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A UNIVERSAL DENSITY PROFILE FOR DARK AND LUMINOUS MATTER?

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Abstract

We explore similarities in the luminosity distribution of early type galaxies and the mass profiles of ΛCDM halos. The spatial structure of these systems may be accurately described by a simple law where the logarithmic slope of the projected density is a power law of radius; the Sérsic law. We show that this law provides a significantly better fit than a three-parameter generalization of the NFW profile and derive the best-fitting Sérsic parameters for a set of high-resolution ΛCDM halos spanning a wide range in mass. The mean Sérsic n values are 3.0 for dwarf- and galaxy-sized halos and 2.4 for cluster-sized halos, similar to the values that characterize luminous elliptical galaxies. We discuss possible reasons why the same law should describe dark and luminous systems that span a range of over seven decades in mass.

Subject headings:

1. INTRODUCTION

The Sérsic (1968) law,

$$\ln(\Sigma/\Sigma_e) = -b\,(X^{1/n} - 1), \qquad (1)$$

relating the 2D (projected or surface) density, Σ, and the dimensionless radius, $X = R/R_e$, is often fit to the luminosity profiles of elliptical galaxies and to the bulges of disk galaxies. The parameters of the fit include the Sérsic index, n, as well as the constant, b, which is normally chosen so that R_e is the radius containing one-half of the projected light; $b = b(n) \approx 2n - 0.324$ (Ciotti & Bertin 1999).

In a recent series of papers, A. Graham and co-workers have shown that the Sérsic law provides a remarkably good fit to the luminosity profiles of stellar spheroids, from dE galaxies to the most luminous ellipticals (Graham 2001, 2002; Graham & Guzman 2003; Graham et al. 2003; Trujillo et al. 2004). The fits apply over 2-3 decades in radius, and often extend down to the innermost resolvable radius. Deviations from the best-fitting Sérsic law are typically of order 0.05 magnitudes rms. The Sérsic index n is found to correlate well with galaxy absolute magnitude,

$$\log_{10} n \approx -0.106\,M_B - 1.52 \qquad (2)$$

(Graham & Guzman 2003), and also with other structural parameters like R_e and Σ_e (Caon, Capaccioli & D'Onofrio 1993; Graham & Guzman 2003). Setting $n = 4$ gives the de Vaucouleurs (1948) law, which is a good fit to luminous elliptical galaxies, and $n = 1$ is the exponential law, which reproduces well the luminosity profiles of dwarf ellipticals.

There are some known limitations to the applicability of equation (1) to the very central regions of some galaxies. In particular, Sérsic's law fails to represent adequately the very central profiles of elliptical galaxies with cores; the pointlike nuclei of some dE galaxies; and the steep power-law density cusps observed in the inner few parsecs of nearby galaxies like M32 and the bulge of the Milky Way. The origin of these features is not well understood, but it is likely that they are the result of dynamical processes, possibly involving single or multiple black holes, which act to modify the pre-existing Sérsic profile in the innermost regions (van der Marel 1999; Milosavljevic et al. 2002; Ravindranath, Ho & Filippenko 2002; Graham 2004; Merritt et al. 2004; Preto, Merritt & Spurzem 2004).

The density profiles of the *dark matter* halos formed in N-body simulations of hierarchical clustering have traditionally been fit to a rather different class of functions, essentially broken power laws (Navarro, Frenk & White 1996, 1997; Moore et al. 1999). However, the most recent simulations (Power et al. 2003; Reed et al. 2004) suggest that halo density profiles are better represented by a function with a continuously-varying slope. Navarro et al. (2004) proposed the fitting function

$$d\ln\rho/d\ln r = -2\,(r/r_{-2})^{\alpha} \qquad (3)$$

where r_{-2} is the radius at which the logarithmic slope of the *space* density is -2, and α is a parameter describing the degree of variation of the slope. The corresponding density profile is

$$\ln(\rho/\rho_{-2}) = -(2/\alpha)\,(x^{\alpha} - 1) \qquad (4)$$

with $x \equiv r/r_{-2}$. Remarkably, this is precisely the same functional form as equation (1) – with the difference that Navarro et al. fit equation (4) to the *space* density of dark matter halos, while equation (1) applies to the *projected* densities of galaxies.

