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Modulation transfer function of image motion due to operator vibration

Venkatraman Nagaswami

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MODULATION TRANSFER FUNCTION OF IMAGE MOTION DUE TO OPERATOR VIBRATION

by

Venkatraman Nagaswami

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the School of Photographic Arts and Sciences in the College of Graphic Arts and Photography of the Rochester Institute of Technology

September 1979

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MODULATION TRANSFER FUNCTION OF IMAGE MOTION
DUE TO OPERATOR VIBRATION
by
Venkatraman Nagaswami

Submitted to the Photographic Science and Instrumentation Division in partial fulfillment of the requirements for the Master of Science degree at the Rochester Institute of Technology.

ABSTRACT

The displacement path of a hand-held photographic camera is studied at low shutter speeds when operated by different observers. Using the Fourier method, the average power spectrum of hand vibration is obtained. The Modulation Transfer Function of image motion for an average observer at various exposure times is theoretically predicted.
I. INTRODUCTION

The traditional means of determining the quality of an optical system or a photographic system is to evaluate its limit of resolution. Several resolution targets consisting of a series of alternating light and dark parallel rectangular bars are available for this purpose. As the width of the bars on the target narrows, a limit is reached where the fine-line structure will be no longer discernible. This then is the resolution limit of the system. We can think of it as a spatial frequency cut-off, where each bright and dark bar pair constitutes one cycle on the object. When it became evident that this procedure of evaluating the performance of an optical system by its resolution limit alone has severe limitations, the need for some figure of merit applicable to the entire operating frequency range was felt.

A square bar target provides an input signal which is a series of square pulses. The contrast in the image is the superposition of contrast variations due to the constituent Fourier components. It has been observed that the optical elements functioning as linear operators transform a sinusoidal input into
an undistorted sinusoidal output, although the input and output irradiance distributions may not be identical. For example, diffraction, aberrations in the lens system, scattering in the film emulsion and other degradation factors reduce the sinusoid's amplitude.

In the science of image evaluation, the object is envisioned as being composed of the Fourier components and Optical Transfer Function (OTF) is defined as the operation which transforms the individual harmonic components of the object into harmonic constituents of the image. Optical Transfer Function is a spatial frequency dependent complex quantity whose modulus is the Modulation Transfer Function (MTF). Modulation Transfer Function is the ratio of image-to-object modulation for sinusoids of varying spatial frequency. It has become a widely accepted means of specifying the performance of all sorts of elements, conditions and systems such as lenses, magnetic tapes, the atmosphere and the eye, to mention but a few.

Apart from the previously mentioned degradation factors, the image formed by a photographic system may be further degraded by the non-stationary film plane during the exposure. For smaller exposure times,
the image motion is usually insignificant. But, when the exposure time is longer, the image motion has been found to introduce an additional spatial filter and band limit the process.

The image motion may be of several types. In aerial photography, linear movement of the image plane is frequently encountered while photographing from high altitude. The film plane may also undergo a simple harmonic vibration. The type of image motion with which we will be particularly concerned in this research, is the random movement of the exposure-plate when an operator uses a hand-held photographic system.

The resolution limit of a photographic system undergoing various types of image motion has been obtained by Paris theoretically and experimentally. Scott, Hendeberg and Welander and several others have obtained mathematical expressions for the transfer function due to image motion and verified these expressions experimentally. Shack and Myszko have given rigorous mathematical treatment and have derived expressions for the Modulation Transfer Function of image motion. But, in these works the authors have considered only the final integrated effect of image motion over the entire exposure time. Yet another
way of studying the image motion when a hand-held photographic system is used, is to devise an experiment dividing the total exposure time into small, finite, equal time intervals, and measure the image displacement in each of these intervals as the exposure is still in progress. From this, the profile of the image displacement in time can be obtained. This profile is nothing but a point spread function of the image motion due to operator vibration. Using this data and Fourier transformation technique, it is possible to obtain the Modulation Transfer Function of image motion. This method is employed in this research to obtain the image motion Modulation Transfer Function for various times of exposure.
II. HISTORICAL BACKGROUND

The effect of image movement unsharpness on the resolution of a photograph has been dealt with earlier by Selwyn and Romer. Selwyn found that a subject could be moved linearly during exposure, a distance of 0.5 times the resolved distance of the corresponding photographs taken with stationary cameras without the resolution limit being affected noticeably. Romer found that the greatest image movement that could be tolerated was affected by the test contrast.

Formerly, the resolving power or resolution limit was used as a criterion for determining the quality of aerial photographs. Later it was realized that such studies are very subjective and the Fourier transform of line image described the image more completely than the resolution limit. Scott was the first to report mathematical expressions for transfer function of image motion. For a rectangular aperture, when the image motion is linear, the transfer function was shown to be a SINC function.

A detailed analysis of the resolving power being affected by image motion has been done by Dieter P. Paris. Three kinds of image motion were considered - linear, periodic and random.
In the case of linear image motion, the relation between the resolving power \( R \) for a certain image motion \( d \) and the static resolution \( R_o \) was found to be

\[
\frac{1}{R^n} = \frac{1}{R^n_o} + \frac{1}{R^n_m}
\]

where \( 1 < n < 2 \). The familiar reciprocal law and the inverse square law between \( R \) and \( R_o \) did not fit the data of Paris. Paris has theoretically shown that the sine-wave response of the system affected by image motion is given by,

\[
T_s(N) = \frac{1}{(1 + \frac{2\pi N}{a})^2} \cdot T_{mot}(N)
\]

where:

- \( N \) – the spatial frequency in lines per unit length
- \( a \) – a parameter describing the scattering in the emulsion
- \( T_{mot} \) – the sine wave response of image motion.

From the work of Scott and Lohman, for linear image motion,

\[
T_{mot}(N) = \frac{\text{SIN}^2(\pi Nd)}{(\pi Nd)^2}
\]

6
where:

\[ d \text{ - the displacement.} \]

For a stationary emulsion, since \( d = 0 \), \( T_{\text{mot}}(N) = 1 \).

For a badly vibrating system, if no structure is detectable, \( T_{\text{mot}}(N) = 0 \). The corresponding limiting value of \( N \) is called the resolution \( R_{\text{mot}} \). Hence,

\[
T_{\text{mot}}(N) = \frac{\text{SIN}(\pi R_{\text{mot}}d)}{(\pi R_{\text{mot}}d)} = 0.
\]

This means:

\[
(\pi R_{\text{mot}}d) = \pi
\]

i.e. \( R_{\text{mot}} = \frac{1}{d} \)

For purely sinusoidal image motion, Scott and Lohman have shown that the sine wave response is equal to the Bessel function of zero order \( J_0(\pi Nd) \). The first zero of the zero order Bessel function is 2.405. The analysis yields \( R_{\text{mot}} = \frac{0.707}{d} \). The relation between \( R \) and \( R_0 \) in this case was found to be

\[
\frac{1}{R^n} = \frac{1}{R_0^n} + \frac{1}{(R_{\text{mot}}/M)^n}
\]

where:

\( M \) - a parameter which is a measure of shift.
By the method of least squares, Paris has found that
\[ N = 2.17 \quad \text{and} \quad m = 1.33. \]

The third kind of motion considered was random motion. The sine wave response for random motion is
equal to the Gaussian function \( \exp[-2(\pi N d)^2] \). \( d \) represents the standard deviation of the random motion. Paris observed that the random motion on the whole,
degrades the resolution more extensively than linear
motion or vibration.

