM case:

$$\Delta r = \frac{-1}{2n \sin^2 \theta} \Delta n$$  \hspace{1cm} (18)

**TE Case**

In the case of the TE case, the Fresnel reflection coefficient is given by:
\[ r = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \]  \hspace{1cm} (19)

Upon substituting in the values mentioned previously in the evaluation of the TM case:

\[
r_x := \frac{(n + \Delta n) \sin(\theta) - n \sqrt{1 - \frac{n^2}{(n + \Delta n)^2} \cos^2(\theta)}}{(n + \Delta n) \sin(\theta) + n \sqrt{1 - \frac{n^2}{(n + \Delta n)^2} \cos^2(\theta)}} \hspace{1cm} (20)
\]

Extracting terms of the first order of \( \Delta n \) from the Taylor series expansion of \( r_x \) about the origin:

\[
\Delta r = \frac{dr_x}{d\Delta n} \bigg|_{\Delta n=0} \Delta n
\]

This leads to the expression:

\[
\Delta r := \left[ \left( \frac{\sin(\theta) - \frac{1}{\sqrt{1 - \cos^2(\theta)}} \cos(\theta) \sqrt{1 - \cos^2(\theta)}}{\frac{1}{\sqrt{1 - \cos^2(\theta)}} \sin(\theta) + \sqrt{1 - \cos^2(\theta)}} \right) \sin(\theta) + \frac{1}{\sqrt{1 - \cos^2(\theta)}} \cos(\theta) \right] \Delta n
\]

The expression simplifies to:

\[
\Delta r := \frac{1}{2} \left( \frac{\cos(\theta)^2 - \sin(\theta)^2}{\sin(\theta)} \right) \Delta n
\]

The expression yields a final result for the infinitesimal reflection in the TE case:

\[
\Delta r = \frac{-\cos 2\theta}{2n\sin^2 \theta} \Delta n
\]

By utilizing the small angle approximation, \( \cos 2\theta \approx 1 \), it can be seen that the resulting approximation formulas from the TE and TM reflection cases possess the same form:
\[ \Delta r \approx \frac{-1}{2n\sin^2 \theta} \Delta n \]  

Returning to the expression required for the integration, the expression for \( \frac{dr}{dx} \) must be determined. By using the expression for the infinitesimal reflectance as a function of infinitesimal index changes, an expression for \( \frac{dr}{dx} \) may be derived:

\[ \frac{dr}{dx} = \frac{dr}{dn} \frac{dn}{dx} = \frac{-1}{2n\sin^2 \theta} \frac{d}{dx} (n - \Delta n_0 \cos(qx - \Phi)) \]  

(26)

\[ \frac{dr}{dx} = \frac{-1}{2n\sin^2 \theta} q\Delta n_0 \sin(qx - \Phi) \]  

(27)

By applying the identity that \( \sin(x) = \frac{e^{ix} - e^{-ix}}{2j} \), the integral for the total reflectance from the Bragg cell becomes:

\[ r = \frac{1}{2} jr' e^{j\Phi} \int_{-L/2}^{L/2} e^{j(2k\sin\theta-q)x} \, dx - \frac{1}{2} jr' e^{-j\Phi} \int_{-L/2}^{L/2} e^{j(2k\sin\theta+q)x} \, dx \]  

(28)

Solving for the first integral:

\[ r_a = \frac{1}{2} jr' e^{j\Phi} \left[ e^{j(2k\sin\theta-q)L/2} - e^{-j(2k\sin\theta-q)L/2} \right] = \frac{1}{2} jr' e^{j\Phi} \left[ \frac{\sin((2k\sin\theta-q)L)}{(2k\sin\theta-q)L} \right] \]  

(29)

\[ r_a = \frac{1}{2} jr' e^{j\Phi} \left[ \frac{\sin((2k\sin\theta-q)L)}{(2k\sin\theta-q)L} \right] \]  

(30)

\[ r_a = \frac{1}{2} \pi(2k\sin\theta-q) \frac{L}{2\pi} \]  

(31)
\[ r_a = j r' e^{j\Phi} \frac{L}{2} \text{sinc}\left[\left(2k \sin \theta - q\right)\frac{L}{2\pi}\right] \]  

(32)

where \( \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \).

Therefore, evaluation of the entire reflectance integral leads to:

\[ r = j r' e^{j\Phi} \frac{L}{2} \text{sinc}\left[\left(2k \sin \theta - q\right)\frac{L}{2\pi}\right] - j r' e^{-j\Phi} \frac{L}{2} \text{sinc}\left[\left(2k \sin \theta + q\right)\frac{L}{2\pi}\right] \]  

(33)

Significance of the Bragg Angle

Since the maximum of the sinc function occurs when the argument of the sinc function is zero, the resulting expression for the total reflectance will be at a maximum when either \( 2k \sin \theta = q \) or \( 2k \sin \theta = -q \). Under the condition when \( 2k \sin \theta = q \), the first term of the expression dominates. This represents the "up shifted" case and corresponds to the operating mode of the Bragg cell used in the optical system operation. An explicit statement of the Bragg condition is given below:

\[ \sin \theta_B = \frac{\lambda}{2\Lambda} \]  

(34)

\[ \sin \theta_B = \frac{\lambda_o}{n_{\text{Bragg}}2\Lambda} = \frac{\lambda_o f_{\text{signal}}}{2n_{\text{Bragg}}v_S} \]  

(35)

Derivation of the Expected Acousto-Optic Cell Output: Frequency Modulation

Another important result that can be seen from the reflectance term occurs when the temporal dependence of the reflectance is reintroduced by letting \( \Phi \rightarrow \Omega t \):

\[ r = \frac{1}{2} j r' L \text{sinc}\left[\left(q - 2k \sin \theta\right)\frac{L}{2\pi}\right] e^{\jmath \Omega t} \]  

(36)

Consider a plane wave incident on the Bragg cell at the Bragg angle. The output electric field is given by the expression:
\[ E_{\text{out}} = r E_{\text{in}} \]  

Suppressing the spatial dependence of the input electric field, the output electric field displays a joint time dependence:

\[ E_{\text{out}} \propto r e^{jo_{\text{optical}}} \]  

Substituting in for the reflectance expression:

\[ E_{\text{out}} \propto \frac{1}{2} jr' L \text{sinc}[(q - 2k \sin \theta) \frac{L}{2\pi}] e^{jo} e^{jo_{\text{optical}}} \]  

Again, suppressing the non-temporal related terms found in the expression:

\[ E_{\text{out}} \propto e^{j(\Omega + \omega_{\text{optical}}) t} \]  

From this result, it is apparent that the output field will be frequency shifted from the input field by the value of the acoustic frequency found in the transducer of the Bragg cell. Thus, if a plane wave is introduced into the Bragg cell at the Bragg angle (for maximum reflected output from the cell,) the acoustic signal traveling in the transducer will be placed upon an optical carrier. The resulting frequency of the output field is given by:

\[ \omega_{\text{TOT}} = \Omega + \omega_{\text{optical}} \]  

**Recovery of Acoustic Signal: Optical Heterodyning**

By the process of optical heterodyne detection, an RF signal that has been placed on an optical carrier is recovered by "beating down" the optical carrier with a non-frequency shifted reference beam. The process of signal recovery can be examined by considering the interference of the two beams at an optical square-law detector.

The signal beam can be characterized as a plane wave of amplitude \( A_s \), optical frequency \( \omega_o \), and an RF frequency \( \omega_{RF} \). Therefore, the signal wave will possess the form:

\[ \Psi_s(t) = A_s \exp[j(\omega_o + \omega_{RF})t + j\Phi_s] \]
\( \Phi_S \) represents a relative phase shift between the signal and the local oscillator beam.

The reference, or local oscillator, wave will also be a plane wave. However it will only exhibit the frequency of the optical carrier:

\[
\Psi_{LO}(t) = A_{LO} \exp[j\omega_s t]
\]

(43)

Since both beams are coherent, the resulting intensity at the detector will be the square of the magnitude of the sum of the waveforms:

\[
I_{Det}(t) = |\Psi_s(t) + \Psi_{LO}(t)|^2
\]

(44)

which may be written as:

\[
I_{Det} = \Psi_s \Psi_s^* + \Psi_{LO} \Psi_{LO}^* + \Psi_s \Psi_{LO}^* + \Psi_{LO}^* \Psi_s
\]

(45)

By employing the well known trigonometric identity:

\[
\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}
\]

(46)

the form of the intensity at the detector is found to be:

\[
I_{Det} = |A_s|^2 + |A_{LO}|^2 + 2\sqrt{|A_s|^2 |A_{LO}|^2} \cos((\omega_o + \omega_{RF}) t - \omega_s t + \Phi_s)
\]

(47)

Upon simplifying, the resulting form is shown below,

\[
I_{Det}(t) = I_s + I_{LO} + 2\sqrt{I_s I_{LO}} \cos(\omega_{RF} t + \Phi_s)
\]

(48)

which indicates that the intensity pattern incident on the detector will vary in time at a rate equal to the RF frequency placed on the signal beam, with a phase delay equal to the phase delay present in the signal beam. Since the electrical signal will be proportional to the intensity incident on the square-law detector, the RF signal will be recovered.

A typical optical setup used in the placement and extraction of an RF signal on an optical beam is shown in Figure 11.
Various means may be used to place the RF signal on an optical carrier. However, in the case of the optical system only a Bragg will be considered. Also, the presence of an optical phase shifter, in the case of the integrated system, is rather complex and will be discussed later.

**The Connection Between Heterodyne Systems and Phased Array Antennas**

From the classic heterodyne optical system, one can see a possible application of technique to phased array antennas. If each radiating element of the antenna were to be attached to an separate heterodyning system, the RF signal to be broadcast could be placed upon the signal beam (utilizing the optical frequency shifter) and then phase shifted the amount needed in order to steer the beam in a particular direction. Some of the problems that exist with such a configuration is the amount of space required for configuration and the high price of the components required. The integrated system is a novel way of realizing multiple heterodyning systems, possessing a high packing density of the delay lines that is not present in the multiple system case.

**Dependence of Output Angle on Acoustic Frequency**

Upon varying the frequency of the acoustic wave in the cell, the deflection angle of the output beam will vary proportionally. This can be seen by applying the small angle
approximation to the Bragg equation and differentiating the output angle, \( \theta \), with respect
to the signal frequency, \( f_{\text{sig}} \).

\[
\Delta \theta = \frac{\lambda \Delta f_{\text{signal}}}{2n_{\text{Bragg}} v_s}
\]  

(49)

**Operation of the Binary Optic Device**

In general, a binary optic is an optical component on which a surface relief pattern
has been etched that will result in phase shifts of the incident beam by values of 0 or \( -\pi \).
The surface relief pattern is governed by the input and desired output field pair. The
binary optic device used in the integrated system has been designed to break an incident
plane wave into 25 equal intensity diverging plane waves, each beam will represent an
individual delay line. In order to confirm the operation of the binary optic, the phase
function of the surface was entered into a MATLAB program and the magnitude squared
of the Fourier transform of the transmission function was found. The is analogous
examining the far field intensity pattern that results when a plane wave is incident on the
Binary Optic. A mesh and contour plot of the resulting intensity pattern are shown
below.

![2-D FFT]^2 of binary optic transmission function

Figure 12. The resulting intensity profile of the Binary optic upon being illuminated with a uniform plane
wave.
Introduction to the Deformable Mirror Device (DMD)

The deformable mirror device is a two-dimensional array of electronically controlled mirrors. Each mirror may be addressed, controlling the X and Y tilt angle of the mirrored surface. Using state-of-the-art technology, it is possible to generate device possess over one million mirror segments in an area of one square centimeter, with an active (reflecting) area greater than 70% of the surface\textsuperscript{10}.

IV. Single Photonic Delay Line Concept

Generation of a True Time Delay (TTD)

The time delays required for the operation of the phased array antenna are obtained by actively selecting, or tapping, points on a Bragg cell illuminated by a collimated beam. The source of the time delays can be seen with the aid of the diagram shown below:

![Diagram showing Bragg cell and time delays](image)

Figure 14. The Bragg cell, indicating the region from which time delays can be selected and a ray fan associated with a particular time delay. The illuminated portion of the Bragg cell defines the region from which time delays can be tapped.

In this case, the Bragg cell is illuminated by a beam of diameter \(d\) and is driven by the desired output signal of the antenna. The availability of signal delays results from the finite time it takes the RF signal to propagate through the AO Cell across the beam diameter. The total theoretical time delay available is given the length of the cell...
illuminated by the source divided by the velocity of sound traveling in the transducer medium or:

\[ t_{\text{dot}} = \frac{D}{v_s \cos \theta_B} = \frac{D}{v_s \cos \theta_B} \]  \hspace{1cm} (50)

In order to maintain some symmetry in the optical system, the "zero" time delay point was chosen to be the center of the incident illumination. Therefore, the possible time delays fall in the range of:

\[ -\frac{D}{2v_s \cos \theta_B} \leq t_d \leq \frac{D}{2v_s \cos \theta_B} \]  \hspace{1cm} (51)

A particular time delay value of the signal may be chosen by "tapping" the output face of the Bragg cell. Tapping the cell may be interpreted as selecting out an infinitesimally small area (equivalent to a delta function) from the illuminated portion of the Bragg cell. By tapping the cell at a particular point, the resulting light leaving the cell from that point will serve as a carrier, carrying the signal frequency as well as the selected time delay. In Figure 14, the Bragg cell is being tapped at a point a distance of \( x_0 \) from the center \( (x = 0) \) of the incident beam. Thus, the signal placed on Bragg cell tap at \( x = x_0 \) will be advanced from the signal placed on a beam tapped at \( x = 0 \) by a value of

\[ t_d = +\frac{x_0}{v_s} \text{ seconds} \]  \hspace{1cm} (52)

As the signal propagating through the Bragg cell varies, the output beam from the AO cell will be deflected accordingly. The highest and lowest frequencies serve to define a ray fan that will be associated with a particular time delay for all RF signal frequencies (as shown in the above figure.)
General Optical System for Tapping the Photonic Delay Line

From the previous section, it is known that each particular time delay for all signals can be looked at as tapping particular points of the Bragg cell. However, the problem of selecting a particular ray fan (time delay) for the purposes of beating down to recover the time delayed signal must be examined.

A novel optical architecture has been proposed by Zmuda and Toughlian in order to select particular ray fans (time delays) in order to recover the delayed signal\textsuperscript{11}. A simplified version (a driver for only one element of the antenna) of the heterodyning system is shown below:

![Architecture of a single photonic delay](image)

Figure 15. Architecture of a single photonic delay, as proposed by Zmuda and Toughlian.

In the above system, the ray fans are transformed into planer waves by the use of a positive lens that is placed such that its focal length is located at the center of the Bragg cell. Thus, the positive lens serves to collimate the fans. Once the fans are collimated, each time delay is associated with a particular plane wave of a unique spatial frequency.

In order to select a particular plane wave (time delay,) a tiltable mirror is introduced into the system. By controlling the tilt angle of the mirror different points on the Bragg cell may be tapped as shown above in Figure 15. Thus, by adjusting the angle of the mirror, a particular location on the AO cell is selected, corresponding to the selection of a particular time delay for all signals. The selected plane wave is then interfered with the local oscillator of the heterodyning system. In this system, the time delay becomes a function of the tilt angle of the mirror and the focal length of the positive lens used. The appropriate mirror tilt angle associated with selecting a particular time delay is given by the formula\(^{12}\), assuming that the tilt angle of the Bragg cell is negligible:

\[
\theta_m = \frac{1}{2} \tan^{-1} \left( \frac{\tau_v S}{F} \right) \tag{53}
\]

This formula is derived from the geometry of the system and considering the reversibility of light. Consider tracing a plane wave back through the system, traveling normal to the face of the detector. Such a plane wave is associated with the desired time delay. Upon tracing the plane wave back through the signal portion in the system, it will be brought to a focus at some point on the Bragg cell, as shown in Figure 16. The point selected on the Bragg cell corresponds to the desired time delay extraction point.

---

In order to cause the signal beam to become parallel with the optical axis, the mirror must be tilted at an angle of 2θ. This is due to the law of reflectivity, \( \theta_r = \theta'_r \).

From the diagram, it is obvious that the following relationship exists:

\[
\tan \theta = \frac{x_o}{F} \quad \text{(54)}
\]

If the center of the cell is assumed to be the zero delay reference point, then the time delay associated with \( x_0 \) is simply the value of \( x_0 \) divided by the velocity of the signal propagating in the transducer, or:

\[
\tan \theta = \frac{v_s t_d}{F} \quad \text{(55)}
\]

where \( t_d \) is the true time delay of the signal. Recalling that the mirror tilt angle should be twice the value of \( \theta \), or \( 2\theta = \theta_m \), yields the result shown previously for mirror tilt angle required to achieve a certain delay:

\[
\theta_m = \frac{1}{2} \tan^{-1} \left( \frac{v_s t_d}{F} \right) \quad \text{(56)}
\]
V. Design of the Integrated Photonic Delay Line System

The Significance of Ray Fans and Delay Lines for System Design

As stated in the previous section, ray fans are generated in the system by considering a fixed point on the Bragg cell and varying the driving frequency of the cell between the highest and lowest RF values in the signal. The highest frequency will result in the largest positive deflection of the beam from the Bragg angle, thus it will define the upper limit of the ray fan as it exits the Bragg cell. The lowest frequency will result in the greatest negative deflection of the beam from the Bragg angle, thus defining the low limit of the ray fan as it exits the Bragg cell. Since a ray fan is associated with a fixed point on the Bragg cell, it is associated with a specific TTD for all RF frequencies.

A photonic delay line, or simply delay line for short, is the superposition of the all possible ray fans along the illuminated region of the Bragg cell. Therefore, a delay line is composed of all possible delay values. A particular delay value is selected by probing the delay line to select out a particular ray fan associated with a particular TTD. The purpose of the optics of the integrated system, and the simple system shown previously, is to create a situation in which the individual rays fans are caused to converge in a manner in which the delay line may be manipulated to extract a particular TTD value. In both the integrated and the simple photonic delay line system all ray fans converge upon a deformable mirror device, which can select a particular ray fan (TTD) for heterodyne detection (conversion of the optical signal to the desired delayed electrical signal) from the delay line by tilting its reflecting surface.
Underlying Values Used for Designing

Before describing how the system is designed, it is important to present some of the factors that will be considered throughout the design. The importance of these values will become apparent in the system design.

The laser source used in the system is an IR source, operating at 1319nm, which has an output beam diameter of 0.6mm.

Unless otherwise stated, all glass components used in the system are fabricated from BK7. At the particular wavelength of the laser to be used in the system, the refractive index of BK7 is 1.50348. The refractive index of air is considered to be 1.0.

The central RF operating frequency of the Bragg cell is $1.3 \times 10^9$ Hz. The bandwidth of the system is $0.2 \times 10^9$ Hz, resulting in the highest driving frequency being $1.4 \times 10^9$ Hz and the lowest driving frequency $1.2 \times 10^9$ Hz.

The index of refraction of the Bragg cell is 3.34. The distance that the active region of the cell occupies along the optical axis of the system is 5 mm. The velocity of sound propagating in the Bragg cell is 5125 m/s. From these parameters, the Bragg angle may be determined:

\[
\sin \theta_B = \frac{(1319 \times 10^{-9} \text{m})(1.3 \times 10^9 \text{ s}^{-1})}{(2)5125 \text{ m/s}} = 0.1672878
\]

\[
\theta_B = 9.63016^\circ = 0.16808 \text{ rad}
\] (58)

The angular spread of a ray fan may be calculated from the bandwidth of the system. The angular difference between the an extreme RF frequency and the central RF frequency is given by:

\[
\Delta \theta = \frac{\lambda_c \Delta f}{v_{\text{sound}}} = \frac{1319 \times 10^{-9} \text{ m}}{5125 \text{ m/s}} (\pm 0.1 \times 10^9 \text{ s}^{-1})
\]

\[
\Delta \theta = \pm 0.0257366 \text{ rad}
\] (60)
Simll: Computer Simulation and Design Tool

In this section, the name Simll will be mentioned several times in the determination of component specifications. Simll is a program, written by the author, in order to carry out a real ray tracing analysis of the integrated system, as well as other optical systems. By using Simll information concerning the relative displacement of rays at particular planes in the surface, i.e. a geometrical estimation of beam diameters, and difference in optical ray slopes after surfaces, i.e. a measurement of the collimation of a beam, the performance of the optical system being designed can be evaluated. Simll has also has the capability to optimize optical component parameters, i.e. the thickness of a component, until the difference in slopes after a user designated surface falls within a given tolerance. Such an optimization is quite useful, removing from the user the need to closely calculate component parameters to achieve well collimated beams. Since the optimization is accomplished using real ray tracing, the error introduced from applying paraxial methods for the calculation has been minimized.

Integrated System Overview

The design of the integrated system is an extension of the single photonic delay line system shown earlier. The extension to the system is done in order to increase the number of optical delay lines available and insure that each new individual optical delay line is a reproduction of the single delay line associated with the "simple" system. The insertion of extra optics between the output of the front end of the system and the DMD is required in order to accomplish the replication of the delay line. Once the delay line has been replicated, additional optics are further required to shape the individual delay lines.

The integrated optical system may be looked at as being composed of four major portions. The primary functions of the first portion of the integrated system, or the "front
end," is to place the RF signal onto an optical carrier via an acousto-optic cell, to break the input beam into reference and local oscillator beams, and to recombine the beams after the RF signal has been placed on the optical carrier for later heterodyne detection. The output of this portion of the system contains the information required for the generation a single delay line.

