Comparison of Two MTF Measurement Methods: Sine-Wave Vs Edge Gradient Analysis

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SINE-WAVE VS EDGE GRADIENT ANALYSIS

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DATE 7/24/85
COMPARISON OF TWO MTF MEASUREMENT METHODS:
SINE-WAVE VS EDGE GRADIENT ANALYSIS

By

John R. Cadou

Submitted to the Center for Imaging Science in partial fulfillment of the requirements for the Master of Science degree at the Rochester Institute of Technology.

ABSTRACT

A comparison of the sine-wave and edge gradient MTF measurement methods, on a photographic black and white film, was performed. A statistical test, a CMT acutance test, and a graphical comparison showed that there was no significant difference between the two measurement methods. This was true for the film processed to have large adjacency effects, as well as for the film processed to have no adjacency effects; however, the agreement was slightly better for the latter process.

The research also showed that aligning the midpoints of the edge traces, normalizing the individual edge traces, and then averaging several edge traces significantly reduced grain noise, and produced a superior representative edge for MTF analysis. A new adaptive damping filter also proved quite successful in the suppression of grain noise without degrading the MTF measurement. The combination of averaging several edge traces, and then using the adaptive filter, produced excellent MTF results from the edge traces.
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The optical transfer function, OTF, has become one of the most important tools in the evaluation of an imaging system.\(^1\) The modulation transfer function, MTF, is the modulus of the OTF, and is the more commonly used measurement, when talking about the performance of a system. Simply stated, the modulation transfer function is the ratio of the output to the input of a system.\(^2\) The main reason for the use of the MTF, is that it enables the determination of the MTF for an entire imaging system, such as camera lenses and film, by multiplying (cascading), frequency by frequency, the MTF's of the individual components of the system. In photographic films, where one or more photosensitive layers may be used to produce the final image, it has become necessary to determine the transfer function of each emulsion layer.

The MTF is a function of spatial frequency, and it can be difficult to measure directly. However, the MTF can also be derived indirectly from either the spread function or the edge response.\(^3\) Measurements taken from photographic emulsions are often complicated by grain noise, and the data obtained from physical experiments are seldom error-free.
Therefore, the problem with the indirect method is to evaluate the transfer function in one domain from noisy records in the other. The methods currently used to measure MTFs, directly or indirectly, all involve some degree of smoothing to minimize error. Attempts to separate the signal from the noise usually result in some degradation of the signal.

The objective of this research was to do a comparative study of the sine-wave and edge gradient methods, with as little smoothing as possible. Historically, the sine-wave method has been the most widely used and accepted method, (4,5). It was also chosen as the standard method of evaluation by the American National Standards Institute (ANSI). However, there are advantages to the edge method that suggest that this method could become the more popular way of evaluating the modulation transfer function.
BACKGROUND INFORMATION

Over the years, there have been many methods devised to measure the modulation transfer function of an imaging component/system. Dainty (3) has grouped the various methods into the following four major categories:

1. sine-wave methods
2. Fourier transformation of the line spread function
3. coherent light processing methods
4. calculation from scattering methods.

These four categories of measurement methods are broad and do overlap to some extent.

Most of the measurement methods may be used to measure either the transmittance, or the effective exposure MTF. However, the latter is more likely to satisfy the condition of linearity, and therefore, the MTF in terms of effective exposure is the more commonly referenced.(3)

In the sine-wave method, a photographic emulsion is exposed to a sinusoidally varying intensity distribution, of known spatial frequency \( f \), and modulation \( b/a \).

\[
I(x) = a + b \cos(2\pi f x)
\]

(eq.1)

If the object varies sinusoidally along one dimension, then the image of the object will also be a sinusoid. The
frequency may be shifted by magnification; however, the modulation and phase will have been changed by the spread function of the system, as shown in figure 1.

![Input and Output Signals](image)

**Fig.1 Sine-Wave MTF Model**

After processing the exposed emulsion, the image is scanned with a microdensitometer, and the densities are transferred back through the characteristic $D$-$\log H$ curve of the film, to give the effective exposure modulation. The ratio of the output effective exposure modulation, to the input exposure
modulation, is the modulation transfer factor at a given spatial frequency. The modulation transfer function curve is then constructed by plotting modulation as a function of spatial frequency, using the measured modulation factors. A best-fit plotting method is usually employed in displaying the results. A model of the sine-wave MTF is shown in figure 2.

Fig. 2 Effective Exposure MTF using Sine-Wave Methods
One of the major problems with sine-wave method is the production of targets that are truly sinusoidal with known modulations. Dainty (3) gives a good review of the methods of production of the sinusoidal targets, as well as alternative methods that vary from "smearing"(6), to using a square-wave target (7) as a substitute for the sinusoidal pattern, or even by calculating the MTF, by using a mathematical formula, credited to Coltman.(8)

For a linear imaging system, the MTF, $M(f)$, is the modulus of the Fourier transform of the line spread function $l(x)$.

$$M(f) = \left| \int_{-\infty}^{\infty} l(x) \exp(-i2\pi fx) dx \right|$$  \hspace{1cm} (eq.2)

In practice, the line spread function is more commonly derived from the image of an edge, rather than from the image of a line. The image on the film will be a degraded version of the original object, due to the scattering of light caused by the grain in the photographic emulsion, and by the imaging system.

The edge method is popular because of the ease with which a target can be made and the spread function determined. This method involves imaging an edge, either by contact or projection printing, onto an emulsion. After developing the
image of the edge, the film is scanned with a microdensitometer. The densities are then transferred back through the D-logH curve of the film, to yield the effective exposure. These exposure values are then differentiated to yield the line spread function, which is then Fourier transformed to yield the optical transfer function. The MTF is then obtained by calculating the modulus of the OTF. Somewhere provisions must be made to remove or minimize noise and instrument effects. Several variations of this basic method have been tried and used. An automated technique by Jones, (9), as well as other alternative schemes, have been devised. (10,11,12)

The edge method is advantageous, because it is more readily adaptable to digital recording, and data processing. (3) Another advantage is that MTF measurements can be made from laboratory test target edges, or from natural scene features in photographs, (i.e. shadows of buildings). The edge method has the additional feature, in that, the Fourier transform process yields an OTF/MTF as a continuous function of spatial frequency. In comparison, the sine-wave method gives the modulation factors for a discrete number of spatial frequencies, depending upon the sine-wave target used.
Scattering in the emulsion of the film is a linear process, but photographic adjacency effects introduced in processing are not linear and cause the effective exposure distribution to have a non-linear response function without a unique MTF. (13,14) However, in the absence of adjacency effects, or if the effects are held to a minimum, different measurement methods will produce similar MTF curves. (3)

The following section will discuss some of the mathematics involved in using the edge method to calculate the modulation transfer function.
MATHEMATICAL CONCEPTS

For the most part, the light used to expose a photographic emulsion is incoherent, and the interaction of this light with the emulsion can be treated as a linear process. (15)

First, consider an infinitesimal ray of light hitting a piece of film. The isotropic scattering of the emulsion (assuming the grain in the film to be randomly distributed) will transform the ray of light into a spatial distribution. This is known as the point spread function, PSF, and is graphically demonstrated in figure 3.

Fig. 3 Point Spread Function
The point spread function is a two dimensional irradiance distribution of the image of an idealized point source and is denoted by \( p(x, y) \). Any two dimensional input can be thought of as a set of closely packed point sources of varying intensities. The resulting degraded image can be calculated, by adding all of the resulting point spread functions, which have been multiplied by their respective intensities. Thus, if \( I(x, y) \) is the intensity at point \( (x, y) \) in the original, the resulting image, modified by scattering in the film \( I'(x, y) \), is given by

\[
I'(x, y) = \iint_{-\infty}^{\infty} I(x-a, y-b)p(a, b) \, da \, db. \tag{eq.3}
\]

which is known as the convolution integral (\( a \) and \( b \) representing dummy variables). The resulting intensity \( I'(x, y) \), is due to light not scattered out of the original \( I(x, y) \), plus all the light scattered into point \( (x, y) \) from the surrounding images.

If this image is depicted in only one dimension, the line spread function, LSF, is obtained. Mathematically, the LSF, is the integral of the PSF in one direction. It is denoted by \( l(x) \) and is shown in figure 4.

\[
l(x) = \int_{-\infty}^{\infty} p(x, y) \, dy \tag{eq.4}
\]
The Fourier transform of the line spread function is the optical transfer function. The OTF is denoted by $L(f)$, with the $f$ representing spatial frequency.

$$ L(f) = \int_{-\infty}^{\infty} l(x) \exp(-i2\pi fx) dx $$  \hspace{1cm} (eq.5)$$

Consider now a plane of light bounded on one edge by a perfectly straight line. This source can be regarded as an infinite array of line sources, each parallel to the edge and
each imaged by the film as the LSF. Since the total irradiance for any line in the image, is the sum of the contributions from the spread functions of all the lines in the source, the irradiance distribution is simply the integral of the LSF's. It is called the edge response function and is shown in figure 5.(16)

In practice, it is easier to obtain the image structure data from edge traces rather than from the spread functions defined earlier.(10) The edge response function, \( e(x) \), is then differentiated to obtain the line spread function, \( l(x) \), which is then transformed to yield the OTF.(17)

\[
l(x) = \frac{d}{dx} e(x)
\]  
\[
(\text{eq.6})
\]

then

\[
L(f) = F[l(x)]
\]  
\[
(\text{eq.7})
\]

where \( F \) denotes the Fourier transform operator, \( L(f) \) is the OTF of the system, and \( f \) is the spatial frequency.

