Mathematics: it's all about language

Elizabeth Barrett

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Mathematics: It's All About Language

Master’s Project

Submitted to the Faculty
of the Master of Science Program in Secondary Education
of Students who are Deaf or Hard of Hearing

National Technical Institute for the Deaf
ROCHESTER INSTITUTE OF TECHNOLOGY

By

Elizabeth Barrett

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It's all about language

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Abstract

Not many years ago, Hillegeist and Epstein, from Gallaudet University, wrote that 'while there is no question that the increase in difficulty of the mathematical concepts is an important factor in mathematical comprehension and there seems to exist an effect on comprehension specifically related to language. The special role of language in mathematics is a factor' in the educational success of deaf students. However, they add that at this point (1987), the nature of the language effect is not completely clear. (Hillegeist & Epstein, 1987). Reading this statement led me to the purpose of this project, which is to review literature that will examine the areas of critical period and language development of deaf children; core knowledge and cognitive development with regard to the impact on deaf students and mathematical instruction; and deaf students' mathematical progress based on the National Council for Teacher's of Mathematics standards (NCTM). Focus is on the critical period for language acquisition, core knowledge and current mathematical instruction methods of deaf students to analyze the consequences for deaf education in the area of mathematics.

The importance of early language acquisition is noted. Alternative ideas and observations from contemporary research related directly to: the unique connection between cognitive development and language acquisition; the construction of knowledge by children, and NCTM standards are summarized regarding learning mathematics. The goal is to synthesize the most current literature and perhaps suggest ways in which educators of the deaf can contribute to the development of the higher-order processing and cognitive skills necessary for their successful post-secondary study of mathematics.
Statement of the problem

Research has been published regarding the critical period and language development of deaf children; the impact of language acquisition on learning for deaf students; core knowledge and cognitive development; the important connection between language development and mathematical instruction; and deaf students' mathematical progress based on the National Council for Teacher's of Mathematics standards (NCTM). However, research from these various venues have not been synthesized in a cohesive manner as to assist professionals teaching mathematics to deaf students.

Importance of the problem

Several studies have revealed that language acquisition and understanding of language will impact the student's ability to learn, especially in the area of mathematics. Related research has also demonstrated that the reading comprehension skills of deaf students remain considerably lower than those of their hearing peers (Bochner, J.H. & Albertini, J.A., 2001). Additionally, in the Master of Science Secondary Education program at the National Technical Institute for the Deaf, which prepares teachers of the deaf for certification, students learn that language development is problematic for deaf students and has the potential to impact learning. Unfortunately, despite all our efforts, deaf students continue to lag behind their hearing peers in language development and mathematics.

Language is crucial for the development of mathematical skills. According to Powers, Gregory & Thoutenhofd, a delay in deaf children's abilities in a number of mathematical concepts has been noted (Powers, Gregory & Thoutenhofd, 1998). Given the findings of recent research regarding the language development of writing across the mathematics curriculum, I believe the synthesized research will show: language development is dependent on input, many deaf students experience English language
development similar to "English as a second language" students, mathematical learning is different for deaf students and an educational program based on language development, and that an approach utilizing the correlation between the spatial nature of American Sign language and the spatial nature of mathematics is essential in order that the mathematical advancement of deaf students be comparable to that of their hearing peers. The research implicates the need to answer the question: Is there a critical period for mathematical development similar to the critical period for language development?
LITERATURE REVIEW

Critical Period for Learning Language

Language is defined in many ways; however, according to Susan Fischer, "language is an abstract system that can use a variety of means of expression and comprehension" (Fischer, S., 1998). Noam Chomsky defines knowing a language as meaning being able to produce an infinite number of sentences never spoken before (Chomsky, 1968). For the purposes of this review, spoken does not necessarily mean oral, but perhaps voiced or spoken through sign language.

Chomsky proposes that the mechanism of language acquisition formulates from "innate processes and all children share the same internal constraints, which characterize narrowly the grammar they are going to construct". According to Chomsky, the "mechanism for language acquisition is innate". Much of his work on language is in relation to the principle that all children go through the same stages of language, regardless of the language they are learning. Furthermore, Chomsky believes that there is a critical age for learning language similar to overall development of the human body (Thomas Jefferson High School, 2000).

Chomsky states that language is essential for the expression of thought. He considers "the study of language to be the study of the essence of human beings", because he considers language a distinctive quality of mind, unique to humans. His beliefs about human essence lead to his beliefs about generative grammar. He believes that "generative grammar must render explicit the implicit knowledge of the speaker". He proposes that individuals spontaneously comprehend that certain combinations of three words make sense. He provides several examples to make his point: "we ate lunch, John eats cake, Jules loves Chloe". Children start with small 2-3 word constructs to build more
meaningful language. Children tend to fine-tune their syntactic structures during their early school years. Their vocabulary will increase and they will develop an evaluation procedure which, given possible grammars, they will be able to select the right one. Throughout a child's early years, researchers have noticed an increase and complexity of their linguistic repertoire and the use of this repertoire for conversation and written narration (Chomsky, 1986).

In addition to mastering new forms of language, Chomsky states that children learn to use these and existing structures to communicate more effectively. The acquisition of language is a process of selecting the best grammar compatible with the available data. The awareness that enables the language user to think about and reflect about language is metalinguistics. As children progress in metalinguistics, their ability to think about language in the abstract is reflected in their reading and writing development (Chomsky, 1986). Language development is developmental in nature and follows a sequence outlined in Table 1.

Citing the work of James Woodward, Susan Fischer states that one indication of a critical period for language acquisition, which includes sign language acquisition, comes from research on the acquisition of American Sign Language (Woodward, J., 1978). Woodward found, and Fischer agreed, that people who learn to sign prior to age six had more constructs with which to work, than people who learn to sign after age six (Fischer, S., 1998, Woodward, J., 1978). If a child's ability to acquire language begins to decline after age six, this could have serious implications for deaf students who do not have effective modes of communication by this age.

Language use, or pragmatics, is the area of our most important linguistic growth during school-age years. Young children do not experience decenteration (the process of moving from rigid, one-
dimensional description of an object) until they are closer to puberty. According to linguists, language development is becoming less flexible by this age. If a child does not experience early language development, their ability to recognize that there are multidimensional, many possible perspectives on any given topic will be severely impaired (Owens, Ed., 1992).

Fischer states that if there exists a critical period for many aspects of language acquisition, then children who are not exposed to a first language by school age will not fully master their first or perhaps any language. Fischer cites two examples for the critical period for the development of behaviors, talents and even ideas but adds that while behavior is predetermined, it must be activated by something in their environment. One of the stories Fischer tells is of ethnologist, Konrad Lorenz’s observation of a flock of graylag geese. When the goslings hatched, their first encounter was Lorenz. They imprinted onto Lorenz and followed him as if he were their mother. Therefore, for graylag geese the critical period is within minutes of hatching (Fischer, 1998).

Although linguists believe children are born with an innate endowment for language, the ability to learn a particular language must be activated early and children must have accessible exposure to that language. Additionally, Fischer states that there is an age after which language will only be learned through explicit instruction (Fischer, S., 1998). Vygotsky agreed, as he saw language and thought to have independent origins. He emphasized the existence of intellect occurring before or without speech. Accordingly, Vygotsky states that "speech occurs without or before intellect", such as when babies babble and parrots imitate (Schutz, 2002). Fischer found that despite an almost total lack of formal instruction, children acquire the core grammar of their first language almost effortlessly by around age five (Fischer, 1998). Therefore, there is an important period, early in life,
that language, in either spoken or signed, must be stimulated for the innate properties to obtain
greatest benefit.

We need to consider both spoken and signed language development in order to discuss deaf
education. As previously stated, early language development is critical. There has been much
discussion about the exact age of the critical period, however, thirty years ago, in response to the
work of Chomsky, Erich Lenneberg argued that the critical period for language acquisition was
approximately around the age of puberty. A numbers of studies Lenneberg reviewed found that
children with little or no language exposure prior to puberty never fully developed any language.
Lenneberg, as Fischer and others have found, noted children with "minimal exposure to language
before puberty could never make up for that initial deficit". Citing the work of Woodward, Fischer
agrees that second language acquisition is less problematic prior to puberty. Fischer states that the
research suggests second-language acquisition is much more available and possible for people who have
a first-language, largely because they have the foundation of the first language on which to build
(Fischer, S., 1998).