The second portion of the system is used to generate the required number of delay lines needed to feed the antenna elements. This is accomplished by breaking the signal and reference beam into 25 identical beams by the use of a binary optic. The delay lines are then fed into an optical system which collimates the individual delay lines with respect to one another. Coming out of this portion of the system are replicated versions of the front end output, each beam possess the information required in the delay line. However, due to the optics required for the collimation of the delay lines with respect to one another, each individual delay line is brought to a focus.

The third portion of the system is designed to collimate the individual delay line beams, which have been brought to a focus by the overall delay line collimator. After this portion of the system, the single delay output from the system front end has been replaced 25 times.

The fourth portion of the system is designed to extract the proper TTD for each element in the antenna by tapping various locations on the AO cell through the use of a deformable mirror device (DMD.) This is exactly analogous to the function of the DMD in the single photonic delay line system, except that the DMD now represents a two dimensional array of deformable mirrors, one for each delay line to be utilized. Since each mirror is independently accessible, each delay line is capable of providing a TTD independent of TTDs tapped by other deformable mirrors.
General Design of the Integrated System

The design concepts and an analysis of the overall functionality of the system may be introduced by considering the entire integrated system. In order to present the functional portion of the system in relatively straightforward manner, complex lens systems have been replaced by simple lens with equivalent effective focal lengths. Ray diagrams of the integrated system are shown in Figures 17 and 18\textsuperscript{13}. In Figure 17, ray fans are traced through the system, representing the "zero" reference TTD (indicated by the ray fan centered about the optical axis) and the maximum possible positive TTD. These fans represent a portion of the delay line, as defined by driving the Bragg cell at all possible frequencies in the operating range.

![Ray diagram of the integrated system](image)

Figure 17. The integrated system, represented by grouping some lensing system into simple positive lenses. In this figure, the ray fans associated with the zero TTD reference point and the maximum TTD point have been traced through the system. The optical axis of the signal beam portion of the system is represented by a dashed line. The ray fans represent only a portion of the total delay line but are sufficient for the design of the integrated system.

Note: The local oscillator has been appears in a more classical sense, in that it is recombined with the signal beam at the detector instead of traveling with the signal beam over the majority of the system. This is done in order to minimize the cluttering of the diagram, since no more useful information will be introduced by it's "co-presence" with the signal beam. In actuality, the reference beam is recombined with the signal beam at the beam splitter immediately before L1.

\textsuperscript{13} Figure used with the permission of Captain Ed Toughian
In Figure 18, all time delays are traced through the system for the central drive frequency of the Bragg cell. This represents an entire delay line associated with a fixed Bragg cell driving frequency. It is important to consider the behavior of both the individual ray fans and the total time delay in order to present a full picture of the operation of the integrated system. This usefulness will become apparent as the discussion of the system develops.

Figure 18. The integrated system, again complex portions of the system have been represented by simple lenses. In this case, a delay line has been traced through the system when the Bragg cell driving frequency is fixed at the central frequency. The entire illuminated portion of the Bragg cell (i.e. all possible time delays) is considered in defining the rays traced through the system, representing an entire delay line. 

Note: The Bragg angle of the cell has been neglected with respect to the output of the Bragg cell in order to simplify the presentation of the information. In the actual system, the optical axis will be along the Bragg angle and the components will be aligned accordingly.

The purpose of L1 is to collimate the ray fans in anticipation of being able to select out a particular TTD by the use of a deformable mirror, just as in the construction of the single photonic delay line shown previously. In order to collimate the ray fans, the focal point of the lens should be at the center of the Bragg cell, the source of the ray fans. To provide system symmetry about the optical axis, it has been selected that the on-axis
focal point of the lens be positioned at the zero TTD reference point. The collimating effect of L1 on the ray fans can be seen in Figure 17.

While the problem generating a single delay lane has been addressed, the problem of replicating the delay line for the ultimate feeding the multiple antenna elements remains. The generation of the multiple delay lines is accomplished by the use of the binary optic, labeled BO in Figures 17 and 18. As has been stated several times before, the binary optic will diffract the incident beam into 25 identical orders, all diverging from the binary optic. Each contains the information required for a complete delay line. However, since the multiple delay lines have been created by the use of a diffractive optic, the delay lines will be diverging. In order to prevent the further divergence of the diffractive orders (and control the spacing of the delay lines,) the lens L2 has been placed in the system. In order to prevent the figures from becoming too cluttered, only the central order to the binary optic output is considered.

L2 collimates the delay lines with respect to one another. Since the diffractive orders are diverging from the center of the binary optic, L2 should be positioned such that the focal point of the lens lies at the center of the binary optic. After the delay lines leave L2, the individual delay lines will be traveling parallel to one another. This does not, however, indicate the collimation the individual delay lines. In fact, the introduction of L2 causes each delay line to come to a focus. This can be seen from Figure 17, where L2 causes the focusing of the ray fans. Since all ray fans will be brought to a focus, the entire delay line will be focused at some point, as shown in Figure 18. Since the ray fans incident on the binary optic are collimated, refraction due to L2 will cause the ray fans to come to a focus at a distance equal to the focal length of L2, "behind" the lens L2.

The individual delay lines are collimated by the insertion of L3 into the system. L3 is represented in the figures as a lens possessing a large clear aperture, but in actuality
there exists one L3 for each delay line. Since only one delay line is being considered, only one L3 has been included. Since L3 is used to collimate the ray fans, it should be placed a focal length away from the position where the fans are focused.

At this point, the optics of the system have generated the multiple delay lines and conditioned each delay line to appear as the original single delay line would as it exited L1. By placing a deformable mirror at the position where the collimated ray fans of each TTD delay converge, the focus of the delay line in Figure 18, it is possible to use each delay line as an independent tap of the Bragg cell output. Hence, it is possible to feed each element of the antenna array with a separate photonic delay line, originating from a single Bragg cell.

**A Closer Look at the Integrated System Front End Design**

In order to provide a meaningful means by which to discuss the components of the system, all elements have been given a designation. Since at this particular time the front end of the system is under discussion, the entire front end of the system, and the corresponding component names, are shown in Figure 19.
Figure 19. The entire front end of the system, presented with names corresponding to the components and spacings found in the system in order to aid in the discussion of the front end design and specifications. Note: CL1 is a cylindrical lens. When viewed from the side, as is the case here, it will appear as a block of glass. All other lenses in this figure are spherical. Gap1 and Gap2 represent an air gap between the transducer of the Bragg cell and the nearest optical surface. The gaps represent space taken up in the system by the housing of the transducer and are not necessarily physical gaps between the components. Some gap has been left to allow for some play in the positioning of the Bragg cell, however the majority of Gap1 and Gap2 is the housing for the Bragg cell's transducer.

As stated previously, the front end of the system places the RF signal on to an optical carrier via an AO cell as well as breaking away the local oscillator and then recombining it. The realization of these requirements is quite apparent from the configuration of the front end. The RF signal is placed upon the signal beam via the Bragg cell located in the "lower" portion of the system. Beam splitting and recombination is accomplished by a pair of beam splitters found at opposite ends of the system. The beam is separated into the signal and local oscillator beams by the beam splitter designated BS1 and then later recombined as the beams prepare to exit the system front end by beam splitter BS2.

The most important factor to consider in the alignment of the "lower" portion of the front end is the requirement that, in order for the acousto-optic cell to function in the Bragg mode, the beam must enter the cell at the Bragg angle and will also exit the cell at
the Bragg angle if the cell is driven at its central RF frequency. However, before the alignment and specifications of the pieces in the signal branch of the system can be completed, the desired behavior of the signal beam portion of the system, as seen from a top view (as opposed to a side view which has been studied until now,) must be examined.

In general, a Bragg cell is constructed such that the active area of the cell (the transducer) is narrow with respect to the length of the cell. Referring to Figure 19, the narrow dimension of the Bragg cell will be perpendicular to the plane of the diagram. Therefore, in order to utilize the cell to its fullest extent, the signal beam of the system should be conditioned such that only the narrow active region of the cell is illuminated. This situation is best shown in Figure 20, which represents a top view of the signal branch of the system just before the Bragg cell. Here CL1 is used to focus the beam down into the active region of the cell. Since the focusing of the beam is only required in this dimension, CL1 is a cylindrical lens. When illuminated by a collimated beam, CL1 will bring the beam to a focus in the narrow dimension of the Bragg cell, with the focus being located at the center of the Bragg cell.

![Figure 20](image.png)

Figure 20. View of a section of the system front end from above. As seen in the figure, the active region of the Bragg cell is quite small in this dimension, requiring the insertion of the cylindrical lens, CL1, which is used to focus the beam down in the desired dimension. This significantly reduces the wasting of needed optical power.
Since the focused illumination of the Bragg cell should be along its entire length, the focusing surface of the lens should be parallel to the face of the Bragg cell. This requirement is shown in Figure 21, in which the face of CL1 is made to lie parallel to the face of the Bragg cell by the introduction of Wedgel into the system.

![Figure 21. Side on view of the first portion of the system front end. The first few components of the signal branch of the front end of the integrated system. The beam splitter is used to break off the reference beam into the upper portion of the front end. The wedge is used to adjust the beam so that it enters the Bragg cell at the Bragg angle. The cylindrical lens, which appears as a block when viewed in this orientation, serves to focus the beam onto the active area of the transducer.](image)

The required wedge angle of Wedgel may be determined by the application of Snell's Law and the requirement that the beam enter the cell at the Bragg angle. The geometry of the configuration is shown in Figure 22, where \( \theta_B \) is the Bragg angle and \( \theta_W \) is the required wedge angle.

![Figure 22. A close-up of the system components and the geometry that must be considered in determining the required wedge angle of Wedgel.](image)
From the figure it is obvious that the wedge angle can be found with an application of Snell's Law at the air/glass boundary of CL1:

\[ n_{\text{glass}} \sin \theta_w = n_{\text{air}} \sin \theta_B \]  

(61)

\[ \theta_w = \sin^{-1}\left( \frac{n_{\text{air}} \sin \theta_B}{n_{\text{glass}}} \right) \]  

(62)

Using the values outlined previously in this section, the wedge angle is found to be:

\[ \theta_w = \sin^{-1}\left( \frac{1.0(0.16745)}{1.50348} \right) \]  

(63)

\[ \theta_w = 0.11149794 \text{ rad} = 6.388361^\circ \]  

(64)

Upon establishing the function of the first few components of the system, it is now possible to assign values to the remaining components. Where possible, components have been selected from "off the shelf" stock components which are inexpensive (when compared to custom fabricated pieces) and readily available. BS1 and BS2 are polarizing beam splitters, measuring 12.7mm on a side. Wedge1 is 12.7mm in height, so that it matches the size of the beam splitter, and 3mm thick at its smallest dimension. CL1 was chosen such that its focal length focused into the center of the transducer while leaving enough space between the transducer housing and surface of the lens for slight manipulation of the Bragg cell. To provide ample space for the positioning of the Bragg cell, Gap1 was chosen to be a least 6mm. From this value, an estimate can be made of the focal length of CL1:

\[ f_{\text{min}} = 6 \text{ mm} + \frac{\text{Transducer Thickness}}{2n_{\text{Bragg}}} \]  

(65)

\[ f_{\text{min}} = 6 \text{ mm} + \frac{2.5 \text{ mm}}{3.34} = 6.749 \text{ mm} \]  

(66)
An "off the shelf" lens was found which possesses a thickness of 5mm and has a radius of curvature of 6.58749mm. Using the thin lens approximation, the focal length of the lens CL1, at 1319nm, is:

\[ f_{\text{CL1}} = \frac{\text{ROC}}{n_{\text{BK7}} - 1} = \frac{6.58749\text{mm}}{1.50348 - 1} = 13.08392\text{mm} \quad (67) \]

which provides adequate room for the transducer housing and Bragg cell manipulation.

Upon fixing the focal length of CL1, the length of Gap1 can be computed by taking into account the focal length of the lens and that the focus of the lens must be at the center of the transducer. The distance Gap1 should be the focal length of CL1 minus one half the apparent physical distance of the transducer:

\[ T_{\text{Gap1}} = f_{\text{CL1}} - \frac{\text{Transducer Thickness}}{2n_{\text{Bragg}}} = 13.08392\text{mm} - \frac{2.5\text{mm}}{3.34} \quad (68) \]

\[ T_{\text{Gap1}} = 12.335413\text{mm} \quad (69) \]

Recalling the design of the simple photonic delay line, the next components in the system, following the Bragg cell, are designed to collimate the ray fan which emerges from the center of the Bragg cell (the "zero" TTD point) upon driving the cell through the entire bandwidth of the RF signal. In the simple delay line scheme, only a positive lens was required for the collimation, however in the integrated system it is desirable to have a length of glass following the Bragg cell and cemented to the positive lens.

After the Bragg cell has deflected the beam, the beam then returns to glass. In order to prevent undesirable effects from refraction in the dimension of the focus of the cylindrical lens, CL1, the face of the glass surface after the Bragg cell should be parallel to the face of the Bragg cell. This equivalent to flipping of the system about the Bragg cell in the side view of the system. Since the face of the next portion of the system is to be parallel to the Bragg cell face, it must all have a wedge angle associated with it equal
to the wedge angle before the Bragg cell, since both portions of the system before and after the Bragg cell are made of the same glass. The wedge angle of Wedge2 is equal to that of Wedge1 by mirroring the system about the Bragg cell. This is obvious from the argument used to determine the angle of Wedge1.

As stated previously, the purpose of L1 is to collimate the ray fans from the Bragg cell. The components used in the collimation of the ray fan are shown in the figure below:

![Figure 23. The remaining components of the signal portion of the front end of the system, including the Bragg cell. In this case, the lens serves to collimate the ray fan associated with the "zero" time delay point in the Bragg cell. The ray fan is defined the driving the Bragg cell at the high and low ends of the expected RF signal.](image)

Recalling the design of the single photonic delay line, it is most important to focus upon the ray fan associated with the zero TTD, as this will be symmetric about the optical axis and, by being the "zero" reference point, should be a prominent factor in optimizing the system specifications. In order to chose a lens for L1, the desired size of the collimated zero TTD ray fan was constrained to a value of about 2mm. Since the angular spread of the ray fan is known, then the focal length of L1 may be estimated, analogous to all other collimation calculations:

Given: Angular Spread of Bragg cell Output \[ \Delta \theta = \pm 0.0257366 \text{rad} \]
\[
\tan(\Delta \theta) = \frac{\text{Desired Spot Radius}}{f_{l1}} \tag{70}
\]

\[
f_{l1} = \frac{1\, \text{mm}}{\tan(0.0257366\,\text{rad})} = 38.85\,\text{mm} \tag{71}
\]

It was decided that an "off the shelf" lens of thickness 3.6mm and radius of curvature of 18.67320mm would be used. Using the thin lens approximation, the focal length of the lens at 1319nm is 37.08827mm. Due to external design considerations, the length of the available for Gap2 and Wedge2 was constrained to a fixed value. SimII was used in a step-wise manner to optimize both components to fit in the given length. The resulting length of Gap2 was determined to be 7.131780220mm and the top thickness of Wedge2 was determined to be 26.54975mm. It should be noted that if this system was designed from scratch, the value of Gap2 and Wedge2 would represent two degrees of freedom in the system design. At this point, the design of the signal beam, or lower, portion of the front end of the integrated system has been completed.

**A Closer Look at the Local Oscillator Portion of the System Front End**

In order for signal and local oscillator beams to heterodyne with a high efficiency, the beams should totally overlap and be as close to parallel as possible. In this system design, the "zero" time delay ray fan (the ray fan which emerges from the center of the beam that illuminates the Bragg cell) is used to define the signal beam for system design purposes. The local oscillator, split off from the incident beam at the leading element of the front end, is sent through the "upper" portion of the front end which serves as a telescope by increasing the local oscillator diameter will maintaining the collimated nature of the beam. Therefore, for the highest heterodyne efficiency, the both the signal
and the reference output from the system front end should both be collimated, with the reference beam diameter large enough to insure a good beam overlap.

To accomplish the expansion of the reference beam, the local oscillator branch of the system is essentially a telescope. A collimated laser beam is introduced into the system at the face of BS1 and expanded to the desired diameter by the lens combination of CL2 and L1. The upper branch of the front end is shown in the figure below:

![Diagram](image)

Figure 24. The upper portion, or reference branch, of the front end of the integrated system. The portion of the incident laser beam is diverted into the upper portion of the system and expanded via the telescope configuration.

Since L1 has already been determined by the requirements placed upon the slower portion of the front end, CL2 and the separation between the CL2 and L1 must be determined. By specifying the desired output beam diameter, the focal length of CL2 and the required separation are fixed. This can be seen from recalling the simple geometrical model of the telescope, as shown in Figure 25. In order to determine the lens required to expand the reference beam, the desired beam diameter magnification, and the focal length of the lens which collimates the signal beam must be supplied. An expression for the approximate focal length may be derived from the basic telescope design shown in Figure 25.
Figure 25. The geometry of a simple telescope. $D_{in}$ is the diameter of the input beam, $D_{out}$ is the desired diameter of the output beam, and $F_1$ and $F_2$ are the focal lengths of the lens used in the telescope.

From similar triangles:

$$\frac{D_{in}}{2} = \frac{D_{out}}{2}$$

From (72)

$$F_2 = \frac{D_{out}}{D_{in}} F_1$$

(73)

It should be noted that the index of refraction between the two lenses need not be taken into account. This can be seen by including the index in the equations as a multiplicative factor on both the focal lengths to account for the apparent optical thickness of the glass, reflecting the apparent increase in the focal lengths due to the presence of a refractive medium. Since this factor will be common to both focal length expressions, it will exactly cancel out.

In this system, it is desired to expand the beam by a factor of 4.5. This ratio insures that CL2 will also be an "off the shelf" lens, as well as expanding the local oscillator to workable value. Therefore, the focal length of CL2 is given by:

$$F_{CL2} = \frac{D_{in}}{D_{out}} F_{L1} = \frac{1}{4.5} F_{L1} = \frac{1}{4.5} (37.08827 \text{mm})$$

(74)

$$F_{CL2} = 8.24184 \text{mm}$$

(75)
Therefore, an "off the shelf" lens has been found which possess a focal length of 8.24184mm and a thickness of 2.5mm. Once the lens pair is established to generate the desired beam expansion, SimII was used in order to determine the length of glass between the lenses by optimizing the difference between the slope of the optical axis and a ray introduced into the upper portion of the system, parallel to the optical axis and initially at a height of 1/2 the input beam diameter (0.3mm.) Since the focal lengths of the lenses are known the distance, in glass, separating the lens may be estimated:

\[
d = n_BK_f (F_1 + F_2) = 1.50348(8.24184\text{mm} + 37.08827\text{mm})
\]

\[
d = 68.15291\text{mm}
\]

As shown in Figure 24, the local oscillator beam must be steered in a manner that when it is finally recombined with the signal beam it will be parallel to it. In the design of the system, it was easiest to visual the upper branch as two spacers: Spacer1 accomplishing the steering of the beam such that it is parallel to the output of the lower portion of the system and Spacer2 providing the optical path length required to expand the beam using the telescope configuration. In the case of Spacer2, the optical path is folded to follow a periscope configuration. In the case of both spacers, the surfaces shown as reflecting the beam have been mirrored to contain the illumination in the system.

As was stated previously, the steering of the local oscillator is accomplished by the reflecting surface of Spacer1. The angle that the reflecting surface makes with the input optical axis may be determined knowing the angular difference in the optical axis as it enters the system front end and exits it. From SimII, the angle the optical axis makes with respect to the input optical axis is:

\[
tan\theta_{oa} = 0.22676723
\]

\[
\theta_{oa} = 12.77672^\circ
\]
To achieve this angle with respect to the input optical axis, the reflecting surface must be oriented such that the angle the incoming ray makes an angle with respect to the surface normal of 0.5*12.77672°, obeying the law of reflection. The geometry of the reflecting surface is shown in Figure 26.

From the diagram it can be seen that the surface normal bisects the angle that is \( \theta_{OA} + 90^\circ \). Therefore, the slope of the surface normal, which is the negative inverse of the slope of the surface, is equal to:

\[
\frac{-1}{\tan \theta_M} = \tan \left( \theta_{OA} - \frac{90^\circ + \theta_{OA}}{2} \right)
\] (80)
\[
\frac{-1}{\tan \theta_M} = \tan \left( 12.77672^\circ - \frac{90^\circ + 12.77672^\circ}{2} \right) = -0.79862
\]

(81)

\[
\theta_M = 51.38836^\circ = 0.89690 \text{ rad}
\]

(82)

Once the required orientation of the reflecting surface of Spacer1 is known, the question of how the spacers should be proportioned must be answered. Since the beam direction may be altered to the desired trajectory by the placing of the reflecting surface of Spacer1 anywhere above BS1, a myriad of possible configurations exist with the only limiting factor being the length of glass required in Spacer2 for the proper spacing of CL2 and L1. Some possible combinations are shown in the figure below:

Figure 27. Possible configurations for the partitioning of the upper optical system into Spacer1 and Spacer2. In this particular figure, 3 possible configurations of the upper branch of the system are being shown. Note that the possibilities are constrained by the required spacing between CL2 and L1 for the telescope configuration. Hence, the "uppermost" configuration provides one limiting case. The other limiting case results from keep the reflecting surface of Spacer1 aligned with the front surface of BS1. This limiting case is shown in "lowermost" configuration.