Equation 6 defines the line spread function as the slope of the edge response. Therefore, a graph of the derivative of an edge, is an image of the line spread function.
By using the Fourier transform derivative theorem, (18,19), another method of obtaining the OTF would be to first transform the edge response directly, and then multiply the transform by \((i2\pi f)\).

\[
L(f) = (i2\pi f) \ast \left( F\left[e(x)\right]\right) \tag{eq.8}
\]

The above mathematics works well for continuous functions, but not very well for finite and sampled functions. Tatian (11) developed a method that allows for a finite sampled edge to be treated to yield the OTF directly.
This was done by approximating the exact Fourier transform in the following manner.

If \( e(n\xi) \) represents a sampled edge trace, where \( n \) is the sample number, and \( \xi \) is the sample interval (\( \Delta x \)), then,

\[
L(f) = i2\pi f \int_{-\infty}^{\infty} e(x) e^{-i2\pi f x} \,dx \quad \text{(eq. 9)}
\]

which can be approximated by,

\[
L(f) = i2\pi f \xi \sum_{n=-\infty}^{\infty} e(n\xi) e^{-i2\pi fn\xi} \quad \text{(eq. 10)}
\]

If \( N \) represents a finite value on the D-min portion, and \( M \) a finite value on the D-max portion of the edge trace, it is normal for the edge trace for, \( n < -N \) to have, \( e(n\xi) = 0 \), and for \( n > M \), to have, \( e(n\xi) = 1 \).

Therefore, eq. 10 can be written as follows

\[
L(f) = i2\pi f \xi \sum_{n=-N}^{M} e(n\xi) e^{-i2\pi fn\xi} + i2\pi f \xi \sum_{n=M+1}^{\infty} e^{-i2\pi fn\xi} \quad \text{(eq. 11)}
\]

since

\[
i2\pi f \xi \sum_{n=-\infty}^{-N-1} e^{-i2\pi fn\xi} = 0 \quad \text{was excluded from above.}
\]
Expanding the above yields,

\[ L(f) = i2\pi f \sum_{n=-N}^{M} e(n)e^{-i2\pi fn} + i2\pi f \sum_{n=M+1}^{\infty} \cos(-2\pi fn) \]

\[ + 2\pi f \sum_{n=M+1}^{\infty} \sin(-2\pi fn) \]  

(\text{eq.12})

breaking the first term into its real and imaginary terms,

\[ i2\pi f \sum_{n=-N}^{M} e(n)e^{-i2\pi fn} = E(f) + IE(f) \ . \]  

(\text{eq.13})

Now, breaking \text{eq.11}, into the real and imaginary terms yields

\[ L(f) = E(f) + 2\pi f \sum_{n=M+1}^{\infty} \sin(-2\pi fn) \]  

(\text{eq.14})

\[ L(f) = i(E(f) + 2\pi f \sum_{n=M+1}^{\infty} \cos(-2\pi fn)) . \]  

(\text{eq.15})

Using the trigonometric identities

\[ \sum_{n=N+1}^{\infty} \sin(nu) = \frac{\cos((N+1/2)u)}{2\sin(u/2)} \]  

(\text{eq.16})

\[ \sum_{n=N+1}^{\infty} \cos(nu) = \frac{\sin((N+1/2)u)}{2\sin(u/2)} \]  

(\text{eq.17})
and substituting eq.16 into eq. 14 the following results,

\[
L(f) = E(f) + \frac{2\pi f \xi}{2 \sin(\pi \xi)} \cos((N+1/2)2\pi f \xi) \tag{eq.18}
\]

and substituting eq.17 into eq.15 yields,

\[
L(f) = E(f) - \frac{2\pi f \xi}{2 \sin(\pi \xi)} \sin((N+1/2)2\pi f \xi). \tag{eq.19}
\]

Using the function \( \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \) \tag{eq.20}

eqs.18 and 19 can be written as follows,

\[
L(f) = E(f) + \frac{\cos((N+1/2)2\pi f \xi)}{\text{sinc}(f \xi)} \tag{eq.21}
\]

\[
L(f) = E(f) - \frac{\sin((N+1/2)2\pi f \xi)}{\text{sinc}(f \xi)}. \tag{eq.22}
\]

Equations 21 and 22 can be combined to yield the total OTF, and the MTF can be obtained, by taking the square root of the squared real and imaginary terms.

The above mathematical concepts are based on the assumption that the photographic system is a linear system.
The theory is true until the film is processed. Apparent non-linearities, introduced when the image is measured by its photographic density, can be eliminated by working back through the sensitometric calibration data, (D-logH curve), as long as chemical adjacency effects are not present.(20) Therefore, adjacency effects will have to be corrected, minimized, or even eliminated.(21) Some other possible sources of error will be covered in the following section.
OTHER SOURCES OF ERROR AND DATA DISTORTION.

The OTF/MTF measurement is affected by the alignment and focus of the scanning microdensitometer (22), as well as the shape (i.e. slit, circle) and size of the aperture used, to scan the image. Another important factor in the measurement is the noise caused by the granular structure of the photographic emulsion, which introduces noise into scanned data, both edges and sinusoids.(23,24) The noisy scans need to be smoothed, or treated in some other way, to obtain a reasonable measure of the OTF/MTF.

The list of MTF measurement methods, and the mathematical concepts presented here, are by no means a complete examination. The concepts and techniques are intended to be sufficient for the understanding of the experimental approach used in this research. The relevance of these methods and the mathematical concepts to the thesis will be discussed in the following section. For a more complete study of the mathematics and measurement techniques, the reader is referred to the reference section of this thesis.

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EXPERIMENTAL

The procedural part of this research can be divided into two main areas. The first is the data collection and the second is the development of computer programs to process the data.

The data collection began when an edge target, consisting of x-ray lines at 5 density levels on a high resolution film, provided by J. Altman of the Eastman Kodak Company, was contact printed, along with a 21 step gray scale, onto Kodak Tri-X Pan black and white film. A Kodak DF sensitometer, modified with a vacuum pump and platen, was used to make the exposure. The sensitometer contained a 3000 degree Kelvin light source. The exposures were made and processed at various levels of filtration and exposure times, until the entire edge target density values fell completely on the straight line portion of the D-logH curve. This requirement was met by exposing for one second, using a 2.10 Inconel filter, and a Kodak Wratten 61 (green) filter. Therefore, the actual exposure used to contact print the edge target onto the film was 1170 lux-sec. For the sine-wave target, a Kodak 60% modulation MTF target was used. The target consisted of a 21 step gray scale, and sinusoidal patterns at
22 different frequencies. The target was exposed onto the same emulsion as the edge target, using a Kodak MTF reduction camera. The MTF target was exposed on each strip of film for a period of .02 seconds, at 6 different exposure levels, established by using Inconel filters, and a Kodak Wratten 61 filter. After the exposures of the edge and sine-wave targets were made, the film strips containing the targets were processed together in a Kodak Versamat V-11 processor with Kodak Duraflo developer. The strips of film were processed at the rate of 5 feet per minute in the developer at a temperature of 26.7 degrees Celsius. This method of processing was used so that a comparison of the sine-wave and edge gradient MTF measuring methods could be made on film with no or minimal adjacency effects.

Another process used in the data collection, was to develop film strips with the sine-wave and edge target exposures in Kodak D-76 developer diluted 1:4 with water, using no agitation. The strips of film were developed in the D-76 at 20 degrees Celsius for a period of 25 minutes, which was the time selected by trial and error. This type of processing should produce enhanced adjacency effects, and would allow for a comparison of the edge and sine-wave methods with large adjacency effects, as well as the comparison with minimal adjacency effects.
After the developed samples were made, the next step in the data collection was to scan the strips of film, using a microdensitometer. A Perkin-Elmer PDS10 microdensitometer, with a 2x200 micrometer scanning slit, and a 0.25 efflux numerical aperture, was used to scan the targets. Density data was collected at a sampling interval of one micrometer. The film samples, with the sine-wave and edge target exposures, were scanned using the same slit, the same optics, and the same sampling interval of one micrometer. The step tablets were also scanned using the same set up, but outputted directly onto chart recorder paper calibrated for densities ranging from 0.0 to 5.0. A chrome edge was also scanned using the same slit and numerical aperture, in order to determine the MTF of the microdensitometer.

Once the data collection procedure was completed, the data had to be reduced. This portion of the experiment consisted of a combination of writing and using existing computer programs, which would convert the microdensitometer deflection readings into modulation values.

The sine-wave portion of the reduction process utilized programs developed by the Photographic Technology Division of the Eastman Kodak Company, and made available to the author. These programs take the scanned density data for each frequency and convert these values to relative exposure
values, using the calibration data from the step tablet scans obtained from the chart recordings. The program then smoothes the noisy data, using a fast Fourier transform routine, and calculates the modulation transfer factor for each frequency. This is done by averaging the peaks and valleys for the number of cycles found for that frequency, into a modulation value. This output modulation value is then divided by the input modulation, yielding the modulation transfer factor for that frequency. This is done for each of the 22 frequencies or until the data is determined to be unusable at the higher frequencies.

