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Manuela Campanelli
Rochester Institute of Technology

Carlos O. Lousto
Rochester Institute of Technology

Yosef Zlochower
Rochester Institute of Technology

David Merritt
Rochester Institute of Technology

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LARGE MERGER RECOILS AND SPIN FLIPS FROM GENERIC BLACK-HOLE BINARIES

Manuela Campanelli[1], Carlos Lousto[2,1], Yosef Zlochower[2,1], David Merritt[1,3]
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ABSTRACT

We report the first results from evolutions of generic black-hole binaries, i.e. a binary containing unequal mass black holes with misaligned spins. Our configuration, which has a mass ratio of 2 : 1, consists of an initially non-spinning hole orbiting a larger, rapidly spinning hole (specific spin $a/m = 0.885$), with the spin direction oriented $-45°$ with respect to the orbital plane. We track the inspiral and merger for ~ 2 orbits and find that the remnant receives a substantial kick of $454\,\mathrm{km\,s^{-1}}$, more than twice as large as the maximum kick from non-spinning binaries. The remnant spin direction is flipped by $103°$ with respect to the initial spin direction of the larger hole. We performed a second run with anti-aligned spins, $a/m = \pm 0.5$ lying in the orbital plane that produces a kick of $\sim 1830\,\mathrm{km\,s^{-1}}$ off the orbital plane. This value scales to nearly $4000\,\mathrm{km\,s^{-1}}$ for maximally spinning holes. Such a large recoil velocity opens the possibility that a merged binary can be ejected even from the nucleus of a massive host galaxy.

Subject headings: black hole physics – galaxies: nuclei – gravitational waves – relativity – gravitation

1. INTRODUCTION

One of the major goals of numerical relativity has been the accurate evolution of generic black-hole binaries from inspiral through merger and ringdown. It is in this non-linear merger regime where most of the gravitational radiation is emitted, including the radiation of linear momentum responsible for large merger recoils that can eject the remnant from the host galaxy. With the recent breakthroughs in numerical techniques (Pretorius 2005; Campanelli et al. 2006a; Baker et al. 2006a) this goal is finally being realized. Within the past 18 months rapid progress has been achieved in our understanding of black-hole-binary mergers. Non-spinning equal-mass binaries were studied in detail, including the last few orbits (Campanelli et al. 2006b; Baker et al. 2006b), the effects of elliptical motion (Pretorius 2006) on the gravitational radiation, and waveforms generated from binaries with large initial separations were successfully matched to post-Newtonian theory with very good agreement (Buonanno et al. 2006; Baker et al. 2006c,d). Non-spinning unequal mass black holes were studied in Campanelli (2005), Herrmann et al. (2006), Baker et al. (2006e) and González et al. (2006), where the recoil velocity of the post-merger remnant was computed. In particular, the accurate calculations of González et al. (2006) indicate that the maximum recoil velocity of non-spinning quasi-circular binaries, with mass ratio $q = m_1/m_2 \approx 1/3$, is $\sim 175\,\mathrm{km\,s^{-1}}$. Simulations of highly-spinning black-hole binaries were introduced in Campanelli et al. (2006c) where it was shown that the direction of the spin (in that case either aligned or counter-aligned with the orbital angular momentum) has a strong effect on the merger time and energy-

momentum radiated to infinity. In Campanelli et al. (2006d) it was found that the non-linear tidal effects were too weak to drive a binary into a corotating state. Finally in Campanelli et al. (2006e) spin precession and spin-flips were studied for equal-mass binaries with individual spins not aligned with the orbital angular momentum (but with individual spins having the same magnitude and direction).

All the previous simulations contained symmetries which suppressed some important astrophysical properties (e.g. precession, recoil, spin-orbit coupling) of generic binary mergers, and in the case of the recoil calculation, spins were neglected entirely. With the knowledge gained from these simulations we can now design and evolve a truly prototypical black-hole binary. In the scenario considered here, a high-mass black hole, with a specific spin of $a/m = 0.885$ (the largest considered thus far), merges with a smaller hole having negligible spin. The mass ratio of the two holes is 1.99, and the initial binary configuration is such that the spin of the larger hole points $45°$ below the orbital plane. This configuration will manifest precession of the spin axis, a significant spin flip of the remnant spin with respect to the initial individual horizon spin, and a significant recoil kick. The simulations that we report in this letter show that the recoil due to the spin can be more than an order of magnitude larger than the maximum recoil due to unequal-masses alone.