An "off the shelf" component was available for Spacer1. The dimensions and shape of the piece are shown below, in Figure 28.
Since the dimensions of Spacer1 are fixed, solving for the dimensions of Spacer2 are straightforward. For convenience, the dimensions of Spacer2 have been labeled in Figure 29. The most obvious dimension to calculate, since Spacer1 is fixed, is the distance $h_0$, which will immediately lead to the value of $h_1$.
The distance, $h_0$, is the perpendicular distance between the optical axis in the upper portion of the system front end and the output optical axis of the lower branch. By overlaying the system on a Cartesian coordinate system, with the origin looked where the input optical axis intersects BS1, it is possible to describe surfaces and rays in terms of equations of lines making it much easier to deal with the geometry of the system. This concept is covered in great detail in the section discussing real ray tracing and its realization using SimIII. By using this concept, the important quantities may be assigned workable values (all distance measurements are in mm):

Equation of the Upper Optical Axis:  
$$ y = 0.22676723x + 31.23146421 $$

Point A:  
$$ (x_A, y_B) = (78.63415, 9.27805) $$

This values were calculated using the SimIII program.

The distance, $h_0$, is the perpendicular distance from point A to the upper optical axis:

$$ h_0 = \frac{|m_x x_A - y_A + b_x|}{\sqrt{m_x^2 + 1}} $$  \hspace{1cm} (83)

$$ h_0 = \frac{|0.22676723(78.63415) - 9.27805 + 31.23146421|}{\sqrt{0.22676723^2 + 1}} $$  \hspace{1cm} (84)

$$ h_0 = 38.8 \text{ mm} $$  \hspace{1cm} (85)

For convenience, the value of $h_2$ was assigned to 12.7mm, trying to keep the limiting "height" of the components to that of the beam splitters. By assigning this value, $h_1$ becomes:

$$ h_1 = h_0 - t_{BS} = 38.8 \text{ mm} - 12.7 \text{ mm} $$  \hspace{1cm} (86)

$$ h_1 = 26.10 \text{ mm} $$  \hspace{1cm} (87)
To find the value of $h_3$, SimII was allowed to optimize that parameter until a collimated input beam of beam, with a diameter equal to the laser diameter, results in a collimated output beam. Upon completion, the value of $h_3$ was found to be 10.474mm.

In order to provide some closer on the dimensions of Spacer2, the optimized value will be compared to the estimated value, as determined in equation 77. Recalling from equation 77:

$$d_{est} = 68.15291\text{mm} \quad (88)$$

In the actual system, the value of $d$ will be considered the distance from the refracting surface of CL2 to the refracting surface of L1. Therefore:

$$d_{act} = t_{CL2} + h_5 + t_{BS} + h_4 + t_{BS} + t_{L1} \quad (89)$$

$$d_{act} = 2.5\text{mm} + 10.474\text{mm} + 12.7\text{mm} + 26.10\text{mm} + 12.7\text{mm} + 3.6\text{nm} \quad (90)$$

$$d_{act} = 68.074\text{mm} \quad (91)$$

In terms of the percentage difference between the optimized value and the paraxial estimated value is:

$$\Delta\% = \left| \frac{d_{act} - d_{est}}{d_{act}} \right| \times 100\% \quad (92)$$

$$\Delta\% = \left| \frac{68.074 - 68.15291}{68.074} \right| \times 100\% \quad (93)$$

$$\Delta\% = 0.116\% \quad (94)$$

which indicates that the estimated value of the optical distance and the optimized distance are in close agreement.

Since there are several degrees of freedom that have been striped away from the system design by the use of an "off the shelf" Spacer1, the design process of the upper portion of the system front end should be examined as if the dimensions of Spacer1 were not fixed. In this case, the design process could proceed along these lines:
Unconstrained Upper System Design Process

1) Since the lens pair CL2 and L1 are known, then the total length of glass between the lenses is known. Partition the length of glass intelligently among the dimensions of Spacer2.

2) Estimate the amount of clearance one would like for the manipulation of CL2, this will tell the amount of air gap required between the refracting surface of CL2 and the face of Spacer2.

3) Find the intersection of the optical axis extended backwards from CL2, towards where the reflecting surface of Spacer1 should be, and the optical axis, pointing "upwards" as it reflects off of the surface of BS1. The intersection point defines a point on the reflecting surface of Spacer1. Since a point is known on the reflecting surface, and the desired slope of the surface is known, \( \tan(\theta_M) \), then the surface has been fully specified.

4) Limits are placed on the piece by the desired dimensions of the piece, i.e. the measurement across a face.

At this point, the entire front end of the system has been designed and the dimensions specified. For completeness the components of the front end are listed in Table 1.
Table 1.

**Specification of Front End Components**

<table>
<thead>
<tr>
<th>Component Name</th>
<th>Component Type</th>
<th>Thickness(^{14})</th>
<th>Index of Refraction</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS1</td>
<td>Beam Splitter</td>
<td>12.7 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td>Wedge1</td>
<td>wedge</td>
<td>3 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td>CL1</td>
<td>Cylindrical Lens</td>
<td>5 mm</td>
<td>1.50348</td>
<td>6.58749 mm</td>
</tr>
<tr>
<td>Gap1</td>
<td>Air Gap</td>
<td>12.335413 mm</td>
<td>1.00000</td>
<td>----</td>
</tr>
<tr>
<td>Bragg Cell</td>
<td>---</td>
<td>5 mm</td>
<td>3.34</td>
<td>----</td>
</tr>
<tr>
<td>Gap2</td>
<td>Air Gap</td>
<td>7.131780220 mm</td>
<td>1.00000</td>
<td>----</td>
</tr>
<tr>
<td>Wedge2</td>
<td>wedge</td>
<td>26.54975 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td>BS2</td>
<td>Beam Splitter</td>
<td>12.7 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td>L1</td>
<td>Spherical Lens</td>
<td>3.6 mm</td>
<td>1.50348</td>
<td>18.67320 mm</td>
</tr>
<tr>
<td>CL2</td>
<td>Spherical Lens</td>
<td>2.5 mm</td>
<td>1.50348</td>
<td>8.24184 mm</td>
</tr>
<tr>
<td>Spacer2</td>
<td>Spacer</td>
<td>h1 = 26.10 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td></td>
<td></td>
<td>h2 = 12.7 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td></td>
<td></td>
<td>h3 = 10.474 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
</tbody>
</table>

The specifications of the components used in the construction of the front end of the integrated system.

Note: Spacer1 is not included in the table as it is fully specified in Figure 28.

\(^{14}\) When the thickness is with reference to a wedge, it indicate the minimum thickness of the piece, or the "top" thickness of the piece.
A Closer Look at the Multiple Delay Line Generation

In order to generate the delay lines to feed the twenty antenna elements, the now collinear signal and reference beams should be broken into several individual beams. Each beam will serve as an individual delay line for feeding a particular element of the antenna array. In order to accomplish this, a binary optic has been introduced into the system directly following the output lens of the front end, L1. The binary optic splits the beam into 25 beams of equal intensity, each representing a potential delay line. A high packing density of the delay lines is maintained by an optical system that has been designed to collimated each delay line with respect to the other delay lines. This entire assembly is the equivalent of L2, as shown in Figures 17 and 18. The behavior of the system was confirmed by the use of the SimII program. The figure below was generated using the SimII program, which is seen here tracing the local oscillator beam through the portion of the system.

Figure 30. The "second" portion of the integrated system, which serves to generate the separate delay lines through the use of a binary optic and an associated delay line collimating system. Each of the beam shown represent an individual delay line. The deflection angle of the orders has been altered in order to make the separation of the delay line more apparent. Note that the delay lines are collimated with respect to one another, but the individual beams themselves are coming to a focus.
The specifications of the system are listed in table 2, which presents the information in a classic optics format. The surfaces are listed from the "back side" of the binary optic to the 5 mm gap following the last surface in the system.

Table 2.

Specification of Delay Line Collimation Components

<table>
<thead>
<tr>
<th>Radius of Curvature (mm)</th>
<th>Index of Refraction, after Surface (at 1319 nm)</th>
<th>Distance to Next Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>----</td>
<td>1.50348</td>
<td>99.45 mm</td>
</tr>
<tr>
<td>∞</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>----</td>
<td>1.50348</td>
<td>8 mm</td>
</tr>
<tr>
<td>65.87743</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>----</td>
<td>1.0000</td>
<td>20.41354 mm</td>
</tr>
<tr>
<td>-341.67405</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>----</td>
<td>1.64653</td>
<td>4.000000 mm</td>
</tr>
<tr>
<td>-80.89038</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>----</td>
<td>1.54944</td>
<td>9.50 mm</td>
</tr>
<tr>
<td>120.35830</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>----</td>
<td>1.00000</td>
<td>5.000000 mm</td>
</tr>
</tbody>
</table>

The specifications for the optical sub-system used for the replication of the single delay line output from the front end of the system. The specification of surfaces begins after the binary optic and ends after the last curved surface in the sub-system. The format of the values is in the form classically used in the specification of optical surfaces.
A Closer Look at the Third Portion of the System: Collimation of the Individual Delay Lines

After the segment of the system described in the previous section of the system, the delay lines as a unit are in the desired state, however, the delay lines themselves are coming to a focus. In order to correct this situation individual optical systems were designed for each beam in order to achieve collimation. The system is composed of a piece of BK7, placed 5mm after the last surface of the second portion of the system, and a positive lens, cemented to the face of the glass. The positive lens to be specified plays the role of L3, as seen in Figures 17 and 18. It should be noted that extensive use of SimII was made in the design of this portion of the system, due to it's flexibility in allowing many different positive lens to be easily analyzed and the ease at which it can optimize the length of glass to collimate the beam.

It was specified that after the introduction of the new system the estimated spot size of the local oscillator beam as it exits the system should be a close to that of the local oscillator beam diameter as it exited the front end of the system, 2.7mm. In order to keep unnecessary clutter from accumulating in the analysis and design of the new portion of the system, only the second portion of the system plus the two new components were used in SimII. The "second portion" of the integrated system refers to the optical system outlined in Table 2. The optical subsystems, and the component designations, are shown in Figure 31.
The "Second Portion" of the Integrated System
or Subsystem 1

Subsystem 2

Figure 31. The optical subsystems that are to be addressed in this section and the designation of the components in subsystem 2. Terminology has been introduced in order to try to limit confusion.

Note: This figure only shows the central order of the binary optic being traced through the system, hence the need for only one SP1. For each order of the binary optic, there will be a SP1, aligned about the central ray of that order.

By looking at the system, one sees that it is basically a telescope system that increases the beam diameter by a factor of 1. The first subsystem may be characterized as a simple positive lens, just by the virtue that it brings a collimated input beam to a focus. Subsystem 2 serves the purpose of the other lens in the telescope system, recollimation of the beam to the desired diameter.

The first step in designing subsystem 2 is to find the back focal length of subsystem 2, this will locate the focus of subsystem 1 with respect to the last surface in subsystem 1 and give a partial indication of the required length of glass between the subsystem 1 and SP1. The values required in the calculation were generated by SimII. In the analysis, a ray was introduced into the system at a height of 1.35mm above the optical axis with a ray angle of 0 degrees with respect to the optical axis (since the beam diameter of the local oscillator exiting the front end of the system was 2.7mm) and the exiting ray angle and height where found. From these values, the bfl may be determined:

\[ bfl = \frac{y_{last}}{\tan(\theta_{last})} = \frac{y_{last}}{m_{last}} \]  

(95)
Since it is known that a piece of glass will be 5mm after subsystem 1, the length of the bfl in glass made found:

$$bfl = 5.0\text{mm} + (63.73512\text{mm} - 5\text{mm}) \cdot 1.50348$$

(97)

$$bfl = 93.30708\text{mm}$$

(98)

At this point, the input portion of the telescope system has been characterized, in a paraxial sense, and the second portion of the system may be specified based on the desired beam magnification of 1. This just requires finding the distance, in glass, where the beam returns to the desired diameter after it has come to a focus. A simple trigonometric relationship defines the additional amount of glass:

Using the slope of the output ray in glass:

$$m_{\text{glass}} = -0.01043029$$

$$\frac{\text{Desired Spot Size}}{2} = \frac{2.7\text{mm}}{|m_{\text{glass}}|} = \frac{2}{|-0.01043029|}$$

(99)

$$\text{additional glass} = 129.43073\text{mm}$$

(100)

Therefore, the total length of glass required for BL1 will be:

$$l_{BL1\text{est}} = (63.73512\text{mm} - 5.0\text{mm}) \cdot 1.50348 + 129.43073\text{mm}$$

(101)

$$t_{BL1\text{est}} = 217.73781\text{mm}$$

(102)

Also, the focal length of the lens required to collimate the beam made to approximated:

$$f_{SP1\text{est}} = \frac{\text{additional glass}}{n_{\text{BK7}}} = \frac{129.43073\text{mm}}{1.50348}$$

(103)

$$f_{SP1\text{est}} = 86.08743\text{mm}$$

(104)

$$\text{ROC}_{SP1\text{est}} = f_{SP1\text{est}}(n_{\text{BK7}} - 1) = 86.08742\text{mm}(1.50348 - 1)$$

(105)

$$\text{ROC}_{SP1\text{est}} = 43.34329\text{mm}$$

(106)
At this point, rough values have been found to specify the parameters of Subsystem 2. Unfortunately, an "off the shelf" lens was not available in the neighborhood of the estimated values, requiring the custom fabrication of SP1. In order to find the ideal values for BL1 and SP1, a two dimensional optimization was carried out using SimII. Multiple values of radius of curvature were intelligently chosen to be tried in SimII and BL1 optimized for each lens to provide beam collimation. This process was repeated until the geometrical spot size of the input, 2.7mm, equaled the geometrical spot size of the output. It was determined that a lens with a radius of curvature of 43.3598mm and a thickness of 3.0mm would be used. In order to collimate the beam, the length of BL1 was optimized to a value of 214.76294835214mm, resulting in an output spot size of 2.7mm.

Since this length of glass is rather unwieldy, it was decided that the optical path be folded into a pentaprism. In order to achieve the required optical path length, the side of the pentaprim must be:\textsuperscript{15}

\begin{equation}
 t_{BL1} = t_{\text{side}}(2+\sqrt{2}) \tag{107}
\end{equation}

\begin{equation}
 t_{\text{side}} = \frac{214.76294835214 \text{ mm}}{(2+\sqrt{2})} = 62.90261 \text{ mm} \tag{108}
\end{equation}

In order to confirm the values obtained from the optimization, a comparison of the paraxial estimates of the thickness of BL1 and the ROC of SP1 will be carried out. The first comparison is between the estimated and optimized values of the ROC of SP1:

\begin{equation}
 \Delta\% = \left| \frac{ROC_{SP1} - ROC_{SP1\text{est}}}{ROC_{SP1}} \right| \times 100\% \tag{109}
\end{equation}

\textsuperscript{15} Formula derived in Warren Smith's 441 Geometrical Optics Class notes from the University of Rochester, using the tunnel diagram technique.
\[ \Delta\% = \left| \frac{43.3598\text{mm} - 43.34329\text{mm}}{43.3598\text{mm}} \right| \times 100\% = 0.03808\% \quad (110) \]

The percent difference will be calculated to express the deviation from the optimized and the estimated value of the thickness of BL1, but first the thickness of BL1 must be altered to take into account the finite width of SP1:

\[ t_{BL1} \rightarrow t_{BL1} + t_{SP1} \quad (111) \]

to provide a basis for the comparison of real ray tracing values to the paraxial estimates.

The percent difference in the thicknesses of BL1 can then be calculated:

\[ \Delta\% = \left| \frac{t_{BL1} + t_{SP1} - t_{BL1est}}{t_{BL1} + t_{SP1}} \right| \times 100\% \quad (112) \]

\[ \Delta\% = \left| \frac{217.76294835214\text{mm} - 217.73781\text{mm}}{217.76294835214\text{mm}} \right| \times 100\% = 0.0116\% \quad (113) \]

As can be seen from the results, the percent difference in the values is on the order of what is expected from comparison of the values in the design of Spacer2, as seen in the result in equation 94.

For convenience, the component specifications are shown in Table 3.
Table 3.
Specification of Individual Delay Line Collimation Components

<table>
<thead>
<tr>
<th>Component Name</th>
<th>Component Type</th>
<th>Thickness(^{16})</th>
<th>Index of Refraction</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Gap</td>
<td>Air Gap</td>
<td>5.00 mm</td>
<td>1.0000</td>
<td>----</td>
</tr>
<tr>
<td>BL1</td>
<td>Block</td>
<td>229.339135 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td>Pentaprisim</td>
<td></td>
<td>62.90261 mm</td>
<td>1.50348</td>
<td>----</td>
</tr>
<tr>
<td>SP1</td>
<td>Spherical Lens</td>
<td>3.11 mm</td>
<td>1.50348</td>
<td>45.76 (\pi)</td>
</tr>
</tbody>
</table>

The specifications of the components found in Subsystem 2, as labeled in Figure 31.

**A Closer Look at the Fourth Section of the System: Extraction of the Proper TTD and Configuration for Detection of the Heterodyne Signal**

By designing the first, second, and third sections of the system, L1, L2, and L3 in Figures 17 and 18 have been specified, respectively. The remaining requirement seen in Figures 17 and 18 is the determination of the distance from SP1 (L3) to the DMD. The DMD is positioned at the intersection of all collimated TTD ray fans, as shown in Figure 17. Before continuing farther into the specification of the SP1 to DMD distance, it should be noted that it was desired to partition the distance up such that a 10mm air gap appears after the SP1 array, for ease in manipulating the lenses. The remaining distance was to the DMD is to be realized in BK7.

Before beginning the calculation, it is helpful to visualize what is occurring at the DMD in terms of the ray fans. Figure 32, shows collimated ray fans intersecting at the

---

\(^{16}\) In the case of a pentaprisms, thickness refers to the length of a side.
DMD surface, each fan representing each an extreme TTD value or the zero TTD reference. Recalling the model of the signal photonic delay line, all ray fans converge upon the mirror so that, by rotating the mirror, a particular time delay ray fan may be steered so that it will coincide with the local oscillator beam. Therefore, in order to find the position of the DMD find the location of the signal beam focus when the cell is driven at a single frequency.

![Collimated Ray Fans](image)

**Figure 32.** Collimated ray fans, representing the maximum, minimum, and zero TTDs, intersecting at the surface of a deformable mirror. It should be noted that for an ideal situation, all rays originating from the cell, which is being driven at only one frequency, will intersect at a single point. This is shown in the figure by the intersection of all 1.2GHz (low Bragg cell driving frequency,) 1.3GHz (central Bragg cell driving frequency,) and 1.4GHz (high Bragg cell driving frequency) rays for three separate TTD values.

SimII was used to determine the values required to quantify the necessary length to the DMD. First, the entire integrated optical system was entered into the program, up to and including SP1. After SP1, a 10mm air gap was inserted, as was described in this section. After the 10mm air gap, a section of BK7 was inserted that was long enough to
contain the location of the DMD. This block of glass shall be designated BL2 for ease in referring to the piece. Once the system was specified, a series of rays were traced through the system, the same rays seen in Figure 32, and the resulting slopes and y intercepts of the rays in the length of BK7 were recorded. Next, a point on the fixed face of the section of BK7 was found along with the slope of the surface. From this information, it is now possible to compute the required length of glass. An example calculation is shown below, followed by a tabulation of all the results.

**Calculation of Required BK7 length to DMD:**

1) Acquire information defining the linear equation describing the fixed surface of the glass block. Recall that all distance measurements in SimII are in millimeters, even if it is not explicitly stated.

   This information is readily obtainable from SimII:

   **Slope of Surface:** -4.40980824
   **Point on Surface (x,y):** (455.84810989, 94.92029620)

   The y intercept value made determined:

   \[ b_s = y_s - m_x \]  \hspace{1cm} (114)

   \[ b_s = 94.92029620 - (-4.40980824)(455.84810989) = 2105.12305 \]  \hspace{1cm} (115)

2) Trace a pair of rays through the system and acquire the linear equations describing the rays' trajectory in the glass block. The rays must be generated by driving the Bragg cell at a fixed frequency but originating from a different point on the Bragg cell. This describes a situation where the Bragg cell is driven at a fixed frequency and different TTD are selected. As an example, consider the pair of rays generated by driving the Bragg cell at 1.2GHz, one ray is associated with the maximum TTD (since the input beam diameter is 0.6mm, this ray is generated by tracing a ray through the system the initially has a 0 slope
with respect to the optical axis and a y intercept value equal to -1/2 the beam diameter) and the other with the 0 TTD point.

