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EFFICIENT MERGER OF BINARY SUPERMASSIVE BLACK HOLES IN NON-AXISYMMETRIC GALAXIES

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Abstract

Binary supermassive black holes form naturally in galaxy mergers, but their long-term evolution is uncertain. In spherical galaxies, N -body simulations show that binary evolution stalls at separations much too large for significant emission of gravitational waves (the “final parsec problem”). Here, we follow the long-term evolution of a massive binary in more realistic, triaxial and rotating galaxy models. We find that the binary does not stall. The binary hardening rates that we observe are sufficient to allow complete coalescence of binary SBHs in 10 Gyr or less, even in the absence of collisional loss-cone refilling or gas-dynamical torques, thus providing a potential solution to the final parsec problem.

Subject headings:

1. INTRODUCTION

When two galaxies containing supermassive black holes (SBHs) merge, a binary SBH forms at the center of the new galaxy. The two SBHs can eventually coalesce, but only after stellar- or gas-dynamical processes bring them close enough together ($\lesssim 10^{-2}$ pc) that gravitational radiation is emitted. There is strong circumstantial evidence that rapid coalescence is the norm. For instance, no binary SBH has ever been unambiguously observed (Merritt & Milosavljevic 2005). Furthermore, in a galaxy containing an uncoalesced binary, mergers would eventually bring a third SBH into the nucleus, precipitating a gravitational slingshot interaction that would eject one or more of the SBHs from the nucleus (Mikkola & Valtonen 1990; Volonteri, Haardt & Madau 2003). This could produce off-center SBHs, and could also weaken the tight correlations that are observed between SBH mass and galaxy properties (Ferrarese & Merritt 2000; Graham et al. 2001; Marconi & Hunt 2003).

Unless the binary mass ratio is extreme, dynamical friction rapidly brings the smaller SBH into a distance $\sim G\mu/\sigma^2$ from the larger SBH, where $\mu \equiv M_1 M_2 / (M_1 + M_2)$ is the binary reduced mass and σ is the 1D velocity dispersion of the stars. At this separation – of order 1 pc – the two SBHs begin to act like a “hard” binary, ejecting passing stars with velocities large enough to remove them from the nucleus. N -body simulations (Makino & Funato 2004; Szell, Merritt & Mikkola 2005; Berczik, Merritt & Spurzem 2005) show that continued hardening of the binary takes place at a rate that depends strongly on the number N of “star” particles used in the simulation. As N increases, the hardening rate falls, as expected if the binary’s loss cone is repopulated by star-star gravitational encounters (Yu 2002; Milosavljevic & Merritt 2003). When extrapolated to the much larger N of real galaxies, these results suggest that binary evolution would generally stall (the “final parsec problem”).

To date, N -body simulations of the long-term evolution of binary SBHs have only been carried out using spherical or

nearly spherical galaxy models. But it has been suggested (Merritt & Poon 2004) that binary hardening might be much more efficient in non-axisymmetric galaxies due to the qualitatively different character of the stellar orbits. Here, we test that suggestion by carrying out the first N -body simulations of massive binaries in strongly non-axisymmetric galaxy models. We find that the hardening rate is *independent* of N for particle numbers up to at least 0.4×10^6 . To the extent that our galaxy models are similar to real merger remnants, these results imply that binary SBHs can efficiently harden through purely stellar-dynamical interactions in many galaxies, thus providing a plausible solution to the final parsec problem.

2. METHOD

Our N -body models were generated from the phase-space distribution function

$$f(E, L_z) = \text{const} \times (e^{-\beta E} - 1) e^{-\beta \Omega_0 L_z} \quad (1)$$

(Lagoutte & Longaretti 1996). Here $E = v^2/2 + \Phi$ is the energy per unit mass of a star, $\Phi(\varpi, z)$ is the gravitational potential in the meridional plane, and L_z is the angular momentum per unit mass in the direction of the symmetry (z) axis; the potential is set to zero at the radius where the density falls to zero. The quantity in parentheses on the right hand side of equation (1) is the energy-dependent King (1966) distribution function. The additional, angular-momentum-dependent factor has the effect of flattening the models and simultaneously giving them a net rotation about the z axis. The degree of flattening can be specified via the dimensionless rotation parameter $\omega_0 \equiv \sqrt{9/(4\pi G \rho_0)} \Omega_0$, with ρ_0 the central mass density of the galaxy. The parameter β determines the central concentration of the model; its value was chosen such that the spherical isotropic model generated from equation (1) had a dimensionless central concentration $W_0 = 6$ (King 1966). Here and below we adopt standard N -body units, i.e. the gravitational constant and total mass of the galaxy are one, and the galaxy’s energy is $-1/4$.

