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A Digital signal processing-based predistortion technique for reduction of intermodulation distortion

Richard James Buckley

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A DIGITAL SIGNAL PROCESSING-BASED PREDISTORTION TECHNIQUE FOR REDUCTION OF INTERMODULATION DISTORTION

by

Richard James Buckley

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of
MASTER OF SCIENCE in Electrical Engineering

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ROCHESTER INSTITUTE OF TECHNOLOGY
ROCHESTER, NEW YORK
NOVEMBER, 1993
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PREFACE

The work presented herein represents original thoughts on my part, the result of combining knowledge gained at Harris Corporation with academic study of DSP techniques at the Rochester Institute of Technology. This thesis is an example of how new technologies can be successfully applied to a mature field of study.

I would like to thank Prof. M. R. Raghuveer for his guidance, patience, and encouragement throughout the course of this work; my interest in this area was sparked by his teaching of DSP-related courses at RIT. Prof. J. D. DeLorenzo and John Lundberg provided insightful comments on the practical applications of this work. Floyd Koontz, J. Christopher Jones, Tom Kenney, Cliff Hessel, Ron Hepler and Douglas Zak were all active listeners when I needed to discuss ideas or problems. Duane Reid, Robert Broccolo and Bruce Florack were extremely helpful in my setting up of DSP experiments, providing the training required to get me started in an unfamiliar field.

I would also like to thank my wife Ann, for her help in proof-reading my draft copies, and for her support in general.

Richard J. Buckley
Linearization of power amplifiers has been the topic of many studies, dating back to the work of H. S. Black in the 1920s. For many applications, the well-documented techniques of feedforward and feedback can be used to design low intermodulation distortion (IMD) amplifiers. However, certain applications, including the design of high-power, radio frequency amplifiers, preclude the use of these techniques. The work herein describes an alternative to presently accepted distortion reduction techniques. In-band IM distortion (multi-tone distortion located close in frequency to the desired signal), is reduced by modifying a baseband input, upconverting this signal to the transmission frequency, then performing the amplification. This allows DSP hardware to be used, resulting in a novel IMD reduction method.

The approach presented is unique in that multiple orders of nonlinearity are reduced using DSP technology, at baseband, through a commonly used method of upconversion. Existing work has addressed mostly third-order, analog solutions applied at the frequency of transmission.

Theoretical work, simulations, and experimental results are used to describe the technique. Advantages and limitations are discussed, as are areas for future work.
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LIST OF SYMBOLS

a₃  Third-order Insertion Level to Exciter
a₃,opt  Optimum value for a₃
a₁  Fundamental Level to Exciter
a₅  Fifth-order Insertion Level to Exciter
A  Single Tone Baseband Input Amplitude
B  Two Tone Baseband Input Amplitude (per tone)
bᵢ  level of iᵗʰ-order Intermodulation Tone
baseband  300 Hz - 3000 Hz information channel
Cₐ,b  constant as a function of a*w₁ and b*w₂
carrier  carrier frequency of transmission (in the HF band)
DSP  Digital Signal Processing
exciter  assembly that upconverts and amplifies
f₀  HF Carrier Frequency
f₁  Input Frequency (lower of the two tones)
f₂  Input Frequency (higher of the two tones)
HF  2 - 30 MHz
i  Order of tone
IF  Intermediate Frequency
IMD  Intermodulation Distortion
ISB  Independent Sideband
kᵢ  Coefficient of Amplifier Transfer Function
LSB  Lower Sideband
R    Load impedance (typically 600 ohms for audio, 50 ohms for HF)
SSB  Single Sideband
u    Baseband Input to Predistorter
upconvert Mix baseband input up to the HF band
USB  Upper Sideband
V_{p-p}  Peak-to-peak voltage in Volts
V_p      Peak voltage in Volts
V_{rms}  Root-mean-square voltage in Volts
w_1     Input Radian Frequency (lower)
w_2     Input Radian Frequency (higher)
W_{av}  Average power in Watts
W_{pep} Pep envelope power in Watts
x_{bb}  Baseband output from predistorter
x      HF input to amplifier
x^i     HF input x to the power of i
y      HF output from amplifier
x(n)   digitized baseband output sample
u(n)   digitized baseband input sample
1. INTRODUCTION

A. Overview

The information presented herein describes a unique approach to power amplifier linearization. This method, developed entirely through the course of this work, uses Digital Signal Processing (DSP) to modify the information transmitted through the amplifier. By altering a baseband signal using DSP, upconverting the result, then applying this signal to the amplifier, a new technique has been developed. This approach is offered as an alternative to the well-documented techniques of feedforward and feedback. In certain applications, including the design of high-power, radio frequency amplifiers, the combination of high power (1 to 50 kW) and wide bandwidth (2 to 30 MHz) preclude the use of feedforward and feedback, since these applications would require the use of components that are either physically unrealizable or are too costly to use in a practical design.

The DSP-based approach presented herein is offered as an alternative to presently accepted distortion reduction techniques. An approach has been devised that is based on the following:

i) An input signal to a non-linear amplifier may be modified such that the output is a linear version of the
original input signal.

ii) To reduce in-band intermodulation distortion (multi-tone distortion located close in frequency to the desired signal), modification of such an input signal may be done at a frequency below the amplifier's specified operating band, then upconverted to the frequency of interest. The significance of this is that DSP hardware can be used to produce a low-cost, high-performance alternative to present IMD reduction techniques.

iii) Generation of such a signal may be accomplished in real time, using digital signal processing to pass the input signal through a power series function. Laboratory tests of back-off IMD and of IMD symmetry will show that a power series suffices to describe amplifier transfer function.

The use of DSP has several advantages over analog techniques:

A. The solution can be adaptive. Most high-power transmitters have a learn, or "tune-up" sequence. The DSP approach could be run in a closed-loop fashion during this learn sequence, then operate without feedback (open-loop) during normal transmission. The solution can thus be updated over time, without concern for stability problems that plague feedback designs.

B. The solution may be applied over a wide frequency range with one circuit. For example, a 2 - 30 MHz amplifier
may be broken into sub-bands, with predistortion coefficients stored for each sub-band. To do this in an analog fashion would require different circuits for each sub-band.

C. Once the hardware is in place, DSP may be used to modify both the amplitude and phase components of the signal, thus allowing for correction of both AM-AM and AM-PM type non-linearities. The circuitry, once designed, would also have application for multiple amplifier designs.

Both theoretical and experimental results described herein demonstrate the validity of this theory. A FORTRAN program was written to model an amplifier that is typically used in high-power, wideband designs. Various situations were simulated, where incoming baseband signals were operated on in a nonlinear fashion, then applied to the input of the amplifier. The resulting IMD was reduced in all cases, when compared to the unprocessed input.

Laboratory testing was performed on a Harris RF-1310 HF Exciter. Using a general purpose DSP board, the baseband input signal was processed, then applied to the input of the HF Exciter. The output of the exciter was measured for IMD performance with and without baseband processing, with improved IMD when the baseband was processed.

Laboratory testing was also performed on a Harris RF-1310 HF Exciter driving a Harris RF-1110C 1 kW Transmitter. In this case, too, there was improved IMD when the baseband
signal was processed.

Conclusions are drawn from the comparison of theoretical and practical results. Limitations of the technique are discussed, as are areas for further investigation in the future.

B. Background

Intermodulation distortion is an unwanted by-product of amplification. IMD is defined as the residual frequency components that are generated by an amplified, multi-tone signal. These components occur due to a non-linear amplifier transfer function, that is, the output $y$ of an amplifier is equal to the input signal, $x$ times the amplifier gain, plus additional terms that are proportional to higher powers of $x$ ($x^2, x^3, \ldots$), and derivatives of $x$:

$$y(t) = f(x(t), \frac{dx(t)}{dt}, \frac{d^2x(t)}{dt^2}, \ldots)$$

(1)

In the analysis of non-linear amplifiers, a less general, more common form is written [9] as

$$y = \sum_i (k_i x^i + k'_i \frac{d^i x}{dt^i})$$

$$i = 1, 2, 3, 4, 5, \ldots$$

(2)

$k_i = \text{constant coefficient}$

$k'_i = \text{constant coefficient}$
This form is used when considering both AM-AM conversion (change of amplifier gain versus level) and AM-PM (change of amplifier phase shift versus level) conversion.

In many applications, the AM-AM component of distortion is far greater than the AM-PM component. In these instances, the input-output relationship may be further simplified [9] to

\[ y = \sum_{i} k_i x^i, \quad i = 1, 2, 3, 4, 5, \ldots \]

(3)

In a typical communications system, the even-order products, and some of the odd-order products, are located far away (in frequency) from the input tone frequencies, and can be filtered. However, some of the odd-order tone frequencies are located very close (less than .01%) in frequency to the original input signals; filtering of these products is extremely difficult.

For example, an HF (2 - 30 MHz) Single Sideband (SSB) amplifier transmits over a relatively narrow (3 kHz) channel. When a SSB signal (transmitted at a 2 MHz carrier frequency) is amplified, the desired signal resides from 2.000 MHz to 2.003 MHz. The even-order distortion products reside near 4 MHz (second harmonic), 8 MHz (4th), 12 MHz (6th), and so on. Some of the odd-order products reside at 6 MHz (3rd), 10 MHz (5th), 14 MHz (7th), and so on. All of these distortion
products are easily filtered away using conventional low-pass filtering techniques. As will be demonstrated, some of the odd-order distortion products reside near 2 MHz, some of which are actually in the information band (2.000 to 2.003 MHz). These products are difficult to filter away.

As a result, the in-band IMD is often tolerated and transmitted along with the desired signal. The highest IMD level that can be tolerated is usually listed as part of the transmitter design specification ("30 dB below one of two equal tones" is a common design goal). From a design standpoint, the IMD level is commonly balanced versus other design goals, such as efficiency, gain, stability, cost, and circuit complexity.

To maximize signal quality, and to minimize the bandwidth of the transmitted signal, it is desirable to reduce IMD to the greatest extent possible. An improvement of ~10 dB without adversely affecting other design requirements would represent a significant improvement in amplifier performance. IMD improvements greater than 10 dB would allow superior IMD performance, or allow a trade-off of improved IMD against other specifications (perhaps a higher efficiency amplifier with good intermodulation performance).
C. Problem Statement

Power amplifier transfer functions frequently exhibit a compressor characteristic as the amplifier approaches saturation. This deviation from linearity (see Figure 1) can be modeled by a power series. If this power series has odd-order terms \((x^3, x^5, x^7, \ldots)\), in-band IMD will result when the amplifier is driven by a multiple tone input.

![RF-1310 Amplifier Model](image)

**Figure 1.** RF-1310 Amplifier: Linear vs. Seventh-order Model

For example, if the amplifier has a transfer function
\[ y = \sum_{i} k_i x^i, \quad i = 1, 3, 5, \ldots \]  

and the input signal is

\[ x = A(\cos w_1 t + \cos w_2 t) \]  

\( y \) may be written as

\[ y = \sum_{i} k_i [A(\cos w_1 t + \cos w_2 t)]^i, \quad i = 1, 3, 5, \ldots \]  

By using the relation [2]

\[ \cos A \cos B = \frac{1}{2} [\cos (A+B) + \cos (A-B)] \]  

\( y \) may be written as

\[ y = \sum_{i} k_i A^i \sum_{a, b} c_{a, b} \cos (aw_1 + bw_2) t, \quad \text{for all } \abs(a) + \abs(b) = i, i-2, i-4, \ldots. \]

\[ i = 1, 3, 5, \ldots \]  

where \( c_{a, b} \) is defined as

\[ c_{a, b} = \text{constant coefficient (function of } aw_1 \text{ and } bw_2) \]
Given this expression for $y$, and a brief examination of a typical communications system, certain properties of the problem at hand may be examined. In a communications system, the allocated transmission band is usually much smaller than the transmission frequency. In the above equation, $w_1$ and $w_2$ represent two tones that reside in the allocated transmission band. For example, in an HF transmission system operating at 2 MHz, the allocated bandwidth for SSB communications would be 3 kHz. For USB operation, the minimum value for $f_1$ or $f_2$ would be 2.000000 MHz, and the maximum value would be 2.003000 MHz. Hence, $w_1 (= 2\pi f_1)$ and $w_2 (= 2\pi f_2)$ are very close to each other on a percentage basis.

The output $y$ contains desired signals, in-band distortion, and out-of-band distortion. In-band distortion terms are located close to the input tones, and are difficult to filter away. Out-of-band distortion terms are located relatively far away (at harmonics of the carrier frequency), and are easily filtered away, using well-known low-pass filtering methods after amplification.

The in-band terms occur when

$$\text{abs}(a + b) = 1$$

(10)

and the out-of-band terms occur when
\begin{equation}
\text{abs}(a + b) \neq 1
\end{equation}

For example, consider the following third-order transfer function:

\[ y = \sum_i k_i A^i \sum_{a,b} C_{a,b} \cos(aw_1 + bw_2) t, \]  \textit{for all } \text{abs}(a) + \text{abs}(b) = i, i-2, i-4, \ldots \hspace{1cm} i = 1, 3 \]

For i=1,

\[ y = k_1 A \left[ C_{1,0} \cos(w_1 t) + C_{0,1} \cos(w_2 t) + C_{-1,0} \cos(-w_1 t) + C_{0,-1} \cos(-w_2 t) \right], \]
\[ C_{1,0} = C_{0,1} = C_{-1,0} = C_{0,-1} = 1/2 \]

and for i=3,

\[ y = k_3 A^3 \left[ C_{1,0} \cos(w_1 t) + C_{0,1} \cos(w_2 t) + C_{-1,0} \cos(-w_1 t) + C_{0,-1} \cos(-w_2 t) + C_{2,1} \cos(2w_1 - w_2) t + C_{1,-2} \cos(w_1 - 2w_2) t + C_{-1,-2} \cos(-w_1 - 2w_2) t \right], \]
\[ C_{1,0} = C_{0,1} = C_{-1,0} = C_{0,-1} = 9/8 \]
\[ C_{3,0} = C_{0,3} = C_{-3,0} = C_{0,-3} = 1/8 \]
\[ C_{2,1} = C_{1,2} = C_{-2,-1} = C_{-1,-2} = 3/8 \]
\[ C_{2,-1} = C_{-2,1} = C_{1,-2} = C_{-1,2} = 3/8 \]

with the definition of in-band terms given above, y may be written as
\[ y = (k_1 A + (9k_3 A^3/4)) \cos w_1 t \cos w_2 t \]
\[ + (3k_3 A^3/4) \cos(w_1 - 2w_2) t \cos(-w_1 + 2w_2) t \]
\[ + \text{out-of-band terms} \] (15)

The problem at hand is to find a practical, effective means of reducing the in-band IMD terms.

D. Scope

The solution presented herein offers an alternative to those presented in the past. With the advent of DSP technology, a problem that has been attacked with analog techniques can now be addressed using digital signal processing.

This work provides the theoretical background for the development of a DSP-based predistorter. The method presented receives an incoming baseband \((300 - 3000 \, \text{Hz})\) signal, predistorts this signal, then passes this signal (through an up-conversion process) to an amplifier. DSP-based methods exist that operate directly on the transfer function. These methods typically use a look-up table that operates on the input waveform; the look-up table output values are a function of the input amplitude and the transfer function of the amplifier. The output that results is a predistorted version of the input signal. The effectiveness of this technique depends largely on the amount of memory used to store the
look-up table; in an adaptive situation, this look-up table must be updated after every learn sequence. In contrast, the technique presented herein requires no memory other than storage of the power series coefficients.

A theoretical analysis of the technique ensues, with bounds placed on the validity of the theory.

Simulation of the predistorter versus a typical amplifier transfer function is presented for a variety of conditions, including overdrive, low drive, tone spacing, and carrier frequency.

Finally, hardware built and laboratory testing will be described. A high degree of correlation between theory and experiment is found.

E. General Approach

By modifying the baseband information applied to the input of the frequency upconverter (commonly referred to as an exciter), the in-band IMD in the output of the amplifier can be reduced. This is proven mathematically in a closed form manner, then demonstrated experimentally.

This distortion reduction technique is known as predistortion. The predistortion is generated by creating a power series that operates on the incoming baseband signal. This operation takes place by passing the baseband input through a digital signal processor. If the power series
coefficients are chosen properly, a solution may be found to minimize either a particular term or the overall IMD level.

Without predistortion, the time-domain waveform that results at the amplifier output is usually an amplified, compressed version of the input waveform. A portion of the (demodulated) envelope of such a waveform is shown in Figure 2. The peak of the output waveform is compressed without predistortion (typical of an amplifier approaching, but not at, saturation). With predistortion added, the waveform at the output more closely represents a pure two-tone envelope.

A significant feature of this technique is that the largest distortion term can be minimized without raising the level of other-order products. This is not the case with many other IMD improvement techniques, such as optimizing bias or load line.

F. Sequence of Presentation

As this topic is a mature one, existing solutions to the problem are discussed, in order to both cite references and provide a historical perspective. These techniques include amplifier parameter optimization, feedback, feedforward, predistortion and postdistortion.

A DSP-based predistortion solution is then discussed in
SHAPE OF ENVELOPE, WITH/WITHOUT PREDISTORTION

\[ A_2 = 42V \]

**Figure 2.** Envelope of Waveform, With and Without Predistortion

further detail. Relative merits of analog and digital predistortion are covered, with several compelling arguments made in favor of digital predistortion. A further distinction is made between digital predistortion techniques described in the literature, and the technique described here, that is, that the method described here requires virtually no memory storage.

The mathematics of IMD are then described, with a distinction made between in-band and out-of-band
predistortion. As this thesis has been developed to solve a problem actually found in industry, a brief discussion of industry-standard test methods and necessary features ensues.