Factors that impair the quality of aerial photographs include image movement, air turbulence and haze. Hendebberg and Welander have treated the effects of
image movement theoretically and experimentally. For
a constant and unidirectional velocity, it was assumed
that the line-image profile in linear measure \( x \), is
the same as the shutter function in time coordinates.
The variable \( x \) is given by
\[ x = \frac{f}{r} vt \frac{10^3}{3.6} \text{ mm} \]
where:
\[ f - \text{focal length of the lens} \]
\[ v - \text{speed of the aircraft in km/hr} \]
\[ r - \text{altitude in m} \]
\[ t - \text{time in sec.} \]
The shutter function was taken as:

\[ \phi(x) = \begin{cases} 
0 & \text{for } x < a \\
\frac{A}{b-a} (x-a) & \text{for } a \leq x \leq b \\
A & \text{for } b < x < c \\
\frac{A}{c-d} (x-d) & \text{for } c \leq x \leq d \\
0 & \text{for } d < x 
\end{cases} \]

where:

\[ a < b < c < d \]
The function $\phi_v(x)$ forms the spread function to be Fourier-transformed in order to obtain the transfer function $T_v(N)$. Assuming the shutter function to be symmetrical it was shown that

$$T_v(N) = \left( \frac{\text{SIN} \pi N (d-c)}{\pi N (d-c)} \right) \left( \frac{\text{SIN} \pi N (d+c)}{\pi N (d+c)} \right).$$

It may be noted that the function in the second parentheses is identical with that published by Scott which gives too high an approximate value. The first term may be thought of as a correction factor due to Hendeberg and Welander.

In the actual experiment, Hendeberg and Welander employed two aircraft in flight, one photographing the test figures on the other. One of the planes had its wings painted with bright lines on dark background. The aircraft flew in the same direction and no movement unsharpness was found. When they flew in opposite directions a distinct difference was observed in the width of the line. The correlation between the theory and experiment of Hendeberg and Welander is shown in the following graph:
Hendeberg and Welander:

\[ |T_v(N)| \]

- Solid line - theoretical
- Dotted line - experimental

Figure 2
The effect of image motion on photographic transfer function has also been studied by Roland V. Shack. In the first section of the paper, Shack derives a general two-dimensional expression for the transfer function of the photographic exposure image including factors such as Optical Transfer Function, scattering in the emulsion, shutter function and motion of the image. The discrete latent image is described by a continuous statistical image. The statistical image is related to the optical image through exposure image. The exposure image is the time integration of the optical image. When the effective exposure time is large enough compared to the opening and closing times of the shutter, the Optical Transfer Function of the system is independent of time. The effective transfer function of image motion has been shown to be given by

\[ \tilde{\phi}_m(\tilde{u}, \tilde{v}) = t_e^{-1} \int_{-\infty}^{+\infty} S \exp[-2\pi i (\tilde{u} u + \tilde{v} v)] \, dt \]

where:

- \( \tilde{u}, \tilde{v} \) - spatial frequency coordinates
- \( u, v \) - parametric expressions describing the path of the motion
- \( \tilde{\phi}_m \) - Transfer Function of the image system
- \( S \) - shutter function
- \( t_e \) - effective exposure time
It is easier to evaluate the integral at some frequency \((\tilde{u}, \tilde{v})\) by doing a rotational transformation through \(x\), so that the new abscissa \(\tilde{u}'\) is in the direction of \((\tilde{u}, \tilde{v})\). In this case,

\[
\tilde{\Phi}_m(\tilde{u}', \psi) = \frac{t_e^{-1} \int_{-\infty}^{\infty} \frac{S}{u'} \exp(-2\pi i \tilde{u}' u') \, du'}{\int_{-\infty}^{+\infty} \frac{S}{u}, \, du'}
\]

This has the form of a normalized Fourier transform and \(\frac{S}{\tilde{u}}\) represents the equivalent image motion spread function.

If \(u\) and \(v\) are constant with respect to time, the image motion is uniform and linear. The magnitude of velocity is given by

\[
\tilde{v} = (\tilde{u}_\lambda^2 + \tilde{\phi}_\lambda^2)^{1/2}
\]

in a direction

\[\phi_\lambda = \text{arc TAN}(\tilde{v}_\lambda / \tilde{u}_\lambda)\,.
\]

If the shutter function is a trapezoid with a rise time \(t_r\), the Transfer function of image motion is given by

\[
\tilde{\Phi}_m = \text{SINC}(\pi \tilde{u}' \tilde{u}_\lambda t_e) \text{SINC}(\pi \tilde{u}' \tilde{u}_\lambda t_r).
\]

Shack has also obtained an expression for \(\tilde{\Phi}_m\) in the case of simple harmonic motion.
In a similar approach, R. Myszko\textsuperscript{9} considers the effect of film platen vibration during exposure from the standpoint of the degradation of the spectral density of the exposure random process.

The image illuminance along some line lying in the image plane is a random process $u'(x)$ which is given by,

$$u'(x) = \int_{-\infty}^{\infty} h(\alpha) u(x-\alpha) \, d\alpha$$

where:

$h(\alpha) =$ spread function

$u(\alpha) =$ object illuminance process

Because of vibration, the incident illuminance process is spatially displaced by an amount $v(t)$, a random process. Therefore,

$$u'(x) = \int_{-\infty}^{\infty} h(\alpha) u(x+v[t]-\alpha) \, d\alpha$$

The exposure at each point in the image plane is given by,

$$i(x,T) = \int_{0}^{T} \int_{-\infty}^{\infty} h(\alpha) u(x+v[t]-\alpha) \, d\alpha \, dt$$

where: $i(x,T)$ is the exposure random process.
The product of the exposure sample functions evaluated at $x_1$ and $x_2$ yields,

$$i(x_1,T) i(x_2,T) = \int_0^T \int_0^T \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\alpha) h(\beta) \{u(x_1+v(t_1)) - a\} u(x_2+v(t_2) - \beta) \, da \, db \, dt_1 \, dt_2.$$ 

Let $u(x)$ be given as the Fourier inverse transform of $g(\eta)$, so that,

$$u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\eta) \exp(jn\eta) \, d\eta.$$ 

Also let $I_1$, $I_2$, $U_1$, $U_2$ and $V_1$, $V_2$ be the random variables defined on the sample spaces of possible values which can be assumed, by the exposure, object illuminance, and vibration processes at points $x_1$ and $x_2$ and time instants $t_1$ and $t_2$.

The expectation $E(I_1 I_2)$ is then shown to be given by,

$$E(I_1 I_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^T \int_0^T h(\alpha) \exp(-jn_1 \alpha) \, da \, \int_{-\infty}^{\infty} h(\beta) \exp(-jn_2 \beta) \, db \, \int_{-\infty}^{\infty} E(U_1 U_2) \exp(-jn_1 \xi_1 - jn_2 \xi_2) \, d\xi_1 \, d\xi_2 \, E(\exp[jn_1 V_1 + jn_2 V_2]) \exp(jn_1 x_1 + jn_2 x_2) \, d\eta_1 \, d\eta_2 \, dt_1 \, dt_2.$$ 

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The illuminance and vibration random processes are assumed to be stationary so that

\[ E(U_1U_2) = R(\zeta_2 - \zeta_1) \]

where: \( R(\ ) \) is the auto-correlation function of the illuminance random process.

With \( \zeta_2 - \zeta_1 = \zeta \), the inner double integral

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(U_1U_2) \exp(-j\eta_1\zeta_1 - j\eta_2\zeta_2) \, d\zeta_1 \, d\zeta_2
\]

reduces to

\[ S_u(\eta_1) 2\pi \delta(\eta_1 + \eta_2) \]

where:

\[ S_u(\eta_1) = \int_{-\infty}^{\infty} R(\zeta) \exp(-j\eta_1\zeta) \, d\zeta \]

which is nothing but the power spectral density of the object illuminance process.

