Size reduction of microstrip antennas using left-handed materials realized by complementary split-ring resonators

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Size Reduction of Microstrip Antennas
Using Left-Handed Materials
Realized by Complementary Split-Ring Resonators

by
Aparna U. Limaye

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in
Electrical Engineering

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Abstract

Recently, metamaterials (MTMs) engineered to have negative values of permittivity and permeability, resulting in a left-handed system, have provided a new frontier for microwave circuits and antennas with possibilities to overcome limitations of the right-handed system. Microwave circuit components such as waveguides, couplers, power dividers and filters, constructed on left-handed materials, have demonstrated properties of backward coupling, phase compensation, reduced sizes, and propagation of evanescent modes. However, there is very limited work to date, on the microstrip antennas with metamaterials. Microstrip antenna is widely used for its low-profile, simplicity of feed and compatibility with planar microstrip circuitry. As the trend towards miniaturization of electronic circuitry continues, antennas remain as the bulkiest part of wireless devices.

There are three primary objectives to the present work:
1. Explore the possibility of miniaturizing microstrip patch antennas using left-handed materials through phase-compensation
2. Achieve negative permittivity using Complementary Split-Ring Resonators (CSRR)
3. Implement CSRR in the ground plane of a rectangular patch antenna, and validate through simulation and measurement

A rectangular patch antenna with a combined DPS-DNG substrate has been analyzed with the cavity model, from which the condition for mode propagation has been derived. Criteria for ‘electrically small’ patch, using phase-compensation have been developed and propagating modes that satisfy these criteria have been obtained.
With an objective to design practically realizable antennas, amongst several available LHM structures, the Complementary Split Ring Resonators (CSRR) has been chosen, primarily for the ease of implementation in the ground plane. CSRRs are periodic structures which alter the bulk effective permittivity of a host medium in which they are embedded. The effective permittivity becomes negative in a certain frequency band defined as a ‘stop-band’. In the present work the frequency response of the CSRR and the ‘stop-band’ has been determined using a full wave solver, from which, effective permittivity of the composite with CSRRs has been obtained by parameter extraction.

Finally, several combinations of patch and CSRR in the ground plane have been designed and constructed in the X-band frequency range. Measurements of input characteristics and directivity have been validated through simulation by Ansoft Designer and HFSS. It has been observed that the best designs are achieved when the ‘stop-band’ of the CSRR corresponds to the desired resonant frequency of the antenna. Under these conditions, a size reduction of up to fifty percent has been achieved and it is noted that the back lobe is negligible and the directivity is comparable to that of a right-handed microstrip antenna.
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Chapter 1

Introduction

1.1 Background:

The trend for technology in recent times is towards miniaturization and the demand for more robust and compact designs has been growing. The revolution in semiconductor manufacturing and device design methodologies has helped achieve very high data rates and compact sizes. However, in a wireless device, the antenna still remains a matter of concern as regards to its size. Microstrip antennas and arrays are extensively used in several applications, however they are limited by their size, despite of their other advantages. Several research groups have, over the years attempted size reduction in these planar antennas for example antennas with shorting pins and several of there variations [1]. The technology of microstrip antenna which was highly promising about two decades ago has reached its limits with respect to size reduction.

A fresh approach to microwave and optical devices presented itself with the interesting breakthrough in the area of metamaterials. Metamaterials which exhibit negative permittivity and permeability (double negative or left-handed materials) show a phenomenon where the phase velocity is anti-parallel to the wave propagation direction. The past few years have been very eventful with respect to the evolution of the concept and implementation of ‘left-handed materials (LHMs)’. After being dormant as an infeasible idea, for nearly three decades, this concept attracted attention when Shelby et al. [2] demonstrated experimental verification of the ‘left-handed’ behavior for some periodic structures in the microwave regime. This opened up various avenues of
applications of these materials to obtain phenomena hitherto unknown. ‘Metamaterials’ are engineered to modify the bulk permeability and/or permittivity of the medium. Examples of metamaterials are single negative materials (SNG) like $\varepsilon$ negative (ENG) which have effective negative permittivity and $\mu$ negative (MNG) which have effective negative permeability, and double negative materials (DNG) realized by placing periodically, structures that alter the material parameters. Split-ring resonators, metal-wire strips and complementary split-ring resonators are currently used to periodically embed the host medium to obtain ‘metamaterials’. The backward couplers [3, 4], phase compensation resulting in electrically small resonators [5], sub-wavelength waveguides with lateral dimensions below diffraction limits [6-10], enhanced focusing [11,12], Čerenkov radiation [13], doppler effect, photon tunneling, and backward wave antennas [14], are some examples of applications and properties studied so far.

Although left-handed materials, their properties and various applications have been in focus for a few years now, comparatively less attention has been directed towards their application to antennas and antenna systems. Some studies pertaining to the radiation pattern and far-fields in a LHM environment [15] have been done. Few attempts have been made for understanding the behavior of antennas on left-handed substrates and their analysis for miniaturization. Theoretical formulation for circular patch on a combined double positive (positive $\varepsilon$ and $\mu$) and double negative (negative $\varepsilon$ and $\mu$) substrate has been attempted [16]. An annular patch with negative material loading is considered in [73]. Transmission line model of a rectangular patch antenna with DPS-DNG substrate has been proposed [74]. Analysis of resonances, propagating and
evanescent modes in such sub-wavelength resonators have also been studied extensively. Antenna using a CRLH transmission line approach is realized in [17, 18]. Radiation and scattering features of antennas with bi-anisotropic substrates [19], and chiral substrates [20], small antennas with ENG superstrates [21] and DNG superstrates [22] [68], have been analyzed. Very little has been explored about the practicable implementation of the microstrip antennas with left-handed materials [23].

Effect of CSRRs in the ground plane on planar circuits are studied by other researchers [24] and it is found that such CSRRs in the ground plane affect the permittivity of the substrate and hence cause the wave to follow the left hand rule. Planar circuits like coupling enhancement in parallel transmission lines [3], coplanar waveguide filters [25] are some of the circuits that are developed using the CSRR in the ground plane also called the Defected Ground Plane (DGP).

1.2 Motivation and Objectives:

Typically, the size of a microstrip patch antenna is dependent on the wavelength corresponding to its resonant frequency. The patch can be treated as a leaky cavity, with its radiating edge dimension approximately half the wavelength. The phase difference for the E field between the radiating edges is a multiple of ‘π’. When a wave travels in a right-handed material (RHM) it experiences a phase change. Consider a wave traveling through two consecutive slabs, the first a DPS and the second a DNG. It would experience a phase change say positive in the DPS material and a negative (opposite) phase change in the DNG material. The slab thicknesses can be adjusted to nullify the
positive and negative phase shifts so that a phase compensated resonator, independent of the \( \lambda/2 \) constraint is obtained. This property of phase-compensation demonstrates the possibility of size reduction for patch antenna on a combined DPS-DNG substrate.

Among several structures that are currently used for the realization of negative permittivity and/ or permeability, the Complementary Split-Ring Resonator (CSRR) is easiest to implement and fabricate. CSRRs placed in the ground plane of a microstrip circuitry can alter the substrate permittivity, to become negative in a certain frequency range. Simple and robust designs of patch antennas on DPS-DNG substrate are possible, where the DNG part of the substrate can be achieved by placing the CSRR in the ground plane.

The first objective of this work is to determine the feasibility of incorporating the concept of phase compensation for size reduction in microstrip antennas. The patch is treated as a 3-D cavity filled with two consecutive slabs, one DPS and other DNG.

The second objective is to analyze the CSRRs for obtaining their frequency response through simulation and to extract the material parameters of the bulk composite host medium by parameter extraction.

The final objective is to design and construct reduced size antenna with CSRR in ground plane, and validate by simulation and measurement. From the analysis of various size CSRRs and patches, an approximate design methodology for size reduction will be attempted.
1.3 Contributions:

The key contributions of the present work are as follows

1. **Analytical Development:** The rectangular microstrip patch antenna with DPS-DNG substrate for two configurations has been investigated. In one the DPS-DNG interface is parallel to the radiating edge and in the other the DPS-DNG interface is normal to the radiating edge. Condition for propagation and criterion for size reduction based on phase compensation have been obtained for each configuration.

2. **Analysis of CSRR:** The effective $\varepsilon$ and $\mu$ due to periodically embedded CSRR have been obtained through simulations and parameter extraction. From the frequency response, the ‘stop-band’ of the CSRR – defined as the frequency range over which $\varepsilon$ becomes negative – is determined for various sizes of CSRRs.

3. **Design and Construction:** Various combinations of X-band patch antennas with CSRR sizes are studied through simulation. Designs have been implemented and antennas have been constructed on Rogers RT5870 substrate.

4. **Validation and Measurement:** Frequency response, standing wave ratio and directivity of the antennas have been measured using the Agilent PNA (E8363B) and Sanders Antenna test set. With respect to combinations of patch size and CSRR geometry, the results have been analyzed for antenna size reduction, back lobe radiation and effect on directivity.

5. **Design Methodology Proposed:** A design methodology based on antennas that show a 50 percent reduction in patch size is proposed. In these cases, the CSRR has a ‘stop-
band’ at the desired resonant frequency, which in turn results in a negligible back lobe and directivity comparable to a patch without CSRRs.

1.4 Organization:

This work is divided into 6 chapters. The first chapter is an introduction and gives an overview of the work. Chapter 2 focuses on the concept of metamaterials. It covers the history of research in metamaterials, defines metamaterials, the different types of metamaterials, their realization and their prominent applications studied so far. It also discusses the SRR and the methods to prove the phenomena of effective permittivity and permeability of the bulk medium embedded by SRRs. Some earlier methods and simulations have been repeated here for validation of the procedure adopted during the course of this work.

Analytical development of criteria for size reduction through phase compensation in microstrip antenna on DPS-DNG substrate is shown in chapter 3. Two configurations for the arrangement of the DPS-DNG substrate combination are analyzed and condition for mode propagation and criterion for size reduction in each case are derived. Modes satisfying both the conditions are obtained for both configurations.

Chapter 4 is a study of Complementary Split-Ring Resonators (CSRRs) for antenna application. Methods for obtaining the frequency response and material parameters of the composite substrate are discussed. Several sizes of CSRR are considered and their ‘stop-bands’ are tabulated.
Chapter 5 discusses the implementation of the antennas with CSRR and the results and proposes a design methodology for size reduction using CSRRs in the ground plane.

Conclusion and future work are given in Chapter 6.

Appendices contain simulation and test data like simulated graphs for $\mu$ and $\varepsilon$, measured input characteristics for all designs.
Chapter 2

Left-Handed Materials – A Review

2.1 History

In 1967, Veselago [26] postulated a negative material and theoretically proved the phenomenon that a uniform plane-wave followed the left hand rule in a medium with negative permittivity ($\varepsilon$) and negative permeability ($\mu$). He concluded that, in such a medium, the phase velocity or wave vector of a monochromatic wave exhibited a direction opposite to that of the Poynting vector, thus exhibiting a backward propagation. Hence the medium was said to support backward waves and could be considered as a medium with negative index of refraction or ‘left handed’ as he termed it. However, these effects could not be experimentally verified since, as Veselago pointed out, substances with negative magnetic permeability ($\mu<0$) were not available.