2. TECHNIQUES

We use the puncture approach (Brandt and Brügmann 1997) along with the TwoPunctures (Ansorg et al. 2004) thorn to compute initial data. We evolve these black hole binary data sets using the LazEv (Zlochower et al. 2005) implementation of the moving puncture approach (Campanelli et al. 2006a) (which is based on the BSSN (Nakamura et al. 1987; Shibata and Nakamura 1995; Baumgarte and Shapiro 1999) formulation). We use the Carpet (Schnetter et al. 2004) mesh refinement driver to provide a 'moving boxes' style mesh refinement. In this approach refined grids of fixed size are arranged about the coordinate

[1] Center for Computational Relativity and Gravitation, School of Mathematical Sciences, Rochester Institute of Technology, 78 Lomb Memorial Drive, Rochester, New York 14623
[2] Center for Gravitational Wave Astronomy, Department of Physics and Astronomy, The University of Texas at Brownsville, Brownsville, Texas 78520
[3] Department of Physics, Rochester Institute of Technology, 85 Lomb Memorial Drive, Rochester, New York 14623

TABLE 1
Initial data parameters for the SP6 (top) and SP2 (bottom) configurations. m_p is the puncture mass parameter of the two holes. SP6 has spins $\vec{S}_1 = (0, S, -S)$ and $\vec{S}_2 = (0, 0, 0)$, momenta $\vec{P} = \pm(P_r, P_\perp, 0)$, puncture positions $\vec{x}_1 = (x_+, d, d)$ and $\vec{x}_2 = (x_-, d, d)$, and masses m_1 and m_2. While SP2 has spins $\vec{S}_1 = -\vec{S}_2 = (0, S, 0)$, puncture positions $\vec{x}_1 = -\vec{x}_2 = (x, 0, 0)$, and momenta $\vec{P}_1 = -\vec{P}_2 = (0, P, 0)$.

m_p/M	0.3185	d/M	0.0012817	m_1/M	0.6680
x_+/M	2.68773	P_r/M	-0.0013947	m_2/M	0.3355
x_-/M	-5.20295	$P_\perp/M$	0.10695	S/M^2	0.27941
m_p/M	0.430213	x/M	3.28413	S/M^2	0.12871
P/M	0.13355	m/M	0.5066		

Fig. 1.— The recoil velocities for the SP6 configurations as measured for an observed at $r = 30M$.

centers of both holes. The Carpet code then moves these fine grids about the computational domain by following the trajectories of the two black holes. We measure the horizon spin (magnitude and direction) using the techniques detailed in Campanelli et al. (2006e).

3. RESULTS

The initial data parameters for our SP6 configuration (i.e. generic binary configuration), which were obtained using the 3PN equations of motion, are given in Table 1. Note that the binary has a small inward radial velocity (that we obtain from the post-Newtonian inspiral.) The initial orbital plane coincides with the xy plane.

We tested our code with mesh refinement by evolving the SP3 configuration of Campanelli et al. (2006e). For this test we evolved SP3 with three different grid configuration with finest resolutions of $M/32$, $M/40$, $M/52$ respectively, and 6 levels of refinement. We placed the refinement boundaries at the same coordinate distance from the punctures for each configuration. We confirmed that the waveforms converge to fourth order and agree with our unigrid SP3 evolution.