Substituting the above expressions and integrating over \( \eta_2 \), it can be shown that,

\[
E(I_1I_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |H(j\eta)|^2 S_u(\eta) M(j\eta, -j\eta; \tau_2 - \tau_1) \exp(j\zeta\eta) \, d\zeta \, d\tau_2 \, d\eta
\]
Assuming the vibration is wide sense stationary,

\[ E(I_1 I_2) = R_\text{I}(\zeta) \]

where: \( R_\text{I}(\cdot) \) is the auto-correlation function of the resultant exposure random process.

Letting

\[
G(\eta, T) = \iint_{0}^{T} M(j\eta, -j\eta; t_2 - t_1) \, dt_1 \, dt_2,
\]

\[
R_\text{I}(\zeta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\eta)|^2 S_u(\eta) \, G(\eta, T) \, \exp(j\zeta\eta) \, d\eta
\]

where: \( |H(j\eta)|^2 \) is the square of the magnitude of the optical system transfer function, \( S_u(\eta) \) is the power spectral density of the object illuminance process, and \( G(\eta, T) \) is the equivalent filter function due to vibration, a function of spatial frequency and exposure time.

The above equations are the key relations derived in the paper by Myzko. The expression gives the correlation function of the exposure process when spatially perturbed or displaced by a random process.

For a Gaussian, or normally distributed vibration random process having a zero mean,

\[
M(j\eta, -j\eta; t_2 - t_1) = \exp(-[\sigma^2 - R_v(t_2 - t_1)]) \eta^2
\]
where:

\[ \sigma^2 \] is the vibration variance

\[ R_v(t_2-t_1) \] is the vibration correlation function

\[ \eta \] is the spatial frequency.

As a model it was assumed that

\[ R_v(\tau) = \sigma^2 \exp(-a|\tau|) \]

where:

\[ \sigma^2 \] is the vibration variance

\[ a \] is some constant

The exponential form of the correlation function might be observed as the result of platen vibration obtained when a camera is housed within a mount having a non-resonant, "low pass", mechanical filter passband characteristic. \( \sigma^2 \) is the variance of the camera platen displacement which may be measured by a suitable instrumentation arrangement.

Using this model, Myszko shows that,

\[ \frac{G(\eta, T)}{T^2} = \exp \left(-\sigma^2 \eta^2 \right) \left\{ 1 + \sum_{k=1}^{\infty} \frac{(\sigma^2 \eta^2)^k}{k!} \frac{(kaT + \exp[-kaT-1])}{(kaT)^2} \right\} . \]
This expression gives the normalized filter function due to vibration having an auto-correlation function of the form \( \sigma^2 \exp(-a|\tau|) \). Some properties of the filter can be obtained by taking the limits as follows:

\[
\lim_{\sigma \eta \to 0} \frac{G[\eta,T]}{T^2} = 1 \quad \text{i.e. "low" frequency components of the power spectral density are not effectively attenuated}
\]

\[
\lim_{\sigma \eta \to \infty} \frac{G[\eta,T]}{T^2} = 0 \quad \text{i.e. "high" frequency components are highly attenuated}
\]

\[
\lim_{aT \to 0} \frac{G[\eta,T]}{T} = 1 \quad \text{i.e. if exposure is small with respect to correlation time, input power spectral density is unaffected}
\]

\[
\lim_{aT \to \infty} \frac{G[\eta,T]}{T^2} = \exp(-\sigma^2 \eta^2) \quad \text{i.e. in the limit, as the exposure becomes large, the filter function becomes Gaussian}
\]

The results of the final equation are schematically shown in the following graph:
Myszko:

\[ \sigma(\eta, T)/\eta^2 \]

\( \sigma T = 0 \)

\( 0.01 \)

\( 0.08 \)

\( 0.32 \)

\( -.12 \)

\( 10.00 \)

\( 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \)

\( \sigma \eta \)

**Figure 3**
In summary, the researchers in the field of image degradation due to motion have shown that the effect of vibration produces a spatial filtering which band limits the process just like the insertion of a lens in an optical system produces a spatial frequency cut-off. The aim of the present investigation is to show, as Shack and Myszko have shown, that the image motion imposes an additional resolution limit on the photographic system. The method of approach, however, will be different. The exact displacement path of the camera will be studied while an operator photographs an object (a circuit board containing an array of Light Emitting Diodes). The earlier researchers, on the other hand, have considered only the time-averaged effect of image displacement. The data will be analyzed by the Fourier method to obtain the Modulation Transfer Function characteristics of image motion.
III. THEORY

The image formed by a photographic system is often degraded by factors such as the lens performance, scattering in the emulsion of the film and shutter effect. Additional contribution to the degradation comes from the image motion, especially when the exposure time is large.

In general, the Fourier transform theory is extensively used in the analysis of images formed by an imaging system. The main reason for using the Fourier method in optical imaging systems is that concepts like convolution in the space domain, which are difficult to perform, can be replaced by a simple numerical product in the frequency domain.

As mentioned previously, the objective of this research is to obtain the Modulation Transfer Function of image motion when an operator uses a hand-held photographic system. Since the Fourier transform theory is an essential tool for this study, the basics of the Fourier transform, its digital computer version and the practical problems in using the computer version will be first presented in this section. This will be followed by a description of the method that will be used to obtain the Modulation Transfer Function of image motion as a function of exposure time.
Basics of the Fourier transform:

The essence of the Fourier transform of a waveform is to decompose or separate the waveform into a sum of sinusoids of different frequencies. If these sinusoids sum to the original waveform then we have determined the Fourier transform of the waveform. The pictorial representation of the Fourier transform is a diagram which displays the amplitude and frequency of each of the determined sinusoids. Mathematically,

\[ H(f) = \int_{-\infty}^{+\infty} h(t) \exp(-2\pi ift) \, dt \]  

(1)

where:

- \( h(t) \) - waveform to be decomposed into a sum of sinusoids
- \( H(f) \) - Fourier transform of \( h(t) \)
- \( i = \sqrt{-1} \)

The Fourier transform is a frequency domain representation of a function. The frequency domain transform contains exactly the same information as that of the original function; they differ only in the manner of presentation of the information.
Discrete Fourier transform:

Because of the numerous applications, the Fourier transform analysis has been extended to the digital computer. The numerical integration of equation (1) can be written as

\[
H(f_k) = \sum_{j=0}^{N-1} h(t_j) \exp(-2\pi i f_k t_j) (t_{j+1} - t_j).
\]  \hspace{1cm} (2)

\[
k = 0, 1 \ldots (N-1)
\]

For those problems which do not yield to a closed form Fourier transform solution, the discrete Fourier transform (2) offers a powerful method of attack. However, if there are \( N \) data points of the function \( h(t_j) \) and if we desire to determine the amplitude of \( N \) separate sinusoids, then computation time is proportional to \( N^2 \), the number of multiplications. Even with high speed computers, computation of the discrete Fourier transform requires excessive machine time for large \( N \).

In 1965 Cooley and Tukey published their mathematical algorithm which became known as the fast Fourier transform. The fast Fourier transform (FFT) is a computational algorithm which reduces the computing time of equation (2) to a time proportional to \( N \log_2 N \). The increase in computing speed is very impressive (fig. 4).
FFT vs direct calculation:

![Graph showing comparison between FFT and direct calculation]
Inverse Fourier transform:

The inverse Fourier transform is defined as

\[ h(t) = \int_{-\infty}^{+\infty} H(f) \exp(2\pi i ft) \, dt \]  \hspace{1cm} (3)

If the functions \( h(t) \) and \( H(f) \) are related by equations (1) and (3), the two functions are termed a Fourier transform pair. A list of familiar Fourier transform pairs is given in Appendix I.