From this mathematical base, predistortion is then added into the equations. As most literature available covers third-order distortion, this topic is covered first, with the model then expanded to fifth and higher-orders. Of particular note is that the technique described here can reduce not only a particular term, but also the overall in-band distortion.

The mathematical models, developed with the assistance of computer tools, were modified in an attempt to put boundaries on the region of applicability. These results show that the solution presented is valid over a wide range of amplifier transfer functions and power levels.

Using commercially available equipment, and a DSP prototype, tests were run to obtain measured data for comparison with the models developed above. Baseband tone spacing, exciter carrier frequency, input level, and number of input tones were parameters under consideration. In all cases, there was measured improvement.

With the main application being in the high-power transmitter field, a high-power transmitter was tested with and without a predistorted signal. The results of this testing show that the third-order IMD of the high-power transmitter is greatly reduced when third-order predistortion
is used.

The details of the DSP hardware and firmware are discussed, with an attempt to correlate theory and practice.

Although the results of the experiments are favorable, shortcomings of the technique are discussed, with solutions proposed as a basis for future work.

Although not particularly troublesome in this case, much of the literature discusses AM-PM conversion, and its effect on IMD. For the sake of completeness, mathematical fundamentals are described.

Finally, overall conclusions are drawn, with a discussion of possible future trends in this area.
2. EXISTING SOLUTIONS TO THE PROBLEM

As the amplifier linearization problem has existed for many decades, there has been exhaustive treatment of the general topic in the technical literature. This section provides a historical perspective, including a chronology of events, and cites pertinent references.

A. Minimization of the Compressor Function

The most common approach to improving the IMD performance of an amplifier is to choose a load line (bias setting and load impedance) that will optimize the amplifier input-to-output transfer function. The linearity of a device changes as a function of operating point and load line; choice of output load impedance and bias are typically made as a tradeoff between linearity and amplifier efficiency.

The theory of choosing optimum operating conditions has been developed in past decades [3]. However, it remains difficult to determine the optimum distortion performance of a device in a particular amplifier; published specifications of device parameters often do not address the issue of IMD performance versus bias setting, load impedance, and power level. If specified at all, the IMD is defined at one bias, load impedance, and power level data point. Although this
information fixes one data point, it merely serves as a starting point for the designer. Hence, intermodulation optimization is frequently a trial-and-error process.

Once the operating conditions have been determined for a given carrier frequency, it is likely that the conditions change as a function of carrier frequency. For instance, it is common to choose a bias setting that provides suitable performance over the entire carrier frequency range. Alternatively, one may design a "frequency sensitive bias" scheme. In either case, these techniques will serve to make the transfer function as linear as possible. At best, there is a fundamental limit imposed by the device and circuit.

When the optimum operating conditions for a particular power level are found, there remains a certain level of intermodulation performance. To achieve better than this performance, additional circuitry is needed.

B. Feedback and Feedforward

The concepts of feedforward and feedback date back to the work of H.S. Black in the 1920s [20]. These techniques are commonly used to improve the IMD performance.

Feedback is a closed-loop technique that samples the output waveform, and subtracts this sample from the waveform input to the amplifier. With this negative feedback technique, distortion present at the output of the amplifier
is used to modify the input waveform. In amplifiers, the output is typically compressed; the peak of the output waveform is proportionally smaller than the rest of the waveform. A sample of the output signal, fed back in the correct amplitude and phase, will accentuate the peak of the input waveform; this allows the amplifier to return this modified input waveform to the desired shape at the output. The IMD present at the output is smaller than without feedback, as the distortion reduction due to feedback is $1/(1+\text{Gg})$ [13,20]. Figure 3 illustrates the feedback concept.

![Feedback Block Diagram](image)

**Figure 3.** Feedback Block Diagram

There are, however, drawbacks to use of feedback in high-power transmitter applications. It is desirable to have a high-gain final amplifier stage; feedback reduces the gain of
the amplifier stage (inside the feedback loop) by \((1+G\beta)\), where \(G\) is the amplifier open loop gain, and beta is a fractional sample of the output. Thus, any reduction in distortion performance comes with an associated reduction in amplifier gain. Often, the IMD improvement is offset by the worsened intermodulation performance of the driver amplifier, which must now supply more power to make up for the loss of gain in the final amplifier.

Stability is also a concern; in a high-power transmitter, oscillation represents either transmission of high levels of unwanted frequencies, or amplifier damage that can be costly to repair. Feedback is most commonly used at the frequency of transmission. For an HF amplifier, the frequency of transmission can be anywhere from 2 to 30 MHz. The control loop, therefore, must be stable over the entire frequency band. In short, the stability criterion is that

\[
(G\beta) < 1 \text{ at the frequency where } \angle (G\beta) = 180^\circ \quad (16)
\]

A variation on the theme is called envelope feedback, where the envelope of the output signal is compared to the envelope of the input signal [4]. In this scheme, the baseband envelope is compared to the baseband envelope of the output. The gain of the amplifier is then changed dynamically, over the cycle of the envelope, to compensate for the compression
of the output waveform. With the control loop frequency being in the kHz range, as opposed to the MHz range, this technique can be achieved in practice without the severity of stability problems that plague rf feedback designs. Envelope feedback works only on the envelope, however; anything in the loop bandwidth can be corrected for (such as in-band intermodulation), but anything outside the bandwidth cannot (such as out-of-band intermodulation or harmonics).

A newer variation of feedback is called phase feedback. The majority of IMD is caused by amplitude non-linearities. However, once the amplitude non-linearities are minimized and/or corrected for, there remains a secondary cause of IMD in the form of phase non-linearity. Recent papers [16] describe phase non-linearity, and solutions are proposed. At least one manufacturer of amplifiers has incorporated this technique in practice [19].

Another distortion reduction method is referred to as feedforward. With this technique, the difference between the output and input is amplified by an amplifier with similar characteristics to the amplifier under consideration [20]. The output of this second amplifier is summed with the output of the primary amplifier in a fashion that cancels the distortion component of the primary amplifier. The final result is a distortion free signal.

There are several advantages to this technique. The
distortion can be completely canceled (theoretically). Also, this method has no closed feedback loop, and is thus unconditionally stable.

Unfortunately, this method is also difficult to implement in a typical high-power transmitter. As the final amplifier device is typically the most expensive component in the transmitter, use of an additional device to improve performance is not usually cost-effective. Also, the second amplifier needs to have a close gain tracking versus the primary amplifier. As this is difficult to assure in a small signal amplifier, it is even more complicated in a large signal application, where effects such as temperature, gain change versus signal level, etc. need to be accounted for. For this particular application, alternative methods need to be explored.

C. Predistortion and Postdistortion

Postdistortion is implemented in some applications. At the output of the transmitter, or at the input to the receiver, an inverse of the transfer function is created, which corrects for the amplifier non-linearity.

In this application, postdistortion is limited in two fundamental ways. The high power transmitter output would require large, costly components to create the inverse transfer function. At the receive end, all receivers tuned to
the transmit frequency would, require special circuits to create the transfer function, and special circuits would be required for each transmitter and frequency. This is clearly not a practical solution except in the most restrictive of cases.

Predistortion retains the benefits of postdistortion, yet circumvents the practical implementation problems associated with postdistortion. Predistortion creates the inverse of the amplifier transfer function at the input to the amplifier. This solution may be implemented with low-power components.

Envelope predistortion is the predistortion counterpart to envelope feedback. It may be advantageous in some instances to generate the inverse transfer function at an envelope rate. This solution would be restricted, though, to in-band distortion components in much the same way that envelope feedback is restricted to in-band components.

As mentioned in the technical literature, predistortion in the best choice under some conditions. Predistortion has been found to be superior to equalizers [12], data predistortion [15], and several other methods of nonlinear compensation [14]. The question with predistortion techniques has been how to create the predistorted signal. Historically, analog techniques have been used, with varying degrees of success. With the recent advances made is DSP technology, there are distinct advantages in DSP-based solutions.
3. A DSP-BASED PREDISTORTION SOLUTION

A. Advantages of Predistortion

One main advantage of predistortion is that low power circuitry can be used. With high-power amplifiers, any technique that involves signal processing at the output requires large, expensive components that maintain their characteristics over the operating band of the transmitter (in this case, 2 - 30 MHz). With predistortion, a small signal circuit can be used to correct a large signal problem [7].

Since the signal processing is done at the input only, the technique is open loop, hence unconditionally stable.

Being a low-level signal, predistortion also lends itself to the use of digital signal processing techniques.

B. Shortcomings of Non-DSP Predistortion

When predistortion is attempted in an analog fashion, the fundamental roadblock is in the generation of the predistorted waveform. Papers have outlined the basis for creating nonlinear circuits using analog components [6], but these techniques are inherently a one-frequency, or narrowband solution. A solution that worked properly over a wide range of frequencies would likely require a great deal of circuitry, and bandswitching to switch the proper circuitry in, depending
of the desired transmission frequency.

The designs often need to be "tweaked" from amplifier to amplifier.

Being an open-loop solution, the IMD is prone to long-term deterioration, due to component aging, part replacement, temperature excursions, line variations, etc.

The required circuit functions are often difficult to implement. Each circuit upgrade (add a term, add phase compensation, etc.) requires an additional circuit design.

C. Advantages of DSP-based Predistortion

Bateman, Wilkinson, and Marvill [1] provide a thorough coverage of the general topic of DSP-based predistortion, citing several advantages described below. Cavers [5] provides further arguments in favor of predistortion, and presents favorable results. These general techniques, and others found during the course of this work, point out several advantages of the use of DSP.

A simple, one board solution can solve simple or complex problems with no additional hardware. For example, a higher-order solution (or phase correction) would perhaps be a simple code change. In an analog solution, it is likely that more complicated circuitry would be required as the order of the predistortion increased.

In digital exciters that presently contain DSP, the
additional hardware is minimal, since the DSP hardware already exists. Alternatively, an entirely new circuit that provides the predistortion is also cost effective in most high-power transmitter applications.

With additional code, the technique can compensate for both envelope and AM-PM non-linearities. In the analog case, the amplitude and phase solutions require entirely different types of circuitry.

Using minimal memory, solutions can be stored for particular "bands" of carrier frequencies. In the sample amplifier studied here, the solution at 2 MHz is somewhat different from the solution at 30 MHz. With DSP, the hardware need not change, nor even the code listing; a simple change of coefficients, as a function of frequency, will effect the change. The solution can be as many or as few bands as required, with no additional circuitry required.

The technique is well-suited for a "learn", or "adaptive" sequence to compensate circuit changes in temperature, aging, antenna impedance changes, or ac line voltage variation. Analog circuits are difficult to change to compensate over time; in fact the analog components themselves may be subject to change under these circumstances.

With a minimum number of adjustments in the design, no tweaking is required if adaptive techniques are used. A common issue with analog predistortion techniques is that of
frequency of adjustment. Often, the analog components need to be periodically adjusted to maintain optimum performance.

The solution is easy to incorporate or bypass (with a command to the control circuitry).

D. Memoryless Predistortion

Most of the literature available discusses the creation of the predistorted waveform in the time domain, by using a look-up table to re-align the transfer function. The method presented herein actually uses the power series to operate on the incoming waveform. In this manner, the only memory storage required is that of the power series coefficients, as opposed to a piecewise mapping of the entire transfer function. This method not only reduces memory requirements, but reduces processor time. As this method lends itself to higher operating frequencies in the future (preferably predistortion at the exciter output), processing time will become an increasingly important factor in the practical implementation of this technique.
4. IMD FUNDAMENTALS

A. Background

A typical HF communications transmitter receives incoming baseband information in a 300 - 3000 Hz bandwidth, and upconverts this information to a carrier frequency in the 2 - 30 MHz frequency range. The final stage of the upconversion process is amplification of the signal to a level suitable for transmission via an antenna system.

For this work, the upconversion and amplification is accomplished with a Harris Corporation RF-1310 HF Exciter (the RF-1310 data sheet is located in the Appendix). The baseband input to the exciter is passed through a series of upconversions to achieve the desired SSB output at a frequency in the 2 - 30 MHz (HF) range. This signal is then amplified to a nominal +20 dBm (100mW) output level, using an output amplifier assembly (see Appendix) that is used as the unit-under-test for the majority of this work.

Practical transmitters use additional amplifier stages between the RF-1310 output and the antenna system, such that the nominal +20 dBm output from the exciter is amplified to +60 dBm or higher. (Chapter 8 discusses the validity of the proposed solution versus multistage amplification).

The nominal operating environment for the experiments
that follow is defined as follows. The amplifier topology and operating point are used in the standard RF-1310 output amplifier (the schematic diagram is located in the Appendix). The output amplifier is driven with a two-tone input of \(0.42\) \(V_{\text{peak}}\) per tone. With a 50 ohm input impedance \((s_{11})\), this level translates to

\[
\frac{0.420V_{\text{peak}}}{\text{per tone}} = \frac{0.297V_{\text{rms}}}{\text{per tone}}
\]

\[
dBm = 10\log\left(\frac{V_{\text{rms}}^2}{R}\right) \cdot 0.001
\]

\[
dBm = 10\log\left(\frac{0.297^2}{50}\right) \cdot 0.001
\]

\[
dBm = 2.46\text{dBm per tone}
\]

The output, nominally terminated into 50 ohms, is terminated into 25 ohms for the course of this work. This tends to degrade the IMD performance, and presents a VSWR (2:1) that is found in practical systems; testing into 50 ohms tends to indicate a best-case situation. The output equations in this 25 ohm case, may be written as

\[
\text{fundamental tones} = 2.75V_{\text{peak}}/\text{tone} = 21.8\text{dBm/tone}
\]

\[
3^{\text{rd}} \text{order im} = 0.0389V_{\text{peak}}/\text{tone} = -15.2\text{dBm/tone}
\]

\[
5^{\text{th}} \text{order im} = 0.0138V_{\text{peak}}/\text{tone} = -24.2\text{dBm/tone}
\]

\[
7^{\text{th}} \text{order im} = 0.0049V_{\text{peak}}/\text{tone} = -33.2\text{dBm/tone}
\]

\[
\text{POWER GAIN} = 21.80 - 2.46 = 19.34\text{dB}
\]

IMD data has been taken on the RF-1310 output amplifier
at power levels ranging from -2 dB to + 2 dB in one dB steps, from the aforementioned nominal case. Figure 4 (IMD before) shows data for the nominal case. The x-axis is the order of the IMD term. The terms at +1 and -1 are the desired amplifier output, and all other terms are unwanted. The +3 and -3 terms correspond to third-order distortion; with \( w_1 \) and \( w_2 \) being the desired frequencies, the third-order distortion terms are located at "\( 2w_1 - w_2 \)" and "\( 2w_2 - w_1 \)". This implies a cube term in the amplifier transfer function. The +5 and -5 terms are fifth-order distortion terms, and so on. The mathematical framework for IMD is developed in the following section.

This data was subsequently fit into a seventh-order power series model, specifically

\[
HF \text{ output } y = 6.352x + 0.693x^3 - 2.510x^5 + 3.900x^7 \tag{19}
\]

This non-linear model for the amplifier holds over the input power range of .42 volts per tone to -5 dB from that level, as shown in Figure 5. The amplitudes of the lower and upper tones of each order are within 1 dB of each other, which indicates that the cause of the non-linearity is primarily amplitude distortion (referred to as AM-AM distortion in much of the literature), as opposed to a phase shift type of non-
Figure 4. Simulated Results, IMD Before and After Fifth-order Predistortion (+00 dB)

linearity, or AM-PM distortion (discussed in Chapter 11).

Although the power series model takes only amplitude distortion into account, it appears to suffice for this particular amplifier; the amount of distortion caused by AM-PM conversion is a secondary effect (Amplifiers that operate at higher carrier frequency/power level combinations typically exhibit more AM-PM distortion than is present here. In these situations, a simple power series would not properly characterize the transfer function).
The amplifier output level is commonly controlled via a closed loop power control scheme. In a typical mode of operation, the output is controlled to within +/- 1 dB.

B. IMD Testing

RF amplifier testing frequently uses a two-tone baseband source as the input signal to the exciter. Although not the most comprehensive evaluation of amplifier non-linearity, it is the test most widely used to describe high-power amplifier...
non-linearity (Low power amplifiers commonly use the third-order intercept point to describe non-linearity, defined as the input or output power level at which the third-order IMD would theoretically equal the level of the desired signal).

In a two-tone test, a baseband source is converted to HF by a low-level, distortion-free (relative to the amplifier distortion) mixer, such that the baseband input

\[ u = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t) \tag{20} \]

where \( f_1 \) and \( f_2 \) are baseband frequency tones, is converted to the HF, SSB input [8]

\[ x = a_1 \cos(2\pi (f_0 + f_1) t) + a_2 \cos(2\pi (f_0 + f_2) t) \tag{21} \]

\[ x = a_1 \cos(w_1 t) + a_2 \cos(w_2 t) \tag{22} \]

where \( f_0 \) is an HF frequency (for example, 2.0 MHz).

This low level HF signal is converted to a larger signal by the output amplifier stage. However, amplifiers also distort the HF signal, which (in the case of a two-tone input) create additional, unwanted output signals. This effect may be described by the equation

\[ y = \sum \frac{1}{1} k_i x^i, \quad i = 1, 3, 5, 7 \tag{23} \]
where the desired component is $k_1 x$. With the two-tone input $x$ applied to the seventh-order transfer function $y$, the output can be described as:

$$
y = b_1 (\cos(w_1) t + \cos(w_2) t) \\
+ b_3 (\cos(2w_1-w_2) t + \cos(2w_2-w_1) t) \\
+ b_5 (\cos(3w_1-2w_2) t + \cos(3w_2-2w_1) t) \\
+ b_7 (\cos(4w_1-3w_2) t + \cos(4w_2-3w_1) t) \\
+ \text{wideband products}
$$ (24)

The wideband products are distortion products that are far removed (in frequency) from the desired "$b_1 \ast (\cos w_1 t + \cos w_2 t)$" term. Figure 6 shows the out-of-band third order terms. For a 2 MHz signal, for instance, these terms are located at 6 MHz, fifth-order tones located at 10 MHz, and seventh order terms at 14 MHz.