This information was determined by SimII:

<table>
<thead>
<tr>
<th>TTD</th>
<th>AO Drive Frequency</th>
<th>Ray Slope</th>
<th>Ray Y Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>1.2GHz</td>
<td>0.22051131</td>
<td>-3.54819445</td>
</tr>
<tr>
<td>Zero</td>
<td>1.2GHz</td>
<td>0.22608274</td>
<td>-7.01986835</td>
</tr>
</tbody>
</table>

3) Find the intersection of the two rays:

\[
x_o = \frac{b_2 - b_1}{m_1 - m_2}
\]  
\[(116)\]

\[
x_o = \frac{-7.01986835 - 3.54819445}{0.22051131 - 0.22608274} = 623.1208
\]  
\[(117)\]

\[
y_o = m_1 x_o + b_1
\]  
\[(118)\]

\[
y_o = 0.22051131(623.1208) + -3.54819445 = 133.85699
\]  
\[(119)\]

4) Find the perpendicular distance from the fixed surface of the intersection point, this will give the required length of the glass to the DMD:

\[
Length = \left| \frac{m_1 x_o - y_o + b_1}{\sqrt{m_1^2 + 1}} \right|
\]  
\[(120)\]

\[
Length = \left| \frac{(-4.40980824)623.1208 - 133.85699 + 2105.12305}{\sqrt{(-4.40980824)^2 + 1}} \right| = 171.74184\text{mm}
\]  
\[(121)\]
### Table 4.
Various Lengths of Glass Used in Specifying DMD Distance

<table>
<thead>
<tr>
<th>TTD1</th>
<th>TDD2</th>
<th>AO Drive Frequency</th>
<th>Required Glass Length to DMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>Zero</td>
<td>1.2 GHz</td>
<td>171.742 mm</td>
</tr>
<tr>
<td>Zero</td>
<td>Min</td>
<td>1.2 GHz</td>
<td>172.527 mm</td>
</tr>
<tr>
<td>Max</td>
<td>Min</td>
<td>1.2 GHz</td>
<td>172.133 mm</td>
</tr>
<tr>
<td>Max</td>
<td>Zero</td>
<td>1.3 GHz</td>
<td>172.657 mm</td>
</tr>
<tr>
<td>Zero</td>
<td>Min</td>
<td>1.3 GHz</td>
<td>172.647 mm</td>
</tr>
<tr>
<td>Min</td>
<td>Max</td>
<td>1.3 GHz</td>
<td>172.647 mm</td>
</tr>
<tr>
<td>Max</td>
<td>Zero</td>
<td>1.4 GHz</td>
<td>171.368 mm</td>
</tr>
<tr>
<td>Zero</td>
<td>Min</td>
<td>1.4 GHz</td>
<td>172.888 mm</td>
</tr>
<tr>
<td>Min</td>
<td>Max</td>
<td>1.4 GHz</td>
<td>172.129 mm</td>
</tr>
</tbody>
</table>

Various lengths of glass generated by using the technique described by equations 114 through 121. A pair of ray, passing through the TTD1 and TTD2 points on the Bragg cell, which is driven at the AO Drive Frequency, were used to determine the length of glass.
As indicated by the table, a variety of glass lengths were produced. It was decided that an average of the lengths be taken, weighting the average towards the value associated with the 1.3GHz ray pairs, as these describe the central operating point of the system. The resulting length of glass was decided to be 172.30157121mm.

Now that the length of glass to the DMD is known, the problem of separating the local oscillator and the signal beam must be addressed. The problem arises from the fact that since the beams are traveling along a common path, the DMD cannot be tilted to select out a particular TTD without altering the trajectory of the local oscillator. Returning to the picture of the simple photonic delay line, one will note that the signal and reference beams are kept separate until they are combined at the detector and hence did not suffer from this problem.

One possible solution to this problem revolves around altering the polarization of the reference beam such that it is orthogonal to that of the signal beam and then employing polarization altering components, i.e. $\lambda/2$ and $\lambda/4$ waveplates, in combination with polarizing beam splitters. In this scheme, the beams splitters BS1 and BS2 are polarizing beam splitters. Upon the introducing a laser beam into the system, BS1 will separate the input laser beam into two beams, one beam polarized orthogonal to the other. A half wave plate may be inserted before BS1 and rotated to determine the amount of light partitioned into each beam.

Since the beams are now polarized orthogonally as they approach the DMD, it becomes trivial to selectively steer the signal beam and leave the reference beam unaltered by introducing a polarized beam splitter into the system, immediately before the DMD. This described configuration is shown in Figure 33, which also depicts the scheme used to recombine the beams for heterodyning.
Figure 33. The portion of the system devoted to the selecting of a particular TTD, by the use of a DMD, and insuring the recombination of the local oscillator and the desired TTD. Figure A shows the path of the signal beam in this portion of the system, in actuality it shows the zero TTD collimated ray fan propagating in the system. Figure B shows the path of the local oscillator through the same components.

Note: The beam diameters are greatly exaggerated in order to make the propagation of the beams as clear as possible. Also, the beams in the diagram are offset upon reflection, making the paths a little more visible. In reality, this does not happen for cases of normal incidence, the reflected beam will return along the same path it had before reflecting.

Upon entering the polarizing beam splitter, BS3, the signal beam passes through the reflecting surface of the beam splitter, through a quarter wave plate (causing the linearly polarized light to become circularly polarized), reflects off of the DMD, and passes back through the quarter wave plate (causing the light to become linearly polarized again, but orthogonal to its orientation upon entering BS3.) Thus, by the time the signal beam returns from the DMD, its polarization has been altered so that it will be reflected off of the reflecting surface of the beam splitter and not return down its original line of flight.
The reference beam, however, will experience reflection from the surface in BS3. The reference beam is deflected towards a different quarter wave plate which is backed by a reflecting surface. The beam passes through the quarter wave plate twice, rotating the polarization of the beam by 90°, allowing the beam to continue directly through BS3 on the same path as the steered signal beam.

An additional half-wave plate is inserted into the system, after BS3, to prevent any reflects that may occur in the later portions of the system from entering the system.

BS4 is used to split the signal/reference beam pair such that 50% of the signal travels to the lens holder, where the beams are focused by a custom made coupling lens into a fiber optic which directs the beams on to a PIN detector and the time delay electrical system is recovered via the heterodyne process. The other signal/reference combination used to provide feedback for a digital control system that monitors the current beam state and adjusts the DMD elements to either maintain the current output of the antenna or redirect the antenna output.

In order to force the orthogonally polarized beams to heterodyne, a polarizer has been inserted into the system, at an angle of 45° with respect to the polarization of the beams. This will cause the common components of the polarization to heterodyne.

Since this latter portion of the system has been fixed in length by the components required, the length of glass in BL2 may finally be computed. It is known that the thickness of BS3 is 35mm, the thickness of a quarter wave plate is 7.5mm, and the total length of glass needed is 172.30157121mm. Therefore, the actual length of glass needed in the piece is:

\[ t_{BL2} = t_{tot} - t_{BS3} - t_{\lambda/4} \quad (122) \]

\[ t_{BL2} = 172.30157\text{mm} - 35\text{mm} - 7.5\text{mm} = 129.80157\text{mm} \quad (123) \]
To create a compact system, this length of glass was also folded into a pentaprism, measuring 38.018mm on a side. (See equation 107.)

**Specification of Coupling Lens for Detection Assembly**

The only portion of the system that remains to be specified are the coupling lens, which will be used in coupling the reference/signal beam into a multimode fiber. While the specific properties of the coupling lens are not known, the only requirement needed in specifying the lens is the Gaussian spot size and location of the reference beam as it exited BS4 into the lens holder.

The Gaussian beam analysis of the local oscillator was carried out using ABCD matrix approach in order to find the final spot size and location. The ABCD law states that the output Gaussian beam, which is characterized by a "q" factor, q2, may be found using the following matrix equation17:

\[ q_2 = \frac{Aq_1 + B}{Cq_1 + D} \]  

(124)

where q1 describes the input Gaussian beam, and A, B, C, and D are parameters used to describe a paraxial optical system when using the ray matrix approach to optics. As stated, the q factor may be used to describe the behavior of a Gaussian beam18. The q factor represents a complex number of the form19:

\[ q = z + jz_o \]  

(125)

where z is the distance the to the beam waist from the current position on the beam axis and z0 is the Rayleigh range. The Rayleigh range can be expressed as20:

---

\[ z_o = \frac{\pi W_o^2}{\lambda} \]  \hspace{1cm} (126)

where \( \lambda \) is the wavelength of the laser and \( W_o \) is the waist radius of the beam.

The waist radius is further defined\(^{21} \) to be (in the small angle approximation):

\[ \theta_o = \frac{\lambda}{\pi W_o} \]  \hspace{1cm} (127)

where \( \theta_o \) is the divergence angle of the beam.

In the analysis of the local oscillator Gaussian beam, the laser manufacturer provided the parameters needed to define the beam as it exits the laser unit. Listed on the specification sheet were the x and y values of beam diameters and the full divergence angles of the output beam, at the aperture of the laser, shown in Table 5.

### Table 5.

**Input Gaussian Beam Characteristics**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Beam Divergence Angle, in x direction</td>
<td>$2\theta_{0x}$</td>
<td>10 mrad</td>
</tr>
<tr>
<td>Beam Diameter, in x direction</td>
<td>$2W_x$</td>
<td>5 mm</td>
</tr>
<tr>
<td>Full Beam Divergence Angle, in y direction</td>
<td>$2\theta_{0y}$</td>
<td>8 mrad</td>
</tr>
<tr>
<td>Beam Diameter, in x direction</td>
<td>$2W_y$</td>
<td>5 mm</td>
</tr>
</tbody>
</table>

Parameter listing describing the Gaussian beam characteristics of the laser source used in the integrated system. These parameters are for the laser beam at the aperture of the laser.
From these parameters, the Gaussian beam characteristics of the laser can be found. The divergence angle is found by halving the full divergence angle given in the laser specifications:

\[
\theta_{Dx} := \frac{10 \cdot 10^{-3}}{2} \text{ radians} \quad \theta_{Dy} := \frac{8 \cdot 10^{-3}}{2} \text{ radians}
\]  

(128)

From the divergence angle of the beam, the beam waist radius may be determined:

\[
W_{ox} = \left( \frac{1}{\pi} \right) \frac{\lambda}{\theta_{Dx}} \quad W_{oy} = \left( \frac{1}{\pi} \right) \frac{\lambda}{\theta_{Dy}}
\]

\[
W_{ox} = 8.397014797528399 \cdot 10^{-5} \text{ m} \quad W_{oy} = 1.04962684969105 \cdot 10^{-4} \text{ m}
\]  

(129)

From the beam waist radius, the Rayleigh range, \(z_0\), may be calculated, completing half the information required in the q factor:

\[
z_{ox} = \frac{\pi \cdot W_{ox}^2}{\lambda} \quad z_{oy} = \frac{\pi \cdot W_{oy}^2}{\lambda}
\]

\[
z_{ox} = 0.016794029595057 \text{ m} \quad z_{oy} = 0.026240671242276 \text{ m}
\]  

(130)

The beam diameter at the aperture of the laser can be used as an estimate of the beam radius, at the aperture:

\[
W_x := \frac{0.5 \cdot 10^{-3}}{2} \text{ m} \quad W_y := \frac{0.5 \cdot 10^{-3}}{2} \text{ m}
\]  

(131)

Knowing the parameters calculated to this point, it is possible to determine the value of \(z\), required for the completion of the q factor:

\[
z_x = z_{ox} \cdot \left( \frac{W_x}{W_{ox}} \right)^2 - 1 \quad z_y = z_{oy} \cdot \left( \frac{W_y}{W_{oy}} \right)^2 - 1
\]

\[
z_x = 0.047095228738805 \text{ m} \quad z_y = 0.056724572918223 \text{ m}
\]  

(132)

At this point, the input Gaussian beam can be fully described by it's q factor:

\[
q_{xin} := z_x + \sqrt{-1} z_{ox} \quad q_{yin} := z_y + \sqrt{-1} z_{oy}
\]

\[
q_{xin} = 0.047095228738805 + 0.016794029595057j \quad q_{yin} = 0.056724572918223 + 0.026240671242276j
\]  

(133)
Since the input Gaussian beam has been characterized, all that remains is the determination of the paraxial ray matrix for the optical system traversed by the local oscillator. For convenience, the indexes of refraction found in the system are grouped into a collective unit, shown below:

$$\begin{align*}
n_{\text{air}} & := 1.0 \\
n_{\text{BK7}} & := 1.503480 \\
n_{\text{Sub1}} & := n_{\text{BK7}} \\
n_{\text{Sub2}} & := 1.646530 \\
n_{\text{Sub3}} & := 1.646530 \\
n_{\text{Sub4}} & := 1.549440 \\
n_{\text{Sub5}} & := 1.549440
\end{align*}$$

The "Sub#" subscript certain indices of refraction indicate that the index is found in the second portion of the optical system, referred to as "Subsystem 1" in a previous section. In order to group the remaining system specifications into a cohesive unit, SimII was used to condense down some of the glass thickness measurements by providing ray/surface intersection point values, which may be used in conjunction with the distance formula to find total lengths of glass. The remaining system parameters are:

The distance from the laser aperture to the optical system:

$$T_{2\text{system}} := 10$$

The distance from the first surface of the system, BS1, the end of Spacer1:

$$\begin{align*}
T_{\text{BK72Air}} & = 6.35 + 32.67143613 - \sqrt{(6.35 - 49.42104736)^2 + (32.67143613 - 42.43853826)^2} \\
T_{\text{BK72Air}} & = T_{\text{BK72Air}} 10^{-3} \\
T_{\text{BK72Air}} & = 0.083186030600064 \text{ m}
\end{align*}$$

It should be noted that in order to find some distances SimII was used to generate the ray/surface intersection point values. By adding the distances between the points together it is possible to make a very quick measurement of the length. Hence, the appearance of the distance formula several times in this section.

The distance from the face of Spacer1 to the face of CL2:
The radius of curvature of CL2:

\[ \text{ROC}_{\text{CL2}} = 4.1496 \times 10^{-3} \text{ m} \]

The length of glass from the surface of CL2 to the planar surface of L1:

\[ T := \sqrt{(51.20796150 - 70.05348537)^2 + (47.11729908 - 9.27801498)^2} \]

\[ T := T + \sqrt{(70.05348537 - 84.82696447)^2 + (47.11729908 - 10.68233215)^2} \]

\[ T_{\text{CL2-L1}} := T + \sqrt{(84.82696447 - 78.63419503)^2 + (10.68233215 - 9.27801498)^2} \]

\[ T_{\text{CL2-L1}} = 0.064473999998025 \text{ m} \]

The thickness and radius of curvature of L1:

\[ T_{\text{L1}} = 3.6 \times 10^{-3} \text{ m} \quad \text{ROC}_{\text{L1}} = -18.6723 \times 10^{-3} \text{ m} \]

The next few values describe the refracting surfaces and distances found in the second portion of the integrated system. The derivation of these values is apparent from Table 2:

\[ T_{\text{Slab}} = 99.45 + 1 \times 10^{-3} \text{ m} \]

\[ T_{\text{Slab}} = 0.10045 \text{ m} \]

\[ T_{\text{Sub1}} = 8 \times 10^{-3} \text{ m} \quad \text{ROC}_{\text{Sub1}} = -65.87742910^{-3} \text{ m} \]

\[ T_{\text{AirGap}} = 20.41 \times 10^{-3} \text{ m} \quad T_{\text{AirGap}} = 0.02041 \text{ m} \]

\[ T_{\text{Sub2}} = 4 \times 10^{-3} \text{ m} \quad \text{ROC}_{\text{Sub2}} = 341.674052 \times 10^{-3} \text{ m} \]

\[ T_{\text{Sub3}} = 0 \text{ m} \quad \text{ROC}_{\text{Sub3}} = 80.89038310^{-3} \text{ m} \]

\[ T_{\text{Sub4}} = 4.75 \times 10^{-3} \text{ m} \quad \text{ROC}_{\text{Sub4}} = 80.89038310^{-3} \text{ m} \]

\[ T_{\text{Sub5}} = 4.75 \times 10^{-3} \text{ m} \quad \text{ROC}_{\text{Sub5}} = 120.358300010^{-3} \text{ m} \]
The air gap between the above system and BL1:
\[ T_{2BL1} = 5.0 \times 10^{-3} \text{ m} \]

The length of BL1:
\[ T_{BL1} = 214.734967910725110 \times 10^{-3} \text{ m} \]

The parameters of the lens SP1:
\[ T_{SP1} = 3.0 \times 10^{-3} \text{ m} \quad \text{ROC}_{SP1} = -43.349628 \times 10^{-3} \text{ m} \]

Distance from SP1 to BL2:
\[ T_{SP12BL2} = 10.00010^{-3} \text{ m} \]

The length of BL2:
\[ T_{BL2} = 129.80310^{-3} \text{ m} \]

The thickness of a beam splitter:
\[ T_{BS} = 35 \times 10^{-3} \text{ m} \]

The thicknesses of a quarter and a half-wave plate:
\[ T_{\lambda/4} = 7.5 \times 10^{-3} \text{ m} \quad T_{\lambda/2} = 13 \times 10^{-3} \text{ m} \]

Now that the parameters of the system are specified and assigned to meaningful variable names, the actual calculation of the ray matrix of the reference beam can begin.

Propagation of the beam to the first surface of the system:
\[ M = \begin{pmatrix} 1 & T_{2\text{system}} \\ 0 & 1 \end{pmatrix} \]

Propagation of the beam through Spacer1:
\[ M = \begin{pmatrix} 1 & T_{\text{Air2CL2}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & n_{\text{BK7}} \\ 0 & n_{\text{air}} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & T_{\text{BK72Air}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & n_{\text{air}} \\ 0 & n_{\text{BK7}} & 0 & 1 \end{pmatrix} \]

Refraction of the beam by CL2:
\[ M = \begin{pmatrix} 1 & 0 \\ n_{\text{BK7}} & n_{\text{air}} \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ n_{\text{BK7}} \end{pmatrix} \]
Transfer of the beam to the refracting surface of L1 and refraction by that surface:

\[
M = \begin{bmatrix}
1 & 0 \\
\frac{n_{air} - n_{BK7}}{n_{air} \cdot ROC_{L1}} & \frac{n_{BK7}}{n_{air}}
\end{bmatrix}
\begin{bmatrix}
1 & T_{CL22L1 + T L1}
\end{bmatrix}
\]

Refraction and transfer of the beam through the second portion of the system:

\[
M = \begin{bmatrix}
1 & 0 \\
0 & \frac{n_{air}}{n_{BK7}}
\end{bmatrix}
\begin{bmatrix}
1 & n_{air} \\
0 & n_{BK7}
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
1 & 0 \\
0 & \frac{n_{air} - n_{BK7}}{n_{air} \cdot ROC_{Sub1}}
\end{bmatrix}
\begin{bmatrix}
1 & T_{AirGap} \\
0 & 1
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & n_{Sub4} - n_{Sub2} \\
0 & n_{Sub2}
\end{bmatrix}
\begin{bmatrix}
1 & T_{Sub4 + T Sub5} \\
0 & 1
\end{bmatrix}
\]

Completion of the refraction by the second portion of the integrated system and transfer of the beam through to the refracting surface of SP1:

\[
M = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & T_{BL1 + T SP1} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & \frac{n_{air} - n_{Sub5}}{n_{Sub5} \cdot ROC_{Sub5}}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & n_{Sub5}
\end{bmatrix}
\]

Refraction by SP1 and transfer of the beam to BL2:

\[
M = \begin{bmatrix}
1 & 0 \\
0 & \frac{n_{air}}{n_{BK7}}
\end{bmatrix}
\begin{bmatrix}
1 & T_{SP12BL2} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & \frac{n_{air} - n_{BK7}}{n_{air} \cdot ROC_{SP1}}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & n_{BK7}
\end{bmatrix}
\]

Propagation of the beam through BL2 to the output face of BS4:

\[
M = \begin{bmatrix}
1 & 0 \\
0 & \frac{n_{BK7}}{n_{air}}
\end{bmatrix}
\begin{bmatrix}
1 & 0.5 \cdot T_{BS} + 0.5 \cdot T_{BS} + 2 \cdot T_{AirGap} + T_{BS} - T_{AirGap} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & \frac{n_{BK7}}{n_{air}}
\end{bmatrix}
\]

Assignment of the elements of the ray matrix to A, B, C, and D values:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = M = \begin{bmatrix}
4.678060458937561 & 0.335340874252063 \\
1.247585447482933 & 0.303195396278654
\end{bmatrix}
\]

(134)
The output Gaussian beam characteristics are determined:

\[
q_{\text{xout}} := \frac{A \cdot q_{\text{xin}} + B}{C q_{\text{xin}} + D} \quad q_{\text{yout}} := \frac{A \cdot q_{\text{yin}} + B}{C q_{\text{yin}} + D}
\]

\[
q_{\text{xout}} = 1.5425638096745 \times 0.12776244581011j
\]

\[
q_{\text{yout}} = 1.622609695113359 - 0.186208477959114j
\]

Recalling the form of the q factor from equation 125, the distance from the output face of BS4 to the waist of the beam will be about 1.58258m behind the surface of BS4, and the output waist size will be:

\[
W_{\text{xout}} := \sqrt{\frac{\text{Im}(q_{\text{xout}})}{\pi}} \lambda
\]

\[
W_{\text{yout}} := \sqrt{\frac{\text{Im}(q_{\text{yout}})}{\pi}} \lambda
\]

\[
W_{\text{xout}} = 2.316056074489476 \times 10^{-4} \text{ m} \quad W_{\text{yout}} = 2.7960645065 \times 10^{-4} \text{ m}
\]

The divergence angles of the beam was found to be:

\[
\theta_{\text{Dx}} := \left(\frac{1}{\pi}\right) \frac{\lambda}{W_{\text{xout}}} \quad \theta_{\text{Dy}} := \left(\frac{1}{\pi}\right) \frac{\lambda}{W_{\text{yout}}}
\]

\[
\theta_{\text{Dx}} = 0.001812783138115 \text{ rad} \quad \theta_{\text{Dy}} = 0.0015015777445397 \text{ rad}
\]

At this point, the design of the optics of the integrated photonic delay is complete.
VI. Real Ray Tracing with Techniques for Computer Realization

General Ray Tracing Technique

Real ray tracing is the process by which the trajectory of a ray through an optical system is determined by the repeated application of Snell's Law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \), at optical surfaces. Real ray tracing will provide exact ray trajectories, as opposed to paraxial ray tracing, which utilizes the first order approximation of Snell's Law, \( n_1 \theta_1 = n_2 \theta_2 \).

Ray analysis of an optical system is accomplished by a series of refraction/transfer operations. The refraction operation is used to determine the ray trajectory as the ray passes from one medium to another. The transfer operation utilizes the ray trajectory to determine the intersection of the ray and the next refracting surface. By repeating the series of operations, the line of flight of the ray through the entire system may be found. The iterative ray tracing process can be best seen in the form of the following flow chart:

![Flow Chart]

Figure 34. Iterative Ray Tracing Process
Background

Basic Philosophy behind "User Friendly" Optical System Design

In order to create an intuitive optical system design environment the use of optical components, instead of optical surfaces, to describe an optical system has been implemented. Under such conditions, the designer of the optical system will be provided with a more visual picture of the optical system being assembled in terms of tangible components, instead of having to contend with manipulating a conglomeration of abstract surface specifications. In order to follow this concept in the ray analysis of an optical system, methods must be devised in order to decompose optical components into their respective optical surfaces. This procedure is, of course, totally transparent to the optical system designer.

