Psychophysics of the measurement of the distance between two negative ticks, two positive ticks, and two edges

Alan Coble

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PSYCHOPHYSICS OF THE MEASUREMENT OF THE DISTANCE
BETWEEN TWO NEGATIVE TICKS, TWO POSITIVE TICKS,
AND TWO EDGES

by

Alan Paul Coble

A thesis submitted in partial fulfillment
of the requirements for the degree of
Bachelor of Science in the School of
Photographic Arts and Sciences in the
College of Graphic Arts and Photography
of the Rochester Institute of Technology

April, 1980

Signature of the Author......................................................... Photographic Science and Instrumentation

Certified by................................................................. Thesis Adviser

Accepted by......................................................... Supervisor, Undergraduate Research
PSYCHOPHYSICS OF THE MEASUREMENT OF THE DISTANCE BETWEEN TWO NEGATIVE TICKS, TWO POSITIVE TICKS, AND TWO EDGES

by

Alan Paul Coble

Submitted to the Photographic Science and Instrumentation Division in partial fulfillment of the requirements for the Bachelor of Science degree at the Rochester Institute of Technology

ABSTRACT

The research that has been done deals with the increase in the precision of measurement under low magnification. There were four factors used in the field of view of a microscope to attempt an increase in precision. The factors were 10X and 20X magnifications, light intensities, and Kodak Wratten filters Nos. 58 and 45A. The precision of measurement was increased in the positive ticks and edges with factors other than high magnification.
ACKNOWLEDGMENTS

Thanks must be expressed to Gary Reif of Photographic Sciences for supplying the glass plates needed for the research.

The observers must be thanked for making the time consuming measurements.

Thanks goes to Professor Carson for his help with ideas and the ordering of materials.

A special thanks to the Central Intelligence Agency for making the research financially possible.

Special thanks must be conveyed to Professor Rickmers for putting up with the daily questioning about statistics.

A very special thanks to Professor Shoemaker for guiding me through the research project.
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INTRODUCTION

My research problem came from Gary Reif of Photographic Sciences Corporation. Mr. Reif made measurements of the distances between two negative ticks, between two positive ticks, and between two edges which were on a glass plate. An example of the glass plate is in Figure 1 in the appendix. These measurements were made on a Nikon comparator with a digital readout with the accuracy of plus or minus one ten thousandths of an inch. The comparator readings were considered to be the "actual" values of the distances. Mr. Reif used thirteen observers for the 7X magnification and eight observers for the 30X magnification. The observers looked through a measuring microscope using the two specified magnifications and measured the distances between the objects. The data was plotted as percent response versus measurement. The data with the 30X magnification, by inspection, indicated a more precise measurement than for the 7X magnification. The analysis was made by inspection due to the small number of observers. Mr. Reif would like to know if there is a way to increase the precision of measurement under low magnifications.
In this experiment, I decided to use four different factors at two different levels. The two levels designate either high and low or with and without depending on the factor. The four factors were magnification, light intensity, and two kinds of filters. The factor magnification has a high level at 20X and a low level at 10X. The light intensities of the Nikon measuring microscope used were the highest setting of the instrument for the high level and the next highest intensity for the low level. The two filters were a Kodak Wratten filter No. 58 and Kodak Wratten filter No. 45A. The levels for the filters are either with the filter in the field of view or without the filter in the field of view. The Kodak Wratten filter No. 58 was chosen because it approximates the photopic sensitivity curve of the eye. The Kodak Wratten filter No. 45A was used because it gives high resolving power in visual microscopy. The intensity of the light was not changed to compensate for the addition of one or both of the filters.

The observers underwent an eye acuity test before measuring the objects with the various factors. After the eye test, the observers made the measurements of the objects with the factors at different levels in the field of view at high and low magnifications. The observers were required to make the measurements with their left eye.
The whole purpose behind the research is to try to increase the precision of measurement under low magnification using the stated factors at two levels. The distance is not what is of major concern, but the placement of the vertical cross-hair as close to the edge or tick as possible, is.
I. THEORY

The research I have done is based on the following factors: magnification, light intensity, and two types of filters. I chose the magnifications 10X and 20X for the research, because the problem was to increase the precision of measurement under low magnification. The two highest light intensities on the measuring microscope were taken because the amount of light is reduced by the filters being in the path of light. In other words, they produce high contrast to be able to find the vertical cross-hair in the negative ticks. Also, when viewing the negative ticks even without filters, it is hard to see the cross-hair with which to measure. Kodak Wratten filter No. 58 approximates the photopic sensitivity curve of the eye and it also makes finding the cross-hair easier with its green background. The other filter used is a Kodak Wratten filter No. 45A. It is a blue-green filter which gives the best resolving power for viewing objects through a microscope.