The programs for the edge data reduction process were created by the author. The creation of these programs was an evolutionary process in which programs were written for each step of the MTF calculation. A program was written to convert the density data into exposure values, again using calibration values obtained from the chart recordings of the scanned step tablet. This was done by working the density data back through the diffuse D-logH curve. (The diffuse density values were obtained by measuring the 21 step gray scale used in the contact printing, along with the edge target, with a MacBeth TD-504 densitometer.) Another program was written which normalized the individual edge traces after
conversion to exposure values, by using the following formula;

\[
\text{Normalized Value} = \text{Exposure Value} - \frac{a}{b - a} \quad \text{(eq.23)}
\]

where \(a\) is the average exposure value from the E-min data, and \(b\) is the average exposure value from the E-max data. These values \((a\) and \(b\)) were obtained by summing and averaging 40 exposure values from each end of the edge trace. The normalizing was done to remove any potential bias that might have occurred if the edge values had been only summed and averaged. Another program was written to calculate the midpoint of each edge. The midpoint was determined by summing and averaging 40 density values from the D-max and D-min ends of the edge trace respectively, then summing the average D-max and D-min values and dividing by 2. The program then searched for the density value that was the closest to the calculated midpoint value. The data files were then adjusted by hand so that the midpoint density value for each trace was in the same position, in each of the data files. This was done so that the normalized exposure values could be summed and averaged, without introducing phase problems. The next program written, reads in the exposure values from data files, sums the normalized and realigned edges, and
calculates the average value and standard deviation for each point along the edge trace. The averaging was done in an attempt to eliminate the unwanted grain noise. The process of aligning the files by their midpoints, normalizing each edge, and then averaging the normalized exposure values is believed to be a new way of attempting to eliminate grain noise. Another program calculates the spread function of the edge by taking the derivative. The program then multiplies the derivative by an adaptive gaussian damping function. This was done to further reduce the unwanted noise on the D-max and D-min portions of the edge, which was not eliminated by the averaging of the edges. The gaussian function was chosen in an attempt to eliminate as much of the noise as possible without altering the edge shape. The damping function was mathematically calculated using the following formula from Gaskill (17);

\[
gaus(x/b) = \exp\left(-\pi(x/b)^2\right)
\]  
\text{(eq.24)}

where \((b)\) is the width of the gaus. The width was visually determined, by using the criteria that the gaussian damping function be approximately twice the width of the edge, and inputted into the program. The program then integrates the damped derivative, to produce an image processed edge. The
last program written takes the image processed edge data, and mathematically calculates the OTF and MTF using a modified version of Tatians' method, which was discussed in the introduction. This program reads in the image processed values, or exposure values without image processing, normalizes them to 1.0, and calculates the OTF by transforming the data into its real and imaginary parts. The modulus of the real and imaginary parts of the OTF is then calculated, producing the MTF.

The programs discussed above were used in the following order and are contained in the appendix of this thesis. The first step was to determine the midpoint of each edge trace. Once the midpoints were determined the data files were aligned by hand, so that the midpoint of each edge was in the same position in each data file. The next step was to convert each edge from density values to exposure values. After the conversions were made, each edge was normalized to eliminate any high or low readings which could possibly bias the resultant edge. After normalization, the edges were then summed and averaged. The derivative of the averaged edge is then calculated and multiplied by the gaussian damping function. The damped derivative is then integrated, producing the image processed edge. The image processed edge is then

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used in the calculation of the MTF, determined by using Tatians' method.

Prior to putting experimental data through the programs, two mathematically determined functions which resemble a scanned edge target, were input, to test the computer programs and to see if their known MTFs resulted.

The results of the data collection, and the data reduction, as well as a comparison of the MTFs obtained from the two methods, will be shown in the following section.
RESULTS

The results of this research will be divided into the theoretical and experimental data sections. The theoretical results, using the mathematically determined functions, will be first. The experimental section will follow, firstly, showing the results of using a single edge without smoothing, and secondly, using the averaging and adaptive gaussian damping function, developed for this research, to calculate the modulation transfer function.

THEORETICAL RESULTS

Before reducing any experimental data, a functional testing of the computer programs written was performed. For this purpose the mathematically determined ramp and gaussian shaped edges shown in figures 6 and 7 were used. These theoretical functions resemble the actual experimental edge trace data, except that they are noise free. The mathematically determined functions were put through the derivative program, and the outputs, figures 8 and 9, did show the expected rectangle and gaussian shaped spread functions.
THEORETICAL RAMP FUNCTION

Fig. 6 Theoretical Ramp Function

THEORETICAL GAUSSIAN FUNCTION

Fig. 7 Theoretical Gaussian Function
Fig. 8 Ramp Spread Function

Fig. 9 Gaussian Spread Function
The results of multiplying the derivative of the ramp and gaussian functions by another gaussian damping function, that is twice as wide as the edge, are shown in figures 10 and 11. Figures 12 and 13 show the image processed edges that result after integrating the damped derivatives. A comparison of figures 12 and 13 with figures 6 and 7 shows that the gaussian damping function does not alter the overall shape of the edges, to any noticeable degree. The results of the program calculating the MTFs of the image processed ramp and gaussian functions, are shown in figures 14 and 15, with the expected sinc and gaussian shaped MTF's being obtained.

The functional testing has demonstrated that the computer programs are operating correctly. These results have also shown that even though the testing was done with noise free theoretical functions, the overall shape of the edge would not be degraded by the adaptive gaussian damping function.
DAMPED RAMP SPREAD FUNCTION

Fig. 10 Dampened Ramp Spread Function

DAMPED GAUSSIAN SPREAD FUNCTION

Fig. 11 Dampened Gaussian Spread Function
Fig. 12 Ramp Function after Image Processing

Fig. 13 Gaussian Function after Image Processing
Fig. 14 MTF: Theoretical Ramp Edge

Fig. 15 MTF: Theoretical Gaussian Edge
EXPERIMENTAL RESULTS

Once the computer programs were tested, the experimental density values were reduced by the programs, and the following graphical and statistical results were obtained.

The first set of graphs show the results of an attempt to obtain a modulation transfer function, using a single edge scan without smoothing. A microdensitometer trace of a sample processed in Duraflo developer, with the edge target image, is shown in figure 16. Figure 16 shows that the combination of using Duraflo developer and a Versamat processor, resulted in no apparent adjacency effects. This graph also shows that the grain noise is very obvious in density space, and the low signal-to-noise ratio makes it difficult to tell where the edge starts and ends. It also shows that there is more variability in the D-max portion of the trace. The sensitometric transfer curve obtained from the diffuse density readings off the MacBeth TD-504 densitometer is shown in figure 17. This curve, and the readings off the chart recorder paper of the scanned image of the step tablet, were used to convert the density values in figure 16 to the relative exposure values shown in figure 18. Figure 18 shows that after converting the density values to relative exposure values, the resultant grain noise appears to have
DENSITY VALUES ACROSS AN EDGE

Fig. 16 A Microdensitometer Trace of an Edge

DENSITY VS LOG EXPOSURE

Fig. 17 Sensitometric D-LogH Curve
increased at the D-max end of the edge trace and decreased at the D-min end. This is an artifact of the logarithmic function. Figure 19 shows the resultant MTF of the single edge trace of figure 18 without smoothing, and shows the basic difficulty with the edge gradient method. Figure 19 demonstrates why some form of smoothing must be applied in order to use the edge trace method for calculating MTFs. Those who have used the edge trace method sometimes smooth the edge scan by hand, or by a combined convolution and differentiating function, prior to calculating the MTF. Smoothing makes it easier to use the edge data, but smoothing may also degrade the image of the edge in such away that the resultant MTF is an apparently smooth function, but a poor measure of the MTF.

The objective of this research was to use as little smoothing as possible, in calculating the MTF. Therefore, rather than trying to use a single edge without smoothing, which was shown in figure 19 to be of very little use in the calculation of the MTF, the approach used in this research was to try to obtain a better image of the edge by averaging several edge traces. The averaging of several edge scans should eliminate or at least minimize the grain noise, yielding a higher signal-to-noise ratio. The averaging procedure used was described earlier in the experimental
Fig. 18 Relative Exposure vs Distance Curve

Fig. 19 MTF: Single Edge
section of this paper. The resultant averaged edge, shown in figure 20, is the result of averaging 20 individual edge traces, but the significance of that number has not been examined. The graph of the averaged edge shows that the unwanted grain noise has been significantly reduced. Figure 20 also illustrates that, since the grain noise has been significantly reduced, the averaged edge gives a clearer picture of where the edge starts and ends, in comparison to the single edge shown in figure 16. Figure 21 shows the result of taking the derivative of the averaged edge. It shows that even though the grain noise was significantly reduced, the noise is still substantial. Therefore, an adaptive gaussian damping function was used to further suppress the unwanted noise, without altering the shape of the edge itself. Figure 22, shows the result of multiplying the derivative of the averaged edge in the space domain, by the gaussian damping function. In this case, the edge in figure 20 appears to be approximately 35 micrometers wide, therefore, the gaussian damping function chosen had a width of 70 micrometers. Figure 23 shows the results of integrating the damped derivative. This figure demonstrates that, even with noisy data, the gaussian damping function still does not alter the shape of the edge. However, the adaptive gaussian damping function does suppress a majority
Fig. 20 Average of 20 Edge Scans (averaged edge)

Fig. 21 Derivative of Averaged Edge
Fig. 22 Dampered Derivative of Averaged Edge

AVERAGED EDGE AFTER IMAGE PROCESSING

Fig. 23 Averaged Edge after Image Processing
of the unwanted noise, as was its purpose. This was significant, because all the values along the entire averaged edge were used in the calculation of the MTF and the damping of the noise should eliminate adding any noise artifacts which would degrade the MTF of the edge itself.