We ran the SP6 run with 7 levels of refinement, with a finest resolution of $M/43.6$. The outer boundaries were placed at $250M$. We tracked the individual horizon spins throughout the evolution and found no significant spin-up of the smaller (initially non-spinning) hole (the value of a/m at merger was $\sim 10^{-4}$). The larger black hole, on the other hand, showed a significant $45°$ angle of spin-precession, with final spin (at merger) $\vec{S}_1^{merger}/M^2 = (-0.262, 0.189, -0.214)$. During the evolution the binary performed ~ 1.8 orbits prior to the formation of the common apparent horizon (CAH). The first common horizon was detected at $t_{CAH}/M = 197.96 \pm 0.07$. The common horizon had mass $M_\mathcal{H}/M = 0.9781 \pm 0.0001$ indicating that $(2.19 \pm 0.01)\%$ of the mass was converted into gravitational radiation. The spin of the remnant horizon was $\vec{S}_{rem}/M^2 = (-0.0397 \pm 0.0005, 0.242 \pm 0.002, 0.4097 \pm 0.0002)$. The ADM angular momentum for this system is $\vec{J}_{ADM}/M^2 = (0, 0.27941, 0.56447)$, thus we predict that the radiated angular momentum is $\vec{J}_{rad}/M^2 = (0.0397 \pm 0.0005, 0.037 \pm 0.002, 0.1548 \pm 0.0002)$. The measured radiated mass and angular momentum, based on the $\ell = 2$ through $\ell = 4$ modes of ψ_4 were $E_{rad}/M = 0.0218 \pm 0.0004$ and $\vec{J}_{rad} = (0.04 \pm 0.01, 0.04 \pm 0.01, 0.16 \pm 0.01)$, which agree well with the remnant horizon parameters. Note that the agreement in $\vec{J}_{rad}$ between the horizon spin and radiation calculation indicates that our method
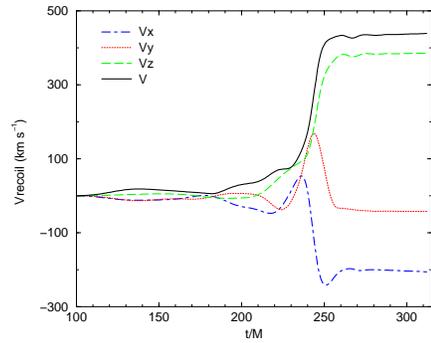
for calculating the spin direction is reasonably accurate for our choice of coordinates. The final horizon spin is flipped by $103°$ with respect to the initial spin of the larger individual horizon and $33°$ with respect to the initial orbital angular momentum.

The remnant hole acquires a significant recoil velocity of $\vec{V}_{recoil} = (-208 \pm 30, -48 \pm 7, 424 \pm 10)\,\mathrm{km\,s^{-1}}$ (see Fig. 1), which makes an angle of $27°$ with respect to the initial orbital angular momentum and $135°$ with respect to the initial spin. We measured this kick by calculating the radiated linear momentum (Campanelli and Lousto 1999) based on the $\ell = 2$ through $\ell = 4$ modes of ψ_4. We extracted these modes at $r = 25M, 30M, 35M, 40M$ and then extrapolated the radiated momenta calculated at these radii to $r = \infty$ using a linear (least-squares) fit (we excluded the initial data burst from this momentum calculation). The quoted errors in $\vec{V}_{recoil}$ are the differences between the linear extrapolation and a quadratic extrapolation. This recoil velocity of $454 \pm 25\,\mathrm{km\,s^{-1}}$ is more than double the maximum recoil velocity found for non-spinning holes (González et al. 2006) even including small eccentricity effects (Sopuerta et al. 2006). Furthermore, the spin-induced recoil in the xy plane might be offset by the mass-difference-induced recoil, potentially implying that a rotation of the spin about the z-axis may lead to a significantly larger in-plane component of the recoil velocity. Further study will be needed to determine if the mass-difference-induced recoil is (partially) aligned or counter-aligned with the spin-induced recoil.

4. DISCUSSION

We studied, for the first time using fully non-linear numerical relativity, a realistic astrophysical configuration of unequal mass, spinning black holes starting from a slightly elliptical orbit, with radial inward velocity as predicted by post-Newtonian theory, for large initial separations. Our main new result is that the spin component to the recoil velocity may produce the leading contribution. This is suggested by the fact that the z-component of the recoil velocity, which is not present for non-spinning binaries, is the dominant component. This also can be seen from the 2nd post-Newtonian expressions for the radiated linear momentum (Kidder 1995)

$$\dot{\vec{P}} = -\frac{8}{15}\frac{\mu^2 m}{r^5}\Big\{4\dot{r}(\vec{v} \times \vec{\Delta}) - 2v^2(\hat{n} \times \vec{\Delta})$$
$$- (\hat{n} \times v)\Big[3\dot{r}(\hat{n} \cdot \vec{\Delta}) + 2(\vec{v} \cdot \vec{\Delta})\Big]\Big\}, \quad (1)$$