Convolution integral:

Convolution of two functions \( x(t) \) and \( h(t) \) is given by

\[ y(\tau) = \int_{-\infty}^{+\infty} x(t) \, h(\tau-t) \, dt = x(t) * h(t) \]  \hspace{1cm} (4)

The convolution of two functions has ample physical implications. For example, the illuminance in the image plane is the convolution of the object illuminance and the spread function response of the optical system.

Convolution theorem:

One of the most important and powerful tools in Fourier analysis is the convolution theorem, which allows the freedom of replacing the convolution in
time domain by a simple multiplication in the frequency domain. The convolution theorem states that if 
\[ y = x * h, \] 
then 
\[ Y(f) = X(f) H(f) \]  
(5)

where:
\[ X(f) \] - Fourier transform of \( x(t) \)
\[ H(f) \] - Fourier transform of \( h(t) \)
\[ Y(f) \] - Fourier transform of \( y(t) \).

Parseval's theorem:

The Parseval's theorem states that the energy in a waveform \( h(t) \) computed in the time domain must equal the energy of \( H(f) \) as computed in the frequency domain. Mathematically,
\[ \int_{-\infty}^{\infty} h^2(t) \, dt = \int_{-\infty}^{\infty} |H(f)|^2 \, df \]  
(6)

For discrete functions, the Parseval's theorem may be stated as
\[ \sum_{k=0}^{N-1} h^2(k) = \frac{1}{N} \sum_{i=0}^{N-1} |H(i)|^2 \]  
(7)

Equation (7) offers the easiest way to verify the calculations while finding the discrete Fourier transform using a digital computer. Most of the computers
now have the fast Fourier transform algorithm stored in the library memories. The computer output would have to be scaled down in most cases and the equation (7) offers the effective method to find the scaling factor.

Sampling theorem:

When the numerical calculation of the Fourier transform of a function is necessary, a decision has to be made on how to sample the function. If the sampling interval is too large, the Fourier transform of the sampled function is distorted, an effect known as "aliasing".

According to the sampling theorem, if the transform of a function $h(t)$ is zero for all frequencies greater than a certain frequency $f_c$ and if the sample spacing $T$ is so chosen that,

$$T = \frac{1}{2f_c},$$

then the continuous function $h(t)$ can be uniquely determined from a knowledge of its sampled values. Frequency $\frac{1}{T} = 2f_c$ is known as the Nyquist sampling rate. Aliasing will not occur if the function is sampled according to Nyquist sampling rate.
Leakage:

If a periodic, band-limited function is sampled and truncated to consist of other than an integral multiple of the period, the resulting discrete and continuous Fourier transform will differ considerably. The time domain truncation is equivalent to multiplying by a rectangular function. This introduces an unwanted convolution with a SINC function in the frequency domain. Hence additional frequency components known as sidelobes will be present in the transform. This effect termed "leakage" can be reduced by proper time domain truncation functions known as "windows".

Windows are weighting functions applied to data to reduce the spectral leakage associated with finite observation intervals. The effect of truncation, by anything other than a multiple of the period, is to create a periodic function with sharp discontinuities. The windowed data are brought to zero at the boundaries so that the periodic extension of the data is continuous in many orders of derivative. Several windows are currently being used such as Hanning window, Hamming window, Chebyshev window, etc., each serving a different purpose.
Hanning window:

The Hanning window is described by the function,

\[ w(t) = 0.5 - 0.5 \cos\left( \frac{2\pi t}{T_c} \right) \quad 0 \leq t \leq T_c \]

where:

- \( T_c \) - truncation interval.

The magnitude of the Fourier transform of the Hanning function is given by,

\[ |W(f)| = 0.5 Q(f) + 0.25 (Q[f + \frac{1}{T_c}] + Q[f - \frac{1}{T_c}]) \]

where:

\[ Q(f) = \frac{\sin(\pi T_c f)}{\pi f} \]

- Figure 5
The Hanning window and its transform are shown in the figures 5 and 6. It may be noted that the frequency function has very small side-lobes.

**Spread functions:**

When an incoherent source of light is imaged by an optical system, each point on the object plane emits light which is processed by the system and the light emerges to form a spot on the image plane. Because of diffraction (and the possible presence of aberrations), this light is smeared out into some sort of blur spot over a finite area on the image plane rather than focused to a point. The spread of the radiant flux is usually described by a mathematical
function known as "point spread function". The one-dimensional counterpart of the point spread function is the "line spread function", which corresponds to the flux-density distribution across the image of a geometrical line source having infinitesimal width. In fact, the image illuminance at a point in the image plane is nothing but the convolution of the object illuminance and the point spread function of the system.

Transfer functions:

A highly useful parameter in evaluating the performance of a system is the contrast or modulation, defined by,

\[
\text{Modulation} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

where:

I is the image illuminance.

As mentioned previously, in the case of linear systems, the image is harmonic if the object is harmonic, regardless of the form of the spread function. In the science of image evaluation, an object may be envisioned as being composed of Fourier components.
The Optical Transfer Function defines the transformation of individual harmonic components of the object into harmonic constituents of the image. The Modulation Transfer Function is the modulus of the Optical Transfer Function. It gives the ratio of image to object modulation for sinusoids of varying spatial frequency.

Image motion transfer function:

The main theory and the equations that will be used to obtain the image motion transfer function will be presented in this section.

Let it be assumed that the displacement path of a hand-held photographic camera used by an observer is recorded in the image plane. The method of obtaining this data and instrumentation required will be described later.

The transverse and longitudinal vibrations of the observer can be considered independently for the analysis. Let \( h_x(t) \) and \( h_y(t) \) be the rectangular components of the image displacement function in the two mutually perpendicular directions. A sample of these two functions is shown in the figure (7):
Image displacement function

\[ h_x(t) \]
\[ h_y(t) \]

Time \((t)\)

Figure 7
It may be noted that if there was no image displacement during the exposure, the functions $h_x(t)$ and $h_y(t)$ would plot in the above graph as a straight line coinciding with x-axis. But, in general when the exposure time is large, the random motion of the observer is significant and the displacement function spreads the point-image leading to degradation of image quality.

Let $H_x(f)$ and $H_y(f)$ be the Fourier transforms of $h_x(t)$ and $h_y(t)$ and let $|H_x(f)|^2$ and $|H_y(f)|^2$ be the respective power spectrums. Since we will be interested in studying how most of the observers vibrate the camera, the average power spectrum of all the experimenters will be considered in the analysis. Let $P_x(f)$ and $P_y(f)$ be the average power spectrum of all the observers. Rather than collecting experimental data using a great many observers, a computer simulation process can be used to generate the required number of operators. The computer simulation uses a uni-variant random noise function. To generate the operators the image displacement function has to be convolved with different uni-variant noise functions. Alternatively, the Fourier transform of the displacement function can be multiplied by the transform of the noise function in the frequency domain. Since the transform of the uni-variant noise function is also
uni-variant, the total power contained in the transform of the displacement function is unaltered by this procedure. The inverse transform of the product yields the displacement function of one of the simulated observers. More observers can be simulated by using different uni-variant random noise functions.