Nevertheless the connection is intriguing and a number of questions spring to mind. Does the Sérsic profile fit the surface density profiles of dark matter halos as well as it fits galaxies? We will show here (§3) that the answer is "yes": the same fitting function provides an equally good description of the projected densities of both dark and luminous spheroids. In §4 we ask whether it is most appropriate to fit the Sérsic law to the space or projected densities of dark matter halos, and whether these functions are better fits than other three-parameter functions. §5 contains some speculations about why a single density law should describe dark and luminous systems over such a wide range in mass.

2. METHOD

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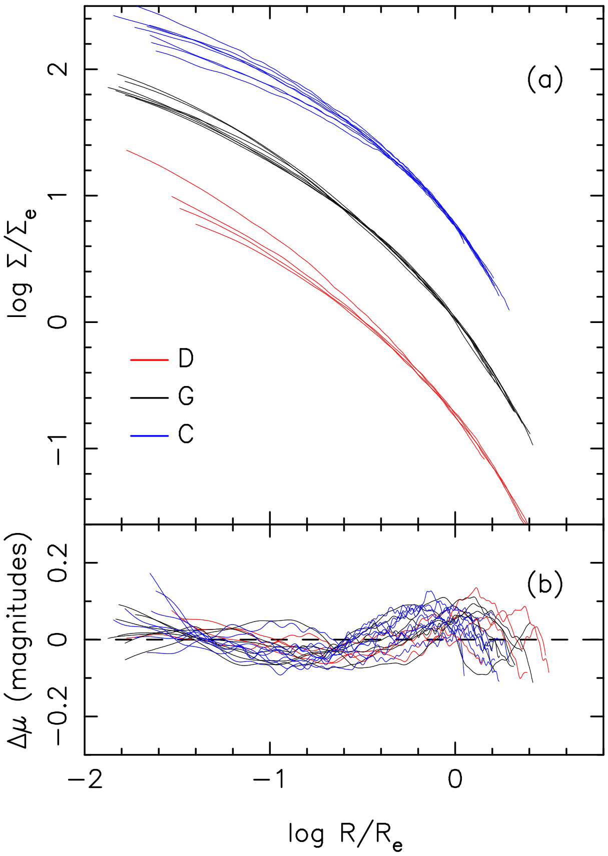

FIG. 1.— (a) Nonparametric estimates of the surface density profiles of the 19 halo models. Profiles of the *D* (*C*) models have been shifted downward (upward) by 0.75 in the logarithm. (b) Deviations of the best-fitting Sérsic model from $\hat{\Sigma}(R)$. Fitting parameters are given in Table 1.

We constructed nonparametric estimates of the space and projected density profiles of the 19 ΛCDM halo models in Navarro et al. (2004), and compared them with a number of fitting functions, including the Sérsic law, equation (1); the deprojected Sérsic law $\Sigma_d(r)$, defined as the spherical density law whose spatial projection is $\Sigma(R)$; and a generalized, three-parameter NFW (1996, 1997) profile, which may be expressed as

$$d\ln\rho/d\ln x = -(\gamma+3x)/(1+x), \qquad (5)$$

with $x = r/r_s$. The NFW profile has $\gamma = 1$ and $r_s = r_{-2}$; the Moore et al. (1999) profile has a similar functional form with inner slope $\gamma = 1.5$.

Details of the numerical simulations are given in Navarro et al. (2004). Four halos are "dwarf" sized ($M \approx 10^{10} M_\odot$), seven are "galaxy" sized ($M \approx 10^{12} M_\odot$), and eight are "cluster" sized ($M \approx 10^{15} M_\odot$). We adopt the notation of that paper (*D*= dwarf, *G*=galaxy, *C*=cluster) in what follows.

Nonparametric estimates of the space and projected density profiles, $\hat{\rho}(r)$ and $\hat{\Sigma}(R)$, were constructed using the spherically symmetrized kernels defined by Merritt & Tremblay (1994) (see e.g. Reed et al. (2004), Appendix A). Each *N*-body point was replaced by a kernel of the form

$$K_\rho(r,r_i,h_i) = \frac{1}{2(2\pi)^{3/2}}\left(\frac{rr_i}{h_i^2}\right)^{-1} e^{-\left(r_i^2+r^2\right)/2h_i^2}\sinh\left(rr_i/h_i^2\right) \qquad (6)$$

$$K_\Sigma(R,R_i,h_i) = \frac{1}{2\pi}e^{-\left(R_i^2+R^2\right)} I_0\left(RR_i/h_i^2\right) \qquad (7)$$

TABLE 1
MODEL FITS TO THE HALO DENSITY PROFILES.