Citing an earlier review with Mayberry, Mayberry and Fischer reviewed a series of studies
that examined the processes of American Sign Language (ASL) by native and non-native signers.
Mayberry and Fischer found that the native signers (native is defined as first language acquired prior
to age five), out performed non-native signers. Fischer also cites the work of Woodward's research
from the early 1970's, in which Woodard states that people who learned to sign prior to age six had
more constraints with which to work when compared to those who learned to sign after age six
(Fischer, S., 1998). Fischer concurs. With what Chomsky, Fischer and Owens are saying about
Language, decentration and descriptors, we need to examine the potential for learning consequences for all children, especially children lacking the educational foundation of a first language.

Language Development of Deaf and Hard of Hearing Students

Fischer asks the next logical question, "what about signed languages"? If there is a critical period for learning sign language as Woodward proposes, then there are a number of consequences facing educators. Besides the work of Woodward, Fischer also cites Mindel and Vernon and Geer and Schick, who state another indication of a critical period for sign language comes from deaf children who learn to sign from their deaf parents. Children who are exposed to signed language from an early age (birth) show enhanced acquisition of English as well as speechreading skills. These children are experiencing early language acquisition (Fischer, 1988).

These findings may have as much to do with all that deaf children and their parents share. They may experience a parent/child relationship more similar to that of hearing children and their hearing parents. From birth, deaf parents are able to communicate with their deaf children. Deaf parents' acceptance of their child's deafness might also enhance the amount of information intake a deaf child will encounter early in life (Fischer, S., 1998). Language is mastered by a wide variety of children raised in a variety of environments. Despite the great diversity of cultures our students come from, and the diversity in patterns of early communication between child and caregiver, almost all children in all cultures master the language to which they are exposed. However, there does seem to be limits on the richness of language depending on early language exposure.

The limit of language enrichment has been evidenced in children brought up by animals. These
children have not been able to develop language spontaneously. Children raised by humans under inhumane circumstances experience similar results. Studies of a girl confined in a small room, without freedom of movement and no human companionship for the first thirteen years of her life, revealed during this period of severe deprivation, language did not develop at all. Although hers was an extreme case, it did provide researchers with evidence for the limitation on the resilience of language development in children (Owens, Ed., 1992).

Not all properties of language seem to be equally robust in the face of variations in environmental conditions. Certain properties of language have been found in environments that differ dramatically from typical language-learning environments, while other properties of language have not. Goldin-Meadow and Mylander (1990) report on a study conducted earlier on the language development of hearing children reared by deaf parents. They considered the hearing children as being exposed to an impoverished model of English, since the first language of many deaf parents is sign language, not spoken English. The importance of their results was that they found the children formed some of the properties of English, but did not develop others. The lack of linguistic input appeared to have had differential effects on their language development (Goldin-Meadow & Mylander, 1990).

Goldin-Meadow & Mylander focused on properties of language whose development can withstand variations in learning conditions. This would describe the language-learning environment of prelingual deaf children born to hearing parents. The children studied by Goldin-Meadow & Mylander were profoundly deaf, and were considered to have a "hearing loss significant enough to impede their development of spoken language". Additionally, their parents chose not to expose them to sign language. However, in spite of their impoverished language learning condition, these deaf
children developed a gestural communication system that stimulated the structure of other communication systems. However, there is a significant difference between language and communication. Of the ten subjects in the study by Goldin-Meadow & Mylander, all subjects combined their gestures into strings that functioned similar to the sentence structure of an early child's language. Goldin-Meadow & Mylander referred to these structures as gestural systems, which "first expressed the semantic relation typically found in early language". The deaf child's predicates were similar to the predicates of early language with underlying frames constructed with as many as three arguments. The gestural sentences produced by deaf children imitated early language development in that they conformed to regularities of two types: ordering regularity and production regularities (Goldin-Meadow & Mylander, 1990). This proves that deaf children can develop gesture systems with structure, at the level of the sentence, with structure identifiable and similar to syntactic structure.

Therefore, children acquiring either a conventional spoken language or a conventional sign language begin to develop structure at the level of the word or sign by their fourth year. As a rule, children begin with an initial period during which they learn the words or signs of their language as unanalyzed wholes or amalgams. During the next period, they begin to develop the understanding that words are made up of morphemes, which are the smallest part of a word that have meaning. At this point, children gain productive control over the parts of the words. By knowing the parts themselves and how they combine to form words, children experience an internal structural development of language. Goldin-Meadow & Mylander, at the very least, indicate that children will seek structure at the word level when developing systems for communication. Children will actively pursue whatever
input is available, conventional or otherwise, to develop linguistic structure (Goldin-Meadow & Mylander, 1990).

**Impact of Language Acquisition on Learning for Deaf Students**

According to M. Virginia Swisher (1989), the primary educational challenges faced by deaf students are the acquisition of the linguistic system and spoken system of English. They are "stuck in a linguistic limbo", hovering between English and some form of sign language. Since most deaf children are born into hearing families who often do not know or learn sign language, they live in a "language-abridged world". Frequently, for deaf and hard of hearing students, language is limited until they enter school. Unlike spoken languages, deaf students more readily acquire communicative competency in a signed language. Swisher states that, "deaf students often function as late-acquired first language learners" (Swisher, 1989).

In order to understand the complex relationship between language and learning, we must review how children learn language and what factors influence their communication with others. A full understanding of language development is muddled by the variation of parental guidance and language modality. Hearing parents who raise their children using sign language do not use that method exclusively or consistently. Therefore, deaf children of hearing parents do not develop a first language and hence do not have access to competent language for either sign or spoken language. This places them at a distinct disadvantage for effective language development and consequently these students are often placed in remedial level English classes with other English as a Second Language (ESL) student. This can happen with or without interpreters. This situation has the potential for being an appropriate placement, if the apposite language interpretation is included in the educational
program. However, regardless of cognitive functioning, deaf students are often relegated to resource classes in a mainstream setting (Marschark, Albertini & Lang, 2000).

Kluwin and Moores (1989) also investigated the effect of placement and attainment in language and mathematics, giving serious consideration to the problems of previous research, which related to how the children were initially selected for such placements. They concluded that placement does contribute to their educational experience. However, they also found that the quality of instruction received is the main factor in achievement. Supportive teachers, regular and extensive language instruction, review of the material with a focus on vocabulary, direct instruction, positive encouragement and high demands on the students were also important factors influencing academic success. Teachers' ability to communicate clearly with students, the students' acquisition of a first language, and mode of communication and effectiveness of instruction are also noted as influential factors (Kluwin and Moores, 1989).

According to Swisher, researchers Goldberg & Bordman found that many of the errors made by deaf students when learning English were similar to those of foreign decent learning English. She reiterates that strong nativists, like Chomsky, assume that access to input is required for language learning. Reduction in both quality and quantity of linguistic input has the potential to have a negative impact on a student's educational success (Swisher, 1989).

Additionally, Swisher provides us with three important factors impacting language learning for deaf students: "the age at which the loss occurred; the quality of residual hearing; and the severity of the hearing loss". Students becoming deaf post-lingually have a better probability of language acquisition. The extent of family involvement in a child's education will also have a substantial impact
on learning and language acquisition. The primary problem of deaf learners is access to input (Swisher, 1989).

