Segmentation of patient motion from other MRI system instabilities

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Segmentation of Patient Motion from other MRI System Instabilities

by

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Date: 11/4/1997
Segmentation of Patient Motion from other MRI system instabilities

by Wayne E. Prentice

Submitted to the Center for Imaging Science in partial fulfillment of the requirements for the Master of Science Degree at the Rochester Institute of Technology.

Abstract

Patient motion causes the same image artifacts or ghosting patterns as system instabilities cause in Magnetic Resonance images (MRI). Misinterpreting a patient-motion-induced artifact as a system instability can cause unnecessary system downtime and expense while a field engineer searches for a non-existent system problem. Although the images may look the same, the original sampled data is different and can be used to determine the cause of the image artifacts. The process for segmentation is to detect the existing instabilities and bulk motion vectors within the sampled frequency-space complex data. Because the imaged object is real, the frequency-space sampled data is Hermitian and redundant information exists. The two halves of the data set are compared and statistics extracted. Vectors are created representing the instability types: magnitude, phase, and echo shift, in addition to frequency encoding and phase encoding bulk motion. Each element of the vector represents how the phase encoding view differed from the complex conjugate view. The vector is then used to correct the data to remove the instability or motion. The order used for segmentation is: magnitude, echo shift, frequency encoding bulk motion, phase, phase encoding bulk motion. Bulk motion in the frequency encoding direction can be segmented from all types of system instability. Bulk motion in the phase encoding direction can be segmented from magnitude and echo shift instabilities, but not from phase instabilities. Other types of motion like scaling and rotation are problematic since the quantized nature of the sampled data precludes their characterization. The methods developed for isolating bulk motion from system instabilities are demonstrated on synthetic data.
Acknowledgments

The author wishes to thank the General Electric Company for the experience in magnetic resonance imaging, especially Phil Steen for teaching me the basics in system analysis, and the “ETC” group for experiencing it. I would also like to thank RIT for providing the background to take my understanding further, and my advisor Dr. Hornak for guiding me through the process. Finally, I would like to thank my wife Linda for providing me with the support required to complete this thesis and my son Evan for giving me the greatest inspiration to finish.
Dedication

This thesis is dedicated to my wife, Linda I. Prentice.
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GLOSSARY

NMR  Nuclear Magnetic Resonance. A physical phenomenon where a nucleus of an atom with an odd numbers of protons and neutrons emit an RF signal in response to an excitatory RF pulse at a specific frequency

MRI  Magnetic Resonance Imaging. An imaging device that uses a spatially encoded NMR signal to produce images

RF  The portion of the electromagnetic spectrum between 30 khz - 300 Ghz

K-Space  The representation of the sampled NMR signal prior to image reconstruction

kx  A coordinate in K-Space along the time or frequency encoding direction.

ky  A coordinate in K-Space along the phase encoding direction.

\[ RECT(x) = \begin{cases} 
1 & |x| < \frac{b}{2} \\
\frac{1}{2} & |x| = \frac{b}{2} \\
0 & |x| > \frac{b}{2} 
\end{cases} \]

SINC  A mathematical function equal to \( SINC(x) = \frac{\sin(m\pi)}{m\pi} \)

Gx  The frequency encoding gradient wave-form.

Gy  The phase encoding gradient wave-form

DC  Literally Direct Current. Used to indicate a signal that is constant over time.

CT  Computed Tomography. An X-Ray based medical imaging modality for imaging cross sectional anatomy.
CRT Cathode Ray Tube.

FFT Fast Fourier Transform. An efficient mathematical algorithm for converting signals from the time domain to the frequency domain.

$B_0$ Symbol used to represent the static magnetic field used in MRI imaging.

TR Repetition time. The time between the initial RF pulses in each phase encoding.

TE Echo time. The time between the 90 and 180 degree RF pulses and the sampled echo.

$T_1$ A time constant representing the time required for longitudinal relaxation or spin-lattice relaxation time.

$T_2$ A time constant representing the time required for transverse relaxation or spin-spin relaxation time.

$k$ A proportionality constant that depends on system gain.

$\phi(x,y,t)$ Phase as a function of position and time.

$s(x,y,t)$ NMR Signal as a function of position and time.

$m(x,y)$ An expression representing the imaged object

$n(t)$ Noise as a function of time.

$rg(t)$ System receive gain as a function of time.

$\delta(t)$ Dirac Delta Function. A mathematical function that is zero everywhere except where $x=t$ when the value becomes infinity.
\[ \frac{d\varphi_x(t)}{dt} \] The rate change of phase over time.

\[ \frac{d\varphi_y(\text{view})}{d(\text{view})} \] The rate change of phase over views.

\( \varphi_{x0} \) Initial X phase.

\( \varphi_{y0} \) Initial Y phase.

★ The symbol used to represent the cross-correlation operation.

\[ F\{ \} \] Used to denote the Fourier transform operation
Chapter 1

INTRODUCTION

The body of literature dedicated to Magnetic Resonance Imaging (MRI) is extensive. Much research has been done to explore subjects such as the efficacy of MRI compared to other modalities, applications, detection of pathology, novel data acquisition techniques, system design, etc. One subject that is central to the success of MRI that is sparsely treated, if not non existent in the literature is the study of MRI system performance, characterization and system analysis. It is often taken for granted that a given scanner always operates at peak performance. Given the complexity of the system, and the specialized nature of the equipment, the reliability of a scanner is remarkable.

Magnetic resonance (MR) image acquisition differs from all other imaging systems. Most imaging systems involve visible light, whereas MRI uses radio-frequency (RF) waves as the illuminate. Nuclear medicine gamma cameras, computed topography (CT), and ultrasound modalities also do not use visible light. The wavelength of the illuminate in MRI is of the order of 2-3 meters, while the resolution of the device is tenths of a millimeter. In modalities using visible light, the wavelength of the illuminate is orders of magnitude
smaller than the resolution of the image. Further, the imaging data is sampled in the frequency domain, instead of the spatial domain, as with other imaging systems. The imaging chain for an MRI scanner is depicted in Figure 1.

![Imaging Chain Diagram](image)

**Figure 1.** MRI imaging system.

The MRI system is complex, using a large magnet which is often superconducting. Specialized equipment is required to keep the superconducting magnetic core at 4 degrees above absolute zero. The system also uses an RF transmitter suitable for a medium-sized FM radio station, audio frequency amplifiers sufficient for a rock concert, specialized waveform signal generators, digital sampling signal detectors, and high-speed computers for image reconstruction and system control. The specialized equipment and the complexity would imply that the MRI system would be unreliable. While that is not the case, the study of system performance, analysis, and characterization is necessary to ensure image quality.
The performance of an MRI imaging system is a function of system design, scan parameter selection, and the current state of system tuning. Maintaining an MRI system and diagnosing a system problem can be daunting; given the complexity of the equipment and the specialized and diverse knowledge required. The difficulty is also compounded by limited system access. The cost of a scanner is typically $1.5-2.5 million. The facilities required to support a system cost an additional $1 million. The cost requires that the scanner be operated as much as 20 hours/day to recover the investment. In many cities, MRI clinics compete for referrals. A referral base is built on superior image quality and reliability. Given these controlling factors, there is a real need for fast and accurate system evaluation, diagnose problems and maintain system performance.