A pair of massive particles representing the two SBHs were introduced into the models at time $t = 0$. The two particles were given equal masses, $M_1 = M_2 \equiv M_\bullet/2$, and were placed on coplanar, circular orbits at distances ± 0.3 from the galaxy center in the equatorial plane. In most of the simulations described below, $M_\bullet = 0.04$. This is rather larger than the typical ratio, $\sim 1 \times 10^{-3}$ (Merritt & Ferrarese 2001), observed

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between SBH mass and galaxy mass; such a large mass for the SBH particles was chosen in order to minimize the rate of relaxation-driven loss cone refilling, which occurs more rapidly for smaller M_\bullet (Berczik, Merritt & Spurzem 2005), and to come as close as possible to the “empty loss cone” regime that characterizes real (axisymmetric) galaxies. In order to estimate the dependence of the binary decay rate on M_\bullet , we carried out a limited set of additional simulations with different values of M_\bullet , as described below.

Integrations of the particle equations of motion were carried out using a high-accuracy, direct-summation N -body code (Berczik, Merritt & Spurzem 2005) on two parallel supercomputers incorporating special-purpose GRAPE (Fukushige, Makino & Kawai 2005) accelerator boards: gravitySimulator⁵ and GRACE.⁶ Integration parameters were similar to those adopted in Berczik, Merritt & Spurzem (2005) and we refer the reader to that paper for details about the performance of the code. Integrations were carried out for various N in the range $0.025 \leq N \leq 0.4 \times 10^6$ and for various values of the galaxy rotation parameter ω_0 in the range $0 \leq \omega_0 \leq 1.8$. In addition, each model was integrated with two choices for the orientation of the binary’s angular momentum, either parallel to that of the galaxy (“prograde”) or counter to it (“retrograde”).

3. RESULTS

After the two SBH particles come close enough together to form a bound pair, the parameters of their relative Keplerian orbit can be computed. Figure 1a shows the evolution of $1/a$, the binary inverse semi-major axis, in a set of simulations with $\omega_0 = 0$ and various N . These spherical, non-rotating models are very similar to the models considered in Berczik, Merritt & Spurzem (2005), and the binary evolution found here exhibits the same strong N dependence that was observed in that study: the hardening rate, $s(t) \equiv (d/dt)(1/a)$, is approximately constant with time and decreases roughly as N^{-1} . This behavior has been described quantitatively (Milosavljevic & Merritt 2003) on the basis of loss-cone theory: stars ejected by the binary are replaced in a time that scales as the two-body relaxation time, and the latter increases roughly as N in a galaxy of fixed mass and size.

When the rotation parameter ω_0 is increased to ~ 0.6 , the initially axisymmetric models become unstable to the formation of a bar, yielding a slowly-tumbling, triaxial spheroid. Figure 2 illustrates the instability via snapshots of the $\omega_0 = 1.8$ model integration. This model is moderately flattened initially, with mean short-to-long axis ratio of ~ 0.46 , and strongly rotating, with roughly 40% of the total kinetic energy in the form of streaming motion. Movies based on the simulation⁷ reveal that the two SBH particles initially come together by falling inward along the bar before forming a bound pair.

The long-term behavior of the binary (Figure 1b) is strikingly different in this rotating model than in the spherical model: not only is the hardening rate high, but more significantly, it shows *no systematic dependence on particle number*. In fact, the two simulations of the $\omega_0 = 1.8$ model with largest N (200k and 400k) exhibit almost identical evolution of the binary.