The atmosphere is relaxed and the observer can stop to rest his eyes at any time. The observer is allowed to re-focus if he or she desires.

From the book Industrial Microscopy In Practice, I
found it is easier for non-skilled viewers to look through monocular microscopes than binocular ones. This is due to the fact in binocular microscopes the head and eyes have to be extremely still. The monocular microscopes require the head and eyes to be fairly still. Since most of my observers have used binocular microscopes but their skills are not sharp from the lack of use, I decided to use monocular vision. I chose arbitrarily to use the left eye to view the objects. Classmates later told me most people can see better with the right eye. This gave me a theory that if I could increase the precision in the left eye measurements maybe the right eye measurements could be increased with the same factors. In this experiment though, the only eye to be tested will be the left eye. In the *Journal of Experimental Psychology*, there was an article called "The Role of Physiological Nystagmus in Monocular Acuity". It gave two problems that related to monocular viewing. One problem was the eye tends to drift and the second problem states the eye has tremors. In both cases there are no ways to counter the problems. Even though these problems are significant, I still went with the monocular vision.
II. METHODOLOGY

The first step of the experiment was to get the glass plate with the objects on it. The first glass plate I received from Gary Reif of Photographic Sciences Corporation was the glass plate they had used. This plate turned out to have the distances between the ticks and edges too large for RIT's Nikon measuring microscope to measure. In other words, the Nikon measuring microscope can only measure 25mm in the x-direction and 25mm in the y-direction. The distance between the marks was about 31mm. I then asked Photographic Sciences Corporation to make another glass plate with the distances less than 25mm.

Photographic Sciences used a glass plate from Kodak. The plate size specified was 4 X 5 inches. It was an Ortholith PF0 glass plate with a 4566 Lith photographic emulsion. The objects were made on a coordinatograph. A coordinatograph is a table about three feet by three feet which has a large circular diffusing screen. There is a light source underneath the diffusing screen. There are two parallel tracks, one on each edge of the table. Both tracks have a linear scale in inches to measure movement in the y-direction of a crossbar perpendicular to the
tracks. The crossbar has an instrument which is a cutting device. Using an attachment and taking the cutting device off, it can be used as a low magnification microscope. The crossbar can be fitted with an etching device. The instrument on the crossbar can be moved in the x-direction. The crossbar also has a linear scale to measure displacement.

After the glass plate is exposed and developed, the objects were cut out and etched out. The edges were made by cutting away the emulsion. A larger space was cut out for the positive ticks to have a clear area around them. Then they were etched out. The negative ticks were etched out with no extra emulsion taken off.

The objects were then measured on a Nikon Comparator. A comparator is an instrument which projects the profile of an object onto a viewing screen. There are cross-hairs on the viewing screen which can be used as a reference to measure thicknesses or distances. There are screens capable of measuring angles by rotating the screen.

The following are the measurements of the objects. For the negative ticks, it was 21.328mm. The distances between the positive ticks and the edges are 21.333mm and 21.308mm respectively.

A Nikon measuring microscope Model 1 was used for the experiment. Its serial number from the manufacturer is 18805 and the RIT number is 123434 for the base. For the
stage, which is an E2 stage, the numbers are 24358 for the manufacturer and 119992 for RIT. The focusing unit's numbers are 60310 for Nikon and 123435 for RIT. The optical head's numbers are 104624 for Nikon and 11993 for RIT. The eyepiece used had a 10X magnification with a cross-hair reticle. The Nikon number for it is Bi HKW10X. The 1X objective's number is EP01 46507 for Nikon. The 2X objective's number is EP01 43703 for Nikon. The eyepiece and the objectives have no RIT numbers.