These products can be filtered with a suitable low pass or bandpass filter, and as such are not particularly troublesome to the amplifier designer.

By contrast, the in-band distortion products listed above ($b_3$, $b_5$, $b_7$) are not easily filtered away, and often allowed to pass through to the antenna system. Figure 7 shows that the in-band terms are located only 1 kHz away from the desired HF signal, with each higher-order term an additional 1 kHz away.
Figure 6. In-band vs. Out-of-Band Intermodulation Products

Unless otherwise specified, the analyses that follow assume that the out-of-band products are eliminated by a suitable filter. Hence, the equations that follow list only in-band products, to provide clarity, and to focus on the products that are to be can be quantified and eliminated.

Given the input \( x \) above, and considering the case of \( a_1 = a_2 \) (the two input test tones are equal in amplitude), the output \( y \) becomes
**Figure 7.** In-band Intermodulation Terms

\[
y = \left( k_1 a_1 + 9 k_3 a_3^3 / 4 + 50 k_5 a_5^5 / 8 + 1225 k_7 a_7^7 / 64 \right) \\
\left( \cos (w_1 t) + \cos (w_2 t) \right) \\
+ \left( 3 k_3 a_3^3 / 4 + 25 k_5 a_5^5 / 8 + 735 k_7 a_7^7 / 64 \right) \\
\left( \cos (2 w_1 - w_2) t + \cos (2 w_2 - w_1) t \right) \\
+ \left( 5 k_5 a_5^5 / 8 + 245 k_7 a_7^7 / 64 \right) \\
\left( \cos (3 w_1 - 2 w_2) t + \cos (3 w_2 - 2 w_1) t \right) \\
+ \left( 35 k_7 a_7^7 / 64 \right) \left( \cos (4 w_1 - 3 w_2) t + \cos (4 w_2 - 3 w_1) t \right) \\
\text{+ wideband products}
\]

With a known input amplitude \( a_1 \), there are four equations
and four unknowns. The unknowns are found by solving the seventh order term for \( k_7 \), then the fifth order term for \( k_5 \), the third order term for \( k_3 \), and finally the first order term to solve for \( k_1 \).

Hence, the HF output \( y \) equals

\[
y = b_1 \cos w_1 t + \cos w_2 t + b_3 \cos (2w_1 - w_2) t + \cos (2w_2 - w_1) t + b_5 \cos (3w_1 - 2w_2) t + \cos (3w_2 - 2w_1) t + b_7 \cos (4w_1 - 3w_2) t + \cos (4w_2 - 3w_1) t + \text{wideband products}
\]  

(26)

where

\[
b_1 = (k_1 a_1 + 9k_3 a_3^3 / 4 + 50k_5 a_5^3 / 8 + 1225k_7 a_7^3 / 64)
\]
\[
b_3 = (3k_3 a_3^3 / 4 + 25k_5 a_5^3 / 8 + 735k_7 a_7^3 / 64)
\]
\[
b_5 = (5k_5 a_5^3 / 8 + 245k_7 a_7^3 / 64)
\]
\[
b_7 = (35k_7 a_7^3 / 64)
\]

(27)

where \( a_1 \) is a known input, and \( k_1 \), \( k_3 \), \( k_5 \), and \( k_7 \) are solved given a knowledge of the amplifier's output spectrum. The resulting equations are:

\[
k_1 = \frac{1}{a_1} (b_1 - 3b_3 + 5b_5 - 7b_7)
\]
\[
k_3 = \frac{4}{3a_1^3} (b_3 - 5b_5 + 7b_7)
\]
\[
k_5 = \frac{8}{5a_1^5} (b_5 - 7b_7)
\]
\[
k_7 = \frac{64}{35a_1^7} (b_7)
\]  

(28)
For the amplifier investigated in this work, the output amplifier of the RF-1310, the amplitudes of \( b_1 \), \( b_3 \), \( b_5 \), and \( b_7 \) have been measured under the following conditions \( (a_1 = 0.42) \):

- Fundamental tones at 21.8 dBm into 25 ohms; \( b_1 = 2.750 \) Vpeak,
- 3\textsuperscript{rd}-order tones at -15.2 dBm into 25 ohms; \( b_3 = 0.0389 \) Vpeak,
- 5\textsuperscript{th}-order tones at -24.2 dBm into 25 ohms; \( b_5 = 0.0138 \) Vpeak,
- 7\textsuperscript{th}-order tones at -33.2 dBm into 25 ohms; \( b_7 = 0.0049 \) Vpeak.

We may then solve for \( k_1 \), \( k_3 \), \( k_5 \), and \( k_7 \). The results are:

\[
\begin{align*}
  k_1 &= 6.352 \\
  k_3 &= 0.693 \\
  k_5 &= -2.510 \\
  k_7 &= 3.900
\end{align*}
\]  
\hspace{1cm}(29)

In summary, the nominal input level is

\[
\text{HF input } x = a_1 \cos w_1 t + a_1 \cos w_2 t \\
= .42 \cos w_1 t + .42 \cos w_2 t
\]  
\hspace{1cm}(30)

and the amplifier under test is characterized as

\[
\text{HF output } y = 6.352x + 0.693x^3 - 2.510x^5 + 3.900x^7
\]  
\hspace{1cm}(31)
5. PREDISTORTION FUNDAMENTALS

It has been demonstrated [9] that it is possible to create a predistortion signal in the form of a power series to reduce the distortion present at the output of an amplifier.

This section will elaborate on the developments of Hecken and Heidt, with expansion to higher-order amplifiers, and higher-order predistortion. When higher than third-order predistortion is used, there is interaction between the predistortion amplitudes; this interaction is discussed herein. The development of the mathematical framework is developed in general, with specific examples given that pertain to the RF-1310 power series model developed earlier, under the nominal conditions of $a_1 = 0.42 \, V$, $f_0 = 2.0 \, MHz$, and a two tone input at 500 and 625 Hz.

An IMD reduction technique is useful in practice only if the total IMD power is reduced. Techniques exist (particularly choice of bias setting) that will improve the third-order intermodulation performance at the expense of higher-order intermodulation, and vice-versa. A true test of usefulness of a technique is one where a particular order of intermodulation can be reduced without raising the level of any other ordered tone, and certainly not raise the overall, or total IMD, defined by
\[ P_{im} = 3^{rd \text{ order IMD}} + 5^{th \text{ order IMD}} + \ldots \]  
\[ (33) \]

\[ P_{im} = \sum_{i=3,5,7,\ldots,\infty} \frac{b_i^2}{R_{load}} \]
\[ (34) \]

By modifying the baseband input to the RF-1310 Output Amplifier, the following cases have been studied to determine whether \( P_{im} \) can be reduced.

A. Third-order Insertion

With a seventh-order power series model, the amplifier is described as

\[ y = \sum_{i} k_i x^i, \quad i = 1,3,5,7 \]
\[ (35) \]

where, for the RF-1310 amplifier under nominal conditions,

\[ k_1 = 6.352 \]
\[ k_3 = 0.693 \]
\[ k_5 = -2.510 \]
\[ k_7 = 3.900 \]  
\[ (36) \]

A four-tone signal can be applied to this amplifier, of the form
\[ x = a_3 \cos(2w_1 - w_2)t + a_1 \cos w_1 t \\
+ a_2 \cos w_2 t + a_4 \cos(2w_2 - w_1)t \]  

where the variables \( a_3 \) and \( a_4 \) are additions to the two-tone input signal, created by passing the input signal through a third-order transfer function. The purpose of the DSP predistorter is to generate these extra terms. This is done by sampling the input waveform, operating on each sample \( (y = k_1 x + k_3 x^3 + ...) \), and sending the result to the exciter.

It will be shown that \( a_3 \) and \( a_4 \) are present in the equations for the distortion terms. If the "inserted" levels of \( a_3 \) and \( a_4 \) can be controlled, there is an opportunity to choose the levels of \( a_3 \) and \( a_4 \) in order to minimize \( P_{\text{IM}} \).

With \( a_1 = a_2 \) and \( a_3 = a_4 \), the output spectrum is symmetric, in the sense that all lower and upper terms of each order are of the same amplitude (characteristic of AM-AM distortion). With this assumption, the output equation in terms of \( a_1 = a_2 \), \( a_3 = a_4 \), \( k_1 \), \( k_3 \), \( k_5 \), and \( k_7 \) has been determined, and is listed in the Appendix. With \( a_1 \), \( k_1 \), \( k_3 \), \( k_5 \), and \( k_7 \) determined by the drive level and amplifier transfer function, all variables are known except for \( a_3 \). This variable may then be chosen to minimize individual distortion components, or the overall IMD level.
B. Cancellation of the Third-Order Term

Frequently, IMD specifications limit the amount of energy in any one tone (for example, "no IMD tone shall be less than 35 dB below the fundamental tone"). Under these circumstances, it would be advantageous to cancel a particular term or terms, usually the third order-term.

If we consider the following baseband input

$$u = a_3\cos(2\pi(2f_1-f_2))t + a_1\cos2\pi f_1 t$$
$$+ a_1\cos2\pi f_2 t + a_3\cos(2\pi(2f_2-f_1))t$$

(38)

that is upconverted to the HF band

$$x = a_3\cos(2w_1-w_2)t + a_1\cos w_1 t$$
$$+ a_1\cos w_2 t + a_3\cos(2w_2-w_1)t$$

(39)

and is inserted into an amplifier with transfer function

$$y = \sum_k k_i x^i, \quad i = 1, 3, 5, 7$$

(40)

it can be shown that the output y is
HF output \( y = b_1 (\cos w_1 t + \cos w_2 t) \)
\( + b_3 (\cos (2w_1 - w_2) t + \cos (2w_2 - w_1) t) \)
\( + b_5 (\cos (3w_1 - 2w_2) t + \cos (3w_2 - 2w_1) t) \)
\( + b_7 (\cos (4w_1 - 3w_2) t + \cos (4w_2 - 3w_1) t) \)
\( + b_9 (\cos (5w_1 - 4w_2) t + \cos (5w_2 - 4w_1) t) \)
\( + b_{11} (\cos (6w_1 - 5w_2) t + \cos (6w_2 - 5w_1) t) \)
\( + b_{13} (\cos (7w_1 - 6w_2) t + \cos (7w_2 - 6w_1) t) \)
\( + b_{15} (\cos (8w_1 - 7w_2) t + \cos (8w_2 - 7w_1) t) \)
\( + b_{17} (\cos (9w_1 - 8w_2) t + \cos (9w_2 - 8w_1) t) \)
\( + b_{19} (\cos (10w_1 - 9w_2) t + \cos (10w_2 - 9w_1) t) \)
\( + b_{21} (\cos (11w_1 - 10w_2) t + \cos (11w_2 - 10w_1) t) \)

\[ (41) \]

plus wideband products, or

\[ \sum_{i=1}^{10} b_{2i-1} (\cos (iw_1 - (i-1)w_2) t + \cos (iw_2 - (i-1)w_1) t), \quad (42) \]

As demonstrated previously, we may express these terms as a function of \( a_1 = 0.42, k_1, k_3, k_5, k_7 \). For example, the third-order term, typically the highest level tone, may be described as (from the Appendix):

\[ b_3 = (1225k_7/64)a_3^7 \]
\[ + (14700k_2a_1^2/64 + 100k_5/16)a_3^5 \]
\[ + (12250k_2a_1^3/64)a_3^4 \]
\[ + (22050k_2a_1^4/64 + 600k_5a_1^2/16 + 9k_3/4)a_3^3 \]
\[ + (11025k_2a_1^5/64 + 300k_5a_1^3/16)a_3^2 \]
\[ + (5145k_2a_1^6/64 + 300k_5a_1^5/16 + 18k_3a_1^2/4 + k_1)a_3 \]
\[ + (735k_2a_1^7/64 + 50k_5a_1^6/16 + 3k_3a_1^3/4) \]

\[ (43) \]

With the exception of \( a_3 \), the third-order insertion
level, all variables in the expressions are known from the RF-1310 amplifier model.

Using the assumption that the addition of the $a_3$ terms are sufficiently small to not affect the $k_1$, $k_3$, $k_5$, and $k_7$ terms (these terms were shown to be constant over +/- several dB, whereas the effect of $a_3$ on the output amplitude is much less than 1 dB), we may express $b_3$ as

$$b_3 = 74.65a_3^7 + 142.33a_3^5 + 55.31a_3^4 + 26.77a_3^3 + 5.29a_3^2 + 7.16a_3 + 0.039$$

(44)

By setting $b_3$ to zero (the desired result), we may solve for

$$a_3 = -0.0055$$

(45)

This indicates that the third-order term, previously fixed at -37 dB below either of the desired tones, can be canceled entirely via a controlled insertion of additional terms at the input to the amplifier (Note that there are seven solutions for $a_3$; $a_3 = -0.0055$ is the solution that reduces $b_3$ to zero, while minimizing the total IM level, $P_{im}$).

The equations for all output intermodulation tones, order 3 through 21, have been determined, and are listed in the Appendix. At the nominal level, Table I lists the terms before and after predistortion.
Table I. IMD terms before, after third-order predistortion.

<table>
<thead>
<tr>
<th>TERM</th>
<th>Before $P_{im} = -36.4$ dB</th>
<th>After $P_{im} = -47.4$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$b_3$</td>
<td>-37</td>
<td>-92</td>
</tr>
<tr>
<td>$b_5$</td>
<td>-46</td>
<td>-48</td>
</tr>
<tr>
<td>$b_7$</td>
<td>-55</td>
<td>-58</td>
</tr>
<tr>
<td>$b_9$</td>
<td>no term</td>
<td>-77</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>no term</td>
<td>-144</td>
</tr>
<tr>
<td>$b_{13}$</td>
<td>no term</td>
<td>-187</td>
</tr>
<tr>
<td>$b_{15}$</td>
<td>no term</td>
<td>---</td>
</tr>
<tr>
<td>$b_{17}$</td>
<td>no term</td>
<td>---</td>
</tr>
<tr>
<td>$b_{19}$</td>
<td>no term</td>
<td>---</td>
</tr>
<tr>
<td>$b_{21}$</td>
<td>no term</td>
<td>---</td>
</tr>
</tbody>
</table>

Note that the overall IM level ($P_{im}$) is reduced from -36.4 dB to -47.4 dB when using third-order predistortion.

Using the seventh-order model listed above, and a two-tone baseband input with third-order tones inserted, a closed-
form solution for the output spectrum has thus been found.

There are several methods of making the predistorted signal. Most of these solutions in the past have been analog in nature, that is, analog circuitry was developed to create the predistortion transfer function. Of the DSP-based solutions, there are two known methods: adjustment of the input signal amplitude as a function of level, and adjustment of the input signal amplitude by running it through a power series. The former method requires a good deal of memory, as it is a technique that usually requires a look-up table. The latter method, used in the course of this work, only requires storage of the power series coefficients \( (k_3, k_5, \ldots) \).

C. Minimization of Overall Distortion Power

With the exception of \( a_3 \), the third-order insertion level, all variables in the expressions are known from the amplifier model. With \( j^{th} \)-order insertion applied to a \( k^{th} \)-order amplifier transfer function, the output has distortion up to order \( (j \times k) \). Thus, with third-order insertion applied to a seventh-order amplifier, the output has distortion up to order \( (3 \times 7) = 21 \). To minimize the output distortion power is to minimize
\[ \min(P_{im}) = \min\left(\frac{V_3^2}{R} + \frac{V_5^2}{R} + \ldots + \frac{V_{21}^2}{R}\right) \]
\[ = \min\left(\sum_i \frac{V_i^2}{R}, \ i=3,5,7,9,\ldots, (j \ k)\right). \]
\[ = \min\left(\sum_i \frac{V_i^2}{R}, \ i=3,5,7,9,\ldots, 21. \right) \tag{46} \]

\( j = \text{insertion order} = 3 \)
\( k = \text{amplifier transfer function order} = 7 \)
\( R = \text{load impedance} = 25 \text{ ohms resistive} \)

The distortion is minimized [18] when

\[ \frac{\partial}{\partial a_3} \left(\sum_i \frac{V_i^2}{R}\right) = 0, \ i = 3,5,\ldots, 21 \tag{47} \]

D. Fifth-order Predistortion

A natural extension is to insert third- and fifth-order predistortion terms. This is done by making

\[ x = a_5 \cos(3w_1 - 2w_2) t + a_3 \cos(2w_1 - w_2) t + a_1 \cos w_1 t \]
\[ + a_2 \cos w_2 t + a_4 \cos(2w_2 - w_1) t + a_6 \cos(3w_2 - 2w_1) t \tag{48} \]

the input to the amplifier. The calculations involved with such a model become quite involved (67\*2 individual terms), so a computer program was written to calculate \( b_1 \) and all of the distortion terms \( (b_3, b_5, \ldots) \). Letting \( a_1 = a_2, \ a_3 = a_4, \ a_5 = a_6, \) the third-order in-band term equals \( (b_1, b_3 \) and higher-order terms are listed in the Appendix)
\[ b_3 = \left(1225k_r/64\right) a_3^7 + (14700k_r a_1^2/64 + 22050k_r a_1 a_5/64 + 14700k_r a_5^2/64 + 100k_s/16) a_3^5 + (12250k_r a_1^3/64 + 36750k_r a_2^2 a_5/64 + 1225k_r a_5^3/64 + 18375k_r a_4 a_5^2/64) a_3^4 + (22050k_r a_1^4/64 + 58800k_r a_3 a_5/64 + 95550k_r a_2^2 a_5^2/64 + 58800k_r a_1 a_5^3/64 + 22050k_r a_5^4/64 + 600k_s a_1^2 a_5^2/16 + 800k_s a_1 a_5/16 + 600k_s a_2^2/16 + 9k_r/4) a_3^3 + (11025k_r a_1^5/64 + 47775k_r a_4 a_5^4/64 + 66150k_r a_1^3 a_5^2/64 + 14700k_r a_1^2 a_5^3/64 + 300k_s a_1^3 a_5^2/16 + 900k_s a_1^2 a_5^2/16 + 300k_s a_1 a_5^2/16) a_3^2 + (5145k_r a_1^6/64 + 17640k_r a_3 a_5^5/64 + 47775k_r a_4 a_5^2/64 + 51k_r a_1^3 a_5^3/64 + 44100k_r a_1^2 a_5^4/64 + 14700k_r a_1 a_5^5/64 + 4900k_r a_5^6/64 + 300k_s a_1^4 a_5^2/16 + 1200k_s a_1 a_5^3 a_5/16 + 200k_s a_1 a_5^3 a_5/16 + 1200k_s a_1^2 a_5^2/16 + 600k_s a_1 a_5^3 a_5/16 + 300k_s a_1^2 a_5^4/16 + 18k_r a_1^2 a_5^2/4 + 18k_r a_4 a_5^4/4 + k_r a_1 a_5^3/4 + 200k_s a_1^2 a_5^2/16 + 200k_s a_1 a_5^3 a_5/16 + 300k_s a_1^2 a_5^4/16 + 3k_r a_1^3 a_5^2/4 + 9k_r a_1^2 a_5^4/4) \]

(49)

Higher-order predistorters require a more elaborate algorithm, since the variables \( b_3 \) and \( b_5 \) interact. In other words, when \( b_5 \) is varied to reduce either the fifth-order or overall IMD level, the value of \( b_3 \) can then be adjusted (or readjusted) to obtain further distortion reduction. Computer simulations and laboratory experiments conducted herein show
that this interaction is small, and that algorithms can be determined that converge to a solution. The IMD products are clearly not independent (see Appendix), since the predistortion terms $a_3$ and $a_5$ appear in multiple output distortion terms, but their dependence is weak enough that individual products can be reduced in an iterative fashion, and yield a minimized $P_{im}$.