Figure 24 shows the resultant MTF obtained from the averaged edge values, for the film with no apparent adjacency effects. The graph also illustrates that the MTF is a continuous curve, which cuts off about 100 cycles per millimeter. Figure 24 also shows the noise or discontinuities in the noise-suppressed edge are starting to obscure the MTF results, and a noise-produced lobe appears between 70 and 80 cycles per millimeter. Figure 24 graphically illustrates that as the signal-to-noise ratio further decreases, and the noise begins to affect the results, the curve moves upward in a linear progression and the MTF results become useless. For the case shown, this occurs at about 100 cycles per millimeter.

The MTF which results from using the sine-wave measuring method is shown in figure 25. The graph shows the 17 modulation factors calculated by the program, and their associated frequencies. The program calculating the modulation factors was terminated at 106.67 cycles because the sinusoidal pattern was obscured by noise.
MTF: AVERAGED EDGE

Fig. 24 MTF: Using the Edge Gradient Method

MTF: SINE-WAVE TARGET (60% MODULATION)

Fig. 25 MTF: Using the Sine-wave Method
This allowed for a total of 17 frequencies and their corresponding modulation values which could be used in a comparison of the two MTF measurement methods.

Figure 26 shows a comparison of the MTFs of the film obtained using the sine-wave and edge gradient measurement methods. (Only the 17 frequencies and their corresponding modulation values are shown for the edge in this graph.) The graph illustrates that the two MTFs are, within experimental error, identical at the lower frequencies and reasonably close at the higher frequencies, up to the point where the noise appears to distort the data. Another observation from figure 26 is that the sine-wave MTF curve seems to yield a slightly higher modulation value at the higher frequencies than the edge method MTF curve. It might be speculated that this is attributable to a higher noise "floor", resulting from less smoothing, in the sine-wave method, so that the noise becomes a significant contributor to the spectral coefficients. Therefore, if the noise floor were subtracted out, or the noise more heavily smoothed in the sine-wave approach, the modulation values for the sine-wave method at the higher frequencies would be reduced, and the two methods would be in even better agreement. Table 1 shows the frequencies, and the high, average, and low modulation factors, along with the standard deviations that were
calculated using the sine-wave method. The average value was calculated by summing and averaging the modulation factors at each frequency, for the 20 film strips.

Table 2 shows the calculated modulation values using the edge gradient method for the same 17 frequencies. A statistical comparison of the average sine-wave modulation factors and the edge modulation values at the corresponding frequencies, using a t-test and the hypothesis that the two methods yield the same MTF results, revealed that the two methods are not significantly different within a 95% confidence limit.
## Sine-wave MTF data with Minimal Adjacency Effects

**Table 1**

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>HIGH MTF</th>
<th>AVERAGE MTF</th>
<th>LOW MTF</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
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</tr>
<tr>
<td>1.67</td>
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<td>0.985</td>
<td>0.969</td>
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<tr>
<td>2.50</td>
<td>1.036</td>
<td>1.013</td>
<td>0.993</td>
<td>0.016</td>
</tr>
<tr>
<td>3.33</td>
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<td>0.997</td>
<td>0.984</td>
<td>0.007</td>
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<td>0.994</td>
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<td>1.000</td>
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<td>0.016</td>
</tr>
<tr>
<td>10.00</td>
<td>1.028</td>
<td>1.001</td>
<td>0.984</td>
<td>0.016</td>
</tr>
<tr>
<td>13.33</td>
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<td>0.968</td>
<td>0.944</td>
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<tr>
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<td>0.311</td>
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<tr>
<td>80.00</td>
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<td>106.67</td>
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</table>

## Edge Gradient MTF data with Minimal Adjacency Effects

**Table 2**

<table>
<thead>
<tr>
<th>FREQUENCY</th>
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<td>1.25</td>
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</tr>
<tr>
<td>5.00</td>
<td>1.000</td>
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<tr>
<td>106.67</td>
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</tr>
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</table>
Figure 27 shows a single edge trace of the film samples obtained by tray processing in Kodak D-76 developer with no agitation. As was expected, large adjacency effects were introduced. Figure 28 shows the result of averaging 20 of these edge traces in the same manner as previously discussed. As can be seen in figures 27 and 28, an edge with adjacency effects cannot be represented by a simple function, such as a gaussian or ramp, as easily as the edge with minimal adjacency effects. Figure 29 graphically shows the comparison of the MTFs obtained using the sine-wave and edge gradient measurement methods for the film samples processed in D-76 with no agitation. This figure shows that the MTFs obtained from the sine-wave and edge gradient measurement methods are not as similar in the presence of adjacency effects as the MTFs are in the absence of adjacency effects. The graph shows an approximate 10% modulation difference at the lower frequencies. This difference can be attributed to the fact that, in the edge method calculation of the MTF, the modulation value at the zero frequency is forced to be 1.0, and the increase to a peak modulation greater than 100% appears as a gradual process, as opposed to the sine-wave method with discretely measured values and an abrupt jump from the 1.0 at the theoretical zero frequency. Despite this difference, the two methods do yield a similar peak
modulation value, at approximately the same frequency. A slightly greater difference in modulation at the higher frequencies also appears in this process, as opposed to the process with minimal edge effects. It might be speculated that this is due to sharper discontinuities at the normalized exposure value of about 1.50, and in the adjacency effect lobe in the averaged edge, shown in figure 28, as compared to the discontinuities in the averaged edge shown in figure 20. Figure 29, again suggests that the noise "floor" tends to raise the modulation values at the higher frequencies as was previously discussed.

An Edge with Adjacency Effects

![Graph showing edge scan with adjacency effects.](image)

Fig. 27 Edge Scan with Adjacency Effects
AVERAGED VALUES ACROSS THE EDGE

Fig. 28 Average of 20 Edge scans with Adjacency Effects

MTFs: Sine-Wave (60% modulation) vs Edge Gradient
With Large Adjacency Effects

Fig. 29 Comparison of MTFs with Large Adjacency Effects
<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>HIGH MTF</th>
<th>AVERAGE MTF</th>
<th>LOW MTF</th>
<th>STANDARD DEVIATION</th>
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</thead>
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<tr>
<td>13.33</td>
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<td>1.194</td>
<td>0.023</td>
</tr>
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<td>0.303</td>
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Sine-wave MTF data with Large Adjacency Effects
Table 3

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<td>0.492</td>
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<tr>
<td>106.67</td>
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</table>

Edge Gradient MTF data with Large Adjacency Effects
Table 4
Comparing the data statistically, again with a T-test, the values from tables 3 and 4 revealed the average sine-wave MTF values, and the corresponding edge gradient MTF values, are not significantly different within a 99% confidence limit. A further analysis showed the sine-wave low MTF values, and the edge MTF values, are within a 95% confidence limit. The result of a t-test, using one standard deviation subtracted from the average sine-wave values, and the edge MTF values, also substantiates the hypothesis that the two MTFs are not significantly different, at the 95% confidence limit.

A cascaded modulation transfer acutance, CMTA, is a number based upon the area under the MTF curve which evaluates sharpness. CMTA was calculated for the different MTFs using the following formula:

\[
CMT = 111 - 21 \log_{10} \sum_{i} (200M_{i}/A_{i})^2
\]  

(eq.25)

where \( M \) is the magnification of a given stage of the system as referenced to the retina of the eye, and \( A \) is the area under the modulation transfer curve of the \( i \)th system component, where the spatial frequency is given in cycles per millimeter.
The CMTA values calculated for the average sine-wave and edge gradient MTF values, and their differences, are shown in table 5.

CMT ACUTANCE VALUES

Minimal Adjacency Effects

<table>
<thead>
<tr>
<th></th>
<th>Sine-wave</th>
<th>Edge</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>35mm Slide</td>
<td>97.98</td>
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<td>35mm Movie</td>
<td>95.71</td>
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</tr>
<tr>
<td>Pocket 110</td>
<td>93.42</td>
<td>92.78</td>
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<td>16mm Movie</td>
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<tr>
<td>Standard View</td>
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<td>99.76</td>
<td>0.01</td>
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</table>

Large Adjacency Effects

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<th>Difference</th>
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<td>0.82</td>
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<td>Super 8</td>
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<tr>
<td>Standard View</td>
<td>102.34</td>
<td>100.75</td>
<td>1.59</td>
</tr>
</tbody>
</table>

CMT Acutance Values for the Sine-wave and Edge Gradient MTF Measurements.

Table 5

According to James (15), but not supported by many others (25), a CMT acutance difference of 1.0 is believed to be a
just noticeable change in subjective sharpness. If this is true, it means that there is a slight sharpness difference between the two MTFs in the presence of large adjacency effects, and also suggests that there is a difference in the statistical analysis comparison and the CMT acutance comparison of the two methods.

The graphical and statistical results presented above have led to the following discussion, and conclusions concerning the sine-wave and edge gradient MTF measurement methods.
In evaluating the sine-wave and edge gradient MTF measurement methods, the edge gradient measurement method seemed easier to work with to this experimenter, especially with computer assistance. Although both methods use sampled data and computer analysis, the impact of the computer is greater on the edge method. The single greatest advantage of the edge method is that laboratory targets are much easier to make, and input modulation measurements are not required. (Of course, this does produce a normalized MTF result.) Another advantage is that edges also appear in nature and can be found in most photographs; therefore, the edge method can theoretically be used to evaluate almost any photograph. The problem with this is that there is no sensitometric data to convert densities to effective exposures. The major problem encountered in using the edge gradient method was the elimination of unwanted noise.