I had two problems with the Nikon measuring microscope. Both of them I could not fix. The first problem is minor. The coarse focusing knob, when it is initially turned slightly, offers no resistance. There should be resistance all the time in this knob. The second problem is major. The cross-hair reticle was not positioned in the correct place for it to be focused properly. In other words, there is a point at which the objective focuses inside the microscope tube. In order for the cross-hair to be in focus, it must be placed at this point. When an observer looked through the microscope, he or she saw the cross-hairs with a spread function. The spread function was a sinc. This means the cross-hairs were the darkest in the field with fainter lines close to the actual cross-hairs. It is like seeing ghost images on both sides of the cross-hairs. The faint images made the measurements harder than they really were. The reticle
also had a chip taken out of it. It did not hamper the viewers' measurements, but there might have been a psychological barrier for some of them.

The measuring microscope was placed on a desk in the research laboratory R21. There were two stools of different height available. The viewer could pick the stool which was more comfortable. I took the stool that was not picked and sat to the right of the observer. This enabled me to read the vernier scale on the movable stage, record the measurements and put in or take out factors.

Before an observer comes in for the observations, the glass plate should be positioned on the stage of the microscope. The experimenter must use the high magnification to position the glass plate. First, the experimenter should place the negative ticks towards the back of the microscope. This matters because of the distance between the edge of the glass plate and the negative ticks is smaller than the distance between the edge of the glass plate and the object edge. The main objective is to position the plate in such a way that by moving the stage in the y-direction you can change to a different object without moving the plate. You will notice the object edges are not in the center of the viewing field. This is due to the distances between the three objects. This is the main reason for having the negative ticks in the position stated. It is easier to
measure the edges that are not centered as compared to the negative ticks. As one will see, the negative ticks are hard to measure when they are centered.

The next thing to do before the observer comes in is to hang the eye chart. The eye chart was taped to the wall in the small room of R21. It was placed so it was parallel to the viewer's body and perpendicular to the viewer's eyes. The viewer would be sitting 20 feet from the chart. This unfortunately meant the observer had to sit in a chair outside of the darkroom. All lights in the darkroom and in the hallway were off. The hallway door was closed. The only light on was in the inner room. About half way through the experimenting, the standard way of measuring eye acuity that I was following was changed to metric. This means the observer undergoing the acuity test would sit four meters away from the chart instead of twenty feet. A person with 20/20 vision will now be classified to have 4/4 vision in metric.

The observer must read the chart three times. Once for the left, right, and for both eyes. The observer is required to cover his eye with his hand and read the chart with the other eye. Then change so he can read the chart with the other eye and then both eyes. Make sure to tell the observer not to press his hand against the eye, because this deforms the eye which causes less acuity for that eye. If the observer reads the 160 line on the
Snellen eye chart the best, then he or she has 160/20 vision. These values should be recorded for each observer.

The lights are then turned on. The observer adapts to the level of light in the room while the experimenter takes down the eye chart, brings the chair back in the room, and checks to see if the microscope has the correct factors in position.

Before the observer views the objects, he or she is told how to measure the objects. The observer will see there is tape on the right eyepiece. This is to block the light from coming up from the stage. During the observations, the observer is required to keep the right eye open. The observer is told that if the vertical cross-hair is not parallel to the objects, then he or she should straighten it. Also, if the observer touches the eyepiece with his face, chances are it moves the vertical cross-hair. This is due to the lack of fit of the eyepiece in the microscope tube.

