9-1-1985

Analysis of vehicle suspension system subjected to random vibration

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ANALYSIS OF VEHICLE SUSPENSION SYSTEM SUBJECTED TO RANDOM VIBRATION

BY

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IN

PARTIAL FULFILLMENT FOR THE DEGREE OF

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

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ROCHESTER, NY 14623

SEPTEMBER 1985

Title of Thesis *Analysis of Vehicle Suspension System Subjected to Random Vibration*

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ACKNOWLEDGEMENT

Boundless gratitude to my advisor, Professor Dr. H. A. Ghoneim; not only for his scientific advices but also for his understanding, cooperation and encouragement.

I also wish to express my gratitude to Dr. Dinesh Shah for his constructive suggestion and support.

ABSTRACT

A two dimensional, two degree of freedom vehicle model is studied. A randomly profiled road is assumed to impart hyperbolic distributed stationary vertical random displacements to the front and the rear wheel. A computer program has been developed to evaluate transfer function matrices. The performance characteristic of the vehicle based on the Power Spectral Density of the vertical acceleration as well as the rattle space, for different values of damping factors, are studied.

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NOMENCLATURE

a, b	Distance of front and rear wheel from CG of the vehicle
A, B	Non dimensional distance of front and rear wheel from CG of the vehicle.
$A_0, A_1, \ldots A_4$	Coefficient of the polynomial D (s).
c_1, c_2	Damping coefficient of the damper of the main suspension system.
$D(\phi)$	Non Dimensional Characteristics equation of the two degree of freedom model.
f_g	Dynamic variation of the ground force.
$H(\Phi)$, $H(\gamma)$	Non Dimensional transfer function between an output and an input.
k_1, k_2	Stiffness of the main suspension system.
K	Non dimensional stiffness rate of the springs of the main suspension and the spring of the absorber, for the two degree of freedom model.
L	Distance between the front and the rear wheel.
m	Unsprung mass of the two degree of freedom model.
n	Spatial Frequency.
no	Reference Spatial Frequency.
R(t)	Autocorrelation function.
s	Laplace operator.
$S_x(w)$	Input displacement spectral density in the time domain.
$S_x(w_{nn})$	Value of input displacement spectral density at w.

$S_x(\Omega)$	Spectral density of terrain surface in spatial domain.
$S_{\ddot{x},x}(v)$	Sprung mass acceleration spectral density for an input displacement spectral density.
t	Time
V	Forward vehicle velocity.
x	Displacement of two degree of freedom vehicle model
x_1	Input displacement to the front wheel
$\dot{x}_1$	Input Velocity of the front wheel
$\ddot{x}_1$	Input Acceleration of the front wheel
x_2	Input Displacement to the rear wheel.
$\dot{x}_2$	Input Velocity of the rear wheel
$\ddot{x}_2$	Input Acceleration of the rear wheel.
X	Non dimensional displacement of two dimensional vehicle model.
X_1	Non dimensional displacement of the front wheel.
$\dot{X}_1$	Non dimensional velocity of the front wheel.
$\ddot{X}_1$	Non dimensional acceleration of the front wheel.
X_2	Non dimensional displacement of the rear wheel.
$\dot{X}_2$	Non dimensional velocity of the rear wheel.
$\ddot{X}_2$	Non dimensional acceleration of the rear wheel.

Xr1, x1	Non dimensional relative displacement of the front wheel suspension system.
Xr2, x2	Non dimensional relative displacement of the rear wheel suspension system.
γ	Non dimensional frequency.
ξ_1	Non dimensional damping factor of the front wheel suspension system of the two degree of freedom vehicle model.
ξ_2	Non dimensional damping factor of the rear wheel suspension system of the two degree of freedom vehicle model.
τ	Shift or lag of one function with respect to another.
τ_0	Time required to travel wheel base distance.
w	Input circular frequency.
Wnn	Design natural frequency of the two degree of freedom model.
max	Maximum
min	Minimum
rms	The root mean square.
PSD	Power spectral density.

INTRODUCTION

1.1 Vehicle Suspension System

Vehicle suspension synthesis can be considered in a part as an application of the mechanical isolation theory. A considerable amount of work has been performed on general vehicle suspension analysis, design, and optimization . Analysis of vehicle suspension has been largely confined to one-dimensional two degree of freedom vehicle model [1,7,9,14]. Some studies have been concerned with one-dimensional three degree of freedom model [15,17] and others have included multidegree of freedom system [3,12,17].

System disturbance are usually considered individually and are caused by guideway irregularities. They cover step input, different pulses, sinusoids [3,9,12,19] and random inputs [1,2,8,13,14,15,17,]. Hedrik and Young have considered the simultaneous effect of guideway and external force (wind gust) disturbances.

System which have been considered are usually linear except for few studies [1,3,8,15,19]. Nonlinear exponential damping and nonlinear exponential elastic restoring elements [15] have been widely used. Sometimes, Coulomb friction [3,15] and nonlinearity due to the loss of contact between tire and terrain [1,3] are considered. Unsymmetric damping has been treated by Thompson, while actual tire characteristic has been investigated by Omata [19].

Most of the previous studies select a behaviour variable as a performance criterion. These selected variables include the relative displacement as a measure of the dynamic excursion of the suspension, body acceleration as a comfort criteria, and ground force (normal force between tire and terrain) as a controllability criteria. However, perceived acceleration has been chosen as a ride comfort criterion [1,8], also, contact frequency has been proposed as guideway clearance criterion [14].

1.2 Scope of study

This study deals with the random vibration of vehicle suspension system [10,11,18]. Herein, we are adopting the technique of input-output relation for spectral densities: knowing the excitation mean square spectral density together with the linear system characteristics, we can obtain the mean square spectral density of any desired vehicle responses.

The response spectral density could serve as a basis for subjective judgement of the vehicle performance. It is also possible to integrate the spectral density to obtain the variance of the output signal. The variance is also a measure of the vehicle performance and can be used for design purposes. When integrating, however, some of the information contained in the response mean square spectral density is lost.

The vehicle model used in this study is a linear two-degree of freedom (2DOF) system. The model is excited at the front and the rear wheel. Concerning the vehicle whole body vibration, only the vertical displacement (lifting) and the pitch movement are studied. The vehicle model thus represents vehicle as well as excitation which is symmetrical with respect to its longitudinal axis. Further, it is assumed that the rear wheel follow the track of the front wheel.

2. PROBLEM FORMULATION AND SOLUTION

2.1 Mathematical Model

The equivalent diagram of an automobile (Fig. 1) shows the individual components which are relevent to vibration investigation. It has ten degree of freedom, the body has six (three in translation and three in rotation) and four degree of freedom for wheel masses which are shown in individual sprungs.

Since such a large number of degrees of freedom complicates the solution, the model is simplified to a two dimensional where only heaves and pitching are considered. In addition, the model is further simplified by neglecting the effect of the tires. Fig. 2, shows the resulting two-dimensional, two-degree of freedom model, where the body is replaced by a sprung mass and the suspension system is represented by massless elements (spring and dashpot) providing forces between the body and the roadway directly.