After 3 decades nearly, Pendry et al [27] proposed structures which demonstrated negative permittivity in certain frequency bands. Shelby et al [2] based on the ideas in Pendry’s paper designed some periodic structures with split-ring resonators and wire strips which exhibited both, negative permittivity and permeability and this led to a great interest in investigation of metamaterials, their properties, methods of realizations and applications. These metamaterials are addressed by various names like Left-Handed Materials (LHM), Double Negative Materials (DNG), and Metamaterials (MTM) etc. The list is given in the Appendix A.
2.2 Defining Left-Handed Materials (LHM)

To understand the idea of a left-handed material we first reiterate the established right hand rule in electromagnetism. It states that when the direction of the E field and the H field are represented by the thumb and the index finger of the right hand respectively, placed at right angles to each other then the middle finger placed at right angle to both the fingers gives the direction of propagation of the wave, which is normal to both the E field and the H field. All electromagnetic waves in nature, including light follow this rule. This law can be stated mathematically as follow in equation (2-1):

$$\vec{E} \times \vec{H} = \vec{S}$$  \hspace{1cm} (2-1)

where

$\vec{E}$ is the electric field

H is the magnetic field and

$\vec{S}$ represents the Poynting vector and the direction of propagation of the energy and the wave

The Maxwell’s equations which describe the electromagnetic wave are:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$  \hspace{1cm} (2-2)

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$  \hspace{1cm} (2-3)

$$\nabla \cdot \vec{D} = \rho$$  \hspace{1cm} (2-4)

$$\nabla \cdot \vec{B} = m$$  \hspace{1cm} (2-5)

The first two equations take the form:
\[ \vec{k} \times \vec{E} = \frac{\omega}{c} \mu \vec{H} \]  \hspace{1cm} (2-6)

\[ \vec{k} \times \vec{H} = -\frac{\omega}{c} \mu \vec{E} \]  \hspace{1cm} (2-7)

where the \( k \) is the wave vector and is along the direction of the phase velocity.

It may be noted here that in a medium where the permittivity and permeability is negative the phase velocity will be anti-parallel to the direction of wave propagation or energy flow. It can be said that the wave has a ‘negative phase velocity’ in that medium. Hence, although, the direction of energy flow is always from the sender to the receiver, the phase moves in the opposite direction. It can be illustrated as in Figure 2.1. Here the \( S \) vector follows the right hand rule, while the \( k \) vector is anti-parallel to the \( S \) vector.

**Right Handed Medium**

**Left Handed Medium**

*Figure 2.1: The wave propagation in a right-handed and left handed system*

Now, the question arises that how can the refractive index be negative for such a left handed system. We know that the refractive index of a material is defined by the square root of the product of the permittivity and permeability.
\[ n = \sqrt{\mu \varepsilon} \]  
\[ \text{(2-8)} \]

where \( n \) is refractive index

\( \mu \) - Permeability of material

\( \varepsilon \) - Permittivity of material

Here it may appear that the negative permeability and permittivity will not cause the refractive index to be negative. However, the refractive index in this case is considered to be the negative square root of the product. It has been proved conclusively by Ziolkowski [28] that the square root choice that leads to a negative index of refraction is mathematically correct. The mathematical proof is briefly provided here. We know that the expressions for the wave number \( k \), the wave impedance and the wave speed \( v \) are as follows in equations (2-9), (2-12) and (2-13):

\[ k = \omega \sqrt{\mu \varepsilon} = k_0 n \]  
\[ \text{(2-9)} \]

\[ k_0 = \omega \sqrt{\mu_0 \varepsilon_0} \]  
\[ \text{(2-10)} \]

\[ n = \pm \sqrt{\mu_r \varepsilon_r} \]  
\[ \text{(2-11)} \]

\[ v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0} \sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{n} \]  
\[ \text{(2-12)} \]

\[ Z = \frac{E_0}{H_0} = \frac{k}{\omega \varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta Z_0 \]  
\[ \text{(2-13)} \]

\[ \zeta = \sqrt{\frac{\mu_r}{\varepsilon_r}} \]  
\[ \text{(2-14)} \]

Here we notice that 'n', by a typical choice of square root would be positive inspite of \( \mu \) and \( \varepsilon \) being negative. But it becomes mathematically evident that the 'n' for a
double negative material (DNG) is negative. Consider the expression for the permittivity and the permeability of a negative medium in terms of magnitude and phase as:

\[
\begin{align*}
\varepsilon_r &= |\varepsilon_r| e^{j\phi_\varepsilon} \quad \phi_\varepsilon \in \left(\frac{\pi}{2}, \pi\right] \\
\mu_r &= |\mu_r| e^{j\phi_\mu} \quad \phi_\mu \in \left(\frac{\pi}{2}, \pi\right]
\end{align*}
\]

Then the wave impedance and the refractive index of the medium can be written as:

\[
\begin{align*}
n &= \sqrt{|\mu_r|/|\varepsilon_r|} e^{j\phi_n} \\
\zeta &= \sqrt{|\mu_r|/|\varepsilon_r|} e^{j\phi_\zeta}
\end{align*}
\]

where the phase for \( n \) and \( \zeta \) would be

\[
\begin{align*}
\phi_n &= \frac{1}{2} (\phi_\mu + \phi_\varepsilon) \in \left(\frac{\pi}{2}, \pi\right] \quad \text{for a positive square root choice} \\
\phi_\zeta &= \frac{1}{2} (\phi_\mu - \phi_\varepsilon) \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \\
\phi_n &= \frac{1}{2} (\phi_\mu + \phi_\varepsilon) - \pi \in \left(-\frac{\pi}{2}, 0\right] \\
\phi_\zeta &= \frac{1}{2} (\phi_\mu - \phi_\varepsilon) + \pi \in \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]
\end{align*}
\]

In either case, irrespective of the choice of the root the refractive index works out to be negative, therefore there is no ambiguity about the choice of the sign of the refractive index for DNG. Upon substitution of this negative expression for ‘\( n \)’ in the wave equation we find that the wave vector and Poynting vector are anti parallel.
In these materials, where the index of refraction is negative, the wave travels from air into the medium and bends to the same side of the normal as the incident ray.

![Diagram](image)

**Figure 2.2 Refraction at a DPS-DNG interface**

This property of the negative materials is stated to be very useful in realizing a ‘perfect lens’ as suggested by Pendry et al [29].

Veselago predicted that it was essential for the material to have both the permeability and permittivity of the material to be negative or positive. However recent research has considered and theoretically proved the realizability and utility of materials with only one of these parameters having a negative real value. These materials are termed as single negative materials (SNG). They are of both kinds-with epsilon negative (ENG) and mu negative (MNG). Some examples for ENG are plasmonic materials like noble metals in the infrared and visible. It has been found that some of these media have exciting properties [30-34]. When cascaded as a combination of ENG, MNG and DNG media slabs, these can exhibit properties like phase compensation, dispersion compensation etc. These properties may not be visible in single slabs but are a feature of the combination of complementary SNG material pairs (ENG-MNG). Some of the
applications are virtual image formation, growing evanescent fields, wave tunneling and evanescent wave displacement in an ENG-MNG slab pair.

2.3 **Realization of Left-Handed Materials**

Metamaterials are artificial materials and are realized by embedding periodic metallic patterns in dielectric substrates. They are complex structures which result in altering the properties of the bulk composite medium. Thus the regular dielectric when embedded with such structures exhibits a negative effective permittivity and/or permeability for a certain frequency band.

Similar types of alteration of a medium were tried earlier, in case of chiral materials, where, there have been examples of altering the plane of polarization by using twisted structures [35]. These chiral materials, in the same way as left-handed materials, affect the permittivity and permeability of the medium. Also, chiral inclusions are variant with inversions (reciprocity) and are typically anisotropic or bi-anisotropic in nature. Metamaterials also show such properties and are dispersive in nature, which is why they are left-handed only in a certain range of frequency. Metamaterials which show pronounced negative parameters in the microwave region have been realized.

Left-Handed materials were first realized by Schultz et al [36] by creating a periodic array of interspaced conducting non-magnetic split-ring resonators and continuous wires, that exhibited a frequency region in the microwave regime with simultaneously negative values of effective permeability $\mu_{\text{eff}}(\omega)$ and permittivity $\varepsilon_{\text{eff}}(\omega)$. Attempts for realization of negative epsilon materials were made earlier. A 3 dimensional
mesh of conducting wires was used as a structure which altered the epsilon of the supporting substrate to give a frequency dependent negative epsilon $\varepsilon_{\text{eff}}(\omega)$. This composite medium supported propagation modes that had a dispersion relation analogous to that of neutral plasma, and was used earlier for studying the plasma frequency in the ionosphere. Pendry et al in 1999 [27] proved that an array of conducting non-magnetic rings can modify the the permeability of the host substrate to give a frequency dependent negative effective permeability $\mu_{\text{eff}}(\omega)$. When the constituent unit cells are made resonant the magnitude of $\mu_{\text{eff}}(\omega)$ is enhanced considerably. This results in the variation of the bulk $\mu_{\text{eff}}(\omega)$ becoming very large positive value at frequency lower than the resonance and a significantly large negative $\mu_{\text{eff}}(\omega)$ at the higher frequency side of the resonance .

Schultz et al [36] combined the two structures and created a mesh of interspaced split-ring resonators and wire strips. The wire strips affect the epsilon and the split-ring resonators (SRRs) alter the mu of the medium thus giving a frequency dependent negative material with both the parameters negative. The wire medium and the SRRs have certain frequency dependence. By combining the two, the following expression (2-19) was derived by them.

$$k^2 = \frac{(\omega^2 - \omega_p^2)(\omega^2 - \omega_b^2)}{c^2 - (\omega^2 - \omega_s^2)}$$  \hspace{1cm} (2-19)

This equation shows that the range of the propagation band ($k$ real) extends from $\omega_o$ to $\omega_b = \omega_o/\sqrt{(1-F)}$. This was formerly the region of the gap of the SRR structure in the absence of the wires. Here, $F$ is the fractional area of the enclosed by the rings and $\omega_0$ and $\omega_b$ are the resonant frequency and magnetic plasmonic frequency respectively.
These periodic structures can be assumed to give a uniform isotropic alteration of the properties of the base material if the size of the unit cell of these periodic structures is considerably smaller than the smallest wavelength in the bandwidth under consideration. As an analogy, Pendry [27] in his paper on effective permeability, states that any conventional material is a composite with periodic arrays of structures called atoms or molecules. Long wavelengths are myopic to such small structures and hence what actually is a composite of periodic structures appears to the wave as a homogeneous medium, with bulk composite parameters which are actually effective parameters. Now, it is evident that there has to be a constraint on the unit cell dimension that would make it appear homogeneous to the wave under consideration. For a typical electromagnetic wave of frequency ‘ω’ the characteristic dimension ‘a’ of the structure should be as follows in (2-20)

\[ a \ll \lambda = 2\pi c_0 \omega^{-1} \]  \hspace{1cm} (2-20)

There have been several other attempts on establishing the theory and building these materials based on the composite material concept [37-44]. Since this concept gives a number of degrees of freedom to design a customized material for a particular application, there are numerous new materials being designed and explored. 2-D and 3-D structures have been attempted and analyzed for their properties and their effects on the bulk composite medium parameters based on several factors like spacing, size, arrangement, shapes, compositions, density or materials used etc [45]. Lumped element models have been made for these materials so their parameters can be studied
[45]. The Figure 2.3 gives a generic view of metamaterials made with periodic inclusions resulting in an effective bulk permittivity and permeability for the composite medium.

![Figure 2.3: Generic view of a host medium with periodically placed structures constituting a metamaterial](image)

Wire strips causing the epsilon to be negative will be briefly discussed below and the split-ring resonators (SRR) will be examined in detail to understand their contributions to the composite medium.