The observer is to place the vertical cross-hair in such a way that it just touches the edge. He is not to put the cross-hair inside the edge. This is done by turning the micrometer handle on the stage. The observer then moves the stage across until he or she finds the other edge and does the same thing. Each time the observer says this is the point where they meet, the experimenter records the measurement from the vernier scale. When the two
edges are found, take the absolute value of the difference between the two values. This is one of the sixteen values in the Yates program. The observer then moves the stage in the y-direction until the positive ticks appear in the center of the field. A positive tick is black in color with a clear background. A tick is just a letter T rotated 90 degrees to the left or right. If it is on the left side of the glass plate, it is rotated 90 degrees to the left. If the tick is on the right side of the plate, it is rotated 90 degrees to the right. The experimenter should move the stage in the x-direction just a little because the edges and both types of ticks are almost the same distance in the x-direction. The observer is now required to place the vertical cross-hair next to the inside of the tick. If you look at the ticks, the vertical lines are what the observer places the cross-hair next to. The side he or she approaches from is determined by which side a horizontal line jets out from the vertical line. Whatever side the horizontal line is on, that is the side of the tick in which the cross-hair is to be placed against the vertical line. Once the observer finds the position closest to the tick, he or she must do the same thing for the other tick. The experimenter must record this data in the same way as before. The observer moves the stage in the y-direction again until a negative tick is observed. The negative ticks are clear with a dark background. The
The experimenter should move the stage in the x-direction after a change to a different object. The observer will probably have a difficult time measuring the negative ticks. The observer should move the stage back and forth until he or she finds the vertical cross-hair. This can be difficult because the only part of the cross-hair observed is in the horizontal line of the tick. The experimenter must emphasize that if the vertical line of the tick's light intensity decreases then they have gone into the tick. This is not allowed because they must touch the side of the tick. Once the position is found and recorded, they must do it to the other tick. The three objects have been measured under the same conditions. Now the condition will be changed and all of the objects will be measured under the new condition. This process continues until all sixteen combinations are done. The experimenter must move the stage in the x-direction after changes in conditions or treatments.

In order to tell which factors are needed to be put in or taken out, the experimenter should use the treatment column of the Yates method. A treatment consists of four digits. The digits are all zeroes or all ones or a combination of them. A zero designates a factor at the low level and a one designates a factor at the high level. The first digit in this experiment was considered to be magnification, the second digit was the Kodak Wratten filter
No. 58, the third was the light intensity, and the fourth was the Kodak Wratten filter No. 45A. Following the treatment number is easy once it is learned. In the experiment, the objects were viewed under low magnification with changes in the other three factors only. Then it was repeated at high magnification. The reason for this is the low magnification requires the focusing mount to be higher than does the high magnification. It would have been very time consuming if the Yates order of treatments was followed exactly. The data taken from each observer had to be put back into Yates order before using the Yates computer program. This gives three sets of data with sixteen data points for each set. The data sets were then put into computer files. The initials of the observer followed by one of the following letters: N for negative ticks, E for edges, or P for positive ticks. Now there are three files for each person. The data files one at a time were run through the Yates and Bstat programs.

After the computer finishes with the Yates method, it prints the treatments, yields, and columns one through four. The fourth column is the calculated total effects, along with the sum of the yields. The first value of the column is the sum of the yields. This value is not plotted. The absolute values of the total effects are taken and then ordered from highest to lowest. The highest value of the
total effects correspond to 15, the next highest is 14 and so on down the list. The values 15 through one are called order numbers. The graphs are plotted as order number versus total effects. The paper used for this is the top half of normal probability paper. A line is drawn from the zero point through the first few points. Whatever is to the right of the line is significant and whatever is to the left is due to error. For an overall view of the observers data, the yields must be added up for each treatment. Then the sixteen points are put into the Yates computer program. The effects are plotted in the same way.

Using the graphs, the estimated standard deviation due to error and its confidence interval can be found. They can only be calculated if there is something significant. Assuming there is something significant so the method can be explained, count the points that lie to the right of the line. The number is then subtracted from 15. The value is the sample size due to error, ne. Fifteen is the total number of total effects. Using the formula \( .68 = \frac{\text{Rank} - .5}{n_e} \), the value for \( n_e \) is plugged in and then the equation is solved for Rank. The number .68 is the area under a normal curve between plus and minus one standard deviation. Take the value for Rank and round up if the decimal part of it is greater than .5. If it is less, round down to the nearest integer. Go to the place
where the total effects are ranked. Read the total effect value that corresponds to the Rank number computed. This value is equal to the estimated standard deviation contrast, \( es_c \). The estimated standard deviation is computed by the formula \( es_e = es_c / \sqrt{n_e + 1} \). The confidence interval is calculated as follows: using the Hahn tables, Figure 2, \( n_e \), and \( es_e \). The formula is \( c_l(es_e) \) is less than \( s_e \) is less than \( c_u(es_e) \). \( c_l \) is the lower limit multiplier. \( s_e \) is the standard deviation due to error of the population. \( c_u \) is the upper limit multiplier. I used 95% confidence values in calculating the confidence interval.
III. HYPOTHESIS

