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WHITE-LIGHT OPTICAL VORTEX CORONAGRAPH

by

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(2010)

A thesis submitted in partial fulfillment of the
requirements for the degree of Master of Science
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CERTIFICATE OF APPROVAL

M.S. DEGREE THESIS

The M.S. Degree Thesis of Prachyathit Kanburapa
has been examined and approved by the
thesis committee as satisfactory for the
thesis requirement for the
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Abstract

An optical vortex is characterized by a dark core of destructive interference in a light beam. One of the methods commonly employed to create an optical vortex is by using a computer-generated hologram. A vortex hologram pattern is computed from the interference pattern between a reference plane wave and a vortex wave, resulting in a forked grating pattern. In astronomy, an optical vortex coronagraph is one of the most promising high contrast imaging techniques for the direct imaging of extra-solar planets.

Direct imaging of extra-solar planets is a challenging task since the brightness of the parent star is extremely high compared to its orbiting planets. The on-axis light from the parent star gets diffracted in the coronagraph, forming a “ring of fire” pattern, whereas the slightly off-axis light from the planet remains intact. Lyot stop can then be used to block the ring of fire pattern, thus allowing only the planetary light to get through to the imaging camera. Contrast enhancements of 10^6 or more are possible, provided the vortex lens (spiral phase plate) has exceptional optical quality.

By using a vortex hologram with a $4\text{ }\mu\text{m}$ pitch, and an $f/300$ focusing lens, we were able to demonstrate the creation of a “ring of fire” using a white light emitting diode as a source. A dispersion compensating linear diffraction grating of $4\text{ }\mu\text{m}$ pitch was used to bring the rings together to form a single white light ring of fire. To our knowledge, this is the first time a vortex hologram based OVC has been demonstrated, resulting in a well-formed white light ring of fire. Experimental results show measured power contrast of $1/515$ when HeNe laser source was used as a light source and $1/77$ when using a white light emitting diode.

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Chapter 1 Introduction

Optical vortices are ubiquitous in nature [1] and can be generated in the laboratory. The topic is currently gaining interest in research due to the advancement in micro-fabrication techniques. Optical vortices may be identified as a dark core of zero magnitude due to the destructive interference of coherent light beam. The OV beam has a helicoid wavefront and has undefined phase at its core (phase singularity). Optical vortices can be created in several ways; most commonly used techniques are Diffractive Optical Elements (DOE), and Computer Generated Holography (CGH).

Due to the unique characteristics of the optical vortices, researchers find many useful applications of optical vortices. Examples are optical tweezers [2], optical spatial filter [3], high-resolution fluorescence microscopy [4], quantum entanglement [5], and of interest in this thesis, extra-solar planet detection [6-9].

In astronomy the optical vortex coronagraph [6,7] is one of the most promising techniques for the direct observation of low luminance companions such as earthlike exoplanets [8,9]. Contrast enhancements of 10^6 or more are possible, provided the vortex lens (spiral phase plate) has exceptional optical quality [10]. In this thesis, we used vortex hologram with a $4\text{ }\mu\text{m}$ pitch, and an $f/300$ focusing lens to demonstrate the creation of a “ring of fire” using a white light emitting diode as a source. A dispersion compensating linear diffraction grating of $4\text{ }\mu\text{m}$ pitch was used to bring the rings together to form a single white light ring of fire. Previously, white light vortices have been formed with such holographic pairs [11,12]. To our knowledge, this is the first time a vortex hologram based OVC has been demonstrated, resulting in a well-formed white light ring of fire.

This thesis is organized as follows; Chapter 2 provides backgrounds and literature review of the optical vortex coronagraph. Chapter 3 presents the optical theory of the optical vortex coronagraph. The creation of the computer generated vortex hologram is discussed in Chapter 4. In Chapter 5, the experiment of the designed white light optical vortex coronagraph is illustrated. The results of the experiment are shown and discussed in Chapter 6. Finally, the conclusion to this thesis is presented in Chapter 7.

Chapter 2 Background

Vortices are characterized by the swirling motion around a center of a fluid. In nature, vortices appear as whirlpools, tornadoes, hurricanes, toroidal vortices produced by dolphin or humpback whales, and many more. In optics, vortices occur as a light with dark core of zero intensity due to the destructive interference at the center of the beam, the phase of this beam has a spiral profile hence the name optical vortices.

The essential feature of optical vortices is the spiral phase profile given in cylindrical coordinates by

$$\phi\{r, \theta, z\} = m\theta \quad (2.1)$$

where m is an integer called the topological charge

The phase of an optical vortex beam is singular at the center because it is undefined hence the term “phase singularity”. The phase increases from 0 to integer multiple (m) of 2π around one circle of the vortex. Due to the spiral phase profile, an optical vortex beam propagates with a helical wave front.

In 1974, Nye and Berry shows in the first comprehensive benchmark paper about optical vortices [13] that a scatter wave train from rough surface contains dislocation much like found in an imperfect crystal. They found theoretically and experimentally that these phase defects could have a form of edge, screw and mixed edge-screw dislocations. Further experiment reported that controllable net optical vortices could be generated by an interference of only three plane waves [14].