Optical Components to be Considered

From the observation of various optical systems, the existence of several fundamental components were found. These components were determined to be spacers (made of either air or glass,) wedge-shaped pieces of glass and planoconcave/planoconvex lenses. From these components it is possible to build other "more complicated" components (i.e. a cemented doublets) occasionally found in optical systems. However, the evaluation of the integrated optical system requires the addition of two non-linear optical components, the acousto-optic cell operating in Bragg mode and the binary optic. Upon determining the optical component "pool," the methodology from the decomposition of the elements into their respective optical surfaces must be determined.
Fundamental Concepts for the Generation of a Ray Tracing Environment

Use of a Global Coordinate System

In order to provide a means of measuring relative distances and orientations in the optical system, the surfaces and rays are superimposed on a global Cartesian coordinate grid. It was chosen that the origin of the coordinate system lie at the midpoint of the first surface of the optical system. Thus, the coordinate system (with a sample optical system superimposed) will appear as:

![Diagram](image)

Figure 35. The Global Coordinate System upon which the optical components are superimposed. The origin of the coordinate system is at the center of the first surface of the optical system.

The Cartesian system provides an adequate basis on which the decomposition of components into their optical surfaces may be carried out.

General Terminology

In order to make the presentation of the methods of surface decomposition as clear as possible several terms, which will be used repeatedly in the descriptions, should be clarified. The term "first surface" is used when referring to the surface of the component that is facing the origin of the global coordinate system. It naturally follows that "second
surface" or "next surface" refers to the surface facing away from the coordinate origin. In general, the system will be assembled and aligned from left to right. With this in mind, the meanings of the terms becomes apparent.

The "height" of a component refers to the length of the component in the y direction, from one side of the component to the other. "Thickness" or "length" refers to the distance measured in the x direction, from one surface to the other. Please see Figure 36 for clarity.

![Diagram](image)

Figure 36. The definitions of measurements and orientations that will be used in the discussion of implementing real ray tracing.

**Representation of Optical Surfaces**

From the component "pool" it was determined that the principle optical surface types that must be considered were either planar (which, for example, arise when considering ray tracing through spacers) or spherical surfaces (which must be contended with when analyzing lenses.) By making use of the global coordinate system, the surfaces may be represented as either linear equations, as in the case of a planar surface, or in the case of a spherical surface, by the equation of a circle.
**Describing Planar Surfaces**

As stated before, the planar surfaces of the system are easily represented by linear equations. Recalling from simple mathematics, the form of a planar surface can be specified in two forms. The first is the classic slope-intercept form:

\[ y = mx + b \]  \hspace{1cm} (135)

where \( x \) and \( y \) are Cartesian coordinates, \( m \) is the slope of the line, and \( b \) is the \( y \) intercept. It should also be noted that the slope of the line is equal to the tangent of the angle made between the line and the positive \( x \)-axis.

The second means of expressing a line is the \( x \)-intercept form:

\[ x = \frac{y}{m} + x_o \]  \hspace{1cm} (136)

where \( m \) is the slope of the line associated with the slope-intercept form of the line, and \( x_o \) is the \( x \)-intercept of the line. Both types of equations must be utilized in order to handle the limiting cases when the planar surface is vertical (\( m = \infty \), in which case the \( x \) intercept is needed in order to fix the surface) or horizontal (\( m = 0 \), in which case the \( y \) intercept must be used to fix the surface.)

**Describing Spherical Surfaces**

Spherical surfaces are easily described by the equation of a circle:

\[ (x-x_o)^2 + (y-y_o)^2 = R^2 \]  \hspace{1cm} (137)

where \( x_o \) and \( y_o \) indicate the center of curvature of the surface and \( R \) indicates the radius of curvature.
Application of Snell's Law...

At an Arbitrary Planar Surface

The application of Snell's law at a planar surface should be addressed in the context of the global coordinate system and the slope-intercept form of a ray. The geometry of the situation is shown below:

Figure 37. The definitions of angles important to the application of Snell's Law at an arbitrary planar surface. The planar surface is specified as a linear equation in terms of the global coordinate system.

In the diagram, the incident ray makes an angle, $\Phi_s$, with the surface normal and an angle, $\Phi_r$, with the x axis of the global coordinate system. The exiting ray makes an angle, $\Phi_s'$, with the surface normal and an angle, $\Phi_r'$, with the x axis of the global coordinate system. The surface possesses a slope of $m$, and therefore makes an angle of $\Phi_m \, ( = \text{atan}(m) )$ with the x axis. The problem to be addressed in the application of Snell's Law is how to determine the slope the exiting ray makes with the optical axis based upon the slope of the refracting surface, the slope of incident ray, and indexes of refraction present.
From the slope of the surface, the angle that the surface makes with the \( x \) axis of the coordinate system is known:

\[
\Phi_m = \arctan(m) \tag{138}
\]

From the slope of the incident ray, the angle that the ray makes with the \( x \) axis is known:

\[
\Phi_r = \arctan(m_r) \tag{139}
\]

From the value of these angles, the angle that the incident ray makes with respect to the surface normal is known:

\[
\Phi_s = \Phi_r - \Phi_m', \text{ where } \Phi_m' = 90^\circ - \Phi_m. \tag{140}
\]

From this angle, the angle the exiting ray makes with the optical axis may be determined by applying Snell's Law:

\[
\sin \Phi_{s'} = \left(\frac{n}{n'}\right) \sin \Phi_s \tag{141}
\]

Once this angle has been determined, the angle that the exiting ray makes with the optical axis is known:

\[
\Phi_{r'} = \Phi_{s'} + \Phi_m' \tag{142}
\]

From the angle the exiting ray makes with the optical axis, it is trivial to find the slope of the exiting ray:

\[
m_{r'} = \tan(\Phi_{r'}) \tag{143}
\]

**At an Arbitrary Spherical Surface**

Application of Snell's Law at a spherical boundary requires the determination of the normal to the surface at the point where the surface and the ray intersect. By considering the spherical surface as part of a full circle, the determination of the normal becomes trivial. The slope of the surface normal is the slope of a line that passes through the center of the circle and the point where the ray intersects the circle. Once the surface
normal has been determined, the method used to determine the ray trajectory at a planar surface can be applied.

Alignment of Optical Components About the Optical Axis

In order to provide a coherent means of assembling the optical system from the system components, a method of active alignment is employed. By the process of active alignment the first surface of an optical element to be aligned is centered about the incident ray/surface intersection point. For the initial alignment of the system, a ray is introduced into the system which possess a zero slope and passes through the origin of the coordinate system. This ray will define the optical axis of the system. By this method, the desired alignment of the integrated system is obtained, as well as providing a means through which most conventional optical systems may be aligned.

A Common Mathematical Operation

When determining the proper placement of surfaces on the coordinate system, it was found that the equation used in determining the intersection(s) point(s) between a line and a circle was invaluable. Since the final result was obtained through simple algebraic manipulations, only the premise and the final result are stated here.

Given the equation of a line, as in equation 135, and a circle, as defined by equation 137, then the resulting solution(s), when it (they) exist are given by:

\[
x_{1,2} = \frac{x_o - mb + my_o \pm \sqrt{-2x_o mb + 2x_o my_o - b^2 + 2by_o - y_o^2 + R^2 - m^2 x_o^2 + m^2 R^2}}{1 + m^2}
\]  

(144)

The corresponding \( y \) solution can be found by simply inserting the obtained \( x \) values into the original linear equation. The usefulness of such a formula will become apparent as this section develops.
Decomposition of Optical Components into Respective Surfaces

It should be noted that when the decomposition of a surface is described, it is assumed that it is with respect to an already existing planar surface.

Spacer Decomposition

The breakdown of a component into it's refracting surfaces is most easily visualized by considering a spacer at it's refracting surfaces. By inspection, a spacer can be looked at as two planar, parallel surfaces which describe the both refracting surfaces of the component. The first surface of the spacer will be collinear with the previous existing planar surface. This corresponds to "butting" the spacer to the previous component. Once this surface has become fixed, a point on the next surface may be found by finding the intersection of a line, which passes through the center of the first surface and is normal to the surface, and a circle which possess a radius equal to the physical thickness of the component. This process is described below:

Figure 38. Decomposition of a Spacer into refracting surfaces. The numbers in circles correspond to procedural steps.
**Procedure for Spacer Decomposition into Surfaces**

1) Locate the edge of the first surface based on the height of the spacer and the location of the optical axis. This is accomplished by utilizing equation 143, where the center and radius of the circle are defined by one-half the height of the spacer and the point of intersection between the optical axis and the surface, respectively, and the line slope and y intercept are given by the parameters of the first surface’s linear equation.

2) Apply equation 143 again, this time the center of the circle is at the point found in step #1 and radius of the circle is equal to the length of the component. In this case, the slope of the line is given as the negative reciprocal of the slope of the first surface (specifying the surface normal) and the y-intercept can be found based on the line passing through the point found in Step #1. The resulting solution in the positive y direction specifies a point on the next surface.

3) Since the component being considered is a block, the sides of the element will be parallel to each other. Therefore, the next surface may be specified by the slope of the first surface and the y intercept, which may be determined from the fact that the second surface of the spacer passes through the point found in Step #2.

For the purposes of surface decomposition, the AO cell and the binary optic may be treated as spacers.

**Wedge Decomposition**

The process of defining the surfaces of the wedge is very similar to that of the spacer. However, in the case of the wedge, the "top thickness," or the smallest thickness of the wedge, must be specified in place of the thickness term used in the spacer case. Once a point on the other surface has been established, the slope of the first surface must be rotated according to the wedge angle. This is illustrated in Figure 39.
Procedure for Wedge Decomposition into Surfaces

Steps #1-#2) Same as Spacer Decomposition case

3) From Step #2, a point on the surface has been determined. The slope of the surface can be found as the tangent of the angle the first surface makes with the +x axis plus the wedge angle. The angle the first surface makes with the +x axis is simply the arc tangent of the line's slope. At this point, since the slope of the surface and a point on the surface are known, the surface is completely specified.
**Lens Decomposition**

**Planoconvex: Oriented with the Curvature Facing Away the Previous Surface**

![Diagram of Lens Decomposition](image)

Figure 40. Decomposition of a Planoconvex lens into refracting surfaces. Numbers in circles correspond to procedural steps.

**Procedure for Planoconvex Decomposition in Surfaces**

1) The vertex of the lens can be found at the intersection of a circle of radius equal to the thickness of the lens and centered about the midpoint of the surface and a line normal to the previous planar surface that passes through the center of the lens.

2) Once the vertex of the lens is determined, the center of curvature of the surface may be found at the intersection of a circle, centered at the vertex of the lens and possessing a radius equal to the radius of curvature of the lens, and a line normal to the previous planar surface and passes through the vertex of the lens. At this point the center of curvature and radius of the spherical surface have been determined, fully specifying the parameters required for the description of the surface.
Planoconvex: Oriented with the Curvature Facing Towards the Previous Surface

Figure 41. Decomposition of a Planoconvex lens into refracting surfaces.

The planoconvex lens, in this particular orientation, will be decomposed into a spherical surface, whose vertex lies on the previous surface, and a planar surface, representing the back of the lens.

Procedure for Planoconvex Lens Decomposition into Surfaces

1) The midpoint of the planar surface of the lens is found at the intersection of a circle, centered at the midpoint of the previous planar surface and possessing a radius equal to the thickness of the lens, and a line, normal to the previous planar surface and passing through the midpoint of the previous surface. Since the slope of the planar surface, which is equal to that of the previous surface, and a point of the surface, just determined, are known, the planar surface component of the element is known.

2) The radius of curvature of the spherical surface is found at the intersection of a circle, centered at the vertex of the lens (the midpoint of the previous surface) and with a
radius equal to the radius of curvature of the surface, and a line, normal to the previous planar surface and passing through the vertex of the lens. Thus, by finding the center of curvature on the global coordinate system and knowing the radius of curvature of the lens, the surface has been specified.

**Planoconcave: Oriented with the Curvature Facing Towards the Previous Surface**

When considering the Planoconcave lens, it is most useful to consider it only as a planoconcave surface, possessing zero axial thickness, and fixed to the previous planar surface.

![Figure 42. Decomposition of a Planoconcave lens into refracting surfaces. Numbers in circles correspond to procedural steps.](image)

**Procedure for Planoconcave Lens Decomposition into Surfaces**

1) The center of curvature of the surface can be found at the intersection of a circle of diameter equal to the radius of curvature and a normal to the planar surface that
passes through the midpoint of the planar surface. Once the location of the center of curvature is known, the surface is specified.

**Planoconvex: Oriented with the Curvature Facing Away the Previous Surface**

![Diagram of Planoconvex lens](image)

Figure 43. Decomposition of a Planoconcave lens into refracting surfaces.

**Procedure for Planoconcave Lens Decomposition into Surfaces**

1) The determination of this surface utilizes the same method as mentioned in the above section.

**Ray Tracing Through the Bragg Cell**

Ray tracing through the Bragg cell presents a problem to many conventional ray tracing programs since it is an active optical component. For purposes of ray tracing, the Bragg cell may be looked upon as a RF frequency dependent mirror, with the mirror oriented normal to the face of the Bragg cell. The explanation for with treatment can be seen from the operation of the AO cell in the Bragg mode.
For proper Bragg mode operation, a ray will enter the cell at the Bragg angle with reference to the surface normal. The resulting output ray will also be at the Bragg angle, assuming that the cell is driven at its central operating frequency. Any deviation in the cell driving frequency will cause the ray deviate about the Bragg angle, hence the Bragg cell can be modeled as a mirrored surface normal to the face of the cell, with the reflecting angle modified according to the formula derived in Section III:

\[
\Delta \theta = \frac{\lambda \Delta f_{\text{signal}}}{2n_{\text{Bragg}} v_{\text{sound}}} \quad (145)
\]

Figure 44. The ray model of the Bragg cell. The Bragg cell is interpreted as a frequency dependent reflecting surface, with the reflecting angle centered about the Bragg angle of the output ray. The modification to the deflection angle is calculated according to equation 145. Note: It is assumed in the figure that the drive range of the Bragg cell is centered about the central driving frequency of the cell. Also note: The angles shown are greatly exaggerated in order to clarify the ideas described.

The ray model of the Bragg cell is shown in the figure above. A ray incident on the Bragg cell is first refracted by the surface of the Bragg cell and point where the ray
intersects the center of the cell is found. The ray is then reflected off of a mirror surface that is oriented normal to the face of the cell and passes through the ray/center of the cell intersection point. The reflection angle is then modified according to the difference between the current cell driving frequency and the central operating frequency of the cell, as specified by equation 145. The resulting ray then exits the cell, after refracting at the exit face of the cell.

**Ray Tracing Through the Binary Optic Device**

Ray tracing through the binary optic is accomplished by modeling the binary optic as a diffraction grating in terms measuring the trajectory of the output diffractive orders. The angular deflection of the output orders of a diffraction grating are governed by the paraxial form of the grating equation:

\[ \theta_q = \theta_i + m\frac{\lambda}{\Lambda} \]  

(146)

where \( \theta_q \) is the resulting output angle of the mth order with respect to the optical axis, \( \theta_i \) is the angle the incident beam makes with the optical axis, \( \lambda \) is the optical wavelength, and \( \Lambda \) is the period between rulings on the grating.

In the case of the binary optic, the paraxial form of the grating equation has been adopted in order to reflect the refined nature of the binary optic over that of a conventional diffraction grating. Since the binary optic does not possess a ruling period, the grating equation has been modified to the form:

\[ \theta_q = \theta_i + m\chi \]  

(147)

where \( \chi \) is a device specific parameter indicating the angular difference between adjacent diffracted orders.

---

VII. SimII: An Overview

Introduction

SimII is a program designed from the propose of using real ray tracing in order to design and evaluate the performance of the integrated optical system by use of the methods in the previous sections. However, due to it's dynamic nature, it allows the user to easily design and evaluate other optical systems.

SimII is capable of allowing the entry of an optical system, manipulation of the individual component parameters, simple optimization of some parameters, and displaying the system on the graphics screen. SimII also allows for multiple rays to be traced through the system and displayed on the graphics screen.

Specifying Components (Units of Measurement)

It should be noted that the component dimensions are measured in millimeters and all angles are measured in radians, unless otherwise specified by the program.

Parameters of Optical Components Used By SimII

All components have a value of height associated with them. This value is equal to the distance across the face of a planar surface of an element, measured from corner to corner. This parameter is included primarily for the program internal manipulation of wedges, as well as providing a basis for which the graphics screen can display the current system.
**Block Parameters**

Blocks are specified by the thickness of the component, the index of refraction of the piece, and whether or not the particular block is a beamsplitter. The beamsplitter status of the block is used only in conjunction with the integrated system analysis. The program will fuse the reference (Local Oscillator) branch of the system to the locations specified by the beam splitters.

**Wedge Parameters**

Wedges are specified by the top thickness of the component, the index of refraction of the component's material, and the wedge angle on the leading face of the piece. The wedge angle is the angular deviation of the surface from the normal block position.

**Lens Parameters**

**General Planoconvex**

All planoconvex, or positive lenses, should ideally be specified by the thickness of the lens, the index of refraction of the lens material, the radius of curvature of the lens, and the orientation of the lens (the orientation of the lens will be mentioned in a following section.) In the case of the planoconvex lens, the focal length (i.e. radius of curvature) must be a positive value. However, special provisions have been made for the entry of lens descriptors in order to allow the user flexibility in the radius of curvature entry. The purpose of these additional parameters is to allow the user to enter lens parameters from, for example a manufacture's lens catalog, in order to define the radius of curvature. These extra parameters are the focal length of the lens at a given design index (and the associated design index) and the focal length of the lens at the operating index.
(and the associated operating index of refraction.) The simple formula is used to
determine the radius of curvature is given by:

$$R = (n-1) f$$  \hspace{1cm} (148)

where R is the radius of curvature, n is either the operating index or the design index, and
f is either the focal length at the operating index or the focal length at the design index.
These parameters are, by necessity of dynamic element entry, redundant to some degree.
Again, it is emphasized that the program user indicate the power of the lens to the
program by entering the radius of curvature that the lens possesses. This is the surest way
of insuring the program is working with the correct lens, due to the varying methods of
power calculation employed by different lens manufactures.

**General Planoconcave**

As was mentioned before, the planoconcave, or negative, lens should be looked
upon as a planoconcave surface, possessing zero axial thickness. Therefore, the value of
thickness that is specified for the lens represents the "edge" thickness of the lens and
NOT the axial thickness. The "edge" thickness is the thickness of the lens material at a
distance equal to one half the height of the component from the optical axis. All other
parameters are analogous to the planoconvex lens except that the power of the lens (i.e.
radius of curvature) should be entered as a negative number.

**Orientation of a Lens**

The orientation of the lenses, a value of either 1 or 2, is used to specify whether
the curvature of the lens is towards or away from the previous element. The orientation
values of the particular lens types are shown in Figure 45.
Acousto-Optic Cell

When indicating the presence of an acousto-optic cell to the program, only the transducer is considered. Therefore, the thickness of the AO Cell is the thickness of the transducer and the index of the material is the index of refraction associated with the transducer. The housing of the transducer should be represented as separate spacers placed on either side of the transducer. The central operating frequency of the AO cell is not stored with the component information, but is stored in the program, itself, and is accessible under the Ray Tracing/Defaults menu.

Binary Optic Device

The binary optic device is specified by thickness of the element, the index of refraction of the material, and the deflection angle associated with each order. The deflection angle is a realization of the diffraction equation that is independent of the wavelength of light illuminating the element. Assuming that light is incident normally on the element, the departing angles of the orders are determined by the order number multiplied by the deflection angle. It should be stressed that the means of realizing the binary optic may be changed to take into account varying realizations of the output deflection of the orders. However this must be done internal to the program.
General Program Usage

Entry and Manipulation of Data

SimII has been designed in order to simple optical system design as simple (a.k.a. painless) and straight-forward as possible. The main program functions are controlled by a series of pulldowns: the file pulldown, output pulldown, graphics pulldown, tracing pulldown, and Simulation pulldown. The main user screen is shown below:

![Main User Screen](image)

Figure 46. The front end of SimII, displaying the possible selections available to the user. This screen appears upon initial program execution.

From the pulldowns, the user has access to several simple "select" boxes. These boxes provide a simple means through which the user can select, for example, to change the thickness of a spacer or to add a lens to the system. An example "select" box is shown in Figure 47.

![Optimize Thickness Parameters](image)

Figure 47. A Typical selection box found in the SimII program. By moving the pointer to the desired entry and pressing the return key, an action will be performed on the selected item.
This particular box allows the user to change the parameters involved with optimizing a component in the optical system. To change a parameter the user positions the cursor (>) before the desired parameter using the up or down arrow keys and then press the enter (return) key. In the case of the select box above, when the return is pressed a "prompt" box will appear. The resulting prompt box is shown below:

\[
\begin{align*}
\text{Bragg cell frequency} \\
\text{Enter Bragg cell frequency [1.4e+09] (Hz):}
\end{align*}
\]

Figure 48. A typical prompt box found in the operation of SimII.

The prompt boxes ask for a new value to be entered for the parameter, as well as displaying the previous value in brackets. In order abort the operation press <return> without entering any information.

**Back-Tracking To Abort Undesired Operations**

It is possible for the user to return from a prompt box or select box without altering any parameters. In the case of the prompt box, simply delete any text that was typed into the box and press the enter key. The value in brackets (if there was one) will remain the value of the parameter. To exit from a select box, press either the end key or the escape key.

