6-1-1974

Design and Construction of Sinusoidal Targets for Resolving Power Cameras

Peter Hertzmann

Follow this and additional works at: http://scholarworks.rit.edu/theses

Recommended Citation

This Thesis is brought to you for free and open access by the Thesis/Dissertation Collections at RIT Scholar Works. It has been accepted for inclusion in Theses by an authorized administrator of RIT Scholar Works. For more information, please contact ritscholarworks@rit.edu.
A set of sinusoidal targets for producing aerial sinusoidal images when combined with a cylindrical lens were designed and constructed. The targets were produced for use in a low power resolving power camera. In addition, a set of calibration patches for relating density, a set of modulation adjusting slides, and a slide for focusing the resolving power camera were produced.
I. INTRODUCTION

The proposed American National Standard Method For Determining The Photographic Modulation Transfer Function (MTF) of Photographic Films\textsuperscript{1} allows for the use of variable area test patterns, incorporated with a cylindrical lens to smear the image, for producing a sinusoidally varying exposure on photographic materials. A variable area test pattern such as shown in Figure 1, consists of (1) an upper portion where the vertical height of the opening is sinusoidally proportional to the horizontal distance, and (2) the lower portion which is of constant height and serves to reduce the modulation of the sinusoidal portion of the test pattern.

In his 1973 Master's Thesis Proposal, Muh-Fa Chen proposed a scheme for a low frequency resolving power camera using variable area patterns in which the modulation of the sinusoidal exposure could be varied by means of a modulation background exposure. Figure 2 shows the schematic arrangement of the resolving power camera.

The target projector contains targets similar to the upper portion of the ANSI standard pattern; the background projector contains neutral density filters to produce an effect similar to the power portion. Images from the two projectors are brought together at a partially reflective surface and mutually smeared by the cylindrical lens. The reduction lens serves to produce the final image at the film plane.
FIGURE 1 - ANSI STANDARD TARGET
II. OBJECTIVES

The objective of this project was to design and construct a set of sinusoidal aerial targets for a resolving power camera. The targets were to be the size of a conventional 35mm slide. The set includes low frequency sinusoidal targets ranging from 0.1 lines/mm to 10 lines/mm in constant increments of change. In addition, a set of modulation adjusting patches, a set of calibration patches, and a focusing slide was designed and constructed.
III. DESIGN

The design of the target and accessories was divided into two aspects: (1) the design of the specifications; (2) the design of the construction process.

There were four types of slides that needed to be designed: (1) sinusoidal target slides; (2) calibration patch slides; (3) modulation adjusting slides; and (4) a focusing slide.

The normal expression for a sinusoidal target is

\[ y = b + m \cos 2\pi f x \]  

where \( f \) is the frequency, \( x \) is a spatial coordinate, and \( b \) and \( m \) are constants. The modulation of a target of this shape is \( m/b \).

The sinusoidal target slide is essentially an opaque slide with a transparent portion. One edge of the transparent portion is the above function, the opposite edge is a straight line corresponding to

\[ y = 0 \]  

The other edges correspond to

\[ x_0 = k \]

and

\[ x_n = \left( \frac{n}{f} \right) + k \]

where \( k \) is a constant phase shift and \( n \) is the number of cycles the target contains. For ease in calculating the values of \( y \) needed in the target, \( x \) is shifted by a value of \( k \) and thus

\[ y = b + m \cos (2\pi fx + k) \]
The modulation of the system is determined by the relationship of the maximum and minimum exposures (illuminances) at the film plane.

\[ M = \frac{E_{\text{max}} - E_{\text{min}}}{E_{\text{max}} + E_{\text{min}}} \]  

The system has both a target source of illumination \( E_o \) and a constant background source of illumination \( E_b \). Thus

\[ E_{\text{max}} = E_o,\text{max} + E_b \]  

and

\[ E_{\text{min}} = E_o,\text{min} + E_b \]  