2.3.1 **Metal Wire Geometry**

Thin metallic wires are found to have properties which can alter the effective permittivity of the host medium, when excited appropriately. In 1998 Pendry [46] evaluated the long wavelength limit for the effective transverse dielectric function of a composite made up of long metallic cylinders embedded in an otherwise homogeneous medium, based on photonic band gap structure theory and calculations. The geometry of a composite medium with periodically placed wire inclusions is shown in Figure 2.4 [46]
They conclusively proved that such a composite can be replaced by an effective homogeneous medium and gave the dispersion relation of the effective epsilon of the composite medium to be derivable from:

$$\varepsilon_{\text{eff}}(\omega) = \frac{k^2c^2}{\omega^2}$$

The effective permittivity can be given by a dispersive relation–Drude dielectric function given by (2-22):

$$\varepsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$$

where $\omega_p$ is the bulk plasma frequency of the metal and $\gamma$ represents an inverse electron relaxation time.

### 2.3.2 Split-Ring Resonator (SRR) Geometry

As discussed earlier, the split-ring resonators are embedded in a host medium to cause the bulk composite permeability of the medium to vary with frequency which becomes negative in a certain frequency band. This band is generally observed to be in the region above the resonant frequency of the SRR. The geometry of the resonator is
fairly simple. The split-ring resonator as the name suggests has a split in its rings. This split causes the ring to resonate at much higher frequencies than a closed ring of the same size would. Typically, a closed ring is a quarter wavelengths in size at its resonant frequency. However with the split in the ring these dimensions reduce to one–tenth of the wavelength, which enables these rings to resonate at frequencies higher than closed rings. Introduction of capacitive elements enhances the magnetic effect produced by the structure. The splits in the rings give the desired capacitive effect. Although the current in the rings of a SRR configuration does not find a closed path due to the split, the current will still flow due to the strong capacitance between the two concentric rings. Since in the SRR the capacitive and inductive effects nullify, the $\mu_{\text{eff}}$ has a resonant form. This happens due to the resonant interaction of the inherent inductance of the structure and the capacitive effect due to the gap and, hence at resonant frequency, electromagnetic energy is shared between the magnetic field and the electrostatic fields within the capacitive structure. Hence we observe that the effective permeability is negative only in a certain frequency band which is around and above the resonance frequency, in general. A typical SRR is shown in the Figure 2.5. In the square SRR ‘a’ is the outer-most dimension, while ‘$r_{\text{ext}}$’ represents the external radius for the circular SRR. The ‘$r_0$’ is the average radius of the two concentric rings. The SRR split has a dimension ‘g’, the width of each ring is ‘c’ and the separation between the two concentric rings is ‘d’.
Figure 2.5 Split-ring resonator (SRR) configurations (a) typical split-ring resonator (b) Circular Split-ring resonator (SRR)

The $\mu_{\text{eff}}$ for such a composite made up of periodic split-ring resonator (SRR) can be given by the expression in (2-23) [27]:

$$
\mu_{\text{eff}}(\omega) = 1 - \frac{F}{1 + \frac{j2\sigma}{\omega \mu_0} - \frac{3}{\pi^2 \mu_0 \omega^2 Cr^3}}
$$

(2-23)

where $F$ is the fractional volume of the cell occupied by the interior of the concentric inner ring of radius ‘r’, if it is a circular SRR and $C$ is the capacitance per unit area between the two concentric rings separated by a distance ‘d’ given by (2-24).

$$
C = \frac{\varepsilon_0}{d} = \frac{1}{dc_0^3 \mu_0}
$$

(2-24)

The $\mu_{\text{eff}}$ begins to diverge from its regular trend at a certain frequency $\omega_0$ which is given by equation (2-25):

$$
\omega_0 = \sqrt{\frac{3}{\pi^2 \mu_0 Cr^3}} = \sqrt{\frac{3dc_0^2}{\pi^2 r^3}}
$$

(2-25)

and the ‘magnetic plasmonic frequency’ ($\omega_{\text{mp}}$) is given by the expression in (2-26)
which means that the plasmonic frequency depends on the fraction not enclosed by the inner ring. The dispersion relation of the permeability of the medium is discontinuous and has gap over the range between $\omega_0$ and $\omega_{mp}$ where the $\mu_{\text{eff}}$ is negative. It is obvious that square geometries will maximize the area effect for the same dimensions; hence the square geometries are used.

In order to understand how the SRR affects the material permeability we need to understand its electromagnetic behavior. Consider an infinitely thin, perfectly conducting (metallic) SRR in an external electromagnetic field $E^0, B^0$. Then the fields scattered by the SRR – $E', B'$ will be given by the fields produced by a magnetic dipole (2-27).

$$m = \alpha_0 (\omega_0^2 / \omega^2 - 1)^{-1} B^0 \hat{z}$$

(2-27)

where $\omega_0$ is the frequency of resonance of the SRR and $\alpha_0$ is a geometrical factor.

### 2.3.3 Composite DNG medium Geometry

In order to fabricate a double negative material with a negative refractive index the metallic wire strips and split-ring resonators are juxtaposed in lattice structure such that both these components are excited strongly, and affect the material parameters. The unit cell of such a composite double negative medium (DNG) with both type of inclusions is shown in Figure 2.6
The wire strips are placed such that the E field passes along their axis and the SRRs are placed such that the H field is parallel to their axis. This is primarily a 1-D anisotropic medium. To make this medium isotropic the following arrangements can be considered:

Several modifications to the SRR have been suggested to obtain variations in the properties of the medium such as spiral SRR for zero cross polarization [45], increased...
number of azimuthal gaps for increased local electrostatic non-linear effects, broad side coupled SRR or non-bi-anisotropic SRR for removing bi-anisotropic behavior avoiding cross polarization [47], double-slit SRR to double the resonant frequency at the same size [45], spiral and double spiral configurations reduction in resonant frequency at the same size [48], orientations for cross polarization of SRRs for frequency selective surfaces, smiling resonators for broader bandwidths [49] and deformed SRRs for size reduction of the unit cell [50] and modified SRR for filter applications [48]. Some of the modifications are as shown in Figure 2.8.

![Figure 2.8](image)

**Figure 2.8:** Variations in the split-ring resonator (a) smiling resonators (b) distorted SRR (c) Modified SRR (d) Broad Side Coupled SRR

### 2.3.4 Complementary Split-Ring Resonator (CSRR):

Among other variations proposed for the SRR the CSRR is an important one. This work uses CSRRs to obtain the effective permittivity for bulk medium. A more involved discussion on CSRRs follows in chapter 4. A few applications of CSRR studied so far by
researchers, are worth a note here. Since the CSRRs were first proposed [51] as potential constituent of a composite medium to obtain negative permittivity (ENG) medium there have been several applications that have been considered and studied. CSRR are found to have a strong potential to design simple planar filters since they have distinct frequency band around their resonant frequency in which the medium has a negative permittivity. Due to this, when a microstrip line is etched on top of an array of CSRRs in the ground plane, it shows stop-band characteristics. In this arrangement the shunt impedance is dominated by the tank inductance of the CSRRs and the arrangement behaves as a one-dimensional effective medium with negative permittivity, in the frequency band above the resonant frequency. The detailed analysis of the CSRR in this configuration in terms of equivalent LC circuits can be found in [27]. To make the filter a pass band instead of a stop band, capacitive gaps need to be etched in the microstrip line to nullify the tank inductance. The structure with capacitive gaps is given below in Figure 2.9.

![Figure 2.9 Pass band filter using CSRR](image)

Coupled line coupler exhibits distinct backward coupling when CSRRs are placed under the coupling arms of the coupler. At the resonant frequency of the CSRR we find
that the power is not coupled into the coupled port but is coupled into the isolated port in the backward direction instead. Figure 2.10 will show the structure.

Figure 2.10: Coupled line coupler (a) with and (b) without CSRR

The transmission coefficient for port 4 and 1 drops down distinctly at the frequency of resonance of the CSRR. Several researchers have studied similar applications which may be found in [52-58],[3].

2.4 Verification of Negative Parameters

Several methods and formulae for analysis of the composite materials and for the determination and evaluation of the dispersive nature of their material parameters have been studied. Pendry [27],[46] in his papers has given formulae for the computation of the material parameters of the host medium, based on electromagnetic analysis of the constituent structures, which are restated as in equation (2-28) and (2-29).

Effective permittivity of due to the metallic wire strips follows the Drude model:


\[ e_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + j\omega\gamma} \]  
(2-28)

Effective permeability of medium due to SRR is:

\[ \mu_{\text{eff}}(\omega) = 1 - \frac{\mu_0 F}{1 + j2\sigma - \frac{3}{\pi^2 \mu_0 \omega^2 Cr^3}} \]  
(2-29)

This method has been studied and implemented as a part of the current work to validate and support the course of progress adopted during the work. A sample plot of the implementation as a Matlab code is shown here in Figure 2.11. The plot is for a SRR with a radius ‘r’ of 2 mm, a lattice spacing ‘a’ of 10 mm, and ‘l’ the vertical spacing of 2 mm. The width of the rings ‘c’ is 1mm and the spacing between the rings ‘d’ is 0.1mm.

![Figure 2.11: Matlab output sample for mathematical expression for effective permeability](image)
Another method has been suggested and implemented by Schultz et al [59] and Ziolkowski et al [60] where a slab of the host medium containing the inclusions is excited by a TEM wave and the transmission coefficient and reflection coefficient are obtained. The characteristic impedance and refractive index expressions are then obtained from the transmission and reflection coefficients. These parameters are then used to compute the dispersive permittivity and permeability of the bulk medium. The set up for this method is shown in Figure 2.12.

![Figure 2.12: Set-up for the material parameter extraction for single and multiple split-rings embedded in a dielectric medium](image)

The above method is typically used to determine the permittivity and permeability of any isotropic dispersive medium. This method has been found to be useful in determining successfully the material parameters even for metamaterials. The equations (2-30), (2-31) and (2-32) are used to determine the intrinsic impedance (z) and imaginary and real parts...
of the refractive index \((n)\) of the composite medium, respectively from the transmission coefficient ‘\(t\)’ and reflection coefficient ‘\(r\)’ [59].

\[
z = \pm \sqrt{\frac{(1 + r)^2 - t^2}{(1 - r)^2 - t^2}}
\]  

\[
\text{Im}(n) = \pm \text{Im}\left(\frac{\cos^{-1}\left(\frac{1}{2r}[1 - (r^2 - t^2)]\right)}{kd}\right)
\]  

\[
\text{Re}(n) = \pm \text{Re}\left(\frac{\cos^{-1}\left(\frac{1}{2r}[1 - (r^2 - t^2)]\right)}{kd}\right) + \frac{2\pi n}{kd}
\]

The \(\mu\) and \(\varepsilon\) of the composite medium are obtained from the equation (2-33)

\[
\mu = \frac{n}{z} \quad \text{and} \quad \varepsilon = n \times z
\]

The sample graph showing the frequency dependent parameters \((\varepsilon\) and \(\mu\)) is as shown in Figure 2.13
Figure 2.13: Material parameters obtained from S-parameters from simulation, plotted as a function of frequency

2.5 Composite Right/Left Handed Materials

Apart from these, interesting attempts have also been made to obtain the left-handed behavior in 1-D and 2-D by cascading unit cells of circuit elements like capacitors and inductors [61,62]. The concept behind these implementations is that a right-handed transmission line can be modeled like an inductor in series a capacitor in parallel, then a transmission line with a dual configuration of the above i.e. capacitor in series and inductor in shunt would potentially behave like a left-handed transmission line. In order to realize such a transmission line an inter-digital capacitor is combined with a shunt inductor to get the left-handed effect. This configuration is however not purely left-handed since the transmission lines are inherently inductive in nature. Hence what results is a composite right/left handed transmission line (CRLH-TL). The transmission line
model is used to analyze the behavior of these structures. The equivalent circuit element diagrams for the right-handed (RH), left-handed (LH) and CRLH transmission lines are as follows [63]:

![Figure 2.14](image.png)

**Figure 2.14:** Equivalent circuit model. (a) Homogeneous RH TL. (b) Homogeneous LH TL. (c) Homogeneous CRLH TL [63].

Coupled line couplers, directional couplers [64], filters, quadrature hybrids, zeroth order resonator antennas [65, 66] and leaky wave antennas [67] have been designed based on the transmission line model using the inter-digital capacitors and stubbed inductors. These studies primarily delve into the one dimensional components. However, there are attempts on similar lines to obtain a 2-D structure [69]. There have also been attempts to model the 2-D and 3-D structure as transmission line networks [70].
The idea of a left handed material as stated earlier is that when a uniform plane wave is launched in such a medium it will have a phase velocity opposite to that of its Poynting vector. When such a double negative material (DNG) is combined with a double positive material (DPS) to obtain a group of alternating DPS and DNG slabs, the wave can travel through this combination without a phase shift if the thickness of these slabs is adjusted to achieve phase compensation. This leads to a concept of a sub-wavelength 1-D resonator proposed in [5], where it is mathematically derived, that a 1-D resonator bound by two parallel conducting plates with DPS and DNG slabs may have sub-wavelength dimensions, since the thickness of the resonator becomes independent of the wavelength and solely depends on the ratio of the material permeabilities. An elaborate discussion on this concept and its applications may be found in [71].