**Examining an Optical System**

An optical system loaded by the program may be viewed by two methods. The first is by using the "sketch window." The sketch window provides a brief description, or sketch, of the components used in the system, as well as a telling the user the important values associated with each component. A sample sketch window is shown in Figure 49.
Figure 49. A sample system description window showing how optical components are presented to the program user.

In and Out indicate the index of refraction the ray is traveling in before intercepting the first surface in the optical system and the index of refraction seen by a ray exiting the optical system, respectively. From the figure above, the representation of the components can be easily understood. This particular system happens to be the front end of the integrated optical system.

The second means by which a system may be viewed is by using the "graphics" screen. Upon selecting "Graphical System" from the "Graphics" pulldown menu, the program will generate a visual representation of the system. From the list of components shown above, the optical system will appear in Figure 50.
Figure 50. The system described in Figure 49 as seen using the SimII Graphics screen.

The resulting image on the graphics screen is scalable and translatable, as well as presenting the components using a color code scheme.

**General Terminology**

Some of the options in the program present the user with choices concerning the "primary" optical system and the "Local Oscillator" system. The primary optical system is the system in which the Bragg cell is used to deflect the beam. The Local Oscillator system is the system in which the beam will not be deflected by Bragg cell, therefore it will serve as the local oscillator of the heterodyning system. To make the point clearer, consider the following diagram of the front end of the system shown in Figure 51.
Figure 51. The terminology used in describing the integrated system. The "primary optical system" refers to the optical components through which the signal beam passes. Likewise, the "Local Oscillator system" refers to the optical components through which the local oscillator beam passes. The "entire optical system" refers to the fusing of both systems.

At some points in the text, the Local Oscillator is also referred to as the "reference branch" of the system, since the local oscillator travels through the optics, and the "upper" branch, due to it's physical location in the figures. Consequently, the primary optical system is sometimes referred to as the "lower" optical system.

Optical System Entry and Manipulation: The File Pulldown

The options under the file pulldown are primarily those associated with the creation, loading, saving, and manipulation the optical system on the component level. The possible options under the pulldown can be seen from the pulldown itself:
**Descriptions of the Pulldown Functions**

*New*  Removes the current optical system from memory and allows the user to input a fresh optical system. This will place the user in macro-alter mode after any previously loaded system has been purged.

*Marco Alter* - Allows the user to add or delete elements in the optical system. The process has been designed to be fairly straightforward by presenting the user with a series of understandable selection boxes. When this option is selected, the master macro-alter box appears:

![Macro Alter Box](image)

Figure 53. The options available under the MacroAlter option of the File pulldown: either the addition or deletion of an optical component in the currently loaded optical system.

**Deleting a Component**

Upon selecting the delete option, the "sketch" window will appear on the screen. In order to delete a component, simply position the prompt so that it points to the component to be deleted, then press the enter key.

**Adding a Component**

Upon selecting the add option, another selection box will appear:

![Macro Alter Box](image)

Figure 54. The components that may be added an existing optical system.
allowing the user to select the type of component to be inserted into the system. In order to select a component, simply position the cursor near the desired component and press the enter key. Upon doing so, another window will appear, asking for the parameters of the component, as shown in Figure 55.

![Figure 55](image)

Figure 55. An example of the parameter that can be set when a component has been inserted into the optical system. In this figure a block has been selected for insertion.

Again, in order to select a parameter to change, simply position the cursor such that it points to the desired value. then press the enter key. Upon doing so, a prompt box will appear, asking for a new value to be inputted. When all the element parameters have been set to the desired values, the "Insert" key should be pressed (as indicated in the selection box title bar.) Once the "Insert" key is pressed, the cursor will appear in the sketch window. To insert the element, first position the cursor at a component that will be an immediate neighbor of the new element and press the enter key. Next, press either the up arrow key, to insert the new element before the component pointed to by the prompt, or the down arrow, to insert the new component after the component pointed to by the prompt. Below is an example of inserting new components before and after an existing component, once the new component parameters have been specified. In this case, a block possessing a thickness of 10 mm and an index of refraction of 1.0 is being added to the system, before and after the existing block.
**MicroAlter** is an option under which the parameters of an existing component may be modified (for example, changing an existing block's thickness from 10 mm to 12 mm.) The method of changing the parameters of the components is almost exactly the same as entering the component parameters using MacroAlter. When this selection is chosen, the sketch window will appear. Simply select the component whose parameters will be altered and press the enter key. A select box, which is associated with the particular element type, displaying all the component's parameters will appear. Select the parameter to change, press enter, and a box will appear asking for a the new value. Enter in the new value (or press Escape to abort.) An example process of changing the wedge angle of an already existing wedge from a value of 0.1 radians to 0.05 radians is shown in Figure 57.
Figure 57. An example of using "MicroAlter" to change the parameters of an existing component. In this case the wedge angle of an existing wedge has been altered from a value of 0.1 radians to 0.05 radians.

Load and Save - carry out the actions suggested by the names. "Load" will load in an optical system that has been saved to the diskette, purging any optical system that was currently in memory. "Save" will save the optical system that is currently in memory to the disk drive. It is suggested that the file be given a "sys" extension.

Quit will end the current optical design session and terminate the program execution.

Ray Tracing Through the System: The Tracing Pulldown

Once an optical system has been entered, it is now possible to trace varying bundles of rays through the system. The trajectory of the rays entering the system is
determined by the user entering the angle that the ray makes with the optical axis as it enters the system (given in degrees) and where the ray intercepts the first surface with respect to the optical axis (given in mm.) When selecting the "Tracing" pulldown, the following choices are presented:

![Figure 58. The options available by selecting the Tracing pulldown.](image)

**Descriptions of the Pulldown Functions**

*Ray Tracing* - This option allows the user to trace one ray though the system. The parameters required are best seen from the "Ray Tracing" selection box:

![Figure 59. The Ray Tracing selection box which indicates the parameters that can be set to describe the conditions on the input ray.](image)

As mentioned before, the ray angle is the angle that the ray makes with the optical axis as it enters the systems, specified in degrees. The ray intercept is the distance from the ray/first surface intersection to the optical axis/first surface intersection. This, and the sign convention utilized, can be best seen with a simple diagram as shown in Figure 60.
Whenever there is a Bragg cell present in the optical system, the frequency that the Bragg cell is being driven at must be specified. This is required in order to determine the deflection of the incident beam by the Bragg Cell.

When ray tracing through an optical system, of which at least one component is a binary optic, the diffracted order to be ray traced can be specified. The convention for numbering the diffracted orders is the normal convention found in the grating equation, characterized in Figure 61.
In order to visualize the result of using the ray trace option, the result is shown below for introducing a ray into the system at an angle of +15 degrees and a zero ray-intercept value:

![Ray Trace Diagram](image)

Figure 62. The graphics screen that result after a ray has been traced through the system using the Ray Tracing option.

**Beam Tracing**  By utilizing the beam tracing option the user may trace multiple rays that enter the system at the same angle with respect to the optical axis (a.k.a. rays of the same slope.) The beam trace window appears as:

![Beam Tracing Parameters](image)

Figure 63. The Beam Tracing parameters window.

The number of rays is determined by setting the "Rays in bundle" value. The term "beam tracing" is derived from the ability of the user to specify the starting distance from the optical axis and an ending distance (a.k.a. a range of y-intercept values), simulating the introduction of a beam into the system possessing a diameter equal to the distance between the two end points. By supplying a range of y-intercepts and the number of rays
In a bundle, the program will equally space the rays throughout the range and trace them through the system.

In order to visualize the result of a beam trace, a beam was introduced into the system shown previously, at an angle of +15 degrees and a diameter of 0.6 mm, and composed of 3 rays, is shown below:

![Beam Trace](image)

Figure 64. The resulting graphics screen after a beam has been traced through the system using the Beam Tracing option.

**Multi-Beam Tracing** As its name implies, it allows the tracing of many beams, where beam is defined in the above section, through the system. Executing this option first causes a dialog box to appear, asking the user to enter the number of beams to trace through the system:

![Multi-beam Trace Dialog Box](image)

Figure 65. The prompt box that appears upon selecting the Multi-Beam option.

Once this value has been entered, the user will be presented with a number of boxes requiring the same parameters as the beam trace box that will be used to define an individual beam. A typical box is shown in Figure 66.
Figure 66. The parameters that can be altered to describe an input beam. In the case of the Multi-Beam option a selection box will appear for each beam to be traced.

In order to show the results of a multi-beam trace, as compared to the other tracing option, three beam where introduced into the previous system at angles of +15, 0, and -15 degrees (all with a diameter of 0.6 mm):

![Figure 67. The resulting graphics screen after the Multi-Beam Trace option has been utilized.](image)

**Defaults** These values represent default values used in either describing the index of refraction found in the reference branch of the optical system or the Bragg Cell operating conditions. The default window is shown below:

![Figure 68. SimII default values.](image)
"Index of refraction of glass" and "index of refraction of air" are used to describe the indexes found in the reference branch of the integrated system. Upon program startup the index of refraction of the glass in the reference branch is set to the index value of BK7 at 1319nm, the operating wavelength of the system.

The operating point of the Bragg cell is defined by the remaining parameters in the default box. The "driving frequency of the Bragg cell" is the central frequency of operation defined in the Bragg deflection equation. The "optical wavelength" is the wavelength of the light passing through the system and the "velocity of sound" is the velocity of an acoustic wave traveling in the AO cell transducer. These parameters are used in defining the Bragg angle of the AO cell and in the calculation of other deflection angles.

**Viewing Generated Data: The Output Pulldown**

By utilizing the output pulldown, the user may chose which information to display about the system and the method that will be used to display it. The output pulldown appears as:

```
Output
 Trace information
 System description
 Refracting surface description
 Options
```

Figure 69. The option available under the Output pulldown.

**Descriptions of the Pulldown Functions**

*Trace information* makes the information about the ray traced using the ray tracing option in the Tracing pulldown. The information concerning the ray is presented, in a box whose contents may be scrolled up or down by the arrow keys, in the format found in Figure 70.
As mentioned previously, the information concerning the ray's trajectory is presented by specifying the equation of a line, in terms of the global coordinate system, that lies beneath the ray. Thus, the values of m and b shown in the box indicate the slope and the y intercept of the ray in terms of the global coordinate system. As noted in the figure, the intercept number (the number of times the trajectory of the ray has been alter) is not necessarily equal to the number of the surface. This occurs primarily due to the presence of ray deflecting elements, such as the binary optic and the Bragg Cell. When ray tracing through the components, intercept #1 will be at the face of the component, intercept #2 is found in the center of the element (where the deflection has been model to occur,) and the third intercept is found where the ray exits the component. Hence, the element only possesses two surface but the ray trace information will be composed of three intercept values. When there exists a question concerning the location of a particular intercept value, the intercept numbers displayed using options under the graphics pulldown. This will be discussed in a later section.
System Description will place the System Description window on the screen. Upon selecting a component and pressing the <return> key, a more detailed description of the component will appear. This is shown in the figure below:

![Detailed Description](image)

Figure 71. A detailed description of a component obtained by selecting System Description from the Output pulldown.

Refracting Surface Description shows the parameters describing the refracting surfaces of the system in terms of their Cartesian forms. For example, the planer surface of a block is described by the slope, x intercept, and y intercept of the line on which the surface lies. The physical limits of the surface are also indicated. Upon selecting a component from the System Description, the two faces of the component are presented, as shown below:

![Planar Surface Parameters](image)

![Spherical Surface Parameters](image)

Figure 72. Viewing the information concerning the refracting surfaces of a particular component in the system. The information on the left represents the surface of the component that is towards the incident ray and the information on the right represents the surface away from the incident ray. In the figure the surfaces of a plano-convex lens have been selected for display.
Options  By setting the option in the "Options" box, one can direct the output of the information to either the screen, a file, and/or to the printer, as well as determine if the information will either be about the primary optical branch and/or the reference branch of the optical system.

![Options Table]

Figure 73. The options available for either selecting the data set to be observed (either the signal or local oscillator branch an optical system) or channeling the data to an desired destination.

In order to toggle an option on or off, simply place the highlight bar over the option desired option and press the enter key.

Viewing the Optical System: The Graphics Pulldown

The options shown under the graphics pulldown allow the user access to the graphics mode of SimII. The options available are:

![Graphics Pulldown]

Figure 74. The Graphics pulldown.

Descriptions of the Pulldown Functions

Graphical System  This is the gateway to the graphics portion of the SimII. By selecting the graphics mode, SimII will display a cross-section of the optical system, as described by the user input and aligned according to it's algorithm. For example, consider a system entered as (numbers and arrows have been included in order to illustrate the correspondence between the sketch window and the graphical representation:)

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When viewed using the graphics mode, the system will appear as:

While in graphics mode, the user can translate and scale the optical system. Translation is accomplished by the use of the arrow keys, i.e. the up arrow key will move the system upward on the screen. The picture may be scaled upward, by pressing the page-up button, or downward, by pressing the page-down key. The scaling of the system is done about the center of the screen. Therefore, in order to achieve a closer view of an
area of the system, place that area near the center of the screen and press the page-up key several times.

**Options** - This option menu controls some of the parameters involved in displaying the image on the graphics screen. The selections are shown below:

```
Show surface numbers [ ]
Draw only refracting surfaces [ ]
Draw primary optical system [x]
Draw reference optical system [ ]
Draw current beam(s) [ ]
Draw current ray(s) [ ]
Label Surface Points [ ]
Flip about y axis [ ]
Flip about x axis [ ]
Current scale factor [ ]
X screen origin [ ]
Y screen origin [ ]
```

Figure 77. The options available for altering the appearance and what is to be displayed on the graphics screen.

**Show Surface Numbers** Displays the number of each surface, on the optical axis, at the surface. It will also display the intercept number if a ray is being shown on the graphics screen. The result of selecting this option is shown below:

![Surface Numbers](image)

Figure 78. The resulting of turning on the "Show Surface Numbers" option of the Graphics Options.
**Draw Only Refracting Surfaces** will only draw the portions of the surfaces that are shown using the Surface Information option available in the Output pulldown.

**Draw Primary Optical System, Draw Reference Optical System** - This options are used to determine which optical system will be shown in the case when the reference branch of a heterodyning system is present.

**Draw Current Beam(s)** This will allow the user to view the last beam traced using either the beam trace or multi-beam trace options under the Tracing menu option. The results of this option can be seen in the figures found in the tracing section.

**Draw Current Ray** - This will display the last ray traced through the system.

**Label Surface Points** - This will label the end points of the surfaces. Due to the nature of the surface alignment, the endpoints labeled may not correspond to the endpoints shown on the graphics screen. An example of the labeled surface points is shown below:

![Diagram](image)

Figure 79. An optical system with its surface endpoints labeled by turning on the Label Surface Points option of the Graphics Options.
Flip about x axis, Flip about y axis - This function does exactly what it states, mirroring the optical system about either the y and/or the x axis. This function is used in some cases to put the optical system into a more intuitive orientation.

Current scale factor, X screen origin, Y screen origin - This allow the user to set the optical system scaling factor for the graphics mode of the system, and the screen coordinates where to place the center of the first surface of the optical system. This functions are used primarily to recover the graphics image of the system if it moves too far off the screen. The origin of the screen coordinate system is at the upper left of the computer screen. The positive x axis is to the right of the screen origin and the positive y axis is down from the origin. Typical maximum screen coordinate values are (640, 480) for a VGA monitor and (320, 200) for EGA monitors.

Display Data file - Allows the user to plot the contents of a data file on the screen. The data is stored in the form of a text file, composed of two columns of numbers. The first column specifies the x coordinate of a data point and the second column the y coordinate. SimII will read in the data file and plot the points on the screen in a graphics mode, labeling the axis endpoints, as well as indicating the maximum and minimum values found in the data. A typical plot appears as shown in Figure 80.

---

Figure 80. An example of a plot that can be generated by using the Plot option in the Graphics pulldown. The limits of the x and y axis are printed on the plot as well as the maximum and minimum values found in the data.

**An Important Note About Using the Graphics Screen for Analysis**

The graphics screen should not be heavily relied upon in the analysis of the system in some circumstances due to the presence of round off errors when the program draws the optical system on the screen. As mentioned, the computer screen is composed of a finite number of pixels, each of which is accessible by a pair of integer numbers. Since the coordinates of the system will be calculated and scaled as real numbers, some rounding errors will occur as the values are truncated to screen coordinates. Under this condition, some line that should appear parallel may not especially if the optical system is being viewed using a small magnification value. This effect is especially noticeable in
the following series of figures. Each figure represents the same system and the same coordinate values, only the scaling has been increased in order to minimize the effects of round off error. The output of the optical system should be 3 rays that represent a collimated beam.

Figure 81. The magnification of the optical system is relatively small, resulting in a noticeable manifestation of the round off error. The lower ray defining the beam appears that it will intersect the ray defining the optical axis, even though the numbers found in the ray data confirm that the rays are indeed parallel.

Figure 82. The magnification of the optical system has been increased, resulting in a less noticeable effects of the round off error. However, in this case it appears that the optical axis is not parallel to the two rays defining the outside of the beam. This graphical optical system appears to be presenting different information than the previous figure, even though they are generated from the same data set.
Figure 83. The system has been further magnified, resulting in almost the complete elimination of the visual effects of the round off error.

Figure 84. This system has been magnified several times, centering of the termination of the output rays. From the figure it can be seen that the effects of round off error now appear to be negligible.

Evaluation of the Integrated System: The Simulation Pulldown

As stated before, the primary purpose of SimII is to aid in the design and evaluation of the integrated optical system. It should be noted that the designation of some component names are different than those assigned to the system in the section describing the design of the integrated system. In SimII, CL3 corresponds to CL2 in the integrated design section. Also, CL4 in SimII corresponds to L1 in the description of the system design. All other components carry the same names. The menu option "Simulation" has been added to expressly deal with components and component positions found in the integrated system. The options available under the Simulation pulldown are:
Descriptions of the Pulldown Functions

**AO Collimation Check**  Selecting this option will cause the program to trace several rays through the system in order to allow the user to evaluate how well collimated a ray fan associated with a particular time delay is as it exits CL4. The option will prompt the user to enter the value of the a y-intercept of an input ray (this will fix a particular delay point on the Bragg Cell) and the upper and lower driving frequencies of the Bragg Cell (in order to define the upper and lower limits of the ray fan.) Once the parameters have been specified and the action executed, the following information will be presented on the screen:

<table>
<thead>
<tr>
<th>Upper Marginal Ray</th>
<th>Exiting ray slope (( \mu )) : 0.226748</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Last Surface x intercept : 88.1256</td>
</tr>
<tr>
<td></td>
<td>Last Surface y intercept : 12.4146</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optical Axis</th>
<th>Exiting ray slope (( \mu )) : 0.226767</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Last Surface x intercept : 88.5328</td>
</tr>
<tr>
<td></td>
<td>Last Surface y intercept : 11.4795</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower Marginal Ray</th>
<th>Exiting ray slope (( \mu )) : 0.226748</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Last Surface x intercept : 88.5495</td>
</tr>
<tr>
<td></td>
<td>Last Surface y intercept : 10.5451</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Estimated Spot Size : 1.91687 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope Difference Upper (( \mu_1 - \mu_0 )) : -0.00008957266</td>
</tr>
<tr>
<td></td>
<td>Slope Difference Lower (( \mu_1 - \mu_0 )) : -0.00008905255</td>
</tr>
</tbody>
</table>

Figure 85. The options available in the Simulation pulldown. Most of this options are specific to the evaluation and design of the integrated optical system.

Figure 86. The output resulting from selecting the AO Collimation check located in the Simulation pulldown.
The upper and lower marginal rays are the rays traced through the system entering at a value equal to the specified y-intercept and experiencing the Bragg cell being driven at the high and low frequencies, respectively. The "Statistics" box gives the estimated spot size of the resulting beam (found by taking the distance between the two rays as they exit the lens CL4) and the difference between the slope of the rays and the optical axis. The goal of this simulation is to allow the user to adjust the system parameters until the slope difference approaches 0, i.e. the rays will be parallel to each other and thus define a collimated beam. For design purposes, ray fans originating at maximum and minimum time delays on the Bragg Cell must also be collimated.

**Lens Displacement** Can be used to slide CL3 along the face of the spacer it is butted against in order to evaluate the amount of beam steering that can occur from the misplacement of CL3. The process can be best visualized by the use of the figure below:

![Diagram of lens displacement](image)

**Figure 87.** The visualization of what SimII is simulating by selecting the Lens Displacement option from the Simulation pulldown.
The parameters that can be set for the simulation are shown in the select box that appears upon choosing the lens-displacement option:

<table>
<thead>
<tr>
<th>Lens-Steering Parameters, F4 to EXE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward lens displacement : 1.00000 mm</td>
</tr>
<tr>
<td>Lower lens displacement : -1.00000 mm</td>
</tr>
<tr>
<td>Number of steps : 50 pts</td>
</tr>
<tr>
<td>Input ray angle : 0.00000°</td>
</tr>
<tr>
<td>Input ray height : 0.00000 mm</td>
</tr>
<tr>
<td>Bragg cell Driving frequency : 1.3e+09 Hz</td>
</tr>
</tbody>
</table>

Figure 88. The parameters involved in the Lens Steering simulation.

The upper and lower limits of the lens displacement are set relative to the optical axis of the local oscillator branch of the system. The resolution of the simulation is controlled by setting the "number of steps," which defines how many evenly spaced lens positions will be evaluated. In order to quantity the amount of beam steering that occurs due to the displacement of CL3, a ray is traced through both the altered local oscillator branch and primary system. The parameters of the ray are defined by the last three options in the box. When the simulation is executed (by pressing the F4 key,) the user will be prompted for a file name. Into this file will be written the value of the lens displacement from the optical axis and the difference in slope between the rays resulting tracing through the altered upper and lower systems as the rays exit CL4. Thus, this information will indicate the amount of misalignment in the beam traveling through the two systems. The information is stored in a text format the is readable by the "plot" option under the "Graphics" pulldown.