where $\vec{x} \equiv \vec{x}_1 - \vec{x}_2$, $r \equiv |\vec{x}|$, $\vec{v} = d\vec{x}/dt$, $\hat{n} \equiv \vec{x}/r$, $\mu \equiv m_1 m_2/m$, $m = m_1 + m_2$, $\vec{\Delta} \equiv m(\vec{S}_2/m_2 - \vec{S}_1/m_1)$, and an overdot denotes d/dt.

Based on this expression we can predict that the maximum recoil velocity is reached for equal mass black holes with opposite (and maximal) spins lying on the orbital plane since all four terms add constructively to the radiated momentum. We performed one additional run, denoted by SP2, with anti-aligned spins of magnitude $a/m \pm 0.5$ lying initially along the y-axis as reported in Table 1. We obtain a $\vec{V}_{\mathrm{recoil}} = (0, 0, 1830 \pm 30)\mathrm{km\,s}^{-1}$. By rescaling this to maximal spins we obtain essentially double those values raising the maximum recoil of spinning holes to almost $4000\,\mathrm{km\,s}^{-1}$.

Equation (1) also allows us to propose an empirical formula for the total recoil velocities [4]

$$\vec{V}_{\mathrm{recoil}}(q, \vec{\alpha}) = v_m\,\hat{e}_1 + v_\perp (\cos\xi\,\hat{e}_1 + \sin\xi\,\hat{e}_2) + v_\parallel\,\hat{e}_z,$$
$$v_m = A \frac{q^2(1-q)}{(1+q)^5} \left(1 + B\,\frac{q}{(1+q)^2}\right),$$
$$v_\perp = H \frac{q^2}{(1+q)^5} \left(\alpha_2^\parallel - q\alpha_1^\parallel\right),$$
$$v_\parallel = K \cos(\Theta - \Theta_0) \frac{q^2}{(1+q)^5} \left(\alpha_2^\perp - q\alpha_1^\perp\right), \quad (2)$$

where $\vec{\alpha}_i = \vec{S}_i/m_i^2$, the index $\perp$ and $\parallel$ refer to perpendicular and parallel to the orbital angular momentum respectively, $\hat{e}_1, \hat{e}_2$ are orthogonal unit vectors in the orbital plane and ξ measures the angle between the unequal mass and spin contribution to the recoil velocity in the orbital plane. The constants H and K can be determined from newly available runs. The angle Θ is defined as the angle between the in-plane component of $\vec{\Delta}$ and the infall direction at merger. We have confirmed this $\cos\Theta$ dependence by evolving a set of runs similar to SP2 but with initial spins rotated by $\delta\Theta = \pi/4$, $\pi/2$, and π. The resulting kicks were well modeled by a $\cos(\Theta - \Theta_0)$ dependence, with $V_z = (1873 \pm 30)\mathrm{km\,s}^{-1} \cos[\delta\Theta - (0.18 \pm 0.02)]$. Note that we measured a maximum kick of $(1830 \pm 30)\mathrm{km\,s}^{-1}$ for $\delta\Theta = 0$ and $\delta\Theta = \pi$, and a minimum kick of $(352 \pm 10)\mathrm{km\,s}^{-1}$ for $\delta\Theta = -\pi/2$. This will be the subject of an upcoming paper by the authors. The total recoil velocity also acquires a correction (Sopuerta et al. 2006) for small eccentricities, e, of the form $\vec{V}_e = \vec{V}_{\mathrm{recoil}}(1 + e)$.

From González et al. (2006) $A = 1.2 \times 10^4\,\mathrm{km\,s}^{-1}$ and $B = -0.93$. From fits to Herrmann et al. (2007) and Koppitz et al. (2007) we find $H = (7.3 \pm 0.3) \times 10^3\,\mathrm{km\,s}^{-1}$ and from SP2 and Gonzalez et al. (2007) (which appeared after this paper had been submitted) we find $K \cos(\Theta - \Theta_0) = (6, -5.3) \times 10^4\,\mathrm{km\,s}^{-1}$ respectively. Note the sign difference showing some of the V_{recoil} dependence on Θ.