This concept of a miniaturized cavity resonator can be extended to a 3-D cavity and further to a microstrip patch antenna through the cavity model. Cylindrical cavities with DPS-DNG loading have been analyzed in [32]. The patch antenna is traditionally modeled as cavity resonator. The size of the radiating edge of the patch is approximately half the wavelength in free space for the resonant frequency of the cavity. The input impedance and the resonant frequency of the patch can be easily determined when it is modeled as a cavity. The rectangular microstrip antenna that has been considered in this work is placed on a DPS-DNG combined substrate. Two configurations of DPS-DNG loading are analyzed. The patch on the DPS-DNG substrate is then modeled as a cavity...
with 2 PEC and 4 PMC walls with two dielectrics. The interface between the two
dielectrics is considered as a dielectric-dielectric boundary.

We will first briefly understand the 1-D cavity resonator and the condition for
miniaturization obtained through phase compensation. We will then describe in detail the
derivations for two configurations of DPS-DNG substrate combination under the patch
using the 3-D cavity model and obtain the size reduction condition for both
configurations.

3.1 1-D Sub-Wavelength Cavity Resonator (SWCR) based on phase compensation
phenomenon:

The ‘1-D sub-wavelength cavity resonator’ (SWCR) as it is termed [5] consists of
two juxtaposed slabs, one DPS and one DNG, between two PEC walls. The propagating
wave is a simple TEM wave through this arrangement in the direction normal to the
walls. When the boundary condition at the slab interface is applied to the field equations
in the two dielectric regions we obtain a transcendental equation. The diagram illustrating
the 1-D cavity resonator with DPS-DNG slabs is shown in Figure3.1.

![Figure 3.1](image_url)

**Figure 3.1**: The 1-D sub-wavelength cavity resonator with a DPS-DNG combination substrate;
note that the phase constant $k_2$ in the second substrate with negative permeability and permittivity
has a direction opposite to that of the Poynting vector.
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The widths of the DPS and DNG slabs are ‘d_1’ and ‘d_2’ respectively. The directions of the phase velocities and Poynting vectors in each medium are given by the arrows in the respective slabs. Observe that the phase vector in the DNG substrate is anti-parallel to the direction of energy flow (Poynting vector). Consider a TEM wave propagating between the two conducting plates (PEC) in the DPS-DNG combined medium. The E field and the H field are in the x and y direction respectively. Then, at the interface of the two dielectrics, the tangential component of E and H propagate continuously. This is expressed as in (3-1) below.

\[
E_{x1} \big|_{z=d_1} = E_{x2} \big|_{z=d_1}, \\
H_{y1} \big|_{z=d_1} = H_{y2} \big|_{z=d_1}
\] (3-1)

The electric and magnetic fields for a TEM wave in the two regions can be written as:

**For Region I (DPS)**

\[
E_{x1} = E_{o1} \sin(n_1k_0z) \\
H_{y1} = \frac{n_1k_0}{j\omega\mu_1} E_{o1} \cos(n_1k_0z)
\] (3-2)

\[
E_{x2} = E_{o2} \sin[n_2k_0(d_1 + d_2 - z)] \\
H_{y2} = -\frac{n_2k_0}{j\omega\mu_2} E_{o2} \cos[n_2k_0(d_1 + d_2 - z)]
\] (3-3)

where the \( k_0 \) is the free space wave number, \( n_1, n_2 \) are the refractive indices of the two regions respectively, and \( \mu_1 \) and \( \mu_2 \) are the permeabilities of the two regions respectively, and \( \omega \) is the angular frequency.

When fields in both the regions are equated at the interface we obtain the following relation in equation (3-4)
For a non-trivial solution for the E field we take the ratio of the two equations in (3-4) to obtain (3-5)

\[
\frac{n_2}{\mu_2} \tan(n_1 k_0 d_1) = \frac{n_1}{\mu_1} \tan(n_2 k_0 d_2)
\]

where 
\( \mu_i = |\mu_i| \) for positive material (DPS)
\( \mu_z = -|\mu_z| \) for negative material (DNG)

In a limiting condition when the \( k_0 \) and ‘\( d_1 \)’ and ‘\( d_2 \)’ values are small, the tangent of the argument becomes equal to the argument itself and hence we obtain equation below which leads to (3-6).

\[
\frac{d_2}{|\mu_2|} = \frac{d_1}{|\mu_1|}
\]

Here, we observe that the 1-D sub-wavelength resonator does not have to satisfy the condition of \( d_1 + d_2 \) approximately equal to \( \lambda/2 \), but needs to satisfy only the inverse ratio between the absolute values of the two permeabilities. Hence the cavity resonator could be much smaller than the wavelength at which it resonates. It can be as small as \( \lambda/10 \) of the wavelength at the resonant frequency. This is a significant size reduction. Hence this condition is termed as the ‘electrically-small size’ condition. A detailed analysis of modes and mode excitation and the FDTD simulations can be found in [71]. However, there can be a few limitations to such a resonator which are discussed in [72].
3.2 Patch antenna using phase compensation – Cavity model

The present work investigates the rectangular microstrip patch antenna with DPS-DNG substrate for two configurations – one with the DPS-DNG interface parallel to the radiating edges as in Figure 3.2 (a) and the other with DPS-DNG interface normal to the radiating edges as in Figure 3.2 (b). The cavity model with two electric walls (PEC) and four magnetic walls (PMC) has been used. The condition for propagation and the modes that can exist have been derived for each configuration.

Figure 3.2: DPS-DNG substrate for a rectangular patch antenna (a) DPS-DNG interface parallel to radiating edge (b) DPS-DNG interface normal to radiating edge

Figure 3.3: 3-D view of DPS-DNG substrate for a rectangular patch antenna (a) DPS-DNG interface parallel to radiating edge
The fields that exist in an open-walled cavity have been considered for the microstrip patch antenna. The derived expressions are, as expected, different in the two dielectric regions. At the interface of the two dielectrics the normal components of the E and H fields are continuous. Using this condition of equality at the interface, an expression is arrived at, which is a non-trivial solution for the E and the H fields in both regions of the substrate, namely region-I (DPS) and region-II (DNG).

In both configurations, the TE and TM modes that can exist have been obtained. It may be noted that the field expressions for TE and TM modes oriented along all three rectangular coordinate directions have been considered and checked for satisfying the boundary condition and the ‘electrically-small size’ condition. It has been found that unlike the conventional microstrip patch antenna, the modes in the above configurations were TM oriented normal to the DPS-DNG slab interface. In the first configuration (Figure 3.2 (a)) the mode that satisfies the ‘electrically-small size’ condition is found to be TM\(^x\), while in the second configuration (Figure 3.2 (b)) it is TM\(^y\). The detailed derivations of the expressions for the E and H fields and the electrically-small size condition are given below for both configurations.

3.2.1 **Configuration 1: DPS-DNG interface parallel to the radiating edge**

For the first configuration shown in figure 3.2(a) the fields for the TM\(^x\) mode expressed in terms of their magnetic vector potential have the expression in (3-7)
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\begin{align}
H_x &= \frac{1}{\mu} \frac{\partial A_y}{\partial y} - j \frac{1}{\omega \mu \epsilon} \frac{\partial^2 A_z}{\partial x \partial z} \\
E_x &= -j \frac{1}{\omega \mu \epsilon} \frac{\partial^2 A_z}{\partial y \partial z} \\
H_y &= \frac{1}{\mu} \frac{\partial A_x}{\partial x} \\
E_y &= -j \frac{1}{\omega \mu \epsilon} \frac{\partial^2 A_z}{\partial y \partial z} \\
H_z &= 0 \\
E_z &= -j \frac{1}{\omega \mu \epsilon} \left( \frac{\partial^2}{\partial z^2} + \beta^2 \right) A_z
\end{align}

(3-7)

where

\[
A_z(x, y, z) = C \sin \beta_x x \cos \beta_y y \sin \beta_z z
\]

where \( A \) is the magnetic vector potential with a non-zero component in the \( z \) direction.

The magnetic vector potential \( A \) in the two regions is shown in (3-8)

\[
A_{z1}(x, y, z) = C_1 \sin \beta_{x1} x \cos \beta_{y1} y \sin \beta_{z1} z
\]

and

\[
A_{z2}(x, y, z) = C_2 \sin \beta_{x2} x \cos \beta_{y2} y \sin \beta_{z2} (d - z)
\]

(3-8)

where \( d_1 + d_2 = d \), where \( d \) is the total thickness of the slabs together and \( d_1 \) and \( d_2 \) are the thicknesses of the DPS and DNG slabs respectively.

The \( H \) and \( E \) fields in both the regions (DPS and DNG) are stated in equation (3-9)

\[
H_{x1} = -\frac{\beta_{x1}}{\mu_1} C_1 \cos \beta_{x1} x \cos \beta_{y1} y \sin \beta_{z1} z
\]

\[
H_{x2} = -\frac{\beta_{x2}}{\mu_2} C_2 \cos \beta_{x2} x \cos \beta_{y2} y \sin \beta_{z2} (d - z)
\]

\[
E_{x1} = -j \frac{\beta_{x1} \beta_{y1}}{\omega \mu_1 \epsilon_1} C_1 \cos \beta_{x1} x \cos \beta_{y1} y \cos \beta_{z1} z
\]

\[
E_{x2} = +j \frac{\beta_{x2} \beta_{y2}}{\omega \mu_2 \epsilon_2} C_2 \cos \beta_{x2} x \cos \beta_{y2} y \cos \beta_{z2} (d - z)
\]

(3-9)

The boundary conditions at the interface are in equation (3-10)

\[
E_{x1} \big|_{z=d_1} = E_{x2} \big|_{z=d_1}
\]

\[
H_{y1} \big|_{z=d_1} = H_{y2} \big|_{z=d_1}
\]

(3-10)
When the boundary conditions at the interface are applied and a non-trivial solution is sought, we obtain the expression in (3-12) using (3-11).

\[
\frac{E_{z1}}{H_{y1}} \bigg|_{z=d_1} = \frac{E_{z2}}{H_{y2}} \bigg|_{z=d_2} \tag{3-11}
\]

\[
\frac{\beta_{z1}}{\varepsilon_1} \tan \beta_{z2} d_2 = -\frac{\beta_{z2}}{\varepsilon_2} \tan \beta_{z1} d_1 \tag{3-12}
\]

This leads to the final expression for the mode TM° for the configuration-I stated in equation (3-13) when arguments of the tangent function are small i.e. $\beta_{z2} d_2$ and $\beta_{z1} d_1$ are small.

\[
\frac{d_1}{d_2} = \frac{\varepsilon_2}{\varepsilon_1} \tag{3-13}
\]

Observe here that the width of each slab is determined by the inverse ratio of their permittivities and is independent of the constraint of $d_1 + d_2 \sim \lambda_g/2$.