The mathematical model considered is linear and about the position of static equilibrium and from rest, this includes the restriction that no separation is allowed between the tire and the terrain. Linearity assumption is not very severe since many systems are inherently quite linear over a wide operational range and certain types of nonlinearities such as Coulomb friction are often undesirable, and should be minimized in any advanced

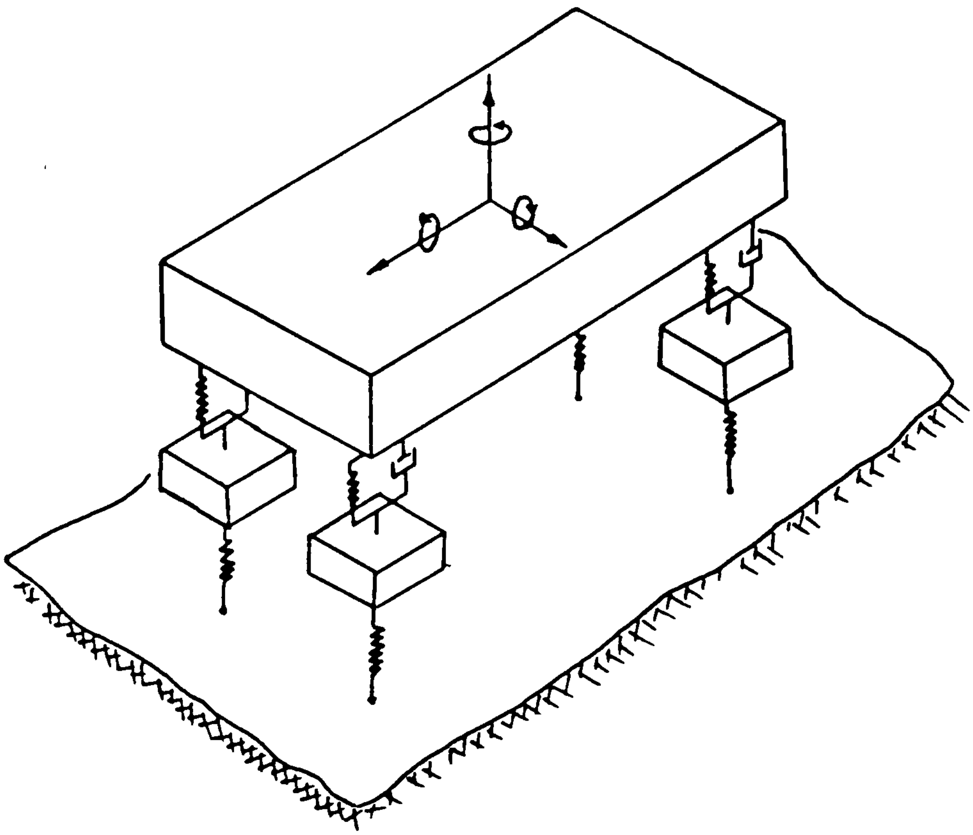

Fig.1. Equivalent diagram of a vehicle

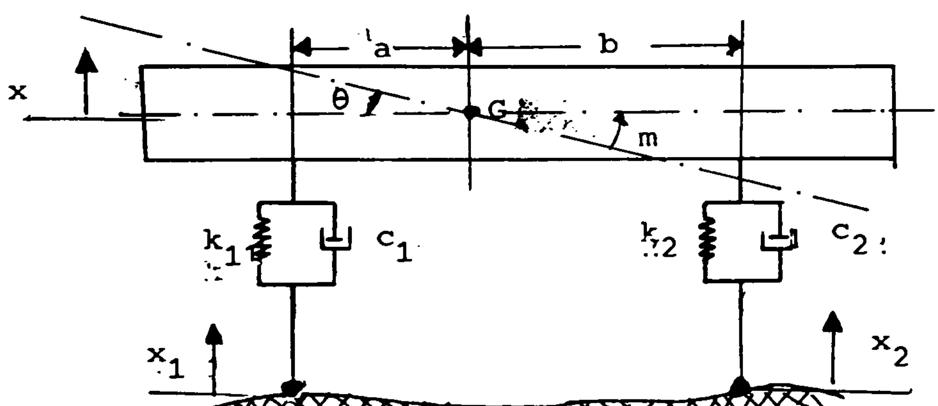

Fig. 2, Two-Dimensional, two degree of freedom vehicle model.

isolator design.

Now the mathematical representation of the model is considered.

The equation of motion of the system shown in Fig. 2 are:

$$m\ddot{x} + (k_1 + k_2)x + (c_1 + c_2)\dot{x} + (c_2 b - c_1 a)\dot{\theta} +$$

$$(k_2 b - k_1 a)\theta = c_1\dot{x}_1 + c_2\dot{x}_2 + k_1 x_1 + k_2 x_2 \qquad (1)$$

$$J\ddot{\theta} + (c_2 b - c_1 a)\dot{x} + (k_2 b - k_1 a)x + (c_2 b^2 + c_1 a^2)\dot{\theta}$$

$$+ (k_2 b^2 + k_1 a^2)\theta = (c_2\dot{x}_2 + k_2 x_2)b -$$

$$(c_1\dot{x}_1 + k_1 x_1)a \qquad (2)$$

For non dimensionalization we consider the following non dimensional design parameters: using t = t*/Wnn:

$$I = J/mx_o^2 \;,\quad W_{nn} = \sqrt{k_1/m} \;,\quad A = a/x_o \;,\; B = b/x_o$$

$$X = x/x_o \;,\quad X_1 = x_1/x_o \;,\quad X_2 = x_2/x_o$$

$$\dot{X} = \dot{x}/x_o W_{nn} \;,\quad \dot{X}_1 = \dot{x}_1/x_o W_{nn} \;,$$

$$\dot{X}_2 = \dot{x}_2/x_o W_{nn}$$

$$\ddot{X} = \ddot{x}/x_o W_{nn}^2 \;,\quad \ddot{X}_1 = \ddot{x}_1/x_o W_{nn}^2 \;,$$

$$\ddot{X}_2 = \ddot{x}_2/x_o W_{nn}^2$$

$$K = k_2/k_1 \;,\quad \xi_1 = c_1/2mW_{nn} \;,\quad \xi_2 = c_2/2mW_{nn} \qquad (3)$$

Where x_o is a length related to the input magnitude and W_{nn} is a natural design frequency.