In the fifth-order insertion case, there are two unknown variables, namely $a_3$ (third-order insertion level) and $a_5$ (fifth-order insertion level). The output has distortion terms up to order $(5 \times 7) = 35$. As above, minimum output distortion is described as

$$\min(P_{im}) = \frac{1}{R} \min(\sum_i V_i^2), i=3,5,7,9,\ldots,35. \quad (50)$$

$$\min(P_{im}) = \frac{1}{R} \min[\sum_i V_i^2(a_3,a_5)], i=3,5,7,9,\ldots,35. \quad (51)$$

This equation is satisfied when

$$\frac{d^2}{da_3 da_5} [\sum_i V_i^2(a_3,a_5)] = 0,$$

$$i = 3,5,\ldots,35 \quad (52)$$

This equation has been plotted versus a matrix of values for
a_3 and a_5 to observe its behavior. In the simulations that follow this section, it has been shown that the following algorithm converges to the optimum solution:

1. With a_5=0, choose a_3 such that

\[
\frac{\partial}{\partial a_3} (V_3^2) = 0
\]  \hspace{1cm} (53)

2. With a_3 so determined, choose a_5 such that

\[
\frac{\partial}{\partial a_5} (V_5^2) = 0
\]  \hspace{1cm} (54)

3. With a_5 so determined, choose a_3 such that

\[
\frac{\partial}{\partial a_3} (V_3^2) = 0
\]  \hspace{1cm} (55)

4. Repeat steps 2 and 3 until convergence.

An alternative algorithm, one that starts by optimizing a_5 with a_3 equal to zero, is also possible. By minimizing the highest-order distortion first, there is a potential to obtain convergence is a minimum number of steps, since the highest-order predistortion is the only term that affects the highest-order distortion (see equations in the Appendix).

This algorithm also lends itself to adaptive procedures using DSP, for example:
1. With $a_5=0$, vary $a_3$ to minimize third-order IMD
2. With $a_3$ determined, vary $a_5$ to minimize fifth-order IMD
3. With $a_5$ determined, vary $a_3$ to minimize third-order IMD
4. Repeat steps 2 and 3 until coefficients change by less than a specified amount.

It is shown in Chapter 7 that starting with the highest-order insertion first has benefits; the method used to generate the highest-order tone insertion also generates lower order insertion levels. By starting with the highest-order tone, the IMD levels may be reduced in a systematic fashion.

The forthcoming simulated results yield several examples of how this adaptation algorithm works, and is shown in the following section. The results show that both the third- and fifth-order IMD can be significantly reduced, under a variety of drive levels, when the correct predistortion levels are chosen. For the nominal ($a_1 = 0.42V$) case, with

$$a_3 = \text{third-order level} = -0.0054$$
$$a_5 = \text{fifth-order level} = -0.0016$$

then $P_{in}$, the total IMD, equals -59.3 dB, relative to one of two equal fundamental tones, which is an improvement over the -36.4 dB with no predistortion ($a_3 = 0$, $a_5 = 0$), and the -47.4 dB in the third-order predistortion case ($a_3 = -0.0055$, $a_5 = 0$). Table II compares the three instances.
Table II. No vs. third- vs. fifth-order predistortion.

<table>
<thead>
<tr>
<th>TERM</th>
<th>Before ( P_{im} = -36.4 \text{ dB} )</th>
<th>Third ( P_{im} = -47.4 )</th>
<th>Third/Fifth ( P_{im} = -59.3 \text{ dB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>-37</td>
<td>-92</td>
<td>-87</td>
</tr>
<tr>
<td>( b_5 )</td>
<td>-46</td>
<td>-48</td>
<td>-89</td>
</tr>
<tr>
<td>( b_7 )</td>
<td>-55</td>
<td>-58</td>
<td>-60</td>
</tr>
<tr>
<td>( b_9 )</td>
<td>no term</td>
<td>-77</td>
<td>-72</td>
</tr>
<tr>
<td>( b_{11} )</td>
<td>no term</td>
<td>144</td>
<td>-110</td>
</tr>
<tr>
<td>( b_{13} )</td>
<td>no term</td>
<td>187</td>
<td>-129</td>
</tr>
<tr>
<td>( b_{15} )</td>
<td>no term</td>
<td>---</td>
<td>-156</td>
</tr>
<tr>
<td>( b_{17} )</td>
<td>no term</td>
<td>---</td>
<td>-176</td>
</tr>
<tr>
<td>( b_{19} )</td>
<td>no term</td>
<td>---</td>
<td>-210</td>
</tr>
<tr>
<td>( b_{21} )</td>
<td>no term</td>
<td>---</td>
<td>-218</td>
</tr>
</tbody>
</table>
6. SIMULATED RESULTS

A. Distortion Minimized versus Amplitude

Using the seventh-order model that was developed earlier, the amplifier performance was optimized at an input level of .42 V_{peak}, and at -2, -1, +1, and +2 dB relative to this nominal level. In all cases, the amplifier distortion could be reduced using either third or fifth-order predistortion. Figures 7 through 11 show the "before and after" plots of IMD, using fifth-order predistortion.

**Figure 8.** IMD Before and After Fifth-order Predistortion (-02 dB)
Figure 9. IMD Before and After Fifth-order Predistortion (-0.01 dB)

Figure 10. IMD Before and After Fifth-order Predistortion (+0.00 dB)
**Figure 11.** IMD Before and After Fifth-order Predistortion (+01 dB)

**Figure 12.** IMD Before and After Fifth-order Predistortion (+02 dB)
It is clear that with fifth-order predistortion, the third- and fifth-order intermodulation terms can be reduced in all cases.

The order of IMD that can be canceled will correspond to the order of the predistortion inserted. This is apparent when one considers that \( i \)-th order insertion into a \( j \)-th order model yields \( \left((i \times j) - 1\right)/2 \) equations with \( (i-1)/2 \) unknowns. By neglecting \( (i \times (j-1))/2 \) equations, there are \( (i-1)/2 \) equations, \( (i-1)/2 \) unknowns, and a closed form solution may be obtained. As a result, the order that may be canceled corresponds to the number of terms inserted; the IMD orders above the insertion order are not necessarily canceled, but are usually lower than the level of the lower orders (prior to cancellation). The practical design tradeoff is IMD performance versus DSP performance in the areas of computational complexity and resolution (number of A/D conversion bits).

A shortcoming of many distortion reduction techniques is that they optimize one order of IMD at the expense of others. For example, amplifier bias can often be adjusted to improve third-order or fifth-order IMD, but not both. The above figures illustrate that DSP predistortion can cancel both 3\(^{rd}\) and 5\(^{th}\) terms; the impact on 7\(^{th}\) and higher order terms is small enough to make this technique a viable one, as the total IMD is lower than without predistortion is all cases shown in
Figures 8 through 12.

B. Distortion Minimized at Nominal Level

Figure 13 is a graph of the improvement possible, versus drive level, when $3^{rd}$ and $3^{rd}/5^{th}$ order predistortion is employed at each specific level. By contrast, Figure 14 shows the distortion level versus drive level, when the predistortion is optimized at $0.42 \, V$ (nominal input drive level) and then left alone.

The IMD is improved at all levels except at $-2 \, \text{dB}$ drive, where it is slightly worse. This indicates that although solutions exist for any power level, the nominal power level should be known prior to operation. This relates to companding theory, where compression followed by expansion has benefits in terms of improved signal-to-noise ratio over a range of power levels (dynamic range) [10]. Expansion followed by compression (i.e., amplifier predistortion) results in an opposite effect, that is, the amplifier drive level should be known, and held constant. In practice, most HF transmitters in industry have power control circuits that achieve this result.
Figure 13. $P_{im}$ vs. Level with No, Third-, and Fifth-order Predistortion

C. Sensitivity of Cancellation

With some distortion reduction techniques, the solution is narrow. There is a critical setting at which the distortion is reduced, but close to that setting there is little or no improvement, and in some cases degradation.

The results simulated herein indicate that the IMD level drops gracefully as a function of the third-order predistortion level. In practice, this means that the solution need not be exact to achieve improvement.
For example, Figures 15 through 19 show that any third-order insertion level between 0 and 200 percent improves the IMD performance. With no predistortion, the IMD level of the nominal case is -36 dB. With third-order distortion (y-axis), the distortion steadily improves to an optimum point of -47 dB at $a_3 = -0.0055$. Moving along the x-axis (fifth-order predistortion), the IMD steadily improves to a point that is very close to the overall optimum solution. As a result, an algorithm that will optimize third- and fifth-order
intermodulation in an iterative fashion converges rapidly to the optimum solution. Alternatively, the function appears to lend itself readily to the well-known gradient search algorithms that exist (steepest descent, for example).

In a practical iterative solution, the output of an amplifier would be sampled and downconverted. This signal would then be applied to a DSP circuit that performs an FFT, then evaluates $P_{im}$. As part of a learn sequence, it has been shown [5] that similar techniques converge to an updated solution in a short period of time (4 milliseconds).
Figure 15. IMD vs. Third- and Fifth-order Predistortion Levels (-02 dB).

Figure 16. IMD vs. Third- and Fifth-order Predistortion Levels (-01 dB)
Figure 17. IMD vs. Third and Fifth-order Predistortion Levels (+00 dB)

Figure 18. IMD vs. Third- and Fifth-order Predistortion Levels (+01 dB)
Figure 19. IMD vs. Third- and Fifth-order Predistortion Levels (+02 dB)
Table III lists the IMD as a function of $a_3$, at the optimum drive level (+00 dB).

**Table III. IMD as a function of $a_3$.**

<table>
<thead>
<tr>
<th>% of $a_{3,\text{opt}}$</th>
<th>$a_3$</th>
<th>IMD (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-36</td>
</tr>
<tr>
<td>50</td>
<td>-0.0025</td>
<td>-43</td>
</tr>
<tr>
<td>100</td>
<td>-0.0050</td>
<td>-47</td>
</tr>
<tr>
<td>150</td>
<td>-0.0075</td>
<td>-43</td>
</tr>
<tr>
<td>200</td>
<td>-0.0100</td>
<td>-36</td>
</tr>
</tbody>
</table>

From the above figures and Table III, it can be seen that inexact optimization yields improvement of total IMD level.

Once $a_{3,\text{opt}}$ is determined, fifth-order predistortion can be added. Table IV shows the IMD as a function of $a_5$, with $a_3$ optimized, and is defined as $a_{5,\text{opt}}|a_{3,\text{opt}}$. 
Table IV. IMD as a function of \( a_5 \), with \( a_3 \) optimized.

<table>
<thead>
<tr>
<th>% of ( a_{5,\text{opt}} )</th>
<th>( a_5 )</th>
<th>IMD (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-47</td>
</tr>
<tr>
<td>50</td>
<td>-0.00075</td>
<td>-52</td>
</tr>
<tr>
<td>100</td>
<td>-0.00150</td>
<td>-58.8</td>
</tr>
<tr>
<td>150</td>
<td>-0.00225</td>
<td>-52</td>
</tr>
<tr>
<td>200</td>
<td>-0.00300</td>
<td>-47</td>
</tr>
</tbody>
</table>

This iterative solution may be extended to the desired degree. For example, Table V lists the progression of the solution, and the resulting IMD.

Table V. IMD improvement versus algorithm iteration.

<table>
<thead>
<tr>
<th>ITERATION</th>
<th>( a_3 )</th>
<th>( a_5 )</th>
<th>IMD (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-36</td>
</tr>
<tr>
<td>1</td>
<td>-0.0050</td>
<td>0</td>
<td>-47</td>
</tr>
<tr>
<td>2</td>
<td>-0.0050</td>
<td>-0.001</td>
<td>-57</td>
</tr>
<tr>
<td>3</td>
<td>-0.0055</td>
<td>-0.001</td>
<td>-58.8</td>
</tr>
<tr>
<td>4</td>
<td>-0.0055</td>
<td>-0.0015</td>
<td>-58.8</td>
</tr>
</tbody>
</table>
7. LABORATORY WORK WITH THE RF-1310

A prototype of the DSP-based predistorter was built to compare theoretical results against actual measurements. A block diagram of the test circuitry is shown in Figure 20.

![Block Diagram of Test Circuitry]

Figure 20. Predistortion Circuitry

The predistorter input comes from a baseband source (in the test case, two audio generators); the output is a baseband signal that is sent to the RF-1310 Exciter. This signal is
upconverted to the HF spectrum in the RF-1310 Exciter, then amplified to the desired output level (nominally +20 dBm, or 100 mW).

The RF-1310 Output Amplifier is a class A design that has a nominal gain of 20 dB. The output amplifier has been identified (and verified) as the predominant cause of distortion in the RF-1310 amplifier; the signal present at the input the amplifier has lower distortion, on all orders, by greater than 10 dB. The transfer function of this amplifier has been described in earlier sections, and formed the basis for the simulated results in the previous section.

In this section, the RF-1310 is tested under a variety of practical conditions, with and without predistortion. The results are compared to the simulated (theoretical) results found previously.

A. Distortion Minimized versus Amplitude

i. Model Validity

The amplifier distortion versus drive level was discussed in Chapter 4. It was determined that the model closely describes the IMD levels for both underdrive and overdrive conditions.

ii. Solution versus Simulated Results

Table VI shows the IMD versus output level, optimized at
each level. The first column shows the output power level relative to the nominal case. The second column lists the amplifier output level per tone in dBm (equal to analyzer level + 29.3 dB). Columns 3 and 4 are the IMD levels, at the output, before and after third-order predistortion, when optimized at each individual level. The HEX code refers to the constant required in the DSP program to achieve optimum cancellation.

These results are compared with the theoretical levels determined previously. In the Table, $P_{\text{im}}$, after is the measured result, as opposed to the theoretical result $P_{\text{im, aft}}$, theory. In the Hex code column, IM, is the measured third-order IM product, as opposed to the theoretical ratio of $a_3/a_1$. The relationship between the hexadecimal code and the third-order level (relative to the fundamental) is developed in Chapter 9. This level, present at the DSP output, is compared to the theoretical relationship between the fundamental and third-order level, at the amplifier input. In other words, the DSP output (amplifier input) IM measured should equal the theoretical amplifier input IM required for minimum output IM.

The results indicate a reasonable correlation between the theoretical predictions, and the results found through laboratory testing. There are several potential reasons that there are differences between theory and practice, including:
distortion in the upconversion process not accounted for, inexact power series characterization, or AM-PM conversion.

Table VI. IMD versus Output Level

| level rel., | level abs., | $P_{in}$ bef., | $P_{in}$ after, | Hex code, $I_{M,3}$ | $P_{in,after}$ theory | $|a_3/a_1|$ theory, dB |
|------------|-------------|----------------|----------------|-----------------|-----------------------|------------------------|
| -02        | 19.8        | -45.7          | -56.0          | 0070H           | -59 dB               | -48 dB                 |
| -01        | 20.8        | -41.6          | -50.2          | 00D0H           | -52 dB               | -42                    |
| +00        | 21.8        | -34.5          | -41.2          | 0260H           | -47 dB               | -38                    |
| +01        | 22.8        | -28.8          | -35.3          | 04C0H           | -42 dB               | -30                    |
| +02        | 23.8        | -25.9          | -33.0          | 07A0H           | -39 dB               | -24                    |

B. Distortion Minimized at One Level

Of interest is what happens to the IMD, versus level, when third-order predistortion is determined at the nominal
power level, then left alone.