### 3.2.2 Configuration 2: DPS-DNG interface perpendicular to the radiating edge

For the second configuration in the Figure 3.2 (b) the field equations for the TM° mode are as in equation (3-14).

\[
H_x = \frac{1}{\mu} \frac{\partial A_y}{\partial z} \quad E_x = -j \frac{1}{\omega \mu \varepsilon} \frac{\partial^2 A_y}{\partial x \partial y} \\
H_y = 0 \quad E_y = -j \frac{1}{\omega \mu \varepsilon} \left( \frac{\partial^2 A_y}{\partial y^2} + \beta^2 \right) A_y \\
H_z = \frac{1}{\mu} \frac{\partial A_y}{\partial x} \quad E_z = -j \frac{1}{\omega \mu \varepsilon} \frac{\partial^2 A_y}{\partial y \partial z} \tag{3-14}
\]

where

\[
A_y(x, y, z) = C_0 \sin \beta_x x \sin \beta_y y \cos \beta_z z
\]
where $\mathbf{A}$ is the magnetic vector potential with a non-zero component in the $y$ direction.

The magnetic vector potential $\mathbf{A}$ in the two regions is shown in (3-15)

$$A_{y_1}(x, y, z) = C_1 \sin \beta_{y_1} x \sin \beta_{y_1} y \cos \beta_{z_1} z$$

and

$$A_{y_2}(x, y, z) = C_2 \sin \beta_{y_2} x \sin \beta_{y_2} (w - y) \cos \beta_{z_2} z$$

where $w_1 + w_2 = w$, and $w$ is the total thickness of the slabs together and $w_1$ and $w_2$ are the thicknesses of the DPS and DNG slabs respectively.

The $H$ and $E$ fields in both the regions (DPS and DNG) are stated in equation (3-16)

$$H_{x_1} = \frac{\beta_{y_1}}{\mu_1} C_1 \sin \beta_{y_1} x \sin \beta_{y_1} y \sin \beta_{z_1} z$$

$$H_{x_2} = \frac{\beta_{y_2}}{\mu_2} C_2 \sin \beta_{y_2} x \sin \beta_{y_2} (w - y) \sin \beta_{z_2} z$$

$$E_{z_1} = j \frac{\beta_{z_1} \beta_{y_1}}{\omega \mu_1 \varepsilon_1} C_1 \sin \beta_{x_1} x \cos \beta_{y_1} y \sin \beta_{z_1} z$$

$$E_{z_2} = -j \frac{\beta_{z_2} \beta_{y_2}}{\omega \mu_2 \varepsilon_2} C_2 \sin \beta_{x_2} x \cos \beta_{y_2} (w - y) \sin \beta_{z_2} z$$

The boundary conditions at the interface are in equation (3-17)

$$E_{z_1} \big|_{y = w_1} = E_{z_2} \big|_{y = w_1}$$

$$H_{x_1} \big|_{y = w_1} = H_{x_2} \big|_{y = w_1}$$

When the boundary conditions at the interface are applied and a non-trivial solution is sought for the fields, we obtain the expression in (3-19) using (3-18).

$$\frac{E_{z_1}}{H_{x_1}} \big|_{y = w_1} = \frac{E_{z_2}}{H_{x_2}} \big|_{y = w_1}$$

(3-18)
\[
\frac{\beta_{y1}}{\varepsilon_1} \tan \beta_{y2} w_2 = -\frac{\beta_{y2}}{\varepsilon_2} \tan \beta_{y1} w_1
\] (3-19)

This leads to the final expression for the mode TM\textsuperscript{y} for the configuration-II stated in equation (3-20) when arguments of the tangent function are small i.e. \(\beta_{y2} d_2\) and \(\beta_{y1} d_1\) are small.

\[
\frac{w_1}{w_2} = \left| \frac{\varepsilon_2}{\varepsilon_1} \right|
\] (3-20)

Observe here that the width of each slab is determined by the inverse ratio of their permittivities and is independent of the constraint of \(w_1 + w_2 \sim \lambda_0/2\).

This leads to a size reduction in the non-radiating edge length \((d_1 + d_2) \ll \lambda_0/2\) in Figure 3.2 (a) and a size reduction in the radiating edge length \((w_1 + w_2) \ll \lambda_0/2\) in Figure 3.2 (b).

It may be noted here, that in each configuration, only the derivation for the modes that satisfy both, the open-wall boundary condition and the ‘electrically-small size’ condition, have been shown. The other modes with TE/TM oriented along other directions do not satisfy both the conditions.
Chapter 4

Analysis of Complementary Split-Ring Resonators for Patch Antenna Application

The split-ring resonators (SRR) are metallic inclusions in the substrate and hence they are not very useful for 2-D planar structures with thin dielectrics. It is important to note that the SRRs need to have a certain orientation with respect to the fields in the medium they are embedded in. The H field of the electromagnetic wave has to be along (parallel) the axis of the SRR in order to induce the current through the rings which gives the desired effect. If SRRs are to be used in planar circuitry, the substrate in the microstrip configuration needs to be thicker and hence the configuration becomes bulkier. This defeats the purpose of small and compact devices, which is an important advantage of microstrip technology. For example in the paper by Ikonen et. al. [75], we observe that the thickness of the substrate is 10mm, which is very large as compared to normal microstrip substrates, usually ranging from 0.5mm to 3mm for typical applications. Hence we need to have periodic structures which will not only be easy to fabricate but will not increase the dimensions of the device. The idea of the complementary split-ring resonator – slots replacing the metallic structures – sounds like an obvious choice, based on the principle of duality in field theory. In this chapter, the geometry of the complementary split-ring resonator (CSRR) is studied and its effects on the bulk composite permittivity of the host medium are understood. Methods for extraction of the
bulk composite material parameters are developed and used. The resonant frequency of the CSRRs of various sizes is also determined.

### 4.1 Complementary Split-Ring Resonator (CSRR) Geometry:

The complementary split-ring resonator (CSRR) or ‘slotted split-ring resonator’ (SSRR) or ‘dual split-ring resonator (DSRR)’ as it is called, comprises of slots, corresponding to SRR structures, in the common metallic plate of a planar device. These slots have the exact dimensions as the corresponding SRR. By the principle of duality the CSRRs have properties that are duals of the properties of the SRRs.

In order to understand how the CSRR affects the material permittivity we need to understand its electromagnetic behavior. Consider an infinitely thin, perfectly conducting (metallic) plate with slots of the shape of SRR. By the well known principle of duality, they are expected to have characteristics which are duals of the properties of SRRs. As discussed earlier in chapter 2, when a SRR is placed in an external magnetic field $E^0, B^0$, the field scattered by the SRR $E^{'}, B^'$ will be given by the field produced by a magnetic dipole as in equation (4-1).

$$\vec{m} = \alpha_0 (\omega_0^2 / \omega^2 - 1)^{-1} \vec{B}^{(0)}_{z,z}$$ (4-1)

where $\omega_0$ - frequency of resonance of the SRR; $\alpha_0$ - geometrical factor. The SRR behaves as a magnetic point dipole and by Babinet’s principle [76] the CSRR behaves as an electric point dipole with negative polarizability. The fields scattered by the CSRR are then given by the fields produced by the electric dipole given as in equation (4-2).
The paper by Falcone et al [77] explains the concept in detail based on wave propagation and reflection due to the CSRR.

Since the CSRR is the dual of the SRR, it is, by the virtue of the duality principle, not excited by the H-field but is excited by the E-field. The E-field has to be parallel to the axis of the CSRR. In planar technology the E field is normal to the ground plane and the patch geometry on the trace. Thus, the CSRRs are etched in the ground plane such that they can be easily excited by an E-field parallel to their axes. This, in fact will not only be easy to fabricate but also will reduce the bulkiness of the substrate. The CSRR, however, will affect the material permittivity, unlike the SRR which affects the material permeability. Figure 4.1 below shows the split-ring resonator and its dual the complementary split-ring resonator.

\[
\bar{p} = \beta_0 (\omega_0^2 / \omega^2 - 1)^{-1} \bar{E}_{cut}^0
\]  

(4-2)

![Figure 4.1: Split-Ring Resonator (SRR) and Complementary Split-Ring Resonator (CSRR)](image-url)
4.2 Determination of Resonant Frequency and Material Parameters ($\mu$ and $\varepsilon$):

The SRR and the CSRR have approximately the same resonance frequency since the dimensions are the same, and may be given by equation (4-3) [27]:

$$\omega_0 = \sqrt{\frac{2}{\pi r C_{p.u.l} L}}$$

(4-3)

where $C_{p.u.l.}$ and $L$ are the capacitance per unit length between the two rings and their inductance respectively. The capacitance and inductance values can be determined depending on the geometry of the CSRR.

4.2.1 Modeling CSRR in Ansoft HFSS:

Experimental methods can also be used to determine the resonant frequency of the CSRR. One such method is to place the CSRR in between 2 parallel PEC and parallel PMC walls to simulate a TEM environment and excite it with the TEM wave. This method is similar to the one discussed in chapter 2. The resultant transmission and reflection coefficients are then used to determine the resonant frequency and the effective permittivity caused by the CSRR. This method, however, is difficult to set-up, for both experimental and simulation environments, since it requires 3-D arrangements.

This method has been adopted to extract the material parameters of the host substrate and the resonant frequency of the CSRR through simulation, in this work. Ansoft–HFSS has been used for this purpose. A TEM environment is built using two parallel PEC walls as top and bottom surfaces and two parallel PMC walls as side surfaces. Two wave-ports are defined at the two remaining surfaces. A dielectric material with permittivity equal to that of the material chosen for fabrication (here RT5870
\( \varepsilon_r = 2.33 \) is placed inside the PEC and PMC walls equidistant from both wave-ports. The volume between the wave-port and the dielectric face is filled with air on either side. The slots of the CSRRs are cut in the top PEC boundary and defined as PMC. This can be done by drawing a metal SRR of very small thickness and subtracting this from the PEC sheet. Then the slotted portion can be selected and assigned as PMC. The diagram illustrating the set-up is shown in Figure 4.2.

A sample S-parameter graph showing the resonant frequency for a CSRR is shown in Figure 4.3. The width between the outer-most edges of the CSRR used in this example is 3mm on all sides. The width of each ring is 0.33mm, the spacing between the rings is 0.33mm and the azimuthal gap (width of split in the rings) is 0.33mm.
The S-parameters are used to obtain the reflection and transmission coefficients, and these are used to determine the material parameters ($\mu$ and $\varepsilon$), using a MATLAB code based on equations relating the coefficients to the $\mu$ and $\varepsilon$ (discussed earlier in chapter 2). The sample extracted parameters as a function of frequency are shown below in Figure 4.4.
4.2.2 Modeling CSRR in Ansoft Designer

Another method to find the resonant frequency of the CSRR is to place the CSRRs in the ground plane under a 50 Ω line on the trace of a planar material with a dielectric constant equal to that of the material used for fabrication (here RT5870 $\varepsilon_r=2.33$). The 50 Ω line is chosen to match the port impedances.

The S-parameters of this set-up give us the transmission and reflection coefficients which could be used to determine the material parameters. The dip in the transmission coefficient gives the resonant frequency right away. This method is simulated using a planar simulation tool (Ansoft-Designer). The set-up is illustrated in Figure 4.3: Parameter Extraction: permittivity and permeability for HFSS model.
Figure 4.5. The CSRR (shown in green) are slots in the ground plane, and the microstrip line (shown in black) is on the trace. The line is excited using edge ports.