The reliability of any system depends on the reliability of the component subsystems. The subsystem most likely to fail is one that is most stressed, such as the high-power sub-systems. The type of failure most common is not complete, but degrades such that the subsystem either performs in a compromised state or its behavior changes during a scan. This type of failure may be called an “instability”. An instability is a condition where the subsystem performance does not necessarily fail, but fluctuate between a good and bad state. System instabilities are very difficult to diagnose in MRI since they tend to be intermittent. System variation within a good operating range can also have a devastating impact on image quality.
The most common instability is not due to the MRI system, but to patient motion. Image-domain data is not a specific indicator of the source of an instability. For example, the images in Figure 2 all result from data corrupted by a simulated instability. The quality of the artifact looks similar; each image suffers from "ghosting" above and below the image.
Figure 2. Comparison of the effect of instabilities on an image.
Image artifacts due to patient motion are indistinguishable from MRI system instabilities. Currently, the only available method for differentiating between these two is to test the MRI scanner for instabilities over a period of time such that one can rule out the MRI system as the instability source. Instabilities due to patient motion occur intermittently. If the frequency of a system instability causing an imaging problem was one every 50 scans (about twice/day), a total of 235 scans would be required to be 95% certain that the instability was not due to patient motion. This test would require 2-3 days to perform. At an estimated revenue of $800/MRI exam and 20 exams per/day, the cost of testing is $40,000 plus the time and costs of the service organization; the total cost can easily exceed $50,000. The goal of this thesis is to explore the mathematics involved to segment system instability from patient motion using the complex sampled data before image reconstruction. The method will be tested for robustness and limitations.

Artifacts in magnetic resonance images arise from many sources, including the aforementioned system instabilities. As stated, it is not possible to view a magnetic resonance image and determine the source of an instability. This is because the information needed to make this determination is lost when the data is reconstructed to form the image. Image data contains only magnitude information; no phase data is retained. Since the mathematics of MRI are well understood, it should be possible to design an algorithm to examine the frequency-space data to provide information about the source of an instability.
It will be shown that sub-system instabilities can alter the sampled nuclear magnetic resonance (NMR) signal in three ways. The effect on the complex-value sampled data (k-space) manifests itself as a magnitude, phase, and frequency shift in the data. A magnitude instability causes the magnitude of the sampled data to be modulated by the system instability. A phase instability creates an alteration of the starting phase of an NMR echo. A frequency instability, is a alteration of the linear phase component in image space. This is caused by a time shift in the sampled echo and is also called echo shift. Frequency instabilities are related to echo shift by the Fourier shift theorem which states a shift in time in one domain results in a linear phase shift in the frequency domain.

This thesis will develop a mathematical model for patient motion and other system instabilities. The model will serve as a basis for, a method to segment patient motion from the other system instabilities. The method assumes that the two-dimensional (2-D) complex frequency-domain data can be obtained. From that data the cause of the instability will be determined. The process for troubleshooting an instability problem is depicted in Figure 3.
Figure 3. Troubleshooting flow.

**Problem Domain**

This thesis will present the mathematics behind segmenting patient motion from other system instabilities. MRI raw data are minuscule sampled RF signals emanating from patient tissues. The data collected on actual MRI hardware contain additive noise and sampling artifacts. The methods derived in this thesis will be demonstrated on synthetic data that does not suffer from these problems. The methods derived are intended to be robust for use with real world data, but other problems must be solved before these techniques can be applied in a clinical situation.

This thesis addresses only motion within a slice. Intra-slice motion can be classified as bulk, scaling, rotational, and flow. “Bulk motion” (sometimes called rigid motion), is shift invariant, meaning that every portion of the image moves the same amount and in the same direction. “Scaling motion” is displacement proportional to the distance from an
origin. "Rotational motion" is due to rotation about an origin and "flow" is displacement of a portion of an image from the rest. This thesis will discuss all types of motion and the issues involved with segmenting from system instabilities. Solutions for only the bulk case will be provided.

Method

This thesis will demonstrate that all instabilities can be represented as one of four types: magnitude, echo shift, phase, and patient motion. Each of these instabilities will be extracted from the image data represented in frequency space. In other words, the sampled data set is the Fourier transform of the image data. The object being imaged is real. Therefore the Fourier-domain data is Hermitian. The technique used is this thesis extracts the statistics of the Fourier-domain data and uses the fact that the second half of the sampled data is the flipped, mirrored, complex conjugate of the first half. The instability types are orthogonal however the impact on image data is not. These interactions will be described. Once the instability data are extracted and the interactions understood, the segmentation process can occur.

The methods will be tested on synthetic data. These test images will be subjected to the various instability types and simulated patient motion. Software employing the segmentation techniques will be used to extract system instability and motion information.
Chapter 2

OTHER RESEARCH

The goal of this thesis is to differentiate motion from other system instabilities. A search of MRI and scientific journals indicates an absence of work to characterize, measure, and classify system instabilities. However, the effect of motion is well represented. For the purposes of this thesis, detection of motion rather, than the more difficult task of correction, is required. However, detection of motion must be derived from the sampled raw data after acquisition. The bulk of papers describe motion correction using navigator pulses,\textsuperscript{11,16} gated acquisitions,\textsuperscript{5,6,7} modified acquisitions,\textsuperscript{8,9,10} gradient moment nulling,\textsuperscript{12} transducer detection,\textsuperscript{15} multiple acquisitions,\textsuperscript{20,21,22} (two- and three-point phase cancellation) and the assumption of periodicity.\textsuperscript{14} A number of papers exist that attempt to correct for motion given the following constraints:

- No motion occurs during frequency encoding
- No \textit{a-priori} knowledge exists about the motion.
- Only bulk transitional motion may exist
The motion is limited to the imaging plane.

Three methods are described in the literature to correct the motion: the phase retrieval algorithm,\textsuperscript{17,18} the spectrum shift algorithm\textsuperscript{16,19} (X motion only), and a method based upon the symmetrical density constraint (Y motion only).\textsuperscript{19}

The phase retrieval algorithm is an iterative method that attempts to estimate the motion parameters by removing the \textit{ghosts} outside the region of support of the imaged object. This is done by clipping the image data and setting the \textit{noise} or \textit{ghost} field to zero. Then return to K-space and subtracting the phase of the original image. A least-squares calculation is performed to estimate the slope of the phase. This slope is then removed and the process is repeated until no further improvement is obtained.

The spectrum shift algorithm is used to correct motion in the X direction. A one-dimensional (1-D) fast Fourier transform (FFT) is applied in the X (or read-out) direction. The edge of the imaged object is detected and maintained throughout each \( k_y \) view. An unaligned \( k_y \) view is shifted back to remove the effect of the motion.

The symmetrical density constraint is used to detect bulk motion in the Y (phase encoding) direction using the following property: if the density distribution of the imaging object is symmetrical along the Y-axis, the phase of its Fourier spectrum is a linear function.
of \( k_y \). To apply this method, there must exist at least one \( Y \) line in the image that is symmetrical. The test for \( Y \) motion is performed on this line. Since the data are symmetrical, the Fourier transform is real valued. If the object is centered the phase component without motion must be constant. If the object is not centered, the phase is a linear function of frequency with slope proportional to the displacement from center (Fourier Shift theorem: The transform of a shifted function is the transform of the original function with a linear phase factor added to the phase of the original transform)\(^1\). Any deviation in phase from a straight line can be associated with motion. This property can be used to restore the linear phase function to correct the motion.
Chapter 3

MATHEMATICS OF MRI PHYSICS

The source of the MRI signal is the phenomenon of nuclear magnetic resonance (NMR). This section will describe the mathematics of signal generation and how the signal encodes image information. The process transforms spatial information into frequency information. The frequency information is carried by a time-domain signal. A 2-D inverse Fourier transform converts the signal back to an image representation. This chapter presents a detailed description of that process.