**Binary optic tracing** - This option will trace the integrated system input beam through the binary optic, generating the five diffracted orders in the plane of the system.

**Skew Optical Axis**  Allows the user to slightly rotate portions of the optical system such that the surface of the component is no longer parallel to the optical axis.
This is accomplished by introducing a component into the system called a "tolerance wedge." By setting the amount of "wedge angle," the optical system can be "skewed." This can be used for finding system tolerances.

Adjust Spacer # 2 Length - Generates a "plot-able" data file that contains the difference in the slope of a rim ray traced through the upper portion of the system and the optical axis after CL4 as a function of the length of spacer 2.

Offset(Tspacer1) - Generates a "plot-able" data file containing the ray offset as the height of Spacer 1 is changed.

Component Elongation - Generates a "plot-able" data file contains the difference in the slope of two arbitrary rays traced through the lower portion of the system after an arbitrary surface as a function of an arbitrary components thickness.

Align Spacer2 - Generates a "plot-able" data file that contains the distance between an arbitrary ray/surface intersection in the upper system and an arbitrary ray/surface intersection traced through the lower portion of the system at an arbitrary surface in the system.

Optimize CL4 (lower) - finds combinations of focal length and thickness of CL4 for which the output of the Bragg Cell is collimated.

Optimize CL4 (upper) - finds combinations of focal length and thickness of CL4 which collimate the rays traveling through the upper branch of the system

Optimize CL3 - for a given CL4, finds combination of focal length and thickness for CL3 to collimate the beam in the upper branch of the system

DMD Operation - If a deformable mirror is present in the system, this option will allow the user to rotate the mirror a specified angle
Proper Splitter Use For the proper analysis of the system performance, some beam splitters must be looked at as either transmitting or reflecting at different times. This option controls the way the rays interact with the beam splitters.

Graphical Overlap - Traces the rays required for a visual analysis of how well the local oscillator overlaps all possible outputs of the Bragg cell. The rays are viewed using the "Graphical System" option.

Adjust CL3 parameters Allows one to adjust the parameters of CL3 (i.e. thickness, focal length, etc.)

True Thickness Optimize Adjusts the thickness of a particular component until the difference in the slopes of 2 arbitrary rays is less than a given tolerance.

```
<table>
<thead>
<tr>
<th>Optimize Thickness Parameters, F4 to EXE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component #</td>
</tr>
<tr>
<td>Enter tolerance</td>
</tr>
<tr>
<td>Test input ray angle</td>
</tr>
<tr>
<td>Test input ray height</td>
</tr>
<tr>
<td>Test Bragg cell Driving frequency</td>
</tr>
<tr>
<td>Compare Ray Slope after Intercept</td>
</tr>
<tr>
<td>Control input ray angle</td>
</tr>
<tr>
<td>Control input ray height</td>
</tr>
<tr>
<td>Control Bragg cell Driving frequency</td>
</tr>
<tr>
<td>Initial Search Step Size</td>
</tr>
</tbody>
</table>
```

Figure 89. The parameters available for governing the optimization of a component in the system.

Method of Optimization

SimII uses a rather simple optimization technique which searches an error surface composed of absolute error values in a binary-type fashion until a local minimum is found. The algorithm begins by evaluating the optimizing quantity at the current component length, at the component length plus the initial search step size, and at the component length minus the step size. Upon deducing which value the smallest compared to the start value, the algorithm will move the next starting point to that value. This process continues until no smaller value can be found. At this point the search step
size is halved and the search process is repeated again. The reduction and halving of the step size continues until the absolute value of error falls below the given tolerance.

It should be noted that the quantity to be optimized may not reach an exact value of zero. This is due to the round off error associated with the computer inability to carry out calculations with infinite precision.
VIII. Calculation of Signal to Noise Ratio

The Form of Optics Related Signal to Noise Ratios

By definition, the signal to noise of a random variable, in this case of the current resulting from the detection of optical radiation, is given by the expression\(^24\):

\[
SNR = \frac{\text{mean}^2}{\text{variance}} = \frac{\overline{i^2}}{\sigma_i^2}
\]

(149)

where \(\overline{i^2}\) is the mean-squared value of the random variable and \(\sigma_i^2\) is the associated variance.

The expression can be made more specific by considering the classic noise sources in an optical detection process the effect the photodetector current. Specifically, the expression for the signal to noise ratio of an optical detection process\(^25\) is given by:

\[
SNR = \frac{\langle i_{\text{seg}} \rangle^2}{\sigma_{i,\text{shot}}^2 + \sigma_{i,\text{dark}}^2 + \sigma_{i,\text{Johnson}}^2}
\]

(150)

where \(\sigma_{i,\text{shot}}^2\) is the noise power due to shot noise, \(\sigma_{i,\text{dark}}^2\) is the noise power due to the dark (or leakage) current in the photodetector circuit, and \(\sigma_{i,\text{Johnson}}^2\) is the noise power due to thermal conditions. The noise power quantities will be detailed in the following sections.

Signal Power

The expected value of the current originating from the detector is equal to the responsivity of the detector times the expected value of the optical power incident on the detector. The responsivity is a figure of merit which describes the efficiency with which

---


photons incident on the detector surface result in electrons being released to form the detector current. Thus, the expected value of the detector (signal) current\(^{26}\) is given by:

\[
\langle i_{\text{sig}} \rangle = \eta \langle P_s \rangle = \frac{\eta q \langle P_s \rangle}{\hbar \omega}
\]  

(151)

where \(\eta\) is the quantum efficiency or the efficiency with which photons result in electrons, \(q\) is the fundamental charge of an electron, and \(\hbar \omega\) is the energy stored in a photon.

In a heterodyne detection scheme, the expression for the mean squared signal current\(^{27}\) becomes:

\[
\langle i_{\text{sig}}^2 \rangle = 2 \langle P_s P_l \rangle \left( \frac{\eta q}{\hbar \omega} \right)^2
\]  

(152)

where \(P_s\) is the power in the signal beam and \(P_l\) is the power in the reference, or local oscillator, beam.

**The Classical Noise Sources**

There exist several noise sources that contribute to the degradation of a signal present in any optical system. These noise sources are called shot noise, Johnson (or thermal) noise, and noise resulting from the detector's dark current. The derivation of the expressions quantifying the noise powers is rather lengthy and not the main thrust of this report. For further understanding, the reader is directed to several references on the subject such as *Optical Radiation Detectors* by Dereniak and Crowe, *Optical Electronics* by Yariv Chapters 10 and 11, and *Fundamentals of Photonics* by Saleh and Teich.

**Shot Noise**

---


Shot noise in the output signal occurs from the impacting of the photons on the surface of the detector. The expression for shot noise\textsuperscript{28} is given by:

\[ \sigma_{i,\text{shot}}^2 = 2q\langle P_s \rangle B \]  

(153)

where \( q \) is the fundamental charge of an electron, \( \langle P_s \rangle \) is the average optical power received, and \( B \) is the bandwidth of the system.

**Johnson Noise**

Johnson noise results from the random movement of carriers in resistive electrical materials due to the excitation of the carrier by thermal energy. The variance of the current, due to finite temperature, in a resistive element\textsuperscript{29} is given by:

\[ \sigma_{i,\text{Johnson}}^2 = \frac{4kTB}{R} \]  

(154)

where \( k \) is Boltzmann's constant. \( T \) is the operating temperature of the system, \( B \) is the system bandwidth, and \( R \) is the resistance found in the circuit.

**Dark Current Noise**

Dark current noise results from the ever present generation of electron/hole pairs in the detector, which are generated in the absence of light due to either thermal conditions or tunneling\textsuperscript{30}. The value of the dark current is generally given on the specification sheet for a particular detector. The resulting noise value associated with the dark current\textsuperscript{31} is given by:

\[ \sigma_{i,\text{dark}}^2 = 2q_iB \]  

(155)

where \( q \) is, again, the fundamental charge of an electron, \( i_d \) is the value of the dark current, and \( B \) is the bandwidth of the system.

Substituting in the explicit expressions for the noise powers into equation 150 and taking into account that the optical system is a heterodyne system, the SNR expression\(^{32}\) becomes:

\[
SNR = \frac{2P_sP_i \left( \frac{\eta q}{\hbar \omega} \right)^2}{2q \left( P_s + P_i \right) B + \frac{4kTB}{R} + 2qi_dB} \tag{156}
\]

Taking into account the gain due to amplification in the circuit and the photodetector gain by allowing \( q \) to become \( q \) multiplied by the product of all system gain\(^{33}\), the SNR expression becomes:

\[
SNR = \frac{2P_sP_i \left( \frac{\eta q G_{\text{amp}} G_{\text{pin}}}{\hbar \omega} \right)^2}{2qG_{\text{amp}} G_{\text{pin}} \left( P_s + P_i \right) B + \frac{4kTB}{R} + 2G_{\text{amp}} G_{\text{pin}} q_i_dB} \tag{157}
\]

Since the form of the signal to noise ratio is established, what remains is determining the amount of power in the signal and reference beam based upon the total power available and the transmission efficiency of the optical components in the system. The expression are derived by multiplying the total optical input into the system and multiplies that by each diminishing optical transmission factor the beam experiences.

The signal beam encounters the half wave plate used in partitioning the power, BS1, the Bragg cell, BS2, the Binary optic, and finally the polarizer before illuminating


the detector. Therefore, the signal power, in terms of the total optical power available will be:

\[ P_s = P_{in} T_s T_{\text{cube}} T_A O T_{\text{cube}} T_{BO} T_p \]  \hspace{1cm} (158)

where TBO represents the total transmission efficiency of the binary optic. This value is equal to:

\[ T_{BO} = BO_s BO_{eff} BO_u \]  \hspace{1cm} (159)

\[ T_{BO} = \left( \frac{1}{15} \right) (0.73)(0.7) = 0.02044 \]  \hspace{1cm} (160)

The values of the transmission factors and the variable definitions are shown in Table 6. Substituting the values from Table 6 into expression 158:

\[ P_s = 150(0.5)(0.95)(0.16)(0.95)(0.02044)(0.5) \text{mW} \]  \hspace{1cm} (161)

\[ P_s = 0.11068 \text{ mW} \]  \hspace{1cm} (162)

The power in the reference beam can then be found by noting that the power in the beam is that portion of the total power which has not been channeled into the signal beam by the half wave plate and that the beam travels through BS1, BS2, the Binary optic, and then finally through the polarizer reaching the detector. Therefore, the power in the reference beam is:

\[ P_r = P_{in} (1-T_s) T_{\text{cube}} T_{\text{cube}} T_{BO} T_p \]  \hspace{1cm} (163)

\[ P_r = 150(1-0.5)(0.95)(0.95)(0.02044)(0.5) \text{mW} \]  \hspace{1cm} (164)

\[ P_r = 0.69177 \text{ mW} \]  \hspace{1cm} (165)

A preliminary value can now be assigned to signal to noise ratio by substituting the value of the signal and reference beam power, as well as those parameters required from Table 6, into equation 157. The resulting value is:

\[ \text{SNR}_{\text{dB}} = 10 \log_{10} \left( 2.227 \times 10^6 \right) = 63.478 \text{dB} \]  \hspace{1cm} (166)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>A property of...</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Resistance</td>
<td>Detection Circuit</td>
<td>$R_L$</td>
<td>50 Ω</td>
</tr>
<tr>
<td>Responsivity of Detector</td>
<td>Detector</td>
<td>$R$</td>
<td>0.76 A/W</td>
</tr>
<tr>
<td>Dark Current</td>
<td>Detector</td>
<td>$i_d$</td>
<td>0.3 x 10^{-6} A</td>
</tr>
<tr>
<td>System Bandwidth</td>
<td>Overall System</td>
<td>$B$</td>
<td>200 x 10^{6} Hz</td>
</tr>
<tr>
<td>Total Output Power of Laser</td>
<td>Laser Source</td>
<td>$P_{in}$</td>
<td>150 mW</td>
</tr>
<tr>
<td>Transmission Efficiency of Bragg Cell</td>
<td>Bragg Cell</td>
<td>$T_{AO}$</td>
<td>0.16</td>
</tr>
<tr>
<td>Transmission Factor of Polarizer Preceding</td>
<td>Optical System</td>
<td>$T_p$</td>
<td>0.5</td>
</tr>
<tr>
<td>Transmission Efficiency of BS1 or BS2</td>
<td>Optical System</td>
<td>$T_{cube}$</td>
<td>0.95</td>
</tr>
<tr>
<td>Fraction of Total Input Power Diverted to</td>
<td>Optical System</td>
<td>$T_S$</td>
<td>0.5</td>
</tr>
<tr>
<td>Signal Beam</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain of the Photodiode</td>
<td>Detector</td>
<td>$G_{nin}$</td>
<td>1</td>
</tr>
<tr>
<td>Gain of the Amplifier</td>
<td>Detection Circuit</td>
<td>$G_{amp}$</td>
<td>0</td>
</tr>
<tr>
<td>Splitting Ratio of Binary Optic (BO)</td>
<td>Optical System</td>
<td>$B_O s$</td>
<td>1/25</td>
</tr>
<tr>
<td>Overall Loss due to Absorption and</td>
<td>Optical System</td>
<td>$B_O eff$</td>
<td>0.73</td>
</tr>
<tr>
<td>Scattering by the BO</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency of Beam to Beam Transfer of BO</td>
<td>Optical System</td>
<td>$B_O u$</td>
<td>0.7</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>Operating Conditions</td>
<td>$T$</td>
<td>300 K</td>
</tr>
</tbody>
</table>
Additional Considerations in SNR: Effects of Non-Ideal Detector Size

An important factor left out of the above SNR is the role that the detector size plays in determining how much of the heterodyne signal may be utilized in the generation of an electrical signal. It has been shown by VanderLugt\textsuperscript{34} that, in the analysis of heterodyning system, the detector size can significantly affect the strength of the heterodyne signal. The effect, in the context of the integrated system operation, must be explored before a final estimate of the signal to noise value can be made\textsuperscript{35}.

In general, the electrical signal for a heterodyne process is proportional to the value derived by integrating the time varying intensity pattern, resulting from the coherent interference between the signal and the reference beam, over the surface of the optical detector\textsuperscript{36}. By varying the detector size, more or less of the interference pattern is captured by the detector, which can result in a significant degradation in system performance\textsuperscript{37}. In the following derivation, an expression is obtained which may be numerically evaluated to determine the percentage of heterodyne power that is converted into an electrical signal, based upon the detector size. This percentage will also be a function of true time delay, as each true time delay has a distinct spatial frequency associated with it (as shown in Section VI) which will according alter the interference pattern between the two beams.

\textsuperscript{35} This derivation is based upon the work presented by Dr. Henry Zmuda concerning the calculation of the signal to noise value for the integrated photonic delay line system.
Before beginning quantifying the effects of the detector size on the SNR, the electric field of the signal beam and that of the reference beam must be determined. Once the expression for the electric fields are found, an expression for the intensity pattern across the detector plane can be determined. Upon integrating the intensity over the area of the detector, an expression for the usable percentage of the incident heterodyne power may be found and incorporated into the calculation of the signal to noise ratio.

**Determination of the Signal and Local Oscillator Electric Fields**

In an effort to simplify the concepts in this section, the behavior of the electric fields will be examined first in the single photonic delay line configuration and then extrapolate to the integrated system. For convenience, the layout of the delay line configuration is shown below:

![Diagram](137)

Figure 90. The configuration of the single photonic delay line system.

**The Signal Beam**

Traced through the system in Figure 90 is a ray fan associated with a particular time delay. Recalling that the tilting mirror is at the location of the intersection of all collimated ray fans, leads to a picture of a cylindrical wave, reflecting off of the mirror.
The cylindrical wave reflecting off of the mirror is composed of a continuum of collimated ray fans forced to converge on the tilted mirror by the lens. The situation can be made clear by referencing the figure below:

![Collimated Ray Fans](image)

**Figure 91.** The convergence of the collimated ray fans on a tiltable mirror. One can see that as the density of ray fans increases, it is apparent that the situation represents a cylindrical wave converging on the mirror surface. Therefore, the signal beam seen in the system, after the mirror, may be expressed as a cylindrical wave, originating at the surface of the mirror.

Therefore, the field propagating from the mirror, assuming that the mirror is not tilted, can be expressed as:

\[
E_s(x, z) = A_s \exp \left[ -j \frac{2\pi}{\lambda z} x^2 + j2\pi(f_o + f_s)t \right]
\]

(167)

where \(z\) is the axial distance from the mirror and \(x\) defines the coordinate axis perpendicular to \(z\), lying in the plane of the diagram. The variables \(f_o\), \(f_s\), and \(\lambda\) stand for the optical frequency, the RF signal frequency (placed on the beam by the Doppler shift at the Bragg cell,\) and the wavelength of the optical source, respectively.

In order to bring into the account the effect of tilting the mirror, a plane wave component is used to modify the expression indicating the steering direction of the beam.
Since the mirror will be altering the path of the beam only in the x direction, the expression of the field becomes:

\[
E_s(x, z) = A_s \exp \left[ -j \frac{2\pi}{\lambda z} x^2 + j2\pi(f_s + f_t)t + j(f_s x + f_z z) \right] \tag{168}
\]

The functional form of the spatial frequency, \( f_x \), may be derived by returning back to the relationship defining the required mirror tilt angle as a function of TTD. It should be noted that the spatial frequency, \( f_z \) will be suppressed and interpreted as only a relative phase shift between the signal and reference beam. This shall become apparent in later sections.

As stated in the section describing the operation of a photonic delay line, the mirror tilt angle of the mirror is related to the true time delay by (using the small angle approximation):

\[
\theta = \frac{vT}{2F} \tag{169}
\]

where \( \theta \) is the mirror tilt angle, \( v \) is the velocity of sound in the AO transducer, and \( F \) is the effective focal length of the lensing system.

The resulting spatial frequency of the plane wave\(^{38} \) is given by (in the small angle approximation):

\[
f_x = \frac{\theta}{\lambda} \tag{170}
\]

where \( f_x \) is the spatial frequency.

By substituting equation 169 into equation 170, the relationship between the TTD and the spatial frequency is apparent:

\[
f_x = \frac{vT}{2\lambda F} \tag{171}
\]

The Reference Beam

The reference beam, as shown in Figure 90, is simply a plane wave, traveling normal to the face of the detector. The form of the local oscillator field is given by:

\[ E_{LO}(z) = A_{LO} \exp\left[j2\pi f_o t + j\frac{2\pi}{\lambda} z + j\Phi\right] \] (172)

where \( \Phi \) is a phase shift with respect to the signal beam.

Determination of Intensity Pattern at Detector Plane

The detector plane is located in the x,y plane at a distance \( D \) from the surface of the tiltable mirror. At this plane, the electric field of the signal can be written as:

\[ E_s(x) = A_s \exp\left[-j\frac{2\pi}{\lambda D} x^2 + j2\pi (f_o + f_s)t + jf_s x + j\Phi'\right] \] (173)

The constant phase term, \( \Phi' \) is generated by fixing the value of \( z \). Likewise, the local oscillator field, at the detector plane, can be written as:

\[ E_{LO} = A_{LO} \exp\left[j2\pi f_o t + j\Phi''\right] \] (174)

where \( \Phi'' \) is a constant phase term representing the phase shift accumulated in the propagation of the field to the detector plane. Both field may be further phase shifted, by choosing a specific moment in time when the phase shift of the signal beam is zero. The resulting expressions are:

\[ E_s(x) = A_s \exp\left[-j\frac{2\pi}{\lambda D} x^2 + j2\pi (f_o + f_s)t + jf_s x\right] \] (175)

\[ E_{LO} = A_{LO} \exp\left[j2\pi f_o t + j\Phi\right] \] (176)

where \( \Phi \) represents the phase shift of the local oscillator field with respect to the signal field.

The intensity pattern from the interference of two electric fields\(^{39}\) is given by:

\[ I = \frac{|E_s + E_{LO}|^2}{2\eta} \]  

(177)

where \( \eta \) is the intrinsic impedance of air and is expressed in terms of power per area.