Sensitivity to amplifier power level was measured as follows. The amplifier was optimized at +00 dB with a two-tone input. The amplifier level was then varied +/- 2 dB. Table VII shows the IMD versus output level, when optimized at the nominal level of $a_2 = 0.42V$.

As above, the IMD levels measured with and without predistortion are compared to the theoretical results derived earlier.
Table VII. IMD versus Output Level, one setting

| level rel., dB | level abs., dBm | $P_{im}$ bef., dB | $P_{im}$ after, dB | Hex code, IM$_3$ | $P_{im,aft}$ theory, dB | $|a_3/a_1|$ theory, dB |
|----------------|-----------------|------------------|------------------|-----------------|---------------------|---------------------|
| -02            | 19.8            | -45.7            | -37.0            | 0200H           | -39                 | .013                |
|                |                 |                  |                  | -37dB           |                     | -38dB               |
| -01            | 20.8            | -41.6            | -39.0            | 0200H           | -44                 | .013                |
|                |                 |                  |                  | -37             |                     | -38                 |
| +00            | 21.8            | -34.5            | -40.1            | 0200H           | -47                 | .013                |
|                |                 |                  |                  | -37             |                     | -38                 |
| +01            | 22.8            | -28.8            | -33.0            | 0200H           | -35                 | .013                |
|                |                 |                  |                  | -37             |                     | -38                 |
| +02            | 23.8            | -25.9            | -28.8            | 0200H           | -25                 | .013                |
|                |                 |                  |                  | -37             |                     | -38                 |

Figure 21 compares no predistortion with predistortion optimized at each power level, and with predistortion optimized at one level only.
Figure 21. Laboratory Results, Third-order Predistortion

C. Tests versus Carrier Frequency

The characteristics of the amplifier change as a function of frequency, due to variation in both frequency-dependent amplifier parameters (such as impedance transformation) and frequency-dependent device characteristics (including s-parameters). It is of interest to determine how predistortion acts as a function of frequency.

i. Model Validity versus Carrier Frequency

To this point, the amplifier model has assumed a carrier
frequency of 2.0000 MHz. Frequently, amplifier IMD will change as a function of carrier frequency. However, for the particular amplifier under test, the IMD was found to be constant over the 2 MHz to 30 MHz range.

Table VIII is a list of transfer function coefficients versus frequency.

Table VIII. IMD vs. Frequency, no predistortion.

<table>
<thead>
<tr>
<th>CARRIER FREQUENCY MHz</th>
<th>level dB</th>
<th>$P_{im}$ dB</th>
<th>$b_3$ dB</th>
<th>$b_5$ dB</th>
<th>$b_7$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>21.8</td>
<td>34.4</td>
<td>-35</td>
<td>-44</td>
<td>-57</td>
</tr>
<tr>
<td>7</td>
<td>21.8</td>
<td>-35.4</td>
<td>-36</td>
<td>-45</td>
<td>-55</td>
</tr>
<tr>
<td>12</td>
<td>21.8</td>
<td>-32.5</td>
<td>-33</td>
<td>-42</td>
<td>-57</td>
</tr>
<tr>
<td>17</td>
<td>21.8</td>
<td>-35.3</td>
<td>-36</td>
<td>-44</td>
<td>-55</td>
</tr>
<tr>
<td>22</td>
<td>21.8</td>
<td>-35.5</td>
<td>-36</td>
<td>-45</td>
<td>-58</td>
</tr>
<tr>
<td>27</td>
<td>21.8</td>
<td>-35.6</td>
<td>-36</td>
<td>-46</td>
<td>-57</td>
</tr>
<tr>
<td>30</td>
<td>21.8</td>
<td>-35.4</td>
<td>-36</td>
<td>-45</td>
<td>-56</td>
</tr>
</tbody>
</table>

ii. Solution versus Carrier Frequency

An optimum third-order solution versus frequency was
found during the laboratory testing; the results are shown in Table IX.

**Table IX. IMD vs. frequency, third-order predistortion.**

<table>
<thead>
<tr>
<th>FREQUENCY MHz</th>
<th>level (dBm)</th>
<th>$P_{im}$ (dB)</th>
<th>$b_3$ (dB)</th>
<th>$b_5$ (dB)</th>
<th>$b_7$ (dB)</th>
<th>Hex Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>21.8</td>
<td>-40.2</td>
<td>-46</td>
<td>-42</td>
<td>-52</td>
<td>0260H</td>
</tr>
<tr>
<td>7</td>
<td>21.8</td>
<td>-42.2</td>
<td>-51</td>
<td>-43</td>
<td>-53</td>
<td>01B0H</td>
</tr>
<tr>
<td>12</td>
<td>21.8</td>
<td>-37.9</td>
<td>-45</td>
<td>-39</td>
<td>-54</td>
<td>02B0H</td>
</tr>
<tr>
<td>17</td>
<td>21.8</td>
<td>-42.2</td>
<td>-52</td>
<td>-43</td>
<td>-54</td>
<td>01E0H</td>
</tr>
<tr>
<td>22</td>
<td>21.8</td>
<td>-40.7</td>
<td>-43</td>
<td>-45</td>
<td>-55</td>
<td>0180H</td>
</tr>
<tr>
<td>27</td>
<td>21.8</td>
<td>-41.3</td>
<td>-44</td>
<td>-45</td>
<td>-55</td>
<td>0160H</td>
</tr>
<tr>
<td>30</td>
<td>21.8</td>
<td>-42.2</td>
<td>-46</td>
<td>-45</td>
<td>-54</td>
<td>0180H</td>
</tr>
</tbody>
</table>

This table shows that predistortion works in the laboratory over the entire 2 - 30 MHz range. The optimum solution tends to be fairly constant versus frequency, a reflection on the fact that the transfer function is also fairly constant over the range.

The ability to have a solution that is carrier frequency dependent is easily accommodated in most exciters available in
industry. A look-up table is commonly used, where the input frequency acts as the input address to a ROM, with the output being the hexadecimal code required to achieve optimum predistortion.

iii. One Solution versus Carrier Frequency

An optimum third-order solution versus frequency was found during the laboratory testing; the results are shown in Table X.

<table>
<thead>
<tr>
<th>CARRIER FREQUENCY</th>
<th>level</th>
<th>$P_{im}$</th>
<th>$b_3$</th>
<th>$b_5$</th>
<th>$b_7$</th>
<th>CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 MHz</td>
<td>21.8</td>
<td>-40.1</td>
<td>-45</td>
<td>-42</td>
<td>-54</td>
<td>0200H</td>
</tr>
<tr>
<td>7</td>
<td>21.8</td>
<td>-40.6</td>
<td>-45</td>
<td>-43</td>
<td>-53</td>
<td>0200H</td>
</tr>
<tr>
<td>12</td>
<td>21.8</td>
<td>-37.8</td>
<td>-42</td>
<td>-40</td>
<td>-55</td>
<td>0200H</td>
</tr>
<tr>
<td>17</td>
<td>21.8</td>
<td>-41.0</td>
<td>-49</td>
<td>-42</td>
<td>-54</td>
<td>0200H</td>
</tr>
<tr>
<td>22</td>
<td>21.8</td>
<td>-39.7</td>
<td>-42</td>
<td>-44</td>
<td>-55</td>
<td>0200H</td>
</tr>
<tr>
<td>27</td>
<td>21.8</td>
<td>-38.8</td>
<td>-41</td>
<td>-43</td>
<td>-55</td>
<td>0200H</td>
</tr>
<tr>
<td>30</td>
<td>21.8</td>
<td>-40.3</td>
<td>-43</td>
<td>-44</td>
<td>-54</td>
<td>0200H</td>
</tr>
</tbody>
</table>
For this amplifier, one solution will improve the IMD at all frequencies. This is the simplest method to implement in practice.

![Graph showing IMD versus frequency](image)

**Figure 22.** Laboratory Results vs. Frequency

D. Tests versus Baseband Tone Spacing

i. Model Validity versus Baseband Tone Spacing

The amplifier model has been developed by testing the amplifier with baseband tones at 500 and 625 Hz. Frequently, amplifier IMD will change as a function of tone spacing. To
determine the extent to which this affects the RF-1310 output amplifier, the IMD was measured at the tone spacings shown in Table XI.

**Table XI. IMD versus Tone Spacing.**

<table>
<thead>
<tr>
<th>AUDIO TONE #1</th>
<th>AUDIO TONE #2</th>
<th>IMD(3\textsuperscript{rd}) Lower</th>
<th>IMD(3\textsuperscript{rd}) Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 Hz</td>
<td>625</td>
<td>-35</td>
<td>-35</td>
</tr>
<tr>
<td>500</td>
<td>800</td>
<td>35</td>
<td>-36</td>
</tr>
<tr>
<td>500</td>
<td>1300</td>
<td>-35</td>
<td>-37</td>
</tr>
<tr>
<td>500</td>
<td>1800</td>
<td>35</td>
<td>-34</td>
</tr>
<tr>
<td>500</td>
<td>2300</td>
<td>-35</td>
<td>-35</td>
</tr>
<tr>
<td>1400</td>
<td>1900</td>
<td>-34</td>
<td>-33</td>
</tr>
</tbody>
</table>

In this case, 300 Hz and 3000 Hz are the baseband channel cutoff frequencies. The measurements taken show that the amplifier IMD is essentially unaffected by tone spacing. A comparison of wide and narrow spacings show a third-order difference of approximately 2 dB (narrow spacing is higher). Fifth-order tones were also approximately 2 dB higher in the narrow case, with the seventh order tones about 1 dB higher. It is possible that the wide spaced tones were producing an
output signal that was slightly lower (.25 to .50 dB) than in the narrow case, due to the ripple in the IF filtering. This would account for the slightly lower IMD tones. Nonetheless, an IMD change of 2 dB or less is small, relative to the change in IMD from unit-to-unit and from the standpoint of IMD reduction. An adaptive scheme for reducing IMD would absorb such tolerances.

ii. Solution versus Baseband Tone Spacing

Table XII shows how the solution improves IMD as a function of tone spacing. In all cases, the amplifier showed a reduction of IMD tones. The amplifier and DSP coefficients were unchanged throughout the test.
Table XII. IMD Improvement versus Tone Spacing.

<table>
<thead>
<tr>
<th>AUDIO TONE #1</th>
<th>AUDIO TONE #2</th>
<th>IMD₃ before Lower</th>
<th>IMD₃ before Upper</th>
<th>IMD₃ after Lower</th>
<th>IMD₃ after Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 Hz</td>
<td>625 Hz</td>
<td>-35</td>
<td>-35</td>
<td>-45</td>
<td>-48</td>
</tr>
<tr>
<td>500</td>
<td>800</td>
<td>-35</td>
<td>-36</td>
<td>-35</td>
<td>-42</td>
</tr>
<tr>
<td>500</td>
<td>1300</td>
<td>-35</td>
<td>-37</td>
<td>-35</td>
<td>-38</td>
</tr>
<tr>
<td>500</td>
<td>1800</td>
<td>-35</td>
<td>-34</td>
<td>-35</td>
<td>-40</td>
</tr>
<tr>
<td>500</td>
<td>2300</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
</tr>
<tr>
<td>1400</td>
<td>1900</td>
<td>-34</td>
<td>-33</td>
<td>-45</td>
<td>-43</td>
</tr>
</tbody>
</table>

During this testing several issues became apparent. First, the effectiveness of the predistortion is significantly reduced as the tone spacing widens. This is due to sharp filtering at the 455 kHz Intermediate Frequency (IF). This issue is discussed at length in Chapter 10; it suffices to say at this point that the predistortion should occur after any 3 kHz wide filtering takes place.

Second, with the processing done at baseband, third- and fifth-order harmonics of the processed baseband signal are potentially present in the 300 - 3000 Hz range. For example, the third harmonic of 500 Hz, located at 1500 Hz, is present
in the baseband spectrum when predistortion is added. This harmonic signal is unwanted. A common solution to this problem is to have a "digital IF", where the absolute tone frequencies are higher; this separates the intermodulation tone frequencies from the harmonic frequencies.

Third, digital noise is a concern. Using a 12-bit A-to-D converter, the signal to noise ratio is theoretically 74 dB, but was not this good in practice. Use of more a more up-to-date processor, A/D converter, and D/A converter would surely reduce the digital noise levels.

E. Number of Input Tones

The RF-1310 Output Amplifier was tested with up to seven tone inputs. IMD performance is superior in all cases of two to seven baseband tones, with no adjustments made whatsoever to the DSP coefficients.

The amplifier was subjected to two- through seven-tone baseband inputs in the following manner. The amplifier was optimized with a two-tone input. The amplifier and DSP coefficients were left alone entirely, while baseband tones were added.

In all cases, the amplifier showed a reduction of IMD tones. Plots were made for all cases of input tones (two through seven); all plots exhibit improvement to similar degree.
F. Laboratory Tests with Fifth-order Predistortion

i. Convergence vs. Algorithm Choice

Tests were run in the lab to determine whether third- and fifth-order predistortion could be reduced simultaneously, using the algorithm developed in Chapter 5.

With the nominal case, the algorithm was simulated by inserting $k_3$ first, then $k_5$, then $k_3$, and so on. The algorithm was then simulated by inserting $k_5$ first, then $k_3$, then $k_5$, and so on. The results are shown in Tables XIII and XIV.

**Table XIII. Simulated Results, $k_5$ optimized first.**

<table>
<thead>
<tr>
<th>$k_3$</th>
<th>$k_5$</th>
<th>$P_{im}$</th>
<th>$P_{im,3rd}$</th>
<th>$P_{im,5th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-36.4</td>
<td>-37</td>
<td>-46</td>
</tr>
<tr>
<td>0</td>
<td>-.0020/ -46</td>
<td>-37.1</td>
<td>-37</td>
<td>-80</td>
</tr>
<tr>
<td>-.0054/ -38</td>
<td>-.0020/ -46</td>
<td>-56.6</td>
<td>-79</td>
<td>-59</td>
</tr>
<tr>
<td>-.0054/ -38</td>
<td>-.0016/ -48</td>
<td>-59.3</td>
<td>-87</td>
<td>-89</td>
</tr>
</tbody>
</table>
Table XIV. Simulated Results, $k_3$ optimized first.

<table>
<thead>
<tr>
<th>$k_3$</th>
<th>$k_5$</th>
<th>$P_{im}$</th>
<th>$P_{im,3rd}$</th>
<th>$P_{im,5th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-36.4</td>
<td>-37</td>
<td>-46</td>
</tr>
<tr>
<td>-.0055/-38</td>
<td>0</td>
<td>-47.4</td>
<td>-94</td>
<td>-48</td>
</tr>
<tr>
<td>-.0055/-38</td>
<td>-.0016/-48</td>
<td>-59.0</td>
<td>-70</td>
<td>-85</td>
</tr>
<tr>
<td>-.0054/-38</td>
<td>-.0016/-48</td>
<td>-59.3</td>
<td>-87</td>
<td>-89</td>
</tr>
</tbody>
</table>

Laboratory measurements were taken in a similar fashion. Tables XV and XVI show the results.

Table XV. Test results, $k_5$ optimized first.

<table>
<thead>
<tr>
<th>$k_3$</th>
<th>$k_5$</th>
<th>$P_{im}$</th>
<th>$P_{im,3rd}$</th>
<th>$P_{im,5th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-34.5</td>
<td>-35</td>
<td>-44</td>
</tr>
<tr>
<td>0/-34</td>
<td>0450H/-48</td>
<td>-45.9</td>
<td>-50</td>
<td>-48</td>
</tr>
<tr>
<td>0/-34</td>
<td>0450H/-48</td>
<td>-45.9</td>
<td>-50</td>
<td>-48</td>
</tr>
</tbody>
</table>
Table XVI. Test Results, $k_3$ optimized first.

<table>
<thead>
<tr>
<th>$k_3$</th>
<th>$k_5$</th>
<th>$P_{im}$</th>
<th>$P_{im,3rd}$</th>
<th>$P_{im,5th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-34.5</td>
<td>-35</td>
<td>-44</td>
</tr>
<tr>
<td>0260H/-36</td>
<td>0/-inf</td>
<td>-41.8</td>
<td>-48</td>
<td>-43</td>
</tr>
<tr>
<td>0260H/-36</td>
<td>0/-inf</td>
<td>-41.8</td>
<td>-48</td>
<td>-43</td>
</tr>
</tbody>
</table>

Note that although the hexadecimal codes do not appear to converge, the levels inserted do in fact converge; this is because the method of $5^{th}$-order tone insertion chosen for this work also generates $3^{rd}$-order tones. The $k_3$ level is then chosen to alter the third-order insertion to the desired level (details on this are presented in Chapter 9).

The important result is that for the method of insertion chosen in this work, the algorithm that operates on the highest-order distortion first (i.e., fifth-order before third-order) is the most straightforward. Since the third-order insertion, for example, does not appreciably alter fifth- or higher-order terms, it should be the variable altered last. Of the algorithms proposed in Chapter 5, the latter of the two, the one that modifies $a_5$ first, is the best choice in this case.
8. MULTIPLE STAGE AMPLIFIER TESTING

The Harris RF-1140A 1 KW Transmitter converts incoming baseband data to 1 KW of HF output power. This amplifier consists of an exciter, followed by a power amplifier; The RF-1310 Exciter converts baseband information to a 100 mW HF signal. This power is converted to 1 KW by an RF-1110C three-stage, solid-state power amplifier.

A typical product specification at this power level would be -35 dB below one of two equal tones. Amplifiers by nature are often overdriven into a state where they are no longer linear. It would be of interest to investigate such an overdriven condition, where the IMD is poorer than -35 dB, and attempt to improve the IMD using predistortion.