Optical vortex can be created in various ways such as computer generated holograms (CGH), diffractive optical elements (DOE), and an astigmatic mode converter.

Beijersbergen et. al. showed experimentally the use of an astigmatic mode converter [15] to transform a Hermite-Gaussian mode (HG_{01}) to a Laguerre-Gaussian mode (LG_{01}) which is basically an optical vortex with a topological charge $m=1$. The converter comprises of two cylindrical lenses with an exploitation of Gouy phase.

Another way to create optical vortices is to use diffractive optical elements [16]. Optical vortex DOE is typically a glass etched such that its thickness increases azimuthally in a spiral pattern (sometimes called spiral phase plate) so that the phase can be described by Eq. (1). Light beam that is transmitted through an optical vortex DOE gain azimuthal phase variation results in phase singularity at its core. This technique is found to be more expensive due to the fabrication challenge than other methods.

The creation of optical vortices that is the most versatile and cheap is by using computer-generated holograms. The vortex hologram pattern can be generated from the interference pattern of a vortex beam and a reference beam. The employment of a spherical [17] and a plane wave [18,19] reference beam has been reported. Using a spherical wave reference beam generate a spiral shape hologram pattern whereas using a plane wave reference beam generates a forking pattern. The creation of the vortex hologram will be described in more detail in Chapter 4.

So far we discussed about what essentially is an optical vortex, in what form it occurs in nature and how can we create them in a laboratory, the following will be about the applications of an optical vortex.

Unique properties of optical vortices allow researchers to realize applications in various fields. Optical vortices have been found applications in optical trapping [20]. The next generation of the optical tweezers is expected to become an important tool for the researcher to reach the microscopic world dexterously and noninvasively [21]. By modifying

the wavefronts of the focused beam, many new kinds of optical traps become realized. This is where the optical vortex plays a role. When beam is modified to have a helicoids wavefront, the beam no longer focuses at a point but instead it converges to form a ring of light. While the reflecting, absorbing or low dielectric value objects tend to be damaged or repel by a focusing of light, the optical vortex has dark focus and thus applicable to these objects. Another main feature of the optical vortex tweezers is that the ring-like optical traps exert both forces and torques on the object, which can be applied to study the photon spin and orbital angular momentum in the field of quantum mechanics [21].

Recent research by Vladlen G. Shvedov et al. shows that the optical vortex can be applied to the optical manipulation of 1000 times higher manipulation distance in which they call a “Giant Optical Manipulation”. In this research, instead of using the weak radiation pressure force like in conventional optical tweezers, the more powerful photophoretic force, which is a radiometric force, is used. This approach is based on using the optical vortex beam to hold the light absorbing particles at the center of the beam and in the meantime the effect of the stochastic nature of the radiation force is minimized. The ring of the beam acts as an optical vortex pipeline to constrain the particle to move along the optical axis and the thermal force exert on the particle push the particle to move along the pipeline. The experimental setup is shown in Fig.6. The moving distance of the particle as far as 1.5m has been reported [22].

An application of optical vortices in astronomy is for the direct imaging of the planet, which resides near its parent star in a search for an earth-like planet. Since the brightness of parent star is extremely high (approximately 10^7 times higher than its planets in infrared and 10^{10} in visible), the adjacent planet is very difficult to detect. To tackle this problem, the ultra high contrast imaging is required for this application, several optical schemes have been reported [23-26] but of interest in this thesis is the optical vortex coronagraph [6-9,27].

The configuration of an optical vortex coronagraph is similar to the Lyot coronagraph scheme [28] where in the optical vortex coronagraph, the occult disk is replaced by an optical vortex lens. Both systems have the Lyot stop at the exit pupil to filter out undesired light. While Lyot coronagraph scheme suffers from the leakage of light through the Lyot stop, optical vortex coronagraph theoretically rejects all the on-axis light thus has zero leakage. Contrast enhancement better than 10^4 has been reported at $4\lambda/D$ angular separation [29].

Using an optical vortex lens in coronagraph configuration results in a striking “ring of fire” diffraction pattern much like a solar eclipse, where there is a pitch black circle surrounded with a circular ring of light. The theoretical explanation of the ring of fire pattern will be described in detail in the next chapter.

A typical optical vortex coronagraph configuration can operate under a highly monochromatic light source. Numerous applications can be further realized once the broadband operation of an optical vortex lens is fully explored. Swartzlander proposed a vortex lens design scheme for generating an optical vortex across a bandwidth as large as 140nm by using two adjacent glass materials having single or multiple helicoids interfaces [30]. Leach and Padgett employed a dispersion compensating glass prism to recombine dispersed vortices created from a spatial light modulator in order to observe chromatic effects near a white light vortex [11]. Mariyenko et. al. applied similar configuration to Leach and Padgett to achieve optical vortex in femtosecond pulses which naturally have a large band width. The objective of the creation is to investigate “the influence of optical orbital angular momentum on intense-field ionization processes”. In their experimental setup, the vortex is created via computer-generated hologram and the dispersed beam is recombined using a compensation grating [12].

