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A Mathematical Model of Juvenile Delinquency in the New York State

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A Mathematical Model of Juvenile Delinquency in the New York State

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Abstract:
This report presents a mathematical model of juvenile delinquency in the New York State. In particular, we develop a juvenile delinquency system of non-linear differential equations using the mathematical epidemiology framework. In constructing this model, we assume that juvenile delinquency can be studied as a socially infectious disease. The stability of the juvenile delinquency-free equilibrium of the model is examined using the standard non-linear dynamical systems theory technique. We carried out a data fitting based on real-life data from the New York State Criminal Justice Services. The research result reveals that the formulated model conforms with the available data and could be useful for major future projections during policy formation for the juvenile population.

Keywords: Mathematical model, juvenile delinquency, data fitting, juvenile justice system, New York State.

1 Introduction

1.1 Juvenile Crime and Juvenile Justice

Juvenile crime and delinquency are becoming the norm around the world [1]. Understanding this growing phenomenon through mathematical modeling could help solve this social problem. According to New York State Law [2], a juvenile delinquent is a child over the age of seven but under the age of eighteen (effective from 2019) who commits an act that would be a crime if committed by an adult. Delinquent children are not detained in adult prisons. The Local Social Services Agency or the New York State Office of Children and Families Services will be contacted by the Court to determine if they require monitoring, treatment, or placement.

The State also considers violent crime, robbery, aggravated assault, simple assault, property crime, burglary, larceny-theft, motor vehicle theft, vandalism, drug abuse violation, driving under the influence, liquor law violation, disorderly conduct, unlawful possession of guns and weapons, and other offenses as juvenile offenses that are publishable by law depending on severity [2]. Some other types of crime can be seen in Figure 1

According to research in [3], peer pressure remains one of the major causative factors responsible for juvenile delinquency. When juveniles are accused of delinquent behavior, they are handled by the juvenile justice system (JJS). The JJS's key objectives include skill development, rehabilitation, attending to treatment requirements, and successfully reintegrating juvenile into society, in addition to ensuring public safety. The juvenile justice system is noticeably more rehabilitative. A successful juvenile case would see the teenager change their future decisions and life trajectory, learn from the experience without even being exposed to the severity of adult jail, and have no further engagement with the juvenile or criminal justice system (CJS).

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The racial and ethnic inequality in juvenile justice proceedings also suggests great concern on whether or not the system is efficient [4]. The available data in [4] indicates that white children are less likely than young people of color to be arrested and wind up in the JSS. Despite a considerable success rate and a nationwide decline in JSS engagement, discrepancies still exist, especially for Black and Indian/Alaska native youths as evident in [4].

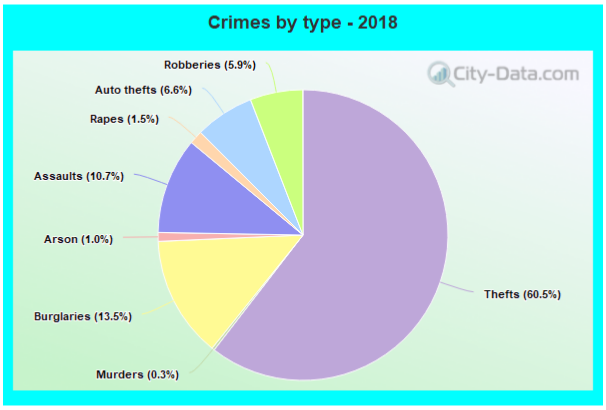

Figure 1: Crime rate in New York. **Source:** https://www.city-data.com/crime/crime-New-York-New-York.html

1.2 Brief Literature Review

Nuno *et al.* (2008) in [5] presented a model that handles the interactions of three social species, referred to as business owners, criminals, and security personnel. Essentially, the phenomenon is referred to as the Triangular model of criminality because criminals are the predators on business owners (prey); security personnel are predators on the criminals (prey) and business owners (prey). According to their bifurcation analysis study, the amount of crimes committed at a time determines the level of recruitment of security personnel, and the more resources available to fight crime, the greater the community of criminals.