Swisher further adds, "three potential input sources for language are: input through lip-reading, input by means of aural pathways (usually with auditory aids) and input using a signed code for English". There is one other vehicle, written input; however, having a first language that can be written (i.e., French, English or Italian) is a prerequisite. Since ASL is not a written language, Swisher focuses on the first three. Lip-reading is limited in two ways. First, fine motor movements of the mouth and tongue are often difficult to distinguish. Second, the person reading must be able to see the face of the speaker. The restrictions with auditory information are two-fold. First the restriction of range and the damage to the inner ear may provide only a distorted signal to the person, which leads to misinterpretations. The second, interconnected issues for aural assistance are probably primary. The damage to neuropathways and level of residual hearing are determinates for auditory success. Signed code for English is the third potential source for representing a spoken language. However, the important factors in the method are vision and access. Both the inability to see (visual impairment) and physical proximity that might inhibit the signer have the potential to restrict overall quality of available input. Second, hearing families of deaf children do not always learn to sign and hence the child may not receive any signed English input at home. When signing is utilized at home, most often, it is only the mother who learns to sign with any degree of competency. Additionally, she may only sign messages intended for the child and thus excluding the child from casual and family conversations. Given the reduced input from the various sources, one could perhaps
predict that deaf students would have problems acquiring the syntax and morphology of a spoken language (Swisher, 1989).

Swisher reports on Gass' framework for describing the mechanisms of second language learning by outlining the five levels of progression from which input can potentially be available to contribute to linguistic output. These levels are:

- **Apperceived input** - first level language data that passes through the learner. Also referred to as incidental language. Quantity and quality of input are critical.

- **Comprehended input** - level at which understanding is achieved. At this level the student is involved with the linguistic input.

- **Intake** - level at which the student is able to go beyond current understanding, to empathetic use and ingestion take place.

- **Integration** - level at which student assimilates new information with old schema. The student is able to transcend the moment, understand past events affect the present and future.

- **Output** - finally, the student is able to express needs and thoughts. Students who do not reach this level are often frustrated with learning and are more likely to lack motivation (Swisher, 1989).

Variations of output depend on the variations of input and level of linguistical accomplishment.

Whatever linguistic development students experience early in life will impact their expressive skills later in life. Students, who have experienced deficits in language input, cannot be assumed to have full linguistic competency. One cannot assume that deaf students have been exposed to sign language or predict what their language attitudes will be. However, the level of their language skill will relate most closely to the severity of their hearing loss and age of onset. The frustration of "language dearth" must also be noted and addressed in the educational setting (Swisher, 1989). However, that is not the focus of this project. Swisher concludes that the difficulties of learning English are many
and thorny. The sociolinguistic environments of deaf students are also complex, depending on their etiology of deafness, the degree of hearing loss, the amount of language input at an early age, and parent/family attitudes toward deafness (Swisher, 1989).

Fischer agrees with Swisher, stating that first and most important, it is imperative that children be given exposure to accessible language as early as possible, or they will lose out both linguistically and experientially. She reiterates, Swisher's reports, that sound is not an easily accessible modality for most deaf children. Fischer adds that speech should be available because each child is wired differently and it is a potential input medium for deaf students. Depending on the residual hearing, aptitude, and incentive each student will acquire speech differently. She adds that according to social, psychological and linguistic experiments, deaf children remain years behind their hearing peers and do not attain a level of literacy comparable to that of their hearing peers. Traditionally, the education of deaf students has included time spent "drilling speech and training hearing" as a substitute for exposing children to information about the world. Next, Fischer states that content information presented to deaf children is inappropriate and often inaccessible to them, due to language deficits and/or modality of presentation. She restates the necessity for early language input and acquisition, as it is difficult, if not impossible, for most deaf students to acquire English or any form of language after the critical period for language acquisition (Fischer, 1988, Swisher, 1989).

Recalling that English is a second language for most deaf students, Bochner & Albertini refer to Bochner's 1982 research, noting "the range of linguistic diversity found in the deaf population probably exceeds that found in any subgroup of the normally hearing population" (Bochner & Albertini,
2001). Gerald Berent echoes Bochner, stating, both nature and nurture impact second language (L2) acquisition. It is often through L2 that deaf students are taught. Since most deaf students learn some form of sign language or gestural system, their first language is typically not English. Since ASL or any sign language is likely to have a structure different than English, understanding the specific transliteration may not be readily available (Berent, 1988). He is concerned with "interlanguage syntax". The ability to draw on certain schema of current linguistic ideas, the concept of overtness in language contributes to the structures necessary for language acquisition sufficient for learning. Berent states that explicit instruction will be necessary for English as L2 learners. He reports that areas of English that present the most difficulty for deaf students are the 'nonclausal' part of the English language. The nonclausal parts are the adjectives, adverbs and prepositional phrases. With respect to figurative language, metaphors, similes, and analogies present another area of difficulty. Multiple meaning words also contribute to the difficulty in interpreting the English language (Berent, 1988).

Subjects' lack of awareness of noun clause consistency and the misunderstanding of sequence-of-tense rules were made apparent in the frequency of errors. However, the least mastered aspects of English, for deaf subjects, are the gerund and infinitive phrases. Therefore, Berent reports, subjects preferred unmistakable expressions of grammatical relations. He concludes that a more principled approach would provide greater insight into the theoretical issues of language acquisition and educational implications (Berent, 1988).

According to Marschark, Lang and Albertini, "critical literacy refers to the ability to use reading and writing to define one's place in history and society". Furthermore, these authors report
that according to the Standards for English Language (1996, p. 3) students should be able to read a variety of literature and written texts, write effectively for different audiences and purposes, be able to use various sources and gather information (research), and use language for their own purpose, including enjoyment and critical analysis. According to Marschark, Lang & Albertini, Walter and Doehrig state "phonological decoding abilities are important for reading with comprehension". Marschark, Lang & Albertini claim that in addition to phonological decoding, deaf students must be able to understand word meaning for comprehension of the written text (Marschark, Lang and Albertini, 2000).

A student's understanding of what they are reading depends largely on context surrounding each word. All students learn through incidental learning. If the student does not have sufficient background information, i.e., foundational language, the lack of vocabulary can have a devastating impact on the student's ability to read.

"The sight recognition knowledge of the 2,000 most frequently used word families of English enables a student to use approximately 84% of words in a wide range of texts. Incidental vocabulary learning is important and influences a child's education; therefore, there are implications for teachers of the deaf" (Huckin & Cody, 1999).

The implications for differential instruction in deaf education are far reaching and will be discussed in further detail in a later section.

**Core Knowledge and Cognition**

Elizabeth Spelke has spent twenty years researching core knowledge systems. Core knowledge systems, according to Spelke, "are mechanisms for representing and reasoning about particular kinds of important entities and events, including those that are inanimate, manipulatable objects and their
motions, persons and their actions, places in the continuous spatial layout and their Euclidean geometrical relations, numerosities and numerical relationships. These systems serve to build representations of objects, persons, places, and numerical understanding that include many abstract concepts and relationships (Spelke, 2000).

The debate about which comes first, language or thought, has been ongoing for many years. Piaget, recall, claims cognition drives language. Theorists, Noam Chomsky and Jerry Fodor, suggest that the development of language and cognition are distinct, depending on different underlying processes and experiences. Chomsky was the first to demonstrate the relationship of rules and derivations within language was sufficient as to account for many developmental and psycholinguistic phenomena that previously had been attributed to language users themselves. According to Chomsky, the modularity hypothesis, "the idea that language and thought are independent components, is evident in our ability to filter out irrelevant and erroneous language examples from the larger group of appropriate language and our ability to identify correct rules from erroneous rules without direct instruction" (Marschark & Everhart, 1997, Chomsky, 1986).

According to Vygotsky, a child's greatest discovery becomes possible only when a high level of thought and speech development has been achieved (Schutz, 2002). Marschark & Everhart conclude from these psychologists that in the "child's ontogenetic development, thought and speech have different roots". Ontogenetic means the development of an individual organism. This implies that our wiring for thought and speech are independent of each other and are able to develop independently of each other. Within cognitive development, Marschark & Everhart establish a prelinguistic phase, and within the speech development of the child, a pre-intellectual state. "Humans
produce it themselves” (Marschark & Everhart, 1997). Marschark & Everhart seem to be agreeing with Spelke, that our core knowledge and pre-wiring will affect our language development.