Upon substitution of equations 175 and 176 into the expression for intensity, the following expression results:

\[ I = \frac{1}{2\eta} \left[ E_{LO}^* E_{LO}^* + E_s^* E_s^* + E_{LO} E_{LO}^* + E_{LO}^* E_s \right] = I_a + I_b + I_c + I_d \]  

(178)

The first term in the expression becomes:

\[ I_a = \frac{1}{2\eta} A_{LO} A_{LO}^* \]  

(179)

This is simply the intensity of the local oscillator beam. In order transform the equation into values that are available in the system design, the intensity expression should be rewritten in terms of the power density. The power density, expressed as terms of power per unit area, is given by\(^\text{40}\):

\[ P_{LO} = \frac{P_{LO}}{\text{Area of illumination}} \]  

(180)

\[ P_{LO} = \frac{P_{LO}}{\pi \left( \frac{d_{LO}}{2} \right)^2} \]  

(181)

where \( P_{LO} \) is the measured power in the local oscillator beam (in this case, the normalized power,) \( d_{LO} \) is the diameter of the local oscillator beam, and \( P_{LO} \) is the associated power density. By seeing the equality of the units for intensity and power density, the intensity of the local oscillator may set equal to the power density:

\[ I_a = P_{LO} \]  

(182)

\(^{40}\) D. Cheng, *Field and Wave Electromagnetics*, (Addison-Wesley Publishing Co., Massachusetts 1985) pg 329-331
Likewise, the second term in the intensity expression can be written as:

$$I_b = P_s = \frac{P_s}{\text{Area of Signal Beam}}$$  \hspace{1cm} (183)

The cross-terms, however, require a little manipulation before being completely simplified. Consider first the expression for $I_c$:

$$I_c = A_{lo} \exp\left[j(2\pi f_s t + \Phi)\right] A_s' \exp\left[j\left(\frac{2\pi}{\lambda D} x^2 - 2\pi (f_0 + f_s) t + f_s x\right)\right]$$  \hspace{1cm} (184)

Simplifying the exponential terms and substituting in the expressions for power density:

$$I_c = \sqrt{P_{lo}} \sqrt{P_s} \exp\left[j\left(\frac{2\pi}{\lambda D} x^2 - 2\pi f_s t + f_s x + \Phi\right)\right]$$  \hspace{1cm} (185)

Since last term in the intensity expression is the complex conjugate of the expression in equation 185:

$$I_d = \sqrt{P_{lo}} \sqrt{P_s} \exp\left[-j\left(\frac{2\pi}{\lambda D} x^2 - 2\pi f_s t + f_s x + \Phi\right)\right]$$  \hspace{1cm} (186)

Therefore, the last two terms in the intensity expression, equation 178, can be combined together to produce:

$$I_c = 2\sqrt{P_{lo}} P_s \cos\left(\frac{2\pi}{\lambda D} x^2 - 2\pi f_s t + f_s x + \Phi\right)$$  \hspace{1cm} (187)

Therefore, the total expression for the intensity pattern that results between the interference of the local oscillator field and the signal field is given by:

$$I = P_{lo} + P_s + 2\sqrt{P_{lo}} P_s \cos\left[2\pi (f_0 x + f_s t) - \frac{\pi}{\lambda D} x^2 + \Phi\right]$$  \hspace{1cm} (188)
**Determination of Percentage of Usable Heterodyne Power**

Now that an expression for the intensity has been formulated, the percentage of total power in the heterodyne signal that can be utilized by the detector must be determined. The total power collected by the detector is equal to the integral of the intensity over the surface of the detector, as shown in equation 189:

\[ P_{TOT} = \iiint_{\text{DetectorArea}} I(x,y) \, dA_{\text{det}} \]  

or:

\[ P_{TOT} = \iiint_{\text{DetectorArea}} p_{LO} + p_S + 2\sqrt{p_{LO}p_S} \cos \left[ 2\pi(f_x x + f_t t) - \frac{\pi}{\lambda D} x^2 + \Phi \right] \, dA_{\text{det}} \]  

It becomes obvious that there exists two distinct components in the solution. The first component is the result of the integration of the temporal independent components over the detector area. Since the result of this integration will be a constant value, as is displayed in equation 191, it is termed the DC component of the resulting signal and serves only to provide a DC offset to the desired signal.

\[ P_{DC} = A_{\text{Det}} (p_{LO} + p_S) \]  

The remaining portion of the integral, or AC signal portion, represents the heterodyne signal portion of the total power incident on the detector. This implies that the temporal dependent signal may be extracted from the integral, resulting in an integral expression times a cosine varying signal. Specifically, the heterodyne amplitude found in the signal to noise ratio calculation multiplied by an additional factor indicating the percentage of the heterodyne power that is utilized in the generation of the electrical signal. The signal power is expressed as:

\[ P_{SIG} = \iiint_{\text{DetectorArea}} 2\sqrt{p_{LO}p_S} \cos \left[ 2\pi(f_x x + f_t t) - \frac{\pi}{\lambda D} x^2 + \Phi \right] \, dA_{\text{det}} \]
from which the temporal dependent portion can be extracted using a cosine identity:

\[ \text{sig} \sim 2^{\text{PloPs}} \cos(2\pi f_0) \int \cos(2\pi f_x - 2\pi N) \cos \left( 2\pi f_x - 2\pi N \right) \text{d}A. \]

This expression can be reorganized into the form of:

\[ \text{Psig} = 2^{\text{PloPs}} \cos \left( 2f_0 + 0.5774 \right) \]

by using the identity:

\[ \cos((a t) + B \sin((f t)) = A^2 + B^2 \cos(at - \arctan(f)) \]

The amplitude factor of the heterodyne signal, \( C_{\text{GQ}} \), becomes:

\[ C_{\text{GQ}} = \text{Psig} \]

The phase factor, \( \beta \), can also be removed from the expression by the use of several trigonometric identities, finally resulting in:

\[ \text{Psig} = 2^{\text{PloPs}} \cos(2\pi f_x - 2\pi N) \int \text{d}A. \]

Therefore, the power collected by the detector can be expressed, in terms of the power densities, as:

\[ \text{Psig} = \text{ADet} \left( \text{Plo} + \text{Ps} \right) + \text{IPloPs} \cos(2\pi f_x - 0.5774) \]

Now that the expression for the amplitude weighting of the signal power has been given a classic heterodyne form, the role it plays in the final signal to noise calculation must be determined. The proper positioning of the term can be determined by walking through the proof of this identity, which has been carried out in Appendix C.
through a proof presented by Yariv\textsuperscript{42} describing the derivation of the signal power term for a heterodyning system.

The current from the optical detector is proportional to the power incident on the detector:

\[ i_d(t) \propto |E_{LO}|^2 + |E_s|^2 + 2E_{LO}E_s\cos(\omega t) \]  (199)

which may also be written as:

\[ i_d(t) \propto P_{LO} + P_s + 2\sqrt{P_{LO}P_s}\cos(\omega t) \]  (200)

Comparing equation 200 to equation 198, it is observed that both equations are of exactly the same form, except for the presence of an AC signal weighting value. This factor is easily included into the calculation, as shown below:

\[ i_d(t) \propto P_{LO} + P_s + 2\alpha\sqrt{P_{LO}P_s}\cos(\omega t) \]  (201)

Taking into account the responsivity of the detector as a measure of the efficiency of the conversion of the optical signal to the electrical signal, the equation can be written as an equality:

\[ i_d(t) = R\left(P_{LO} + P_s + 2\alpha\sqrt{P_{LO}P_s}\cos(\omega t)\right) \]  (202)

Assuming that the power in the local oscillator beam is much greater than that of the signal beam:

\[ i_d(t) = R\left(P_{LO} + 2\alpha\sqrt{P_{LO}P_s}\cos(\omega t)\right) \]  (203)

Since only the time dependent portion of the detector current is considered the signal, the mean-square detector signal current is given by:

\[ \overline{i_d^2} = R^2P_{LO}^2\left(2\alpha\frac{P_s}{P_{LO}}\cos(\omega t)\right)^2 \]  (204)

\[
\overline{i_d^2} = R^2 P_{LO}^2 \left( 4\alpha^2 \frac{P_s}{P_{LO}} \frac{1}{2} \right) = 2\alpha^2 P_s P_{LO} R^2
\]

(205)

which agrees with the final expression given by Yariv and shows that the signal amplitude weighting factor scales the signal power in the signal to noise expression by its square. Therefore, the signal to noise expression, taking this effect into account, becomes:

\[
SNR = \frac{2\alpha^2 P_s P_i \left( \eta q G_{amp} G_{pin} / h\omega \right)^2}{2qG_{amp}G_{pin} (P_s + P_i) B + \frac{4kTB}{R} + 2G_{amp}G_{pin} q_i q B}
\]

(206)

In order to determine the relationship between \(a\) and \(C_{SIG}\), the AC signal portion of the analysis, as seen in equation 194, must be altered to the form of the AC signal portion of equation 201, which was used to determining the final effect on the signal to noise ratio. The difference in the equations is that equation 194 is expressed in terms of power density, where equation 201 is expressed in terms of the power in the optical signal. The values of the optical power can be restored to equation 194 by substituting in expressions 181 and 183:

\[
P_{SIG} = 2 \sqrt{P_{LO} P_s} C_{SIG} \cos(2\pi f_s t - \Phi_{SIG})
\]

(207)

Equating this to equation 201, the value of the amplitude weighting factor can be seen to be:

\[
\alpha = \frac{C_{SIG}}{\sqrt{A_{LO} A_S}}
\]

(208)
Quantification of the Effect of Detector Size on SNR

It is important to note that the expression for $C_{SIG}$, equation 197, is dependent upon the desired TTD due to the presence of the spatial frequency, $f_x$, in the integral expression, which can directly be related to tapping the proper TTD. In order to provide a better understanding of the effect of the detector size on the behavior of the signal to noise ratio, the AC amplitude weighting factor, $\alpha$, was evaluated numerically using MathCad 4.0 and plotted for time delay values ranging from 0 to 98ns.

However, before the computation can be made, several values in the expression of $C_{SIG}$ must first be determined from the integrated system design.

A trivial value to find is $D$, the distance from the mirror to the detector area. The apparent propagation distance is found in the section concerning the design of the integrated system and is equal to 90.5mm divided by the index of refraction of the glass, or 60.19368mm.

Another value of paramount importance is the effective focal length of the system, for use in the determination of the spatial frequency, $f_x$, for each time delay. This value can be estimated from the thin lens model of the integrated system. Consider the diagram of the system shown below:

![Figure 92. A condensed ray diagram of the integrated system indicating the transference of a central ray of a TTD ray fan through the system. The distance, d, represents a particular point on the Bragg cell. The angle, b, is maintained through L2 by the nature of the system (see ray diagram in discussion of system design.)](image-url)
From the figure, the following relationship are apparent:

\[
\frac{DBO}{F_3} = \tan(2\theta_m) \quad (209)
\]

\[
\frac{DBO}{F_2} = \tan(\beta) \quad (210)
\]

\[
\frac{d}{F_1} = \tan(\beta) \quad (211)
\]

From equations 210 and 211:

\[
DBO = d \frac{F_2}{F_1} \quad (212)
\]

From equations 209 and 212:

\[
d = \frac{F_1(F_3)}{F_2} \tan(2\theta_m) = \frac{2\theta_m F_1(F_3)}{F_2} \quad (213)
\]

\[
\Delta T = \frac{d}{v} \approx \frac{2\theta_m F_1(F_3)}{v F_2} \quad (214)
\]

Comparing this expression to the expression for the TTD in terms of the focal length and mirror tilt in equation 169, one sees that the effective focal length is given by:

\[
F_{eff} = \frac{F_1(F_3)}{F_2} \quad (215)
\]

From the system specifications, \( F_1 \) can be estimated from the ROC of \( L_1 \) and the index of refraction of the glass:

\[
F_1 = \frac{\text{ROC}_{L_1}}{n_{BK7} - 1} = \frac{0.01865412 \text{ m}}{1.50348 - 1} = 0.03705 \text{ m} \quad (216)
\]

\( F_3 \) can be estimated from the ROC of \( L_3 \) and the index of refraction of the glass:

\[
F_3 = \frac{\text{ROC}_{L_3}}{n_{BK7} - 1} = \frac{45.76 \text{ mm}}{1.50348 - 1} = 0.09089 \text{ m} \quad (217)
\]
F2 can be estimated by using SimII to determine the paraxial focal length of the subsystem. Upon doing so, the focal length was found to be 0.0860935m. Thus, the effective focal length is:

\[ F_{\text{eff}} = \frac{0.03705(0.09089)}{0.0860935} \text{m} = 0.03911 \text{m} \quad (218) \]

The only remaining factors to be defined are the area of the detector, the area of the local oscillator beam, and the area covered by the signal beam. The detector is considered to be the coupling lens assembly at the output of the system and that all light falling on the lens can be utilized by the detector. For convenience in the integration, it is assumed that the detector is a 2x2mm square.

The area of the local oscillator is known as a design parameter for the integrated system. At the detector plane, the local oscillator possesses a diameter of 2.7mm.

The size of the signal beam at the detector plane can be estimated using SimII to trace a delay line through the system. It was found that the diameter of the beam was also approximately 2.7mm.

At this point, all parameters have been assigned values from the integrated system. The resulting MathCad calculation of \( \alpha \) is shown on the following page.
Definition of parameters:

\[ F_{eff} = 0.03911 \text{ m} \]

Effective Focal Length of the System

\[ \lambda := 1.319 \times 10^{-6} \text{ m} \]

Laser Illumination Wavelength

\[ D := \frac{90.5 \times 10^{-8}}{1.50348} \text{ m} \]

Apparent Distance from DMD to Detector

\[ v := 5125 \text{ m/s} \]

Velocity of Sound in AO Cell

\[ A_s := \pi \left( \frac{2.7 \times 10^{-3}}{2} \right)^2 \text{ m} \]

Area of the Signal Beam at Detector

\[ A_{LO} := \pi \left( \frac{2.7 \times 10^{-3}}{2} \right)^2 \text{ m} \]

Area of local oscillator at Detector

\[ T := 0.98 \text{ ns} \]

Desired True Time Delay

\[ f_T = \frac{v \cdot T \times 10^{-9}}{2 \cdot \lambda \cdot F_{eff}} \]

Spatial Frequency Corresponding to a TTD value

The integral to be evaluated in the determination of \( \alpha \). Note that since the detector defines that smallest area, it defines the limits of integration. Also note that since the function being integrated does not depend on the \( y \) coordinate, the \( y \) dimension of the detector may be removed form the integral.
The resulting plot $a$ versus the desired true time delay is shown below:

![Graph showing $a_T$ versus $T$]

Figure 93. MathCad generated plot of the AC amplitude weighting factor as a function of desired true time delay.

When calculating the signal to noise ratio, it is common practice to consider the worst case scenario for the operation of the system. In the time delay range shown, which defines the anticipated operating range, the minimum value of $\alpha^2$ was found to be 0.00819. Upon inserting this value into the signal to noise equation, formula 206, it is found that the signal to noise ratio drops to a value of 42.611dB.
Bibliography

Works Cited


Additional References


Appendix A
SimII Source Code

SimII is a DOS based program written and compiled using the Borland C++'s compiler and Integrated Design Environment. The executable form of SimII maybe recreated by compiling and linking the separate object files using the huge memory model as well as including the object files for the appropriate graphic monitor support and the Little font supplied by Borland. In order to conserve space, the actual listings of the program have been excluded. It should be noted that this program is copyrighted 1991, 1992, 1993 by Kevin Baldwin (some portions maybe copyrighted by Borland.) Any modifications, further augmentations, the use of sections of the code, and the distribution of the code (and executable) to other than the designated receivers can only be done with the express permission of the author.

The basic structure of the program code is:

<table>
<thead>
<tr>
<th>Program Section</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data.c</td>
<td>Generates the storage space required for the optical system and associated information. It has been kept separate for the remainder of the code due to the 64K page limit of the Intel processor architecture.</td>
</tr>
<tr>
<td>File.c</td>
<td>Code for all the functions found under the &quot;File&quot; pulldown (see Manual)</td>
</tr>
<tr>
<td>Graphics.c</td>
<td>Code to drive the graphics portion of SimII</td>
</tr>
<tr>
<td>Help.c</td>
<td>Contains &quot;help&quot; information</td>
</tr>
<tr>
<td>Math.c</td>
<td>Code for repeated complex mathematical functions</td>
</tr>
<tr>
<td>Opt2.c</td>
<td>Code for functions found under the &quot;Simulation&quot; pulldown (see Manual)</td>
</tr>
<tr>
<td>Optimize.c</td>
<td>Code for functions found under the &quot;Simulation&quot; pulldown (see Manual)</td>
</tr>
<tr>
<td>Output.c</td>
<td>Code for functions found under the &quot;Output&quot; pulldown (see Manual)</td>
</tr>
<tr>
<td>Plot.c</td>
<td>Code to drive the &quot;Plot&quot; function under the &quot;Graphics&quot; pulldown</td>
</tr>
<tr>
<td>Sim19.c</td>
<td>Main Program</td>
</tr>
<tr>
<td>Tracing.c</td>
<td>Code for carrying all ray tracing and surface conversion functions</td>
</tr>
<tr>
<td>Window.c</td>
<td>Code to drive the text window portion of SimII</td>
</tr>
</tbody>
</table>
Appendix B
Analysis of the Binary Optic

The entire surface relief pattern of the Binary optic is generated by tiling a single cell across its entire surface. The cell is a 32x32 pixel phase function, as shown in Figure 1B, composed of either 0 or -π phase shifts.

Figure 1B. The phase function associated with one tile of the Binary Optic. The gray regions indicate a phase shift of -π radians and the white regions a phase shift of 0 rad.

As stated, the entire surface relief pattern can be approximated by replicating the tile. This is shown in Figure 2B, where single cell has been replicated 64 times.

Figure 2B. A portion of the entire surface relief pattern on the Binary optic. This is merely the replication of the phase function shown in Figure 1B.
It is assumed that the Binary Optic will be inserted into the integrated system at the point where the light incident on its surface may be considered a plane wave. The resulting output electric field may be found using the Fraunhofer diffraction integral. The resulting output electric field will be given by:

\[ E_{\text{out}}(x', y') = \sqrt{\frac{j}{\lambda D}} e^{-jkD} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_{\text{in}}(x, y) T(x, y) e^{-j\left(\frac{2\pi}{\lambda D}(x'x + y'y)\right)} \, dx \, dy \] (219)

where \( T(x, y) \) is the transmission function of the Binary optic, \( E_{\text{in}}(x, y) \) is the electric field incident on the Binary Optic, \( D \) is the distance from the Binary optic to the observation plane, and \( x', y' \) are the coordinates of a point on the observation plane. Since \( E_{\text{in}}(x, y) \) is assumed to be a plane wave incident normally on the Binary Optic, its presence will only contribute a constant phase factor. Noting this, the Fraunhofer integral takes on the form of a Fourier transform of the transmission function of the Binary Optic:

\[ E_{\text{out}}(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T(x, y) e^{-j2\pi(f_x x + f_y y)\right)} \, dx \, dy \] (220)

where \( f_x = \frac{x}{\lambda D} \) and \( f_y = \frac{y}{\lambda D} \) are the spatial frequencies. The resulting intensity pattern is found by squaring the magnitude of the output electric field. Therefore, the intensity pattern generated when the Binary optic is illuminated by a plane wave is proportional to the squared magnitude of the Fourier transform of the transmission function of the Binary Optic.

In order to numerically find the electric field after the Binary Optic the FFT of the transmission function can be taken. The output intensity distribution after the field passes through the Binary Optic is found by squaring the magnitude of the resulting field.

A MATLAB file was written in order to carry out the calculations. The phase function of a single tile of the Binary optic was entered, replicated to an area of 256x256
pixels, and the squared magnitude of its FFT was found. The result intensity pattern is
shown in the Background section in the discussion of the Binary Optic.

The MATLAB code used in the calculation is shown below:

```matlab
% Matlab file for Determination of Binary Optic Behavior
% Written by Kevin Baldwin

clear;
% Describe the binary nature of the device
bmask = [1 1 1 1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 1 1;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0;
         0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0;
0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0;]

% Generate the transmission function
bmask = exp(sqrt(-1)*pi*bmask);

% Replacate the transmission function 64 times (8x8 transmission functions)
% so that the current transmission function is 256x256 points
bmask(33:64,:) = bmask(1:32,:);
bmask(65:128,:) = bmask(1:64,:);
bmask(129:256,:) = bmask(1:128,:);

bmask(:,33:64) = bmask(:,1:32);
bmask(:,65:128) = bmask(:,1:64);
bmask(:,129:256) = bmask(:,1:128);

% Take the FFT and shift to center the DC component
Output = fftshift(abs(fft2(bmask)).*abs(fft2(bmask)));
```
mesh(Output);
title('2-D FFT|\textsuperscript{2} of binary optic transmission function');
pause;

contour(Output);
title('2-D FFT|\textsuperscript{2} of binary optic transmission function');
Proof:

\[ \cos(a) + 5\sin(a) = \sqrt{a^2 + b^2} \cos(a - \tan^{-1}(4)) \]

Let \( f(t) = \Re[Ae^{it} + Be^{it}] \), since \( M\cos(\omega t) = \Re[Me^{j\omega t}] \)

\( f(t) = \Re[4\cos(a) + 5\sin(a) + 5\sin(a) - 2\cos(a)] \)

Let \( v(t) = 4\cos(a) + 5\sin(a) + 5\sin(a) - 2\cos(a) \)

Examining the magnitude of \( v(t) \):

\[ |v(t)| = \sqrt{a^2 + b^2} \]

\[ |v(t)| = \sqrt{a^2 + b^2} \]

Examining the phase of \( v(t) \):

\[ \phi(t) = \tan^{-1}(\sin(a) - \cos(a)) \]

\[ \phi(t) = \tan^{-1}(\sin(a) - \cos(a)) \]
\[ \angle y(t) = \tan^{-1}\left( \frac{\tan(\omega t) - \frac{B}{A}}{1 + \frac{B}{A} \tan(\omega t)} \right) \] ; recalling the

formula for the tan of the sum of two angles

\[ \angle y(t) = \tan^{-1}\left( \tan(\omega t - \tan^{-1}\left( \frac{B}{A} \right) \right) \]

\[ \angle y(t) = \omega t - \tan^{-1}\left( \frac{B}{A} \right) \]

Upon reassembling the expression for \( f(t) \):

\[ f(t) = \text{Re}\left[ |y(t)| e^{ij\angle y(t)} \right] \]

\[ f(t) = \text{Re}\left[ \sqrt{A^2 + B^2} e^{j(\omega t \tan^{-1}(\frac{B}{A}))} \right] \]

\[ f(t) = \sqrt{A^2 + B^2} \cos(\omega t - \tan^{-1}(\frac{B}{A})) \]

Hence:

\[ A \cos(\omega t) + B \sin(\omega t) = \sqrt{A^2 + B^2} \cos(\omega t - \tan^{-1}(\frac{B}{A})) \]
Appendix D

Block Diagram of the Integrated System