An atom with an odd number of protons or neutrons exhibits, a nuclear magnetic moment due to the uneven pairing of the protons or neutrons in their respective nuclear orbitals. The hydrogen atom possesses a strong nuclear magnetic moment since each nucleus contains an unpaired proton. In the presence of a strong magnetic field, \( \mathbf{B} \) the nuclear magnetic moment will rotate some of the protons into alignment with the magnetic field. These protons will rotate about the magnetic field axis (precess) at a frequency \( \omega \) defined by the Larmor equation:
\[ \omega = \gamma \cdot B \] (1)

The proportionality constant of \( \gamma \) is called the gyromagnetic ratio and has a value of

\[ 2 \cdot \pi \cdot 42.58 \frac{MHz}{Tesla} \] for hydrogen.

If an RF field is present with a magnetic component having a frequency equal to the Larmor frequency, the protons will absorb the energy and rotate out of alignment with the static magnetic field. Once the field is removed, the protons will return to their relaxed state, thus generating a signal. The strength of the signal depends on the number of signal generating protons excited by the RF pulse and two time constants: the spin-lattice relaxation time \( (T_1) \) and the spin-spin relaxation time \( (T_2) \). \( T_1 \) is a measure of the time required for the protons to realign with the static magnetic field. When the net magnetization vector is rotated by \( \pi/2 \) \( (90^\circ) \) radians away from the applied \( B_0 \) field, the signal is maximized. All magnetization vectors start out with a common phase after this rotation. The individual magnetization vectors begin to precess about the direction of the applied field \( B_0 \), and the protons lose coherence due to small variations in local magnetic field strengths. The \( T_2 \) time constant is derived from the amount of time required for the population of protons in the volume to lose a coherent phase.
The NMR signal is generated when protons are rotated into the transverse plane. The net magnetization vector preseses, creating a small RF signal. That signal induces a current in an RF coil, which is subsequently sampled.

The most commonly used imaging sequence is called a spin-echo sequence. It consists of two RF pulses and a signal from the precession of the net magnetization vector after being rotated into the x-y plane. Figure 4 depicts the timing diagram for a spin-echo sequence. The first signal generated is called a free induction decay (FID), the second is an echo.

![Figure 4. FID and echo generation.](image-url)
The underlying physics occurring at the shaded portion of the timing will be depicted in subsequent figures.

Prior to the 90° pulse, a subset of the protons align with the static magnetic field. No signal is created since there is no component of the net magnetization vector in the transverse plane (Figure 5).
A 90° pulse is applied creating a FID. The net magnetization vector is large since the protons are mostly in phase. The signal generated is maximized since the entire vector lies in the transverse plane (Figure 6).

Figure 6. Spin echo sequence during 90 degree pulse.
The FID decays due to de-phasing. The NMR signal decays due to the incoherence of the combined signal from the protons (Figure 7).

Figure 7. Spin echo sequence after 90 degree pulse.
The FID decays due to de-phasing.

The net magnetization vector approaches zero magnitude as the signals no longer in phase so they do not add constructively (Figure 8).
Reverse the de-phasing with a 180° Pulse

A 180° pulse is applied to create an echo. The occurs when the spins are reversed and the de-phasing process becomes a re-phasing process (Figure 9).
Re-phasing Protons generate a signal
The Net Magnetization Vector magnitude increases as phase becomes coherent

Figure 10. Spin echo sequence after a 180 degree pulse.

The signals return to the maximum as the signal re-phases (Figure 10).
The signal reaches a maximum as the spins acquire the same phase. Net Magnetization Vector at maximum

The re-phasing process continues past the peak signal where de-phasing eventually eliminates the signal (Figure 11).
$T_1$ decay occurs as the small magnetization vectors realign with the static field (Figure 12).
After the $T_1$ relaxation, the protons return to the quiescent state (Figure 13).

The relationship between maximum signal amplitude and the two exponential time constants $T_1$ and $T_2$ is:
\[ m(TE, TR) = \kappa \cdot \rho \cdot e^{-\frac{TE}{T_2}} \left( 1 - e^{-\frac{TR}{T_1}} \right) \]  

(2)

where \( \kappa \) is a proportionality constant that describes the receiving coil sensitivity, \( \rho \) is the proportion of signal-generating protons from the population. TE and TR are scan parameters which are the contrast mechanisms for MRI.

An image is made up of picture elements (pixels). Each pixel \( I(x,y) \) represents a particular area element \( dx,dy \) of the object being imaged (Figure 14). Each pixel represents the signal generated from a three-dimensional (3-D) volume \( dx \times dy \times \) slice thickness. This volume is called a voxel.
A property of the magnetic resonance signal is that the frequency of the signal is proportional to the magnetic field strength at each pixel according to the Larmor equation.

The signal from each voxel is encoded by a set of gradient magnetic fields, which are spatially linear fields.
Let $G_x$ (Figure 15) be the slope of the $x$ gradient field including the constant gyromagnetic constant $\gamma$. The frequency of precession along the $x$ axis can be described by:

$$\omega(x) = \gamma \cdot B(x) = G_x \cdot x$$  \hspace{1cm} (3)$$

The phase along the $x$ axis at any point in time is:

$$\omega(t) = \frac{d\varphi_x}{dt} = x \cdot G_x$$  \hspace{1cm} (4)$$

$$d\varphi(x,t) = x \cdot G_x \cdot dt$$  \hspace{1cm} (5)$$

$$\varphi(x,t) = \int x \cdot G_x \cdot dt$$  \hspace{1cm} (6)$$

Assuming that the object position is constant over time,
During a scan, a slice is selected (i.e., the only signal present is from a 2-D slice of the object being imaged).

The phase in the y dimension is derived in the same manner as x. In the two dimensional case:

$$\varphi(x,y,t) = x \cdot \int G_x \cdot dt + y \cdot \int G_y \cdot dt$$

(8)

Spatial encoding in the x-direction will be called “frequency encoding”, and that in the y-direction will be called “phase encoding”. The reason will become clear in a moment.

The signal from a differential area element is the product of the amplitude $I(x,y)$ and the phase:

$$S(x,y,t) = I(x,y) \cdot e^{\frac{2\pi i \varphi(x,y,t)}{\lambda}}$$

(9)

The signal from every element emanates at the same time. The total signal is the spatial integral of the signal from each area element.
This relationship can be used to spatially encode the NMR signal. In a typical MRI acquisition, \( S(t) \) is not continuous. Each \( S(t) \) of a ensemble is acquired for constant Gy with \( c_{py} \) changing between members in the ensemble. 

\[
S(t) = \int \l(x,y)e^{i2Kt^2}e^{-2Ki\tau}dxdy
\] 

(10)

In an MRI acquisition, \( S(c_j, T) \) represents the ensemble of waveforms, where \( c_j \) represents the time variable within each member of the ensemble and \( r) \) describes the member of the ensemble (Figure 16). This relationship now will be demonstrated.
For each member of the ensemble, $\varphi_x$ changes linearly, and $\varphi_y$ is constant. This array of data is called a “view”. This encodes spatially in the $x$ direction (frequency encoding). For each view, $\varphi_y$ changes linearly, spatially encoding the $y$ direction (phase encoding).

A set of gradient waveforms can be described that phase encodes the image. For all members of the ensemble, let:

$$\varphi_{0x} = -\pi$$
$G_x(t)$ is a gradient field in the $x$ direction.

Figure 17. Gradient as a function of time.

Figure 18. Phase response to a gradient field.

Encoding in this direction is “frequency encoding”.