Let \( T \) be the time-interval at which the displacement functions are sampled. Let \( h(o), h(T), h(2T), \ldots \) be the magnitude of displacement at these intervals of time. Considering the first two samples \( h(o) \) and \( h(T) \), the exposure time involved is \( 2T \). The Modulation Transfer Function for this exposure time may be written as

\[
M_1(f) = 0.5 \left\{ |e^{i2\pi f h(o)} + e^{i2\pi f h(T)} | \right\}
\]

Considering the first four samples \( h(o), h(T), h(2T) \) and \( h(3T) \), the exposure time involved is \( 4T \). The Modulation Transfer Function for this exposure is

\[
M_2(f) = 0.25 \left\{ |e^{i2\pi f h(o)} + e^{i2\pi f h(T)} + e^{i2\pi f h(2T)} + e^{i2\pi f h(3T)} | \right\}
\]

The MTF for other exposure times may be similarly obtained. This procedure can be employed for the horizontal as well as vertical components of image displacement.
IV. EXPERIMENTAL

When an observer uses a hand-held photographic camera there are mainly two sources of vibration causing the image motion. When the shutter in the camera is triggered, various inherent mechanical motions produce vibration. Some of these motions include shutter movement, mirror relocation in a SLR device and other such operational mechanics. But by careful engineering the present day cameras have minimized this vibration. The other primary source of vibration is the inability of the observer to hold the camera still when the exposure is in progress. This source of vibration which may be called "image motion due to operator vibration" is insignificant when the exposure time is small. But for exposure times greater than 1/50 second the image motion due to operator vibration causes extensive degradation of image quality.

Initially the image displacement during exposure vs. time will be plotted. Next, the Fourier technique will be used to obtain the Modulation Transfer function of image motion.

The apparatus used in this project was made available by the department of the Photographic Science and Instrumentation. It was designed and constructed by R. Hube and J.F. Carson.
Description of the equipment:

The equipment consists of an array of Light Emitting Diodes (LEDs) mounted on a board. The board is of dimensions 2 1/4' x 2 1/4' mounted vertically on a stand. The LEDs form a perfect matrix of 8 x 8. The LEDs are less than 5mm in diameter and are separated by 8 cms. The LEDs are Monsanto's MV5354 rated at 10 mcd. A synchronizing cord connects the circuit board with the camera.

When the operator photographs the array of LEDs from a distance, the circuit is activated as the camera shutter is released. The circuit has been so designed that now the LEDs flash in a sequence twice. The first flashing sequence is so fast that even a badly vibrating system will image all the LEDs in a perfect matrix. The second flashing sequence is rather slow during which the lights flash with a small but definite time intervals between them. This slower sequence suffers from camera motion of the operator giving rise to image displacement. In the processed film a set of two dots will be obtained corresponding to the position of each LEDs (Figure 8).
Displacements of LED images

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*Figure 8*
The distance between these dots can be measured, which gives the displacement of the image at various instants of time during the exposure.

Procedure:

The LED circuit board was set up in a room with the distance between the object plane (LED array) and the image plane (film plane of the camera) standardized for all observers. The distance was 15' ± 0.2'. A Nikon F-2 camera with a 135mm lens was used. In order to provide maximum contrast between the LEDs and the background, the room was kept sufficiently dark. The shutter speed in the camera was conveniently set at B (open aperture), since the room was dark and there was no possibility of any stray exposure after the scans were completed. The film used was Tri-X. The aperture was set at f/8 which was determined by preliminary experiment. The exposed film was processed in Versamat and the negatives were mounted on precession glass slide mounts.

The slides were projected on a wall with a magnification of 40 x (±0.1). A grid was set up coinciding with the correct positions of the LEDs. The horizontal and vertical components of displacement were directly measured. Appropriate signs were introduced for these displacements (positive or negative), depending on the
displacement being in the positive or negative direction with respect to the origin. These displacements divided by the magnification yielded the exact displacements in the image plane.

To measure the time interval between the LED flashings, the LED circuit board was photographed at various shutter speed settings: 1 sec, 1/2 sec, 1/4 sec, 1/8 sec and 1/15 sec. These were repeated several times. From this photograph, the average number of dots scanned for each shutter speed setting was determined. The shutter speed setting divided by the number of dots scanned gives the time interval between each LED flash. This time interval was found to be \((0.019 \pm 0.001)\) sec. For each observer, the X and Y components of image displacement vs. time was plotted. The Zeta plotting system and the Sigma-9 computer facility of the Rochester Institute of Technology were used for this analysis.
V. ANALYSIS

The displacements of the positions of LEDs on the film obtained using different observers were first stored in the computer memory.

To find the Fourier transform of image displacement functions several subroutines were available such as in IBM scientific package, IMSL library etc. Several factors had to be considered before using the software. The first and foremost problem was, that in the transformed space, the amplitude at zero frequency will be equal to the area under the curve in the time domain. Hence if the image displacement graph has a significant area under it, an excessively large peak will be obtained in the transform at zero frequency which might suppress the information available at high frequencies. To prevent this, the mean displacement for each observer was determined and was subtracted from the displacement data of the respective observer.

The second problem was due to the truncation in time domain. While taking the discrete Fourier transform, the functions are modified so that they become periodic functions. Hence if the truncation in time domain consist of other than the integral multiple of the waveform of image motion, sidelobes will be present in the transform. This has been explained in the
section on "leakage." The data was brought to zero smoothly at the boundaries by weighting with the Hanning function. But the weighting with the Hanning function unsymmetrically distorts the data. This might result in the data having a non-zero mean again. So, the mean was once again found out and was subtracted from the data.

This procedure was accomplished using the program "Hanning" shown in Appendix II. The results were plotted using the Zeta plotting system of the Rochester Institute of Technology. The image displacement graph was plotted for all the twenty observers. The graphs showed a similarity in their profile. A representative of the plots are shown in the following pages (Fig. 9-18):
IMAGE MOTION
DISPLACEMENT VS TIME
Observer: 1
x component

WEIGHTED BY HANNING FUNCTION
(MEAN-SUBTRACTED)

Figure 9
IMAGE MOTION

DISPLACEMENT VS TIME

WEIGHTED BY HANNING FUNCTION
(MEAN-SUBTRACTED)

Observer: 2
x component

Figure 10
IMAGE MOTION
DISPLACEMENT VS TIME

WEIGHTED BY HANNING FUNCTION (MEAN-SUBTRACTED)

observer: 3
x component

Figure 11
IMAGE MOTION
DISPLACEMENT VS TIME Observer: 5
x Component

WEIGHTED BY HANNING FUNCTION
(MEAN-SUBTRACTED)

TIME (X 0.019SEC)

Figure 12
IMAGE MOTION

DISPLACEMENT VS TIME x component

WEIGHTED BY HANNING FUNCTION (MEAN-SUBTRACTED)

Figure 13
IMAGE MOTION
DISPLACEMENT VS TIME
HEADED BY HANNING FUNCTION
(MEAN-SUBTRACTIONED)

Observer: 3
y component

Figure 14
IMAGE MOTION

DISPLACEMENT VS TIME

Observer: T

y component

WEIGHTED BY HANNING FUNCTION
(MEAN-SUBTRACTED)

Figure 15
IMAGE MOTION

DISPLACEMENT VS TIME

WEIGHTED BY HANNING FUNCTION (MEAN-SUBTRACTED)

Observer: B

y component

Figure 16
IMAGE MOTION

DISPLACEMENT VS TIME

WEIGHTED BY HANNING FUNCTION
(MEAN-SUBTRACTED)

Observer: 10

y component

Figure 17
IMAGE MOTION

DISPLACEMENT VS TIME

y component

WEIGHTED BY HANNING FUNCTION
(MEAN-SUBTRACTED)

Observer: 12

Figure 18
The subroutine "FOUR 1" (Appendix III) was used to find the discrete Fourier transform of the image displacement functions. The subroutine was called using the program "FFT" listed in Appendix IV. The computer output was divided by the total number of data points (scaling factor) to obtain the amplitude of the transform. The power spectrum was obtained by storing the square of the amplitudes of the transform in the computer memory. The Zeta plotting system was used to plot this data vs. frequency.