In this thesis, we employed similar concept of creating white light optical vortex as Leach and Padgett, and Mariyenko et al. to the optical vortex coronagraph configuration to demonstrate experimentally for the first time to our knowledge a white light ring of fire.

Chapter 3 Theory

The Fourier transform (FT) is a remarkable tool in an analysis of imaging systems. Here we shall adopt the Fourier transform in the analysis of the optical vortex coronagraph system.

In most of the optical systems, optical components are circularly symmetric in 2-D. A mathematical tool that is more suitable in the analysis of a circularly symmetric function is the *Hankel transform* or the *Fourier-Bessel transform* [31].

If we have an input light field represents by a function $f[x,y]$ in front of a Fourier transform lens, the light field at the distance f behind the lens can be expressed by the Fourier transform of the input function:

$$F.T.\{f[x,y]\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f[x,y] e^{-i2\pi(\xi x + \eta y)} dx dy \quad (3.1)$$

where x,y represents spatial coordinates in front of the lens and ξ,η is the inverse spatial coordinates at the FT plane.

For the 2-D circularly symmetric system $f[x,y]$ can be rewritten as $f(r)1(\theta)$, the function varies only in r direction and is constant in θ direction. The Fourier transform of the circularly symmetric system can be conveniently converted to the zero order Hankel transform [31] given by:

$$F.T.\{f[x,y]\} = \int_0^{\infty} f(r) 2\pi r J_0[2\pi \rho r] dr = H_0\{f(r)\} \quad (3.2)$$

where r represents radial spatial coordinates in front of the lens and ρ is the radial inverse spatial coordinates at the FT plane.

Similarly the n^{th} order Hankel transform is given by:

$$F.T.\{f[x, y]\} = \int_0^{\infty} f(r) 2\pi J_v[2\pi r \rho] dr = H_v\{f(r)\} \quad (3.3)$$

The simple OVC configuration (Fig 3.1) composes of two FT lens separated by a distance $2f$ and at a distance f from both lens, there is a vortex hologram which in this case we adopt the vortex hologram with a topological charge (m) of 2. Here we will prove mathematically that given a uniform circular plane wave as an input $f(r, \theta)$, the output field $g(r, \theta)$ will be a ring of fire pattern.

Fig 3.1 Simple optical vortex coronagraph configuration

The incoming light beam to the system is assumed to be a plane wave thus, the field at L1 is basically a pupil function of a lens which is a cylindrical function with a diameter D given by:

$$f(r, \theta) = CYL\left(\frac{r}{D}\right) \quad (3.4)$$

The light field at the plane VH can then be computed from the zeroth order Hankel transform of the cylindrical function:

$$H_0 \left\{ CYL \left(\frac{r}{D} \right) \right\} = 2\pi \int_0^\infty CYL \left(\frac{r}{D} \right) r J_0 [2\pi r \rho] dr \quad (3.5)$$

or equivalently,

$$H_0 \left\{ CYL \left(\frac{r}{D} \right) \right\} = 2\pi \int_0^{D/2} r J_0 [2\pi r \rho] dr \quad (3.6)$$

Let $u=2\pi r \rho$, $dr=du/2\pi \rho$, substitution gives:

$$H_0 \left\{ CYL \left(\frac{r}{D} \right) \right\} = 2\pi \int_0^{\pi D \rho} \frac{u}{2\pi \rho} J_0 [u] \frac{du}{2\pi \rho} \quad (3.7)$$

Rearranging the terms gives:

$$H_0 \left\{ CYL \left(\frac{r}{D} \right) \right\} = \frac{1}{2\pi \rho^2} \int_0^{\pi D \rho} u J_0 [u] du \quad (3.8)$$

By using the finite integral formula [32],

$$\int_0^x r J_0 (r) dr = x J_1 (x) \quad (3.9)$$

The expression of Eq. (3.8) becomes,

$$H_0 \left\{ CYL \left(\frac{r}{D} \right) \right\} = \frac{D}{2\rho} J_1 [\pi D \rho] \quad (3.10)$$

which in fact can also be written as a scaled sombrero function:

$$H_0 \left\{ CYL \left(\frac{r}{D} \right) \right\} = \frac{\pi D^2}{4} SOMB(D\rho) \quad (3.11)$$

The field at the VH plane can then be computed by multiplying the expression in Eq. (3.10) with the vortex phase function Eq. (2.1):

$$F(\rho, \phi) = \exp(im\phi) \frac{D}{2\rho} J_1 [\pi D \rho] \quad (3.12)$$

The field at the L2 plane can then be computed from the inverse Fourier transform of $F(\rho, \phi)$ and the equivalent m^{th} order hankel transform:

$$g(r, \theta) = F.T.\{F(\rho, \theta)\} = (-i)^2 \exp(im\theta) H_m\{F(\rho, \phi)\} \quad (3.13)$$

where m is a topological charge of the vortex hologram.