A model explaining the dynamics of teenage gangs was presented in the work of Lee and Do (2011) [6]. According to the research in [6], peer pressure was observed to have a role in juvenile misbehavior. The number of arrests and sanctions required to stop the growth of youth gangs was calculated using a simulation study on the threshold condition.

Sooknanan and Comissiong (2018) in [7] formulated a mathematical model for the treatment of delinquent behavior (public health approach). The model presents a medical approach to tackling youth delinquency through intervention and diversion programs. The research findings suggests that the spread of juvenile delinquency can be curtailed if the treatment intervention quantity can attain a desirable threshold.

1.3 Question:

Sequel to the literature reviewed, we were able to identified a gap that has not been captured. In particular, we are interested in asking the open research question: How does peer pressure impact juveniles delinquency tendency in the New York State population?

1.4 Aim and Objectives

The aim of this research is to formulate a juvenile delinquency model for the New York State. Our objectives are to:

i. Formulate a deterministic model for the study of juvenile delinquency using insight from mathematical epidemiology.

ii. Analyze the model using qualitative and quantitative approaches.

iii. Interpret and validate our result to inform policy formulation in New York State.

2 Mathematical Modeling

In this section, we set out to employ the concept of mathematical epidemiology to model the social problem. We do this by treating juvenile offense and crime as a socially infectious disease which can be spread from persons to persons physically or via online platforms while peer influence is taking into account.

2.1 Model Assumption

In setting up the juvenile model, we have to take into account certain assumptions that would help us concentrate on answering our desired question. The list of some major assumptions of the model is given as follows:

i. We assumed that individual can learn delinquent behavior through their peers physically or on social media.

ii. Individual can exit the system by simply aging out or as a result of natural death.

iii. We also assume that all government interventions are centralized, hence, we would not be breaking the intervention compartment into subcategory.

It is important to note that the consequence of the assumption iii is that we have localized the model to the New York State only. Also in iii, we believe that a more decentralized government intervention might have made the model more robust.

2.2 Mathematical Model Formulation

In this present section, we present the juvenile delinquency model following the epidemiological approach and the assumption given herein. The time (t) dependent model is given by

$$\left.\begin{aligned} \dot{J}_S = & \quad \Lambda - \beta J_S J_D - \mu J_S + p\omega J_R \\ \dot{J}_D = & \quad \beta J_S J_D - (\alpha + \mu) J_D + (1-p)\omega J_R \\ \dot{J}_R = & \quad \alpha J_D - (\mu + \omega) J_R, \end{aligned}\right\} \tag{2.1}$$

where, $N_J(t) = J_S(t) + J_D(t) + J_R(t)$.

The first equation of 2.1, $\dot{J}_S$, represents the change in the susceptible juvenile population; the second equation of 2.1, $\dot{J}_D$, implies the change in the delinquent juvenile population; the third equation of 2.1, $\dot{J}_R$, denotes the change in the recovered delinquent juvenile population, with reference to t. The addition of these population yield the total number of the juvenile population denoted by N_J. The parameter μ denotes the removal rate (exit rate) from the juvenile population among the compartments. The parameter Λ represents the recruitment rate (dependent on birthrate) into the juvenile population, J_S. Since we have assumed that the influence rate is treated as an infection, then the contact rate between individuals in J_S and in J_D will result to an influence and the manifestation of delinquent behaviors at rate β.

Individuals exhibiting the delinquent behaviors may be law-enforced by the police and the court of competent jurisdiction at rate α. Since our question of interest is to assess the impact of the State's intervention on the delinquent juvenile population, we measure the release rate with ω. A fraction (p) of individuals in J_R maybe released back to the susceptible population J_S or the delinquent juvenile population (J_D) having in mind that the condition, $0 \leq p \leq 1$ holds. When p is 0, we assumed that the intervention fails and when p is 1, we assume that the intervention is effective. The pictorial representation of the situation under discussion is given in Figure 2.