The sampling interval in time domain was

$$\Delta t = 0.019 \text{ sec.}$$

The number of data points used was

$$N = 64.$$ 

From the sampling theorem, sampling interval in frequency domain is

$$\Delta n = \frac{1}{64 \times 0.019}$$

$$= 0.83 \text{ cycle/sec.}$$

The power spectrum of all the observers were plotted which showed a remarkable similarity in their profile. But, at a given frequency the amplitudes of the different
observers were different. The average power of all the observers was found out at every frequency and the power spectrum of an 'average observer' was obtained.

The power spectrums of the representative image displacement functions presented in the previous pages are shown in figures 19-28. The power spectrum of the average observer is plotted in figures 29 and 30. The same result when plotted using logarithmic scale looks as shown in figures 31 and 32.
IMAGE MOTION
POWER SPECTRUM

Observer: 1
X-Component

Figure 19
IMAGE MOTION
POWER SPECTRUM

Observer: 2
X-Component

Figure 20
IMAGE MOTION

POWER SPECTRUM

X-Component

Observer: 3

Figure 21
IMAGE MOTION
POWER SPECTRUM

Observer: 5
X-Component

Figure 22
IMAGE MOTION
POWER SPECTRUM

Observer: 7
X-Component

Figure 23
IMAGE MOTION
POWER SPECTRUM

Observer: 3
Y-Component

Figure 24
<table>
<thead>
<tr>
<th>FREQUENCY, V (CYCLES/SEC) x 0.83</th>
</tr>
</thead>
<tbody>
<tr>
<td>POWER, P(V)</td>
</tr>
<tr>
<td>0.00</td>
</tr>
<tr>
<td>0.04</td>
</tr>
<tr>
<td>0.08</td>
</tr>
<tr>
<td>0.12</td>
</tr>
<tr>
<td>0.20</td>
</tr>
<tr>
<td>0.24</td>
</tr>
<tr>
<td>0.28</td>
</tr>
</tbody>
</table>

**Figure 25**

**Observer:** 7

**Y-Component**

**IMAGE MOTION POWER SPECTRUM**
IMAGE MOTION
POWER SPECTRUM

Observer: B
Y-Component

Figure 26
Figure 27
IMAGE MOTION
POWER SPECTRUM

Figure 28
IMAGE MOTION
POWER SPECTRUM

(Frequency - Component - Average of all the Observers)

Figure 29
IMAGE MOTION POWER SPECTRUM
(Y-Component - Average of all the Observers)

Figure 30
IMAGE MOTION POWER SPECTRUM

(Z-Component - Average of all the Observers)

Figure 31
IMAGE MOTION
POWER SPECTRUM

(Y-Component - Average of all the Observers)

Figure 32
The Modulation Transfer function of image motion was obtained using the program MTF listed in Appendix V.

As explained previously one-hundred random univariate noise functions were simulated and stored in the computer memory. For this purpose the subroutine "RANDU" from the IBM scientific package was made use of. The discrete Fourier transform of these functions were determined using the subroutine "FOUR 1." The variance of the transform was also unity. The product of the square root of the average power spectrum and the transform of the noise spectrum was obtained. The subroutine "FOUR 1" was once again used to find the inverse Fourier transform of the product.

Considering the first two samples \( h(o) \) and \( h(T) \), since the exposure time involved is \( 2T \), the Modulation Transfer function at this exposure time is given by

\[
M(f) = 0.5 \left| e^{i\pi f} h(o) + e^{i2\pi f} h(T) \right|
\]

Since the sampling interval was 0.019 sec., the above equation gives the Modulation Transfer function for exposure time 0.038 sec.
The equations used for finding the Modulation Transfer function at various exposure times are listed below:

<table>
<thead>
<tr>
<th>Exposure Time (sec.)</th>
<th>Modulation Transfer function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.038</td>
<td>$M_1(f) = 0.5 {</td>
</tr>
<tr>
<td>0.076</td>
<td>$M_2(f) = 0.25 {</td>
</tr>
<tr>
<td>0.152</td>
<td>$M_3(f) = 0.125 {</td>
</tr>
<tr>
<td>0.304</td>
<td>$M_4(f) = 0.0625 {</td>
</tr>
<tr>
<td>0.608</td>
<td>$M_5(f) = 0.03125 {</td>
</tr>
</tbody>
</table>

Figures 33-42 show the Modulation Transfer Function for various exposure times. The vertical bars represent the standard deviation in the measurement.

The family of curves presented in figures 43 and 44 summarize the final result of the research.
MTF of image motion - X component

Exposure time = 0.038 sec
MTF of image motion: +X component

Exposure time = 0.076 sec

Figure 34
MTF of image motion - X component

Exposure time = 0.152 sec

Figure 35
MTF of image motion - X component

Exposure time = 0.304 sec

Figure 36
MTF of image motion - X component

Exposure time = 0.608 sec

Figure 37
MTF of image motion - Y component
Exposure time = 0.038 sec

Figure 38
MTF of image motion \( \times \) Y component

Exposure time = 0.076 sec

Figure 39
MTF of image motion - Y component

Exposure time = 0.152 sec

Figure 40
MTF of image motion - Y component

Exposure time = 0.304 sec

Figure 41
MTF of image motion - Y component

Exposure time = 0.608 sec

Figure 42
Modulation Transfer Function of image motion

X component

Figure 43
Modulation Transfer Function of image motion

$y$ component

Figure 44
VI. CONCLUSION

Using the data obtained from real observers, the power spectrum of image motion of the observers has been determined. The x-component and y-component of the motion have been independently considered. The power spectrums showed a similarity in profile. Using the average power spectrum of the real observers, more observers have been generated by computer simulation. The Modulation Transfer function of an average observer has been determined for the exposure times of 0.038 sec, 0.076 sec, 0.152 sec, 0.304 sec and 0.608 sec.

At low exposure times, the MTF profiles of x and y components are nearly identical (figures 43 and 44). As exposure time increases, the MTF of the x-component cuts high frequencies off more sharply than the y-component. This may be the result of the spontaneous reaction of the observers to off-set the downward displacement of the camera while the horizontal displacement is uncompensated.

In general, the effect of operator random vibration has been found to be equivalent to the insertion of a spatial filter. The attenuation of the filter has been observed to be a function of the exposure time and frequency.
VII. RECOMMENDATION FOR FUTURE WORK

The method of measuring the displacement needs refinement. In the present method, the slides were projected on the wall, the positions of the LEDs were noted on a piece of paper and long and monotonous hours were spent in using the drafting equipment for measurement. In future, the slides may be printed on a piece of paper of about 10" x 12" dimensions and the x-y plotter as available at Eastman Kodak Company can be used. This will reduce the burden of measurement.

In the present work twenty observers were used to find the power spectrum of an average observer. The power contained at various frequencies showed excessively large standard deviations. More observers may be used in future to find the average power spectrum.

The observers in this work used a Nikon F-2 camera with a Tamron 135 mm lens. Studying the image displacement profile when an operator uses different kinds of equipment may be a worthwhile future project.

The displacement function can be sampled at shorter intervals. This can be accomplished by increasing the flashing frequency of the LEDs during the second sequence of scan.
VIII. REFERENCES


7. Schneider, J. and Bodenstein, B., Mod. Phot. 90 (May 1971).


## Appendix I

### Fourier Transform Pairs

<table>
<thead>
<tr>
<th>Time Domain</th>
<th>Frequency Domain</th>
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<tbody>
<tr>
<td>$h(t) = A$</td>
<td>$H(f) = \frac{2AT}{\pi T} \sin(2\pi f) \left( \frac{1}{2} \right)$</td>
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<table>
<thead>
<tr>
<th>$h(t) = 2A_0 \delta(f-f_0)$</th>
<th>$H(f) = A$</th>
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| $h(t) = K$                  | $H(f) = KH(f)$ |

| $h(t) = k\delta(t)$        | $H(f) = K$   |

| $h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ | $H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$ |

| $h(t) = A \cos(2\pi f_0 t)$ | $H(f) = \frac{A}{2} \delta(f - f_0) + \frac{A}{2} \delta(f + f_0)$ |

| $h(t) = A \sin(2\pi f_0 t)$ | $H(f) = -\frac{A}{2} \delta(f - f_0) + \frac{A}{2} \delta(f + f_0)$ |
Appendix I (Contd.)