Substituting (9) and (10) into (8) and reducing, \( M \) becomes

\[ M = \frac{(E_o,\text{max} - E_o,\text{min})}{(E_o,\text{max} + E_o,\text{min} + 2E_b)} \]  

Thus the modulation of the target slide has to be greater than or equal to the maximum modulation required for the system. Since \( E_{o,\text{min}} \) cannot be less than zero, the modulation of the target has to be less than or equal to 1.0. If the modulation was to equal 1.0 then the maximum system modulation would be 1.0. Unfortunately, it is impossible to draw a target with \( M = 1.0 \) since the value of \( y \) at \( E_o,\text{min} \) would be 0.0. Because of this and the fact that values
of modulation above 65% usually are useless to the researcher, the
modulation of the target slides was set at $M = 0.95$.

By inserting the parameter $m/b = 0.95$ into equation (5)
and normalizing $y$,

$$y = a [0.51282 + (0.48718)\cos(2\pi f x + k)]$$  \hspace{1cm} (12)

the constant "a" sets the maximum height of the target.

In order that the target starts and begins midway between
a maxima and a minima, $k$ must be $\pi/2$, and equation (12) becomes

$$y = a [0.51282 + (0.48718)\cos(2 f x + \pi/2)]$$  \hspace{1cm} (13)

This is the final form for expressing $y$ where $x$ is variable and
$a$ and $f$ are constants.

The main factors involved in the determination of the
frequencies was their intended use and the space available for the
target. Since the targets were being designed for a system whose
maximum frequency is about 30 lines/mm and has a reduction of about
3.5 times, a maximum target frequency of about 9 lines/mm was re-
quired. For simplicity, 10 lines/mm was adopted as the maximum
target frequency.

The minimum target frequency is limited by the amount of
space available for the target. The minimum number of periods for
any target is three. Thus the length of the minimum frequency
target would be

$$L = \frac{3}{f}$$  \hspace{1cm} (14)

Since the target is mounted on a 35mm slide where $L = 30mm$ approxi-
mately, $f_{min}$ is 0.1 lines/mm.
The proposed ANSI Standard on MTF Cameras states: "the increments between frequencies shall be no more than a factor of 1.6." The log at 1.6 = 0.20412. If a log difference between frequencies of 0.2 was used, the factor would be 1.58; which satisfies the Standard.

Thus eleven frequencies were chosen from 0.1 to 10.0 lines/mm with increments between frequencies of a factor of 1.58. The frequencies are 0.100, 0.158, 0.251, 0.398, 0.631, 1.00, 1.58, 2.51, 3.98, 6.31, and 10.0 lines/mm.

The targets were designed at various original sizes to be reduced photo-mechanically to their final size. The processes used are described in Section IV - Construction. Table 1 is a tabulation of the statistics of the targets at various stages.

From Table 1, the reader will note that the final target height is not constant. The effect of this is that the higher the target, the greater the maximum illuminance at the image plane. In order to make the maximum illuminance constant for all the targets a neutral density filter was sandwiched with the target. Using the principle of Large Area Transmission Density (LATD), the neutral density required for matching a target to the target with the minimum height is

$$ND_t = \log \left( \frac{H}{H_{\text{min}}} \right)$$

(15)