**Cognitive Development, Language and Deaf Learners**

Marschark & Everhart make valid points of the importance of understanding the role of a deaf child’s language competencies and the role they play with regard to cognitive development. They report on the research of Hans Furth, who conducted pioneering research on the cognitive development of deaf children. Furth carried out a number of studies to explore Piagetian stages of development in deaf children by looking to understand the relationship between thought and language by examining children whom he considered without language. Furth found a delay in deaf children’s ability to apply conservation concepts. In conservation tasks, the appearance of a liquid or solid is changed without varying the quantity. For example, water is poured from a short flask into a tall, skinny one and students are asked to determine whether the amount remains the same. Deaf students showed a delay, compared to hearing pupils, in understanding that the amount remains the same. According to Marschark & Everhart, the work of Furth has enlightened us regarding cognitive development, primarily because it was conducted with children who did not acquire functional language competence, although these children received years of intensive training. Marschark & Everhart conclude that “the failure of researchers to recognize that deaf children have language, even if it is not spoken, tends to lead educators down the garden path to believe that in absence of language, deaf children do not develop cognitively” (Marschark & Everhart, 1997).

Marschark and Everhart bring additional connections to light. They reiterate that there has long been established a primary link between language and cognitive processing. These psychological
processes begin at birth, if not prenatally. Additionally, Marschark & Everhart state that there exists an early intertwining of social and cognitive development, with more varying communication skills for deaf students. Often a simple view of language and cognition is to consider them synonymous (Marschark & Everhart, 1997).

Diane Lillo-Martin, another supporter of the modularity hypothesis, makes a case for cognitive development being independent of any particular language or language modality. She argues that the language module is largely amodal and that differences between sign language and spoken language are largely superficial. People have the misconception that individuals who have superlative language will have superlative thought, as evidenced by children with Asberger's Syndrome. This has been proven to not necessarily be the case (Marschark & Everhart, 1997).

At the very least, Marschark & Everhart acknowledge that individuals who lack coherent, rule-governed language would have cognitive processes that vary from persons who have an acquired, integrated language. This language does not have to be a formal language; however, certain components of an arbitrary symbolic system, internal coding and manipulation as to allow a full range of mental life, must be present. Marschark & Everhart want to leave open the idea, the potential that a person without a symbolic language of some sort would have cognition that would be imperfect or deficient in some way. The deficiency may be very subtle. These authors propose that developmental cognition that occurs prior to the emergence of language would be of a different sort, and in some ways, be less complex than standard language. Without the emergence of symbolic language, which often happens in profoundly deaf children with hearing parents, the nature of cognition would be
lacking in some of the more subtle, and even not so subtle, aspects of standard language (Marschark & Everhart, 1997).

Considering this, Marschark & Everhart cite Gopnik and Meltzoff, who noted at "least three areas of semantic development that impacts cognitive development: general language constraints; which concepts and rules apply to specific words; and the influence of specific language structures on conceptual development". Specific language patterns used by adults to interact with children can affect conceptual development. Parents tend to label objects according to how they perceive their child's cognitive abilities. These dimensions can be observed in the prosodic, lexical and grammatical features of language that are used by parents when communicating the labeling of objects. In this case, language seems to be clearly influencing cognitive development. What children hear apparently influences what they say. More importantly, "information input through language influences what children do and think" (Marschark & Everhart, 1997).

Another perspective, more frequently associated with the views of Jean Piaget, suggests that cognitive development should be primary for young children. This observation states that children bring a set of cognitive, not language universals, to their development. Piagetians conclude that cognition drives and structures language. However, Marschark & Everhart question the validity of this perspective. They state that even if cognitive development drives language development in some ways, it does not determine it since there are several factors determining both language and thought. Both cognitive and language growth could be driven by qualitatively different kinds of experiences. Another possible origin for both language and cognitive development could be the amount of processing capacity available to children (Marschark & Everhart, 1997).
As children develop greater thinking and attention capacities, they are better able to assimilate and accommodate information important to cognitive and linguistic expansion. Marschark & Everhart say it best, "there exists a connection between cognitive and language growth, and experience drives growth in both domains. At a more global level, the common basis of cognitive and linguistic growth is not completely within the child, but inherent in the structure of the world" (Marschark & Everhart, 1997).

Cognitive and linguistic development follow two different lines, independent of each other up to a certain point, at which time they are inter-dependent on each other. Clearly, there are constraints on cognition and language, both internal and external. These constraints influence the course and timing of development along the way. One element frequently omitted from the development of cognition-language debate is the importance of environment. The importance of experiential factors might include individual-psychological variables, such as cultural. Timing is also an important factor because the timing of events is crucial especially since there are times when a child is more susceptible to particular kinds of input that will influence future growth. Various language-cognition experiences will be affected by limited input, but may have the potential to develop later, however, only with great laborious efforts. They conclude that "experiences drive both cognitive and linguistic growth", and they question which has the greatest impact on the overall development. Marschark & Everhart raise, once again, the nature vs. nurture debate (Marschark & Everhart, 1997).

Regardless of our beliefs regarding the nature vs. nurture debate, our views on implicit impact on language and cognition, Marschark & Everhart state that, "plasticity in development provides for
later resilience as language skills accelerate in more contextually appropriate environments. Early developmental mechanisms, such as discrimination and generalization, are forerunners of classification skills and category knowledge. The internal mechanisms also interact with a child's perceptions and concepts of who they are and who/what is significant in the world around them (Marschark & Everhart, 1997).

Impact of Cognitive Development on Learning Mathematics

Spelke's research provides evidence that "infants build representations of objects as complete, connected, solid bodies that persist even over occlusion, and maintain their identity through time" (Spelke, E. & Van de Walle, 1993). Furthermore, core knowledge systems appear to serve as building blocks for later development of numerical concepts and calculation skills (Spelke, E., 2000). This indicates that we are building on knowledge we already have, which is not synonymous with incidental learning. More specifically regarding mathematical development, Spelke adds, "human's tendency to navigate the flexibility avenues (necessary for the study of abstract mathematics) must come from the core system for object representation". Spelke seems to be indicating that our ability to navigate abstract mathematics may arise from the coordination of the core systems. Our ability to represent objects in a new geometrical or spatial way and our ability to problem solve, arises not from reinventing the wheel, but by bringing together building block representational systems that have existed in us since infancy (Spelke, E., 2000).

Spelke reports that children's core knowledge systems tend to be similar to other animal constructs, hinting that there has been an evolutionary history in the development of these systems. She adds, however, that core knowledge systems are limited in a number of ways, such that they are
domain specific, task specific and encapsulated. To be domain specific means the tasks represent a small subsystem of things and events perceivable by infants. Task specific means each system functions to solve a limited set of problems. Encapsulated systems operate with a certain amount of independence from other cognitive systems. Core systems serve as building blocks for the development of new cognitive skills. When children or adults develop new skills to use tools, to perform symbolic mathematical calculations, to read, to navigate by map reading, or to reason about the people and world around them, they do so by using their core knowledge systems as a building block (Spelke, 2000).

Spelke concludes that core knowledge systems found in infants contribute to later cognitive functioning in two ways. "First, core knowledge exists in older children and adults, giving them the ability to use their experiences (and language) to approach domain or task-specific problems. Second, Spelke states that core systems serve as building blocks for the development of new cognitive skills" (Spelke, E., 2000). What impact does this have on learning, especially for deaf students who may have limited linguistic input? What happens to deaf students who don't have their core knowledge pot stirred early?

**Goals of Mathematical Instruction**

The most recent publication of the NCTM Standards places emphasizes on writing across the math curriculum. Additionally, "various teaching methods should be included to cultivate a student's ability to investigate, to make sense of and make their own conjectures, and to construct meaning out of new situations". The standards summarized expected changes in instructional practices. These changes include increased attention to student active involvement in constructing and applying
mathematical ideas, problem solving as a means of instruction (not just the end), small learning groups, exploratory learning, use of technology, and student ability to communicate mathematical ideas orally and in writing (NCTM, 2000). It isn't just the last that requires language, but all goals of the NCTM require a language base from which to operate.