For each member of the ensemble, $G_y(t)$ is constant.
Let:

\[ \xi = \frac{d\phi_x(t)}{dt} - \pi, \quad \eta = \frac{d\phi_y(\text{view})}{d\text{view}} - \pi \]  \hspace{1cm} (14)

\[ S(\xi, \eta) = \int\int_{x\ y} m(x, y) \cdot e^{-2\pi i \xi x} \cdot e^{-2\pi i \eta y} \, dx \, dy \]  \hspace{1cm} (15)

\( S(\xi, \eta) \) is the Fourier transform of \( m(x, y) \). The complete sequence of gradient pulses is shown in Figure 19.
Figure 20. Typical spin echo imaging sequence.

The RF waveform is used to excite the slice. The shorter pulse is called a 90° pulse and the longer pulse is called a 180° pulse. The slice select gradient is used to excite only the particular slice to be imaged. The first $G_x$ pulse is used to set $\phi_{0x} = -\pi$ radians. The second pulse is shown as the $G_x(t)$ described in Figure 17. $\phi_{xo}$ is set to $-\pi$ so that the center of the sampled window occurs where $\phi_{0x} = 0$. $G_x$ is zero through the sampling process. The starting phase $\phi_{xo}$ is changes each view according to Figure 19. This pulse sequence is called a spin-echo sequence.
Spatial encoding due to $G_x$ is called frequency encoding since encoding is due to phase changes over time: \( \frac{d\phi(t)}{dt} \) or frequency (Figure 21)

![Diagram of frequency encoding]

Figure 21. Frequency encoding.

Encoding due to $G_y$ is called phase encoding since each view is stepped by a constant delta phase.

**MRI: The Good, Bad, and Ugly**

The MRI process works best when the system accurately produces the waveforms described in Figure 20. A more complete equation can be used to describe a real-world MRI
acquisition process. The MRI imaging chain is depicted in Figure 3. The focus of this thesis is on the imaged object (patient), physical phenomenon, and detection.

The received signal is a function of system noise, receive gain, patient position, starting phase and gradient fidelity.
<table>
<thead>
<tr>
<th>Item</th>
<th>Represents</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m(x,y) )</td>
<td>Cross Section</td>
<td>Constant of Physical object</td>
</tr>
<tr>
<td>( n(t) )</td>
<td>Noise</td>
<td>Low Level, uniformly distributed</td>
</tr>
<tr>
<td>( r_g(t) )</td>
<td>Receive Gain</td>
<td>Constant</td>
</tr>
<tr>
<td>( 5(t) )</td>
<td>Patient Position</td>
<td>Constant</td>
</tr>
<tr>
<td>( \Delta x { t } )</td>
<td>( \Delta y { \text{view} } )</td>
<td>( \Delta v { 0 } )</td>
</tr>
<tr>
<td>( x )</td>
<td>Gradient</td>
<td>Constant</td>
</tr>
<tr>
<td>( y )</td>
<td>Gradient</td>
<td>Constant</td>
</tr>
<tr>
<td>( \Delta s { 0 } )</td>
<td>Starting Phase</td>
<td>Constant</td>
</tr>
</tbody>
</table>

\[
S(x,y,t) = r_g(t)m(x,y)n(t) + r_g(t)m(x,y)n(t)
\]

Where:
\( c, = -t + x_0 + \Delta s \)
\( y = -y + \Delta s \)
Types of System Instabilities

For the purpose of this thesis, a system instability is defined the inability of the MR imager to reproduce a desired waveform or accurately sample the NMR signal. By this definition, a noisy image or a shaded can occur without a system. A noisy image can result from either high noise or low signal. Both of these conditions can be caused by a stable system. Low signal might be caused by creating incorrect waveforms, but if the wave-forms are reproduced consistently, then an instability does not exist. A shaded image can be caused by a spatial distortion of the RF transmit field, RF coil sensitivity, or static magnetic fields. While all of these conditions are undesirable, all can occur even though the imager is consistently producing the correct waveforms. Any system instability will change a quality of the sampled data. Changes are possible in magnitude, phase, and echo shift. Each of these instabilities are measured based its effect on the sampled data.
Let $k_x$ and $k_y$ be our sampled data domain. The sampled signal is then: $S(k_x, k_y)$. 

Figure 22: Sampled data layout
An assumption will be made that all instabilities are functions of $k_y$ only. This assumption is valid since $k_x$ typically lasts ~8 msec. Sampling in the $k_y$ direction typically requires 0.5 to 2 seconds.

The remainder of this section will provide the mathematical description of the instability types and how the instabilities relate to physical phenomena. Also it will be
shown that the three instability types (magnitude, phase and echo shift) can be used to describe all possible system instabilities.

Define $S(kx, ky)$ as the ideal data set, and $S'(kx, ky)$ as signal corrupted by an instability.

“Magnitude Instability” is an effect that causes each line of data to be multiplied by a real value, $Mag(ky)$.

$$S'(ky) = |S(ky)| \cdot Mag(ky)$$  \hfill (17)

“Phase instability” is an effect that causes each line of data to be phase shifted by a number of radians, $Phase(ky)$.

$$S'(ky) = S(ky) \cdot e^{i \cdot Phase(ky)}$$  \hfill (18)

Echo shift is an effect that causes each line of data to be shifted in time by a real value, $EchoShift(ky)$. 
\[ S'(ky) = S[ky + EchoShift(ky)] \] (19)

All system instabilities will result in one or more of these effects. This can be shown by going back and rewriting equation 16 in terms of \( k_x \) and \( k_y \) without the motion term. Also note that starting phase terms \( \varphi_{x0} \) and \( \varphi_{y0} \) have been combined into \( \varphi_0(ky) \)

\[ S(kx,ky) = \int \int r_g(ky) \cdot m(x,y) \cdot e^{i \varphi_0(ky)} \cdot e^{i kxz} \cdot e^{i kxy} \, dx \, dy \] (20)

where:

\[ kx = \int G_x \cdot dt \quad \text{and} \quad ky = \int G_y \cdot dview \]

Magnitude instabilities can result from either a receiver gain instability or an instability in the RF transmitter. The effect of transmitter gain can be seen directly from Equation 21. The magnitude signal generated by \( m(x,y) \) depends upon its NMR properties and the amount of transverse magnetization. The signal is optimized when the two RF pulses in Figure 20 are \( \pi/2 \) followed by \( \pi \). These values represent the amount of rotation of the net magnetization vector.

The magnitude of the signal from \( m(x,y) \) also depends on slice thickness, which in turn slice is determined by \( G_z \). Instabilities in \( G_z \) will result in a magnitude instability.
An unstable RF pulse can also cause a phase error. The starting phase of the detected signal is determined by the phase of the RF excitation pulse at the time the pulse terminates. An instability in the RF transmit pulses in terms of either the length or the phase of the pulse has a direct effect on the initial phase of the detected signal.

Phase instabilities can also occur from spatial encoding gradients. In equation 4:

\[ \phi(x, y, t) = x \cdot \int G_x \cdot dt + y \cdot \int G_y \cdot dt \]  

phase was shown to be an integrator of gradient amplitude over time. If the waveforms in Figure 20 are not reproduced such that the integral under each of the x and y gradients are not constant, a phase error will occur.

The NMR echo occurs when: \[ \int G_x \cdot dt = 0 \]. This should occur half way into the second \( G_x \) pulse. Remember that the direction of rotation is reversed by the 180° pulse, so effectively the first gradient pulse is negative. If the shape or duration of \( G_x \) becomes a function of \( ky \), the instability will result in an echo shift.

To summarize, RF instabilities cause both phase and magnitude instabilities. Instabilities in \( G_y \) also cause phase instabilities. Instabilities in \( G_x \) where the area of the first pulse is not equal the area of the first half of the second pulse, result in an echo-shift
instability or a phase instability. Finally, Gz instabilities can cause magnitude instabilities since this gradient determines the thickness of the slice.
Chapter 4

MATHEMATICS OF PATIENT MOTION

Patient motion can be classified into several types: shift invariant or bulk motion, scaling motion, rotational motion, and flow.

