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Heuristics for selecting gray scale morphological structuring elements

Paul Fetter
Title: Heuristics for Selecting Gray Scale Morphological Structuring Elements

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Date: July 29, 1994
ABSTRACT

This thesis explores some heuristics for choosing 8 bit gray scale morphological structuring elements for reducing noise. The variables of size, shape and volume that enter into the choice of structuring elements create a very large number of possible structuring elements. Some general heuristics to guide the choice of an appropriate structuring element will make the task easier. Both the absolute error of the image and the appearance of the image will be used to judge the results. The experiments were performed on 3 images. Each of the images had noise added before processing; one set of data had 10 percent of the pixels disturbed by noise, the other had 20 percent of the pixels disturbed by noise. The resulting 6 images were then filtered with 10 different structuring elements and the resulting images were then compared against the respective baseline image. The conclusions were guided by the resulting absolute error values.
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Borland C                              Borland International

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# Table of Contents

THESIS RELEASE PERMISSION FORM .................................................. ii

ABSTRACT ............................................................................................ iii

TABLE OF CONTENTS ............................................................................... v

LIST OF FIGURES ................................................................................... viii

LIST OF TABLES ................................................................................... xxii

LIST OF EQUATIONS ............................................................................ xxiii

GLOSSARY ........................................................................................... xxiv

1. BINARY MORPHOLOGY ................................................................. 1

  1.1. Input ............................................................................................ 1

  1.2. Dilation ....................................................................................... 2

  1.3. Erosion ....................................................................................... 2

  1.4. Open ........................................................................................... 4

  1.5. Close ........................................................................................... 4

2. GRAY SCALE MORPHOLOGY .......................................................... 5

  2.1. Input ........................................................................................... 5
List of Figures

FIGURE 2-1 GRAPH OF AN ARBITRARY FUNCTION ............................................. 6
FIGURE 2-2 GRAPH OF AN UMBRA ................................................................. 7
FIGURE 3-1 BASELINE COKE IMAGE ............................................................... 16
FIGURE 3-2 COKE IMAGE WITH 10 PERCENT ERROR ..................................... 17
FIGURE 3-3 COKE IMAGE WITH 20 PERCENT ERROR ..................................... 18
FIGURE 3-4 BASELINE GIRL IMAGE ............................................................... 19
FIGURE 3-5 GIRL IMAGE WITH 10 PERCENT ERROR ...................................... 20
FIGURE 3-6 GIRL IMAGE WITH 20 PERCENT ERROR ...................................... 21
FIGURE 3-7 BASELINE ROY IMAGE ............................................................... 22
FIGURE 3-8 ROY IMAGE WITH 10 PERCENT ERROR ....................................... 23
FIGURE 3-9 ROY IMAGE WITH 20 PERCENT ERROR ....................................... 24
FIGURE 4-1 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 3 ............................. 31
FIGURE 4-2 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 5 ............................. 31
FIGURE 4-3 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 7 ............................. 31
FIGURE 4-4 10 PCT. ERR. DILATED BY FLAT SE OF SIZE 3 ............................. 31
FIGURE 4-5 10 PCT. ERR. DILATED BY FLAT SE OF SIZE 5 ............................. 32
FIGURE 4-6 10 PCT. ERR. DILATED BY FLAT SE OF SIZE 7 ............................. 32
FIGURE 4-7 20 PCT. ERR. ERODED BY FLAT SE OF SIZE 3 ............................. 32
FIGURE 4-8 20 PCT. ERR. ERODED BY FLAT SE OF SIZE 5 ............................. 32
FIGURE 4-75 20 PCT. ERR. OPENED BY CONCAVE SE OF SIZE 7 .................43
FIGURE 4-76 10 PCT. ERR. OPENED BY CONCAVE SE OF SIZE 7 .................43
FIGURE 4-77 20 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 5 ................44
FIGURE 4-78 10 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 5 ................44
FIGURE 4-79 20 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 7 ................44
FIGURE 4-80 10 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 7 ................44
FIGURE 4-81 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 3.....................45
FIGURE 4-82 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 5.....................45
FIGURE 4-83 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 7.....................45
FIGURE 4-84 10 PCT. ERR. DILATED BY FLAT SE OF SIZE 3.....................45
FIGURE 4-85 10 PCT. ERR. DILATED BY FLAT SE OF SIZE 5.....................45
FIGURE 4-86 10 PCT. ERR. DILATED BY FLAT SE OF SIZE 7.....................45
FIGURE 4-87 20 PCT. ERR. ERODED BY FLAT SE OF SIZE 3.....................46
FIGURE 4-88 20 PCT. ERR. ERODED BY FLAT SE OF SIZE 5.....................46
FIGURE 4-89 20 PCT. ERR. ERODED BY FLAT SE OF SIZE 7.....................46
FIGURE 4-90 10 PCT. ERR. ERODED BY FLAT SE OF SIZE 3.....................46
FIGURE 4-91 10 PCT. ERR. ERODED BY FLAT SE OF SIZE 5.....................46
FIGURE 4-92 10 PCT. ERR. ERODED BY FLAT SE OF SIZE 7.....................46
FIGURE 4-93 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 3.....................47
FIGURE 4-94 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 5.....................47
FIGURE 4-95 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 7.....................47
FIGURE 4-96 10 PCT. ERR. OPENED BY FLAT SE OF SIZE 3.....................47
FIGURE 4-97 10 PCT. ERR. OPENED BY FLAT SE OF SIZE 5.........................47
FIGURE 4-98 10 PCT. ERR. OPENED BY FLAT SE OF SIZE 7.........................47
FIGURE 4-99 20 PCT. ERR. CLOSED BY FLAT SE OF SIZE 3..........................48
FIGURE 4-100 20 PCT. ERR. CLOSED BY FLAT SE OF SIZE 5.........................48
FIGURE 4-101 20 PCT. ERR. CLOSED BY FLAT SE OF SIZE 7.........................48
FIGURE 4-102 10 PCT. ERR. CLOSED BY FLAT SE OF SIZE 3..........................48
FIGURE 4-103 10 PCT. ERR. CLOSED BY FLAT SE OF SIZE 5..........................48
FIGURE 4-104 10 PCT. ERR. CLOSED BY FLAT SE OF SIZE 7..........................48
FIGURE 4-105 20 PCT. ERR. DILATED BY PYRAMID SE OF SIZE 3....................49
FIGURE 4-106 20 PCT. ERR. DILATED BY PYRAMID SE OF SIZE 5....................49
FIGURE 4-107 20 PCT. ERR. DILATED BY PYRAMID SE OF SIZE 7....................49
FIGURE 4-108 10 PCT. ERR. DILATED BY PYRAMID SE OF SIZE 3....................49
FIGURE 4-109 10 PCT. ERR. DILATED BY PYRAMID SE OF SIZE 5....................49
FIGURE 4-110 10 PCT. ERR. DILATED BY PYRAMID SE OF SIZE 7....................49
FIGURE 4-111 20 PCT. ERR. ERODED BY PYRAMID SE OF SIZE 3....................50
FIGURE 4-112 20 PCT. ERR. ERODED BY PYRAMID SE OF SIZE 5....................50
FIGURE 4-113 20 PCT. ERR. ERODED BY PYRAMID SE OF SIZE 7....................50
FIGURE 4-114 10 PCT. ERR. ERODED BY PYRAMID SE OF SIZE 3....................50
FIGURE 4-115 10 PCT. ERR. ERODED BY PYRAMID SE OF SIZE 5....................50
FIGURE 4-116 10 PCT. ERR. ERODED BY PYRAMID SE OF SIZE 7....................50
FIGURE 4-117 20 PCT. ERR. OPENED BY PYRAMID SE OF SIZE 3....................51
FIGURE 4-118 20 PCT. ERR. OPENED BY PYRAMID SE OF SIZE 5....................51
FIGURE 4-119 20 PCT. ERR. OPENED BY PYRAMID SE OF SIZE 7