$H$ is the target height and $H_{\text{min}}$ is the height of the smallest target, in this case 4.8 mm. Thus
<table>
<thead>
<tr>
<th>Frequency (f)</th>
<th>No. of Cycles (n)</th>
<th>Original Target(in) Ht. Width</th>
<th>$R_1$</th>
<th>Intermediate Target(in) Ht. Width</th>
<th>$R_2$</th>
<th>Total R</th>
<th>Final Target(mm) Ht. Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100</td>
<td>3</td>
<td>20 26.09</td>
<td>1.74</td>
<td>11.50 15.00</td>
<td>12.7</td>
<td>22.09</td>
<td>23.00 30.00</td>
</tr>
<tr>
<td>0.158</td>
<td>5</td>
<td>20 27.13</td>
<td>1.74</td>
<td>11.50 15.58</td>
<td>12.7</td>
<td>22.09</td>
<td>23.00 31.55</td>
</tr>
<tr>
<td>0.251</td>
<td>8</td>
<td>20 27.69</td>
<td>1.74</td>
<td>11.50 15.93</td>
<td>12.7</td>
<td>22.09</td>
<td>23.00 31.85</td>
</tr>
<tr>
<td>0.398</td>
<td>12</td>
<td>20 26.21</td>
<td>1.74</td>
<td>11.50 13.11</td>
<td>12.7</td>
<td>22.09</td>
<td>23.00 30.14</td>
</tr>
<tr>
<td>0.631</td>
<td>12</td>
<td>20 28.53</td>
<td>3.00</td>
<td>6.67 9.50</td>
<td>12.7</td>
<td>38.10</td>
<td>13.33 19.00</td>
</tr>
<tr>
<td>1.000</td>
<td>12</td>
<td>20 18.00</td>
<td>3.00</td>
<td>6.67 6.00</td>
<td>12.7</td>
<td>38.10</td>
<td>13.33 12.00</td>
</tr>
<tr>
<td>1.58</td>
<td>12</td>
<td>28 22.20</td>
<td>5.86</td>
<td>4.78 3.79</td>
<td>12.7</td>
<td>74.46</td>
<td>9.55 7.57</td>
</tr>
<tr>
<td>2.51</td>
<td>12</td>
<td>28 14.00</td>
<td>5.86</td>
<td>4.78 2.39</td>
<td>12.7</td>
<td>74.46</td>
<td>9.55 4.78</td>
</tr>
<tr>
<td>3.98</td>
<td>12</td>
<td>28 17.58</td>
<td>11.67</td>
<td>2.40 1.51</td>
<td>12.7</td>
<td>148.17</td>
<td>4.80 3.01</td>
</tr>
<tr>
<td>6.31</td>
<td>12</td>
<td>28 11.09</td>
<td>11.67</td>
<td>2.40 0.95</td>
<td>12.7</td>
<td>148.17</td>
<td>4.80 1.90</td>
</tr>
<tr>
<td>10.0</td>
<td>12</td>
<td>28 7.00</td>
<td>11.67</td>
<td>2.40 0.60</td>
<td>12.7</td>
<td>148.17</td>
<td>4.80 1.20</td>
</tr>
</tbody>
</table>

TABLE 1
\[ \text{ND}_t = (\log H) - 0.68 \]  \quad (16)

The four target heights used are 23mm, 13.33mm, 9.55mm, and 4.8mm, and their corresponding \(\text{ND}_t\) values are 0.68, 0.44, 0.30, and 0.0.

Since the proposed ANSI Standard allows for 10% non-fundamental harmonic content, critical alignment of the slides with the cylindrical lens is not required.

The design of the modulating adjusting slides starts by simply solving equation (11) for \(E_b\).

\[
E_b = \frac{E_{o,\text{max}} - E_{o,\text{min}}}{2} - \frac{E_{o,\text{max}} + E_{o,\text{min}}}{2}
\]  \quad (17)

which is the same as

\[
E_b = \frac{m}{M} - b
\]  \quad (18)

The value \(E_b\) is the ratio of the illuminance at the image plane due to the background source to the maximum illuminance at the image plane due to the target.

Five different modulations were chosen: 25%, 35%, 45%, 55%, and 65%. The modulation required for the proposed ANSI Standard is 35% ±5%.

Since as \(E_b\) increases, the modulation decreases, the limiting value of \(E_b\) is when the modulation is 25%. At that point \(E_b = 1.436E_{o,\text{max}}\). In order that no filter is required at the background source for 25% modulation, the target source must produce 0.696 times less illuminance than the background source.
(The MTF Camera has to be designed accordingly.) Assuming this to be the case, equation (18) becomes

\[ E_b = (0.69643)\left(\frac{m}{M} - b\right) \]  (19)

Using the values for \( b \) and \( m \) from equation (13)

\[ E_b = \frac{0.33929}{M} - 0.35714 \]  (20)

the ND filter required at the background source is then

\[ ND_b = -\log\left(\frac{0.33929}{M} - 0.35714\right) \]  (21)

The values obtained from equation (20) are 0.0 at \( M=25\% \), 0.21 at \( M=35\% \), 0.40 at \( M=45\% \), and 0.59 at \( M=55\% \), and 0.78 at \( M=65\% \). The ND filters were purchased and mounted in slide mounts.