Measurements were taken on one half of this multi-stage amplifier at 2 MHz, at a power level of 560 W_{pp}. Baseband tones of 500 and 625 Hz were applied to the DSP box, with the DSP box output connected to the RF-1140A amplifier. The output of the amplifier was measured for IMD under a variety of predistortion conditions: none, third-order AM/AM, third-order and fifth-order AM/AM, third-order AM/AM and AM/PM, and finally third-order AM/AM and AM/PM, with fifth-order AM/AM correction. The various results are considered as follows.

For this particular amplifier (an engineering model), the
third-order IMD was -27 dB with no predistortion. With third-order insertion $k_3 = 0700\text{Hex}$, the IMD improved to -32 dB. At this juncture, it was noticed that the lower third-order IM tone was considerably lower than the upper third-order IM tone. The phase relationship between fundamental insertion and third-order insertion was modified, resulting in an AM/PM correction (see Chapter 11) that balanced the lower and upper third-order tones to -35 dB each with a one sample (or 42 microsecond) delay. Subsequent testing with third-order AM/AM, third-order AM/PM, and fifth-order AM/AM resulted in still further improvement. The results are recounted in Figure 23.

In summary, multiple stage amplifier performance also benefits from predistortion. As this is the most practical use of predistortion, it is a significant result. Another interesting result is that AM/PM conversion has a more significant impact on the high-power amplifier testing than just the RF-1310 Exciter alone; this result reconciles several claims in the literature that AM/PM effects are more prevalent at higher power levels.
Figure 23. RF-1140A: IMD versus Predistortion Cases.
9. DSP DETAILS

A. Hardware Description

A DSP board and code were used to achieve the predistortion amplitude and phase shift requirements. The DSP board converts the incoming baseband signal to a 12-bit digital word, cubes each sample, multiplies the fundamental and cubed terms by constants, and inserts a delay between the fundamental \( (x) \) and the cubed term \( (k_3 x^3) \). For fifth-order predistortion, the \( k_5 x^5 \) term is added.

The DSP hardware is straightforward; a schematic of the assembly is located in the Appendix. An existing DSP board was modified to run as a stand-alone microprocessor (no co-processor was used). The input to the board contains a 12-bit A/D converter. The output of this A/D converter is a digital word that represents one tenth of the input signal. After being operated on, the D/A converter turns the digital word into an analog signal that represents ten times the value of the digital word. If these levels prove to be too small, scaling of the levels are accomplished at the processor by left shifting prior to the power series operation, then right shifting afterward to restore the signal back to its original level. In the test results that follow, a scale factor of 16 (shift of four bits) was used.
The sampling rate of the A/D and D/A converters need to meet the Nyquist criteria, not for the highest frequency of the input signal, but for the highest frequency of the predistorted output signal. The highest frequency of the output signal is usually higher than that of the input, as the high-order predistortion terms reside (in frequency) both above and below the input signal. Consequently, the higher the order of the predistortion, the higher the sampling rate required.

B. Firmware Description

The DSP code was written in Texas Instruments TMS32010 assembly language. The code is approximately 100 lines long, with an algorithm that runs as shown in Figure 24.
Figure 24. Firmware Flowchart
C. Theory and Practice Compared; Third-Order Case

A nominal input level of -7 dBm is chosen for this work. This level is a convenient one, since the incoming audio is usually low-distortion (harmonics less than 60 dBC), yet is large enough to use a significant portion of the A/D converter range. This power corresponds to a voltage of

\[ -7 \text{ dBm} = 10 \log \left( \frac{V^2}{R} \right) / .001 \]

\[ V = 0.345 V_{\text{rms}} \]

\[ V = 0.489 V_{\text{peak}} \]

where R = 600 ohms, a standard audio load impedance.

i. Single Tone Case

For the single tone case, the input to the predistorter is

\[ u = A \cos \omega_1 t \]

(57)

and the predistorter output is

\[ x = k_1 u + k_3 u^3 \]

\[ = k_1 (A \cos \omega_1 t) + k_3 (A \cos \omega_1 t)^3 \]

\[ = k_1 (A \cos \omega_1 t) + k_3 A^3 \cos^3 \omega_1 t \]

(58)

Since [2]

\[ (\cos \omega_1 t)^3 = .75 \cos \omega_1 t + .25 \cos 3\omega_1 t \]

(59)
then

\[ x = k_1 A \cos w_1 t + k_3 A^3 (.75 \cos w_1 t + .25 \cos 3w_1 t) \]
\[ = (Ak_1 + .75A^3k_3) \cos w_1 t + .25A^3k_3 \cos 3w_1 t \]

(60)

In the no gain case \((k_1=1)\), and with \(A = 0.4890\), the equation reduces to

\[ x = (0.4890 + .0877k_3) \cos w_1 t + .0292k_3 \cos 3w_1 t \]

(61)

Tests were run in the lab to correlate theory and practice. A block diagram of the test set-up is shown in Figure 25. As shown on the block diagram, the audio generator output is set to -10.2 dBm, which results in -7 dBm being applied to the input of the DSP board. When the signal passes through the DSP board without predistortion, the output is also -7 dBm, which appears as -17.7 dBm on the spectrum analyzer (due to the 10.7 dB attenuation attributed to the 600 to 50 ohm in-line attenuator).

For reference, the baseband input signal was measured (with no predistortion added); the result is plotted in Figure 26. The distortion term is small, and is similar to the distortion of the baseband signal applied to the input of
Figure 25. Block Diagram of the Predistortion Breadboard

Next, predistortion was added. When compared to the theoretical results (see Figure 27), the measured output distortion tracks the theoretical result closely.

Also measured was the validity of the response over the baseband frequency range. Figure 28 shows that the third-order insertion generates a harmonic level that is quite
Figure 26. Baseband Input, No Predistortion

constant over the baseband frequency range.

ii. Two-tone Case

For the two-tone case, an input to the predistorter is

\[ u = B \cos \omega_1 t + B \cos \omega_2 t \]  \hspace{1cm} (62)

and the predistorter output would be
Figure 27. Baseband Harmonic Level vs. $k_3$, Single Tone Case

$$x = k_1u + k_3u^3$$

$$= k_1(B\cos w_1t + B\cos w_2t) + k_3(B\cos w_1t + B\cos w_2t)^3$$  \hspace{1cm} (63)$$

$$= k_1B(\cos w_1t + \cos w_2t) + k_3B^3(\cos w_1t + \cos w_2t)^3$$

$$x = (k_1B + 9k_3B^3/4)(\cos w_1t + \cos w_2t)$$
+ $3k_3B^3/4 [(\cos (2w_1 - w_2)t + \cos (2w_2 - w_1)t)]$
+ $3k_3B^3/4 [(\cos (2w_1 + w_2)t + \cos (2w_2 + w_1)t)]$
+ $k_3B^3/4 [\cos 3w_1t + \cos 3w_2t]$  \hspace{1cm} (64)$$

The ALC of the a typical exciter (including the RF-1310) maintains the peak-envelope-power such that the peak voltage level is held constant. It can be shown that the result is
Figure 28. Baseband Harmonic Level vs. Audio Frequency

\[ B \cos \omega_1 t + B \cos \omega_2 t, \quad B = \frac{0.489}{2} = 0.2445 \]  \quad (65)

or
\[ x = (0.2445 + 0.0329k_3)(\cos w_1 t + \cos w_2 t) + 0.0110k_3[(\cos (2w_1 - w_2) t + \cos (2w_2 - w_1) t] + 0.0110k_3[\cos (2w_1 + w_2) t + \cos (2w_2 + w_1) t] + 0.0036k_3[\cos 3w_1 t + \cos 3w_2 t] \]  

Figure 29 shows a plot of harmonic level as a function of \( k_3 \), third-order insertion factor.

Figure 29. Baseband Harmonic Level vs. \( k_3 \), Two Tone Case
D. Theory and Practice Compared; Fifth-Order Case

As in the third-order case, the fifth-order insertion of tones was tested and compared with theoretical results.

i. Single Tone Case

For the single tone case, the input to the predistorter is

\[ u = A \cos \omega_1 t \]  \hspace{1cm} (67)

and the predistorter output is

\[ x = k_1 u + k_3 u^3 + k_5 u^5 \\
= k_1(A \cos \omega_1 t) + k_3(A \cos \omega_1 t)^3 + k_5(A \cos \omega_1 t)^5 \]  \hspace{1cm} (68)

\[ = k_1(A \cos \omega_1 t) + k_3A^3(\cos \omega_1 t)^3 + k_5A^5(\cos \omega_1 t)^5 \]

Since

\[ (\cos \omega_1 t)^3 = \frac{3}{4} \cos \omega_1 t + \frac{1}{4} \cos 3\omega_1 t \]  \hspace{1cm} (69)

\[ (\cos \omega_1 t)^5 = \frac{5}{8} \cos \omega_1 t + \frac{5}{16} \cos 3\omega_1 t + \frac{1}{16} \cos 5\omega_1 t \]

then

In the no gain case \( k_1 = 1 \), and with \( A = 0.4890 \), the equation reduces to
\[ x = k_1 A \cos w_1 t + k_3 A^3 (\frac{3}{4} \cos w_1 t + \frac{1}{4} \cos 3w_1 t) + k_5 A^5 (\frac{5}{8} \cos w_1 t + \frac{5}{16} \cos 3w_1 t + \frac{1}{16} \cos 5w_1 t) \]

Test the equation: 

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 A )</td>
<td>1</td>
<td>( k_3 A^3 )</td>
</tr>
<tr>
<td>( k_5 A^5 )</td>
<td>( \frac{5}{8} )</td>
<td>( \frac{5}{16} )</td>
</tr>
</tbody>
</table>

Tests were run with -7 dBm being applied to the input of the DSP board. When compared to the theoretical results (see Figure 30), the measured output distortion tracks the theoretical result closely.

Note that fifth-order tones inserted in this manner also insert third-order tones. When optimizing both third- and fifth-order IMD, the fifth-order insertion (selection of \( k_5 \)) should be performed first (to cancel the fifth-order IMD); the third-order insertion (selection of \( k_3 \)) should then be performed to alter the third-order insertion level that exists to the level desired to cancel the third-order IMD.
RELATIVE THIRD AND FIFTH HARMONIC LEVELS VERSUS K5
-7 dBm, single tone audio input

**Figure 30.** Baseband Harmonic Level vs. $k_5$, Single Tone Case

ii. Two-tone Case

For the two-tone case, an input to the predistorter is

$$u = A\cos \omega_1 t + A\cos \omega_2 t$$  \hspace{1cm} (72)

and the predistorter output would be
\[ x = k_1 u + k_3 u^3 \]
\[ = k_1 (A \cos \omega_1 t + A \cos \omega_2 t) + k_3 (A \cos \omega_1 t + A \cos \omega_2 t)^3 \]
\[ + (A \cos \omega_1 t + A \cos \omega_2 t)^5 \]  
(73)
\[ = k_1 A (\cos \omega_1 t + \cos \omega_2 t) + k_3 A^3 (\cos \omega_1 t + \cos \omega_2 t)^3 \]
\[ + k_5 A^5 (\cos \omega_1 t + \cos \omega_2 t)^5 \]

As in the third-order case, \( B = .2445 \), which reduces in-band portion of the equation to

\[ x = (k_1 B + 9k_3 B^3/4 + 25k_5 B^5/4) (\cos \omega_1 t + \cos \omega_2 t) \]
\[ + (3k_3 B^3/4 + 25k_5 B^5/8) \left[ \cos (2 \omega_1 - \omega_2) t + \cos (2 \omega_2 - \omega_1) t \right] \]
\[ + (5k_5 B^5/8) \left[ \cos (3 \omega_1 - 2 \omega_2) t + \cos (3 \omega_2 - 2 \omega_1) t \right] \]  
(74)

or

\[ x = (0.2445 + 0.0329 k_3 + 0.0055 k_5) (\cos \omega_1 t + \cos \omega_2 t) \]
\[ + (0.0110 k_3 + 0.0027 k_5) \left[ \cos (2 \omega_1 - \omega_2) t + \cos (2 \omega_2 - \omega_1) t \right] \]
\[ + 0.0005 k_5 \left[ \cos (3 \omega_1 - 2 \omega_2) t + \cos (3 \omega_2 - 2 \omega_1) t \right] \]  
(75)

Figure 31 shows a plot of harmonic level as a function of \( k_5 \), fifth-order insertion factor under two-tone conditions.
RELATIVE THIRD AND FIFTH IMD LEVELS VERSUS K5
–7 dBm, two tone audio input

Figure 31. Baseband Harmonic Level vs. k₅, Two Tone Case
10. SHORTCOMINGS OF APPROACH PRESENTED

A. Intermediate Frequency (IF) Filtering

In the RF-1310 exciter, baseband information is upconverted to a 455 kHz intermediate Frequency (IF), filtered, then passed on to further upconversion circuitry that generates the 2-30 MHz output signal. The IF filter typically has a passband that corresponds to the information channel to be transmitted. In this case, the channel is roughly 3000 Hz wide, and resides just above or below the 455 kHz carrier (depending on whether the transmission mode is lower sideband or upper sideband). Therefore, any predistortion generated prior to the IF upconversion will be filtered away at the IF, unless the frequency of predistortion resides in the channel 3000 Hz wide channel. In other words, any predistortion that is outside the passband is filtered away. While this is not a problem during testing, it is a practical limitation to the effectiveness of the proposed solution.

A straightforward solution to this problem would be to process the signal after filtering is done. Two methods of achieving this are to: 1) operate the DSP circuitry at a higher frequency, or 2) do the filtering at a lower frequency. The former solution is thought to be impractical
at this time; however, improvements in DSP technology will perhaps make this a viable alternative in the future.

The latter solution, to filter at a lower frequency, has been successfully implemented in several "digital exciters" that are new to the industry, and as a result, lends themselves well to the use of DSP-based predistortion. In fact, most digital exciters have a digital IF frequency, that is higher than baseband, but much lower than 455 kHz (commonly in the 10 to 20 kHz range).

Operation of the predistorter at higher frequencies would also allow for a more straightforward algorithm when operating on a sum of baseband signals. Certain modes of operation, including 2ISB (Independent Sideband), require the combination of two baseband signals. As this combination is typically done at an IF frequency (455 kHz, for example), a DSP the ran at this frequency could process the composite signal. At baseband, the method of predistortion prior to the combination of the two signals is unclear.

B. Baseband Harmonic Generation

With the processing done at baseband, the third and fifth harmonics of the processed baseband signal are present in the 300 - 3000 Hz range. For example, the third harmonic of 500 Hz, located at 1500 Hz, is present in the baseband spectrum. This harmonic signal is unwanted.
By using a digital IF, as described above, this problem no longer exists, due to the fact that the harmonic energy no longer resides in the band of interest, and is easily filtered.

C. Digital Noise

It is important to minimize the amount of digital noise transmitted. Using a 12-bit A-to-D converter, the signal to noise ratio is [11]

\[
\text{SNR (dB)} = (6.02 \times \text{number of bits}) + 1.76
\]

or in this case, 74 dB (12-bit A/D converter). This level is lower than a typical 3rd-order IMD level by approximately 40 dB. For high-performance amplifiers, the number of A-to-D conversion bits could be increased.

D. Higher Order Predistortion

Higher than 5th order distortion requires significant mathematical manipulation in the DSP. The move to a floating point processor is perhaps in order at this time.
11. AM-PM CONSIDERATIONS

A. Background

Several papers on amplifier linearization discuss the importance of AM-PM conversion. The topic has been addressed in a variety of mathematical frameworks (noise/no noise, random inputs/deterministic inputs, single tone/multi-tone) [16,17]. The result of these papers is that, to correct for AM-PM distortion, the predistortion orders inserted need to be in a specific phase relationship to the fundamental.

Analog systems have been presented that achieve third-order AM-PM predistortion, with an analog "cuber" followed by a phase shifter that is added to the fundamental [9]. These circuits are narrowband in nature, working best at one carrier frequency. These circuits also limit the user to third-order predistortion, and do not lend themselves to adaptive type systems. A DSP-based approach would solve these problems.

B. Amplifier Testing

To determine if AM-PM conversion was a significant problem for the RF-1310 amplifier, IMD was tested at several power levels. Table XVII shows that at nominal power, the upper and lower third-order IM tones can both be canceled.
Table XVII. IMD versus Drive Level

<table>
<thead>
<tr>
<th>DRIVE LEVEL (0=nominal)</th>
<th>IMD₃ lower</th>
<th>IMD₃ lower</th>
<th>IMD₃ upper</th>
<th>IMD₃ upper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>before</td>
<td>after</td>
<td>before</td>
<td>after</td>
</tr>
<tr>
<td>0</td>
<td>-35</td>
<td>-47</td>
<td>-34</td>
<td>-49</td>
</tr>
<tr>
<td>+1</td>
<td>-30</td>
<td>-40</td>
<td>-30</td>
<td>-48</td>
</tr>
<tr>
<td>+2</td>
<td>-27</td>
<td>-37</td>
<td>-27</td>
<td>-49</td>
</tr>
</tbody>
</table>

At +1 dB, however, the cancellation of the upper term is greater than that of the lower. At +2 dB, the difference between the upper and lower terms becomes even greater. Figure 31 shows the signals (before and after AM-AM correction) at +2 dB.