The hypothesis that I tried to prove is: there will be an increase in the precision of measurement when the magnification is 20X, the high level of light intensity is used, and the Kodak Wratten filter No. 58 is in the field of view.
IV. RESULTS

Looking at Figure 3, the edge for the first observer shows the treatment AD is significant. The treatment AD is high magnification, Kodak Wratten filter No. 45A, and the other factors at the low level. One will notice the line drawn is not through zero. This is because of an error due to another factor that is not defined in the experiment. I do not know what the error is except it has a value of about 0.266. The three factors A, B, and D are not affected by the same amount as the rest of the treatments.

Figure 4 shows the positive ticks for observer 001. There are eight significant treatments and they are ABC, BC, ABCD, AD, BCD, D, C, and A. The letters shown are the factors at the high level and the letters not shown are at the low level. This will be the case for all of the figures with significant treatments. It shows that treatments other than high magnification increases the precision of measurement.

In Figures 5 and 6, there are no significant treatments but the unknown treatment is present.

The graph in Figure 7 shows 4 out 7 significant
treatments are at the high level of magnification while the rest are at the low level.

In Figure 9, the treatments ABCD and A are significant. You wouldn’t think the treatment ABCD would be significant because of all the treatments it reduces the intensity of light the most.

The only significant treatment in Figure 10 is the treatment A. This is not unusual because you would expect higher magnification to be significant most of the time.

Figure 11 shows high magnification and high magnification with high light intensity to be significant.

Figure 13 shows the treatments D and A to be significant. The treatment D is all factors at the low level except for Kodak Wratten filter No. 45A.

Figure 14 illustrates the same results as Figure 13. High magnification is the only treatment significant in Figure 16.

High magnification by itself and with high intensity of light are shown to be significant in Figure 17.

In Figure 18, there are many treatments of significance. Most of the treatments have the factor magnification at the high level. The unknown factor’s error is present.

The treatments A, ABC, and BC are significant in Figure 19. This observer’s precision of measurement was increased with low magnification, filter No. 58, and high
light intensity.

Figure 20 shows high magnification to be the only significant treatment.

In Figure 21, the treatments of significance are B, AB, D, and A. The two filters by themselves at the high level and the other three factors at the low level show an increase in the precision of measurement.

The significant treatment in Figure 22 is high magnification.

Figure 23 shows the filter No. 45A by itself to be significant. Treatment A is also significant.

In Figure 24, seven treatments are significant. They are AD, D, BD, AB, ABD, ACD, and B. The Wratten filter No. 45A seems to be dominant along with magnification at the high level and filter No. 58 that increase the precision of measurement for the edge for all of the observers.

In Figure 25, the total effects for all of the observers for the positive ticks are graphed. There are five treatments which are significant. They are A, ABC, BC, BCD, and ABCD. There are three treatments with high magnification and two with low magnification.

The only significant treatment for the negative ticks of the seven observers in Figure 26 is high magnification.
V. CONCLUSIONS

My hypothesis was true only for the positive ticks. The precision of measurement of the negative ticks was increased by high magnification only. The precision of measurement was increased in both the edges and positive ticks.
LIST OF REFERENCES
LIST OF REFERENCES


Reif, Gary, Interview by Alan Coble at Photographic Sciences Corporation.