FIGURE 4-120 10 PCT. ERR. OPENED BY PYRAMID SE OF SIZE 3

FIGURE 4-121 10 PCT. ERR. OPENED BY PYRAMID SE OF SIZE 5

FIGURE 4-122 10 PCT. ERR. OPENED BY PYRAMID SE OF SIZE 7

FIGURE 4-123 20 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 3

FIGURE 4-124 20 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 5

FIGURE 4-125 20 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 7

FIGURE 4-126 10 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 3

FIGURE 4-127 10 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 5

FIGURE 4-128 10 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 7

FIGURE 4-129 20 PCT. ERR. DILATED BY DOME SE OF SIZE 5

FIGURE 4-130 20 PCT. ERR. DILATED BY DOME SE OF SIZE 7

FIGURE 4-131 10 PCT. ERR. DILATED BY DOME SE OF SIZE 5

FIGURE 4-132 10 PCT. ERR. DILATED BY DOME SE OF SIZE 7

FIGURE 4-133 20 PCT. ERR. ERODED BY DOME SE OF SIZE 5

FIGURE 4-134 20 PCT. ERR. ERODED BY DOME SE OF SIZE 7

FIGURE 4-135 10 PCT. ERR. ERODED BY DOME SE OF SIZE 5

FIGURE 4-136 10 PCT. ERR. ERODED BY DOME SE OF SIZE 7

FIGURE 4-137 20 PCT. ERR. OPENED BY DOME SE OF SIZE 5

FIGURE 4-138 20 PCT. ERR. OPENED BY DOME SE OF SIZE 7

FIGURE 4-139 10 PCT. ERR. OPENED BY DOME SE OF SIZE 5

FIGURE 4-140 10 PCT. ERR. OPENED BY DOME SE OF SIZE 7
FIGURE 4-141 20 PCT. ERR. CLOSED BY DOME SE OF SIZE 5 ..........55
FIGURE 4-142 20 PCT. ERR. CLOSED BY DOME SE OF SIZE 7 ..........55
FIGURE 4-143 10 PCT. ERR. CLOSED BY DOME SE OF SIZE 5 ..........55
FIGURE 4-144 10 PCT. ERR. CLOSED BY DOME SE OF SIZE 7 ..........55
FIGURE 4-145 20 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 5 ........55
FIGURE 4-146 10 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 5 ........55
FIGURE 4-147 20 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 7 ..........56
FIGURE 4-148 10 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 7 ..........56
FIGURE 4-149 20 PCT. ERR. ERODED BY CONCAVE SE OF SIZE 5 ........56
FIGURE 4-150 10 PCT. ERR. ERODED BY CONCAVE SE OF SIZE 5 ........56
FIGURE 4-151 20 PCT. ERR. ERODED BY CONCAVE SE OF SIZE 7 ..........56
FIGURE 4-152 10 PCT. ERR. ERODED BY CONCAVE SE OF SIZE 7 ..........56
FIGURE 4-153 20 PCT. ERR. OPENED BY CONCAVE SE OF SIZE 5 ..........57
FIGURE 4-154 10 PCT. ERR. OPENED BY CONCAVE SE OF SIZE 5 ..........57
FIGURE 4-155 20 PCT. ERR. OPENED BY CONCAVE SE OF SIZE 7 ..........57
FIGURE 4-156 10 PCT. ERR. OPENED BY CONCAVE SE OF SIZE 7 ..........57
FIGURE 4-157 20 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 5 ..........57
FIGURE 4-158 10 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 5 ..........57
FIGURE 4-159 20 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 7 ..........58
FIGURE 4-160 10 PCT. ERR. CLOSED BY CONCAVE SE OF SIZE 7 ..........58
FIGURE 4-161 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 3 ..........59
FIGURE 4-162 20 PCT. ERR. DILATED BY FLAT SE OF SIZE 5 ..........59
FIGURE 4-207 10 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 5 ................. 66
FIGURE 4-208 10 PCT. ERR. CLOSED BY PYRAMID SE OF SIZE 7 ................. 66
FIGURE 4-209 20 PCT. ERR. DILATED BY DOME SE OF SIZE 5 .................... 67
FIGURE 4-210 20 PCT. ERR. DILATED BY DOME SE OF SIZE 7 .................... 67
FIGURE 4-211 10 PCT. ERR. DILATED BY DOME SE OF SIZE 5 .................... 67
FIGURE 4-212 10 PCT. ERR. DILATED BY DOME SE OF SIZE 7 .................... 67
FIGURE 4-213 20 PCT. ERR. ERODED BY DOME SE OF SIZE 5 ..................... 67
FIGURE 4-214 20 PCT. ERR. ERODED BY DOME SE OF SIZE 7 ..................... 67
FIGURE 4-215 10 PCT. ERR. ERODED BY DOME SE OF SIZE 5 ..................... 68
FIGURE 4-216 10 PCT. ERR. ERODED BY DOME SE OF SIZE 7 ..................... 68
FIGURE 4-217 20 PCT. ERR. OPENED BY DOME SE OF SIZE 5 ...................... 68
FIGURE 4-218 20 PCT. ERR. OPENED BY DOME SE OF SIZE 7 ...................... 68
FIGURE 4-219 10 PCT. ERR. OPENED BY DOME SE OF SIZE 5 ...................... 68
FIGURE 4-220 10 PCT. ERR. OPENED BY DOME SE OF SIZE 7 ...................... 68
FIGURE 4-221 20 PCT. ERR. CLOSED BY DOME SE OF SIZE 5 ...................... 69
FIGURE 4-222 20 PCT. ERR. CLOSED BY DOME SE OF SIZE 7 ...................... 69
FIGURE 4-223 10 PCT. ERR. CLOSED BY DOME SE OF SIZE 5 ...................... 69
FIGURE 4-224 10 PCT. ERR. CLOSED BY DOME SE OF SIZE 7 ...................... 69
FIGURE 4-225 20 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 5 ................. 69
FIGURE 4-226 10 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 5 ................. 69
FIGURE 4-227 20 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 7 ................. 70
FIGURE 4-228 10 PCT. ERR. DILATED BY CONCAVE SE OF SIZE 7 ................. 70
FIGURE 5-32 COKE IMAGE ERODED BY 7X7 FLAT SE SHIFTED UP 55 LEVELS 87
FIGURE 5-33 COKE IMAGE OPENED BY 3X3 FLAT SE ........................................... 88
FIGURE 5-34 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 3 ................................. 88
FIGURE 5-35 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 3 NARROW GRAPH 88
FIGURE 5-36 COKE IMAGE OPENED BY 7X7 FLAT SE ......................................... 89
FIGURE 5-37 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 7 ................................. 89
FIGURE 5-38 20 PCT. ERR. OPENED BY FLAT SE OF SIZE 7 NARROW GRAPH 89
# List of Tables

<table>
<thead>
<tr>
<th>Table Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE 1-1</td>
<td>EXAMPLE OPERATIONS</td>
<td>1</td>
</tr>
<tr>
<td>TABLE 1-2</td>
<td>EXAMPLE OF BINARY MORPHOLOGY</td>
<td>3</td>
</tr>
<tr>
<td>TABLE 2-1</td>
<td>EXAMPLE OF GRAY SCALE MORPHOLOGY</td>
<td>11</td>
</tr>
<tr>
<td>TABLE 3-1</td>
<td>TABLE DECODING</td>
<td>28</td>
</tr>
<tr>
<td>TABLE 5-1</td>
<td>DECODING GRAPH LABELS</td>
<td>74</td>
</tr>
</tbody>
</table>
# List of Equations

| EQ. 1-1 | 2 |
| EQ. 1-2 | 2 |
| EQ. 1-3 | 2 |
| EQ. 1-4 | 2 |
| EQ. 1-5 | 3 |
| EQ. 1-6 | 4 |
| EQ. 1-7 | 4 |
| EQ. 2-8 | 8 |
| EQ. 2-9 | 10 |
| EQ. 2-10 | 13 |
| EQ. 2-11 | 13 |
### Glossary