The power series that was developed for AM-AM correction does not take asymmetry into account. Hence, for AM-PM distortion, equations have been developed that contain the first derivative [9], that is
Figure 32. AM/PM Remains after AM/AM Distortion is Canceled

\[ y = \sum_{i} k_i x^i + k'_i \frac{d^i x}{dt^i} \]
\[ i = 1, 2, 3, 4, 5, \ldots \]  
(76)  
\[ k_i = \text{constant coefficient} \]
\[ k'_i = \text{constant coefficient} \]

where the derivative terms account for the AM-PM conversion. The derivative terms account for the asymmetry between the upper and lower distortion terms. These terms also indicate a phase difference between the fundamental and higher-order
Thus, an AM-PM corrector must control the phase relationship between the fundamental and the higher-order insertion levels. Using DSP, this translates into delaying the third-order (or higher) insertion levels by a certain number of samples. With a sample rate of 24 kHz, a one sample delay in the DSP predistorter prototype corresponds to 41.7 microseconds. With a nominal-case beat frequency of 400 Hz in the two-tone test (1200 and 1600 Hz), this corresponds to a phase shift of 6 degrees per sample, relative to the input frequency.

At +2 dB drive, and input frequencies of 1200 and 1600 Hz, delays of 0, 1, and 2 samples were attempted. Table XVIII shows that with no delay, the lower third is higher than the upper, and at a 2 sample shift, the situation reverses. With a one sample delay, the tones are approximately equal.
Table XVIII. IMD versus Third-Order Delay

<table>
<thead>
<tr>
<th>DELAY # OF SAMPLES</th>
<th>IMD&lt;sub&gt;3&lt;/sub&gt; lower before</th>
<th>IMD&lt;sub&gt;3&lt;/sub&gt; lower after</th>
<th>IMD&lt;sub&gt;3&lt;/sub&gt; upper before</th>
<th>IMD&lt;sub&gt;3&lt;/sub&gt; upper after</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-27</td>
<td>-33</td>
<td>-27</td>
<td>-46</td>
</tr>
<tr>
<td>1</td>
<td>-27</td>
<td>-37</td>
<td>-27</td>
<td>-38</td>
</tr>
<tr>
<td>2</td>
<td>-27</td>
<td>-45</td>
<td>-27</td>
<td>-30</td>
</tr>
</tbody>
</table>

An algorithm to correct for both AM-AM and AM-PM distortion would be:

1. Adjust highest-order insertion level to minimize $P_{im}$.
2. Adjust highest-order insertion phase to minimize $P_{im}$.
3. Repeat steps 1 and 2 until $P_{im}$ is minimized.
4. Adjust next highest-order level to minimize $P_{im}$.
5. Adjust next highest-order phase to minimize $P_{im}$.
6. Repeat steps 3 and 4 until $P_{im}$ is minimized.
7. Repeat steps 4 through 6 down to third-order.
8. Repeat steps 1 through 7 until $P_{im}$ is minimized.
12. CONCLUSIONS

The primary contribution of this work is to show that DSP predistortion can be applied to transmitter systems, and that it is a viable alternative to previously reported techniques. DSP predistortion should be viewed as one of many solutions to a particular distortion problem, and used where it is cost-effective to do so; the simplicity of the DSP hardware required lends itself particularly well to recurring cost considerations. Under such circumstances, DSP-based predistortion will likely be of most value when there is high frequency, a wide frequency range of transmission available (to make use of the adaptive nature of the design), and high power (which makes other solutions difficult to implement).

As time passes, digital exciters will make implementation of DSP-based predistortion extremely cost-effective. Faster A/D converters and DSP chips will make this technique more useful with time, as the technique could likely be used directly at an IF (or higher) frequency.
Appendix A: IMD Tones

IMD tones fifth-order insertion, 1 through 35 (Note, for third-order insertion equation, set \(a_5=0\)).

\[
\begin{align*}
  b_1 &= (4900k_7a_1/64 + 3675k_7a_5/64) a_3^6 \\
  &+ (7350k_7a_2^2/64 + 14700k_7a_1a_5/64 \\
  &+ 3675k_7a_5^2/64) a_3^5 \\
  &+ (22050k_7a_3^3/64 + 44100k_7a_1^2a_5/64 \\
  &+ 47775k_7a_2a_5^2/64 + 300k_5a_1/16 + 200k_5a_5/16) a_3^4 \\
  &+ (18375k_7a_4^4/64 + 63700k_7a_1^3a_5/64 \\
  &+ 66150k_7a_1a_5^2/64 + 44100k_7a_1a_3^3/64 \\
  &+ 4900k_7a_4^4/64 + 300k_5a_1^2a_5^2/16 \\
  &+ 600k_5a_1a_5/16 + 610k_5a_2a_5^2/16) a_3^3 \\
  &+ (15435k_7a_5^5/64 + 44100k_7a_4^4a_5/64 \\
  &+ 95550k_7a_1^3a_5^2/64 + 66150k_7a_1a_5^2a_3^3/64 \\
  &+ 44100k_7a_1a_5^2/64 + 7350k_7a_2^2/64 + 600k_5a_1a_5^3/16 \\
  &+ 900k_5a_2^2a_5^2/16 + 1200k_5a_1a_5^2/16 \\
  &+ 300k_5a_1^2a_5^2/16 + 18k_3a_1/4 + 9k_3a_5/4) a_3^2 \\
  &+ (5145k_7a_5^6/64 + 22050k_7a_1^5a_5/64 \\
  &+ 36750k_7a_2^5a_5/64 + 72k_5a_1^3a_5^3/64 \\
  &+ 22050k_7a_1a_5^2a_5^2/64 + 14700k_7a_1a_5^2/64 \\
  &+ 250k_5a_1^4a_5^2/16 + 800k_5a_1^3a_5^2/16 \\
  &+ 600k_5a_2^2a_5^2/16 + 600k_5a_1a_5^2/16 \\
  &+ 9k_3a_1^2/4 + 18k_3a_1a_5/4) a_3 \\
  &+ (1225k_7a_6^7/64 + 1715k_7a_5^6a_5/64 \\
  &+ 14700k_7a_1^5a_5^2/64 + 3675k_7a_4^4a_5^3/64 \\
  &+ 22050k_7a_1^3a_5^4/64 + 4900k_7a_1a_5^6/64 \\
  &+ 100k_5a_6^5/16 + 50k_5a_4^4a_5/16 + 600k_5a_1^3a_5^2/16 \\
  &+ 300k_5a_1a_5^4/16 + 9k_3a_1^3/4 + 18k_3a_1a_5^2/4 + k_1a_1)
\end{align*}
\]
\[ b_3 = (1225k_r/64) a_3^7 \\
+ (14700k_r a_1^2/64 + 22050k_r a_3/64 + 14700k_r a_5^2/64 \\
+ 100k_s/16) a_3^5 \\
+ (12250k_r a_1^3/64 + 36750k_r a_1 a_5^2/64) a_3^4 \\
+ (22050k_r a_1^4/64 + 58800k_r a_1 a_5^3/64 \\
+ 95550k_r a_1 a_5^4/64 + 58800k_r a_1 a_5^5/64 \\
+ 22050k_r a_1 a_5^6/64 + 600k_s a_5^2/16 \\
+ 800k_s a_1 a_5/16 + 600k_s a_5^2/16 + 9k_s/4) a_3^3 \\
+ (11025k_r a_1^5/64 + 47775k_r a_1 a_5^4/64 \\
+ 66150k_r a_1 a_5^5/64 + 66150k_r a_1 a_5^6/64 \\
+ 14700k_r a_1 a_5^7/64 + 300k_s a_1^3/16 \\
+ 900k_s a_1^2 a_5/16 + 300k_s a_1 a_5^2/16) a_3^2 \\
+ (5145k_r a_1^6/64 + 17640k_r a_1 a_5^5/64 \\
+ 47775k_r a_1^6 a_5^2/64 + 51k_r a_1 a_5^3/64 \\
+ 44100k_r a_1^2 a_5^4/64 + 14700k_r a_1 a_5^5/64 \\
+ 4900k_r a_1^6/64 + 300k_s a_1^4 a_5/16 + 600k_s a_1^3 a_5/16 \\
+ 1200k_r a_1^2 a_5^2/16 + 600k_s a_1 a_5^3/16 \\
+ 300k_s a_5^4/16 + 18k_s a_1^2/4 + 18k_s a_4 a_5/4 + 18k_s a_2^2/4 + k_1) a_3 \\
+ (735k_r a_1^7/64 + 3675k_r a_1^6 a_5/64 \\
+ 7350k_r a_1^5 a_5^2/64 + 14700k_r a_1^4 a_5^3/64 \\
+ 7350k_r a_1^3 a_5^4/64 + 7350k_r a_1^2 a_5^5/64 \\
+ 50k_s a_1^2/16 + 200k_s a_1 a_5/16 + 200k_s a_1 a_5^2/16 \\
+ 300k_s a_1 a_5^3/16 + 3k_s a_1^3/4 + 9k_s a_1 a_5^3/4) \]
\[ b_5 = \left( 3675k_7a_1/64 + 4900k_7a_5/64 \right) a_3^6 + \left( 7350k_7a_1^2/64 + 7350k_7a_1a_5/64 \right) a_3^5 + \left( 14700k_7a_1^3/64 + 47775k_7a_1^2a_5/64 \right) a_3^4 + \left( 44100k_7a_1a_5^2/64 + 22050k_7a_5^3/64 + 200k_5a_1/16 + 300k_5a_5/16 \right) a_3^3 + \left( 15925k_7a_1^4/64 + 44100k_7a_1^3a_5/64 + 66150k_7a_1^2a_5^2/64 + 19600k_7a_1a_5^3/64 + 300k_5a_1/16 + 200k_5a_5/16 \right) a_3^2 + \left( 8820k_7a_1^5/64 + 47775k_7a_1^4a_5/64 + 66150k_7a_1^3a_5^2/64 + 36750k_7a_1^2a_5^3/64 + 14700k_7a_1a_5^4/64 + 300k_5a_1/16 + 1200k_5a_5/16 + 900k_5a_1^2a_5/16 + 600k_5a_1a_5^2/16 + 9k_3a_1/4 + 18k_3a_5/4 \right) a_3 \]
\[ b_7 = (3675k_7a_1/64 + 1470k_7a_5/64) \, a_5^6 \\
+ (3675k_7a_1^2/64 + 14700k_7a_1a_5/64 \\
\quad + 7350k_7a_5^2/64) \, a_5^5 \\
+ (14700k_7a_1^3/64 + 29400k_7a_1^2a_5/64 \\
\quad + 29400k_7a_1a_5^2/64 + 3675k_7a_5^3/64 \\
\quad + 200k_5a_1/16 + 50k_5a_5/16) \, a_5^4 \\
+ (8575k_7a_1^4/64 + 49000k_7a_1^3a_5/64 \\
\quad + 58800k_7a_1^2a_5^2/64 + 49000k_7a_1a_5^3/64 \\
\quad + 14700k_7a_5^4/64 + 100k_5a_5^2/16 \\
\quad + 600k_5a_1a_5/16 + 300k_5a_5^2/16) \, a_5^3 \\
+ (7350k_7a_1^5/64 + 33075k_7a_1^4a_5/64 \\
\quad + 66150k_7a_1^3a_5^2/64 + 51450k_7a_1^2a_5^3/64 \\
\quad + 22050k_7a_1a_5^4/64 + 300k_5a_1^3/16 \\
\quad + 600k_5a_1^2a_5/16 + 600k_5a_1a_5^2/16 \\
\quad + 9k_3a_1/4) \, a_3^3 \\
\quad + (1470k_7a_1^6/64 + 16170k_7a_1^5a_5/64 \\
\quad + 29400k_7a_1^4a_5^2/64 + 72k_7a_1^3a_5^3/64 \\
\quad + 29400k_7a_1^2a_5^4/64 + 14700k_7a_1a_5^5/64 \\
\quad + 3675k_7a_5^6/64 + 50k_5a_1^4/16 \\
\quad + 600k_5a_1^3a_5/16 + 600k_5a_1^2a_5^2/16 \\
\quad + 600k_5a_1a_5^3/16 + 200k_5a_5^2/16 \\
\quad + 18k_3a_1a_5/4 + 9k_3a_5^2/4) \, a_3 \\
\quad + (35k_7a_1^7/64 + 3675k_7a_1^6a_5/64 \\
\quad + 3675k_7a_1^5a_5^2/64 + 14700k_7a_1^4a_5^3/64 \\
\quad + 4900k_7a_1^3a_5^4/64 + 7350k_7a_1^2a_5^5/64 \\
\quad + 200k_5a_1^4a_5/16 + 100k_5a_1^3a_5^2/16 \\
\quad + 300k_5a_1^2a_5^3/16 + 9k_3a_1^2a_5/4) \]
\[ b_9 = (735k_r/64) a_3^7 \\
+ (7350k_r a_1^2/64 + 16170k_r a_1 a_5/64 + 7350k_r a_2^2/64 + 50k_r a_3^2/16) a_3^5 \\
+ (6125k_r a_1^3/64 + 18375k_r a_1^2 a_5/64 + 22050k_r a_1 a_5^2/64 + 4900k_r a_5^3/64) a_3^4 \\
+ (3750k_r a_1^4/64 + 44100k_r a_1^3 a_5/64 + 51450k_r a_1^2 a_5^2/64 + 44100k_r a_1 a_5^3/64 + 200k_r a_1 a_5^4/16 \\
+ 600k_r a_1 a_5/16 + 200k_r a_5^2/16 + 3k_r/4) a_3^3 \\
+ (3675k_r a_1^5/64 + 22050k_r a_1^4 a_5/64 + 58800k_r a_1^3 a_5^2/64 + 36750k_r a_1^2 a_5^3/64 + 600k_r a_1 a_5^4/64 + 3675k_r a_5^5/64 + 100k_r a_3^3/16 \\
+ 300k_r a_1^2 a_5/16 + 600k_r a_1 a_5^2/16 + 100k_r a_3^2/16) a_3^2 \\
+ (245k_r a_1^6/64 + 14700k_r a_1^5 a_5/64 + 18375k_r a_1^4 a_5^2/64 + 14700k_r a_1^3 a_5^3/64 + 600k_r a_1^2 a_5^4/64 + 600k_r a_1 a_5^5/64 + 18k_r a_1 a_5^6/4) a_3 \\
+ (1470k_r a_1^6 a_5/64 + 7350k_r a_1^5 a_5^2/64 + 14700k_r a_1^4 a_5^3/64 + 14700k_r a_1^3 a_5^4/64 + 3675k_r a_1^2 a_5^5/64 + 50k_r a_1 a_5^6/16 + 300k_r a_1^3 a_5^2/16 + 200k_r a_1^2 a_5^3/16 + 9k_r a_1 a_5^4/4) \]
\[ b_{11} = \left( 1470 k_7 a_1 / 64 + 3675 k_7 a_5 / 64 \right) a_5^6 \\
+ \left( 3675 k_7 a_1^3 / 64 + 1470 k_7 a_1 a_5 / 64 \right) a_5^4 \\
+ \left( 3675 k_7 a_1^2 a_5 / 64 + 29400 k_7 a_1^2 a_5 / 64 \\
+ 29400 k_7 a_1 a_5^2 / 64 + 14700 k_7 a_5^3 / 64 \\
+ 50 k_5 a_1 / 16 + 200 k_5 a_5 / 16 \right) a_3^4 \\
+ \left( 4900 k_7 a_1^4 / 64 + 19600 k_7 a_1^2 a_5 / 64 \\
+ 36750 k_7 a_1^2 a_5^2 / 64 + 14700 k_7 a_1 a_5^3 / 64 \\
+ 1225 k_7 a_5^4 / 64 + 100 k_5 a_1^2 / 16 \right) a_3^3 \\
+ \left( 735 k_7 a_5^5 / 64 + 22050 k_7 a_1 a_5^3 / 64 \\
+ 44100 k_7 a_1^2 a_5^2 / 64 + 51450 k_7 a_1^2 a_5^3 / 64 \\
+ 29400 k_7 a_1 a_5^4 / 64 + 7350 k_7 a_5^5 / 64 \\
+ 600 k_5 a_1^2 a_5^2 / 16 + 600 k_5 a_1 a_5^2 / 16 \\
+ 300 k_3 a_5^2 / 16 + 9 k_5 a_1 / 4 \right) a_3^2 \\
+ \left( 7350 k_7 a_1^5 a_5 / 64 + 19375 k_7 a_1^4 a_5^2 / 64 \\
+ 48 k_7 a_1^3 a_5^3 / 64 + 14700 k_7 a_1^2 a_5^4 / 64 \\
+ 7350 k_7 a_1 a_5^5 / 64 + 200 k_5 a_1^3 a_5^2 / 16 \\
+ 300 k_5 a_1^2 a_5^2 / 16 + 200 k_5 a_1 a_5^3 / 16 \right) a_3 \\
+ \left( 245 k_7 a_1^6 a_5 / 64 \\
+ 7350 k_7 a_1^5 a_5^2 / 64 + 1225 k_7 a_1^4 a_5^3 / 64 \\
+ 14700 k_7 a_1^3 a_5^4 / 64 + 3675 k_7 a_1^2 a_5^5 / 64 \\
+ 300 k_5 a_1^3 a_5^2 / 16 \\
+ 200 k_5 a_1 a_5^4 / 16 + 9 k_5 a_1 a_5^2 / 4 \right) \]
\[b_{13} = \left(1470 k_7 a_1 / 64 + 245 k_7 a_5 / 64 \right) a_3^6 + \left(735 k_7 a_1^2 / 64 + 7350 k_7 a_1 a_3 / 64 + 7350 k_7 a_5^2 / 64 \right) a_3^5 + \left(3675 k_7 a_1^3 / 64 + 14700 k_7 a_1^2 a_3 / 64 + 7350 k_7 a_1 a_5^2 / 64 + 50 k_5 a_1 / 16 \right) a_3^4 + \left(1225 k_7 a_1^4 / 64 + 14700 k_7 a_1^3 a_5 / 64 + 44100 k_7 a_1^2 a_5^2 / 64 + 29400 k_7 a_1 a_5^3 / 64 + 14700 k_7 a_5^4 / 64 + 200 k_5 a_1 a_3 / 16 + 300 k_5 a_5^2 / 16 \right) a_3^3 + \left(14700 k_7 a_1^4 a_5 / 64 + 22050 k_7 a_1^3 a_5^2 / 64 + 36750 k_7 a_1^2 a_5^3 / 64 + 3675 k_7 a_1 a_5^4 / 64 + 300 k_5 a_1^2 a_5 / 16 \right) a_3^2 + \left(1470 k_7 a_1^5 a_5 / 64 + 22050 k_7 a_1^4 a_5^2 / 64 + 24 k_7 a_1^3 a_5^3 / 64 + 29400 k_7 a_1^2 a_5^4 / 64 + 7350 k_7 a_1 a_5^5 / 64 + 600 k_5 a_1^2 a_5^2 / 16 + 200 k_5 a_1 a_5^3 / 16 + 200 k_5 a_5^4 / 16 + 9 k_3 a_5^2 / 4 \right) a_3 + \left(3675 k_7 a_1^5 a_5^2 / 64 + 4900 k_7 a_1^4 a_5^3 / 64 + 4900 k_7 a_1^3 a_5^4 / 64 + 3675 k_7 a_1^2 a_5^5 / 64 + 100 k_5 a_1^2 a_5^2 / 16 + 100 k_5 a_1 a_5^3 / 16 \right) (7) \]
\[ b_{15} = (245k_7/64) a_3^7 \\
+ (1470k_7a_1^2/64 + 7350k_7a_1a_5/64 \\
+ 1470k_7a_1^3/64 + 10k_s/16) a_3^5 \\
+ (1225k_7a_1^4/64 + 3675k_7a_1^2a_5/64 \\
+ 14700k_7a_1a_5^2/64 + 7350k_7a_5^3/64) a_3^4 \\
+ (14700k_7a_1^2a_5/64 + 22050k_7a_1^2a_5^2/64 \\
+ 14700k_7a_1a_5^3/64 + 200k_s a_1 a_5/16) a_3^3 \\
+ (3675k_7a_1^4a_5/64 + 22050k_7a_1^3a_5^2/64 \\
+ 29400k_7a_1^2a_5^3/64 + 18375k_7a_1a_5^4/64 \\
+ 7350k_7a_5^5/64 + 300k_s a_1 a_5^2/16 \\
+ 200k_s a_5^3/16) a_3^2 \\
+ (14700k_7a_1^4a_5^2/64 \\
+ 24k_7a_1^3a_5^3/64 + 18375k_7a_1^2a_5^4/64 \\
+ 300k_s a_1^2a_5^3/16) a_3 \\
+ (735k_7a_1^5a_5^4/64 + 7350k_7a_1^4a_5^5/64 \\
+ 7350k_7a_1^2a_5^6/64 + 735k_7a_5^7/64 \\
+ 200k_s a_1^2a_5^3/16 + 50k_s a_5^5/16 + 3k_s a_5^3/4) \]