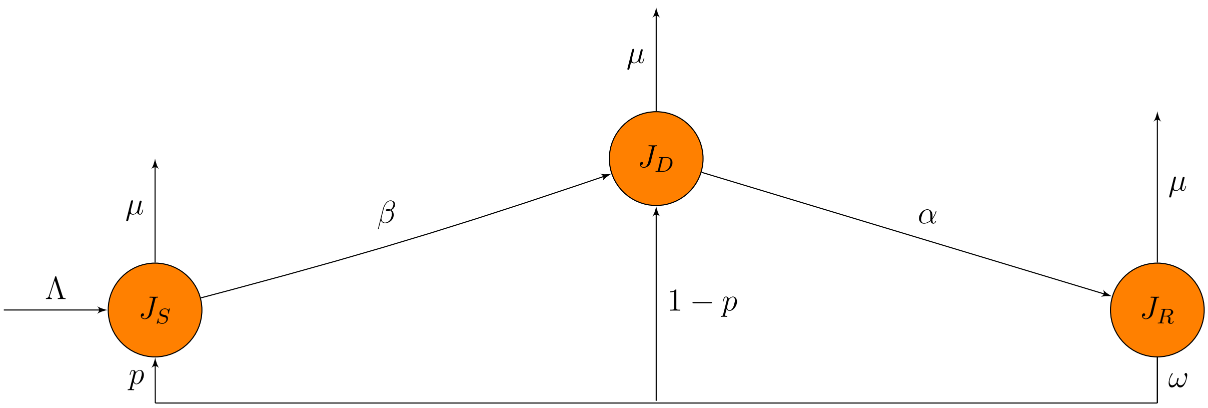

Figure 2: The juvenile delinquency compartmental model.

The summary of the parameters and variables discussed thus far are given on Table 1.

Table 1: Description of variables and parameters.

Variables/Parameters	Interpretation	Unit	Dimension
t	Time	day	T
N_J	Juvenile population in New York State	people	P
J_S	Susceptible juvenile population	people	P
J_D	Delinquent juvenile population	people	P
J_R	Recovered juvenile population	people	P
Λ	Recruitment rate (birthrate)	people per day	PT^{-1}
β	Contact and influence rate	people per day	PT^{-1}
μ	Removal rate (natural death rate/exist rate)	per day	T^{-1}
ω	Released rate from the recovery center	per day	T^{-1}
p	Fraction of released juvenile from the recovery center	per day	D
α	Arrest and prosecution rate of juvenile offenders	per day	T^{-1}

People: P; Time: T; D: Dimensionless.

3 Stability Analysis

3.1 Dimension Check

From the system 2.1, we check to ensure that the left hand side and the right hand side of the system is consistent. We do this by substituting the dimension of each variable/parameters in to system 2.1. This yields

$$\left.\begin{aligned} PT^{-1} &= PT^{-1} - PT^{-1} - PT^{-1} + PT^{-1} \\ PT^{-1} &= PT^{-1} - PT^{-1} - PT^{-1} + PT^{-1} \\ PT^{-1} &= PT^{-1} - PT^{-1} - PT^{-1}. \end{aligned}\right\} \quad (3.1)$$

Observing the result in 3.1, we have shown that the system 2.1 is dimensionally consistent. That is, they are demensionally homogeneous.

3.1.1 Model well-posedeness

Here, we present in brief, the necessary and sufficient condition that ensure the well-posedness of the proposed juvenile delinquency model (JDM). This we establish by summing the right hand side of 2.1 to yield

$$\frac{dN}{dt} = \Lambda - \mu\left(J_S + J_D + J_R\right).$$

At the steady state, $\frac{dN}{dt} = 0$ then we have

$$\frac{dN}{dt} = \Lambda - \mu\left(J_S + J_D + J_R\right).$$

This then yield

$$\Lambda = \mu N_J.$$

This then follows that

$$\frac{\Lambda}{\mu} \leq N_J. \tag{3.2}$$

Equation 3.2 establishes the fact that regardless of the recruitment into or exit from the system, the population will still be bounded and will not grow beyond the total population N_J. Furthermore, we want to ensure that none of the trajectories goes below zero since we are dealing with human population. Provided that the initial data for the population of the juvenile population is positive such that