Rewrite the Hankel transform in Eq. (3.13) to an integral form:

$$g(r, \theta) = (-i)^m \exp(im\theta) \int_0^\infty 2\pi\rho J_m[2\pi\rho r] \frac{D}{2\rho} J_1[\pi D\rho] d\rho \quad (3.14)$$

Taking ρ independent terms out of the integral and substitute $m=2$:

$$g(r, \theta) = (-i)^2 \pi D \exp(i2\theta) \int_0^\infty J_2[2\pi\rho r] J_1[\pi D\rho] d\rho \quad (3.15)$$

An integral of this form can be proved to have the following expression [32]:

$$\int_0^\infty J_\nu(\alpha x) J_{\nu-1}(\beta x) dx = \begin{cases} \frac{\beta^{\nu-1}}{\alpha^\nu}; [\beta < \alpha] \\ \frac{1}{2\beta}; [\beta = \alpha] \\ 0; [\beta > \alpha] \end{cases} \quad (3.16)$$

Employing the expression in Eq. (3.16) to Eq. (3.15) gives the final expression:

$$g(r, \theta) = \exp(i2\theta) \begin{cases} \frac{D^2}{4r^2}; [r > D/2] \\ 0; [r < D/2] \end{cases} \quad (3.17)$$

which is essentially a ring of fire pattern of radius D . From Eq. (3.17), the field is zero inside the radius $D/2$ and is decreasing by $1/r^2$ outside the radius.

Chapter 4 Vortex Hologram Generation

The vortex hologram is the intensity of the interference pattern between plane wave and a vortex wave. Plane wave tilted by an angle ψ with respect to an optical axis is represented by,

$$E_p = \exp\left(\frac{i2\pi x}{\Lambda}\right) \quad (4.1)$$

where $\Lambda = \frac{\lambda}{\sin\psi}$ is the spatial period of the plane wave in transverse axis and will also appear as a spatial period of the interference pattern.

Vortex wave with a topological charge m is,

$$E_v = \exp(-im\theta) \quad (4.2)$$

where $\theta = \arctan\left(\frac{y}{x}\right)$

The intensity of the interference between plane wave (E_p) and vortex wave (E_v) is then represented by,

$$I = |E_p + E_v|^2 \quad (4.3)$$

The intensity pattern is plotted as shown in Fig 4.1 for the case $m=2$. The fork shape in the center of the profile is the characteristic of the vortex hologram.

In the fabrication of the mask, the pattern has to be in binary, either letting the light through or completely blocking the light. Thus, the actual printed pattern will appear as a threshold version of the intensity plot as shown in Fig 4.2.

Fig 4.1 Vortex hologram intensity plot ($m=2$)

Fig 4.2 Vortex hologram ($m=2$) in binary version

To create a Graphic Data System (GDS) file representing this hologram, we have to specify coordinates forming a polygon for all the stripes (Fig 4.3), either black or white. Larger number of coordinate points will result in more detail of a pattern. Matlab was used to calculate all the coordinates representing all the stripes to be interpreted into GDS file format. Next, we will explain how the code was written to accomplish this.

Fig 4.3 The dots illustrate points forming one of the black stripe, GDS file format requires coordinates forming a polygon to represent each stripe of the pattern. The task is to write a programming code to find all the points forming all the stripes pattern.

The programming procedure is described as follows. First, the pattern calculated is subtracted by a DC value such that its magnitude oscillates between -1 and 1. This

process is done in order to make use of the *fzero* built-in function in Matlab. The function *fzero* can find the exact transition points of the pattern. The x-coordinate of the transition points along the horizontal line is recorded line by line along the y-direction (for-loop in y-direction).

Fig 4.4 and Fig 4.5 shows the first two iteration of the code where *fzero* is implemented line by line and the transition point coordinates are recorded. Once the loop runs through all the y value, we will have all the transition point and then we connect these points to form a polygon representing all the stripes by writing the coordinates to the ASCII text file. The precision of the polygon approximating the actual stripes is determined by the precision in the y-direction sampling. Since the size of the mask is 10mm and the grating period is $4\mu\text{m}$, a large amount of memory to store the coordinates to achieve a decent resolution of the desired pattern is required. However, this problem can be solved by customizing the y-sampling interval such that the sampling rate is high near the core of the vortex and is low further away from the center. The text file is then translates to the GDS file via Owlvision software. The resulting GDS file is shown in Fig 4.6 and Fig 4.7.

Fig 4.4 The example of how the coordinates of the stripe boundary are computed and stored, first loop, Matlab function *fzero* is implemented on the vortex pattern at fixed y-value, points recorded shown in black dots: $(x_1, -10), (x_2, -10) \dots (x_{12}, -10)$

Fig 4.5 The example of how the coordinates of the stripe boundary are computed and stored, second loop, Matlab function *fzero* is implemented on the vortex pattern at fixed y-value, points recorded shown in black dots: $(x_1, -8)$, $(x_2, -8)$ $(x_{12}, -8)$, the loops repeat like this for all values of y.