\[ (8) \]
\[ b_{17} = (245k_7a_1/64+1470k_7a_5/64) a_5^6 \\
+ (735k_7a_1^2/64) a_3^5 \\
+ (7350k_7a_1^2a_5/64 \\
+ 14700k_7a_1a_5^2/64 + 3675k_7a_5^3/64 \\
+ 50k_5a_5/16) a_3^4 \\
+ (4900k_7a_1a_5^2/64 + 7350k_7a_1^2a_5^2/64 \\
+ 14700k_7a_1a_5^3/64 + 3675k_7a_5^4/64) a_3^3 \\
+ (22050k_7a_1a_5^2/64 \\
+ 14700k_7a_1^2a_5^2/64 + 14700k_7a_1a_5^4/64 \\
+ 300k_5a_1a_5^2/16) a_3^2 \\
+ (3675k_7a_1a_5^2/64 \\
+ 48k_7a_1^3a_5/64 + 7350k_7a_1^2a_5^4/64 \\
+ 7350k_7a_1a_5^3/64 + 1470k_7a_5^5/64 \\
+ 200k_5a_1a_5^3/16 + 50k_5a_5^4/16) a_3 \\
+ (4900k_7a_1a_5^3/64 \\
+ 1225k_7a_1^3a_5^4/64 + 3675k_7a_1^2a_5^5/64 \\
+ 100k_5a_1^2a_5^3/16) \]
\[ b_{19} = (245k_7a_1/64) \, a_3^6 \\
+ (1470k_7a_1a_5/64 + 3675k_7a_5^2/64) \, a_3^5 \\
+ (3675k_7a_1^2a_5/64) \, a_3^4 \\
+ (14700k_7a_1^2a_5^2/64 + 14700k_7a_1a_5^3/64 \\
+ 4900k_7a_5^4/64 + 100k_5a_5^2/16) \, a_3^3 \\
+ (7350k_7a_1^3a_5^2/64 + 7350k_7a_1a_5^4/64) \, a_3^2 \\
+ (1200k_7a_1^3a_5^3/64 + 3675k_7a_1a_5^5/64 \\
+ 7350k_7a_1a_5^5/64 + 200k_5a_1a_5^3/16) \, a_3 \\
+ (1225k_7a_1^4a_5^3/64 + 3675k_7a_1^3a_5^4/64 \\
+ 1470k_5a_1a_5^6/16 + 50k_5a_1a_5^4/16) \tag{10} \]

\[ b_{21} = (35k_7/64) \, a_3^7 \\
+ (1470k_7a_1a_5/64) \, a_3^6 \\
+ (3675k_7a_1a_5^2/64 + 4900k_7a_5^3/64) \, a_3^5 \\
+ (7350k_7a_1^2a_5^2/64) \, a_3^4 \\
+ (14700k_7a_1^2a_5^3/64 + 7350k_7a_1a_5^4/64 \\
+ 3675k_7a_5^5/64 + 100k_5a_5^2/16) \, a_3^3 \\
+ (400k_7a_1^3a_5^3/64 + 3675k_7a_1^2a_5^4/64 \\
+ 1470k_7a_1a_5^6/64) \, a_3 \\
+ (3675k_7a_1^3a_5^4/64 \\
+ 1470k_7a_1a_5^6/64 + 50k_5a_1a_5^4/16) \tag{11} \]
\[ b_{23} = (245k_{7} / 64) a_{3}^{6} + (3675k_{7} a_{1} a_{5}^{2} / 64) a_{3}^{4} + (4900k_{7} a_{1} a_{5}^{3} / 64 + 3675k_{7} a_{5}^{4} / 64) a_{3}^{3} + (7350k_{7} a_{1}^{2} a_{5}^{3} / 64) a_{3}^{2} + (7350k_{7} a_{1}^{2} a_{5}^{4} / 64 + 1470k_{7} a_{1} a_{5}^{5} / 64 + 1470k_{7} a_{5}^{6} / 64 + 50k_{2} a_{5}^{5} / 16) a_{3} + (1225k_{7} a_{1} a_{5}^{3} / 64 + 735k_{7} a_{1}^{2} a_{5}^{4} / 64) a_{3}^{2} + (7350k_{7} a_{1} a_{5}^{5} / 64 + 1470k_{7} a_{5}^{6} / 64 + 10k_{5} a_{5}^{5} / 16) \] (12)

\[ b_{25} = (735k_{7} a_{5}^{2} / 64) a_{3}^{5} + (4900k_{7} a_{1} a_{5}^{3} / 64) a_{3}^{3} + (3675k_{7} a_{1} a_{5}^{4} / 64 + 1470k_{7} a_{5}^{5} / 64) a_{3}^{2} + (3675k_{7} a_{1}^{2} a_{5}^{4} / 64) a_{3} + (1470k_{7} a_{1}^{2} a_{5}^{5} / 64 + 245k_{7} a_{5}^{6} / 64 + 10k_{5} a_{5}^{5} / 16) \] (13)

\[ b_{27} = (1225k_{7} a_{3} / 64) a_{3}^{4} + (3675k_{7} a_{1} a_{5}^{4} / 64) a_{3}^{2} + (1470k_{7} a_{1} a_{5}^{5} / 64 + 245k_{7} a_{5}^{6} / 64) a_{3} + (735k_{7} a_{1}^{2} a_{5}^{5} / 64) \] (14)

\[ b_{29} = (1225k_{7} a_{4} / 64) a_{3}^{3} + (1470k_{7} a_{1} a_{5}^{5} / 64) a_{3} + (245k_{7} a_{1} a_{5}^{6} / 64) \] (15)
\[ b_{31} = (735k, a_5^5/64) a_3^2 + (245k, a_1 a_5^6/64) \]  \hspace{1cm} (16)
\[ b_{33} = (245k, a_5^6/64) a_3 \]  \hspace{1cm} (17)
\[ b_{35} = 35k, a_5^7/64 \]  \hspace{1cm} (18)
APPENDIX B: DATA SHEETS, DIAGRAMS, CODE LISTINGS

RF-1310A Data Sheet
RF-1310A Simplified Block Diagram
RF-1310A Output Amplifier Block Diagram
RF-1310A Output Amplifier Schematic
RF-1140A Data Sheet
DSP Board Schematic
DSP Code Listing
The RF-1310A is a high performance, fully synthesized, independent sideband exciter. It utilizes microprocessor control techniques to provide many operational features which meet the needs of the future. Yet it is programmed for ease of operation. It provides a nominal 100 mW rf output from 405 kHz to 30 MHz in 10 Hz increments.

Features
- 405 kHz to 30 MHz
- 10 Hz Tuning
- 2-Channel ISB
- Built-In Test
- Preset Channel Memory
- Keyboard Tuning
- Integral FSK Keyer
- Remotely Controllable
- High-Stability Frequency Standard
- Vacuum Fluorescent Displays
- Signal Generator Test Mode

The RF-1310A has been designed to replace the RF-1310 Exciter in Harris's high-performance transmitters, such as the RF-130-01, RF-745-02, RF-755, RF-765 and RF-1130-01 Series. The RF-1310A performs all of the functions of the RF-1310. In addition, the microprocessor control provides more front-panel controllable features: FSK shift, ALC, and VOX.

A built-in test capability diagnoses and isolates malfunctions to the module level within the exciter. Output power from the power amplifier can be operator controlled in 1 dB increments from the full power level to a level 50 dB below full power.

The RF-1310A provides 100 preset channels in which frequency, mode, and output power level may be stored. Modes include A1A (CW), H2A (MCW), R3E, H3E (AME), J3E (USB), J3E (LSB), B8E (2-ISB), F1B (FSK), and F3E (FM).

Options
- Delay-Compensated Filters—RF-1311-02
- Internal Postselector—RF-1317A
- 4-Channel ISB—RF-1314A
- Internal AFSK—RF-1318
- Remote Control Modem—RF-1313
The RF-1140A Series offers a new generation of high performance 1 kW HF Transmitters, incorporating the RF-1310A Microprocessor-Controlled Exciter. The transmitter is extremely lightweight and compact. The power amplifiers use the latest MOSFET technology for high efficiency and excellent spectral purity. The RF-1140A is designed for use in fixed station, shipboard, and transportable applications. Complete coverage of the 1.5 to 29.9999 MHz frequency range is provided in 10 Hz increments. Modes of operation include USB, LSB, AME (compatible AM), CW, reduced carrier, FSK, and two-channel ISB. Delay-compensated filters are available as options for high-speed data applications including LINK-11 operation.

The redundant modular concept designed into the RF-1140A transmitter provides operational reliability. It allows the unit to continue functioning at reduced power in the event one of the identical final amplifier or power supply modules malfunctions.

Remote control capability is provided by an internal microprocessor-based system capable of accepting asynchronous serial data in accordance with data standards MIL-STD-188C, EIA RS-232C or RS-422. Controllable functions include frequency, mode, keying, power output level, clipper on/off, channel programming/selection, and BITE.

A built-in test capability (BITE) diagnoses and isolates malfunctions to the module level. For fast switching systems, such as ARQ, a high-speed T/R switch is also available.

The RF-1140A Series HF-ISB Transmitters are fully compatible with the RF-2601A Fast-Tune Antenna Coupler which provides for matching 15- to 35-foot whip antennas and long-wire antennas with the optional RF-625A Long-Wire Adapter.

The RF-1140A is enclosed in a standard 19-inch metal rack cabinet.

Features
- 1 kW PEP and Average
- 1.5 to 29.9999 MHz
- Frequency Agile Tuning
- 100 Preset Channel Capacity
- Built-In Redundancy (Fail Soft)
- Remote Control Capability
- Remote BITE Indications, with Performance Monitor

Accessories
- Remote Control RF-7405 or RF-7700 Series
- 4-ISB Option
IDT 'GENER2'

R. BUCKLEY 1/12/92, THESIS PROJECT

35H = 53D = k3(X(t))
36H = 54D = k3(X(t-1))
37H = 55D = k3(X(t-2))
38H = 56D = k3(X(t-3))
39H = 57D = k3(X(t-4))
3AH = 58D = k3(X(t-5))
3BH = 59D = k3(X(t-6))
3CH = 60D = k3(X(t-7))
3DH = 61D = k3(X(t-8))
3EH = 62D = k3(X(t-9))
3FH = 63D = k3(X(t-10))
40H = 64D = k3(X(t-11))
70H = 112D = 0800 (SHIFT)
71H = 113D = INPUT MASK
72H = 114D = MASKED INPUT FROM POT
73H = 115D = INPUT SIGNAL
74H = 116D = SHIFTED OUTPUT SIGNAL
75H = 117D = OUTPUT SIGNAL
79H = 121D = IN1
7AH = 122D = IN2 (POTENTIOMETER)
7BH = 123D = IN3
7DH = 125D = SHIFTED INPUT SIGNAL
7EH = 126D = SHIFTED INPUT 2
7FH = 127D = SHIFTED INPUT 3

B INIT * INITIALIZE AT INIT *

B ISR * INTERRUPT ALWAYS BRANCHES TO 0002 *

ROVM
LDPK 0
LARK 0.143
ZAC
LARP 0
SACL *
BANZ LOOP
EINT
NOP

B START * INTERRUPT BRANCHES HERE FROM ISR *

SFT DATA >0800 * SHIFT CONSTANT = 0800 *
OFF DATA >OFFF * MASK FOR POT INPUT *

LACK SFT * PROGRAM STARTS HERE *
LACK KON * LOAD CONSTANTS INTO DATA MEMORY *
TBLR 112
TBLR 113
0054 001  F89  ZAC
0055 001B 4073  IN  115, PA0  * INPUT DATA *
0056 0019 4179  IN  121, PA1  * 0016 TIMING *
0057 001A 427A  IN  122, PA2  * WAS STATUS, NOW IS POT INPUT *
0058 001B 437B  IN  123, PA3  * INTERRUPT TIMING *
0059  *
0060 001C 0073  ADD  115, 0
0061 001D 1070  SUB  112, 0  * SHIFT BY 0800H *
0062 001E 507D  SACL  125, 0  * X *
0063 001F 7F89  ZAC
0064 0020 7F80  NOP
0065  *
0066 0021 8000  MPYK  0
0067 0022 6A7D  LT  125
0068 0023 6D7D  MPY  125
0069 0024 7F8E  PAC
0070 0025 5C7E  SACH  126, 4  * X SQUARED INTO 70 *
0071 0026 7F89  ZAC
0072 0027 7F80  NOP
0073  *
0074 0028 8000  MPYK  0
0075 0029 6A7D  LT  125
0076 002A 6D7E  MPY  126
0077 002B 7F8E  PAC
0078 002C 5C7F  SACH  127, 4  * X CUBED INTO 71 *
0079 002D 7F89  ZAC
0080 002E 7F80  NOP
0081  *
0082 002F 247A  LAC  122, 4  * READ POTENTIOMETER & STORE *
0083 0030 7F71  AND  113  * MASK HIGH ORDER BITS *
0084 0031 5072  SACL  114, 0  * THIS IS THE K3 WE WANT *
0085 0032 7F89  ZAC
0086 0033 7F80  NOP
0087  *
0088 0034 8000  MPYK  0
0089 0035 6A72  LT  114  * LOAD K3 *
0090 0036 6D7F  MPY  127
0091 0037 7F8E  PAC
0092 0038 5C35  SACH  53, 4  * K3(X^3) INTO 74 *
0093 0039 7F89  ZAC
0094 003A 7F80  NOP
0095  *
0096 003B 7040  LARK  ARQ, 64  * START AT 64D *
0097 003C 699B  DMOV  *--  * MOVE 64 INTO 63 *
0098 003D 6998  DMOV  *--  * MOVE 63 INTO 62 *
0099 003E 6998  DMOV  *--  * MOVE 62 INTO 61 *
0100 003F 6998  DMOV  *--  * MOVE 61 INTO 60 *
0101 0040 6998  DMOV  *--  * MOVE 60 INTO 59 *
0102 0041 699B  DMOV  *--  * MOVE 59 INTO 58 *
0103 0042 699B  DMOV  *--  * MOVE 58 INTO 57 *
0104 0043 699B  DMOV  *--  * MOVE 57 INTO 56 *
0105 0044 699B  DMOV  *--  * MOVE 56 INTO 55 *
0106 0045 699B  DMOV  *--  * MOVE 55 INTO 54 *
0107 0046 699B  DMOV  *--  * MOVE 54 INTO 53 *
0108 0047 699B  DMOV  *--  * MOVE 53 INTO 52 *
0109 0048 7F80  NOP
0110  *
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<th>Register 2</th>
<th>Comment</th>
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<td>53</td>
<td>0</td>
<td>MINUS K3(X^3)</td>
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<td>004B5074</td>
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<td>116</td>
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<td>PA0</td>
<td>SEND IT OUT (A/D TIMING)!</td>
</tr>
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<td>OUT</td>
<td>117</td>
<td>PA1</td>
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NO ERRORS, NO WARNINGS
REFERENCES


BIBLIOGRAPHY