To determine the dependence of the binary’s evolution rate on the properties of the galaxy model, a suite of N -body in-

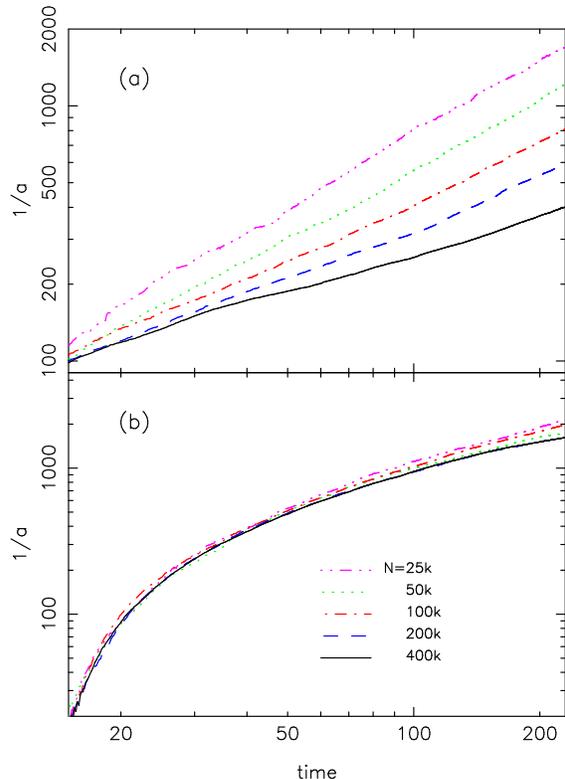


FIG. 1.— Evolution of the binary inverse semi-major axis, $1/a$, in N -body simulations with various N . (a) Spherical, nonrotating galaxy model ($\omega_0 = 0$). (b) Flattened, rotating galaxy model ($\omega_0 = 1.8$). At $t \approx 10$, this model forms a triaxial bar (cf. Figure 2).

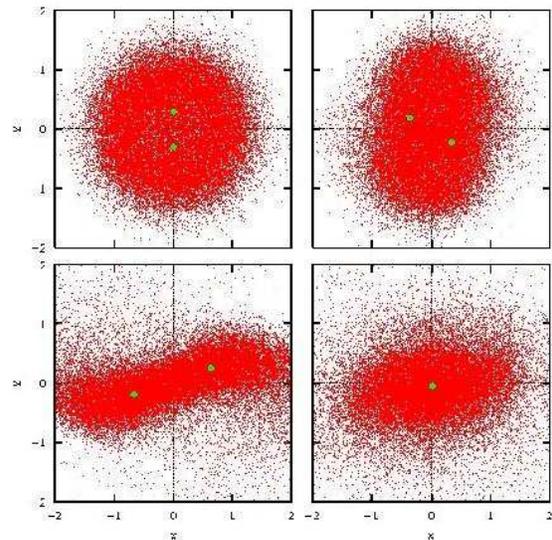


FIG. 2.— Snapshots at four times ($t = 0, 5, 10, 30$) of the particle positions, projected onto the (x, y) plane, in an N -body integration with $\omega_0 = 1.8$ and $N = 200k$. The SBH particles are indicated in green. The evolution of the binary semi-major axis in this integration is shown in Figure 1b.

tegrations were carried out for various (ω_0, N) and for $M_\bullet = 0.04$. Axis ratios of the galaxy were computed using its moment of inertia tensor, as described in Dubinski & Carlberg (1991). The results are summarized in Figure 3. After a strong bar forms at $t \approx 10$ in the unstable models, it evolves gradually toward rounder shapes, but the system maintains