The calibration patches are similar to the target slides in that they are variable-height patches, except that they are rectangular. The proposed ANSI Standard states that the largest patch must produce twice the illuminance of \( E_{\text{max}} \) and the smallest patch one-half the illuminance of \( E_{\text{min}} \). In this case \( E_{\text{max}} = E_{o,\text{max}} + E_b \) and \( E_{\text{min}} = E_{o,\text{min}} + E_b \). Unfortunately there is no patch size which will produce an illuminance corresponding to \( (0.5)(E_{\text{min}}) \) because the smallest patch is one of no height and at that point \( E_{\text{min}} = E_b \); \( E_b \) is always large with respect to \( E_{o,\text{min}} \). The patches range from twice \( (E_{o,\text{max}} + E_b) \) to \( (0 + E_b) \).

The proposed ANSI Standard states that the log difference between patches be 0.1.

The height of the patches were calculated using LATD.
similar to before. The patches are calculated for a modulation of 65% because as the effect of the background source increases the log difference between patches becomes less than 0.1. The maximum height required is twice the sum of $E_{\text{max}}$ or

$$H_{\text{max}} = 2(E_{\text{o, max}} + E_{\text{b, max}}) \tag{22}$$

$E_{\text{o, max}}$ and $E_{\text{b, max}}$ are the equivalent heights of $E_{\text{o, max}}$ and $E_{\text{b, max}}$ in this case

$$H_{\text{b, max}} = 1.436E_{\text{o, max}} \tag{23}$$

or

$$H_{\text{max}} = 4.872E_{\text{o, max}} \tag{24}$$

Not all the illuminance at the image plane is due to the target source so

$$H_{\text{max}} = 4.872E_{\text{o, max}} - H_{\text{b, max}} \tag{25}$$

or

$$H_{\text{max}} = 3.436E_{\text{o, max}} \tag{26}$$

The value of the $H_{\text{b}}$ at any modulation is simply

$$H_{\text{b}} = 1.436E_{\text{o, max}} 10^{-N_{\text{D}}b} \tag{27}$$

Using (26) and (27), the height ($H$) of any calibration patch will be

$$H = (H_{\text{max}} + H_{\text{b}})(10^{-d}) - H_{\text{b}} \tag{28}$$

d is the log difference between the largest patch and ($H + H_{\text{b}}$).
Equations (26), (27), and (28) were used, with the computer, to plot the patches. $HE_{o,\text{max}}$ is 4.8mm, $ND_b$ is 0.8 and $d$ is variable. There can be seven patches per slide, each is $H$ high by 3mm in width, 0.33mm apart.

Since the relative density of any calibration patch will be different for each modulation, it is necessary to calculate each density by re-arranging equation (28), so that

$$d = -\log(H + HE_b) - \log(H_{\text{max}} + HE_b).$$

A tabulation of these values can be seen in Table 2.

A focusing slide with edges at different angles will make focusing of an MTF Camera easier. This slide has edges at $-2^\circ$, $-1^\circ$, $0^\circ$, $+1^\circ$, $+2^\circ$ and two sets of lines at these same angles. The camera can then be focused by adjusting the apparatus until a sharp edge and two sharp lines are obtained.
**TABLE 2**