In the late 1980s, the National Council for Teachers of Mathematics (NCTM) set forth new standards involving curriculum development and implementation (NCTM, 1989). The goal of the Curriculum and Evaluation Standards for School Mathematics was that all students would gain "mathematical power", which is defined as the ability to explore, to make conjectures, and to reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems. The NCTM involved teachers in all steps of curriculum development. The standards encourage integration of mathematical ideas from algebra to geometry, bringing them together for graphical representations (NCTM, 1989).

More specifically, the proposed Principles and Standards for School Mathematics, drafted in 2000, state that school mathematics education bears increasing responsibility in a data-rich era. We are embarking on the age of technological advances and no student should be left behind. Mathematics instruction should provide each student access to a variety of mathematical ideas and should promote their ability to reason analytically. We are a society saturated with quantitative information ranging from global climate change to political polls and consumer reports. Having the mathematical skills will help students understand a variety of information, make informed decisions about such information, and aid them in influencing their world, both at home and abroad. School mathematics should contribute to the development of students' public awareness and make them
capable of determining the social and economic consequences of their own decisions as well as those made by elected representatives on their behalf (NCTM, 2000).

The NCTM Standards outline important instructional principles that should be emphasized in mathematical instruction. The "Equity Principle" strikes at the heart of our goal as teachers (especially teachers of deaf and hard-of-hearing students). The Equity Principle states that "mathematical instructional programs should promote the learning of mathematics by all students". The emphasis on mathematics for all is important because of the role that school mathematics has historically played in educational inequity. A student's mathematical proficiency is often used as a basis for decisions regarding future schooling and job opportunities. Moreover, mathematics has been one of the school subjects often associated with tracking, a practice whereby students are sorted into various instructional groups who follow instructional sequences based on assumptions about a student's abilities (NCTM, 2000). The problem with this method is that it often results in inequitable educational opportunities and outcomes for students, especially those of lower socio-economic means, minority groups and students with disabilities.

An emphasis on mathematics for all is also important because it challenges a pervasive belief among some members of society that a great number of students are not capable of mathematical proficiency. This belief is quite different from the equally pervasive view that students can learn to read and write in English. Verbal literacy has been expected of all students, and school programs that do not produce literate students are regarded as failures. According to the NCTM Standards, the discrepancy between English literacy and mathematical literacy and proficiency is incredible. The education system has been expected to succeed with only the mathematically able. The standards
unequivocally state that the all students can learn to think mathematically. This underscores the need for high-quality mathematical instruction for all students. Including:

- Students who have previously been denied access to any educational opportunities, as well as those who have not;
- Students who are African American, Hispanic, American Indian, and other minorities, as well as those considered part of the majority;
- Students who are female as well as male;
- Students who have not previously been successful as well as those who have (NCTM, 2000).

Adopting the belief that all students can learn mathematics is critical. But it will take more than good intentions to enact equitable mathematics instructional programs. We must overcome the assumption that mathematics is not for all students, that only some students are capable of succeeding mathematically. Low expectations are problematic because they are not random; certain populations are targeted, such as students for whom English is a second language (as related above, this often includes deaf students), students who are non-white, and students who live in poverty. Lower expectations can manifest themselves in obvious and subtle ways. Patterns in the classroom often express these lower expectations. Additionally, enrollment of students in low-level courses with minimal objectives, advising students away from advanced placement or more demanding electives or career objectives are a few of the more subtle ways students are undermined (NCTM, 2000). Placing deaf students who are mainstreamed in resource courses is another example of sending a subtle message of lower expectations.

The standards point out that mathematical content and processes are important to school mathematics instructional programs. The concepts most fundamental to deeper mathematical study are: place value, function, scaling and similarity, structure in the number system and rate of change.
Mathematics curriculum should include content and processes that make up a comprehensive set of instructional goals. A comprehensive curriculum should achieve an appropriate balance of conceptual knowledge and procedural competence in important areas. Another aspect of a comprehensive curriculum is how well that curriculum provides students with opportunities to learn about the nature and practice of mathematics. Students need to experience the humanness, and the tremendous power of mathematics. Although mathematics is often abstract, students need to understand the connection between mathematics and real-world phenomena. Students need to see that mathematics is based in rigorous systems of definitions, axioms, and theorems. They need to come to understand, through experiencing mathematics first hand, that reasoning often requires considerable time and experimentation with arguments that fail before finding one that succeeds (NCTM, 2000).

Mathematics is developed through human activities such as problem solving, reasoning, representing, communicating, and making connections. From a very young age, children are interested in mathematical ideas (whether they recognize them as such or not), and they gradually develop a more complex set of informal ideas about numbers, patterns, shapes, data and size through their experiences in everyday life. Therefore, children learn many mathematical ideas quite naturally before they enter school (NCTM, 2000). This is often referred to as incidental learning.

A major goal of school mathematics instructional programs should be to create autonomous
learners. Learning mathematics should include developing students' abilities to perceive mathematics in a powerful way and building on previous knowledge to look at new situations. Students' mathematical dispositions are manifested in the way they approach mathematical learning. We need to foster students' confidences in their ability to problem solve and encourage them to be flexible when exploring mathematical ideas. They need to be allowed to try new ways to solve problems, create their own conjectures and persevere when approaching challenging tasks. Classrooms must foster their abilities to think independently, exchange ideas and deepen conceptual understanding. Students must be actively engaged with mathematics. Active engagement with mathematics is best fostered through problems that are motivating and challenging. Their realization that their present knowledge is insufficient may provide further incentive and motivation to explore problems with interesting or real-world contexts (NCTM, 2000).

The standards include spatiality in their recommendations for optimal mathematical instructional methods. The standards state that mathematics instruction programs should include attention to geometry and spatial sense so that all students are able to:

- Analyze characteristics and properties of two- and three-dimensional geometric objects;
- Select and use different representational systems, including coordinate geometry and graph theory;
- Recognize the usefulness of transformations and symmetry in analyzing mathematical situations;
- Use visualization and spatial reasoning to solve problems both within and outside mathematics (NCTM, 2000).

"Much of mathematics should be learned through activity, with physical models, drawings, and dynamic software as learning tools. Well-designed activities, availability of appropriate tools, and teacher support can enable students to make conjectures about geometric structure, explore the conjectures of others, and reason about mathematical and geometrical ideas. The eventual goal is for students to gain experience utilizing a variety of visual and coordinate representations, and to be able
to use these representations to analyze problems and study mathematics in the world around them” (NCTM, 2000).

The NCTM standards state that it will be important for our future that our students graduate from high school able to demonstrate the meaning of numbers, the possible operations that can be performed on numbers, and be able to analyze and solve more complex problems (NCTM, 2000).

Mathematical Instruction and Measurement of Deaf students' Progress

According to Pagliaro and Ansell, the reformed NCTM standards also apply to deaf and hard-of-hearing (D/HH) students, as the emphasis on mathematics for all was the hallmark of the Curriculum and Evaluation Standards (NCTM, 1999; Pagliaro, C.M. & Ansell, E., 2002). Even though “70% of teachers of the deaf seem to be incorporating problem solving in their curriculum, test results show that D/HH students continue to make little progress in this area” (Pagliaro, C.M. & Ansell, E., 2002). Earlier research by Marschark and Everhart state that it is important to understand that deafness changes the nature of an individual’s experience with the world. They add that deaf children will have both qualitatively and quantitatively different experiences from their hearing peers, especially during the critical period for learning language. This is generally because of hearing loss and the relatively language-poor environment in which they spend their early (critical) years (Marschark, M. & Everhart, V., 1995).

These same authors state there are a variety of cognitive consequences of hearing loss that will affect and limit the deaf child’s interactions with the world around them. The loss of hearing will have the most significant impact on the 90% of deaf children who are born to hearing parents. Because the parents are English speaking and are not likely to know sign language, these children do not have an effective channel for communicating. Since these children do not have an effective mode
of communication during the early (critical) years, their educational experiences are less likely to be as rich as their hearing peers (Marschark, M. & Everhart, V., 1995).