<table>
<thead>
<tr>
<th>Time domain</th>
<th>Frequency domain</th>
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<tbody>
<tr>
<td>$h(t) = -\frac{A^2}{2Ts} t + A^2$</td>
<td>$H(f) = A^2 \sin^2(2\pi T_0 f)$</td>
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<td>t</td>
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<td>t</td>
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$\phi(t) = A \cos(2\pi f_0 t)$

$H(f) = -\frac{A^2}{2Ts} (f + f_0) + \frac{1}{2} A \cos\left(\frac{\pi f}{T_0}\right)$

$Q(f) = \sin(2\pi T_0 f)$

$\phi(t) = \sin(2\pi f_0 t)$

$H(f) = -\frac{A^2}{2Ts} (f + f_0) + \frac{1}{2} A \cos\left(\frac{\pi f}{T_0}\right)$

$Q(f) = \sin(2\pi T_0 f)$

$\phi(t) = \frac{\sin(2\pi f_0 t)}{2\pi}$

$H(f) = -\frac{A^2}{2Ts} (f + f_0) + \frac{1}{2} A \cos\left(\frac{\pi f}{T_0}\right)$

$Q(f) = \sin(2\pi T_0 f)$

$\phi(t) = \exp(-\frac{t^2}{2\sigma^2})$

$H(f) = \frac{\sigma^2}{\pi^2 + 4\pi^2 f^2}$

$\phi(t) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^{1/2} \exp(-\frac{t^2}{\sigma^2})$

$H(f) = \exp\left(-\frac{\sigma^2 f^2}{2}\right)$
THIS PROGRAM TAKES THE INPUT DATA, FINDS THE MEAN, SUBTRACTS THE MEAN FROM THE DATA, WEIGHTS THE DATA WITH HANNING FUNCTION AND SUBTRACTS THE MEAN AGAIN.

WRITTEN BY VENKAT NAGASWAMI ON MARCH 30, 1979

DIMENSION A(64), B(64), C(64), D(64), E(64)
READ (105, 100) (A(I), I=1, 64)

100 FORMAT (BF10.4)
WRITE (108, 110)
110 FORMAT (1X, // 'DATA' //)
WRITE (108, 120) (A(I), I=1, 64)

120 FORMAT (1X, BF10.4)
SUM = 0.0
DO 10 J = 1, 64, 2
  M = J
  N = J + 1
  SUM = SUM + A(M) + A(N)
  AVG = SUM / 64.0
10  SUM = SUM + A(M) + A(N)

130 FORMAT (1X, // 'SUM=' F10.4, 'AVG=' F10.4)
DO 20 K = 1, 64
  B(K) = A(K) - AVG
20  B(K) = A(K) - AVG
WRITE (108, 140)
140 FORMAT (1X, // 'DATA (MEAN-SUBTRACTED)' //)
WRITE (108, 150) (B(K), K=1, 64)

150 FORMAT (1X, BF10.4)
C(1) = 0.0
DO 30 J = 1, 63
  C(J+1) = 0.5 - 0.5 * COS(FLOAT(6.283185 * J / 64., )
30  C(J+1) = 0.5 - 0.5 * COS(FLOAT(6.283185 * J / 64., )
DO 40 J = 1, 64
  D(J) = B(J) * C(J)
40  D(J) = B(J) * C(J)
WRITE (108, 160)
160 FORMAT (1X, // 'MEAN-SUBTRACTED AND WEIGHTED DATA' //)
WRITE (108, 161) (D(J), J=1, 64)

161 FORMAT (1X, BF10.4)
SUM2 = 0.0
DO 50 J = 1, 64, 2
  M = J
  N = J + 1
50  SUM2 = SUM2 + D(M) + D(N)

170 FORMAT (1X, // 'SUM2=' F10.4, 'AVG2=' F10.4)
DO 60 K = 1, 64
  E(K) = D(K) - AVG
60  E(K) = D(K) - AVG
WRITE (108, 180)
180 FORMAT (1X, // 'WEIGHTED DATA (MEAN-SUBTRACTED)' //)
WRITE (108, 190) (E(K), K=1, 64)

190 FORMAT (1X, BF10.4)
WRITE (109, 200) (E(K), K=1, 64)
200 FORMAT (1X, F10.4)
STOP
END
SUBROUTINE FOUR1 (DATA, N, ISIGN)

C THE COOLEY-TUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN.
C NOTE—IT SHOULD NOT BE NECESSARY TO CHANGE ANY STATEMENT IN THIS
C PROGRAM SO LONG AS THE FORTRAN COMPILER USED STORES REAL AND
C *IMAGINARY PARTS ADJACENTLY IN STORAGE.
C TRANSFORM(K) = SUM(DATA(J)*EXP(ISIGN*2*PI*SQRT(-1)*(J-1)*(K-1))
C /N), SUMMED OVER ALL J AND K FROM 1 TO N. DATA IS A ONE-
C DIMENSIONAL COMPLEX ARRAY (I.E., THE REAL AND IMAGINARY PARTS ARE
C ADJACENT IN STORAGE, SUCH AS FORTRAN IV PLACES THEM) WHOSE LENGTH
C N=2**K, K. GE.0 (IF NECESSARY, APPEND ZEROES TO THE DATA). ISIGN
C IS +1 OR -1. IF A-1 TRANSFORM IS FOLLOWED BY A +1 ONE (OR VICE
C VERSA) THE ORIGINAL DATA REAPPEAR, MULTIPLIED BY N. TRANSFORM
C VALUES ARE RETURNED IN ARRAY DATA, REPLACING THE INPUT. THE TIME
C IS PROPORTIONAL TO N*LOG2(N), RATHER THAN THE NAIVE N**2.
C ACCURACY IS ALL GREATLY IMPROVED, THE RMS RELATIVE ERROR BEING
C BOUNDED BY 6*SQRT(2)*LOG2(N)*2**(-B) WHERE B IS THE NUMBER OF
C BITS IN THE FLOATING POINT FRACTION. WRITTEN BY NORMAN BRENNER OF
C MIT LINCOLN LABORATORY, JULY 1967. THIS IS THE SHORTEST VERSION
C OF THE FFT KNOWN TO THE AUTHOR. FASTER PROGRAMS FOUR2 AND FOURT
C EXIST THAT OPERATE ON ARBITRARILY SIZED MULTIDIMENSIONAL ARKAYS.
C SEE—IEEE AUDIO TRANSACTIONS (JUNE 1967), SPECIAL ISSUE ON FFT.
C
DIMENSION DATA(1)
IP0=2
IP3=IP0*N
I3REV=1
DO 50 13=1,IP3,IP0
IF(I3-I3REV)10,20,20
10 TEMPR=DATA(I3)
TEMPI=DATA(I3+1)
DATA(I3)=DATA(I3REV)
DATA(I3+1)=DATA(I3REV+1)
14 DATA(I3REV)=TEMPR
DATA(I3REV+1)=TEMPI
20 IP1=IP3/2
30 IF(I3REV-IP1)5050,40
40 I3REV=I3REV-IP1
IP1=IP1/2
50 IF(IP1-IP0)50,30,30
50 IP1=IP0
60 IF(IP1-I3REV)70,100,100
70 IP2=IP1*2
THETA=6.283185307*FLOAT(ISIGN*IP2/IP0)
SINTH=SIN(THETA/2.)
WSTFR=-2.*SINTH*SINTH
WSTPI=SIN(THETA)
WR=1.
WI=0.
DO 90 11=1,IP1,IP0
DO 80 13=11,IP3,IP2
I2A=I3
I2B=I2A+IP1
TEMPR=WR*DATA(I2B)-WI*DATA(I2B+1)
TEMP1=WR*DATA(I2B+1)+WI*DATA(I2B)
DATA(I2B)=DATA(I2A)-TEMPR
DATA(I2B+1)=DATA(I2A+1)-TEMP1
DATA(I2A)=DATA(I2A)+TEMPF
80 DATA(I2A+1)=DATA(I2A)+TEMPF
TEMPR=WR
WR=WR*WSTFR-WI*WSTPI+WR
90 WI=WI*WSTFR+TEMR*WSTPI+WI
IP1=IP2
GO TO 60
100 RETURN
END
FFT