Taking the Laplace transform of (1) and (2) and using the nondimensionalized quantities, we obtain (See Appendix I):

$$(\phi^2 + 2(\xi_1 + \xi_2)\phi + K + 1)X + ((2(\xi_2 B - \xi_1 A)\phi + KB - A)\theta + (2\xi_1\phi + 1)X_1 + (2\xi_2\phi + K)X_2 \tag{4}$$

$$(J\phi^2 + (2\xi_2 B^2 + 2\xi_1 A^2)\phi + KB^2 + A^2)\theta + (2(B\xi_2 - A\xi_1)\phi + KB - A)X = (2\xi_2\phi + K)BX_2 - (2\xi_1\phi + 1)AX_1 \tag{5}$$

Where

$$\phi = s/Wnn$$

Combining (4) and (5) in matrix form, we obtain:

$$[A(\phi)]Y_1 = [Q(\phi)]Y] \tag{6}$$

Where

$$[A(\phi)] = \begin{bmatrix} \phi^2 + 2(\xi_1 + \xi_2) + K + 1 & (2B\xi_2 - 2A\xi_1) + KB - A \\ (2B\xi_2 - 2A\xi_1) + KB - A & I\phi^2 + 2(B^2\xi_2 + A^2\xi_1) + KB^2 + A^2 \end{bmatrix} \tag{6a}$$

$$[Q(\phi)] = \begin{bmatrix} (2\xi_1\phi + 1) & (2\xi_2\phi + K) \\ -(2\xi_1\phi + 1)A & (2\xi_2\phi + K)B \end{bmatrix} \tag{6b}$$

$$Y_1 = \begin{bmatrix} X \\ \theta \end{bmatrix}, \qquad Y = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (6c)$$

2.2 Vehicle Disturbance

In actual enviourment, vehicle suspension systems are subjected to multiple input-disturbances. However, in this study, emphasis is placed on guideway disturbances resulting from terrain irregularity. This guideway disturbance is treated as random and is described by its Power Spectral Density (PSD). Experimental data shows that, for wide range of surfaces the spectrum may be well approximated by a hyperbolic displacement density function [10,11]:

$$S(\Omega) = A/\Omega^2 \qquad (7)$$

where

A is roughness parameter (m)

and

Ω is the wave number (rad/m), representing the number of waves in unit distance [16]

However applying this form of spectrum, it should be kept in mind that there will be some situation in which (A/Ω^2) would not fit the real roadway spectrum very well. Also for a very long or short wave length (A/Ω^2) may represent extrapolation of data, which may not very well be justified. However, the use of such form has the great advantage that all roadways - smooth or rough are represented by a single parameter A, thus rather general preliminary design studies may be made which should have a wide application.

If a vehicle traverses the surface with a constant forward velocity V, and if w is the circular frequency in time, the height of roadway under the vehicle [18] may be described by a random process in time, i.e.,

$$\Omega V = w \tag{8}$$

The spectral density in spatial domain may be converted to a spectral density in the time domain [11] :

$$Sx(\Omega)d = Sx(w)dw \tag{9}$$

From (7) and (8) we obtain :

$$Sx(w) = AV/w^2 \tag{9a}$$

The nondimensional form of (9a) is

$$Sx(\gamma) = Sx(w)/Sx(Wnn) = 1/\gamma^2 \tag{9b}$$

where

$$\gamma = w/W_{nn}$$

and

$$S_x(W_{nn}) = AV/W_{nn}^2$$

It might be worth mentioning that by using a similar expression in (9), it can be shown that for a hyperbolic displacement, input spectral density is

$$X_o = \sqrt{AV/W_{nn}} \qquad (10)$$

More discussion of the vehicle disturbance is presented in section 3.

2.3 Behaviour Variable Representation

Since the input to the vehicle is considered as a random excitation, the behaviour variables of the response are expected to be random as well, consequently, the response can only be described in terms of any of the following statistical parameters

% Autocorrelation function

% Vibration spectrum, continuous in frequency, expressed as spectral density.

% RMS value

% Amplitude probability distribution, expressed as a probability density.

2.3.1 Autocorrelation

The autocorrelation function for a random process x(t) is defined as the average value of the product $x(t)x(t+\tau)$. The process is sampled at time t and then again at time $t+\tau$, (Fig. 3), and the average value of the product, $E[x(t)x(t+\tau)]$ is calculated for the ensemble [18], provided that the process is stationary, the value of $E[x(t)x(t+\tau)]$ will be independent of absolute time t and will depend only on time seperation so that we may have :

$$R_x(\tau) = E[x(t)x(t+\tau)] \tag{11}$$

where

$R_x(\tau)$ is the autocorrelation function for the random process x(t)

If the time lag τ is brought to zero, then

$$R_x(o) = E[x^2(t)] \tag{12}$$

2.3.2 Spectral Density

The autocorrelation function provides information concerning properties of a random process in the time domain. The spectral density is the Fourier transform of the autocorrelation function [16], which means that the spectral density provides information in the frequency domain. It provides measure of amplitude of the response over the frequency domain.