⁵ <http://www.cs.rit.edu/~grapecluster/clusterInfo/grapeClusterInfo.shtml>

⁶ <http://www.ari.uni-heidelberg.de/grace>

⁷ <http://www.cs.rit.edu/~grapecluster/BinaryEvolution>

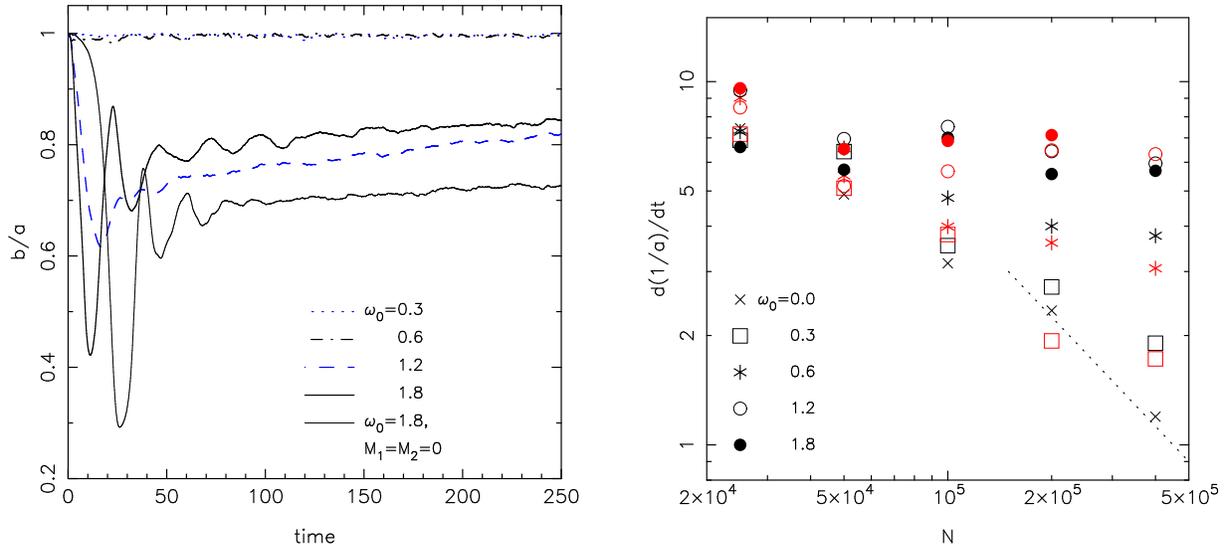


FIG. 3.— (a) Evolution of the intermediate-to-long axis ratio b/a of the galaxy in N -body integrations with $N = 200k$ and various values of the rotation parameter ω_0 ; $M_1 = M_2 = 0.02$. For $\omega_0 > 0.6$, the galaxy is unstable to non-axisymmetric deformations and forms a slowly-tumbling triaxial spheroid. The length c of the short axis is nearly constant with time and is determined by the initial flattening of the model, i.e. by ω_0 . The thin black line shows the result of an integration that omitted the two SBH particles. (b) Hardening rate of the binary measured at $t = 150$. The value of ω_0 is indicated via the key on the lower left; black symbols are “prograde” integrations, i.e. the binary revolves in the same sense as the galaxy, and red symbols are “retrograde” integrations. The dashed lines shows the N^{-1} dependence that is characteristic of a collisionally-resupplied loss cone.

a significant triaxiality until the end of the simulation. This slow evolution appears similar to that in the simulations of Theis & Spurzem (1999), where two-body relaxation was identified as the driving mechanism. The presence of a massive binary in our simulations might also tend to destroy the triaxiality (Athanasoula, Lambert & Dehnen 2005), although an integration excluding the binary showed a similar degree of evolution; see Figure 3a.

The steep, $\sim N^{-1}$ dependence of the binary hardening rate in the spherical model changes to an essentially constant hardening rate for $\omega_0 \geq 1.2$ (Figure 3b). Even the $\omega_0 = 0.6$ model – which, Figure 3a suggests, is close to marginal stability – yielded a hardening rate that was substantially larger than in the spherical models, consistent with suggestions (Merritt & Poon 2004) that even slight departures from axisymmetry could significantly influence the binary’s evolution. The hardening rate was found not to depend on the initial sense (prograde vs. retrograde) of the binary orbit (Figure 3b).