<table>
<thead>
<tr>
<th><strong>Binary Morphology</strong></th>
<th><strong>Binary Morphology</strong> is Morphology performed on bi-level images. See “Binary Morphology” on page 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Close</strong></td>
<td>The closing is a morphological operation that is composed of an erosion followed by a dilation. The same structuring element must be used with both the erosion and the dilation. See “Close” page 4.</td>
</tr>
<tr>
<td><strong>DC Shift</strong></td>
<td>The entire image is shifted in amplitude. It can be thought of as shifting the color (gray) of the image.</td>
</tr>
<tr>
<td><strong>Dilation</strong></td>
<td>The dilation is a dual to erosion. It is computed as eroding the complement of the image. See “Dilation” on page 1.</td>
</tr>
<tr>
<td><strong>Erosion</strong></td>
<td>In the simplest case the erosion is a subset operation. See “Erosion” on page 2.</td>
</tr>
<tr>
<td><strong>Gray Scale Morphology</strong></td>
<td>Gray-scale Morphology is Morphology performed on gray scale images. See “Gray Scale Morphology” on page 5.</td>
</tr>
<tr>
<td><strong>Open</strong></td>
<td>The opening is a morphological operation that is composed of a dilation followed by an erosion. The same structuring element must be used with both the dilation and the erosion. See “Open” on page 3.</td>
</tr>
<tr>
<td><strong>Mathematical Morphology</strong></td>
<td>The type of nonlinear image processing discussed in this paper.</td>
</tr>
<tr>
<td><strong>Morphing</strong></td>
<td>The image processing that is used in Michael Jackson music videos, automobile commercials. It has nothing to do with this paper.</td>
</tr>
<tr>
<td><strong>Morphology</strong></td>
<td>From the dictionary A branch of Biology dealing with the form and structure of organisms. In other words the shape.</td>
</tr>
<tr>
<td><strong>Umbra</strong></td>
<td>The shadow of a function. See “Umbra Transform” on page 5.</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>The image used. The images used were 256 by 256 pixels.</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>The structuring element used. Structuring elements of 3 by 3 pixels, 5 by 5 pixels, and 7 by 7 pixels were used.</td>
</tr>
<tr>
<td><strong>a,b</strong></td>
<td>A specific pixel in the image or structuring element respectively.</td>
</tr>
<tr>
<td><strong>⊕</strong></td>
<td>The dilation operator.</td>
</tr>
<tr>
<td><strong>⋆</strong></td>
<td>The erosion operator. This notation is not common to the literature, but was made because the standard notation could not be inserted into this document.</td>
</tr>
<tr>
<td><strong>◦</strong></td>
<td>The open operator.</td>
</tr>
<tr>
<td><strong>•</strong></td>
<td>The close operator.</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>The complement of the set, used in the binary case. The pixels that are set are cleared, and the pixels that are not set are set.</td>
</tr>
<tr>
<td><strong>∪</strong></td>
<td>The set union operator.</td>
</tr>
<tr>
<td><strong>∩</strong></td>
<td>The set intersection operator.</td>
</tr>
<tr>
<td><strong>(\nabla)</strong></td>
<td>Denotes rotation of the structuring element (B) about the origin.</td>
</tr>
<tr>
<td><strong>−</strong></td>
<td>Also denotes rotation of the structuring element about the origin.</td>
</tr>
</tbody>
</table>
1. **Binary Morphology**

1.1. **Input**

In this paper the input images that are operated on are square matrices of finite size. Binary Morphology operates on 2-valued pixels. The table in the glossary on page xxv describes the notation used in this paper.

The following table provides some examples of the operations used. The cell that is italicized is the origin of the image.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 1 1 0 0</td>
</tr>
<tr>
<td></td>
<td>1 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>$A^c$</td>
<td>0 0 0 1 1</td>
</tr>
<tr>
<td></td>
<td>0 1 1 1 1</td>
</tr>
<tr>
<td></td>
<td>1 1 1 1 1</td>
</tr>
<tr>
<td></td>
<td>1 1 1 1 1</td>
</tr>
<tr>
<td></td>
<td>1 1 1 1 1</td>
</tr>
<tr>
<td>-A</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0 0 1</td>
</tr>
<tr>
<td></td>
<td>0 0 1 1 1</td>
</tr>
</tbody>
</table>

*Table 1-1 Example Operations*
1.2. Dilation

Dilation is dual to erosion because it is found by eroding the complement of an image. This formulation can be expressed as

\[ A \oplus B = [A^c \ast (-B)]^c \]

Eq. 1-1

(Dougherty, eq 1.10, p.7)

The character \( \ast \) represents morphological erosion. This substitution was made because of limitations for characters to represent the operation. Another formulation for dilation is

\[ A \oplus B = \bigcup [A + b : b \in B] \]

Eq. 1-2

(Dougherty, eq 1.12, p.7)

This is a union of the input image shifted by all elements in the structuring element.

Dilation is associative and commutative

1.3. Erosion

Erosion is the other primary operation in morphology. As seen earlier, dilation can be expressed as a dual to erosion. One formulation for erosion using sets is:

\[ A \ast B = [x : B + x \subset A] \]

Eq. 1-3

(Dougherty, eq. 1.3, p. 3)

Another formulation for erosion which will be used later is:

\[ X \ast B = \bigcap [X_b] : b \in B \]

Eq. 1-4

(Serra, p. 43)
The intersection is composed of $X$ translated by the elements of $B$. Each of those elements, $b \in B$, will be added to the point $x \in X$, the sum $x + b$ is in $X$ iff the point $x$ is in $X_b$. The resulting set of the erosion can be expressed as:

$$
Y = \{x: Bx \subseteq X\} = \bigcap_{y \in B0} X - y
$$

$$
= \bigcap_{-y \in B0} X_y = X \ast B
$$

where $\tilde{B}$ is $B$ reflected about the origin. This reflection about the origin comes about as a result of the need for opening and closing (which are defined later) to be idempotent. If neither erosion, nor dilation used a rotated structuring element, a shift would be built into the image for those cases using a non-symmetrical structuring element. As an example:

<table>
<thead>
<tr>
<th>Origin is Underlined</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
</tr>
<tr>
<td>I</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>$\tilde{S}$</td>
</tr>
<tr>
<td>$I \oplus S$</td>
</tr>
<tr>
<td>$I \ast S^*$</td>
</tr>
<tr>
<td>$I \ast S$</td>
</tr>
<tr>
<td>open*</td>
</tr>
<tr>
<td>close*</td>
</tr>
<tr>
<td>close</td>
</tr>
</tbody>
</table>

Table 1-2 Example of Binary Morphology
1.4. Open

The morphological opening is an operation composed of erosion followed by dilation.

\[ A \circ B = (A * B) \oplus B \]

Eq. 1-6

(Dougherty, eq. 2.1, p. 17)

Opening an image with a structuring element is used to truncate protrusions on the outside of the image. The initial erosion will “cut off” those external features that the structuring element cannot fit into. The dilation will expand the image to nearly the original size.

1.5. Close

The morphological closing is an operation composed of dilation followed by erosion.

\[ A \bullet B = [A \oplus (-B)] * (-B) \]

Eq. 1-7

(Dougherty, eq. 2.4, p. 18)

Closing an image will fill in gaps on the inside of the image that the structuring elements can bridge. The initial dilation will fill in the internal gaps and the subsequent erosion will return the image to nearly the original shape.
2. Gray Scale Morphology

2.1. Input

Gray scale morphology is an extension of binary morphology to images with shades of gray, or in other words from two to three dimensions. Eight-bit per pixel images, 256 shades, are a common form, but that is not a theoretical restriction. Minus infinity is also a value that the images can take, and is reserved to indicate the lack of a value. Minus infinity is used when the value is undefined (not in the domain of the image.) The dilation operator can create values for those pixels (expand the domain), since it is defined as a maximum. The erosion operator does not expand the domain of the image.