THIS PROGRAM FINDS THE DISCRETE FOURIER TRANSFORM
OF THE INPUT DATA AND PRINTS OUT THE REAL PART,
THE IMAGINARY PART, THE MODULUS, THE PHASE AND
THE POWER OF THE TRANSFORM. THE INPUT DATA
SHOULD BE REAL. IT USES A SUBROUTINE CALLED
'FOUR1' WRITTEN BY NORMAN BRENNER.

WRITTEN BY VENKAT NAGASUAMI ON APRIL 8, 1979

DIMENSION DATA(2,64),POWER(64),MODULUS(64),PHASE(64)
REAL MODULUS
DO 10 I=1,64
READ(105*100)DATA(1,I)
100 FORMAT(F11.4)
10 DATA(2,I)=0.0
WRITE(108,110)
110 FORMAT(dX,//'WEIGHTED DATA (MEAN-SUBTRACTED)'//)
WRITE(108,120)(DATA(1,I),DATA(2,I),I=1,64)
120 FORMAT(dX,2F20.4)
CALL FOUR1(DATA,64,-1)
DO 20 J=1,64
POWER(J)=DATA(1,J)*DATA(1,J)+DATA(2,J)*DATA(2,J)
MODULUS(J)=POWER(J)**0.5
20 PHASE(J)=ATAN(DATA(2,J)/DATA(1,J))
WRITE(109,130)
130 FORMAT(IX,'REAL',11X,'IMAG',10X,'POWER',8X,'MODULUS',
110X,'PHASE',//)
WRITE(109,140)(DATA(1,K),DATA(2,K),POWER(K),MODULUS(K),
1PHASE(K),K=1,64)
140 FORMAT(1X,5F15.4)
WRITE(109,150)(POWER(I),I=1,64)
150 FORMAT(F10.4)
STOP
END
MTF

This program finds the MTF of image motion due to operator vibration as a function of exposure time.

DIMENSION DATA(2,64),MOD(64),SQRTPOWX(64),N(12),
1MTF1(12),MTF2(12),MTF3(12),MTF4(12),MTFS(12),
1X(5),Y(5),C(32),D(32)

REAL MOD,N,MTF1,MTF2,MTF3,MTF4,MTFS

READ(101,100) (N(I),I=1,12)
READ(102,100) (SQRTPOWX(I),I=1,64)
100 FORMAT(F11.4)
DO 10 I=1,64
READ(103,100) (DATA(1,I),I=1,64)
10 DATA(2,I)=0.0
C WRITE(108,150)
C150 FORMAT(/16X,'UNI-VARIANCE NOISE DATA'/)
C WRITE(108,160) (DATA(1,1),DATA(2,1),SQR'T'PDWa (J)
1=1,64)
C160 FORMAT ( 1X,2F20.4,F40.4)
C
FOURIER TRANSFORM OF THE NOISE SPECTRUM
CALL FOUR1(DATA,64,-1)
DO 15 I=1,64
DATA(1,I)=DATA(1,I)/64.0
15 DATA(2,I)=DATA(2,I)/64.0
C WRITE(108,170)
C170 FORMAT(/16X,'NOISE TRANSFORM',34X,'SQ. ROOT OF POWER'/)
C WRITE(108,175)(DATA(1,I),DATA(2,I),I=1,64)
C175 FORMAT(1X,2F20.4,F40.4)
C
PRODUCT OF THE SQUARE ROOT OF THE AVG POWER SPECTRUM
AND THE TRANSFORM OF THE NOISE SPECTRUM
DO 20 I=1,64
DATA(1,I)=(DATA(1,I)*SQRTPOWX(I))**2.0
20 DATA(2,I)=(DATA(2,I)*SQRTPOWX(I))**2.0
C WRITE(108,180)
C180 FORMAT(/16X,'SQ. ROOT OF AVG POWER SPECTRUM * NOISE
1TRANSFORM'/)
C WRITE(108,200)(DATA(1,I),DATA(2,I),I=1,64)
C200 FORMAT(3F20.4)

PI=3.141592654

Contd.
Appendix V (Contd.)

```
DO 40 J=1,12
DO 50 I=1,32
C(I)=COS(2.0*PI*N(J)*DATA(I,I))
50 D(I)=SIN(2.0*PI*N(J)*DATA(I,I))
X(1)=C(1)+C(2)
X(2)=C(3)+C(4)+X(1)
X(3)=X(2)
DO 60 K=5,8
60 X(3)=X(3)+C(K)
X(4)=X(3)
DO 70 K=9,16
70 X(4)=X(4)+C(K)
X(5)=X(4)
DO 80 K=17,32
80 X(5)=X(5)+C(K)
Y(1)=D(1)+D(2)
Y(2)=D(3)+D(4)+Y(1)
Y(3)=Y(2)
DO 85 K=5,8
85 Y(3)=Y(3)+D(K)
Y(4)=Y(3)
DO 90 K=9,16
90 Y(4)=Y(4)+D(K)
Y(5)=Y(4)
DO 95 K=17,32
95 Y(5)=Y(5)+D(K)
C WRITE(108,205)
( C(I),D(I),I=1,32)
C205 FORMAT(8F10.4)
C WRITE(108,210)
( X(I),Y(I),I=1,5)
C210 FORMAT(1X,F10.4)
MTF1(I)=0.5*(((X(1)**2.0+Y(1)**2.0)**0.5)
MTF2(I)=0.25*(((X(2)**2.0+Y(2)**2.0)**0.5)
MTF3(I)=0.125*(((X(3)**2.0+Y(3)**2.0)**0.5)
MTF4(I)=0.0625*(((X(4)**2.0+Y(4)**2.0)**0.5)
40 MTF5(I)=0.03125*(((X(5)**2.0+Y(5)**2.0)**0.5)
WRITE(108,250)
250 FORMAT(/3X,'N(C/MM)','@X','MTF1','@X','MTF2','@X',
'@X','MTF3','@X','MTF4','@X','MTF5')
WRITE(108,260)(N(I),MTF1(I),MTF2(I),MTF3(I),MTF4(I),MTF5(I),
I=1,12)
260 FORMAT(6F10.4)
WRITE(111,270)(MTF1(I),I=1,12)
WRITE(112,270)(MTF2(I),I=1,12)
WRITE(113,270)(MTF3(I),I=1,12)
WRITE(114,270)(MTF4(I),I=1,12)
WRITE(115,270)(MTF5(I),I=1,12)
270 FORMAT(1X,F10.4)
STOP
END
```