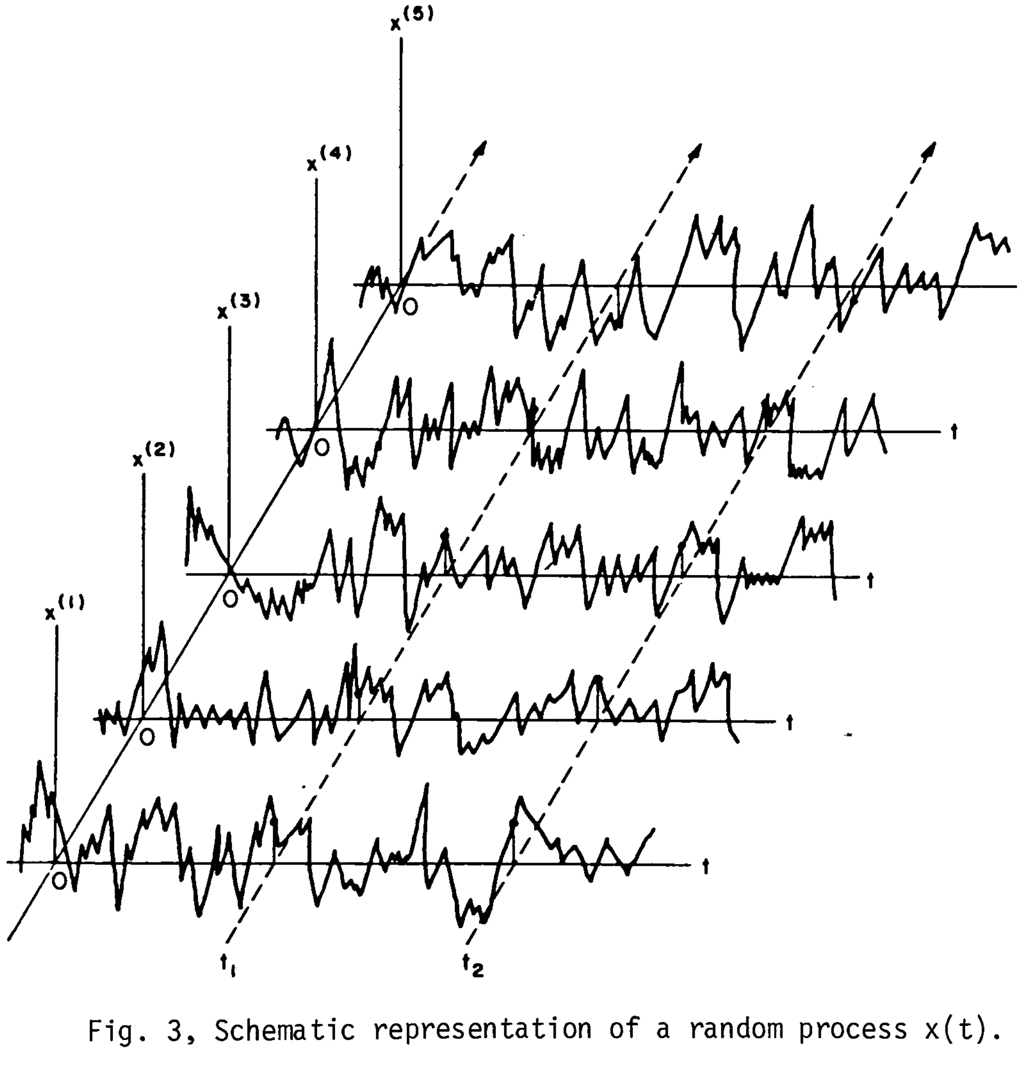

Fig. 3, Schematic representation of a random process x(t).

For stationary ergodic random process, ensemble averages and sample average are same, Hence from (11), we obtain :

$$R_x(\tau) = E[x(t)x(t+\tau)] = \lim_{T\to\infty} \frac{1}{T}\int_{t=-T/2}^{t=T/2} x(t)x(t+\tau)]dt \tag{13}$$

Taking Fourier transform of $R_x(\tau)$, we obtain :

$$S_x(w) = 1/2\pi \int_{-\infty}^{\infty} R_x(\tau)e^{iw\tau}\, d\tau \tag{14}$$

and

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(w)e^{iw\tau}\, dw \tag{15}$$

where

S(w) is the spectral density of the random process x(t) and

For time lag $\tau = 0$, relation (15) transform into]

$$R_x(0) = \int_{-\infty}^{\infty} S_x(w)dw \tag{16}$$

Combining (12) and (16), we obtain :

$$E[x^2(t)] = \int_{-\infty}^{\infty} S_x(w)dw \tag{17}$$

2.3.3 RMS Value

The mean square value of a random variable provides a measure of the energy associated with the vibration described by the variables. The positive square root of the mean square value is known as the root mean square or RMS.

RMS magnitude may be considered as the most convenient statistical parameter that can be selected for the behaviour variable represention, since it is the only statistical quantitative parameter and also because, generally, all other statistical parameters can be expressed in terms of the RMS.

However, when the behaviour variable is strongly frequency dependent, we therefore need to know more about the frequency distribution, i.e., it would be advisable to study the PSD together with the RMS value as representatives of the behaviour variables.

2.4 Vehicle Performance

The vehicle performance and vibration characteristics ingeneral, are obtained based on the following evaluation criteria.

% Wheel Controllability and Ride Comfort

% Ride Comfort

% Allowable Dynamic Excursion

2.4.1 Wheel Controllability and Ride Comfort

The dynamic variation of the normal force between the tire of the wheel and the terrain (ground force), in general, determines the tire terrain contact area during normal operation, consequently, the vehicle controllability can be indicated by the tire-terrain normal force.

Also, since the increase of the variation of the tire-terrain dynamic force relative to the static force would increase the change in wheel load, and consequently the possibility of the wheel to leave the road, thus the ground force variation of the the static wheel force could be considered as well a good indicator of safety.

It should be pointed out that in this study considering the safety is irrelevant since the tires are neglected.

2.4.2 Ride Comfort

This is one of the basic goal to be provided by vehicle suspensions, particularly in case of passenger's cars. The use of integral square of vehicle body acceleration in case of deterministic inputs, such as, isolated bumps and obstacles, has the advantage of giving greater weight to the large values acceleration experienced during the subsequence decay of the oscillation. In addition, it has been shown that force transmitted to the human body is, in the case of vertical vibration, an approximate measure of discomfort. At frequency upto 5Hz the force transmitted is about the same as if human body were replaced by a pure mass. In this range, therefore, the acceleration of the passenger may be taken as proportional to the force transmission and hence as a measure of discomfort. However, human vibration sensitivity depends not only on the amplitude of vibratory acceleration, but also on its frequency. Consequently, the PSD of the vehicle acceleration would have the same weight as the RMS value.

2.4.3 Dynamic Excursion

This indicates the allowable clearance space between suspension components. Large clearance space are not only undesirable from the volume economy point of view, but also because they often amplify moments arising from multidirectional forces.

The RMS relative displacement between the suspension components is chosen as a suitable behaviour variable to represent the suspension clearance space.

3. EVALUATION OF VEHICLE RESPONSES.

3.1 Vehicle Excitation

The road surfaces overrun by a vehicle is always more or less irregular. Measurements of the surfaces show that its profile can randomly be described in statistical terms. Measurement from larger number of roads have been complied in [6]. It is found that irregularities of many roads approximately can be described by the vertical amplitude mean square spectral density S(n), the form of S(n) as a function of the spatial frequency variable n (n > 0) is taken as [4,5,7]:

$$S(n) = S(n_o)(n/n_o)^{-\beta} \tag{18}$$

where

$S(n_o)$ is the vertical amplitude mean square density (roughness coefficient) at the reference spatial frequency n_o ($n_o = 1/2\pi$ cycles/m).

The measurement for typical road surfaces can be classed into various groups and the parameter values for each groups are given in Table 1 [6,7]. The exponent β has two values, depending on whether $n < n_o$ or $n > n_o$. It is found that $\beta = 2.0$ and $\beta = 1.5$

TABLE 1

Classification of roads [5] *based on road spectra presented by MIRA* [4]

$$n_0 = \frac{1}{2\pi} \text{ cycle/m}$$

Road class		$\mathscr{S}(n_0)$† Range	w_1 Mean	w_1 Standard deviation	w_2 Mean	w_2 Standard deviation
Motorways	Very good	2–8	1·945	0·464	1·360	0·221
	Good	8–32				
Principal roads	Very good	2–8				
	Good	8–32	2·05	0·487	1·440	0·266
	Average	32–128				
	Poor	128–512				
Minor roads	Average	32–128				
	Poor	128–512	2·28	0·534	1·428	0·263
	Very poor	512–2048				

† $\mathscr{S}(n)$ measured in units of 10^{-6} m^3/cycle.

are reasonable values for many road surfaces as shown in Table 1. It has been assumed that the random process giving S(n) is stationary.

When travelling with the speed V on a road described by the mean square spectral density according to [18], the vehicle input spectral density will be [5,6]

$$S(f) = S(n)/V \tag{19}$$

where

$f = Vn$ (frequency in Hz)

Introducing the angular frequency $w = 2\pi f$, one finds that the vehicle excitation has the two-side mean square spectral density

$$S(w) = S(w_o)(w_o/w)^2 \tag{20}$$

for $|w| \leq w_o = 2\pi n_o$ (20a)

and

$$S(w) = S(w_o)(w_o/w)^{1.5} \tag{20b}$$

for $|w| \geq w_o = 2\pi n_o$ (20c)

where

$$S(w_o) = S(n_o)/4\pi V$$

It should be pointed out that, as mentioned before, β is taken as 2 throughout the entire spectra.