2.1.1. Umbra Transform

The Umbra Transform was used for the early development of gray scale morphology. Although newer literature does not use it much, it still provides an easily visualized way of looking at gray scale morphology. The Umbra can be thought of as the shadow created by the function (Dougherty, pp. 107-109).
Looking at the function, it only defines the surface. The umbra includes the volume underneath the surface all the way to minus infinity. One important distinction between binary and gray-scale morphology is that the concept of the complement of a picture needs to be examined very closely, since in the umbra context the complement does not fit.
2.2. Morphological Dilation

Gray scale dilation can be expressed as

\[ y = f(x) \]
\[ d(x, y) = \max_{i,j} [a(x-i, y-j) + b(i, j)] \quad \text{Eq. 2-8} \]

\[
\begin{aligned}
A &= m \times m \\
B &= n \times n \\
x &= 0 \ldots m \\
y &= 0 \ldots m \\
i &= \frac{-n}{2} \ldots 0 \ldots \frac{n}{2} \\
j &= \frac{-n}{2} \ldots 0 \ldots \frac{n}{2} \\

= A \oplus B
\end{aligned}
\]

To determine the value of each pixel in the resulting image, it is necessary to take the sum of the corresponding element from the structuring element and the pixel from the input image that is overlaid. For example given the following input image \( A \)

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 \\
6 & 7 & 8 & 9 & 10 \\
11 & 12 & 13 & 14 & 15 \\
16 & 17 & 18 & 19 & 20 \\
21 & 22 & 23 & 24 & 25 \\
\end{array}
\]

\[
a(0,0) = 1 \\
a(0,4) = 5 \\
a(2,2) = 13 \\
a(4,0) = 21 \\
a(4,4) = 25 \\
\]

and the structuring element centered about the middle element.

\[
\begin{array}{cccc}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
\end{array}
\]

The value of the element \((1,1)\) would be determined from the maximum of the sums
1+1 2+2 3+3
6+4 7+5 8+6
11+7 12+8 13+9

In this case the value of the pixel (1,1) would be 22. The pixel (1,1) would take the sum of the pixel from the structuring element (2,2) and from the input image (2,2). A special case is the handling of the edge conditions where part of the structuring element is not over the domain of the input image. Since the image is treated as minus infinity when not in its domain, the out of domain values from the input image are just set to minus infinity. The value of the pixel (0,0) would be

-∞ + 1  -∞ + 2  -∞ + 3
-∞ + 4  1 + 5  2 + 6
-∞ + 7  6 + 8  7 + 9

or 16. The origin of the structuring element is the center of interest. Another important case is where the pixel of interest is out of the domain of the input image, but at least one pixel is in the domain of the input image. One example would be the value of the pixel (5,5)

25+1  -∞ + 2  -∞ + 3
-∞ + 4  -∞ + 5  -∞ + 6
-∞ + 7  -∞ + 8  -∞ + 9

In this case the value would be 26. The only restriction on this is some real implementations. In some implementations, the domain is restricted such that the domain of the resultant image cannot exceed the size of the input image, which includes minus infinity pixels in the image. This restriction is an implementation restriction that is not imposed by morphology.
2.3. **Morphological Erosion**

Gray scale erosion can be expressed as

\[
d(x, y) = \min_{i,j} [a(x-i, y-j) - b(-i, -j)]
\]

\[
eq A * B
\]

Eq. 2-9

(Dougherty, eq. 6.6, p. 97)

One important point that can be seen from the above equation is that the structuring element is rotated around the origin. This is important to remember for implementation, although in many of the common cases (structuring elements that are symmetrical around the origin) it will not change any of the results.

The following is a 1 dimensional example of what happens in the cases where the structuring element is not rotated. The rows that are marked with the superscript asterisk are where the structuring element was not rotated about the origin. The incorrect definition does not have the idempotence property. Opening and closing are both idempotent operations; no openings or closings after the first opening or closing respectively will produce a change in the image.
Erosion reduces the domain in the boundary cases where the offset of the structuring element does not include an element in the domain of \( f \).

Using the image and structuring element that was used earlier the pixel \((1,1)\) of the output image would be

\[
\begin{array}{ccccccc}
1 & -9 & 2 & -8 & 3 & -7 \\
6 & -6 & 7 & -5 & 8 & -4 & = -8 \Rightarrow -\infty \\
11 & -3 & 12 & -2 & 13 & -1
\end{array}
\]

In this case, we have a number of values less than zero which map to \(-\infty\). In another case, the value of the output pixel \((3,3)\) would be
Since erosion uses the minimum of the differences the result would be 13 - 9 = 4. Another case is where the structuring element extends past the domain of the input image. The pixel (0,0) would be

\[19 - 9 \quad 20 - 8 \quad -\infty - 7\]
\[24 - 6 \quad 25 - 5 \quad -\infty - 4\]
\[-\infty - 3 \quad -\infty - 2 \quad -\infty - 1\]

Since there are a number of values at -\infty the output pixel will be -\infty.

2.4. Morphological Opening

In the gray scale case, the basic definition of morphological opening is the same: erosion followed by dilation (Dougherty, p. 111.) The effects are also similar. The initial erosion will remove those protrusions on the surface of the image that the structuring element cannot fit into, the edges of the image will become smaller as the erosion will set the domain boundary pixels to -\infty. The dilation will add back to the image both the boundary pixels and some of the “height” of the pixels.

2.5. Morphological Closing

The morphological closing is dilation followed by erosion (Dougherty, p. 111.) The dilation fills in some voids on the image and expands the domain of the image if it can. Some implementations restrict the size of the resulting image to its initial size. The erosion then makes the image nearer to its initial shape, but the internal voids that were filled in by the dilation cannot be restored. One special case for some implementations is where the dilation...
could not expand the domain because of an image size limit and the erosion reduces the domain of the image.

### 2.6. Common Transforms and filters

Two of the common transforms are the top-hat \( f - (f \text{ open } g) \) (Dougherty, pp. 119-120,) and valley detector \((f \text{ close } g) - f\) (Dougherty, p. 120.) One special case is when the structuring element is flat with a value of 0 along its domain. Then \(-g = g\) and the duality of open and closing becomes \((f \text{ close } g) - f = -f - [(-f) \text{ open } g]\). One way to detect both the peaks and valleys is to use \((f \text{ close } g) - (f \text{ open } g)\).

Two other filters are the iteration of opening followed by closing or closing followed by opening. The filters are referred to, respectively as: CLOSEOPEN, and OPENCLOSE.

\[
\text{CLOSEOPEN}(f) = (f \text{ close } (-g)) \text{ open } g
\]

\[
\text{Eq. 2-10}
\]

(Dougherty, eq. 7.5, p. 127)

\[
\text{OPENCLOSE}(f) = (f \text{ open } g) \text{ close } (-g)
\]

\[
\text{Eq. 2-11}
\]

(Dougherty, eq. 7.4, p. 127)

One simplification to the above is to restrict these filters to flat structuring elements with value 0 so \( g = -g \).

Another type of filtering that can be used is to start with a small structuring element and alternate opening and closings and then increase the filter size to remove successively larger noise particles. It is important to note that in the digital case that the order in which the opening and closings are done is important. Since a closing reduces the domain and an opening may increase
the domain if possible in a digital setting, the opening needs to be done before the closing to avoid diminishing the domain.
3. Experimental Procedure

3.1. Basics
This experiment was set up to judge the results of different sizes of structuring elements and different shapes on restoring images (noise reduction.) The procedure was to take an image, add noise to it, and then use the noised image as the starting point. The basic operations (dilate, erode, open, or close) were then used and the effect of the processing on the image could be calculated. It was also observed that the minimal error for a given image and structuring element was not necessarily given by the original processing. In many cases the error was minimized by shifting the entire image by a constant amount or DC shift.

3.2. Noise
The noise that was applied to the image was uniformly distributed with a range of ±64. In the cases where the noise would have extended beyond the available gray scale range, the resulting value was truncated at the boundary.

3.3. Input Images
The following figures are the images that were used for the experiments. The images are 256 by 256 pixels. The three base images were chosen to provide a variety of input sources. The coke image has the sharply defined lines and
curves. The image of the girl has the softer shadings of a face and hair. The image of Roy combines some of both with the background providing some sharp edges.