Since $M_\bullet/M_{gal} = 0.04$ is considerably larger than the ratio $\sim 1 \times 10^{-3}$ observed in real galaxies (Merritt & Ferrarese 2001; Marconi & Hunt 2003), we carried out an additional set of simulations in order to evaluate the M_\bullet -dependence of the binary hardening rate. These integrations used $\omega_0 = 1.8$, $N = 0.1 \times 10^6$, and $0.01 \leq M_\bullet \leq 0.08$. We found that the hardening rate increased with decreasing M_\bullet . At $t = 100$, the hardening rates were $s = (20.0, 13.0, 8.2, 3.4)$ for $M_\bullet = (0.01, 0.02, 0.04, 0.08)$. These results should be interpreted with caution since we did not vary N and therefore can not state with certainty whether the s values are N -independent. The linear size of the binary’s loss cone is proportional to M_\bullet , making it easier for two-body scattering to affect the hardening rate as M_\bullet is decreased. But the $M_\bullet = 0.04$ integrations are clearly *not* in the collisionally-repopulated regime (Figure 1b) and so the differences that we observe between the hardening rates for $M_\bullet = 0.04$ and 0.08 are likely to be robust. Based on these results, it is reasonable to conclude that binary

evolution would be substantially more rapid than implied by Figure 3b in the case $M_\bullet \approx 10^{-3}M_{gal}$.

4. DYNAMICAL INTERPRETATION

A hardening rate that is independent of N implies a collisionless, i.e. relaxation-independent, mode of loss-cone refilling. Just such a mode is expected in triaxial galaxies: the lack of an axis of symmetry implies that stellar orbits need not conserve any component of the angular momentum, hence they can pass arbitrarily close to the center after a finite time and interact with a central object (Norman & Silk 1983; Gerhard & Binney 1985).

A full derivation of the expected rate of supply of stars to the binary in these models would be very difficult, but we can do an approximate calculation. The rate per unit of orbital energy at which centrophilic orbits supply mass (i.e. stars) to a region of radius r_t at the center of a galaxy is

$$\dot{M}(E)dE = r_t A(E) M_c(E) dE \quad (2)$$

where $A(E)d$ is the rate at which a *single* star on a centrophilic (e.g. box or chaotic) orbit of energy E experiences near-center passages with pericenter distances $\leq d$, and $M_c(E)dE$ the mass in stars on centrophilic orbits with energies from E to $E + dE$ (Merritt & Poon 2004). Setting $r_t = Ka$, with $K \approx 1$, gives the mass flux into the binary’s sphere of influence; the implied hardening rate is (Berczik, Merritt & Spurzem 2005)

$$s \equiv \frac{d}{dt} \left(\frac{1}{a} \right) \approx \frac{2\langle C \rangle}{aM_\bullet} \int \dot{M}(E) dE. \quad (3)$$

Here, $\langle C \rangle \approx 1.25$ is the average value of the dimensionless energy change during a single star-binary encounter, $C \equiv [M_\bullet/2m_\star](\Delta E/E)$.

Our N -body models have density $\rho \sim r^{-2}$ beyond the core

radius $r_c \approx 0.25$. In a $\rho \propto r^{-2}$ galaxy,

$$A(E) \approx \frac{\sigma}{r_h^2} e^{-(E-E_h)/\sigma^2}, \quad M_c(E) = f_c(E) \times \frac{2\sqrt{6} r_h}{9 G} e^{(E-E_h)/2\sigma^2} \quad (4)$$

with $r_h = GM_\bullet/\sigma^2$, $E_h = \Phi(r_h)$, and $f_c(E)$ the fraction of the orbits at energy E that are centrophilic (Merritt & Poon 2004). The implied binary hardening rate is

$$s \approx \frac{4\sqrt{6} \langle C \rangle K_f \bar{f}_c}{9 \sigma r_h^2} \int e^{-(E-E_h)/2\sigma^2} dE \approx 2.5 \bar{f}_c \frac{\sigma}{r_h^2}. \quad (5)$$

Here \bar{f}_c is an energy-weighted, mean fraction of centrophilic orbits, and the lower integration limit was set to E_h ; the latter can only be approximate since the true density of our galaxy models departs from r^{-2} at $r < r_c \approx r_h$. Substituting $\sigma \approx 0.47$ and $r_h \approx 0.18$ from the galaxy models gives $s \approx 40\bar{f}_c$. By comparison, the hardening rates in the N -body models reach a peak value at $t \approx 20$ of $s \approx 16$, consistent with the derived expression if $\bar{f}_c \approx 1/2$. The gradual drop observed in the hardening rate at later times, $s(t) \approx 16 - 5.2 \ln(t/20)$, $20 \leq t \leq 250$, suggests that the number of centrophilic stars is becoming smaller, due to depletion by the binary and to the gradual change in the galaxy's shape.