Since the front and rear wheels follows the same track, they have the same excitation which means

$$S_F(w) = S_R(w) = S(w) \tag{21}$$

When studying responses to multiple excitation, also cross - spectral densities $S_{k,l}(w)$, $k \neq l$, are needed. It can be shown that ($i = \sqrt{-1}$)

$$S_{12}(w) = S(w)e^{-iw\tau_0} \tag{22}$$

$$S_{21}(w) = S(w)e^{iw\tau_0} \tag{22a}$$

where

$$\tau_0 = L/V \tag{22b}$$

is the time needed to travel the wheel base distance L

Combining (20) and (22), the excitation of two wheel vehicle model can be summarized in matrix form as

$$[S(w)] = S(w)\begin{bmatrix} 1 & e^{-iw\tau_0} \\ e^{iw\tau_0} & 1 \end{bmatrix} \tag{23}$$

In nondimensional form, we have

$$[S(\gamma)] = \frac{1}{\gamma^2}\begin{bmatrix} 1 & e^{-i\gamma\gamma_0} \\ e^{i\gamma\gamma_0} & 1 \end{bmatrix} \tag{23a}$$

where

$$\gamma_0 = LW_{nn}/V$$

3.2 Acceleration Response

When using the technique of input-output relation for spectral densities, harmonic transfer functions are needed. These harmonic transfer functions are calculated as follows :

$$[S(\gamma)] = [H(\gamma)][S(\gamma)][H^{*}(\gamma)]^{T} \tag{24}$$

where

$[H(\gamma)]$ is the nondimensional transfer function matrix between the output and the input excitation, and is expressed in matrix forem as :

$$[H(\gamma)] = \begin{bmatrix} H_{xx1} & H_{xx2} \\ H_{\theta x1} & H_{\theta x2} \end{bmatrix} \tag{24a}$$

$[S(\gamma)]$ is the nondimensional input spectral density,

$$[S(\gamma)] = \begin{bmatrix} S_{xx}(\gamma) & S_{x\theta}(\gamma) \\ S_{\theta x}(\gamma) & S_{\theta\theta}(\gamma) \end{bmatrix} \tag{24b}$$

and

$[H^*(\gamma)]$ is the nondimensional conjugate transfer function matrix of $[H(\gamma)]$

Taking (4) into consideration and after some manipulation, (24) can be expressed as :

$$S_{xx} = H_{xx1} S_{xx} H^*_{xx1} + H_{xx1} S_{x\theta} H^*_{\theta x1} + H_{\theta x1} S_{\theta x} H^*_{xx1} + H_{\theta x1} S_{\theta\theta} H^*_{\theta x1} \tag{24c}$$

$$S_{x\theta} = H_{xx1} S_{xx} H^*_{xx2} + H_{xx1} S_{x\theta} H^*_{\theta x2} + H_{\theta x1} S_{\theta x} H^*_{xx2} + H_{\theta x1} S_{\theta\theta} H^*_{\theta x2} \tag{24d}$$

$$S_{\theta x} = H_{xx2} S_{xx} H^*_{xx1} + H_{xx2} S_{x\theta} H^*_{\theta x1} + H_{\theta x2} S_{\theta x} H^*_{xx1} + H_{\theta x2} S_{\theta\theta} H^*_{\theta x1} \tag{24e}$$

$$S_{\theta\theta} = H_{xx2} S_{xx} H^{*}_{xx2} + H_{xx2} S_{x\theta} H^{*}_{\theta x2} + H_{\theta x2} S_{\theta x} H^{*}_{xx2} + H_{\theta x2} S_{\theta\theta} H^{*}_{\theta x2} \tag{24f}$$

where

$$H_{xx1} = \frac{(1 + i2\xi_1\gamma)(KB(A+B) - I\gamma^2 + i2\xi_2 B(A+B)\gamma)]}{D(\gamma)}$$

$$H_{\theta x1} = \frac{(K + i2\xi_2\gamma)(A(A+B) - I\gamma^2 + i2\xi_1 A(A+B)\gamma)]}{D(\gamma)}$$

$$H_{xx2} = \frac{-(1 + i2\xi_1\gamma)(K(A+B) - A\gamma^2 + i2\xi_2(A+B)\gamma)]}{D(\gamma)}$$

$$H_{\theta x2} = \frac{(K + i2\xi_2\gamma)(A + B - B\gamma^2 + i2\xi_1(A+B)\gamma)]}{D(\gamma)}$$

$$S_{xx} = S_x(\gamma) \quad , \quad S_{\theta\theta} = S_x(\gamma)$$

$$S_{x\theta} = S_x(\gamma)e^{-i\gamma\gamma_0} \quad , \quad S_{\theta x} = S_x(\gamma)e^{i\gamma\gamma_0}$$

and

$$D(\gamma) = (A_4 r^4 - A_2 r^2 + A_0) + i(A_1 r - A_3 r^3) \tag{24h}$$

where

$$A_4 = I$$

$$A_3 = 2[\xi_2(B^2 + I) + \xi_1(A^2 + I)]$$

$$A_2 = 4\xi_1\xi_2(A + B)^2 + I(K + 1) + KB^2 + A^2$$

$$A_1 = 2[(K\xi_1 + \xi_2)(A + B)^2]$$

$$A_0 = K(A + B)^2$$

(24j)

Once the S_{xx}, $S_{\theta\theta}$ are determined, the corresponding PSD of the acceleration, $S_{\ddot{x}\ddot{x}}$ and $S_{\ddot{\theta}\ddot{\theta}}$, can be directly obtained, for different parameter values, as follows :

$$S_{\ddot{x}\ddot{x}} = \gamma^4 S_{xx} \tag{25}$$

$$S_{\ddot{\theta}\ddot{\theta}} = \gamma^4 S_{\theta\theta} \tag{26}$$

3.3 Rattle Space Response

The rattle space is defined as the relative distance between the suspension components. For acceptable design, namely space requirements and mechanical stability, the RMS rattle space must be below a certain limit. From (4), we obtain:

$$Y_1 = [H(\gamma)]Y$$

and the rattle spaces, $X_{r1,x1}$ and $X_{r2,x2}$ are,

$$X_{r1,x1} = [(X - A\theta) - X_1]$$

$$X_{r2,x2} = [(X + B\theta) - X_2] \qquad (27)$$

where

$$X = (H_{xx1} X_{x1} + H_{\theta x1} X_2)$$

$$\theta = (H_{xx2} X_{x1} + H_{\theta x2} X_2) \qquad (28)$$

Combining (27) and (28) in matrix form, we get:

$$[X_{r1,x1} \quad X_{r2,x2}]^T = [H(\gamma)][X_1 \quad X_2]^T \qquad (29)$$

where

$$[H(\gamma)] = \begin{bmatrix} H_{xx1} - AH_{xx2} - 1 & H_{\theta x1} - AH_{\theta x2} \\ H_{xx1} + BH_{xx2} & H_{\theta x1} + BH_{\theta x2} - 1 \end{bmatrix} \qquad (29a)$$