$$\left.\begin{aligned} \dot{J}_S =& \quad \Lambda - \beta J_S J_D - \mu J_S + p\omega J_R \geq 0 \\ \dot{J}_D =& \quad \beta J_S J_D - (\alpha + \mu) J_D + (1-p)\omega J_R \geq 0 \\ \dot{J}_R =& \quad \alpha J_D - (\mu + \omega) J_R, \end{aligned}\right\} \tag{3.3}$$

then the solutions $J_S \geq 0, J_D \geq 0, J_R \geq 0$. This is so because when dealing with human population, negative values are not reported as we can not have a negative population. Hence, we claim the following.

Lemma 1. *System 2.1 has been shown to be bounded and possess positive definite trajectories. Hence, the JDM (2.1) is mathematically and sociologically well-posed.*

3.2 Juvenile Delinquency Free Equilibrium and Local Asymptotic Stability

Now, we consider the delinquency free equilibrium (DFE) denoted by E_J. This we obtained by solving the system 2.1with the understanding that that some parameter values will go to zero. That is, when DFE exist, we assume no delinquent behavior ($J_D = 0$). Hence, $\dot{J}_S = \dot{J}_D = \dot{J}_R = J_D = J_R = \beta = p = \omega = \alpha = 0$, except for J_S. Thus,

$$E_J : (J_S, J_D, J_R) = \left(\frac{\Lambda}{\mu}, 0, 0\right). \tag{3.4}$$

Next, we linearize the system 2.1 by employing tool from dynamical systems theory, with a guidance from [8] and [9]. This we do by obtaining the spectral radious of the system 2.1. That is,

$$\rho\left(FV^{-1}\right). \tag{3.5}$$

The full description and proof of this formula can be found in Van den Driessche and Watmough [9], where

$$F = \begin{pmatrix} \beta J_S & 0 \\ 0 & 0 \end{pmatrix}, \tag{3.6}$$

and

$$F(E_D) = \begin{pmatrix} \frac{\Lambda\beta}{\mu} & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.7}$$

Based on trivial linear algebra calculation, the third row and column have been read-off. Then

$$V = \begin{pmatrix} \alpha+\mu & 0 \\ 0 & \omega \end{pmatrix} \tag{3.8}$$

$$V(E_D) = \begin{pmatrix} \alpha+\mu & 0 \\ 0 & \omega \end{pmatrix} \tag{3.9}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\alpha+\mu} & 0 \\ 0 & \frac{1}{\omega} \end{pmatrix}. \tag{3.10}$$

Let

$$FV^{-1} = \left(\begin{pmatrix} \frac{\Lambda\beta}{\mu} & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\alpha+\mu} & 0 \\ 0 & \frac{1}{\omega} \end{pmatrix}\right). \tag{3.11}$$

Then, the reproduction number (R_J) of the JDM is given by

$$R_J = \rho \begin{pmatrix} \frac{\Lambda\beta}{\mu(\alpha+\mu)} & 0 \\ 0 & 0 \end{pmatrix}, \tag{3.12}$$

and can be written as

$$R_J = \frac{\Lambda\beta}{\mu(\alpha+\mu)}. \tag{3.13}$$

Lemma 2. *The JDM has a reproduction number, R_J and it is locally asymptotically stable (LAS) when less than unity and unstable when greater than unity.*

Here, we have just set up a threshold quantity that is essential in handling juvenile delinquency. Basically, this threshold quantity helps us to measure the possibility of a delinquent juvenile to influence a susceptible juvenile. The main goal of government and critical stakeholders in security institutions is to ensure that all parameters are kept under control to ensure that the $R_J < 1$. Drawing inspiration from mathematical epidemiology, we can tie the result from this threshold quantity to the New York State given some realistic data. Qualitatively speaking, provided that the $R_J < 1$, juvenile delinquency activity will always be under control in the New York State.