Fig 4.6 10mm x 10mm Vortex mask GDS file

Fig 4.7 10mm x 10mm Vortex mask GDS file zoom in at the center

The vortex hologram GDS file was then sent to the DWL2000 laser lithography machine (0.6 μm resolution). A chrome pattern (1mm x 1mm) was printed on the 2mm thick fused silica. Fig 4.8 shows the actual 100x microscopic image of the vortex hologram at a center.

Fig 4.8 Microscopic image of the vortex hologram

Chapter 5 Experiment

The experiment part can be divided into two steps. First, we pretest the performance of the fabricated vortex hologram and the designed coronagraph system by using HeNe Laser as a light source. Second, we switched to our main concern, to experiment the system with the broadband source which in this project, we used white light emitting diode (LED) as a light source.

5.1. Experiment I: HeNe Laser Input source ($\lambda=633\text{nm}$)

To predetermine the workability and the performance of the setup designed, HeNe Laser was used as an input light source. The setup is shown in Fig 5.1.

To approximate a plane wave, the HeNe laser first is passed to the beam expander consists of lens L1 and L2 generating approximately uniform circular collimated red laser beam. The mirror M1 then reflects the collimated laser beam to the focusing lens L3 where the pinhole of a diameter 1mm is placed in front of it. The role of the pinhole is to reduce the size of the incoming beam hence the airy disk size is increased according to

$$r = 1.22\lambda f/D \quad (5.1)$$

where r is the airy disk radius and f and D is the focal length and diameter of the lens L3 respectively.

For the 1mm diameter input beam, focused airy disk has $183\mu\text{m}$ diameter covering 91 fringes.

Fig 5.1 White-light optical vortex coronagraph optical setup (Laser source) M is for mirror, L for Lens and H for the vortex hologram. L1 and L2 together form a beam expander, L3, L4, L5 and L6 are 1 inch diameter, $f/300$ achromatic doublets. H1 is the charge 2 vortex hologram whereas H2 is the normal grating. Both grating has $4\mu\text{m}$ spatial period.

At the distance f behind L3 is the $4\mu\text{m}$ period vortex hologram of charge 2 (H1), the focused beam is diffracted into multiple orders. For this experiment we interest in only the first mode ($n=+1$) only, lens L4 is then placed in alignment with the first order diffracted beam at a distance f to separate and collimate the desired order. Lens L4 and L5 functionality is basically to relay the desired order beam, also note that the distance between L4 and L5 is $2f$. Lens L5 focus the beam to the vortex hologram of charge 0 or equivalently a diffraction grating of the same period as H1. Again, the beam is diffracted into several orders and for this time, the opposite diffraction order ($n=-1$) is selected.

5.2. Experiment II: White light source

In the previous section, we demonstrate experimentally the operation of the designed optical vortex coronagraph under the monochromatic source. For this section white LED will be used as a light source to the coronagraph system. The schematic drawing of the setup are shown in Fig 5.2 and Fig 5.3. The experimental procedure can be divided into two steps. First, HeNe Laser was used to align the coronagraph system (Fig 5.2) and second, white LED was switched on instead of Laser (Fig 5.3) and the images were recorded.

Fig 5.2 Optical setup of the white light coronagraph experiment first step; aligning using Laser, AP for aperture, V for vortex hologram and L for Lens

Fig 5.3 The optical setup of the white light coronagraph experiment second step; using the white light LED, AP for aperture, V for vortex hologram and L for Lens

The first step is similar to the previous section where we have a laser beam expanded and sent to the coronagraph system except that now we introduce the pellicles to the system. Note that before aligning the coronagraph system, we have to make sure that the pellicles are in the right orientation where both white LED light going through the pellicles and the Laser reflecting from the pellicles are aligned with each other. The reason for this is that the power of the collimated planar white LED light is extremely

low which makes the alignment of the coronagraph by the white LED light alone become extremely difficult therefore, red Laser was used to align the system prior to the using of the white LED light.

The plane wave white LED light was created by first focusing the collimated LED to the 25 μ m diameter pinhole (AP1). The light coming out of the pinhole represents a point source, which was then collimated using another lens giving the close approximation of the collimated planar white light. Due to the existence of the pellicles, 1:1 optical relay was introduced to relay the desired beam to the telescope objectives (AP2).

One thing in the consideration of the design of the setup was the fact that the light source is a polychromatic LED source and the design must be made such that the collimated white LED at the plane AP2 has a substantial spatial coherence. In this case, the coherence area of the beam must be at least of a circle with diameter of 1mm (the same size as the diameter of AP2). The coherence area can be calculated from:

$$s = \frac{\lambda}{\theta} \quad (5.2)$$

where θ is the angular size of the pinhole as seen from the center of the collimating lens, therefore the equation can be rewritten as follow:

$$s = 1.22 \frac{\lambda f}{d} \quad (5.3)$$

We chose the collimating lens of focal length (f) 60mm to ensure considerable amount of light to be collected, using the smallest λ in the visible region of 0.4 μ m and the desired coherence area of 1mm (AP2), we can calculate the maximum diameter of the

pinhole to be $29.28\mu\text{m}$. In this setup, we use the AP1 pinhole diameter size of $25\mu\text{m}$ to ensure the required coherence area.