This arrangement will give a reasonably accurate value for the resonant frequency. The slight shift in resonances as compared to the HFSS simulation model could be attributed to the fact that a microstrip configuration has a quasi-TEM wave instead of a TEM wave and the solver being a full-wave solver based on method of moments. The 50 Ω line is chosen to match the port impedances.

Various sizes of CSRRs have been considered in simulation to determine their resonant frequency and the effective bulk parameters of the composite medium as a function of frequency. The second method (Method-II) has been used for this purpose since it closely resembles the set-up of the microstrip patch antenna desired. Seven sizes of CSRRs have been studied. Their sizes are 1.5mm, 2mm, 3mm, 4mm, 5mm, 6mm, 9mm (dimension between outer-most edges). The following observations have been made:
• The resonant frequency is inversely related to the size – an approximate trend showing that if size was doubled frequency become half, is observed

• Upon comparison of S21 graph with the graphs of real \( \mu \) and \( \varepsilon \), it has been observed that the stop bands in the S21 graph around the resonant frequencies correspond with the frequency range in which either real \( \mu \) and/ or real \( \varepsilon \) is negative

An example S-parameter graph for a 3mm CSRR (dimensions same as in Method-I) from the planar simulation set-up is shown in Figure 4.6. It may be noted that the stop band is observed around 10 GHz which is the resonant frequency of the CSRR.

![S-parameter graph](image.png)

**Figure 4.6:** S-parameter output of planar set-up for resonant frequency of CSRR

The stop band at 10 GHz for the 3mm CSRR corresponds to a negative permittivity in the frequency range around it, as seen in the Figure 4.6 where the real \( \mu \) and real \( \varepsilon \) are plotted against the frequency.
The details of the dimensions for the seven different CSRRs studied are tabulated in the Table 4.1 below.

<table>
<thead>
<tr>
<th>CSRR #</th>
<th>Outer width of CSRR</th>
<th>Spacing between rings</th>
<th>Width of rings</th>
<th>Azimuthal gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>2</td>
<td>5.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>2.00</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>1.50</td>
<td>0.10</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>9.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>6.00</td>
<td>0.30</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>7</td>
<td>4.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 4.1: Dimensions of CSRR used for the determination of the resonant frequency
It may be noted that the values of ‘d’, ‘c’ and ‘g’ have been chosen at random relative to the value of ‘a’. However, an effort is made to maintain consistency in the scaling of all dimensions when different sizes of CSRRs are studied.

As mentioned earlier the stop-band in case of each CSRR corresponds to either negative real permittivity (in some cases also, negative real permeability). The resonant frequencies and corresponding signs of the $\varepsilon$ and $\mu$ are tabulated in Table 4.2.

<table>
<thead>
<tr>
<th>Outer width of CSRR (h mm)</th>
<th>Stop Band from S21 (GHz)</th>
<th>Permittivity</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>18-19.5</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>2.00</td>
<td>14.5-15</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>15-17</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>3.00</td>
<td>9.9-11</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>4.00</td>
<td>6.5-7.5</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>7.5-10</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>5.00</td>
<td>5.7-6.2</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>6.00</td>
<td>3.7-5.0</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>5-5.5</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>9.00</td>
<td>3-3.8</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td></td>
<td>7-9.5</td>
<td>Positive</td>
<td>Negative</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of resonant frequency of CSRR and corresponding real $\mu$ and $\varepsilon$

The graphs showing the S21 and the real $\mu$ and $\varepsilon$ as a function of frequency for each CSRR studied are shown in the Appendix B.

It may be recalled from chapter 3 that the condition for patch miniaturization is dependent on the ratio of the absolute values of the material permittivities of the DPS and DNG slabs. Also, note that CSRRs tend to alter the permittivity of the host medium to obtain negative $\varepsilon$ in the region around its resonant frequency. Hence CSRRs have been chosen to partially load the patch area along the radiating edge away from the feed, to
obtain an effective negative permittivity. The application of CSRRs to patch antennas and the novel designs for patch size reduction are dealt with, in the next chapter.
Chapter 5

Antennas with CSRR – Design, Construction and Validation

This chapter deals with the design and construction of the practical planar antennas with CSRR in the ground plane for reduction in size and experimental verification of the designs and validation of the predicted results.

5.1 Antenna Design and Construction:

5.1.1 Design through Simulation:

Several combinations of arrangements and sizes for rectangular microstrip patch antennas with CSRRs in the ground plane have been considered. The DNG part of the substrate in each configuration is realized by placing the CSRRs in the ground plane under a partial area of the patch. Both the configurations of substrate combinations shown in Figure 3.2 (a) and (b) are realized as shown in Figure 5.1 (a) and (b) respectively. From the frequency response and radiation patterns it is seen that the configuration, where the DPS-DNG interface is parallel to the radiating edge corresponding to Figure 5.1 (a) the results are encouraging. As for the configuration where the interface is perpendicular to the radiating edge corresponding to Figure 5.1 (b) the antenna has a very large undesired back lobe radiation although it does show a size reduction. Hence this work focuses on the first configuration (Figure 5.1 (a)). For completeness, the simulation results for the second configuration (Figure 5.1 (b)) are given in Appendix F.
Chapter 5: Antennas with CSRR

Figure 5.1 Planar layout of (a) configuration I and (b) configuration II with CSRR slotted on the ground plane only in one half of the patch

For the configuration I, (Figure 3.2 (a)) several combinations of CSRR of outer dimension ‘r’ and patch size ‘a x b’ have been considered for simulation (Table 5.1). The substrate used is Rogers RT5870 ($\varepsilon_r=2.33$ and $t=31$ mils). An example of layout of the antenna in the planar simulation tool (Ansoft’s Designer) is illustrated in Figure 5.1 (a).
The frequency response for each antenna with and without CSRR has been obtained and a shift in resonant frequency is calculated and listed in Table 5.1. It can be noted that the frequency shifts to a lower value for the same size of the patch when it has CSRRs in the ground plane. The simulation graphs showing the return loss and SWR of each antenna in Table 5.1 are given in Appendix E.

<table>
<thead>
<tr>
<th>Antenna #</th>
<th>Size of patch</th>
<th>Outer size of CSRR</th>
<th>Number of CSRRs</th>
<th>Frequency without CSRR (GHz)</th>
<th>Frequency with CSRR (GHz)</th>
<th>Shift in Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a (mm)</td>
<td>b (mm)</td>
<td>h (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.50</td>
<td>7.00</td>
<td>3</td>
<td>13.0</td>
<td>9.80</td>
<td>3.20</td>
</tr>
<tr>
<td>2</td>
<td>10.90</td>
<td>6.50</td>
<td>3</td>
<td>14.0</td>
<td>9.90</td>
<td>4.10</td>
</tr>
<tr>
<td>3</td>
<td>12.00</td>
<td>7.50</td>
<td>3</td>
<td>12.5</td>
<td>9.72</td>
<td>2.78</td>
</tr>
<tr>
<td>4</td>
<td>14.98</td>
<td>9.29</td>
<td>3</td>
<td>10.0</td>
<td>8.40</td>
<td>1.60</td>
</tr>
<tr>
<td>5</td>
<td>14.98</td>
<td>9.29</td>
<td>2</td>
<td>10.0</td>
<td>8.70</td>
<td>1.30</td>
</tr>
<tr>
<td>6</td>
<td>13.80</td>
<td>8.5</td>
<td>1.5</td>
<td>11.0</td>
<td>10.10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 5.1: Shift in frequency for a fixed patch size with and without CSRR in the ground plane

For example, the antenna # 4 in Table 5.1 resonates at 8.4 GHz with the CSRRs, whereas, at 10 GHz without the CSRR. This is as shown in Figure 5.2 where the return loss has been plotted as a function of frequency.

Figure 5.2 Shifted resonant frequency for constant dimension patch with and without 3mm CSRRs in the ground plane (simulation)
Chapter 5: Antennas with CSRR

The shift in the resonant frequency to a lower value when CSRR are placed is promising because it suggests a reduction in patch size. In order to quantitatively determine the percentage reduction in patch size, antennas resonating in the frequency range 8-12 GHz are analyzed and compiled in Table 5.2. For a desired resonant frequency of 10 GHz in Table 5.2 the patch size without the CSRR is 139.16 sq.mm and with the CSRR it is 70.85 sq. mm resulting in a 49 percent size reduction. It should be noted that the correlation between the number and size of CSRR and percentage reduction in size is yet to be determined.

<table>
<thead>
<tr>
<th>Desired frequency</th>
<th>Size of CSRR</th>
<th>Size of patch without CSRR</th>
<th>Size of patch with CSRR</th>
<th>Percent Reduction in patch Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>(GHz)</td>
<td>(mm)</td>
<td>a (mm)</td>
<td>b (mm)</td>
<td>(sq. mm)</td>
</tr>
<tr>
<td>8.5</td>
<td>3</td>
<td>17.65</td>
<td>11.03</td>
<td>194.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.98</td>
<td>9.29</td>
<td>139.16</td>
</tr>
<tr>
<td>8.5</td>
<td>2</td>
<td>17.65</td>
<td>11.03</td>
<td>194.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.98</td>
<td>9.29</td>
<td>139.16</td>
</tr>
<tr>
<td>8.5</td>
<td>3</td>
<td>17.65</td>
<td>11.03</td>
<td>194.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.50</td>
<td>8.00</td>
<td>100.00</td>
</tr>
<tr>
<td>9.5</td>
<td>3</td>
<td>15.78</td>
<td>9.80</td>
<td>154.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.00</td>
<td>7.50</td>
<td>90.00</td>
</tr>
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<td>9.9</td>
<td>3</td>
<td>14.98</td>
<td>9.29</td>
<td>139.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.90</td>
<td>6.50</td>
<td>70.85</td>
</tr>
<tr>
<td>10.5</td>
<td>3</td>
<td>14.28</td>
<td>8.82</td>
<td>125.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.50</td>
<td>7.00</td>
<td>80.50</td>
</tr>
</tbody>
</table>

Table 5.2: Percent reduction in size of patch for a desired frequency- with CSRR as compared to without CSRR in ground plane

The above analysis of input characteristics has to be supported with a study of radiation characteristics. The CSRRs are basically slots in the ground plane and hence may behave as radiators and therefore the back lobe radiation needs to be addressed. Table 5.3 shows a few examples of patch size and CSRR size combinations, on a Rogers RT5870 substrate, where the radiation patterns have been obtained using Ansoft’s Designer. Some of these have been shown in Figures 5.3 through 5.6 to illustrate the size of the back lobe
relative to the front lobe. From this analysis the acceptability of the back lobe has been listed in Table 5.3 relative to each antenna.