4 Numerical Result

This section presents the implementation strategy, New York State juvenile data, and the numerical result.

4.1 Data Description

We obtain a real life data from the Division of Criminal Justice Services (DCJS) Computerized Criminal History (CCH) system [10]. Basically, the data contains arrests for offenses where fingerprints are required to be taken (Criminal Procedure Law 160.10). It is given bellow.

Table 2: New York State Juvenile Arrest Data obtained from the DCJS [10].

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Data	46,876	46,547	42,834	38,414	33,085	29,926	26,851	24,465	21,340	15,422

The data tables contain arrests of individuals who were between 13 and 17-year-olds charged under the Criminal Procedure Law 1.20.44 [11].

4.2 Implementation Strategy

In this subsection, we present the strategy adopted in obtaining our numerical results. Since we believe that mathematical modeling is an enviable approach that could help us understand real-life situation, we were curious to know whether or not the developed model conform with the real life data.

A few steps were made. Firstly, we had to rely on some realistic data with verifiable source of the New Your State. For instance, the life expectancy in New State is set at 81.4 in 2019 [13]. The natural death rate in New York State will be the reciprocal of the life expectancy. That is, $1/81.4 = 0.01228501228/\text{year}$. According to the 2021 United States Census Bureau [14], the New York State population is estimated at 19,835,913. Since the Juvenile population (based on the available data for those under 18 years in [14]) makes the 20.7% of the New York State population, we can estimate the total Juvenile population, $N_J = 19{,}835{,}913 \times (20.7/100) = 4106034$ individuals.

Having achieved the standard data, then opted to fit the model by combining ode15s (to solve the differential equation), least square and fminsearch (matlab function) for minimization routine that helps the least square formula. Essentially, we begin with an initial guess. However, we were guided by making some educational guesses before we could obtain a reasonable fit.

4.3 Result and discussion

We then implement the procedure and fit the model against the data following the idea in [12]. The result is given by

Table 3: Implementation result. $N_J = 4,106,034$.

Parameters	Values	Reference
μ	0.01228501228/year	[13]
Λ	$(0.01228501228N_J)\text{year}^{-1}$	[13]
β	$0.000000356670046\text{year}^{-1}$	Fitted
α	$1.483336189914413\text{year}^{-1}$	Fitted
ω	0	Assumed
p	0	Assumed
R_J	0.979191341172331	Fitted

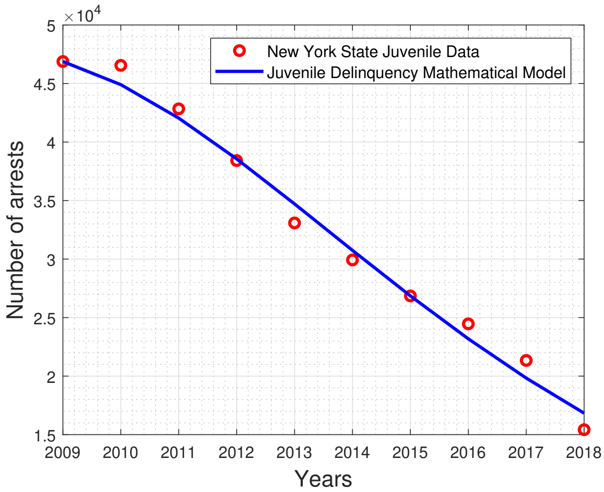

Figure 3: The fit for the DJM and the New York State Juvenile Data.