![Baseline Coke Image](image)

*Figure 3-1 Baseline Coke Image*
Figure 3-2 Coke Image with 10 Percent Error
Figure 3-3 Coke Image with 20 Percent Error
Figure 3-4 Baseline Girl Image
Figure 3-5 Girl Image with 10 Percent Error
Figure 3-6 Girl Image with 20 Percent Error
Figure 3-7 Baseline Roy Image
Figure 3-8 Roy Image with 10 Percent Error
Figure 3-9 Roy Image with 20 Percent Error
3.4. Structuring Elements

The structuring elements used for the experiment can be divided into 4 categories; flat, pyramidal, concave, and dome shaped. All structuring elements used were square; 3, 5, or 7 units on a side. Each category of structuring elements has 2 or 3 different sizes. There are 3 flat structuring elements of sizes 3, 5, and 7. The values of the pixels are all zero valued.

The flat structuring element of size 3 has the values:

```
 0 0 0
 0 0 0
 0 0 0
```

The flat structuring element of size 5 has the values:

```
 0 0 0 0 0
 0 0 0 0 0
 0 0 0 0 0
 0 0 0 0 0
 0 0 0 0 0
```

The flat structuring element of size 7 has the values:

```
 0 0 0 0 0 0 0
 0 0 0 0 0 0 0
 0 0 0 0 0 0 0
 0 0 0 0 0 0 0
 0 0 0 0 0 0 0
```

The pyramidal structuring element of size 3 has the values:

\[
\begin{array}{ccc}
0 & 1 & 0 \\
1 & 2 & 1 \\
0 & 1 & 0 \\
\end{array}
\]

The pyramidal structuring element of size 5 has the values:

\[
\begin{array}{cccc}
0 & 0 & 1 & 0 \\
0 & 1 & 2 & 1 \\
1 & 2 & 3 & 2 \\
0 & 1 & 2 & 1 \\
0 & 0 & 1 & 0 \\
\end{array}
\]

The pyramidal structuring element of size 7 has the values:

\[
\begin{array}{cccccc}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 \\
0 & 1 & 2 & 3 & 2 & 1 \\
1 & 2 & 3 & 4 & 3 & 2 \\
0 & 1 & 2 & 3 & 2 & 1 \\
0 & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{array}
\]

The concave structuring element of size 5 has the values:

\[
\begin{array}{ccc}
0 & 0 & 1 \\
0 & 0 & 1 \\
1 & 1 & 3 \\
0 & 0 & 1 \\
0 & 0 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
0 & 0 & 1 \\
0 & 0 & 1 \\
1 & 1 & 3 \\
0 & 0 & 1 \\
0 & 0 & 1 \\
\end{array}
\]
The concave structuring element of size 7 has the values:
0 0 0 1 0 0 0
0 0 0 1 0 0 0
0 0 1 2 1 0 0
1 1 2 4 2 1 1
0 0 1 2 1 0 0
0 0 0 1 0 0 0
0 0 0 1 0 0 0

The dome structuring element of size 5 has the values:
0 0 1 0 0
0 2 3 2 0
1 3 3 3 1
0 2 3 2 0
0 0 1 0 0

The dome structuring element of size 7 has the values:
0 0 0 1 0 0 0
0 0 1 2 1 0 0
0 1 3 4 3 1 0
1 2 4 4 4 2 1
0 1 3 4 3 1 0
0 0 1 2 1 0 0
0 0 0 1 0 0 0
3.5. **Naming Scheme**

The results generated later in this document often label the results with a code that identifies the image used, the amount of noise added to the baseline image, and the morphological operation applied. In section 4 the traces in the graphs have a naming scheme that includes the operation. The naming scheme for the traces is explained in the following table.

<table>
<thead>
<tr>
<th>Character</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The input image. This can be either ‘c’ for the image of the coke can, ‘g’ for the image of the young girl, ‘r’ for the image of Dr. Czernikowski in the engineer hat.</td>
</tr>
<tr>
<td>2</td>
<td>The amount of noise applied to the image. ‘1’ for 10 percent error, or ‘2’ for 20 percent error.</td>
</tr>
<tr>
<td>3</td>
<td>The morphological operation applied. The values can be ‘c’, ‘d’, ‘e’, ‘o’ for close, dilate, erode, open respectively.</td>
</tr>
<tr>
<td>4</td>
<td>The type of structuring element applied. The values can be ‘c’, ‘d’, ‘f’, ‘p’ for concave, dome, flat, pyramid respectively.</td>
</tr>
<tr>
<td>5</td>
<td>The size of the square structuring element. This can take the values of 3, 5, 7. The concave and dome structuring elements are restricted to sizes of 5 and 7.</td>
</tr>
<tr>
<td>6-7</td>
<td>The trace on the graph. This takes the value ‘v1’ or ‘v2’ which is not significant of itself. The trace labeled ‘v1’ will be above or equal to the trace labeled ‘v2.’ The ‘v1’ trace is the absolute error for the whole image at the various DC offsets, the ‘v2’ trace is the absolute error for a subset of the entire image.</td>
</tr>
</tbody>
</table>

Table 3-1 Table Decoding
4. Results

The rest of this section is divided into 3 sections that contains graphs that show the initial results of the investigation. Each section focuses on a specific input image and the results that the different structuring elements provide. One parameter of specific interest is what DC shift will minimize the absolute error over the entire image. A DC shift is just a shift of the entire image by a constant value. Each graph is composed of the absolute error of the image at the different DC shifts.

The two traces on the graphs are the absolute error for two cases. The first case is the absolute error computed using the entire resultant image against the entire baseline image. The second trace on the graph is the absolute error computed with a border of three pixels around both the input image and the resultant image.

The border was chosen so that the edge effects could be ignored for erosion and closing. As described earlier, the reduction in the domain for erosion and closing will frame the image in black (0 valued) pixels. Reducing the domain of the absolute error allows for comparing the results without that problem, although the graphs in this section do not show the domain reduction as a great problem.

The title of each graph tells more of the details of how each image was processed. The first part of the graph title will either say 20 Pct Err, or 10 Pct Err. Those two phrases indicate how much error was introduced into the base image. The rest of the graph title will indicate what morphological operation was performed on the image and with what structuring element. For example
Dilated by Flat SE of Size 3, the operation was dilation and the structuring element (SE) was a square flat structuring element 3 units on each side.

The x-axis of the results graphs is centered at a DC offset of 0. The label i-20 at this point is an artifact of the creation of the graphs. The data points were read out of a file and were labeled starting at 0. The i-20 term is an offset to get the correct x-axis values. The marks under the -20 and 20 on the x-axis are also artifacts of the package used to create the graphs.
4.1. Coke Images

Figure 4-1 20 Pct. Err. Dilated by Flat SE of size 3

Figure 4-2 20 Pct. Err. Dilated by Flat SE of size 5

Figure 4-3 20 Pct. Err. Dilated by Flat SE of size 7

Figure 4-4 10 Pct. Err. Dilated by Flat SE of size 3
Figure 4-5 10 Pct. Err. Dilated by Flat SE of size 5

Figure 4-8 20 Pct. Err. Eroded by Flat SE of size 5

Figure 4-6 10 Pct. Err. Dilated by Flat SE of size 7

Figure 4-9 20 Pct. Err. Eroded by Flat SE of size 7

Figure 4-7 20 Pct. Err. Eroded by Flat SE of size 3

Figure 4-10 10 Pct. Err. Eroded by Flat SE of size 3
Figure 4-11 10 Pct. Err. Eroded by Flat SE of size 5

Figure 4-12 10 Pct. Err. Eroded by Flat SE of size 7

Figure 4-13 20 Pct. Err. Opened by Flat SE of size 3

Figure 4-14 20 Pct. Err. Opened by Flat SE of size 5

Figure 4-15 20 Pct. Err. Opened by Flat SE of size 7

Figure 4-16 10 Pct. Err. Opened by Flat SE of size 3
Figure 4-17 10 Pct. Err. Opened by Flat SE of size 5

Figure 4-18 10 Pct. Err. Opened by Flat SE of size 7

Figure 4-19 20 Pct. Err. Closed by Flat SE of size 3

Figure 4-20 20 Pct. Err. Closed by Flat SE of size 5

Figure 4-21 20 Pct. Err. Closed by Flat SE of size 7

Figure 4-22 10 Pct. Err. Closed by Flat SE of size 3
Figure 4-23 10 Pct. Err. Closed by Flat SE of size 5