Relative Densities of Calibration Patches

<table>
<thead>
<tr>
<th>Step Number</th>
<th>Percent Modulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.073</td>
</tr>
<tr>
<td>3</td>
<td>0.141</td>
</tr>
<tr>
<td>4</td>
<td>0.204</td>
</tr>
<tr>
<td>5</td>
<td>0.262</td>
</tr>
<tr>
<td>6</td>
<td>0.314</td>
</tr>
<tr>
<td>7</td>
<td>0.360</td>
</tr>
<tr>
<td>8</td>
<td>0.400</td>
</tr>
<tr>
<td>9</td>
<td>0.435</td>
</tr>
<tr>
<td>10</td>
<td>0.465</td>
</tr>
<tr>
<td>11</td>
<td>0.490</td>
</tr>
<tr>
<td>12</td>
<td>0.512</td>
</tr>
<tr>
<td>Background</td>
<td>0.531</td>
</tr>
</tbody>
</table>

Note: Values are good only when background illuminance is 1.436 times maximum target illuminance i.e. 4.8mm high slit.
IV. CONSTRUCTION

The target outline was drawn by a computer plotting device using the four equations: (6), (7), (13), and

\[ y = 0 \]  

(See Figure 3.) Values used for \( a \), \( f \), and \( n \) are shown in Table 1; \( f \) was converted to lines per inch for plotting purposes. A standard subroutine for plotting bars was used for plotting the calibration patches.

The computer plots were attached to heavy cardboard and the outlines "heavied" with black ink. The space between the outlines was filled in with a red self-adhering film normally used by printers for such purposes. The targets were then reduced until they were 12.7 times their final size. At this point, a calibration mark was added and the targets were checked for accuracy; then reduced to their final size.

The targets were then mounted in slide mounts with their appropriate filters.
\[ y = a \left[ 0.12582 + 0.48772 \cos(2\pi f x + \pi/2) \right] \]
V. ANALYSIS

Since the degradation of the sinusoidal pattern would increase as the frequency increases, the highest frequency, 10 lines/mm, was printed with a horizontal enlarger (see Figure 4). A visual comparison was made between the original drawn pattern and the print. Since the difference appeared small, no further comparison was made. The frequency was measured with a measuring microscope and found to be within 1% of the predicted value.

The actual values of the neutral density filters used were 0.68 for the 23mm high targets; 0.38 for the 13.33mm targets; 0.27 for the 9.55mm targets. Using equation (16), it can be seen that all the targets are within one-quarter of a f/stop.

The actual values of the modulation adjusting slide densities are 0.18, 0.38, 0.58, 0.77. Thus using equation (20), the modulation values possible are 25%, 33%, 44%, 55%, and 64%.

The relative densities can also be recalibrated. The new densities, expressed as relative exposures, can be found in Appendix A.
VI. CONCLUSION

Fifteen slides were produced; they are:

A. Seven target slides containing eleven variable area test patterns which vary in vertical height sinusoidally proportional to the horizontal distance. The eleven patterns range in frequency from 0.1 lines/mm to 10 lines/mm in increments of 1.58 times ($\log 1.58 = 0.2$). All patterns require the same exposure. All patterns have a nominal modulation of 95%.

B. Two slides containing twelve variable area calibration patches in log exposure increments of 0.10 when operated with the background illumination adjusted for 65% modulation. The exposures range from less than the minimum exposure possible with a test pattern to twice the maximum exposure possible.

C. Five slides for adjusting the test pattern modulation to 25%, 33%, 44%, 55%, or 65% by use of background illumination.

D. One slide for focusing the cylindrical lens in the camera system.
Fig. 5 - Facsimiles of Typical Targets

- Focusing Guide
- Calibration Patches
- Variable Area Pattern
VII. ACKNOWLEDGEMENTS

The following people provided significant aid at various stages of this project:

Peter Engeldrum of Xerox Corporation: For support and aid during the design and analysis of the targets.

Steve Shore of Motorola Corporation, and Carl Nielsen and Bob Rowe of Eastman Kodak: For help in producing the slides.

Mohamed Abouelata and Gerhard Schumann of Rochester Institute of Technology: For their advice and friendship.

John Friauf, Linda Diamond, and Pamela Hertzmann: For their aid in producing this report.