<table>
<thead>
<tr>
<th>Size of patch</th>
<th>Size of CSRR</th>
<th>Antenna fr</th>
<th>CSRR Stop Band</th>
<th>Back Lobe Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (mm)</td>
<td>b (mm)</td>
<td>h (mm)</td>
<td>(GHz)</td>
<td>(GHz)</td>
</tr>
<tr>
<td>10.9</td>
<td>6.5</td>
<td>1.5</td>
<td>12.2</td>
<td>18-19.5</td>
</tr>
<tr>
<td>10.9</td>
<td>6.5</td>
<td>3.0</td>
<td>9.9</td>
<td>9.9-11.0</td>
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<tr>
<td>10.9</td>
<td>6.5</td>
<td>5.0</td>
<td>8.8</td>
<td>5.5-6.2</td>
</tr>
<tr>
<td>11.5</td>
<td>7.0</td>
<td>1.5</td>
<td>11.5</td>
<td>18-19.5</td>
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<tr>
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<td>7.0</td>
<td>3.0</td>
<td>10.5</td>
<td>9.9-11.0</td>
</tr>
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<td>12.5</td>
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<td>6.0</td>
<td>9.97</td>
<td>3.7-5.5</td>
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<td>12.0</td>
<td>7.5</td>
<td>5.0</td>
<td>9.2</td>
<td>5.5-6.2</td>
</tr>
</tbody>
</table>

Table 5.3: Back lobe radiation in antennas with CSRRs– relation with size of CSRR (simulation)

Figure 5.3 Radiation pattern with small back lobe for antenna #4 in table 5.3
Figure 5.4 Radiation pattern with small back lobe for antenna # 3 in table 5.3

Figure 5.5 Radiation pattern with small back lobe for antenna # 10 in table 5.3
The back lobe level could be related to the frequency response of the CSRR. As an example, antenna # 3 in Table 5.3 has a resonant frequency of 9.9 GHz which corresponds to the stop band frequency of the 3 mm CSRR (CSRR # 1 in Table 4.2) that is placed in its ground plane. It can be concluded that the back lobe radiation could be reduced significantly if the CSRR used under the patch has a stop-band corresponding to the desired resonant frequency of the antenna.

5.1.2 Construction:

The antennas designed were fabricated through a chemical etching process on a Rogers RT5870 duroid material with $\varepsilon_r=2.33$ and $t=31$ mils. The designs were first imported into CAM350 software for PCB layout as gerber data from Ansoft Designer planar tool. The two layers, one the top copper layer and the second, the ground plane, were created as two separate composites in the CAM350. The composite files are sent for
chemical etching where the copper on both sides of the duroid material is removed except in desired areas, thus giving the shape the antennas and CSRRs in the ground plane. SMA connectors are connected to the antennas to be able to feed power into the antenna.

5.2 Experimental Set-up and Measurements

Based on the above analysis of resonant frequency and minimized back lobe radiation some selected antennas have been constructed and tested. The front and the back view of an example antenna with CSRR in the ground plane are shown in Figure 5.7.

Figure 5.7: Back and Front View of the antenna with CSRR in ground plane (The black line indicates the border of the antenna on the front side)
5.2.1 Validation of Shift in Resonant Frequency for Antennas with CSRRs

For each antenna the return loss, standing wave ratio and impedance have been measured in the frequency range 6-15 GHz, using the Agilent PNA E8363B network analyzer shown in Figure 5.8.

![Network Analyzer](image)

**Figure 5.8:** Set-up for testing using a Network Analyzer (PNA E8363B Agilent)

Screen captures showing the measured S-parameter for antenna # 3 (Table 5.1) with and without the CSRRs are illustrated in Figures 5.9 (a) and (b), which has a good agreement with simulation results. It is noted that the back lobe radiation is small for this antenna as shown in Table 5.3 (antenna # 10).
Figure 5.9: Measurement using Network Analyzer
(a) Patch with CSRR resonating at 9.7 GHz (b) Patch without CSRR resonating at 12.4 GHz
(c) simulated frequency for (a) and (d) simulated frequency for (b)

Table 5.4 compares the desired, simulated and measured resonant frequencies for same
patch antennas with CSRR in the ground plane, as tabulated in Table 5.1. It can be seen
that there is very good agreement between predicted and measured values.
Chapter 5: Antennas with CSRR

### Table 5.4: Comparison of desired, measured and simulated frequencies for a patch with CSRR in ground plane

<table>
<thead>
<tr>
<th>Antenna #</th>
<th>Size of patch with CSRR</th>
<th>Size of CSRR</th>
<th>Simulated Frequency (GHz)</th>
<th>Measured Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a (mm)</td>
<td>b (mm)</td>
<td>(mm)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.50</td>
<td>7.00</td>
<td>3</td>
<td>9.8</td>
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<td>2</td>
<td>10.90</td>
<td>6.50</td>
<td>3</td>
<td>9.9</td>
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<tr>
<td>3</td>
<td>12.00</td>
<td>7.50</td>
<td>3</td>
<td>9.7</td>
</tr>
<tr>
<td>4</td>
<td>14.98</td>
<td>9.29</td>
<td>3</td>
<td>8.4</td>
</tr>
<tr>
<td>5</td>
<td>14.98</td>
<td>9.29</td>
<td>2</td>
<td>8.7</td>
</tr>
<tr>
<td>6</td>
<td>13.80</td>
<td>8.50</td>
<td>1.5</td>
<td>10.1</td>
</tr>
</tbody>
</table>

The return loss graphs and impedance smith charts for all the prototypes tested are in the Appendix D. Appendix C contains some comparison graphs for measured resonant frequency shifts for design with and without CSRRs.

To illustrate the reduction in patch size two antennas from Table 5.2 with a desired resonance at 10 GHz are shown in Figure 5.10, where (a) has CSRR in the ground plane and has a dimension of 10.9 x 6.5 mm and (b) does not have CSRR in the ground plane and has a dimension of 14.98 x 9.29 mm. Experimental results shown in Figure 5.11, confirm that they both resonate at 9.9 GHz and validate the claim that there is a 49 percent reduction in antenna size.
Figure 5.10: Comparison of the patch sizes for (a) Antenna with CSRR (reduced size patch) and (b) Antenna without CSRR both resonating at 9.9 GHz.

Figure 5.11: Measured Return Loss (S11) in dB for patch sizes with and without CSRR (shown in Figure 5.10) which both resonate at 9.9 GHz.
5.2.2 Directivity Measurement

As a final confirmation for the feasibility of the LHM antennas as acceptable radiators the measurements of directivity and back lobe radiation have been done. From the Table 5.4 three antennas (antenna #1,2,3) have been chosen, which have small back lobe and are tested for radiation directivity (antenna #7,3,10 respectively from Table 5.3).

The measurements have been done using the Sanders antenna test-bench (Type 6452) set-up shown below in Figure 5.12. The antennas (with and without CSRR) were compared with a standard dipole-fed paraboloid antenna with a gain of 27.4 dB. In the set-up the paraboloid or the test antenna is the transmit antenna and the receive antenna is a standard horn antenna. The received power is measured, when each of the three, namely the paraboloid and the two test antennas acts as a transmitter. The received power for both test antennas is compared to that of the paraboloid and the gain is obtained by the equation in (5-1).

![Figure 5.12: Set-up for directivity measurement](image)
The gain formula obtained by comparing the power of the test antenna with the paraboloid is as follows.

\[ G_A = G_P \times \frac{P_P}{P_A} \]

or

\[ G_A|_{dB} = G_P|_{dB} + \frac{P_P}{P_A} dB \]  

(5-1)

where \( G_A \) is the gain of the microstrip antenna under test and \( G_P \) is the gain of the standard paraboloid. \( P_P \) and \( P_A \) are the power in milli-watts measured on the power meter for the paraboloid and the antenna under test, respectively.

Three antennas with CSRR (reduced size) from the table 5.1 (antennas # 1, 2, 3) are tested and compared to an antenna without CSRR for directivity and also the back lobe radiation. The directivity and the front to back lobe ratio for the antennas with CSRR obtained in each case is given in the Table 5.5 below. The front to back lobe ratio is computed using a standard Yagi antenna equation for F to B power ratio. The formula for this is given in equation as below:

\[ FtoB = \frac{P_f}{P_b} \]

\[ FtoB(dB) = 10\log P_f - 10\log P_b \]  

(5-2)

This formula is called the 180- degree front to back lobe power ratio where \( P_f \) – front lobe power \( P_b \) – back lobe power 180 degrees away from front lobe.
Chapter 5: Antennas with CSRR

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Patch Size</th>
<th>Paraboloid Power</th>
<th>Test Antenna Power</th>
<th>Back Lobe power</th>
<th>Directivity</th>
<th>Front to Back Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without CSRR</td>
<td>a (mm)</td>
<td>b (mm)</td>
<td>(mW)</td>
<td>(mW)</td>
<td>(mW)</td>
<td>(dB)</td>
</tr>
<tr>
<td>With CSRR</td>
<td>14.98</td>
<td>9.49</td>
<td>2.3</td>
<td>0.034</td>
<td>-</td>
<td>9.09</td>
</tr>
<tr>
<td>With CSRR</td>
<td>10.9</td>
<td>6.5</td>
<td>2.3</td>
<td>0.022</td>
<td>0.0015</td>
<td>7.206</td>
</tr>
<tr>
<td>With CSRR</td>
<td>11.5</td>
<td>7.00</td>
<td>2.3</td>
<td>0.031</td>
<td>0.0015</td>
<td>8.696</td>
</tr>
<tr>
<td>With CSRR</td>
<td>12.0</td>
<td>7.5</td>
<td>2.3</td>
<td>0.030</td>
<td>0.0015</td>
<td>8.5536</td>
</tr>
</tbody>
</table>

Table 5.5: Directivity and front to back lobe ratio measurement for fabricated antennas

From the table 5.5 it may be observed that the directivity of the antennas with CSRR is comparable to antennas without CSRR. Also it may be noted that the front to back lobe ratio in case of each antenna with CSRR, is found to be above 10 dB which is good for a single patch antenna. This may improve when a microstrip patch antenna array is used.

5.3 Proposed Design Methodology

Finally, upon successful design, construction and validation through simulation an measurement, of reduced size antennas with left-handed materials a design methodology has been proposed. The steps in the methodology are summarized below.

1. Identifying a CSRR which has a stop-band at/around the desired resonant frequency of the antenna with CSRRs in the ground plane
   - Determining the resonant frequency of the CSRR and hence the stop-band
   - Determining if it has negative material properties in the stop-band
2. Choosing patch dimensions such that it resonates at a frequency approximately 25 percent higher than the desired frequency. Since there are no standard methods or formulae to determine the patch dimensions for a defected-ground-plane-patch (patch with CSRRs in ground plane) this is a ballpark value to start with.

3. Loading the patch with CSRRs along the radiating edge opposite the feed. This would emulate the patch as being placed on a DPS-DNG substrate and fed at the positive side.

4. Simulating the design and running a few iterations to optimize design

Examples, from the current work which illustrate this design methodology are given in Table 5.2. Consider antenna # 3 in Table 5.2. The CSRR chosen (CSRR # 1 in Table 4.2) has a stop band around 10 GHz which is the desired resonant frequency of the antenna with CSRR (reduced size). The patch size is chosen such that it would resonate approximately at 14 GHz without the CSRRs in the ground plane. The CSRRs are loaded along the radiating edge away from the feed. The resultant antenna resonates at the desired frequency (10 GHz for this example).
Chapter 6

Conclusion and Future Work

The present work addresses size reduction of microstrip antennas using left-handed materials. This has been achieved through phase-compensation by implementing a combined DPS-DNG substrate. In order to achieve negative permittivity CSRRs have been placed in the ground plane. Rectangular microstrip patch antennas have been constructed and validated through analytical formulation, simulation and measurement.

6.1 Primary contributions of present work

The following summarizes the analytical, simulated and experimental contributions of this work.

1. Two configurations of microstrip patch with a DPS-DNG substrate have been considered. In the first one, the interface between the two regions is parallel to the radiating edges and in the second configuration, normal to the radiating edges. The ‘electrically-small size’ condition for each case is determined, from which, in conjunction with the boundary condition at the interface, the propagating modes have been determined. For the configuration where the DPS-DNG interface is perpendicular to the radiating edge is has been shown through simulation that the antenna is a poor radiator. The present work focuses on the case where the interface is parallel to the radiating edge.