Figure 4-26 20 Pct. Err. Dilated by Pyramid SE of size 5

Figure 4-24 10 Pct. Err. Closed by Flat SE of size 7

Figure 4-27 20 Pct. Err. Dilated by Pyramid SE of size 7

Figure 4-25 20 Pct. Err. Dilated by Pyramid SE of size 3

Figure 4-28 10 Pct. Err. Dilated by Pyramid SE of size 3
Figure 4-29 10 Pct. Err. Dilated by Pyramid SE of size 5

Figure 4-30 10 Pct. Err. Dilated by Pyramid SE of size 7

Figure 4-31 20 Pct. Err. Eroded by Pyramid SE of size 3

Figure 4-32 20 Pct. Err. Eroded by Pyramid SE of size 5

Figure 4-33 20 Pct. Err. Eroded by Pyramid SE of size 7

Figure 4-34 10 Pct. Err. Eroded by Pyramid SE of size 3
Figure 4-35 10 Pct. Err. Eroded by Pyramid SE of size 5

Figure 4-36 10 Pct. Err. Eroded by Pyramid SE of size 7

Figure 4-37 20 Pct. Err. Opened by Pyramid SE of size 3

Figure 4-38 20 Pct. Err. Opened by Pyramid SE of size 5

Figure 4-39 20 Pct. Err. Opened by Pyramid SE of size 7

Figure 4-40 10 Pct. Err. Opened by Pyramid SE of size 3
Figure 4-41 10 Pct. Err. Opened by Pyramid SE of size 5

Figure 4-44 20 Pct. Err. Closed by Pyramid SE of size 5

Figure 4-42 10 Pct. Err. Opened by Pyramid SE of size 7

Figure 4-45 20 Pct. Err. Closed by Pyramid SE of size 7

Figure 4-43 20 Pct. Err. Closed by Pyramid SE of size 3

Figure 4-46 10 Pct. Err. Closed by Pyramid SE of size 3
Figure 4-47 10 Pct. Err. Closed by Pyramid SE of size 5

Figure 4-48 10 Pct. Err. Closed by Pyramid SE of size 7

Figure 4-49 20 Pct. Err. Dilated by Dome SE of size 5

Figure 4-50 20 Pct. Err. Dilated by Dome SE of size 7

Figure 4-51 10 Pct. Err. Dilated by Dome SE of size 5

Figure 4-52 10 Pct. Err. Dilated by Dome SE of size 7
Figure 4-53 20 Pct. Err. Eroded by Dome
SE of size 5

Figure 4-54 20 Pct. Err. Eroded by Dome
SE of size 7

Figure 4-55 10 Pct. Err. Eroded by Dome
SE of size 5

Figure 4-56 10 Pct. Err. Eroded by Dome
SE of size 7

Figure 4-57 20 Pct. Err. Opened by Dome
SE of size 5

Figure 4-58 20 Pct. Err. Opened by Dome
SE of size 7
Figure 4-59 10 Pct. Err. Opened by Dome
SE of size 5

Figure 4-62 20 Pct. Err. Closed by Dome
SE of size 7

Figure 4-60 10 Pct. Err. Opened by Dome
SE of size 7

Figure 4-63 10 Pct. Err. Closed by Dome
SE of size 5

Figure 4-61 20 Pct. Err. Closed by Dome
SE of size 5

Figure 4-64 10 Pct. Err. Closed by Dome
SE of size 7
Figure 4-65 20 Pct. Err. Dilated by Concave SE of size 5

Figure 4-66 10 Pct. Err. Dilated by Concave SE of size 5

Figure 4-67 20 Pct. Err. Dilated by Concave SE of size 7

Figure 4-68 10 Pct. Err. Dilated by Concave SE of size 7

Figure 4-69 20 Pct. Err. Eroded by Concave SE of size 5

Figure 4-70 10 Pct. Err. Eroded by Concave SE of size 5
Figure 4-71 20 Pct. Err. Eroded by Concave SE of size 7

Figure 4-72 10 Pct. Err. Eroded by Concave SE of size 7

Figure 4-73 20 Pct. Err. Opened by Concave SE of size 5

Figure 4-74 10 Pct. Err. Opened by Concave SE of size 5

Figure 4-75 20 Pct. Err. Opened by Concave SE of size 7

Figure 4-76 10 Pct. Err. Opened by Concave SE of size 7
Figure 4-77 20 Pct. Err. Closed by Concave SE of size 5

Figure 4-79 20 Pct. Err. Closed by Concave SE of size 7

Figure 4-78 10 Pct. Err. Closed by Concave SE of size 5

Figure 4-80 10 Pct. Err. Closed by Concave SE of size 7
4.2. Girl Images

Figure 4-81 20 Pct. Err. Dilated by Flat SE of size 3

Figure 4-84 10 Pct. Err. Dilated by Flat SE of size 3

Figure 4-82 20 Pct. Err. Dilated by Flat SE of size 5

Figure 4-85 10 Pct. Err. Dilated by Flat SE of size 5

Figure 4-83 20 Pct. Err. Dilated by Flat SE of size 7

Figure 4-86 10 Pct. Err. Dilated by Flat SE of size 7
Figure 4-87 20 Pct. Err. Eroded by Flat SE of size 3

Figure 4-88 20 Pct. Err. Eroded by Flat SE of size 5

Figure 4-89 20 Pct. Err. Eroded by Flat SE of size 7

Figure 4-90 10 Pct. Err. Eroded by Flat SE of size 3

Figure 4-91 10 Pct. Err. Eroded by Flat SE of size 5

Figure 4-92 10 Pct. Err. Eroded by Flat SE of size 7
Figure 4-93 20 Pct. Err. Opened by Flat SE of size 3

Figure 4-94 20 Pct. Err. Opened by Flat SE of size 5

Figure 4-95 20 Pct. Err. Opened by Flat SE of size 7

Figure 4-96 10 Pct. Err. Opened by Flat SE of size 3

Figure 4-97 10 Pct. Err. Opened by Flat SE of size 5

Figure 4-98 10 Pct. Err. Opened by Flat SE of size 7
Figure 4-99 20 Pct. Err. Closed by Flat SE of size 3

Figure 4-100 20 Pct. Err. Closed by Flat SE of size 5

Figure 4-101 20 Pct. Err. Closed by Flat SE of size 7

Figure 4-102 10 Pct. Err. Closed by Flat SE of size 3

Figure 4-103 10 Pct. Err. Closed by Flat SE of size 5

Figure 4-104 10 Pct. Err. Closed by Flat SE of size 7
Figure 4-105 20 Pct. Err. Dilated by Pyramid SE of size 3

Figure 4-108 10 Pct. Err. Dilated by Pyramid SE of size 3

Figure 4-106 20 Pct. Err. Dilated by Pyramid SE of size 5

Figure 4-109 10 Pct. Err. Dilated by Pyramid SE of size 5

Figure 4-107 20 Pct. Err. Dilated by Pyramid SE of size 7

Figure 4-110 10 Pct. Err. Dilated by Pyramid SE of size 7
Figure 4-111 20 Pct. Err. Eroded by Pyramid SE of size 3

Figure 4-114 10 Pct. Err. Eroded by Pyramid SE of size 3

Figure 4-112 20 Pct. Err. Eroded by Pyramid SE of size 5

Figure 4-115 10 Pct. Err. Eroded by Pyramid SE of size 5

Figure 4-113 20 Pct. Err. Eroded by Pyramid SE of size 7

Figure 4-116 10 Pct. Err. Eroded by Pyramid SE of size 7
Figure 4-117 20 Pct. Err. Opened by Pyramid SE of size 3

Figure 4-118 20 Pct. Err. Opened by Pyramid SE of size 5

Figure 4-119 20 Pct. Err. Opened by Pyramid SE of size 7

Figure 4-120 10 Pct. Err. Opened by Pyramid SE of size 3

Figure 4-121 10 Pct. Err. Opened by Pyramid SE of size 5

Figure 4-122 10 Pct. Err. Opened by Pyramid SE of size 7
Figure 4-123 20 Pct. Err. Closed by Pyramid SE of size 3