In comparison, the difficulty in making a truly sinusoidal target with known modulation makes the sine-wave method difficult to use. Another problem with the sine-wave method is that in most cases the target is too large to be contact printed onto 35mm film and therefore must be
projection printed. This introduces the added problem of bringing lens MTFs into the calibrations. (Of course this could be the same for a large edge gradient target as well.) Another problem is that sinusoids are very rarely, if at all, found in nature.

The above discussion is not meant to imply that the edge method is the better way of evaluating the MTF. It is only an opinion from the limited work performed by this experimenter.

The conclusions drawn from the research work will be discussed in the following section. The conclusions are solely based on this research work. The limitations to the conclusions are that only one type of black and white film was tested, and the film processing was limited to two methods. However, it is the opinion of this researcher that these results could be extended to other types of photographic film and processing combinations.
CONCLUSIONS

This comparison of the sine-wave and edge gradient analysis measurement methods, showed that statistically, and on the basis of photographic sharpness, the two methods yield nearly identical results. However, the agreement is slightly better for the comparison without any apparent adjacency effects. The graphical comparison showed the two methods produced nearly identical results for the samples processed to have no adjacency effects, but did show a visual difference at the lower frequencies for the film samples with large adjacency effects. The differences in the two methods may show up at frequencies near zero due to the basic difference between the indirect, functional, edge method, and the direct, discretely measured, sine-wave method. Also the differences in handling noise and data smoothing may introduce differences at higher frequencies where the signal-to-noise ratio is low and grain noise obscures the data.

The technique of aligning several edge traces by their midpoints, normalizing the individual edge traces and then averaging the edge traces appears to be a new technique which
produces a significantly superior representative edge for analysis.

The adaptive gaussian damping filter, introduced in this research, and applied to the edge data, seems to significantly reduce noise, even after averaging, without distorting the basic shape of the edge as long as it is carefully chosen.
RECOMMENDATIONS FOR FUTURE WORK

The results reported in this thesis are based on one black and white film and two processes. It is recommended that future work be extended to color films, other black and white films, and to some other process variations to further substantiate the results of this thesis.

The methods for eliminating noise seem sound, but need to be further tested. In particular, the tests for comparison of the two techniques need to be examined, and the potential problems introduced by excessive or incorrect noise smoothing need to be examined, for both the edge gradient and sine-wave methods.

Future work into the methods of averaging edge data could also be investigated. A suggestion would be to make sure the imaginary term in the OTF is set to zero in the low frequency region. This should be done prior to averaging, rather than using the simple averaging scheme used in this research. The removal of the low frequency imaginary term can be accomplished by multiplying each component of the OTF, by the proper linear phase dispersive correction filter, prior to averaging, and doing the averaging in the frequency domain rather than the space domain.
REFERENCES


25. Granger, E.M., Personal communication
APPENDIX

The computer programs developed by the author and user for this thesis are included in this section. The programs were written, in FORTRAN for the IBM/CMS system. Subroutines unique to the Photographic Technology Division of the Eastman Kodak Company, Rochester, N.Y., and the DISSPLA and TELLAGRAF plotting packages, developed by Integrated Software Systems Corporation of San Diego, California, were used. A short description of the programs written by this experimenter and the actual programs follow.

Programs

---

MIDPNT - Calculates the midpoint.

XPOSR - Converts density to exposure.

TRY20 - Averages 20 edges to yield a single edge.

DERIV - Calculates the derivative, multiplies the derivative by a Gaussian damping function, and integrates to yield an Image processed edge.

TMTF - Calculates the MTF using Tatians' formula.

AVG - Calculates averages from the ends of the edges and normalizes the data.

- 62 -
NRMLYS - Normalizes the edge to a max value of 1.0 or a max area of 1.0.

PLT - Plotting routine using the DISSPLA plotting package.

PLOT - Subroutine using DISSPLA subprograms.

FIXUP - Subroutine to put $ in character string.

DEVICE - Subroutine for outputting plots.
THIS PROGRAM READS IN DENSITY DATA AND DETERMINES THE MIDPOINT OF
THE EDGE. THIS IS DONE BY USING SUBROUTINE AVG WHICH AVERAGES SO MANY
VALUES FROM EACH SIDE OF THE EDGE TRACE AND USES THOSE AVERAGES TO
NORMALIZE THE DENSITIES. THE NORMALIZED VALUES ARE THEN USED TO
DETERMINE THE VALUE CLOSEST TO THE MIDPOINT, THE FILE NUMBER, AND THE
NORMALIZED VALUE.

REAL DENS(256),D(256)
CHARACTER*30 FILNAM
CHARACTER*1 ANS

99 WRITE(6,*)'WHAT IS THE NAME OF THE FILE WITH DENSITY DATA?
READ (5,'(A)') FILNAM

WRITE(6,*),'WHAT IS THE NAME OF THE FILE WITH DENSITY DATA?
READ (5,'(A)') FILNAM
WRITE (6,*)'HOW MANY DATA POINTS IN DENSITY FILE?
READ (5,*) N
CALL CMSFVS('FILEDEF 1 DISK '//FILNAM//' DATA AT.NRC)

* READ IN THE DENSITY VALUES FROM A DATA FILE.
READ(1,*)(D(I),I=1,N)

IF (D(1).GT. D(N)) THEN
   DO 1 I=1,N
      DENS(I) = D(N+1-I)
   1 CONTINUE
ELSE
   DO 5 I=1,N
      DENS(I) = D(I)
   5 CONTINUE
ENDIF

WRITE(6,*),'HOW MANY VALUES FROM THE LEFT SIDE OF EDGE DO YOU
WRITE(6,*)'WANT TO AVERAGE?
READ(5,*)K

WRITE(6,*),'HOW MANY VALUES FROM THE RIGHT SIDE OF EDGE DO YOU
WRITE(6,*)'WANT TO AVERAGE?
READ(5,*)M

* USE SUBROUTINE AVG TO DETERMINE THE AVERAGE AND NORMALIZED VALUES.
CALL AVG (DENS,N,K,M)

- 64 -
* DETERMINE THE APPROXIMATE MIDPOINT USING NORMALIZED VALUES.

    DO 10 I=1,N
        IF (DENS(I) .GE. 0.5) GOTO 15
    10 CONTINUE

* OUTPUT MIDPOINT FILE NUMBER, DENSITY VALUE, AND NORMALIZED VALUE.

    15 WRITE(6,*)' '
    WRITE(6,*)' '
    WRITE(6,*)' MIDPOINT=',(I-1),DENS(I-1),D(I-1)
    CLOSE(1)
    WRITE(6,*)' '
    WRITE(6,*)' WOULD YOU LIKE TO FIND MIDPOINT OF ANOTHER FILE?(Y/N)'
    READ(5,'(A)') ANS

    IF (ANS .EQ.'Y') GOTO 99

STOP
END

- 65 -
THIS PROGRAM WILL TAKE DENSITY VALUES AND CONVERT THEM INTO EXPOSURE VALUES BY TRANSFERRING THE DENSITY VALUES BACK THROUGH A D-LOGE CURVE AND TAKING THE ANTILOG.

```fortran
REAL X(21),Y(21),DENS(256),XLOGE(256)
REAL XAXIS(256),YAXIS(256),YY(256),YYY(256)
CHARACTER*1 ANS,ANSR,ANSWR
CHARACTER*20 FILNAM,FILNM

* RELATIVE LOG EXPOSURE OFF STEP TABLET.
  DATA X /0.00,0.16,0.30,0.44,0.57,0.73,0.87,1.02,1.18,1.33,1.48,
     1.63,1.78,1.93,2.09,2.24,2.40,2.56,2.70,2.85,3.00/

* THESE ARE THE EXPOSURE FOR THE 20 EDGES.
  DATA Y /0.55,0.57,0.63,0.70,0.78,0.90,1.02,1.15,1.30,1.43,
     1.57,1.68,1.80,1.87,1.97,2.07,2.17,2.27,2.35,2.44,2.53/

* THESE ARE THE EXPOSURE VALUES FOR THE D76 EDGES.
  DATA Y /0.39,0.42,0.45,0.48,0.55,0.63,0.73,0.83,0.92,1.03,
     1.12,1.21,1.29,1.38,1.45,1.53,1.64,1.74,1.82,1.88,1.97/

999 WRITE(6,*)'WHAT IS THE NAME OF THE FILE WITH DENSITIES IN IT?'
   READ(5,'(A)')FILNAM
   WRITE(6,*)' '
   WRITE(6,*)'HOW MANY DENSITY VALUES IN DATA FILE?'
   READ(5,*)N
   CALL CMSFVS('FILEDEF 1 DISK //FILNAM// DATA A1',NRC)

* READ IN DENSITY VALUES TO BE CONVERTED.
   READ(1,*)(DENS(I),I=1,N)
   CLOSE(1)
   WRITE(6,*)''
   WRITE(6,*)'WHAT IS THE NAME OF OUTPUT FILE YOU WANT?'
   READ(5,'(A)')FILNM
   CALL CMSFVS('FILEDEF 3 DISK //FILNM// DATA A1',NRC)
   DO 50 I=1,N
       Q=DENS(I)
   WRITE(9,*)I,Q
```