This leads me to think about incidental learning. Children learn through interacting with the world around them (Bochner, J. & Albertini, J. 2001). They hear about time, elapsed time problems, and other mathematical concepts through daily interactions with family and friends. Marschark and Everhart seem to agree as they state that the wealth of conceptual knowledge that children normally obtain from indirect teaching is important to their future educational success (op. cit.). However, Marschark and Everhart are unsure whether the etiology of deafness affects the extent of their linguistic (or perhaps non-linguistic) experiences. They conclude that "the etiology of deafness is not any more significant for deaf children's cognitive development than the innumerable other factors that influence both deaf and hearing learners". Marschark & Everhart note, "the relative unavailability of effective communication for young deaf children reduces the diversity of their life experiences". This has had and will have an affect on their ability to function in educational (including mathematics) settings, which might normally require flexibility and creativity (Marschark & Everhart, 1995).

Literacy is important in education (Bochner, J.H. & Albertini, J.A., 2001). This seems to mean to me that students must have a mode of communication, and we must start early communicating with them for educational success to be a realistic goal. The acquisition and the understanding of language use have been proven to impact the student's ability to learn and studies have demonstrated that the reading comprehension skills of deaf students remain considerably lower than their hearing peers (Bochner, J.H. & Albertini, J.A., 2001).
According to Powers, Gregory & Thoutenhofd, there are general implications that can be drawn between language and mathematical thinking. We all experience patterns of learning and attaining information. Therefore, Powers, Gregory & Thoutenhofd ask the stirring question: “how do deaf children solve mathematical problems?” They explore the nature of the difficulties deaf children experience.

Powers, Gregory & Thoutenhofd report on an early study carried out by the National Council of Teachers of the Deaf, which used the Schonell Arithmetic Test, testing 246 students, obtained differences of approximately 2.5 years in mathematical instruction between deaf students and their hearing peers. Although this study was conducted in the mid 1950’s, Gregory states that “later studies yielded similar results”. While deaf students lagged behind their hearing peers with regard to their mathematical ages, the delay was not as great for mathematics as for English and reading (literacy level) (Powers, Gregory & Thoutenhofd, 1998).

Kelly & Mousley report that although deaf students are testing on-par with hearing students in computational arithmetic, when it comes to abstract language of higher level mathematics, deaf student lag behind even further (Kelly & Mousley, 1999). According to Powers, Gregory & Thoutenhofd, in a study conducted by Wood and his colleagues in 1986, a correlation, albeit small, was drawn between hearing loss and mathematical test results. This study reported a marginally decreased mathematical ability as degree of hearing loss increased. The relationship between depth of hearing loss and mathematical age was more significant for girls than boys. This implies that boys have an increased ability to think spatially and mechanically, and could therefore account for the disparity between the genders (Powers, Gregory & Thoutenhofd, 1998).
Powers, Gregory & Thoutenhofd further add that when considering school placement, Wood et al., reported little difference between students educated in special schools and those educated in units. An educational unit could be compared to a cluster, whereby students with similar diagnoses, are clustered and taught in close-knit groups. Powers, Gregory & Thoutenhofd found that "the lack of significant difference between the mathematical ages of deaf students reflects the weak relationship between hearing loss and mathematical attainment within the hearing impaired sample". Additional studies have reported similar results regarding deaf students lagging behind their hearing peers. Concurrently, they report Zwibel and Allen's attempt to account for the disparity by controlling for communication skill and intellectual potential in comparing the groups. All of these researchers recognize the reality that other factors such as home background, teaching ability, communication skills, teacher and even parental expectations may also play a part. Many of the affective variables pointed out in Powers, Gregory & Thoutenhofd's article can also apply to the general population of students taking standardized tests.

By looking to understand the relationship between thought and language, Powers, Gregory & Thoutenhofd point out that "words are critical for developing mathematical understanding, and many of the words with which deaf children have the most difficult are logical connectives such as if and because". There are many ways in which language and symbols are used in mathematical problems, and these terms and symbols often have very specific meaning. The very nature of the vocabulary of mathematics is special and has the potential to create problems for the English as a second language learner (Powers, Gregory & Thoutenhofd, 1998). Mathematics involves seeing patterns, thinking logically, thinking geometrically, recognizing relationships between numbers, and having a sense of
whether or not a solution to a problem is logical. Mathematical thinking also involves developing strategies for solving novel mathematical problems through the use of language (NCTM, 2000).

Mathematics involves a way of thinking, which is considered by some to be a specific type of intelligence. Spelke and others contend that we are all created equal with regard to core knowledge, which includes this intelligence. However, whether it is as strong as one's language abilities or physical abilities varies from one person to the next (Spelke, 2000). Clearly, there are several factors that contribute to the difficulty experienced by deaf students when it comes to problem solving. According to Mousley & Kelly, since literacy development is critical to metacognitive skill development, by virtue of their language deficits, deaf students suffer greater difficulties than their hearing peers. They argue that by employing systematic, precise methods and allowing students to fully explore and focus on relevant information, we can go beyond rote or trial and error methods of educating deaf students (Mousley & Kelly, 1998).

Mousley & Kelly reported that deaf college students enrolled in first- and second-year mathematics courses at NTID demonstrated a substantial diversity when it came to problem solving. The variability included: impulsive responses, lack of transfer of previously acquired knowledge, an inability to organize relevant information presented and a misunderstanding of the goals of the problem (what do they need to know based on what is presented in the problem). Furthermore, they reflect on research that has shown that the cognitive performance of deaf students can be influenced with the "appropriately structured mediated learning experiences". Mediated learning refers to those human interactions that generate the capacity of individuals to change, to modify themselves in the direction of greater adaptability and toward the use of higher mental processes. With mediated
learning, students are guided through increasing their mathematical prowess using previous knowledge and their environmental experiences to develop systematic and generalized processes to solve unfamiliar mathematical problems. This method results in a blending of solutions that allows and encourages student self-correction. Through the structured process, students develop skills needed to generalize and set-up problems independently. However, rigorous attention to concepts and language are important (Mousley & Kelly, 1998).

Interestingly, in another article, Kelly and Mousley have shown that the performance of deaf and hearing students were similarly affected as the complexity of problems increased. The data from thirty-seven deaf and ten hearing students indicated that, regardless of hearing status, all students' performances declined as problem complexity and difficulty increased. Furthermore, they state that given the similarity in performance with the graphic problems and the least complex word problems, the data suggests that the deaf students' computation and solving skills for these kinds of problems were comparable to their hearing peers (Kelly & Mousley, 1999).

Perhaps, as Kelly and Mousley point out, often it is the case that students, deaf and hearing, do not understand the value of developing analytical skills needed to problem solve. They provide suggestions made by Salomon & Perkins in 1989, for developing mathematical skills of students. The suggestion they make could apply to a diverse student population. These suggestions include the provision of multiple opportunities for varied practices; provide opportunities to solve real-world problems that students are likely to encounter later in life; and teach them not to memorize formulae, but to incorporate general rules, strategies or schema to solve a variety of similar problems. They conclude that teachers must provide multiple opportunities, varied problem solving experiences and
enhance the development, fluency for problem solving and transference of book learning to real-world solutions, especially those they may encounter in the world of work. (Kelly & Mousley, 1999). Their comments are in agreement with the NCTM standards.

Pagliaro adds that teachers whose practices reflect mathematical reform increase the amount of problem-solving activities in their instruction, use a multi-method approach, and integrate content across topics, disciplines, and grade levels. Additionally, reform-minded teachers allow sufficient planning time for their mathematics courses. They plan with other teachers to encourage an integrated approach, which incorporates mathematics across the curriculum. Unfortunately, Pagliaro reports that statistics from this study indicate that out of almost two hours of planning, teachers spent approximately eight minutes of planning time was spent planning mathematics with teachers from other areas (Pagliaro, 1998a).