Halo	Σ		ρ					
	n	$\Delta\mu$	n_d	$\Delta\mu$	n	$\Delta\mu$	γ	$\Delta\mu$
D1	3.04	0.043	3.47	0.047	5.58	0.054	1.34	0.071
D2	2.63	0.043	2.89	0.024	4.47	0.029	0.89	0.088
D3	3.91	0.018	4.19	0.041	6.94	0.039	1.51	0.041
D4	2.84	0.067	3.33	0.059	5.26	0.065	1.23	0.090
G1	2.94	0.030	3.17	0.036	5.38	0.038	1.23	0.047
G2	3.21	0.055	3.47	0.056	5.63	0.052	1.25	0.072
G3	2.87	0.050	3.44	0.042	5.98	0.049	1.33	0.047
G4	3.30	0.040	3.70	0.022	6.13	0.015	1.36	0.027
G5	2.95	0.047	3.03	0.077	4.91	0.064	1.03	0.054
G6	2.93	0.063	3.57	0.059	6.10	0.069	1.39	0.064
G7	2.82	0.051	3.09	0.087	5.04	0.097	1.22	0.110
C1	2.49	0.062	3.36	0.048	6.36	0.046	1.26	0.059
C2	2.41	0.028	2.68	0.047	4.65	0.040	1.06	0.048
C3	2.50	0.031	2.92	0.026	5.02	0.031	1.18	0.029
C4	2.19	0.064	3.11	0.101	5.72	0.111	1.29	0.091
C5	2.53	0.041	2.61	0.092	4.33	0.077	0.91	0.066
C6	2.16	0.044	2.62	0.050	4.49	0.065	1.10	0.064
C7	2.79	0.047	3.99	0.040	7.44	0.042	1.41	0.038
C8	1.99	0.069	2.62	0.083	4.67	0.095	1.13	0.087

with h_i the width of the kernel associated with the ith particle and I_0 the modified Bessel function. The projected radii R_i were obtained from the N-body radii r_i by assigning each particle a random position on the sphere of radius r_i. Density estimates were computed on a grid of 100 radial points spaced logarithmically from r_{conv} to r_{200} (these radii are defined below). We followed standard practice (Silverman 1986) and first computed a pilot estimate of the density via a nearest-neighbor scheme, then allowed the h_i to vary as a power δ of this pilot density.

When fitting one of the parametric functions defined above to $\hat{\rho}$ or $\hat{\Sigma}$, we computed the density estimates on a grid in (h_0,δ) (h_0 is the geometric mean of the h_i) to see which choice of kernel parameters minimized the residual of the fit; typically there was a broad range of (h_0,δ) values over which the best-fit parameters and their residuals were nearly constant. The residual was defined as the rms over the radial grid of $\log(\hat{\rho}_j/\rho(r_j))$ with $\hat{\rho}_j$ the density estimate at grid point r_j and ρ the parametric fitting function; this is identical to how most observers define the residual. Below we state the rms deviation between the "measured" profile and the best-fitting parametric model in terms of magnitudes, denoted by $\Delta\mu$.

We followed the practice in Navarro et al. (2004) of only constructing density estimates in the radial range $r_{conv} \le r \le r_{200}$, where r_{conv} is the radius beyond which the halo mass distribution is considered robust to errors or approximations associated with the simulations (particle softening, relaxation etc.) and r_{200} is the virial radius, i.e. the radius within which the mean density contrast is 200 times the critical density. Table 2 of Navarro et al. (2004) gives values of r_{200} and r_{conv} for all halo models.