- 66 -
DO 40 J=1,21
   IF(Q.EQ.Y(J)) THEN
      XLOGE(J)=X(J)
      GOTO 30
   ELSE
      IF(Q.LT.Y(J).AND.Y(J-1).LT.Q) GOTO 25
   ENDIF
40 CONTINUE

* COMPUTES THE LOG EXPOSURE VALUE.
25 XLOGE(J)=X(J)+(Q-Y(J))/((Y(J+1)-Y(J))*(X(J+1)-X(J))

* OUTPUTS THE EXPOSURE VALUES.
30 WRITE(8,*)10**XLOGE(J)
    WRITE(3,*)I,10**XLOGE(J)
    WRITE(7,*)I,DENS(I),XLOGE(J),10**XLOGE(J)
    YY(I)=10**XLOGE(J)
50 CONTINUE
CLOSE(8)

100 WRITE(6,'(A)')'DO YOU HAVE ANOTHER SET OF DENSITIES TO CONVERT?'
    READ (5,'(A)')ANSWR
    IF(ANSWR.EQ.'Y') GOTO 999
    WRITE(6,'(A)')''
    WRITE(6,'(A)')'YOUR OUTPUT(X,DENSITY,EXPOSURE) IS IN EXPOSE DATA A1'
    WRITE(6,'(A)')'YOUR VALUES FOR PLOTTING ARE IN DENSITY DATA A1'
    WRITE(6,'(A)')'YOUR EXPOSURE VALUES ARE IN XPOSUR DATA A1'
    GO TO 201
201 STOP
END
TRY2D

THIS PROGRAM FINDS THE AVERAGE VALUE FOR POINTS ACROSS THE EDGE. IT WILL TAKE IN THE DENSITY VALUES FOR 2D EDGES AND AVERAGE THE VALUES AT EACH DISTANCE POINT. IT WILL OUTPUT THE MEAN, THE VARIANCE, AND THE STANDARD DEVIATION FOR EACH POINT.

REAL A1(256),A2(256),A3(256),A4(256),A5(256),XX(256),YY(256)
REAL A6(256),A7(256),A8(256),A9(256),A1D(256),A11(256),TMEAN(256)
REAL A12(256),A13(256),A14(256),A15(256),A16(256),A17(256)
REAL A18(256),A19(256),A2D(256),A(2DD),B(2DD),VAR(256),SDEV(256)
CHARACTER*1 ANS,ANSR

REAL A12(256),A13(256),A14(256),A15(256),A16(256),A17(256)
REAL A18(256),A19(256),A2D(256),A(2DD),B(2DD),VAR(256),SDEV(256)
CHARACTER*1 ANS,ANSR

N=2DD

* READ IN DATA FROM EXPOSURE FILES.

READ (1,*) (A1(I),I=1,N)
READ (2,*) (A2(I),I=1,N)
READ (3,*) (A3(I),I=1,N)
READ (4,*) (A4(I),I=1,N)
READ (7,*) (A5(I),I=1,N)
READ (8,*) (A6(I),I=1,N)
READ (9,*) (A7(I),I=1,N)
READ (10,*) (A8(I),I=1,N)
READ (11,*) (A9(I),I=1,N)
READ (12,*) (A1D(I),I=1,N)
READ (13,*) (A11(I),I=1,N)
READ (14,*) (A12(I),I=1,N)
READ (15,*) (A13(I),I=1,N)
READ (21,*) (A14(I),I=1,N)
READ (22,*) (A15(I),I=1,N)
READ (23,*) (A16(I),I=1,N)
READ (24,*) (A17(I),I=1,N)
READ (25,*) (A18(I),I=1,N)
READ (26,*) (A19(I),I=1,N)
READ (27,*) (A2D(I),I=1,N)

WRITE(6,*), "HOW MANY VALUES FROM THE LEFT SIDE OF EDGE DO YOU WANT TO AVERAGE FOR NORMALIZING?"
READ (5,*)K

WRITE(6,*), "HOW MANY VALUES FROM THE RIGHT SIDE OF EDGE DO YOU WANT TO AVERAGE FOR NORMALIZING?"
READ (5,*)M
* NORMALIZE THE FILES BEFORE AVERAGING.

CALL AVG(A1,N,K,M)
CALL AVG(A2,N,K,M)
CALL AVG(A3,N,K,M)
CALL AVG(A4,N,K,M)
CALL AVG(A5,N,K,M)
CALL AVG(A6,N,K,M)
CALL AVG(A7,N,K,M)
CALL AVG(A8,N,K,M)
CALL AVG(A9,N,K,M)
CALL AVG(A10,N,K,M)
CALL AVG(A11,N,K,M)
CALL AVG(A12,N,K,M)
CALL AVG(A13,N,K,M)
CALL AVG(A14,N,K,M)
CALL AVG(A15,N,K,M)
CALL AVG(A16,N,K,M)
CALL AVG(A17,N,K,M)
CALL AVG(A18,N,K,M)
CALL AVG(A19,N,K,M)
CALL AVG(A20,N,K,M)

X=20.0

* CALCULATE AVERAGE, VARIANCE, AND STANDARD DEVIATION.

DO 10 I=1,N
+ A20(I)
+ A8(I)**2+A9(I)**2+A10(I)**2+A11(I)**2+A12(I)**2+A13(I)**2+
+ A14(I)**2+A15(I)**2+A16(I)**2+A17(I)**2+A18(I)**2+A19(I)**2+
+ A20(I)**2

TMEAN(I) = SUM/X
VAR(I)=V/X - ((SUM**2)/X**2)
SDEV(I)=SQRT(ABS(VAR(I))*(X/(X-1.0)))

WRITE(30,*)TMEAN(I)
WRITE(31,*)I,TMEAN(I),VAR(I),SDEV(I)

10 CONTINUE

IF (TMEAN(I).GT. TMEAN(N)) THEN
   DO 15 I=1,N
      XX(I) = I

   15 CONTINUE
YY(I) = TMEAN(N+1-I)
WRITE(32,*)I,YY(I)

15      CONTINUE
ELSE
    DO 20 I=1,N
    XX(I) = I
    YY(I) = TMEAN(I)
    WRITE(32,*)I,YY(I)
20      CONTINUE
ENDIF

* OUTPUTS ARE IN THE FOLLOWING FILES.

WRITE(6,'')
WRITE(6,'"YOUR AVERAGE STATISTICS ARE IN DNSTAT DATA A1"
WRITE(6,'"YOUR AVERAGE EXPOSURE DATA IS IN AVGDEN DATA A1"
WRITE(6,'"YOUR AVG EXPOSURE DATA (PLOTTING)IS IN UEDGE DATA A1"

STOP
END
C
C THIS PROGRAM WILL TAKE THE DERIVATIVE OF THE EFFECTIVE OR RELATIVE
C EXPOSURE VALUES AND MULTIPLY THE DERIVATIVE BY A GAUS OF CHOSEN
C WIDTH TO DAMP THE DATA. IT WILL THEN INTEGRATE THE DAMPED DATA TO
C YIELD THE IMAGE PROCESSED EDGE USED TO CALCULATE THE MTF.

REAL YY(256),Y(256),L(256),LS(256),GAUS(-127:128),EDGE(256),E(256)
CHARACTER*40 FILNAM

PI = 3.141592654

WRITE(6,*)'WHAT IS THE NAME OF FILE WITH DENSITIES IN IT?'
READ (5,'(A)')FILNAM

WRITE (6,*)'HOW MANY VALUES IN DATA FILE?'
READ (5,*) N

CALL CMSFVS('FILEDEF 1 DISK '///FILNAM//' DATA A1',NRC)

* READ IN THE DATA.

READ (1,*) (YY(I),I=1,N)

IF (YY(1).GT.YY(N)) THEN
   DO 1 I=1,N
      Y(I) = YY(N+1-I)
   1 CONTINUE
ELSE
   DO 5 I=1,N
      Y(I) = YY(I)
   5 CONTINUE
ENDIF

WRITE(6,*)'WHAT IS THE SAMPLING INTERVAL BETWEEN DATA POINTS?'
READ (5,*) DX

WRITE(6,*)'WHAT IS THE MIDPOINT OF THE DATA?'
READ (5,*) MIDPNT

* TAKE THE DERIVATIVE, INPUT GAUS WIDTH (TWICE THE WIDTH OF EDGE)

DO 10 I=1,N-1
   L(I) = (Y(I+1) - Y(I)) / DX
10 CONTINUE

CALL NRMLYS(L,N-1,1,DX)
DO 15 I=1,N-1
WRITE(2,*)I,L(I)
15 CONTINUE

WRITE(6,*)'WHAT IS THE WIDTH OF THE GAUS YOU WANT?'
WRITE(6,*)'IT SHOULD BE APPROXIMATELY 2X WIDTH OF EDGE.'
READ (5,*) B

* CALCULATE GAUSSIAN DAMPING FUNCTION

DO 20 I=-MIDPNT,N-(MIDPNT-1)
   GAUS(I)=EXP((-PI*((I/B)**2)))
   WRITE(4,*)I+(MIDPNT +1),GAUS(I)
20 CONTINUE

* MULTIPLY DERIVATIVE BY GAUSSIAN DAMPING FUNCTION

DO 30 I=1,N-1
   LS(I)=L(I)*GAUS(I-MIDPNT)
   WRITE(3,*)I,LS(I)
30 CONTINUE

* INTEGRATE TO GET SMOOTHED EDGE.