Additionally, it was discovered that ninety-three percent of teachers scored in the neutral range on the MTSI-A, a test, which measures teaching style to standard-like behavior. Although previously teachers had reported utilizing the NCTM Standards when developing curriculum and lesson plans, in fact only seventeen percent were very familiar with NCTM Curriculum & Evaluations Standards (1989). Data pertaining to individual strategies and practices indicated a low-to-moderate number of teachers made use of activities suggested in the standards. Pagliaro states that these statistics are somewhat skewed since only nineteen percent of teachers limited the use of drill and practice. In addition, Pagliaro adds that teachers must present more open-ended challenges, must integrate mathematics across topics, disciplines and grade levels. Pagliaro further adds a prescription for professionals within deaf education.
First, she states, "teachers of the deaf should increase their own familiarity with mathematics reform". Second, education professionals should work together for continuous improvement of their school's mathematics program. Third, time spent planning mathematics with other teachers should be increased in order to promote integration of mathematics across all content areas and foster a cohesive curriculum. Fourth, technology should be taught and available for all students as a tool to expand instruction and enhance understanding and knowledge. Finally, the field of deaf education must recognize the significance of mathematics knowledge and how this knowledge relates to the real world. Pagliaro concludes that the results of her survey reveal a need for increased attention to mathematics reform in deaf education if we expect deaf students to compete in the world work market in the future" (Pagliaro, 1998b).

Considering deaf students typically enter the language forum later than their hearing peers, Mousley and Kelly report that deaf and hard of hearing students could solve manual problems, calculations and computations, since reading levels did not significantly influence their performance in this arena. However, several aspects of the students' performance in solving both puzzles and math word problems were indeed associated with reading levels. The five students in the study who read on the 10.3-11.1-grade level were more proficient at articulating their strategies for solving either kind of problem than the students with reading scores between levels 6.9-9.7. The ability of Group A (the higher reading level group) to explain in sign language as opposed to written form was 58.2% vs. 49.8% experienced successful communication of the problem and solution. The ability of Group B (the lower reading level group) for explaining the problem and solution, using sign language as opposed to written form, was 63% successful vs. 33.4% unsuccessful explaining the problem and solutions. Similar results were reported for signed vs. written explanations of math word problems. Group A out scored Group B by twenty six percent when it came to signed explanations of math word problems. Clearly, reading level has a significant impact on word problem understanding (Mousley & Kelly, 1998).
Similarly, it would seem to follow that students with increased communication skills will provide more clear explanations of their mathematical understanding and process by which they problem-solve.

**The Spatial Connection**

Parasnis, Samar, Gettger & Sathe, when investigating whether deafness contributes to enhancement of visual spatial cognition independent on knowledge of a sign language, cite Parasnis, & Samar (1995), who found deaf college students to be flexible in their abilities to redirect their attention in a spatial manner. When language is added to the educational mix, findings indicated that users of ASL tended to exhibit superior performances in some of the visuospatial areas. The findings of Parasnis, Samar, Gettger & Sathe support the suggestion that enhanced visuospatial skills do not represent sensory compensation, but rather exposure to a visual-spatial language results in an ability to organize problems in a way that has some broader cognitive advantages (Parasnis, Samar, Gettger & Sath, 1996).

There also seems to be a kinesthetic advantage for students who use a visual-spatial language. Beyond sign language being a visual language, it is also has kinesthetic advantages for literacy. Children who experience a variety of language exposure may encode information in qualitatively different ways. With lists of simple stimuli, such as words, pictures or numbers, deaf students rely heavily on visuospatial short-term memory codes, as opposed to hearing students who rely on verbal-sequential coding, or aural input. The results of several studies reported that temporal and spatial coding of short sequences may lead to equivalent short-term memory performance (Marschark and Lukomski, in press). This has the potential to have a direct impact on a spatial course such as mathematics.
Deaf students are very much a diverse group of learners. No single method of instruction could be applied to any population. Each group of deaf students is different, just as hearing students are heterogeneous. If we are to understand the depths and diversity of skills and knowledge underlying the cognitive process and academic performance among deaf students, we must understand the underlying similarities between deaf and hearing students’ knowledge development. Yet, we must recognize the heterogeneity of these two different student populations, including mode of communication.

Summary:

As the most highly developed mammal, the human being has the most highly developed language or ability to communicate. Communication has been extremely important to the intellectual development of mankind and our society. However, communication and language are not synonymous, but interdependent in the educational setting.

Language is defined in many ways. Chomsky considers the study of language the essence of human beings. Fisher agrees with Chomsky, however adds that human involvement is important for linguistic development. Children learn to master their world through language exposure, what they hear, see, touch, and experience. Therefore, it is important that children have exposure to language early in life. Chomsky, Fischer and others refer to a human’s early life as the critical period for language development. Although there is some question as to the exact age assigned to the critical period, children begin to develop language structure by age four. Children who begin to learn and acquire language by age five, and certainly if they develop language prior to puberty, they will have more flexible constructs with which to develop deep, meaningful language.
Among the Piagetians, there is a commonly held view that language has a thought structuring property or that language produces thought. This implies that language is intrinsically necessary for the development of independent ideas. While Chomsky considers language essential to who we are, he considers language and thought to be distinct developments. Marschark & Everhart establish a primary link between language and cognition. There seems to be little argument that language and cognition are intertwined early in life, with social development and meaningful, linguistic input, as two of the greatest influential factors. Adults who interact with children early in life are influential in the language development process since children observe the world around them pre-lingually, therefore, developing the building blocks for learning (Marschark & Everhart, 1997). From my observations, children also learn much about the world around them from the way we talk and interact with them. Children learn from the lullabies we sing and the nursery rhymes we read (sign) to them. Fischer discusses in class how we experience how things and relationships relate long before we can assign linguistical elements. Children learn much about the passage of time, their relationship to their physical world, and how language is structured from their incidental experiences early in life. Their ability to understand what they learn from incidental learning, the level to which they are able to transfer their linguistic input for empathetic use and ingestion, and their understanding of how past events affect the present and future will affect their ability to express their own needs and thoughts.

Researchers suggest that second-language acquisition, i.e., deaf students learning English, is much more available to people who have a first-language as a foundation. This implies that English acquisition will be easier for deaf students who have a first-language base in ASL. Often the
conclusion is drawn that deaf students who have a first-language in ASL, will develop educationally similar to other English as a Second-Language (ESL) students in all aspects of their language development. Consequently, they are often placed in remedial level English classes, possibly without appropriate communication support. This situation places deaf students at a distinct disadvantage. Unfortunately, they are likely to remain in remedial or resource classes throughout their education.

Swisher describes the five levels of progression that students need to accomplish certain mechanisms of second language learning. These five levels of input affect linguistic output. Swisher states that these five level progressions are similarly important for all language learners. Variations of output depend on the variations of input and level of linguistic accomplishments. Like the progression of learning to rollover, to crawl, to stand, accomplish balance and walk are similar to the five levels of language development. This process could also be compared to Maslows' levels of self-actualization, from survival needs to self-expression and actualization. Any assumptions made about exposure to language (the quantity vs. quality issue) are multifarious, adding to the complexity of learning English.

Entrenched deep in the heart and soul of our linguistic development, according to Spelke, is the idea that infants build complete, connected representations that persist through many changes and other exposures throughout life. Like Chomsky, she believes we are hard-wired for language and other conceptual relationships. Our core knowledge systems are what make us unique, unlike any other, similar to DNA. This unique wiring allows us to build representations about such issues as numerology, abstract concepts and reasoning. Unlike incidental learning, our core knowledge indicates that we are building on knowledge we already have. Our ability as humans to represent objects in a
new way, spatial or otherwise, arise from bringing together internal representational systems that are innate and have existed within since infancy. Our core knowledge systems seem to be more like other animal constructs, suggesting that these systems experience an evolutionary process. Furthermore, these systems operate independently, serving as building blocks for developing new cognitive skills. Spelke's ideas about core knowledge give rise to further evidence that children require early intellectual stimulation to maximize their potential. Regardless of our internal structure, mathematical instruction is important for all students.

NCTM standards have been designed for all students. Deaf and hard-of-hearing, blind, cognitively challenged, developmentally delayed and gifted students are all included in the goals of the NCTM standards. Reading and writing are fundamental to the skills necessary for mathematical power, according to the NCTM standards. In addition to reading and writing, skills such as problem solving, exploratory learning, and use of technology are fundamental. Nonetheless, all of the goals of the NCTM standards require a language base from which to operate. However, fair and equal access is often a problem.