3. DARK MATTER HALOS AS SÉRSIC MODELS

With few exceptions, modelling of the luminosity profiles of galaxies is done in projected space. We therefore began by analyzing the surface density profiles of the dark halos. Nonparametric estimates of $\Sigma(R)$ for the 19 halos are shown in Figure 1a, and Figure 1b plots the deviations from the best-fitting Sérsic model, equation (1); Table 1 gives the best-fitting n and $\Delta\mu$. The mean Sérsic index is 3.11 ± 0.49 (D),

3.00 ± 0.17 (G), 2.38 ± 0.24 (C), possibly indicating a (weak) trend toward decreasing curvature (lower n) in the profiles of halos of increasing mass.

The $\Delta\mu$ values average 0.043 (D), 0.048 (G), 0.048 (C). For comparison, Caon, Capaccioli & D'Onofrio (1993) find $\Delta\mu \approx 0.05$ in a sample of 45 E and S0 galaxies, and Trujillo et al. (2004) find a mean $\Delta\mu$ of 0.09 in a sample of 12 elliptical galaxies without cores. The radial range over which luminous galaxies are fit varies from ~ 1.5 to ~ 3.5 decades, comparable on average with the ~ 2 decades characterizing our dark-matter halos. While the noise properties are different for the two types of data, the particle numbers in our halo models are small enough ($\sim 10^6$) to contribute nonnegligibly to $\Delta\mu$. We conclude that the Sérsic law fits dark halos as well as, and possibly even better than, it fits luminous galaxies.

The residuals in Figure 1b appear to show some structure. We will return in a future paper to the question of whether a modification of the Sérsic law might reduce these residuals even further.

4. WHICH FUNCTION FITS THE SPACE DENSITY BEST?

Navarro et al. (2004) showed that equation (4) provides a good fit to the *spatial* density profiles of dark halos. We showed above (§3) that the Sérsic law (1) is a good fit to the *surface* density profiles of dark halos. An obvious inference is that a *deprojected* Sérsic law should provide a good fit to the space density. Here we ask which function – equation (4), or a deprojected Sérsic law – gives a better fit to $\rho(r)$. We also consider the quality of fit of another three-parameter function, the generalized NFW profile presented in equation (5).

When fitting deprojected Sérsic profiles to the dark halos, we define n_d to be the Sérsic index of the projected function; hence n_d should be close to the index n derived when fitting a Sérsic law to the surface density (and the two would be equal if the halo's surface density were precisely described by Sérsic's law). When reporting fits to $\rho(r)$ with equation (4) we define $n \equiv \alpha^{-1}$, with α the shape parameter of equation (3).

The results are shown in Figure 2 and Table 1. Mean values of $\Delta\mu$ for the three fitting functions (deprojected Sérsic, eq. (4), generalized NFW) are $(0.043, 0.047, 0.073)$ for the dwarf halos, $(0.054, 0.055, 0.060)$ for the galaxy halos, and $(0.061, 0.063, 0.060)$ for the cluster halos. Thus, the two Sérsic functions are almost indistinguishable in terms of their goodness of fit: at least over the radial range available, a deprojected Sérsic profile with index $2.5 \lesssim n_d \lesssim 3.5$ can be well approximated by a Sérsic profile with n in the range $4.5 \lesssim n \lesssim 7.5$. Both functions provide a significantly better fit to $\rho(r)$ than the generalized NFW profile in the case of the dwarf halos, and the two Sérsic functions perform at least slightly better than NFW for the galaxy halos. No single function is preferred when fitting $\rho(r)$ for the cluster halos. [1]

Another way to compare the halo density profiles with Sérsic's law is via the radial dependence of the slope. Figure 3 shows nonparametric estimates of the logarithmic slope, $d\log\rho/d\log r$, for the dark halos; slopes were computed via direct differentiation of the kernel density estimates, using a larger kernel width to compensate for the greater noise generated by the differentiation. Equation (3) predicts a straight line on this plot. That is a reasonable description of Figure 3. The value of $d\log\rho/d\log r$ in the G and C halos reaches ~ -1 at the innermost radii, consistent with the asymptotic power-law inner behavior of an NFW profile. No obvious convergence to a power law (constant logarithmic slope) is seen in Figure 3, and it is likely that simulations of improved resolultion may lead to even shallower slopes at smaller radii, as pointed out by Navarro et al. (2004).