To calculate spectral density of the rattle space, similar procedures to that used for evaluating S_{xx} and $S_{\theta\theta}$ are adopted :

$$S_{rx1} = H_{r,xx1} S_{xx} H^{*}_{r,xx1} + H_{r,xx1} S_{x\theta} H^{*}_{r,\theta x1} + H_{r,\theta x1} S_{\theta x} H^{*}_{r,xx1} + H_{r,\theta x1} S_{\theta\theta} H^{*}_{r,\theta x1} \qquad (30)$$

$$S_{rx2} = H_{r,xx2} S_{xx} H^{*}_{r,xx2} + H_{r,xx2} S_{x\theta} H^{*}_{r,\theta x2} + H_{r,\theta x2} S_{\theta x} H^{*}_{r,xx2} + H_{r,\theta x2} S_{\theta\theta} H^{*}_{r,\theta x2} \qquad (31)$$

where

$$H_{r,xx1} = H_{xx1} - AH_{xx2} - 1$$

$$H_{r,\theta x1} = H_{\theta x1} - AH_{\theta x2}$$

$$H_{r,xx2} = H_{xx1} + BH_{xx2}$$

$$H_{r,\theta x2} = H_{\theta x1} + BH_{\theta x2} - 1 \qquad (31a)$$

From (30) and (31), the PSD of the rattle space, as in the case of acceleration, can be evaluated for different values of the design parameters.

3.4 Numerical Method to Evaluate Vehicle Responses

The previous analysis of evaluating the PSD's can be completely accomplised numerically. The outline of numerical procedure are presented in this section :

From (4), we obtain :

$$[A(\phi)]Y_1 = [Q(\phi)]Y$$

or, in the nondimensional frequency domain ($\gamma = i\phi$):

$$[A(\gamma)]Y_1 = [Q(\gamma)]Y \tag{32}$$

The matrix $[A(\gamma)]$ as well as matrix $[Q(\gamma)]$ is complex. In order to obtain the transfer function matrix $[H(\gamma)]$, which is also a complex matrix, determine the inverse of the matrix $[Q(\gamma)]$ using IMSL-Routine LEQTIC. Then multiply $[Q(\gamma)]^{-1}$ matrix by $[A(\gamma)]$)], to obtain [M], i.e.,

$$[Q(\gamma)]^{-1}[A(\gamma)]Y_1 = [M]Y_1 = Y \tag{33}$$

where

$$[M] = [Q(\gamma)]^{-1}[A(\gamma)] \tag{33a}$$

Relation (33) represents a system of complex equations. Solution to these equations gives the transfer function.

In order to find transfer functions, H_{xx1}, $H_{\theta x1}$, H_{xx2}, and $H_{\theta x2}$, the IMSL Routine LEQTIC is used once again to solve the system of complex equations (33). This routine is used twice to get four transfer functions :

(1) with $Y^T = [1,0]$ to get H_{xx1} and $H_{\theta x1}$

and

(2) with $Y^T = (0,1)$ to get H_{xx2} and $H_{\theta x2}$

This technique is very effective, since it reduces much of the tedious analytical work, and can be extended to solve systems with higher degrees of freedom. The computer program is given in Appendix.

4 RESULTS AND DISCUSSION

Following are the system parameters which are use in this paper for a calculation of spectral density:

Generalised mass m = 1000 Kg.

Spring Stiffness k1 = k2 = 50,000 N/m

Damper Stiffness c1 = c2 = c = 2000 N s/m, 4000 N s/m ,6000 N s/m

Distances a = 1.2 m

b = 1.3 m

L = 2.5 m

Forward Vehicle Velocity V = 20 m/Sec.

The power spectral density (PSD) of the acceleration Sxx, for different values of damping factor, C, are obtained, plotted vs γ, and shown in Fig. 4. Ingeneral, the PSD is consisting of two parts :

(1) " Bell Shaped ", for γ < 3.0, and

(2) a fluctuating part (demonstrated in Fig. 5). The bell shaped portion of the plot is due to the characteristic of the selected two degree of freedom vehicle model, and the fluctuating

characteristic is due to the delay time between the two correlated inputs. From Fig. 3, it is clear that increasing C flattens out the first peaks at the vicinity of the resonant frequency, but increases the peaks of fluctuation at higher modes of frequency. Fig. 6, depicts the two peaks of the corresponding resonant frequency. These two peaks are not distinguished in Fig.3, because of higher values of C.

The PSD of the rattle space $S_{r1,x1}$ and $S_{r2,x2}$, are shown in Fig. 7 and Fig. 8 respectively. It is clear that increasing C, decreases the peaks at the vicinity of the resonant frequency. Ingeneral, increasing C, reduces the rattle space requirement at the vicinity of the resonant frequency. However, over the whole spectrum of frequencies, another criterion such as the Root Mean square (RMS) is needed to find out the effect of C.