Figure 1

Objects
Table II
FACTORS FOR CALCULATING TWO-SIDED 95 PERCENT PROBABILITY INTERVALS FOR A NORMAL DISTRIBUTION

<table>
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<tr>
<th>Number of Given Observations</th>
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Two-sided 95 percent Interval is $c_L(n, 0.95)s$ to $c_U(n, 0.95)s$ where $c_L(n, 0.95)$ and $c_U(n, 0.95)$ are the appropriate tabulated factors for obtaining lower and upper limits respectively and $s$ is the standard deviation of the given sample of size $n$. 
Figure 3
Figure 4
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Figure 26
DIMENSION F(16)
DIMENSION V(16), W(16), X(16), Y(16), Z(16)
F(1)=0000
F(2)=1000
F(3)=0100
F(4)=1100
F(5)=0010
F(6)=1010
F(7)=0110
F(8)=1110
F(9)=0001
F(10)=1001
F(11)=0101
F(12)=1101
F(13)=0011
F(14)=1011
F(15)=0111
F(16)=1111
J=1
10 READ (105, 20) V(J)
20 FORMAT (F7.3)
   IF (J.EQ.16) GO TO 30
   J=J+1
   GO TO 10
30 K=1
   M=1
   N=2
   P=9
35 W(K)=V(M)+V(N)
   W(P)=V(M)-V(N)
   P=P+1
   K=K+1
   M=M+2
   N=N+2
   IF (N.EQ.18) GO TO 40
   GO TO 35
40 K=1
   M=1
   N=2
   P=9
45 X(K)=W(M)+W(N)
   X(P)=W(M)-W(N)
   P=P+1
   K=K+1
   M=M+2

Figure 27
N = N + 2
IF (N .EQ. 18) GO TO 50
GO TO 45

50  K = 1
     M = 1
     N = 2
     P = 9

55  Y(K) = X(M) + X(N)
     Y(P) = X(M) - X(N)
     P = P + 1
     K = K + 1
     M = M + 2
     N = N + 2
     IF (N .EQ. 18) GO TO 60
     GO TO 55

60  K = 1
     M = 1
     N = 2
     P = 9

66  Z(K) = Y(M) + Y(N)
     Z(P) = Y(M) - Y(N)
     P = P + 1
     K = K + 1
     M = M + 2
     N = N + 2
     IF (N .EQ. 18) GO TO 65
     GO TO 66

65  WRITE (108, 70)
70  FORMAT (',', 'TREAT 4X'YIELD'7X'I'8X'II'7X'III'8X'IV')

DO 100 I = 1, 16
    WRITE (108, 90) F(I), V(I), W(I), X(I), Y(I), Z(I)
90  FORMAT (',', 14, 5F10.3)

100 CONTINUE
STOP
END
BSTAT PROGRAM

DIMENSION DATA(16)
SUMDAT=0
SUMSQD=0
I=1
5 READ (105, 10) DATA(I)
10 FORMAT (F7.3)
WRITE (108, 12) DATA(I)
12 FORMAT (' ', F10.3)
IF (I .EQ. 16) GO TO 15
I=I+1
GO TO 5
13 BC=0
DO 134 J=1,16
BC=(DATA(J)-XMEAN)**2+BC
134 CONTINUE
STD2=BC/15
STD=STD2**.5
GO TO 25
15 DO 20 I=1,16
SUMDAT=SUMDAT+DATA(I)
20 CONTINUE
A=16*SUMSQD
B=SUMDAT**2
C=240
D=(A-B)/C
STD=ABS(D)**.5
XMEAN=SUMDAT/16
GO TO 13
25 FORMAT (108,30) STD
30 FORMAT (' ', 'STANDARD DEVIATION ' F7.3)
W1SIGP=XMEAN+STD
W1SIGN=XMEAN-STD
W2SIGP=XMEAN+2*STD
W2SIGN=XMEAN-2*STD
W3SIGP=XMEAN+3*STD
W3SIGN=XMEAN-3*STD
WRITE (108, 40) W3SIGP
WRITE (108, 50) W2SIGP
WRITE (108, 60) W1SIGP
WRITE (108, 70) XMEAN
WRITE (108, 80) W1SIGN
WRITE (108, 90) W2SIGN
WRITE (108,100) W3SIGN
40 FORMAT (' ', 'PLUS 3 STANDARD DEVIATIONS IS ' F7.3)

Figure 28
50 FORMAT (' ', 'PLUS 2 STANDARD DEVIATIONS IS 'F7.3)
60 FORMAT (' ', 'PLUS 1 STANDARD DEVIATION IS 'F7.3)
70 FORMAT (' ', 'MEAN IS 'F7.3)
80 FORMAT (' ', 'MINUS 1 STANDARD DEVIATION IS 'F7.3)
90 FORMAT (' ', 'MINUS 2 STANDARD DEVIATIONS IS 'F7.3)
100 FORMAT (' ', 'MINUS 3 STANDARD DEVIATIONS IS 'F7.3)
STOP
END

Figure 28a