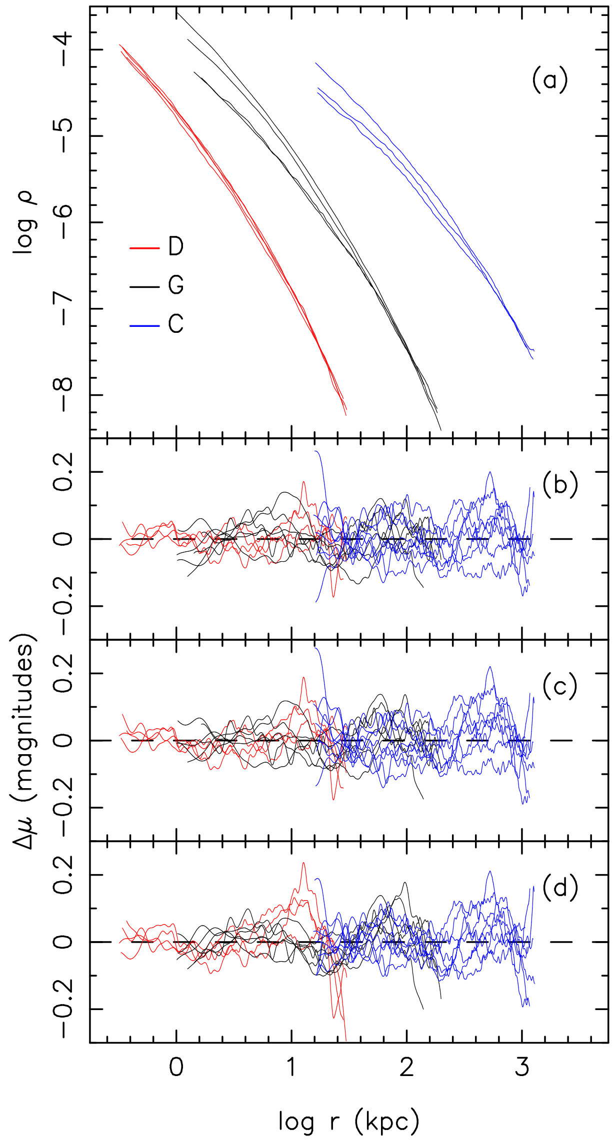

FIG. 2.— (a) Nonparametric estimates of the space density of the 19 dark halos. Vertical normalization is arbitrary. (b-d) Deviations in magnitudes of three parametric models from $\hat{\rho}(r)$: (b) deprojected Sérsic model; (c) equation (4); (d) generalized NFW model, equation (5). Best-fit parameters are given in Table 1.

5. WHAT DOES IT MEAN?

Figure 4 shows Sérsic's n (derived from fits to the surface density) as a function of mass for our dark halos and for a sample of early-type galaxies. There is overlap at $M \approx 10^{10} M_\odot$, the mass characteristic of "dwarf" halos and giant ellipticals. However the galaxies exihibit a much wider range of n values, extending to $n < 0.5$ in the case of dwarf ellipticals. A natural interpretation is that n is determined by the degree to which (dissipationless) merging has dominated the evolution. The

[1] Also of interest are the mean γ-values in the fits to the generalized NFW profile. We find $\langle\gamma\rangle = (1.24, 1.26, 1.17)$ for dwarf, galaxy and cluster halos respectively. We note that these are significantly shallower than the steep inner slope ($\gamma = 1.5$) proposed by Moore et al. (1999) and, as discussed by Navarro et al. (2004), are best interpreted as upper limits to the inner aymptotic behavior of the profile.

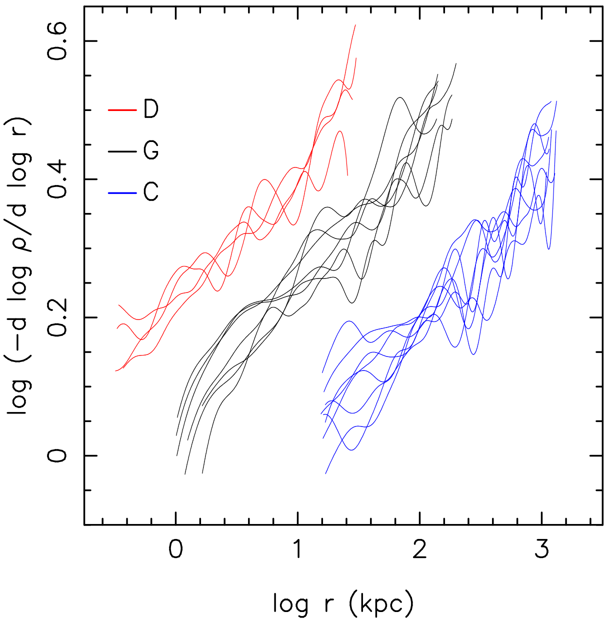

FIG. 3.— Nonparametric estimates of the logarithmic derivative of the space density for the 19 halo models.