Chapter 6 Results and Analysis

For the ease of an explanation, the experimental setup is simplified to the diagram shown in Fig.12. The alignment system of the setup is ignored, showing only the planar white light input to the optical vortex coronagraph system.

Fig 6.1 Broadband optical vortex coronagraph four lenses (L1, L2, L3 and L4) are achromatic doublet ($f=300\text{mm}$ $D=1\text{inch}$). V0 and V2 are vortex hologram of charge zero and two respectively

6.1. Laser source

For the monochromatic light source the ring of fire pattern can be observed at both L2 and Lyot stop (L4) plane (refer to Fig 6.1). The “ring of fire” images taken from the CCD camera are shown in Fig 6.2.

To assess the performance of the setup and the vortex mask fabricated, power ratio (η) is calculated from the following equation:

$$\eta = \frac{P_v - P_0}{P_T - P_0} \quad (6.1)$$

where P_v is the measured power inside the ring of fire, P_T is the measured total power and P_0 is the ambient power measured when the laser source is turned off.

The power is measured and the contrast power ratio is calculated at two locations L2 and L4 planes.

The results are shown in Table 6.1. P_v was measured using power meter by placing 0.5mm diameter pinhole at the center of the ring of fire. P_T was measured in the exact same condition except the pinhole was removed. And P_0 is the ambient power measured. The pinhole size is also illustrated as a dashed circle in Fig 6.2.

	P_v	P_T	P_0	η
L3	1.20 nW	1.10 μ W	3.5 $\pm$ 0.7 pW	1/9174
Lyot stop	0.175 nW	91.0 nW	3.5 $\pm$ 0.7 pW	1/514.6

Table 6.1 The power ratio of the ring of fire pattern calculated at the plane L2 and the Lyot stop plane, P_v is the power measured when the 0.5mm diameter pinhole is placed in front of the power meter, P_T is the total power measured when the pinhole is removed and P_0 is the ambient power measured when the laser is turned off.

The power ratio decreased from around nine thousandth at the first image plane (L3) to only five hundredth at the Lyot stop. This result suggests that the optical relay decreases the performance of the coronagraph significantly. The images of the ring of fire taken at these 2 locations are shown in Fig 6.2. Note that the integration time used to take

Fig 6.2(b) is 10 times longer than for Fig 6.2(a) so the brighter background intensity is observed in Fig 6.2(b).

Fig 6.2 Images of ring of fire taken at (a) first image plane (L3), (b) Lyot stop, the dashed circle inside the ring of fire shows the pinhole size ($d=0.5\text{mm}$) in the power measurement

6.2. *White light source*

To demonstrate the visible spectrum dispersion and recombination, the images of the rings of fire were recorded using Nikon D50 digital camera at both the plane of lens L3 (see Fig 6.3(a)) where the light is dispersed, and at the plane of L4 where a white light image of the ring of fire or pupil are formed (see Fig. 6.3(b) and (c)). The color images have a yellow hue because the blue band of the LED is strongly diffracted out of the system. The white light pupil shown in Fig. 6.3(c) is formed when the center of the vortex lens is displaced from the centroid of the focal spot. The black hole in the center of the ring of fire is indicative of high contrast afforded by this device across a broad band of wavelengths.

(a)

(b)

(c)

Fig 6.3 (a) Dispersed rings of fire in the first image plane of the pupil. (b) The white light ring of fire at the second image of the pupil. (c) The re-imaged pupil in the same plane as (b) with the vortex lens axis displaced from optical axis.

More accurate image was then taken using astronomy camera SBIG SME402 as shown in Fig 6.4. The yellow circle indicates the area which the normalized integrated intensity is plotted shown in Fig 6.5 in comparison with the radial profile plot from the

laser source case. In theory, the profile of the radial plot should be a sharp edge from zero to the maximum value at the edge of the black hole inside the ring of fire. However, the all result shows the gradually increasing in intensity. The error is due to the imperfect alignment, optical quality and the mask fabrication error. As expected, the radial plot from the laser case in the first image plane (Fig 6.5(a)) appears to be the closest to the theory, since the probability of error is less than the results taken from the final Lyot stop plane.

Fig 6.4 “Ring of fire” image from an astronomy camera, the yellow circle shows the area where the radial plot was taken.

Fig 6.5 Normalized integrated intensity verses radius in pixel plot (a) Laser source at the first image plane (L3 plane in Fig 6.1), (b) Laser source at the Lyot stop plane (L4 plane in Fig 6.1) and (c) White light source at the Lyot stop plane

To quantify the performance of the white light coronagraph system, the power contrast is calculated the same way reported in the earlier section where the Laser source is used. The power ratio (η) is the ratio between the power inside the black hole and the total power. For this experiment, we placed a 0.5mm diameter pinhole at the center of the ring of fire and recorded the power using the power meter. However, due to the low power of the white light the signal to noise ratio is too low for the power recorded to be meaningful unlike the Laser case where the power is extremely higher.