Although the political and legal environment of the educational system are not the focus of this project, the Equity Principal strikes at the heart of instruction. The Equity Principal, as stated in the NCTM standards, states that mathematical instruction should promote equal access to mathematical instruction for all students. However, there can be no denying that mathematics is one school subject often with tracking. Tracking is one of the most inequitable educational practices, which can have injurious results, especially for minority groups. The standards state unequivocally that all students can learn to think mathematically.
Questions often asked by educators and researchers, include why are we not making greater strides in deaf education, and why do deaf students continue to lag behind their hearing peers? We need to examine the requirements for teacher certification in this area, for it is this author's opinion that the research conclusively shows a misunderstanding of students' needs and a lack of appropriate preparation for teachers in this area. Mathematical instruction should include the use of visuo-spatial tools, explicit instruction of the language of mathematics and include the same content information requisite for all students. Without teaching the language of mathematics, we risk not achieving the balance of conceptual knowledge and procedural competence in many important areas, such as Algebra and Calculus.

Using the visual and spatial aspects of ASL can be advantageous when teaching the more abstract concepts of mathematics (i.e., Algebra and Calculus). Examples of visuo-spatial instruction from my lesson plans include teaching the real number line, by line dancing. The students stand in line and dance (move) the appropriate number of spaces left for negative numbers and right for positive numbers. The idea is to involve the student, body and mind in the lesson. Involving our students allows them the opportunity to gain true understanding and incorporate mathematical ideas across the curriculum. As teachers, we can make mathematics relevant to other subject areas, such as science, economics, music and art. We must relate mathematics to the world around them.

We must take advantage of teachable moments. Classrooms need to foster students' ability to think, problem-solve, make representations, communicate their ideas and reasoning. Students must have the language base from which to operate in order to develop more complex ideas about numbers, patterns, shapes, data representations and statistical information. The NCTM standards clearly
indicate a need to include spatiality in our instruction for optimal mathematical understanding. Analyzing, representing, recognizing other connections and use of visualization and spatial reasoning to solve problems both within and outside mathematics should be the goal of all students, according to NCTM.

Although Spelke contends that we are all created equal with regard to core knowledge, Marschark & Everhart state, it is equally important for teachers to remember the uniqueness of deafness and how deafness changes the individual's life experiences. Additionally, it is this unique experience that will impact deaf students' use of language. Since language impacts all aspects of education, we need to be especially sensitive to this and develop strategies for presenting the terms and symbols unique to mathematics. Pagliaro adds that teachers must present more open-ended challenges, integrate mathematics into our discussions, and increase their own familiarity with mathematics reform.

Part of mathematics reform is appropriate communication support. Great strides have been made to provide educational communication support through certified interpreters. A supplementary tool available to students and interpreters is the book, *Signs for Science and Mathematics: A Resource Book for Teachers and Students*, developed at NTID (Caccamise & Lang, 1996). This wonderful resource has many signs for science and mathematics; however, there are not always signs that convey the mathematical meaning. Mousley & Kelly thoroughly explore six alternative strategies that enhance problem solving. Then in a subsequent presentation, Kelly & Mousley discuss the importance of specific transfer, the ability to transfer the mathematical rules and formulae to problem solving. However, because there are everyday words in general use in the English language, which when used
in mathematics context may have a specific meaning, different from the general use, there may be a sign for terms that can be misused in a mathematical setting. Another reason for the difficulty of mathematics for deaf children is the use of symbolic language, alone or in mathematical sequences such as $\Sigma$, $\Pi$, and $\delta$. The interpretation for more exact use of such terms as similar, difference, dividend and derivative will rely on any particular interpreter's mathematical knowledge and transliteration ability.

Spelke and others contend that we all have mathematical potential, and that rather than mathematical variance between genders being a nature problem, this is instead a nurture problem. Considering the minimal statistical difference between deaf boys and girls, Powers, Gregory & Thoutenhoefd conclude that the results suggested mathematics problems involve spatial thinking, which is available to all of us to some degree. If boys and girls are equally capable of spatial thinking, then what attitudes do we bring to the classroom that continue to influence students' exposure to mathematics and the teachers' methods of instruction?

Vince Daniele said it best ten years ago, "preparing students for the workplace is only one goal of literacy". Educators face the reality that deaf students have traditionally struggled with qualitative literacy, therefore we may disarm the myth that deaf students have more difficulty with mathematics than their hearing peers if they have the tools (i.e., language). Daniele concludes that "mathematics can be characterized by the language, and we use the language to quantify concepts and express various relationships". Therefore, based on Chomsky's critical period for language development and Spelke's core knowledge research, there is no doubt earlier is better. Additionally, language is such an integral part of mathematics it appears to this author that in contrast to what
Hillegeist and Epstein report in 1987, we now know that the nature of the language effect is completely clear. A mathematics student must know more than only the specific mathematical meanings to acquire the language of mathematics and to be a proficient problem solver. However, the culmination of this research indicates a question remains unanswered. Is it possible that there exists a critical period, during which if students are not exposed to the language of mathematics, that the opportunity will be lost for full mathematical acquisition?
REFERENCES


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## Summary of School-age Child's Language Development

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Syntax/ Morphology</th>
<th>Semantics</th>
<th>Phonology</th>
<th>Pragmatics</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Produces short direct requests</td>
<td></td>
<td></td>
<td>Uses mostly passives (lost, left, broken) Repeats for repair Begins to use gender topics</td>
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<tr>
<td>6</td>
<td>Comprehends parallel embedding, imperative commands, -man, suffix (-er)</td>
<td>Identifies syllables Acquires rule for (/z/, /ts/, /tz/)</td>
<td>Responds to indirect hints</td>
<td>Repeats with elaboration for repair Uses adverbial conjuncts (now then, so, though)</td>
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<tr>
<td></td>
<td>Uses <em>many</em> with plural nouns</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>7</td>
<td>Comprehends 'because' Follows adult ordering of adjectives</td>
<td>Uses left/right and front/back</td>
<td>Is able to manipulate sound units to rhyme</td>
<td>Uses &amp; understands most deictic terms or produce stems</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Recognizes unacceptable sound sequences.</td>
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<tr>
<td>8</td>
<td>Uses full passives (80% of children)</td>
<td></td>
<td>Is able to produce all English sounds</td>
<td>Sustains concrete topics Begins considering others' intentions</td>
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<tr>
<td>9</td>
<td>Comprehends and uses <em>tell</em> and <em>promise</em></td>
<td>Has generally undergone complete syntagmatic-paradigmatic shift Begins to interpret psychological states described with physical terms (<em>cold</em>, <em>blue</em>), but misinterprets</td>
<td></td>
<td>Sustains topics through several terms Addresses perceived source of breakdown in repair</td>
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<tr>
<td>Age (years)</td>
<td>Syntax/Morphology</td>
<td>Semantics</td>
<td>Phonology</td>
<td>Pragmatics</td>
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<tr>
<td>10</td>
<td>Comprehends &amp; uses ask</td>
<td>Comprehends in and on used for temporal relations</td>
<td>Comprehends most familial terms</td>
<td></td>
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<tr>
<td>11</td>
<td>Comprehends if and though Creates much w/ mass nouns</td>
<td>Creates abstract definitions</td>
<td>Uses conventional form definitions</td>
<td>Sustains abstract topics</td>
</tr>
<tr>
<td>12</td>
<td>Uses stress contrasts</td>
<td>Uses stress contrasts</td>
<td>Uses stress contrasts</td>
<td></td>
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<tr>
<td>13-15</td>
<td>Comprehends unless Comprehends all embedding</td>
<td>Comprehends proverbs</td>
<td>Comprehends at used for temporal relations</td>
<td></td>
</tr>
<tr>
<td>16-18 (high school)</td>
<td>Uses vowel-shifting rules (divine-divinity)</td>
<td>Uses sarcasm &amp; double meanings</td>
<td>Uses sarcasm &amp; double meanings</td>
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<td></td>
<td>(high school)</td>
<td></td>
<td></td>
<td>Deliberate use of metaphors</td>
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<td></td>
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<td>Can take others perspective &amp; accepts others' knowledge differs</td>
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