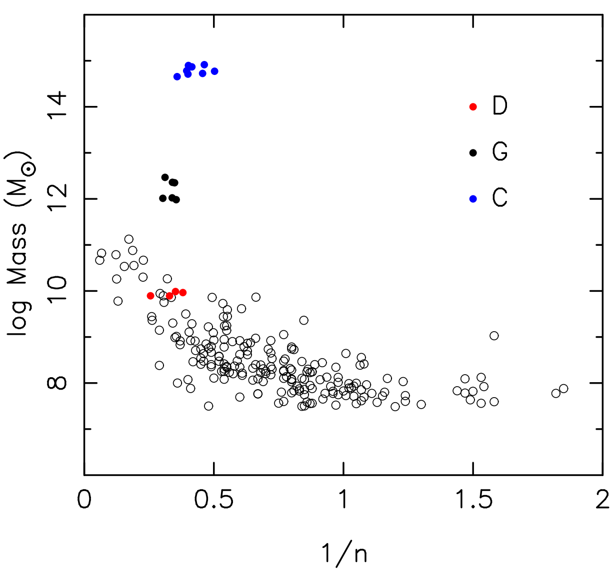

FIG. 4.— Sérsic index (derived from fits to the surface density) versus mass for galaxies (open circles) and dark halos. Galaxy points are taken from Binggeli & Jerjen (1998); Stiavelli et al. (2001); Graham & Guzman (2003); Caon, Capaccioli & D'Onofrio (1993); D'Onofrio, Capaccioli & Caon (1994). Halo masses are M_{200} from Navarro et al. (2004). Galaxy masses were computed from total luminosities assuming the Magorrian et al. (1998) mass-to-light ratio, with $H_0 = 70$ km s^{-1} Mpc^{-1}.

nearly exponential ($n \approx 1$) profiles of dE galaxies are similar to those of disk galaxies, suggesting that dissipation played a critical role in their formation. Luminous ellipticals are the end products of many mergers, the most recent of which are likely to have been gas-poor, and have de Vaucouleurs-like profiles ($n \approx 4$). This view is supported by numerical simulations (Scannapieco & Tissera 2002; Eliche-Moral et al. 2005) that show how exponential profiles are converted into de Vaucouleurs-like profiles via repeated mergers.

A thornier question is: Why should a law like Sérsic's fit dark or luminous spheroids in the first place? Sérsic's law with $2 \lesssim n \lesssim 4$ has an energy distribution that is roughly Boltzmann, $N(E)dE \sim e^{\beta E}dE$, and it is sometimes loosely argued that this "maximum-entropy" state is a result of the mixing that accompanies violent relaxation or merging (Binney 1982; Merritt, Tremaine & Johnstone 1989; Ciotti 1991). With regard to dark halos, Taylor & Navarro (2001) have shown that the dependence of phase-space density on radius is well approximated by a power law whose corresponding inner density profile has the shallowest slope. This can again be interpreted as an indication that the halos are well mixed. While our study does not shed a great deal of light on this question, it does suggest that the scale-free property of Sérsic's law, $d\ln\rho/d\ln r \propto r^{\alpha}$, is the feature that links dark and luminous spheroids and that this property may be a hallmark of systems that form via gravitational clustering.

We have shown that the fitting function that best describes luminous galaxies, the Sérsic law, is an equally good fit to dark halos. We have not shown that the Sérsic law is a good fit in an *absolute* sense to either sort of system. But given that dark and luminous density profiles are not pure power laws, a three-parameter law like Sérsic's is as parsimonious a description as one can reasonably expect. Future work should explore whether other, three-parameter fitting functions can describe dark and/or luminous systems better than Sérsic's law.

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