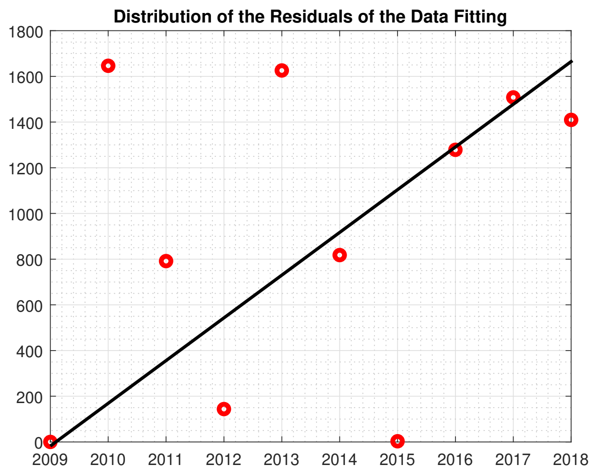

Figure 4: The residual of the fit for the DJM and the New York State Juvenile Data.

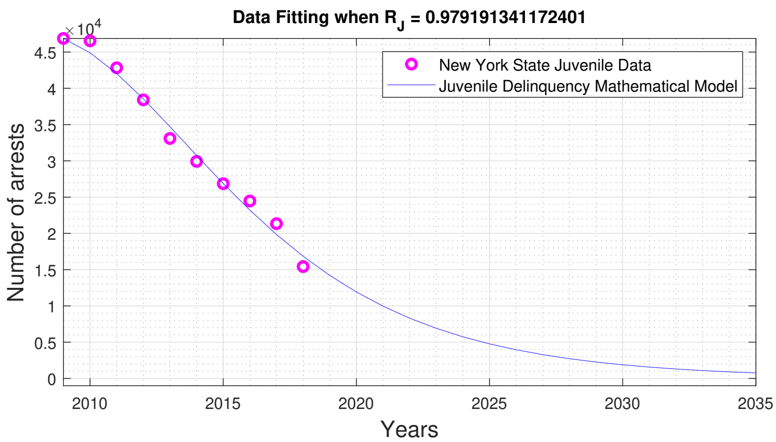

Figure 5: The projection of the number of juvenile arrest in the New York State.

Remark 1: The Table 3 presents the parameters used during the simulation while Figure 3 is the corresponding plot. The residual plot (4) is the difference between the fit. The last figure, Figure 5 is the projection of the number of juvenile arrest in the New York State.

4.4 Discussion

From Table 3, we observed, in particular, the $\beta = 0.000000356670046\text{year}^{-1}$ value which is very close zero. This is in agreement with the report in [11] and shows that the peer influence in New York State is relatively minimal. The arrest and prosecution rate $\alpha = 1.483336189914413\text{year}^{-1}$ is seen to be moderately high, which could be the main reason juvenile delinquencies is under control in the New York State.

Further more, Table 3 presented other parameters and the threshold quantity, R_J. It is evident that the fitted values yielded the value of the reproduction that is less than Unity. This corroborates Lemma 1 and 2. That is, the solutions of the model is positive definite and bounded; the system is locally asymptotically stable. The result presented in Figure 3 implies that the proposed model fitted well with the real life data. To ascertain that out fit is good, the residuals of the fit as seen in Figure 4 is randomly distributed. That implies that our fit is reasonably fine and that our proposed model could be useful.

It is important to note that the model fitting has some limitations. One of such is that we have not been able to obtain good fit with the presence of the parameter ω. Logically speaking, one may think that further educative guess may lead us to a good fit. We also would like to see different scenario that can be obtained form the fitted data values We also believe that model presented in 2.1 can be extended by incorporating more parameters that encodes more realistic pattern of the juvenile delinquency.

Finally, Figure 5 represents the future projection of the JDM given the available real-life data. In particular is the population of the Juvenile Delinquent Individuals in New York State. It is observed that the projection looks great as the trajectory reaches its steady state.

4.5 Conclusion

We have proposed a mathematical model for the study of juvenile delinquency in the New York State. The qualitative properties of the model indicates that DFE of the system is locally asymptotically by establishing that the reproduction number $R_J < 1$. This result is in agreement with the current New York State's Crime Data which show a decline in the population of arrested juveniles. Additionally, we have also shown that this mathematical model could be useful in the event of future increase in juvenile delinquency.

4.6 Acknowledgement

This is to acknowledge Prof. K. Maki for her mentorship during the research.

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