The in-plane recoil velocity for our SP6 configuration is consistent with Eq. (2) with $\xi \approx 88°$, however our simulation shows strong precession of the spin near merger, where a large fraction of the recoil velocity is built up, hence it is difficult to accurately determine the spin parameters $\vec{\alpha}$ and Θ to be used in Eq. (2).

[4] After completion of this work we become aware of a new paper Baker et al. (2007e) also modeling $v_\perp$

5. ASTROPHYSICAL IMPLICATIONS

A number of arguments (Shapiro 2005; Gammie et al. 2004) suggest that spins of supermassive black holes (SMBHs) are close to maximal, $a/m \gtrsim 0.8$, and perhaps as great as 0.99 (Reynolds et al. 2005). Mass ratios of binary SMBHs are poorly constrained observationally, but the luminous galaxies known to harbor SMBHs are believed to have experienced a few to several major mergers (mass ratios $0.3 \lesssim q \lesssim 1$) over their lifetimes (Haehnelt and Kauffmann 2002; Merritt 2006); mergers with $q \approx 1$ were more common in the past. Together with the results discussed above, these arguments suggest that recoil velocities for binary SMBHs in galactic nuclei are often of order $\sim 10^3\,\mathrm{km\,s}^{-1}$. Here, we consider some consequences of such large recoil velocities.

Ejection: Central escape velocities from giant elliptical galaxies and spiral galaxy bulges are $450\,\mathrm{km\,s}^{-1} \lesssim v_e \lesssim 2000\,\mathrm{km\,s}^{-1}$, dropping to $\lesssim 300\,\mathrm{km\,s}^{-1}$ in dwarf elliptical (dE) and dwarf spheroidal (dSph) galaxies (Merritt et al. 2004). Recoil velocities as large as $10^3\,\mathrm{km\,s}^{-1}$ would easily eject SMBHs from dE and dSph galaxies, and in fact there is little evidence for SMBHs in these galaxies. However, we note that the mass dependence of spin-dominated kicks, $V_{\mathrm{recoil}} \sim q^2$, implies that recoil velocities might only infrequently be as large as in the equal-mass case. If the tight empirical relations between SMBH mass and luminous galaxy properties (Ferrarese and Merritt 2000; Marconi and Hunt 2003) are to be maintained, peak recoil velocities are constrained to be $\lesssim 500\,\mathrm{km\,s}^{-1}$ (Libeskind et al. 2006); the upper limit on V_{recoil} is relaxed if most of the merger-induced growth of SMBHs took place at low redshifts ($z \lesssim 2$) when potential wells were deeper (Libeskind et al. 2006). Ejection of SMBHs from shallow potential wells at high redshift implies a maximum z at which the progenitors of present-day SMBHs could have started merging (Merritt et al. 2004), and the existence of bright quasars at $z \approx 6$ is difficult to reconcile with recoil velocities $\gtrsim 10^2\,\mathrm{km\,s}^{-1}$ unless their SMBHs grew very quickly via accretion (Haiman 2004). However we note that these and similar conclusions are based on an assumed mass ratio dependence for the kicks that is invalid if recoil velocities are dominated by spin effects.

Displacement: The long return time for a SMBH ejected near escape velocity implies a substantial probability of finding a displaced SMBH in a luminous E galaxy, especially if the latter was the site of a recent merger (Merritt et al. 2004; Madau and Quataert 2004; Vicari et al. 2006). A recoiling SMBH carries with it material that was orbiting with velocity $v \gtrsim V_{\mathrm{recoil}}$ before the kick; the size of the region containing this mass is

$$\frac{GM_\bullet}{V_{\mathrm{recoil}}^2} \approx 1\ \mathrm{pc}\ M_8 \sigma_{200}^{-2} \left(\frac{v_e}{V_{\mathrm{recoil}}}\right)^2 \quad (3)$$

where M_8 is the SMBH mass in units of $10^8 M_\odot$ and σ_{200} is the nuclear velocity dispersion in units of $200\,\mathrm{km\,s}^{-1}$. This radius is sufficient to include the inner accretion disk and the broad-line region gas, implying that a kicked BH can continue shining for some time as a quasar. Plausibility of models that explain the "naked" quasar HE 0450-2958 as an ejected SMBH (Haehnelt et al. 2006) are enhanced by the larger kick velocities found here, since

v_e from the nearby galaxy is $\gtrsim 500$ km s^{-1}; however the presence of spectral features associated with narrow emission line region gas in the quasar is still difficult to reconcile with the recoil hypothesis (Merritt et al. 2006).