Central Intelligence Agency: For financial support.
VIII. REFERENCES

1. ANSI Standard PH2.39-Proposed (1/74), "Method for Determining the Photographic Modulation Transfer Function (MTF) of Photographic Films."


The following appendix is the operating manual completed as part of the production of the slides.
This set includes the following fifteen slides:

Variable Area Test Patterns
1. 0.100 lines/mm
2. 0.16 lines/mm
3. 0.25 lines/mm
4. 0.40 lines/mm
5. 0.63, 1.0 lines/mm
6. 1.6, 2.5 lines/mm
7. 4.0, 6.3, 10 lines/mm

Calibration Patches
8. Steps 1 through 7
9. Steps 8 through 12

Modulation Adjusting Slides
10. 64%
11. 55%
12. 44%
13. 33%
14. 25%

Miscellaneous
15. Focusing slide
The use of these slides requires a camera set-up as shown in the following diagram.

Focus the system with the focusing slide. When the system is focused, the center bar and the two center lines, only, are in focus. If any of the other bars or lines are in focus, the cylindrical lens and the slide are not aligned. The lines represent one degree angles of alignment.

The illuminance levels due to the individual projectors, when measured at the film plane with all optics in position, should be such that the illuminance from the background projector is 3.15 times * the illuminance from the target projector if:

1) the system has been focused;
2) the 0.40 lines/mm slide is in position in the target project;
3) the 25% modulation slide is in the background projector;
4) an area equivalent to 8 to 10 cycles is measured for both projectors.

Expose the test film as per camera instructions.

When evaluating the processed film it will be necessary to know the relative exposures at various modulations and steps on the calibration patches. These values can be found in the following table.

* The LATD of a target is 0.301, the base plus fog of the target is 0.04, and the illuminance at the film plane due to the background projector is 1.436 times illuminance due to the target projector.
Relative Exposures of Calibration Patches

<table>
<thead>
<tr>
<th>Step Number</th>
<th>25</th>
<th>33</th>
<th>44</th>
<th>55</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.845</td>
<td>0.834</td>
<td>0.813</td>
<td>0.802</td>
<td>0.795</td>
</tr>
<tr>
<td>3</td>
<td>0.723</td>
<td>0.701</td>
<td>0.665</td>
<td>0.645</td>
<td>0.633</td>
</tr>
<tr>
<td>4</td>
<td>0.625</td>
<td>0.596</td>
<td>0.547</td>
<td>0.521</td>
<td>0.503</td>
</tr>
<tr>
<td>5</td>
<td>0.547</td>
<td>0.513</td>
<td>0.453</td>
<td>0.422</td>
<td>0.401</td>
</tr>
<tr>
<td>6</td>
<td>0.486</td>
<td>0.446</td>
<td>0.379</td>
<td>0.343</td>
<td>0.319</td>
</tr>
<tr>
<td>7</td>
<td>0.437</td>
<td>0.394</td>
<td>0.320</td>
<td>0.281</td>
<td>0.255</td>
</tr>
<tr>
<td>8</td>
<td>0.398</td>
<td>0.352</td>
<td>0.273</td>
<td>0.231</td>
<td>0.203</td>
</tr>
<tr>
<td>9</td>
<td>0.367</td>
<td>0.319</td>
<td>0.236</td>
<td>0.192</td>
<td>0.162</td>
</tr>
<tr>
<td>10</td>
<td>0.343</td>
<td>0.292</td>
<td>0.206</td>
<td>0.160</td>
<td>0.130</td>
</tr>
<tr>
<td>11</td>
<td>0.323</td>
<td>0.271</td>
<td>0.183</td>
<td>0.135</td>
<td>0.104</td>
</tr>
<tr>
<td>12</td>
<td>0.308</td>
<td>0.255</td>
<td>0.164</td>
<td>0.116</td>
<td>0.084</td>
</tr>
<tr>
<td>Background</td>
<td>0.295</td>
<td>0.241</td>
<td>0.148</td>
<td>0.099</td>
<td>0.066</td>
</tr>
</tbody>
</table>