SUM=0.0
DO 40 I=1,N-1
   SUM=SUM + LS(I)
   EDGE(I)=SUM
40 CONTINUE

CALL NRMLYS(EDGE,N-1,1,DX)

DO 50 I=1,N-1
   WRITE(7,*)I,EDGE(I)
   WRITE(8,*)EDGE(I)
50 CONTINUE

* OUTPUTS IN THE FOLLOWING FILES.

WRITE(6,*)' '
WRITE(6,*)'YOUR DERIVATIVES ARE IN DERIV DATA AT'
WRITE(6,*)'YOUR SMOOTHED DERIVATIVE IS IN SDERIV DATA AT'
WRITE(6,*)'YOUR SMOOTHED EDGE FOR PLOTTING IS IN SMTHEG DATA AT'
WRITE(6,*)'YOUR SMOOTHED EDGE FOR MTF DATA IS IN SMTEDG DATA AT'

STOP
END
THIS PROGRAM WILL TAKE EXPOSURE VALUES FROM A DATA FILE AND CALCULATE THE MTF OF A SYSTEM USING A MODIFIED VERSION OF TATIAN'S METHOD FOR CALCULATING AN MTF.

REAL XPOS(256), OTF(500), OTF(500), MTF(500), E(500)
REAL XX(500), YY(500), G(17), FASE(500), MTFMD(500), Z(500)
CHARACTER*1 ANZ
CHARACTER*40 FILNAM

INPUT FREQUENCIES FOR 60% TARGET

DATA G /1.25, 1.667, 2.5, 3.333, 5.0, 6.667, 10.0, 13.333, 16.667, 20.0, 26.667, 33.333, 40.0, 53.333, 66.667, 80.0, 106.667/

WRITE(6,*) 'HOW MANY VALUES IN THE EXPOSURE DATA FILE?'
READ (5,*) N
WRITE(6,*) '
WRITE(6,*)' WHAT IS THE SAMPLING INTERVAL IN MILLIMETERS?'
READ (5,*) DX

* READ IN EXPOSURE VALUES FROM DATA FILE
READ (1,*) (E(I), I=1,N)

* NORMALIZE / SCALE EXPOSURE VALUES

IF (E(1).GT.E(N)) THEN
   DO I=1,N
      XPOS(I) = E(N+1-I)
   1  CONTINUE
ELSE
   DO 5 I=1,N
      XPOS(I) = E(I)
  5  CONTINUE
ENDIF

WRITE(6,*) '
WRITE(6,*)' DO YOU WANT TO
WRITE(6,*) 1. AVERAGE VALUES BEFORE SCALING (NOISY DATA)
WRITE(6,*) 2. NORMALIZE VALUES TO 1.0 (SMOOTH TRACE)
WRITE(6,*) INPUT 1 OR 2'
READ (5,*) ANSWER
IF (ANSWER.EQ. 1) THEN
WRITE(6,*), ''
WRITE(6,*), 'HOW MANY VALUES FROM THE LEFT SIDE OF EDGE DO YOU'
WRITE(6,*), 'WANT TO AVERAGE FOR NORMALIZING?'
READ(5,*), K

WRITE(6,*), ''
WRITE(6,*), 'HOW MANY VALUES FROM THE RIGHT SIDE OF EDGE DO YOU'
WRITE(6,*), 'WANT TO AVERAGE FOR NORMALIZING?'
READ(5,*), L

CALL AVG(XPOSR,N,K,L)
ELSE
CALL NRMLYS(XPOSR,N,1,DX)
ENDIF

* DETERMINE APPROXIMATE MIDPOINT OF EDGE

DO 10 I = 1,N
   IF (XPOSR(I).GE. 0.5) GOTO 15
10 CONTINUE

15 J = I - 1
X0 = - (J+1) * DX
M = N - J
PI  = 3.141592654
FMAX = 1.0 / (2.0 * DX)

WRITE(6,*), 'WHAT KIND OF FREQUENCY SAMPLING WOULD YOU LIKE?'
WRITE(6,*), 1. LOGARITHMIC (5TH ROOT OF 10)'
WRITE(6,*), 2. CARTESIAN (EQUAL INCREMENTS)'
WRITE(6,*), INPUT 1 OR 2'
READ (5,*), ANS

IF (ANS .EQ. 1) THEN
   NF - 17
ELSE
   WRITE(6,*), 'WHAT IS THE DELTA F YOU WANT TO USE?'
   READ (5,*), DF
   NF = INT(FMAX / DF)
ENDIF

DO 20 I = 1,NF
   IF (ANS .EQ. 1) THEN
      F=G(I)
   ELSE
      F=(REAL(I) - 1.0) * DF
   ENDIF
20 CONTINUE
40 IF (F .GT. FMAX) GOTO 200

ARG = 2.0 * PI * DX * F
D = 2.0 * PI * F
OTFR(I) = 0.0
OTFI(I) = 0.0

* CALCULATE OTF BY ITS REAL AND IMAGINARY PARTS.

DO 50 J=1,N
    OTFR(I) = OTFR(I) + (XPOSR(J) * SIN(D * (XO + (J*DX))))
    OTFI(I) = OTFI(I) + (XPOSR(J) * COS(D * (XO + (J*DX))))
50 CONTINUE
    OTFR(I) = (OTFR(I) * ARG) + COS((M-0.5)*ARG) / SINC(DX*F)
    OTFI(I) - (OTFI(I) * ARG) - SIN((M-0.5)*ARG) / SINC(DX*F)

* CALCULATE MTF OF SYSTEM

MTF(I) = SQRT((OTFR(I)**2) + (OTFI(I)**2))

20 CONTINUE

DO 55 I=1,NF
    IF(ANS .EQ. 2) THEN
        READ(2,*)Z(I),MTFMD(I)
    ELSE
        READ(3,*)Z(I),MTFMD(I)
    ENDIF
55 CONTINUE

* OUTPUT RESULTS

WRITE(6,'(A)') 'WHAT IS THE NAME OF THE OUTPUT FILE YOU WANT?'
READ (5,'(A)') FILNAM
CALL CMSFVS('FILEDEF 15 DISK ',FILNAM,' DATA A1',NRC)

GOTO 300

200 NF = NF - 1

300 DO 60 I=1,NF
    IF (ANS .EQ. 1) THEN
        F = G(I)
    ELSE
        F = (REAL(I) - 1.0) * DF
75
EN0IF

* MTFMD IS THE MTF FOR THE CHROME EDGE (USED FOR MICRO-D MTF).

400   WRITE(12,*) F, MTF(I)
     WRITE(15,*) F, MTF(I)/SINC(.002*F)/((-1.0/909.0909)*F +1.0)
     WRITE(11,*) F, MTF(I)/MTFMD(I)

* FOR PLOTTING PURPOSES ON LOG SCALE.

    IF (F .LT. 0.9) F = 1.0
    XX(I) = F
    IF (MTF(I) .LT. 0.01) THEN
        YY(I) = 0.01
    ELSE
        YY(I) = MTF(I)
    EN0IF

60 CONTINUE

STOP
EN0
SUBROUTINE AVG (XPOS, N, K, M)

C THIS SUBROUTINE READS IN THE VALUES, AVERAGES SO MANY POINTS FROM EACH
C END OF THE EDGE TRACE AND NORMALIZES THE DATA. THE PROGRAM ALSO
C CALCULATES THE VARIANCE AND STANDARD DEVIATION OF THE VALUES BEING
C AVERAGED.

REAL XPOS(N)

C WRITE(6,*),'HOW MANY VALUES FROM THE TOP DO YOU WANT TO AVERAGE?'
C READ (5,*) K

SUM1= 0.0
Q=0.0

DO 10 I=1,K
   SUM1 = SUM1 + XPOS(I)
   Q=Q+1.0
10 CONTINUE

A = SUM1 /Q

SUMDIF=0.0

DO 15 I=1,K
   SUMDIF = SUMDIF + (XPOS(I) - A)**2
15 CONTINUE

VAR1 = SUMDIF / (Q-1)
SDEV1 = SQRT(VAR1)

C WRITE(6,*),'HOW MANY VALUES FROM THE LAST DO WANT TO AVERAGE?'
C READ (5,*) M

SUM2 = 0.0
R=0.0

DO 25 I=N-M,N
   SUM2 = SUM2 + XPOS(I)
   R=R+1.0
25 CONTINUE

B = SUM2 / R
C = B - A

WRITE(6,*) A,B

SOMDIF = 0.0
DO 30 I=N-M,N
SOMDIF = SOMDIF + (XPOSR(I)-B)**2

CONTINUE

VAR2 = SOMDIF / (R-1)
SDEV2 = SQRT(VAR2)

DO 40 I=1,N
   XPOSR(I) = (XPOSR(I) - A)/ C
40 CONTINUE

* OUTPUTS THE AVERAGE VALUES, VARIANCE, AND STANDARD DEVIATIONS.

WRITE(39,*)'AVERAGE VALUE FOR THE FIRST',K,' POINTS=',A
WRITE(39,*)
WRITE(39,*)'VARIANCE=',VAR1,' STANDARD DEVIATION=',SDEV1
WRITE(39,*)
WRITE(39,*)'AVERAGE VALUE FOR THE LAST',M,' POINTS=',B
WRITE(39,*)
WRITE(39,*)'VARIANCE=',VAR2,' STANDARD DEVIATION=',SDEV2

* RETURN NORMALIZED VALUES TO MAIN PROGRAM.