In shift invariant motion a shifted copy of an object $m(x,y)$ is a function of $k_y$:

$$m'(x, y) = m(x + x_0(ky), y + y_0(ky))$$  \hspace{1cm} (21)

The Fourier Shift Theorem states that:

$$F\left\{ f[x \pm x_0] \right\} = |F[\xi]| \cdot e^{i|\xi| \cdot 2\pi \cdot x_0}$$  \hspace{1cm} (22)

Translation of the object in the spatial domain results in a linear phase component in the frequency domain.

$$S(kx, ky) = \int\int m(x, y) \cdot e^{i\cdot kx \cdot x} \cdot e^{i\cdot ky \cdot y} \, dx \, dy$$  \hspace{1cm} (23)
\[ S'(k_x, k_y) = \int \int m'(x, y) \cdot e^{i k_x x} \cdot e^{i k_y y} \, dx \, dy \]  \hspace{1cm} (24)

\[ = \int \int m(x + x_0(k_x), y + y_0(k_y)) \cdot e^{i k_x x} \cdot e^{i k_y y} \, dx \, dy \]  \hspace{1cm} (25)

\[ = \int \int m(x, y) \cdot e^{i k_x x + 2 \sigma k_x x_0(k_x)} \cdot e^{i k_y y + 2 \pi k_y y_0(k_y)} \, dx \, dy \]  \hspace{1cm} (26)

\[ = \int \int m(x, y) \cdot e^{2 \pi k_x x_0(k_x) + 2 \pi k_y y_0(k_y)} \cdot e^{i k_x x} \cdot e^{i k_y y} \, dx \, dy \]  \hspace{1cm} (27)

\[ = e^{2 \pi k_x x_0(k_x)} \cdot e^{2 \pi k_y y_0(k_y)} \int \int m(x, y) \cdot e^{i k_x x} \cdot e^{i k_y y} \, dx \, dy \]  \hspace{1cm} (28)

\[ = e^{2 \pi k_x x_0(k_x)} \cdot e^{2 \pi k_y y_0(k_y)} \cdot S(k_x, k_y) \]  \hspace{1cm} (29)

Scaling motion where displacement is proportional to the distance from an origin or:

\[ m'(x, y) = m\left(\frac{x}{a(k_y)}, \frac{y}{b(k_y)}\right) \]

According to the Fourier Scaling theorem:

\[ F\left\{ f \left[ \frac{x}{b} \right] \right\} = |b| \cdot F(b \cdot \xi) \]  \hspace{1cm} (30)

As the size of the imaged object increases in the spatial domain the size of the Fourier transform becomes smaller.
\[ S'(k_x, k_y) = \int \int_{xy} m \left( \frac{x}{a(k_\nu)}, \frac{y}{b(k_\nu)} \right) e^{i k_x x} e^{i k_y y} dxdy \] (31)

\[ S'(k_x \cdot a(k_\nu), k_y \cdot b(k_\nu)) \cdot |a(k_\nu) \cdot b(k_\nu)| = \int \int_{xy} m(x, y) e^{i k_x x} e^{i k_y y} dxdy \] (32)

\[ S'(k_x \cdot a(k_\nu), k_y \cdot b(k_\nu)) \cdot |a(k_\nu) \cdot b(k_\nu)| = S(k_x, k_y) \] (33)

Rotational motion describes an object rotating about an origin. The Fourier transform rotates by the angle:

Given:

\[ x = r \cdot \cos \theta \quad y = r \cdot \sin \theta \quad u = \omega \cdot \cos \phi \quad v = \omega \cdot \sin \phi \] (34)

and

\[ F \{ f(x, y) \} = F(u, v) \] (35)

Then:

\[ F \{ f(r, \theta) \} = F(\omega, \phi) \] (36)

\[ F \{ f(r, \theta + \theta_0) \} = F(\omega, \phi + \theta_0) \] (37)

A rotation in the spatial domain is preserved in the frequency domain.
\begin{align*}
S(k_x, k_y) &= \int \int m(r, \theta) \cdot e^{ik_x x} \cdot e^{ik_y y} \, dx \, dy \quad (38) \\
S'(k_x, k_y) &= \int \int m(r, \theta + \theta(k_x)) \cdot e^{ik_x x} \cdot e^{ik_y y} \, dx \, dy \quad (39) \\
S'(\omega, \phi + \theta(k_x)) &= S(k_x, k_y) \quad (40)
\end{align*}

Flow is displacement of a portion of an image from the rest. Flow typically does not remain within a given slice for length of a scan. Flowing material usually contains spatial information from adjacent slices. This thesis is limited to intra-slice motion, so flow motion will not be discussed.
Summary:

Table 2: Motion effect on k-space.

<table>
<thead>
<tr>
<th>Motion Type</th>
<th>Frequency Domain Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bulk Motion</strong></td>
<td>Linear phase shift</td>
</tr>
<tr>
<td>Object moves by p samples in x</td>
<td>Phase increased by $p\pi \cdot k_x$</td>
</tr>
<tr>
<td><strong>Scaling</strong></td>
<td>Inverse Scaling</td>
</tr>
<tr>
<td>Spatial domain object becomes larger by 25%</td>
<td>Dimension of FFT becomes smaller by 25%</td>
</tr>
<tr>
<td><strong>Rotation</strong></td>
<td>Rotation</td>
</tr>
<tr>
<td>Object rotates 20 degrees</td>
<td>Frequency domain data rotates by 20 degrees</td>
</tr>
</tbody>
</table>
Chapter 5

SEGMENTATION OF PATIENT MOTION FROM SYSTEM INSTABILITIES

Method

Earlier chapters described the mathematics behind system instabilities and patient motion. The methods selected to distinguish between patient motion and system instabilities have been based upon robustness with real world data. To that end, the methods used do not compare sampled values directly. Rather statistics are extracted from the $k_x, k_y$ data set that are based upon an entire $k_x$ line.

No *apriori* knowledge is assumed to exist about the images analyzed. Stability and motion statistics are extracted by comparing $k_x$ lines from $k_y$ and $-k_y$. The raw data is expected to be Hermitian since the imaged object is real. Therefore,
\[ S(k_x, k_y) = S^*(-k_x, -k_y) \quad (41) \]

**Extraction of System Instabilities from k-space**

**Extraction of Magnitude Instabilities**

In equation 18, Magnitude instabilities were described as: \( S^*(k_y) = |S(k_y)| \cdot Mag(k_y) \)

The Magnitude instability is calculated as follows:

\[
Mag(k_y) = 100 \cdot \frac{\sum_{k_x} |S(k_x, k_y)| - \sum_{k_x} |S(-k_x, -k_y)|}{\sum_{k_x} |S(k_x, k_y)|} 
\quad (42)
\]

This process will calculate the magnitude sum across \( k_x \) for each \( k_y \). In a stable system \( \sum_{k_x} |S(k_x, k_y)| \) should equal \( \sum_{k_x} |S(-k_x, -k_y)| \), in which case \( Mag(k_y) = 0 \). Magnitude error units are calculated as a percent of the \( S(k_y) \) lines.

**Extraction of Phase Instabilities**

In equation 19 Phase instabilities were described as: \( S^*(k_y) = S(k_y) \cdot e^{\text{Phase}(k_y)} \)

Phase is calculated as follows:
1. "Unwrap" the phase in the $k_x$ direction

2. Perform a linear regression on $\Phi\left[S(k_x,k_y)\right]$ and $\Phi\left[S^*\left(-k_x,-k_y\right)\right]$ to calculate the slope $m(k_x)$, $m'(k_x)$ and intercept $b(k_y)$, $b'(k_y)$ of the phase as a function of $k_y$.