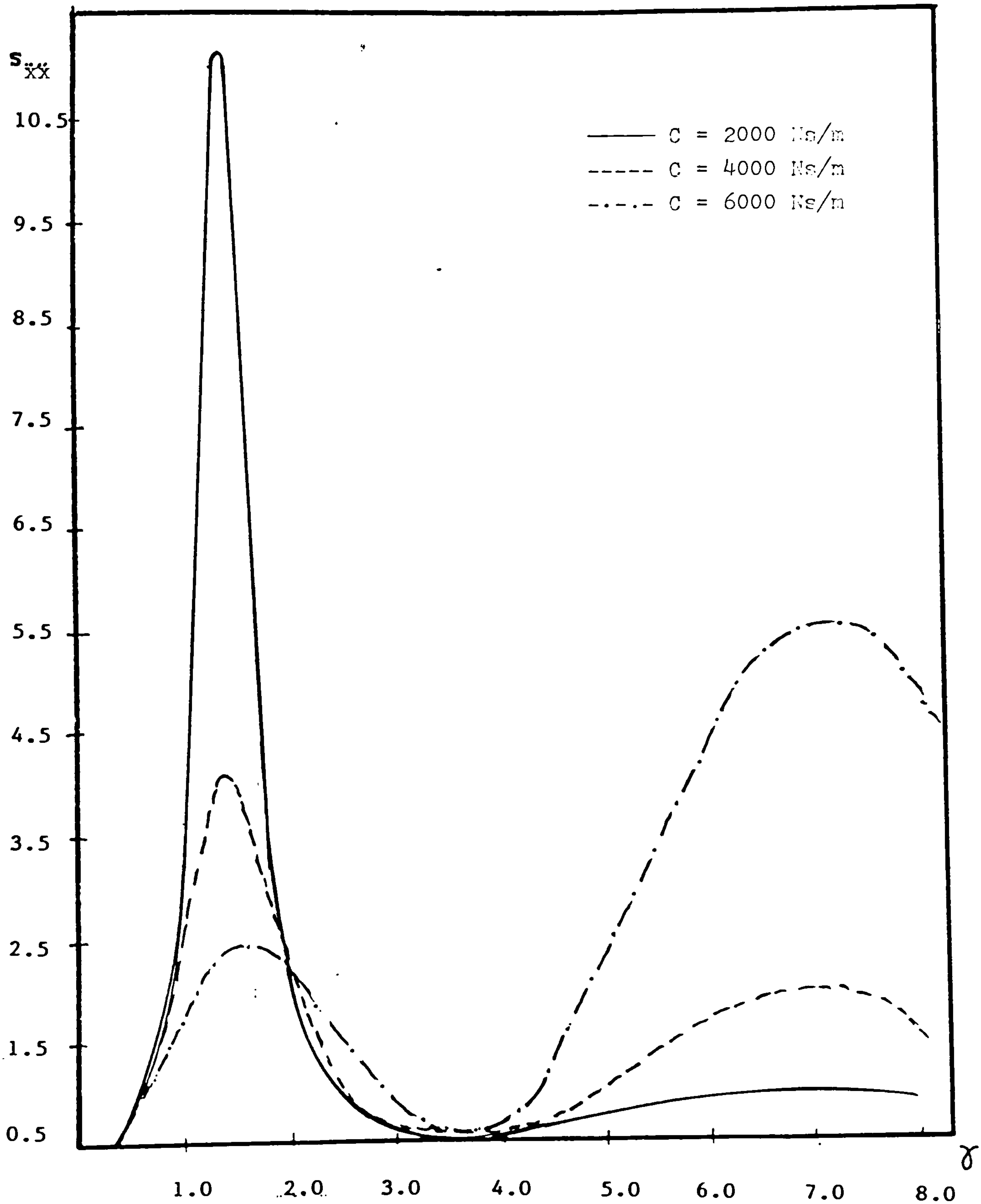

Fig. 4. Spectral Density of Acceleration $S_{\ddot{x}\ddot{x}}$

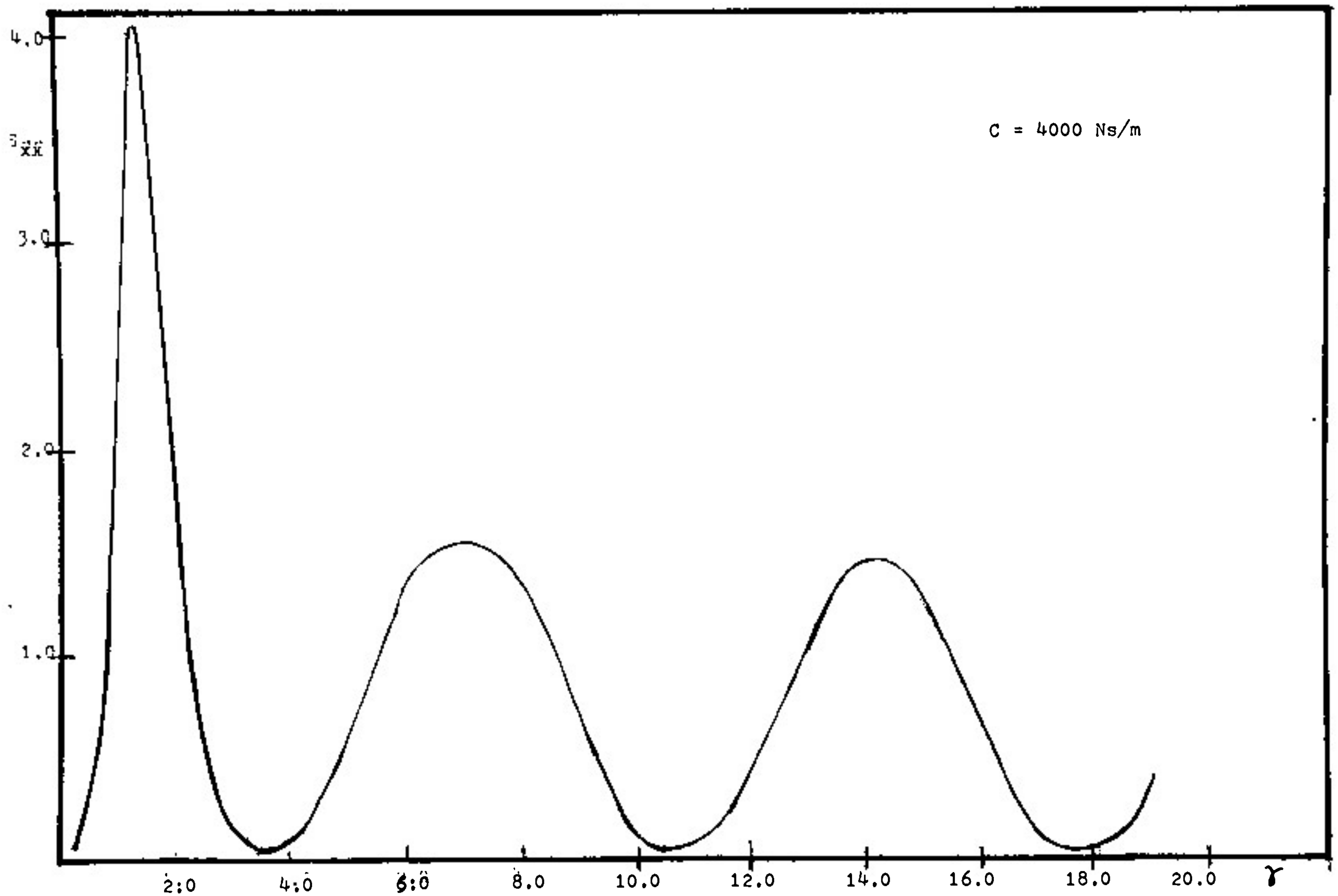

Fig. 5. Spectral Density of Acceleration S_{xx}