To circumvent the power problem, instead of direct power measurement we then opted to calculate the power ratio from the image taken using the astronomy camera. The pixel size of the camera is $9\mu\text{m}$, pixels covering circle of diameter $500/9 \approx 56$ pixels at the core of the black hole are used as a 0.5mm diameter circular area for the integrated density when the pinhole is placed (P_v) to be calculated (The integrated density was calculated using imageJ software built-in function and note that the same notation P is used in this case only to conform with the previous result from the laser case. Thus, P in this case does not represent power but rather the integrated density). The total integrated density (P_T) was calculated in a similar ways as P_v but instead using larger computed circular area ($d=2.5\text{mm}$). P_{0v} denotes the background power for this case because it was calculated using the same circular area as the P_v calculation ($d=0.5\text{mm}$). The value reported is the average value of the computed background integrated density from the randomly selected ten locations outside the ring of fire. Thus, the power ratio calculated from the following expression:

$$\eta = \frac{P_v - P_0}{P_T - \frac{A_T P_{0v}}{A_v}} \quad (6.2)$$

where A_v and A_T represents the calculated circular area in the calculation of P_v and P_T which is 0.5mm and 2.5mm respectively.

The calculated results are shown in comparison with the laser source as shown in Table 6.2. Note that for the white light case, the power ratio is not calculated at plane L3 since in this case light are dispersed and the ring of fire appears as a bar instead. At the Lyot plane on the other hand, dispersed light is brought back together and the power ratio can be calculated to be 1/76.7 which is in agreement with the intuition, when the white light is used, numerous source of errors are prone to happen such as chromatic aberration and the misalignment of the compensation grating.

HeNe laser light source				
	P_v	P_T	P_0	η
L3	1.20 nW	1.10 μ W	3.5 $\pm$ 0.7 pW	1/9174
Lyot stop	0.175 nW	91.0 nW	3.5 $\pm$ 0.7 pW	1/514.6
White LED light source				
	P_v	P_T	P_{0v}	η
L3*	-	-	-	-
Lyot stop	766021	38197035	267861	1/76.7

*In the white light case, the ring of fire at L3 plane is dispersed forming a bar as shown in Fig 6.2(a) hence the power contrast at this plane is not calculated

Table 6.2 The power ratio of the ring of fire pattern calculated at the plane L3 and the Lyot stop plane of the white light source case, P_v is the integrated density within the

0.5mm diameter circle at the core of the ring of fire, P_T is the integrated density within 2.5mm diameter circle covering the whole ring of fire and P_{0v} is the background integrated density computed using 0.5mm diameter circle coverage area in the region outside the ring of fire (average from ten locations)

6.3. *Walk-off problem analysis*

One significant concern owing to the imperfection of an attempt to bringing the dispersed beam back together using the diffraction grating with the same period as the vortex hologram is the walk-off problem. In this case, the holograms are 2mm thick fused silica printed with chrome pattern on one side (Fig 6.6). The concern is deciding on which way the hologram should be faced with the incoming beam for both vortex hologram and the grating to minimize the walk-off.

Fig 6.6 Schematic drawing of the vortex hologram

6.3.1 Grating equation

Fig 6.7 Amplitude grating

The grating equation can be derived from the constructive interference condition in which the intensity peak occurs when the Optical Path Difference (OPD) is equal to the integer multiple (N) of the wavelength of the light (λ). According to Fig 6.7 the condition can be written as:

$$n_1 x_1 + n_2 x_2 = N\lambda \quad (6.3)$$

$$n_1 \sin \theta_1 + n_2 \sin \theta_2 = N\lambda \quad (6.4)$$

6.3.2 Normal incident walk-off

For the grating that is a printed pattern on glass with some defined thickness, walk-off problem is unavoidable but choosing the right face where the light impinges could

minimize the effect of it. In this section, we will theoretically analyze both light impinge on front and back face for the normal incident to the grating surface case.

6.3.2.1 Monochromatic light

For the front face incident, at the grating, equation (6.4) can be rewritten as (consider only the +1 order diffraction):

$$n_1 \sin \theta_1 = \lambda \quad (6.5)$$

$$\theta_1 = \arcsin\left(\frac{\lambda}{n_1}\right) \quad (6.6)$$

Fig 6.8 Schematic drawing depicting the ray diagram for the normal incident monochromatic light source for the chrome pattern in the front (upper) and in the back (lower) face

And at the refraction interface, Snell's law is applied:

$$n_1 \sin \theta_1 = n_0 \sin \theta_2 \quad (6.7)$$

Substitute equation (6.7) into equation (6.6) gives:

$$\theta_2 = \arcsin \left(\frac{\lambda}{n_0} \right) \quad (6.8)$$

And for the back face incident case, light impinge to the refraction surface at a right angle thus, there is no angle change in the first interface, we just have to use the grating equation at the grating interface:

$$n_0 \sin \theta_2 = \lambda \quad (6.9)$$

and again we have the same equation as for the first case:

$$\theta_2 = \arcsin \left(\frac{\lambda}{n_0} \right) \quad (6.10)$$

It can be concluded that, in both light impinging from the front and the back of the grating cases, light coming off from the grating will have the same angle. The only difference to these two cases is that the light coming off is displaced by a distance:

$$h = t \cdot \tan \theta_1 \quad (6.11)$$

6.3.2.2 Polychromatic light

Fig 6.9 Schematic drawing depicting the ray diagram for the normal incident polychromatic light source for the chrome pattern in the front (upper) and in the back (lower) face