3. The calculated phase error becomes:

$$Phase(k_y) = \left| b(k_y) - b'(k_y) \right|$$  \hspace{1cm} (43)

**Extraction of Echo Shift Instabilities**

Echo shift instabilities were described in equation 20 as $S'(k_y) = S[k_y + EchoShift(k_y)]$

The echo shift is calculated as follows:

1. From $S(k_y)$, calculate the cross correlation in the $k_x$ direction only:

$$\left|S(k_x,k_y)\right| \star \left|S(k_x,-k_y)\right| = \int \left|S(\alpha,k_y)\right| \left|S^*(\alpha - k_x,-k_y)\right| d\alpha$$

2. Find the maximum $k_x$ for each $k_y$ line. If multiple maxima values occur, select the value closest to the origin.
EchoShift\((k_x) = \text{Max}(k_y)\) 

Echo shifts are detected by a matched filter. The maximum value of the cross correlation of the magnitude occurs at the location where the two signals are most similar. This is done to make the result independent of the phase data.

**Extraction of Patient Motion from k-space**

Patient motion like system instabilities will be extracted assuming that motion is only a function of \(k_y\).

\[S'(k_x, k_y) = e^{2\pi k_x \cdot \phi_0(k_y)} \cdot e^{2\pi k_y \cdot \phi_0(k_x)} \cdot S(k_x, k_y)\] 

(45)

**The procedure for extraction of Bulk X motion.**

1. Calculate the Fourier transforms of data in the vertical \((k_y)\) direction to produce: \(S(x, k_y)\)

2. Calculate \(\left|S(x, k_y)\right| \ast \left|S*(-x,-k_y)\right|\). The cross correlation in the \(k_x\) direction only.

3. Find maximum \(x\) for each \(k_y\) line. If multiple maximum values occur, select the value closest to the origin.

4. \(BulkX(k_y) = \text{Max}(k_y)\)
This method transforms the $k_x, k_y$ into $x, ky$. In this space, a shift in $x$ is simply a shift in $x$. The cross correlation used in the detection of echo shift is applied.

**Extraction of Bulk Y motion.**

The method used in Bulk x extraction cannot be applied to Bulk y since complete $x$ location information exists in a line of $k_y$ data. $Y$ bulk motion must be extracted using a less robust method for measuring the average phase slope as a function of $k_y$.

1. Unwrap the phase of $\text{PhaseDiff}(k_x, k_y) = \Phi \left[ \frac{S(k_x, k_y)}{S^*(-k_x, -k_y)} \right].$

2. Sum across $k_x$: $\text{PhaseDiff}(k_y) = \sum_{k_x} \text{PhaseDiff}(k_x, k_y)$

3. The bulk motion in the $y$ direction is the derivative with respect to $k_y$.

$$\text{BulkY}(k_y) = \frac{d \text{PhaseDiff}(k_y)}{dk_y}$$

**Extraction of Scaling in Y**

Scaling motion was described in equation 33 as:

$$S'(k_x \cdot a(k_y), k_y \cdot b(k_y)) \cdot |a(k_y) \cdot b(k_y)| = S(k_x, k_y)$$

In the case of $y$ scaling:
\( S(k_x,k_y) \neq S^*(-k_x,-k_y) \) because the \( k_y \) line will be displaced depending upon the value of \( b(k_y) \). The following expression can be used:

\[
S(k_x,k_y) = |b(k_y)| \cdot S^*(-k_x,-k_y \cdot b(k_y)) \tag{46}
\]

\( b(k_y) \) is determined by:

1. For a given pair of \( k_y \) lines: \( S(k_x,k_{Test}) \) and \( S^*(-k_x,-k_{Test}) \), calculate

\[
Test(region) = S(x,k_{Test}) \star S^*(-x,-k_{Test} - region)
\]

where region is an area about \( k_y_{Test} \) used to locate the displaced \( k_y \) line.

2. Find \( Max(region) \)

3. \[
b(k_y) = \frac{k_{y_{Test}}}{k_{y_{Test}} - region} \tag{47}
\]

**Extraction of scaling in X**

The method applied for scaling in \( y \) cannot be applied to scaling in \( x \) due to the assumption that motion is only a function of \( k_y \). To extract scaling in \( x \), one must enter a "scaling-independent" space. The Mellin transform\(^2\) provides one means to make such a transform. The Mellin transform requires that \( k_x \) be re-sampled at intervals determined from
log(k_x). The resampling would require that the amplitudes following coordinates be interpolated from k_x in a 256-sample array of k_x.

The errors introduced by re-sampling would prevent detecting a(k_y).

![Sampled k_x vs log(k_x)](image)

**Figure 24.** Log sampling of k_x.

**Segmentation Process**

The methods presented above describe how to extract several qualities as functions of k_y: magnitude instability, phase instability, echo shift, X bulk motion, Y bulk motion, and Y scaling motion. This section will describe the order that should be used to create the instability plots.
Table 3. Instability and motion interactions.

<table>
<thead>
<tr>
<th>Instability</th>
<th>Complex Component</th>
<th>Effect on Fourier Domain Data</th>
<th>Detection Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude</td>
<td>Magnitude</td>
<td>Multiplier for each (k_y) line</td>
<td>Compare average (k_x) magnitude for complex conjugate (k_y)</td>
</tr>
<tr>
<td>Phase</td>
<td>Phase</td>
<td>Additive Phase shift for each (k_y) line</td>
<td>Compare intercept (k_x) phase for complex conjugate (k_y)</td>
</tr>
<tr>
<td>Echo Shift</td>
<td>Phase and Magnitude</td>
<td>Time shift for each (k_y) line</td>
<td>Find maximum location of (k_x) cross correlation with complex conjugate (k_y)</td>
</tr>
<tr>
<td>X Bulk Shift</td>
<td>Phase</td>
<td>Additive linear (k_x) phase shift for each (k_y) line</td>
<td>Find maximum location of (x) cross correlation of magnitude image with complex conjugate (k_y)</td>
</tr>
<tr>
<td>Y Bulk Shift</td>
<td>Phase</td>
<td>Additive linear (k_y) phase shift for each (k_y) line</td>
<td>Derivative of the (k_x) average phase difference between (k_y) and complex conjugate (k_y)</td>
</tr>
<tr>
<td>Y Scaling Motion</td>
<td>Magnitude and Phase</td>
<td>(k_y) displaced in (k_y)</td>
<td>Attempt to locate complex conjugate (k_y) by match filter detection.</td>
</tr>
</tbody>
</table>

The order is based upon the relative robustness of the detection methods and correction methods. The more robust methods are calculated first. The data are then corrected for the measured instability to improve the detection of subsequent instabilities.
X-bulk motion affects the k-space phase data, however the detection is done on the magnitude of the image after the horizontally Fourier transform. This is to reduce the dependency of x-motion detection on other phase phenomena.

The phase unwrapping is inexact. The phase of a signal with magnitude less than the noise threshold is impossible to determine. The current phase unwrapping algorithm assumes that the phase of unknown data varies linearly between known values. Unknown phase values at coordinates "outside" known phase are extrapolated from the known values. Phase unwrapping is used to detect phase instabilities and bulk-y motion. Also notice extraction of that phase and bulk y shifts are difficult since they both modify the phase of \( k_y \). Y-bulk motion and phase instability are both additive phase that is a function \( k_y \). Each \( k_y \) line is independent in terms of phase instability and bulk-y motion; therefore, it is not possible to segment these two classes. Though each can be detected in the absence of the other.