Galaxy cores: The kinetic energy of a displaced SMBH is transferred to the stars in a galactic nucleus via dynamical friction, lowering the stellar density and enlarging the core before the hole returns to its central location (Merritt et al. 2004; Boylan-Kolchin et al. 2004). "Damage" to the core is maximized for $V_{\rm recoil}/v_e \approx 0.7$ (Merritt et al. 2004), hence the effect could be large even in the brightest E galaxies. Observed core masses are mostly in the range $0.5 - 1.5 M_\bullet$, consistent with the cores having been generated by binary SMBHs without the help of kicks (Merritt 2006); however a significant fraction have core masses exceeding $2 M_\bullet$ suggesting an additional contribution from recoils. Anomalously large cores in BCG's ("brightest cluster galaxies") might be explained in this way, since these galaxies have experienced the largest number of mergers; this explanation would lessen the necessity for alternatives that require BCGs to contain hypermassive BHs (Lauer et al. 2006).

Jet Directions: We measure both a significant pre-merger spin precession of $\sim 45°$ and a change in excess of $90°$ between the initial and final spin vectors, verifying the spin-flip phenomenon first discussed by Merritt and Ekers (2002). Thus, our simulation represents a possible model for the merger process responsible for generating radio sources with changing jet directions. In particular, the highly-spinning large mass black hole merging with the smaller mass non-spinning hole is a possible model for both the gradual semi-periodic deviations of the jet directions from a straight line (Komossa 2006) due to precession of the spin (and hence jet) direction, and the abrupt change in jet direction forming X-shaped patterns (Parma et al. 1985; Leahy and Parma 1992).

REFERENCES

M. Ansorg, B. Brügmann, and W. Tichy, Phys. Rev. D **70**, 064011 (2004), gr-qc/0404056.

J. G. Baker, J. Centrella, D.-I. Choi, M. Koppitz, and J. van Meter, Phys. Rev. Lett. **96**, 111102 (2006a), gr-qc/0511103.

J. G. Baker, J. Centrella, D.-I. Choi, M. Koppitz, and J. van Meter, Phys. Rev. D **73**, 104002 (2006b), gr-qc/0602026.

J. G. Baker, J. R. van Meter, S. T. McWilliams, J. Centrella, and B. J. Kelly (2006c), gr-qc/0612024.

J. G. Baker et al. (2006d), gr-qc/0612117.

J. G. Baker et al. (2006e), astro-ph/0603204.

J. G. Baker et al. (2007e), astro-ph/0702390.

T. W. Baumgarte and S. L. Shapiro, Phys. Rev. D **59**, 024007 (1999), gr-qc/9810065.

M. Boylan-Kolchin, C.-P. Ma, and E. Quataert, ApJL **613**, L37 (2004), astro-ph/0407488.

S. Brandt and B. Brügmann, Phys. Rev. Lett. **78**, 3606 (1997), gr-qc/9703066.

A. Buonanno, G. B. Cook, and F. Pretorius (2006), gr-qc/0610122.

M. Campanelli and C. O. Lousto, Phys. Rev. D **59**, 124022 (1999), gr-qc/9811019.

M. Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower, Phys. Rev. Lett. **96**, 111101 (2006a), gr-qc/0511048.