Figure 4-124 20 Pct. Err. Closed by Pyramid SE of size 5

Figure 4-125 20 Pct. Err. Closed by Pyramid SE of size 7

Figure 4-126 10 Pct. Err. Closed by Pyramid SE of size 3

Figure 4-127 10 Pct. Err. Closed by Pyramid SE of size 5

Figure 4-128 10 Pct. Err. Closed by Pyramid SE of size 7
Figure 4-129 20 Pct. Err. Dilated by Dome
SE of size 5

Figure 4-132 10 Pct. Err. Dilated by Dome
SE of size 7

Figure 4-130 20 Pct. Err. Dilated by Dome
SE of size 7

Figure 4-133 20 Pct. Err. Eroded by Dome
SE of size 5

Figure 4-131 10 Pct. Err. Dilated by Dome
SE of size 5

Figure 4-134 20 Pct. Err. Eroded by Dome
SE of size 7
Figure 4-135 10 Pct. Err. Eroded by Dome SE of size 5

Figure 4-136 10 Pct. Err. Eroded by Dome SE of size 7

Figure 4-137 20 Pct. Err. Opened by Dome SE of size 5

Figure 4-138 20 Pct. Err. Opened by Dome SE of size 7

Figure 4-139 10 Pct. Err. Opened by Dome SE of size 5

Figure 4-140 10 Pct. Err. Opened by Dome SE of size 7
Figure 4-141 20 Pct. Err. Closed by Dome
SE of size 5

Figure 4-144 10 Pct. Err. Closed by Dome
SE of size 7

Figure 4-142 20 Pct. Err. Closed by Dome
SE of size 7

Figure 4-145 20 Pct. Err. Dilated by Concave SE of size 5

Figure 4-143 10 Pct. Err. Closed by Dome
SE of size 5

Figure 4-146 10 Pct. Err. Dilated by Concave SE of size 5
Figure 4-147 20 Pct. Err. Dilated by Concave SE of size 7

Figure 4-150 10 Pct. Err. Eroded by Concave SE of size 5

Figure 4-148 10 Pct. Err. Dilated by Concave SE of size 7

Figure 4-151 20 Pct. Err. Eroded by Concave SE of size 7

Figure 4-149 20 Pct. Err. Eroded by Concave SE of size 5

Figure 4-152 10 Pct. Err. Eroded by Concave SE of size 7
Figure 4-153 20 Pct. Err. Opened by
Concave SE of size 5

Figure 4-154 10 Pct. Err. Opened by
Concave SE of size 5

Figure 4-155 20 Pct. Err. Opened by
Concave SE of size 7

Figure 4-156 10 Pct. Err. Opened by
Concave SE of size 7

Figure 4-157 20 Pct. Err. Closed by
Concave SE of size 5

Figure 4-158 10 Pct. Err. Closed by
Concave SE of size 5
Figure 4-159 20 Pct. Err. Closed by Concave SE of size 7

Figure 4-160 10 Pct. Err. Closed by Concave SE of size 7
Roy Images

Figure 4-161 20 Pct. Err. Dilated by Flat SE of size 3

Figure 4-162 20 Pct. Err. Dilated by Flat SE of size 5

Figure 4-163 20 Pct. Err. Dilated by Flat SE of size 7

Figure 4-164 10 Pct. Err. Dilated by Flat SE of size 3

Figure 4-165 10 Pct. Err. Dilated by Flat SE of size 5

Figure 4-166 10 Pct. Err. Dilated by Flat SE of size 7
Figure 4-167 20 Pct. Err. Eroded by Flat SE of size 3
Figure 4-170 10 Pct. Err. Eroded by Flat SE of size 3

Figure 4-168 20 Pct. Err. Eroded by Flat SE of size 5
Figure 4-171 10 Pct. Err. Eroded by Flat SE of size 5

Figure 4-169 20 Pct. Err. Eroded by Flat SE of size 7
Figure 4-172 10 Pct. Err. Eroded by Flat SE of size 7
Figure 4-173 20 Pct. Err. Opened by Flat SE of size 3

Figure 4-174 20 Pct. Err. Opened by Flat SE of size 5

Figure 4-175 20 Pct. Err. Opened by Flat SE of size 7

Figure 4-176 10 Pct. Err. Opened by Flat SE of size 3

Figure 4-177 10 Pct. Err. Opened by Flat SE of size 5

Figure 4-178 10 Pct. Err. Opened by Flat SE of size 7
Figure 4-179 20 Pct. Err. Closed by Flat
SE of size 3

Figure 4-180 20 Pct. Err. Closed by Flat
SE of size 5

Figure 4-181 20 Pct. Err. Closed by Flat
SE of size 7

Figure 4-182 10 Pct. Err. Closed by Flat
SE of size 3

Figure 4-183 10 Pct. Err. Closed by Flat
SE of size 5

Figure 4-184 10 Pct. Err. Closed by Flat
SE of size 7
Figure 4-185 20 Pct. Err. Dilated by Pyramid SE of size 3

Figure 4-186 20 Pct. Err. Dilated by Pyramid SE of size 5

Figure 4-187 20 Pct. Err. Dilated by Pyramid SE of size 7

Figure 4-188 10 Pct. Err. Dilated by Pyramid SE of size 3

Figure 4-189 10 Pct. Err. Dilated by Pyramid SE of size 5

Figure 4-190 10 Pct. Err. Dilated by Pyramid SE of size 7
Figure 4-191 20 Pct. Err. Eroded by Pyramid SE of size 3

Figure 4-194 10 Pct. Err. Eroded by Pyramid SE of size 3

Figure 4-192 20 Pct. Err. Eroded by Pyramid SE of size 5

Figure 4-195 10 Pct. Err. Eroded by Pyramid SE of size 5

Figure 4-193 20 Pct. Err. Eroded by Pyramid SE of size 7

Figure 4-196 10 Pct. Err. Eroded by Pyramid SE of size 7
Figure 4-197 20 Pct. Err. Opened by Pyramid SE of size 3

Figure 4-198 20 Pct. Err. Opened by Pyramid SE of size 5

Figure 4-199 20 Pct. Err. Opened by Pyramid SE of size 7

Figure 4-200 10 Pct. Err. Opened by Pyramid SE of size 3

Figure 4-201 10 Pct. Err. Opened by Pyramid SE of size 5

Figure 4-202 10 Pct. Err. Opened by Pyramid SE of size 7
Figure 4-203 20 Pct. Err. Closed by Pyramid SE of size 3

Figure 4-204 20 Pct. Err. Closed by Pyramid SE of size 5

Figure 4-205 20 Pct. Err. Closed by Pyramid SE of size 7

Figure 4-206 10 Pct. Err. Closed by Pyramid SE of size 3

Figure 4-207 10 Pct. Err. Closed by Pyramid SE of size 5

Figure 4-208 10 Pct. Err. Closed by Pyramid SE of size 7
Figure 4-209 20 Pct. Err. Dilated by Dome
SE of size 5

Figure 4-212 10 Pct. Err. Dilated by Dome
SE of size 7

Figure 4-210 20 Pct. Err. Dilated by Dome
SE of size 7

Figure 4-213 20 Pct. Err. Eroded by Dome
SE of size 5

Figure 4-211 10 Pct. Err. Dilated by Dome
SE of size 5

Figure 4-214 20 Pct. Err. Eroded by Dome
SE of size 7
Figure 4-215 10 Pct. Err. Eroded by Dome
SE of size 5

Figure 4-216 10 Pct. Err. Eroded by Dome
SE of size 7

Figure 4-217 20 Pct. Err. Opened by Dome
SE of size 5

Figure 4-218 20 Pct. Err. Opened by Dome
SE of size 7

Figure 4-219 10 Pct. Err. Opened by Dome
SE of size 5

Figure 4-220 10 Pct. Err. Opened by Dome
SE of size 7
Figure 4-221 20 Pct. Err. Closed by Dome
SE of size 5

Figure 4-224 10 Pct. Err. Closed by Dome
SE of size 7

Figure 4-222 20 Pct. Err. Closed by Dome
SE of size 7

Figure 4-225 20 Pct. Err. Dilated by Concave SE of size 5

Figure 4-223 10 Pct. Err. Closed by Dome
SE of size 5

Figure 4-226 10 Pct. Err. Dilated by Concave SE of size 5
Figure 4-227 20 Pct. Err. Dilated by Concave SE of size 7