2. The CSRR has been analyzed through simulation using Ansoft’s Designer and HFSS. From the frequency response the stop band has been determined, where the
permittivity becomes negative. From the S matrix the effective permittivity of the composite material has been determined using parameter extraction. Various sizes of CSRRs have been considered and in each case the ‘stop band’ has been determined.

3. Microstrip antennas using complementary split ring resonators (CSRR) in the ground plane for several size combinations have been designed and constructed. Measurements of input characteristics and directivity have been performed. It is observed that the certain designs have a significant patch size reduction (up to fifty percent) and have a negligible back lobe in the radiation pattern. It has been observed that the best designs are for cases where CSRRs’ ‘stop band’ correspond to the resonant frequency of the antenna. In these cases size reduction upto fifty percent has been achieved. In addition, the back lobe radiation is negligible and the directivity is comparable to the traditional microstrip antennas, whose substrate is positive.

4. A design methodology has been proposed, which takes into account the size, and number of CSRRs used in the ground plane and the size of the patch for a desired resonant frequency.

6.2 Future Scope

The following are the proposed extensions to the present work:

1. Theoretical study of the CSRR needs to be addressed. Analytical expression should be developed for the effective permittivity of the host medium containing CSRRs. This would enable the design of CSRRs for desired negative permittivity
values and stop band frequency, that can be incorporated in microstrip patch antenna of reduced size.

2. Several variables contribute to the resonant frequency (stop-band) of the CSRR. An extensive study of the variable dependence will enable a designer to choose the appropriate CSRR for a particular patch antenna. A design criteria needs to be developed, where an antenna designer can obtain a set of ballpark design values based on the relations between the various dimensions of the CSRR and patch. Methods to remove or reduce back lobe radiation need to be explored. A possible solution could be the use of microwave absorbers.

3. The use of composite right/left handed medium (CRLH) could be explored for dual-frequency microstrip patch antennas, where one resonance is in the right-hand region and the other in the left hand region.
References


Publication Resulting from Present Work

## Appendix A

### Abbreviations and Nomenclatures

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Name</th>
</tr>
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<tbody>
<tr>
<td>BW</td>
<td>Backward Wave Medium</td>
</tr>
<tr>
<td>CLL</td>
<td>Capacitively loaded loop</td>
</tr>
<tr>
<td>CSRR</td>
<td>Complementary Split Ring Resonator</td>
</tr>
<tr>
<td>DGP</td>
<td>Defected Ground Plane</td>
</tr>
<tr>
<td>DNG</td>
<td>Double Negative Material</td>
</tr>
<tr>
<td>DPS</td>
<td>Double Positive Material</td>
</tr>
<tr>
<td>DSR</td>
<td>Double Spiral Resonator</td>
</tr>
<tr>
<td>DSRR</td>
<td>Distorted/Dual Split Ring Resonator</td>
</tr>
<tr>
<td>DSSR</td>
<td>Double Slit SRR</td>
</tr>
<tr>
<td>EBG</td>
<td>Electromagnetic Band Gap structured material</td>
</tr>
<tr>
<td>ENG</td>
<td>Epsilon Negative Material (Permittivity)</td>
</tr>
<tr>
<td>LHM</td>
<td>Left-Handed Material</td>
</tr>
<tr>
<td>MNG</td>
<td>Mu Negative Material (Permeability)</td>
</tr>
<tr>
<td>MSRR</td>
<td>Modified Split Ring Resonator</td>
</tr>
<tr>
<td>MTM</td>
<td>Metamaterial</td>
</tr>
<tr>
<td>NB SRR</td>
<td>Non-Bi-anisotropic SRR</td>
</tr>
<tr>
<td>NIM</td>
<td>Negative Index of Refraction Material</td>
</tr>
<tr>
<td>NIR</td>
<td>Negative Index of Refraction Material</td>
</tr>
<tr>
<td>NPV</td>
<td>Negative Phase Velocity Material</td>
</tr>
<tr>
<td>NRM</td>
<td>Negative Refractive Index Material</td>
</tr>
<tr>
<td>PBG</td>
<td>Photonic Band Gap Material</td>
</tr>
<tr>
<td>SNG</td>
<td>Single Negative Material</td>
</tr>
<tr>
<td>SR</td>
<td>Spiral Resonator</td>
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<tr>
<td>SRR</td>
<td>Split Ring Resonator</td>
</tr>
<tr>
<td>SSDR</td>
<td>Single Split Double Ring</td>
</tr>
<tr>
<td>SSRR</td>
<td>Slotted Split Ring Resonator</td>
</tr>
<tr>
<td>SWCR</td>
<td>Sub-Wavelength Cavity Resonator</td>
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Appendix B

Frequency Response and Parameter extraction graphs for CSRR

The sizes of CSRR chosen are tabulated in Table 4.1 which is reprinted here.

<table>
<thead>
<tr>
<th>CSRR #</th>
<th>Outer width of CSRR</th>
<th>Spacing between rings</th>
<th>Width of rings</th>
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<td>h (mm)</td>
<td>d(mm)</td>
<td>c (mm)</td>
<td>g (mm)</td>
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</tr>
<tr>
<td>4</td>
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<tr>
<td>7</td>
<td>4.00</td>
<td>0.50</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Dimensions of CSRR used for the determination of the resonant frequency

The graphs of frequency response showing stop-band for each CSRR and the corresponding frequency dependent material parameters (real $\mu$ and $\varepsilon$) graphs are given below.
CSRR 1: h=3mm

Figure B1: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real \( \varepsilon \) and \( \mu \) values as a function of frequency
CSRR 2: h=5mm

Figure B2: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real $\varepsilon$ and $\mu$ values as a function of frequency
Appendix B  

CSRR 3: h=2mm

![Graph of S-parameter (dB) vs frequency for CSRR 3](image)

**(a)**

![Graph of real permittivity and real permeability vs frequency](image)

**(b)**

Figure B3: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real $\varepsilon$ and $\mu$ values as a function of frequency
CSRR 4: h=1.5mm

Figure B4: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real ε and μ values as a function of frequency
CSRR 5: $h=9\text{mm}$

Figure B5: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real $\varepsilon$ and $\mu$ values as a function of frequency
CSRR 6: h=6mm

Figure B6: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real \( \varepsilon \) and \( \mu \) values as a function of frequency
Appendix B

CSRR 7: h=4mm

Figure B7: Stop-band and material parameters (a) Frequency Response of CSRR (b) Real \( \varepsilon \) and \( \mu \) values as a function of frequency
Appendix C

Measured Shift in Resonant Frequency

For some examples from Table D1, the measured shift in resonance frequency is illustrated for two antennas of the same patch size, one with the CSRR in ground plane and one without.

Figure C1: Comparison of Antenna #21 and Antenna #15 in Table D1
Appendix C

Figure C2: Comparison of Antenna #19 and Antenna #12 in Table D1

Figure C3: Comparison of Antenna #20 and Antenna #14 in Table D1
Figure C4: Comparison of Antenna #22 and Antenna #17 in Table D1
Appendix D

Experimental Results for Input Characteristics

In all 22 antennas were fabricated and tested. The Table D1 below shows the dimensions of the patch and CSRRs used for each antenna.

<table>
<thead>
<tr>
<th>Antenna #</th>
<th>Size of Patch</th>
<th>Outer Dimension of CSRR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a (mm)</td>
<td>b (mm)</td>
</tr>
<tr>
<td>1</td>
<td>14.98</td>
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<td>2</td>
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</table>

Table D1: Fabricated antenna dimensions (Patch Size and CSRR sizes)
Given below are the measured graphs for Return Loss (S11(dB)), SWR and Input impedance, for the highlighted in table D1 fabricated antennas. These antennas are the antennas stated in table 5.1.

Figure D1: Return Loss (dB) for Antenna # 3 in table D1
Figure D2: SWR for Antenna # 3 in table D1

Figure D3: Input Impedance for Antenna # 3 in table D1
Figure D4: Return Loss (dB) for Antenna #4 table D1

Figure D5: SWR for Antenna #4 table D1
Figure D6: Input Impedance for Antenna # 4 table D1
Figure D7: Return Loss (dB) for Antenna #12 Table D1

Figure D8: SWR for Antenna #12 Table D1
Figure D9: Input Impedance for Antenna # 12 table D1
Figure D10: Return Loss (dB) for Antenna # 14 table D1

Figure D11: SWR for Antenna # 14 table D1
Figure D12: Input Impedance for Antenna #14 table D1
Figure D13: Return Loss (dB) for Antenna # 15 table D1

Figure D14: SWR for Antenna # 15 table D1
Figure D15: Input Impedance for Antenna # 15 table D1
Figure D16: Return Loss (dB) for Antenna # 17 table D1

Figure D17: SWR for Antenna # 17 table D1
Figure D18: Input Impedance for Antenna #17 table D1
Appendix E

Simulated Results for fabricated antennas (S-parameters)

All antennas in the table below are fabricated and the results validated through simulation. Simulated frequency response and SWR are given for the antennas highlighted.

<table>
<thead>
<tr>
<th>Antenna #</th>
<th>Size of Patch</th>
<th>Outer Dimension of CSRR</th>
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<tr>
<td></td>
<td>a (mm)</td>
<td>b (mm)</td>
</tr>
<tr>
<td>1</td>
<td>14.98</td>
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<td>14.98</td>
<td>9.29</td>
</tr>
<tr>
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<td>9.29</td>
</tr>
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<td>14.98</td>
<td>9.29</td>
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<td>14.98</td>
<td>9.29</td>
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<td>14.98</td>
<td>9.29</td>
</tr>
<tr>
<td>7</td>
<td>14.98</td>
<td>9.29</td>
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<td>9.29</td>
</tr>
<tr>
<td>9</td>
<td>12.50</td>
<td>8</td>
</tr>
<tr>
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<td>7.5</td>
</tr>
<tr>
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<td>12.00</td>
<td>7.5</td>
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<tr>
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<td>7.5</td>
</tr>
<tr>
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<tr>
<td>14</td>
<td>11.50</td>
<td>7</td>
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<tr>
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<tr>
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<td>8.5</td>
</tr>
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<tr>
<td>22</td>
<td>13.80</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table D1: Fabricated antenna dimensions (Patch Size and CSRR sizes)
Figure 1(a): Return Loss (S11(dB)) for Antenna #3

Figure 1(b): SWR for Antenna #3
Figure 2(a): Return Loss (S11(dB)) for Antenna # 4

Figure 2(b): SWR for Antenna # 4
Figure 3(a): Return Loss (S11(dB)) for Antenna #12

Figure 3(b): SWR for Antenna #12
Figure 4(a): Return Loss (S11(dB)) for Antenna # 14

Figure 4(b): SWR for Antenna # 14
Figure 5(a): Return Loss (S11(dB)) for Antenna # 15

Figure 5(b): SWR for Antenna # 15
Figure 6(a): Return Loss (S11(dB)) for Antenna # 17

Figure 6(b): SWR for Antenna # 17
Appendix F

DPS-DNG substrate configuration with interface perpendicular to radiating edge

For the second configuration in (Figure 3.2 (b)) simulation and experimental results showed a significant back lobe. The snap shot of the simulation set-up is shown in Figure F1. The patch along with the feed is on the trace while the CSRR are placed in the ground plane.

![Diagram of configuration II with CSRR slots cut out in the ground plane]

Figure F1 Planar layout of configuration II with CSRR slotted on the ground plane only in one half of the patch along non-radiating edge

The resonance frequency shifted to a lower value (Figure F2) than a patch with same dimensions without CSRRs in the ground plane. However the radiation pattern was omni-directional and poor as can be seen in Figure F3.
Figure F2 Resonant Frequency of the patch with configuration II of the substrate combination

Figure F3 Poor Radiation pattern at the Resonant Frequency of the patch with configuration II of the substrate combination