M. Campanelli, C. O. Lousto, and Y. Zlochower, Phys. Rev. D **73**, 061501(R) (2006b).

M. Campanelli, Class. Quant. Grav. **22**, S387 (2005), astro-ph/0411744.

M. Campanelli, C. O. Lousto, and Y. Zlochower, Phys. Rev. D **74**, 041501(R) (2006c), gr-qc/0604012.

M. Campanelli, C. O. Lousto, and Y. Zlochower, Phys. Rev. D **74**, 084023 (2006d), astro-ph/0608275.

M. Campanelli, C. O. Lousto, Y. Zlochower, B. Krishnan, and D. Merritt (2006e), gr-qc/0612076.

L. Ferrarese and D. Merritt, ApJL **539**, L9 (2000), astro-ph/0006053.

C. F. Gammie, S. L. Shapiro, and J. C. McKinney, ApJ **602**, 312 (2004), astro-ph/0310886.

J. A. González, U. Sperhake, B. Bruegmann, M. Hannam, and S. Husa (2006), gr-qc/0610154.

J. A. Gonzalez, M. D. Hannam, U. Sperhake, B. Brugmann, and S. Husa (2007), gr-qc/0702052.

M. G. Haehnelt and G. Kauffmann, MNRAS **336**, L61 (2002), astro-ph/0208215.

M. G. Haehnelt, M. B. Davies, and M. J. Rees, MNRAS **366**, L22 (2006), astro-ph/0511245.

Z. Haiman, ApJ **613**, 36 (2004), astro-ph/0404196.

F. Herrmann, D. Shoemaker, and P. Laguna (2006), gr-qc/0601026.

F. Herrmann, I. Hinder, D. Shoemaker, P. Laguna, and R. A. Matzner (2007), gr-qc/0701143.

L. E. Kidder, Phys. Rev. D **52**, 821 (1995), gr-qc/9506022.

S. Komossa, Memorie della Societa Astronomica Italiana **77**, 733 (2006).

M. Koppitz et al. (2007), gr-qc/0701163.

T. R. Lauer, S. M. Faber, D. Richstone, K. Gebhardt, S. Tremaine, M. Postman, A. Dressler, M. C. Aller, A. V. Filippenko, R. Green, et al., ArXiv Astrophysics e-prints (2006), astro-ph/0606739.

J. P. Leahy and P. Parma, in *Extragalactic Radio Sources. From Beams to Jets*, edited by J. Roland, H. Sol, and G. Pelletier (1992), pp. 307–+.

N. I. Libeskind, S. Cole, C. S. Frenk, and J. C. Helly, MNRAS **368**, 1381 (2006), astro-ph/0512073.

P. Madau and E. Quataert, ApJL **606**, L17 (2004), astro-ph/0403295.

A. Marconi and L. K. Hunt, ApJL **589**, L21 (2003), astro-ph/0304274.

D. Merritt and R. D. Ekers, Science **297**, 1310 (2002), astro-ph/0208001.

D. Merritt, ApJ **648**, 976 (2006), astro-ph/0603439.

D. Merritt, M. Milosavljevic, M. Favata, S. A. Hughes, and D. E. Holz, Astrophys. J. **607**, L9 (2004), astro-ph/0402057.

D. Merritt, T. Storchi-Bergmann, A. Robinson, D. Batcheldor, D. Axon, and R. Cid Fernandes, MNRAS **367**, 1746 (2006), astro-ph/0511315.

T. Nakamura, K. Oohara, and Y. Kojima, Prog. Theor. Phys. Suppl. **90**, 1 (1987).

P. Parma, R. D. Ekers, and R. Fanti, aaps **59**, 511 (1985).

F. Pretorius, Phys. Rev. Lett. **95**, 121101 (2005), gr-qc/0507014.

F. Pretorius (2006), gr-qc/0602115.

C. S. Reynolds, L. W. Brenneman, and D. Garofalo, ApSS **300**, 71 (2005), astro-ph/0410116.

E. Schnetter, S. H. Hawley, and I. Hawke, Class. Quantum Grav. **21**, 1465 (2004), gr-qc/0310042.

S. L. Shapiro, ApJ **620**, 59 (2005), astro-ph/0411156.

M. Shibata and T. Nakamura, Phys. Rev. D **52**, 5428 (1995).

C. F. Sopuerta, N. Yunes, and P. Laguna (2006), astro-ph/0611110.

A. Vicari, R. Capuzzo-Dolcetta, and D. Merritt, ArXiv Astrophysics e-prints (2006), astro-ph/0612073.

Y. Zlochower, J. G. Baker, M. Campanelli, and C. O. Lousto, Phys. Rev. D **72**, 024021 (2005), gr-qc/0505055.