Given the rules specified above, the segmentation order becomes:

1. Detect then correct, magnitude instability
2. detect, then correct, echo shift,
3. detect, then correct, x-bulk shift,
4. detect y bulk shift,
5. detect, phase Instability, and
6. detect y scaling motion.

With the exception of phase and y bulk motion, the other classifications are orthogonal. Therefore one can determine the source of the ghosting based upon what was detected in those cases.

Correction was prescribed for magnitude, echo shift, and x-bulk motion. To correct the instabilities, one must know the instability function. The calculations used to extract magnitude, shift, and x-bulk motion do not measure the instability function, but rather measure the error between two k_y lines. While the error is known, how much error to attribute to each of the two lines is not known. For the purpose of this thesis, it will be assumed that the second half of k_y is in error. Though this is not a real-world solution, it will make it possible to demonstrate the principles outlined in this chapter.
RESULTS

This section presents the results of detecting patient motion using simulated images and instabilities. The simulations were computed on a PC using the 32 bit MRImage program\textsuperscript{24} which is written by the author in C++ for Microsoft Windows 3.1\textsuperscript{®}, Windows NT\textsuperscript{®} and Windows95\textsuperscript{®}. This PC-based image analysis package performs real and complex image calculations and data extraction. This program was used to create simulated images and extract of instability data from them. A representative screen dump from an MRImage session is depicted in Figure 25.
Each of the instabilities were be tested individually; some selected cases with multiple instabilities also were tested. The instability function is the product of a step function with a uniformly distributed random data. This function is selected for its
simplicity and because it is easily recognizable. The detection method determines the differences between the data in the two halves of k-space. As a result, the detected function will have half as many samples as the instability function. The first value of each instability plot compares the center two views and continues onto the high-frequency views.

The image used is a simulated abdomen (Figure 26) and contains high contrast edges, large constant areas and low-contrast regions.

![Simulated image of abdomen](image.png)

Figure 26. Simulated image of abdomen for instability testing

The instability function used will be a step function, that is 0 in the region of 0-150. It then will use a uniformly distributed random variable to simulate the instability. An example is displayed in Figure 27.
Figure 27. Single source instability function starting at 150.

**Magnitude Only Instability Detection**

The magnitude instability function will effectively multiply the last 106 lines of k-space by a random uniformly distributed numbers between 0 and 5. The effect of the instability is seen in Figure 28. The instability causes “ringing” and “ghosting” artifacts above and below and through out the simulated abdomen.
The detected instability function shows a detected magnitude instability at view number 22 (Figure 29). This result is expected since the center view is 128. The instability does not affect the data until view 150. Therefore, the first difference should occur at row \((150-128)=22\).
Figure 29. Detected magnitude instability with a magnitude instability present.

It was stated in the last section that magnitude instabilities have no effect on the other instability detections. This can be seen in Figures 30-33.

Figure 30. Detected phase instability with a magnitude instability present
Figure 31. Detected echo center instability with a magnitude instability present.

Figure 32. Detected X Bulk instability with a magnitude instability present.
The magnitude instability tested was severe. The amplitude of last 56 lines of data were 500% greater than nominal. Despite this severe instability, there was no instability measured on phase, echo shift, x-bulk, or y bulk motion.
Phase Only Instability Detection

The phase instability function will effectively add a phase offset generated from a uniformly distributed random variable between 0 and $\frac{\pi}{4}$ to the last 106 lines of k-space. A plot of the specific instability is plotted in Figure 34.

![Phase Instability Function](image)

Figure 34. Phase instability function.
The effect of this instability is seen on the simulated abdomen image is depicted in Figure 35. The detected instability functions can be seen in Figures 36-40.

Figure 35. Phase instability image

Figure 36. Detected phase instability in the presence of a phase instability.
Figure 37. Detected magnitude instability in the presence of a phase instability.

Figure 38. Detected echo shift instability in the presence of a phase instability.
As predicted, y-bulk motion will be erroneously detected when a phase instability exists. Also, note that the detected phase function differs from the instability function, although it becomes non-zero when the instability function becomes non-zero. This is because the phase is detected by calculating the linear regression intercept from the phase unwrapped-data. This will not return the original instability function, but it is expected to be...
more robust since it uses a trend in a population of data rather than calculates individual phase differences.

*Echo Shift Only Instability Detection*

The echo shift instability function will shift the k-space data in the $k_x$ direction according to the instability function depicted in Figure 41.

![Figure 41. Echo shift instability function.](image)

The effect of this instability on the abdomen image is seen in Figure 42. Figures 42-46 present the detected echo shift, magnitude, phase, x motion, and y motion instabilities in the presence of an echo shift instability.
Figure 42. Echo shift instability image.

Figure 43. Detected echo shift instability in the presence of an echo shift instability.
Figure 44. Detected magnitude instability in the presence of an echo shift instability.

Figure 45. Detected phase instability in the presence of an echo shift instability.
The echo-shift detection was able to reproduce the instability function. However, some instability was also detected on the phase, magnitude, and y-shift tests. The phase detection method calculated the intercept of the unwrapped phase. Echo shifts effectively cause a shift in the origin of the line of data. When the echo shifts, the location of the intercept also shifts. This causes the phase instability. Since phase and y-shift are linked,
the y-shift detection is also affected. The segmentation order specifies correcting echo shift before performing the y-shift or phase instability test.

A magnitude instability was also detected (see Figure 44.). The values are small and can be explained as follows. When the data are added, zeros are shifted into the row. The magnitude of the data shifted out will impact the measured magnitude instability to the extent that they differ from zero.

**X-Bulk Motion Only Instability Detection**

The x-bulk motion instability function is a uniformly distributed random variable between 0 and 10. This will simulate a patient moving a random distance in the x-direction, between 0 and 10 pixels in the last 106 lines of k-space. Assuming a 32-cm field of view, a 10-pixel motion is equivalent to 1.25 cm. The instability function is plotted in Figure 48.

![Figure 48. X bulk motion instability function.](image)
The effect of this x-bulk motion instability on the abdomen image is depicted in Figure 49. The detected x-bulk motion magnitude, phase, echo shift and y-bulk motion instability in the presence of an X bulk motion instability are depicted in Figures 50-54.

Figure 49. X motion instability image
Figure 50. Detected x-Bulk motion instability in the presence of a x-bulk motion instability.

Figure 51. Detected magnitude instability in the presence of a x-bulk motion instability.
Figure 52. Detected phase instability in the presence of a x-bulk motion instability.

Figure 53. Detected echo shift instability in the presence of a x-bulk motion instability.
The x-motion instability detection was specific to x-motion. Again phase instability detection was problematic. X-motion causes a linear phase shift in k-space data. The intercept calculation is affected by this change in slope. This should not create a problem for the general case since the algorithm corrects for x-motion prior to the phase instability detection.
The y-bulk motion instability function is a uniformly distributed random variable between 0 and 10. This will simulate a patient moving a random distance in the Y direction between 0 and 10 pixels in the last 106 lines of k-space. The instability function is depicted in Figure 55.

Figure 55. Y bulk motion instability function.
The effect of this y-bulk motion instability on the abdomen image is depicted in Figure 56. The detected x-bulk motion magnitude, phase, echo shift and x-bulk motion instability in the presence of an x-bulk motion instability are depicted in Figures 54-58.

Figure 56. Y motion instability image.

Figure 57. Detected y-bulk motion instability in the presence of a y-bulk motion instability.
Figure 58. Detected Magnitude motion instability in the presence of a y-bulk motion instability.

Figure 59. Detected Phase motion instability in the presence of a y-bulk motion instability.
Figure 60. Detected Echo Shift motion instability in the presence of a y-bulk motion instability.