Figure 4-230 10 Pct. Err. Eroded by Concave SE of size 5

Figure 4-228 10 Pct. Err. Dilated by Concave SE of size 7

Figure 4-231 20 Pct. Err. Eroded by Concave SE of size 7

Figure 4-229 20 Pct. Err. Eroded by Concave SE of size 5

Figure 4-232 10 Pct. Err. Eroded by Concave SE of size 7
Figure 4-233 20 Pct. Err. Opened by Concave SE of size 5

Figure 4-234 10 Pct. Err. Opened by Concave SE of size 5

Figure 4-235 20 Pct. Err. Opened by Concave SE of size 7

Figure 4-236 10 Pct. Err. Opened by Concave SE of size 7

Figure 4-237 20 Pct. Err. Closed by Concave SE of size 5

Figure 4-238 10 Pct. Err. Closed by Concave SE of size 5
Figure 4-239 20 Pct. Err. Closed by
Concave SE of size 7

Figure 4-240 10 Pct. Err. Closed by
Concave SE of size 7
5. Conclusions

5.1. Numerical Results

This section details the numbers that were produced by the experiments. The results can be judged in two ways, both a strictly numeric judgment, as well as a visual judgment. The numeric metrics, absolute error for instance, are useful because they provide a way to automatically rate the goodness of the processing.

The results generated by the previous graphs are summarized in the following graphs. Two sets of graphs are provided for Dilation and Erosion. The graphs Figure 5-2, Figure 5-4, Figure 5-6, and Figure 5-8 were generated by calculating the Absolute Errors for the offsets from -90 to 90 with a step size of 5, (-90, -85, ... 0, 5, ... 90.) These additional results were generated because some of the graphs in section 4 did not reach a minima in the middle of the graph.

The results are provided for each operation. Each graph differs in the operation graphed as well as the noise percentage that was in the image processed. Ten structuring elements were used. The structuring elements vary both in shape and size. The shapes used were flat, pyramid, concave, and dome. The flat and pyramid structuring elements were used in sizes of 3, 5, and 7. The concave and dome structuring elements were used in sizes of 5 and 7. The best results were provided by the smallest structuring elements; these conclusions may not hold for more correlated noise such as scratches. Figure 5-11 demonstrates the benefit of the smaller structuring elements. The coke image results show that the absolute errors for a given size of structuring element are roughly the same. The coke image has the highest error with the structuring elements of size 7, although Figure 5-12 shows
some interesting results. The results show that the relationship between the absolute errors changes for different size structuring elements. In particular, the pyramid and dome structuring element results for p3n2 and f3n2 show the coke image has a lower minimum absolute error than that of the girl image. The results generated for p7n2 and f7n2 in Figure 5-12 show the opposite results. This seems to show that both the shape of a structuring element and the size of the structuring element can interact with the image being processed.

The following table decodes the labels for the graphs.

<table>
<thead>
<tr>
<th>Character</th>
<th>Naming Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The type of structuring element applied. The values can be ‘c’, ‘d’, ‘f’, ‘p’ for concave, dome, flat, pyramid respectively.</td>
</tr>
<tr>
<td>2</td>
<td>The size of the square structuring element. This can take the values of 3, 5, 7. for structuring elements of sizes 3 by 3, 5 by 5, 7 by 7 respectively. The concave and dome structuring elements are restricted to sizes of 5 and 7.</td>
</tr>
<tr>
<td>3-4</td>
<td>The amount of noise applied to the image. ‘n1’ for 10 percent error, or ‘n2’ for 20 percent error.</td>
</tr>
</tbody>
</table>

*Table 5-1 Decoding Graph Labels*
**Figure 5-1 Dilation Results 10 Percent Error**

**Figure 5-2 Wide Dilation Results 10 Pct. Err.**

**Figure 5-3 Dilation Results 20 Percent Error**

**Figure 5-4 Wide Dilation Results 20 Pct. Err.**
Figure 5-9 Open Results 10 Percent Error

Figure 5-11 Close Results 10 Percent Error

Figure 5-10 Open Results 20 Percent Error

Figure 5-12 Close Results 20 Percent Error
5.2. Visual Results

The visual results of an image processing technique provide the final judgment on the goodness of the technique. This section will provide a look at some of the results that were obtained.

This experiment uncovered that using larger structuring elements induced a blockiness in the resulting image. This blockiness is objectionable to the eye. An example of the blockiness induced can be seen by comparing Figure 5-13 Image Closed by 3x3 Flat SE to Figure 5-16 Coke Image Closed by 7x7 Flat SE. Some blockiness is visible in both images, but the larger structuring element makes the blockiness more pronounced. The numerical results did show an increase in the absolute error, but did not indicate how objectionable the results would be. In this case the visual results provide a good check on the numeric results.

The larger structuring elements also require a larger DC shift to obtain the minimum absolute error. A Shift of minus 60 (-60) levels was required to minimize the error from the base image shown in Figure 5-22 Coke Image Dilated by 7x7 Flat SE. The image shown in Figure 5-25 Coke Image Dilated by 7x7 Flat SE Shifted Down 60 Levels has the minimum absolute error for this comparison, but this was done by the DC shift only, darkening the image. The blockiness is still in the image, but the color is truer to the original shown in Figure 3-1 Baseline Coke Image. Originally, it was thought that a DC shift range of -20 to 20 would be sufficient to cover the whole range. When the results were generated, it was discovered that this was not the case. The images in this section also show the old narrow graphs and the new wider graphs. The wider graphs have the range form -90 to 90, with the absolute error evaluated every 5 levels (-90, -85, ..., 0, ..., 90.) The interpretation of the wider graphs is otherwise the similar.
The decision on which of the processed images looks better is subjective, but the numerical results provide some help, although they do not provide a complete answer.
Figure 5-13 Image Closed by 3x3 Flat SE

Figure 5-14 20 Pct. Err. Closed by Flat SE of size 3

Figure 5-15 10 Pct. Err. Closed by Flat SE of size 3 Narrow Graph
Figure 5-16 Coke Image Closed by 7x7 Flat SE

Figure 5-17 20 Pct. Err. Closed by Flat SE of size 7

Figure 5-18 10 Pct. Err. Closed by Flat SE of size 7 Narrow Graph
Figure 5-19 Coke Image Dilated by 3x3 Flat SE

Figure 5-20 20 Pct. Err. Dilated by Flat SE of size 3

Figure 5-21 20 Pct. Err. Dilated by Flat SE of size 3 Narrow Graph
Figure 5-22 Coke Image Dilated by 7x7 Flat SE

Figure 5-23 20 Pct. Err. Dilated by Flat SE
of size 7

Figure 5-24 20 Pct. Err. Dilated by Flat SE
of size 7 Narrow Graph
Figure 5-25 Coke Image Dilated by 7x7 Flat SE Shifted Down 60 Levels
Figure 5-26 Coke Image Eroded by 3x3 Flat SE

Figure 5-27 20 Pct. Err. Eroded by Flat SE of size 3

Figure 5-28 20 Pct. Err. Eroded by Flat SE of size 3 Narrow Graph
Figure 5-29 Coke Image Eroded by 7x7 Flat SE

Figure 5-30 20 Pct. Err. Eroded by Flat SE of size 7

Figure 5-31 20 Pct. Err. Eroded by Flat SE of size 7 Narrow Graph
Figure 5-32 Coke Image Eroded by 7x7 Flat SE Shifted Up 55 Levels
Figure 5-33 Coke Image Opened by 3x3 Flat SE

Figure 5-34 20 Pct. Err. Opened by Flat SE of size 3

Figure 5-35 20 Pct. Err. Opened by Flat SE of size 3 Narrow Graph
Figure 5-36 Coke Image Opened by 7x7 Flat SE

Figure 5-37 20 Pct. Err. Opened by Flat SE of size 7

Figure 5-38 20 Pct. Err. Opened by Flat SE of size 7 Narrow Graph
6. Literature Cited