RETURN
END
SUBROUTINE NRMLYS (ARRAY,N,K,DX)

C SUBROUTINE TO NORMALIZE AN ARRAY OF DATA TO
C MAX VALUE = 1.0 IF K=1
C AN AREA = 1.0 IF K=2

REAL ARRAY(N)
AMAX =0.0
AREA = 0.0

DO 10 I=1,N
   IF (AMAX .LT. ARRAY(I)) AMAX =ARRAY(I)
      AREA = AREA + ARRAY(I) * DX
10 CONTINUE

DO 20 I=1,N
   IF(K .EQ. 1) THEN
      ARRAY(I) = ARRAY(I) / AMAX
   ELSE
      ARRAY(I) = ARRAY(I) / AREA
   ENDIF
20 CONTINUE

RETURN
END
C PLT
C
C THIS IS A USER FRIENDLY PLOTTING ROUTINE USING THE DISSPLA PLOTTING
C PACKAGE AND THE SUBROUTINE PLOT.

CHARACTER*20 FILNM1,FILNM2
CHARACTER*1 ANSWER,ANS
REAL Y(512), X(512), A(512), B(512)

WRITE(6,'')
WRITE(6,'WHAT IS THE NAME OF THE FILE WITH THE X AND Y DATA ?'
READ(5,'(A)') FILNM1

WRITE(6,'')
WRITE(6,'HOW MANY POINTS IN THE DATA FILE ?'
READ (5,*) N

CALL CMSFVS('FILEDEF 1 DISK //FILNM1// DATA A1',NRC)

DO 5 I=1,N
    READ (1,*) X(I),Y(I)
    CONTINUE

WRITE(6,'')
WRITE(6,'DO YOU WANT TWO PLOTS ON THE SAME GRAPH ? (Y/N)'
READ (5,'(A)') ANS

IF (ANS.EQ. 'Y') THEN

WRITE(6,'')
WRITE(6,'WHAT IS THE NAME OF THE SECOND FILE WITH DATA ?'
READ (5,'(A)') FILNM2

WRITE(6,'')
WRITE(6,'HOW MANY POINTS IN THE SECOND FILE'
READ (5,*) M

CALL CMSFVS('FILEDEF 2 DISK //FILNM2// DATA A1',NRC)
    READ(2,*) (A(I),B(I), I=1,M)
ELSE
    M=1
ENDIF

CALL PLOT(X,Y,A,B,ANS,N,M)
STOP
END
SUBROUTINE PLOT(XX,YY,A,B,ANS,N,M)

C THIS SUBROUTINE IS USED ALONG WITH THE MAIN PROGRAM PLOT AND THE
C DISPLA PLOTTING PACKAGE.

REAL XX(N), YY(N), A(M), B(M)
CHARACTER*60 XLABEL, YLABEL, TITLE
CHARACTER*1 ANS, ANSR, ANZ
COMMON DY(5020)
WRITE(6,'')
WRITE(6,'WHAT IS THE TITLE FOR THE PLOT? ( 60 CHARs)'
READ (5,'(A)') TITLE
CALL FIXUP(TITLE)
WRITE(6,'')
WRITE(6,'WHAT IS THE LABEL FOR THE X AXIS? ( 60 CHARs)'
READ (5,'(A)') XLABEL
CALL FIXUP(XLABEL)
WRITE(6,'')
WRITE(6,'WHAT IS THE LABEL FOR THE Y AXIS? ( 60 CHARs)'
READ (5,'(A)') YLABEL
CALL FIXUP(YLABEL)

XMAX = XX(1)
XMIN = XX(1)
YMAX = YY(1)
YMIN = YY(1)

DO 1 I=1,N
   IF (XX(I) .GT. XMAX) XMAX = XX(I)
   IF (XX(I) .LT. XMIN) XMIN = XX(I)
   IF (YY(I) .GT. YMAX) YMAX = YY(I)
   IF (YY(I) .LT. YMIN) YMIN = YY(I)
CONTINUE
1

IF (ANS .EQ. 'Y') THEN
   AMIN = A(1)
   AMAX = A(1)
   BMIN = B(1)
   BMAX = B(1)
DO 5 I=1,M
   IF (A(I) .GT. AMAX) AMAX = A(I)
   IF (A(I) .LT. AMIN) AMIN = A(I)
   IF (B(I) .GT. BMAX) BMAX = B(I)
5 CONTINUE
IF (B(I) .LT. BMIN) BMIN = B(I)
CONTINUE
IF(AMAX .GT. XMAX) XMAX = AMAX
IF(AMIN .LT. XMIN) XMIN = AMIN
IF(BMAX .GT. YMAX) YMAX = BMAX
IF(BMIN .LT. YMIN) YMIN = BMIN
ENDIF
WRITE(6,*)
WRITE(6,*)'DO YOU WANT TO PLOT A SMOOTH CURVE (Y/N)?'
READ(5,'(A)') ANZ
WRITE(6,*)
WRITE(6,*)'WHAT TYPE OF GRAPH AXIS WOULD YOU LIKE?'
WRITE(6,*)' 1. X LINEAR, Y LINEAR (CARTESIAN)'
WRITE(6,*)' 2. X LOG, Y LINEAR (XLOG)'
WRITE(6,*)' 3. X LOG, Y LOG (LOGLOG)'
READ (5,*) Z
WRITE(6,*)
WRITE(6,*)'DO YOU WANT A GRID (Y/N)?'
READ (5,'(A)') ANSR
CALL DEVICE
CALL RESET('ALL')
CALL PAGE (10.5,8.5)
CALL HEIGHT (0.20)
CALL INTAXS
CALL XTICKS (5)
CALL YTICKS (5)
CALL XNAME ((XLABEL),100)
CALL YNAME ((YLABEL),100)
CALL AREA2D (B,,6.)
CALL HEADIN ((TITLE),100,1.25,1)
CALL THKFRM (.02)
IF (Z .EQ. 1) THEN
  CALL GRAF (XMIN,'SCALE',XMAX,YMIN,'SCALE',YMAX)
ELSE
  IF (XX(1) .LT. 1.0) XX(1) = 0.999
  IF (A(1) .LT. 1.0) A(1) = 0.999
  IF (Z .EQ. 2) THEN
    CALL XLOG (1.0,2.9,0.0,.25)
  ELSE
    IF (Z .EQ. 3) THEN
      DO 15 I=1,N
      IF (YY(I) .LT. 0.01 ) YY(I)=0.01
      15 CONTINUE
IF (B(I) .LT. 0.01) B(I)=0.01
15 CONTINUE
CALL LOGLOG (1.0,2.9,.01,2.0)
ENDIF
ENDIF
ENDIF
IF (ANSR .EQ. 'Y') THEN
   CALL OASH
   CALL GRID (2,2)
   CALL RESET ('OASH')
ENDIF
CALL THKCRV (0.01)

IF (ANZ .EQ. 'Y') THEN
   DO 20 J=1,N
       DY(J)=0.2
   CONTINUE
20 CONTINUE
   CALL SMOOTH
   CALL SETCLR ('BLUE')
   CALL MARKER (16)
   CALL CURVE (XX,YY,N,1)
ELSE
   CALL SETCLR ('RED')
   CALL MARKER (16)
   CALL CURVE (XX,YY,N,0)
ENDIF

IF (ANS .EQ. 'Y') THEN
   CALL SETCLR ('GREEN')
   CALL MARKER (2)
   CALL CURVE (A,B,M,0)
ENDIF

CALL ENOPL (0)
CALL ODNEPL
RETURN
END
SUBROUTINE FIXUP(STR)

* PUTS THE $ SIGN IN THE STRING.

CHARACTER(*) STR

ILEN = LEN(STR)
DO 10 I=ILEN,1,-1
10 IF (STR(I:I) .NE. ' ') GOTO 20
20 CONTINUE
    ILEN = I+1
    STR(ILEN:ILEN) = '$'
RETURN
END

SUBROUTINE DEVICE

* TELLS WHERE TO OUTPUT PLOT.

WRITE(6,*)' '
WRITE(6,*)' WHERE DO YOU WANT THE PLOT?'
WRITE(6,*)' '
WRITE(6,*)' 1. TERMINAL SCREEN'
WRITE(6,*)' 2. HARDCOPY FROM HP PLOTTER'
WRITE(6,*)' 3. PRINTOUT FROM VAX PRINTER'
READ (5,*) L
    IF (L .EQ. 1) CALL IBM79
    IF (L .EQ. 2) CALL HP7221
    IF (L .EQ. 3) CALL PPNTNX
RETURN
END
VITA

The author of this paper is currently a Captain in the United States Air Force (USAF) attending Rochester Institute of Technology on an Air Force Institute of Technology sponsored program.

Captain Cadou was born in Jamaica, New York on January 13, 1949. After completion of high school, he worked a year for the U.S. Postal Service before entering the USAF. Upon entering the military, he was trained to be a weather observer and spent 10 years at that job before being selected for an education and commissioning program. The USAF sent him to school for a Bachelor of Science degree in Meteorology obtained from the University of Utah in 1979. After spending 3 years as a weather forecaster, he was selected to attend RIT to obtain a Master of Science degree in Imaging and Photographic Science for which this thesis was written as a partial fulfillment of.

In August, 1985 Captain Cadou will be reassigned to the USAF Space Division in Los Angeles, California where he will be putting to use the education he has received courtesy of the USAF.