Figure 61. Detected x-bulk Shift motion instability in the presence of a y-bulk motion instability.

The detected y-motion function does not match the y-instability function since the detected function is calculated from a derivative of the unwrapped phase function. While the motion was detected, the instability function was not reproduced. Motion in the Y direction will cause a change in the linear phase shift. The motion simulation moves the image an integer number of pixels. As a result, the phase function is not continuous, and
therefore the derivative will exhibit "spikes" at the discontinuities. This can be seen more easily using a simpler instability function depicted in Figure 62.

![Graph of function \( x^2(x>0)/4 \)](image)

**Figure 62.** Example linear phase function.

This function has a linear slope of 0.25 pixels/k_y for the second half of the data. The result of the division between the k-space image and the complex conjugate produces the phase plot shown in Figure 63.

![Graph of quadratic phase function](image)

**Figure 63.** Quadratic phase function resulting from k-space complex conjugate division.
After unwrapping phase, the plot seen in Figure 63 becomes the plot depicted in Figure 64.

![Vertical Plot](image)

**Figure 64.** Phase unwrapped quadratic phase function.

The motion function is the derivative of the plot in Figure 60 after being re-scaled into pixel units. The unscaled plot can be seen in Figure 65.

![Vertical Plot](image)

**Figure 65.** Motion function before scaling.
After rescaling angle to pixel shifts, Figure 66 is obtained.

![Figure 66. Motion function with phase discontinuity.](image)

The plot in Figure 62 shows the bottom extent being the position function. The top extent is due to the phase discontinuity. This is not a problem for correcting the y-bulk motion since both the motion and discontinuity errors must be corrected. As predicted, there is an error in the detected phase when a y-bulk motion exists. There was no detected magnitude or echo shift due to y-bulk motion.

**Combined Instability Detection**

For the combined instability detection, separate instability functions will be used to modify the data. Then each will be extracted according to the prescription described in the last chapter. The instabilities are depicted in Figures 64-67.
Figure 67. Magnitude instability function used for combined instability detection.

Figure 68. Echo shift instability function used for combined instability detection.
The simulated abdomen image resulting from all four instabilities is depicted in Figure 70.
Figures 70-76 display the result after systematically removing one instability at a time. The abdomen image and resulting detected echo shift instability after magnitude correction are shown respectively in Figures 70 and 71.
The abdomen image and resulting detected x-motion instability after echo shift correction are depicted respectively in Figures 72 and 73.
Figure 75. Combined instability image after magnitude, and echo shift correction.

Figure 76. Detected x-motion instability with combined instability.

The abdomen image at this point is seen in Figure 74. The detected y-motion instability after the x-motion correction in seen in Figure 75.
Figure 77. Combined instability image after magnitude, echo shift, and x-bulk motion correction.

Figure 78. Detected y-motion instability with combined instability.

The abdomen image after the final instability correction (y-motion) can be seen in Figure 76.
This sequence of results demonstrates the stability/motion segmentation in the presence of magnitude, echo shift, x-bulk, motion and y-bulk motion instabilities. Phase instability is not represented in this example. When phase and y-bulk motion are both present it was demonstrated that it is not possible to segment the two. When phase is present instead of y-bulk motion, the results for magnitude, echo shift, and x-bulk motion are the same. Figure 77 depicts the phase instability with the detected phase instability presented in Figure 78.
The phase detection suffers from an artifact due to phase unwrapping. Note that the
detection errors are more severe when the phase instability is an extreme.
Each of the instability functions were reproduced in the combined instability at least as well as in the individual instability cases with the exception of the phase instability function. Table 4 summarizes the findings.
<table>
<thead>
<tr>
<th>Detected Instability</th>
<th>Instability function present in K-Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase</td>
<td>Phase: No Crossover Detection. Original function not returned, but correlates well with instability function. Echo Shift: Some detection due to linear regression intercept calculation. No detection after echo shift correction. Bulk x: Some detection due to linear phase shift applied to the data. Bulk y: Problematic, cannot differentiate from y-Bulk motion instability.</td>
</tr>
<tr>
<td>Bulk x</td>
<td>Bulk x: No Crossover Detection. Phase: No Crossover Detection when X Bulk correction used before phase detection. Echo Shift: No Crossover Detection. Bulk x: Good detection, but quantized. Bulk y: No Crossover Detection.</td>
</tr>
</tbody>
</table>
Chapter 7

CONCLUSIONS

Real-World Issues

The methods in this thesis describe techniques that can be used to measure system stability and patient motion. A few limitations remain before these methods can be applied in a robust fashion in real-world MRI sampled data. The issues that prevent immediate application include sampling errors and noise. The example images used for the simulation were 256 x 256 pixels in size. This system guaranteed that the DC component of the image was exactly on 128th sample of the 128th view. The MRI imager samples continuous signals. As a result, there will be typically a small sampling where the DC component can be shared between samples. The method used in this thesis compared ky to -ky. Due to sampling error, the comparison may be invalid. The y-gradient is used to select ky lines during the imaging process. The integral of G_y between the 90° and 180° RF pulses determines which ky line is sampled. Current MRI systems cannot guarantee that ky lines will be equally spaced. If lines of ky where highly correlated and continuous, the sampling error would be negligible. However, if one looks at the population of MRI images, they
typically are "RECT-like". The Fourier domain representation would then be a SINC function, whose magnitude is discontinuous. This problem may be mitigated by resampling the Fourier domain data so that \( k_y = 0 \) occurs exactly on one sample. One method of accomplishing this goal is the following: A shift in the frequency domain corresponds to a linear phase shift in the image domain. The linear phase shift can be measured by unwrapping the phase in the image domain, and then calculating the slope of the average \( k_x \) phase. The linear phase can then be removed.

![Phase Image](image1.png) ![Linear Phase Corrected](image2.png)

The other issues to be considered is noise. The noise level must be estimated to determine what phase values are valid. As the noise level increases, the phase unwrapping algorithm has fewer valid values available to create a phase map.
Additional Applications

In addition to the application of segmenting system instabilities from patient motion, the methods presented here could be used to improve image quality and diagnose other system problems.

Part of the process is to correct the errors found in the sampled data. The author acknowledges that the motion detection corrections cannot correct all types of motion; but it can be used to correct the motions it does detect. The amount of error, but not how much error to attribute to each of the of the two points. Despite this, applying the correction will minimize the motion error. Note that this will not improve image quality on a good working system if there is no patient motion, but it will help reduce the instability artifacts until they can be diagnosed and repaired. The motion detection schemes mentioned in Chapter 3 could be used to correct for existing motion. Magnitude and echo shift errors could be corrected in a similar fashion, however the technique for that method has yet to be developed.

The process detects more than patient motion; it also detects system instabilities. The plots generated provided clues to sub-system faults. This is very helpful in the cause of very intermittent problems. An instrument that provides an autopsy of pathological image
data by performing the type of analysis described would be a valuable tool for a service organization.

Conclusion

This thesis has derived the mathematics behind MRI spatial encoding from the underlying physical principles. The types of instabilities that can occur in the pulse sequences were discussed. The mathematics that describe the instabilities were derived. It was demonstrated that all system instabilities could be described as magnitude, phase, and echo shift. Expressions to describe the types of patient motion in terms of the sampled MRI signal were derived. The methodology for extracting motion data and system instabilities was described. And finally the goal of this thesis to describe how to segment between system instabilities and patient motion was presented. The methods were tested using synthetic data and the results shown.

The findings show that there is a good mathematical basis for segmenting x-bulk motion from other system instabilities, however, y-bulk motion and phase instabilities are not separable at this time.
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