Here red and green beam represents the polychromatic light. Note that the derivation of the output angle is the same as in the previous section of the monochromatic source except that now we have two beams with two different wavelengths. The additional

interesting value we could compute from the polychromatic source case is the displacement difference between the red and green beam dh , which is given by:

$$dh = h_r - h_g = t \cdot (\tan\theta_{1r} - \tan\theta_{1g}) \quad (6.12)$$

where

$$\theta_{1r} = \arcsin\left(\frac{\lambda_r}{n_0}\right) \quad (6.13)$$

$$\theta_{1g} = \arcsin\left(\frac{\lambda_g}{n_0}\right) \quad (6.14)$$

It could be observed that when light incident on the front face of the grating, spectrum of light coming off from the grating appear as though they are not originating from the same point due to the walk off inside a dispersing material. Orienting the grating such that the chrome pattern is on the back face can eliminate this effect.

6.3.3 Walk-off analysis in the vortex coronagraph system

Previous section deals with the walk-off problem for the normal incident case in particular. However, in the actual vortex coronagraph optical system of interest, light incident on the grating with an angle and the system comprises of two such grating as well as the other optical components that take part in the analysis. To circumvent the complication, the more convenient ray tracing analysis is adopted. Note that the vortex coronagraph system has the maximum angle subtending the optical axis of around 8 degrees, which ensure that the system is in the paraxial regime. This approach is appropriate with an error in the order of 10^{-4} .

The ray tracing simulation was done using all four combinations of the grating orientation possible; Front-Front (FF) means the vortex hologram having the grating chrome pattern on the front face facing the incoming beam, Front-Back (FB), Back-Back (BB) and Back-Front (BF) similarly. The simulation was done according to the actual setup where the optical axis is aligned with red wavelength (633nm).

The table summarizing the simulation results is shown in Table 6.3. We simulated two input beams of two wavelength; red beam ($\lambda=633\text{nm}$) and green beam ($\lambda=550\text{nm}$). Since the optics was setup to align with red wavelength, red beam experiences little to no deviation from the walk-off problem. Only the change in the beam size is observed on the order of one-hundredth of millimeters. Of more importance is how the green beam shifted from the red beam, shown in red in Table 6.3 are the shifts of the center of the beam of the green beam with respect to where it should be (fully overlap with red beam) without the walk-off problem. Here we can see that the lowest shift occurs in the FB case ($13\mu\text{m}$), which is the vortex hologram front face and the normal grating back face. FF, BB, and BF show $37\mu\text{m}$, $120\mu\text{m}$ and $140\mu\text{m}$ shift respectively.

	RED				GREEN			
	Beam Center		Beam Diameter		Beam Center		Beam Diameter	
	no walk-off	with walk-off	no walk-off	with walk-off	no walk-off	with walk-off	no walk-off	with walk-off
F								
F	0	0	1	1.01	0	0.0371	1	1.0064
F								
B	0	0	1	1.0185	0	0.0134	1	1.0153
B								
B	0	0	1	1.027	0	0.1155	1	1.0205
B								
F	0	0	1	1.0185	0	0.1386	1	1.0117

*All units in mm

Table 6.3 Beam center and beam diameter results from the ray tracing simulation F (B) indicates chrome pattern on Front (Back) face. Thus, FF means both holograms has chrome pattern on front face. The first and second letter indicates the vortex hologram and the grating orientation respectively.

The ray tracing results quite contradict with the preliminary analysis of the single grating, where the light incident on back face result in lower deviation. This is because, unlike the first grating where the incident beam is almost perpendicular to the grating, the light incident to the second grating has high angle and the walk-off problem is unavoidable and has a prominence effect regardless of the face of the grating impinged upon. Therefore, to cancel this effect, instead of minimizing the walk-off in the first grating we should introduce some degree of walk-off to it.

Chapter 7 Conclusion

In conclusion, we were able to demonstrate for the first time the creation of a broadband “ring of fire pattern” using a vortex hologram and an f/300 focusing lens. The optical vortex coronagraph configuration was adopted with an addition of a dispersion compensation grating to recombine the dispersed light beams.

To analyze the coronagraph performance quantitatively, we calculated the power contrast ratio which is a ratio between the light power inside the ring of fire’s dark core and the total power of the whole ring of fire. The power contrast ratio at the Lyot stop plane when using white LED source was approximately $1/77$ whereas, when using laser source the power contrast was $1/517$. The inferior performance when using white light source is due to the chromatic aberrations, optical alignment imperfection, optics quality, and the walk-off problem.

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