Taken at face value, equation (5) implies $s \propto M_\bullet^{-2}$; however for small M_\bullet , $r_h \ll r_c$ and the assumption that $\rho \sim r^{-2}$, $r > r_h$ breaks down. In any case, the observed dependence of s on M_\bullet is slightly weaker, $s \sim M_\bullet^{-1}$ (§3).

5. IMPLICATIONS

The time scale for gravitational wave emission by a binary black hole is (Peters 1964)

$$t_{gr} = \frac{5}{16^4 F(e)} \frac{G\mu^3 c^5}{\sigma^8 M_\bullet^2} \left(\frac{a}{a_h} \right)^4. \quad (6)$$

Here $M_\bullet = M_1 + M_2$, $\mu = M_1 M_2 / M_\bullet^2$ is the reduced mass of the binary, σ is the 1D central velocity dispersion of stars in the nucleus, and $a_h \equiv G\mu/4\sigma^2$ is the semi-major axis of the binary when it first becomes ‘‘hard,’’ i.e. tightly bound; the factor $F(e)$ depends on the binary's orbital eccentricity and $F(0) = 1$. In order that gravitational-wave-driven coalescence take place in less than 10^{10} yr, an equal-mass, circular-orbit binary with $M_\bullet = 10^8 M_\odot$ must first reach a separation $a \lesssim 0.05 a_h$ (Merritt & Milosavljevic 2005). This is just achieved in our simulations: $a_h \approx 1.1 \times 10^{-2}$, and the final value of a in the bar-unstable models is $\sim 6 \times 10^{-4}$. This

⁸ <http://lisa.jpl.nasa.gov/>

is a conservative interpretation since (1) for reasonable scalings of our galaxy model to real galaxies (e.g. total mass = $10^{11} M_\odot$, half-mass radius = 10^3 pc), an elapsed time of 250 in N -body units corresponds to $\lesssim 1$ Gyr; (2) the binary is continuing to harden at the final time-step in our simulations (Figure 1b); (3) our experiments with different M_\bullet found $s \sim M_\bullet^{-1}$, implying substantially more rapid hardening in the case $M_\bullet/M_{gal} \approx 10^{-3}$; (4) the binary had nonzero eccentricity in our simulations. In addition, gas is a significant component of disk galaxies and, in many mergers, the final hardening of the binary would be accelerated by gas-dynamical torques (Escala et al. 2005; Dotti et al. 2005).

Our simulations of binary evolution are substantially more realistic than existing ones based on spherical or nearly-spherical galaxy models. Even more realistic simulations, which follow both the early and late stages of a merger between two galaxies, are probably beyond the capabilities of current algorithms and hardware due to the need to accurately treat both large (~ 10 kpc) and small (~ 0.01 pc) spatial scales. However, our galaxy models (slowly-tumbling triaxial spheroids) are similar to those produced in full merger simulations (Bournaud, Jog & Combes 2005; Naab, Khochfar & Burkert 2006), suggesting that our results for the long-term evolution of the binary are probably fairly generic in spite of the rather artificial initial conditions.

Uncertainties about the resolution of the ‘‘final parsec problem’’ have been a major impediment to predicting the frequency of SBH mergers in galactic nuclei, and hence to computing event rates for proposed gravitational wave interferometers like LISA.⁸ If binary coalescence rates are assumed to be similar to galaxy merger rates, gravitational wave events integrated over the observable universe could be as frequent as 10^2 yr^{-1} (Haehnelt 1994; Sesana et al. 2004). Our results, combined with the indirect evidence that binary SBH coalescence is efficient, suggest that such high event rates should be taken seriously.

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