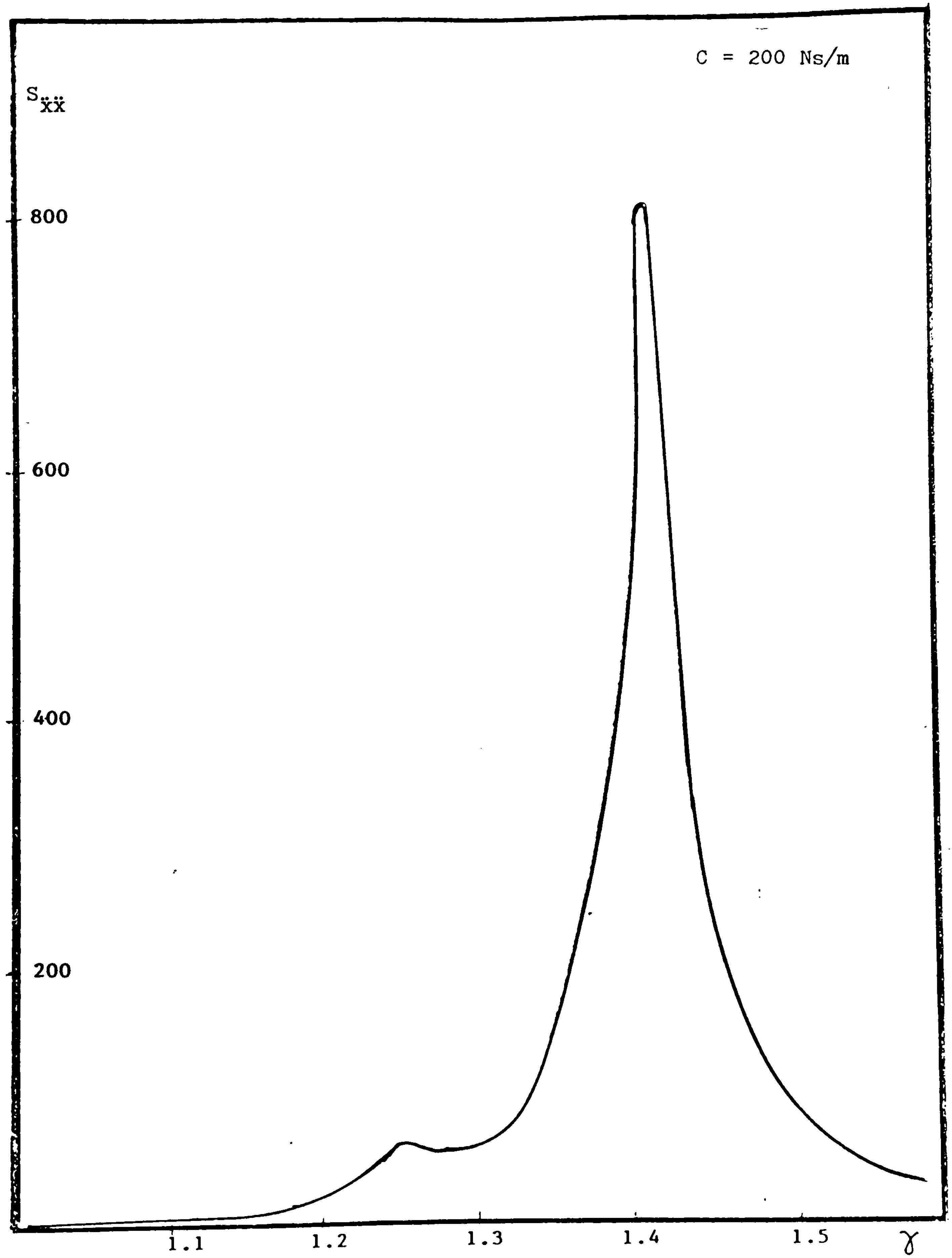

Fig. 6, Spectral Density of Acceleration $S_{\ddot{x}\ddot{x}}$

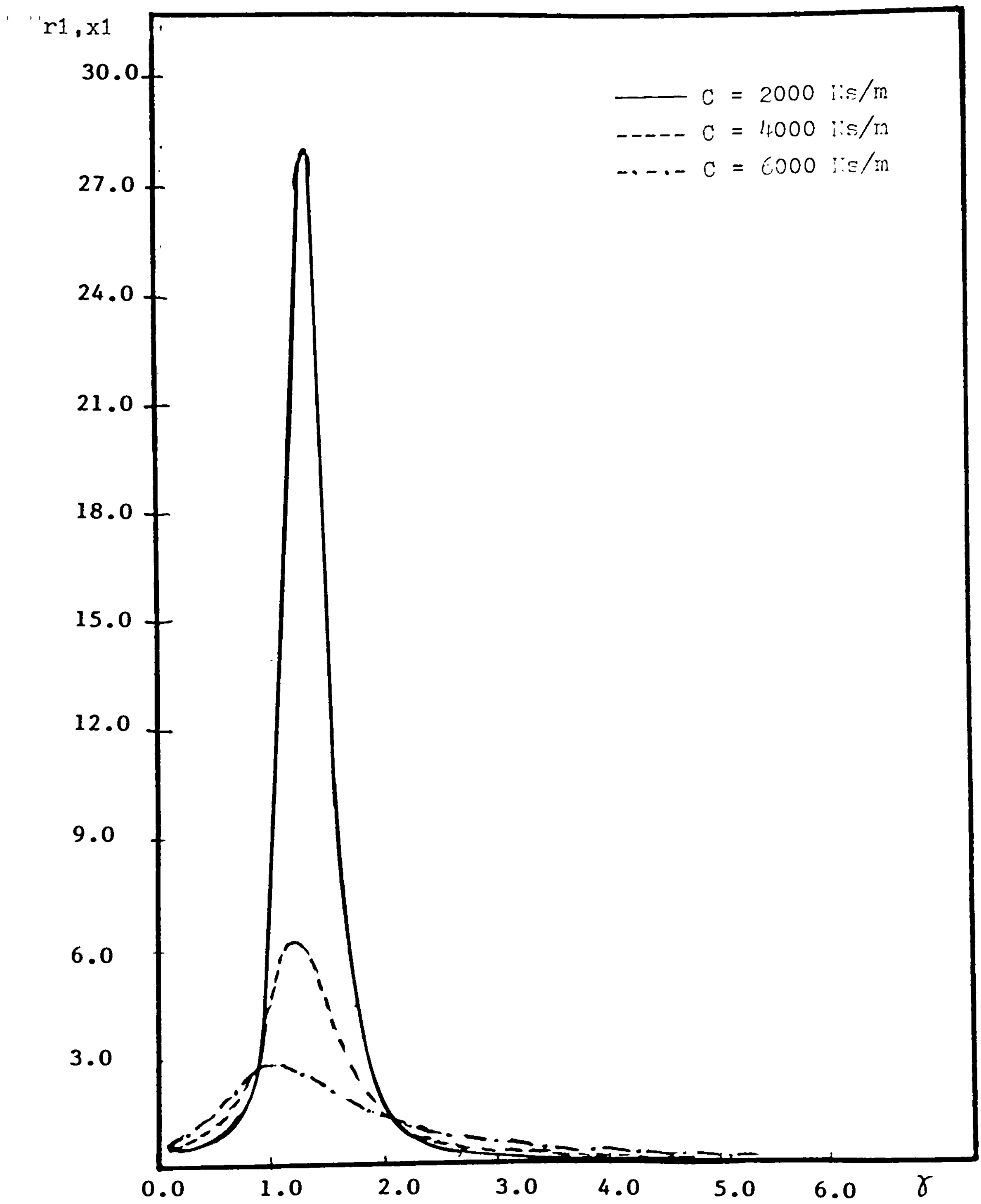

Fig. 7, Spectral Density of Rattle Space $S_{r1,x1}$

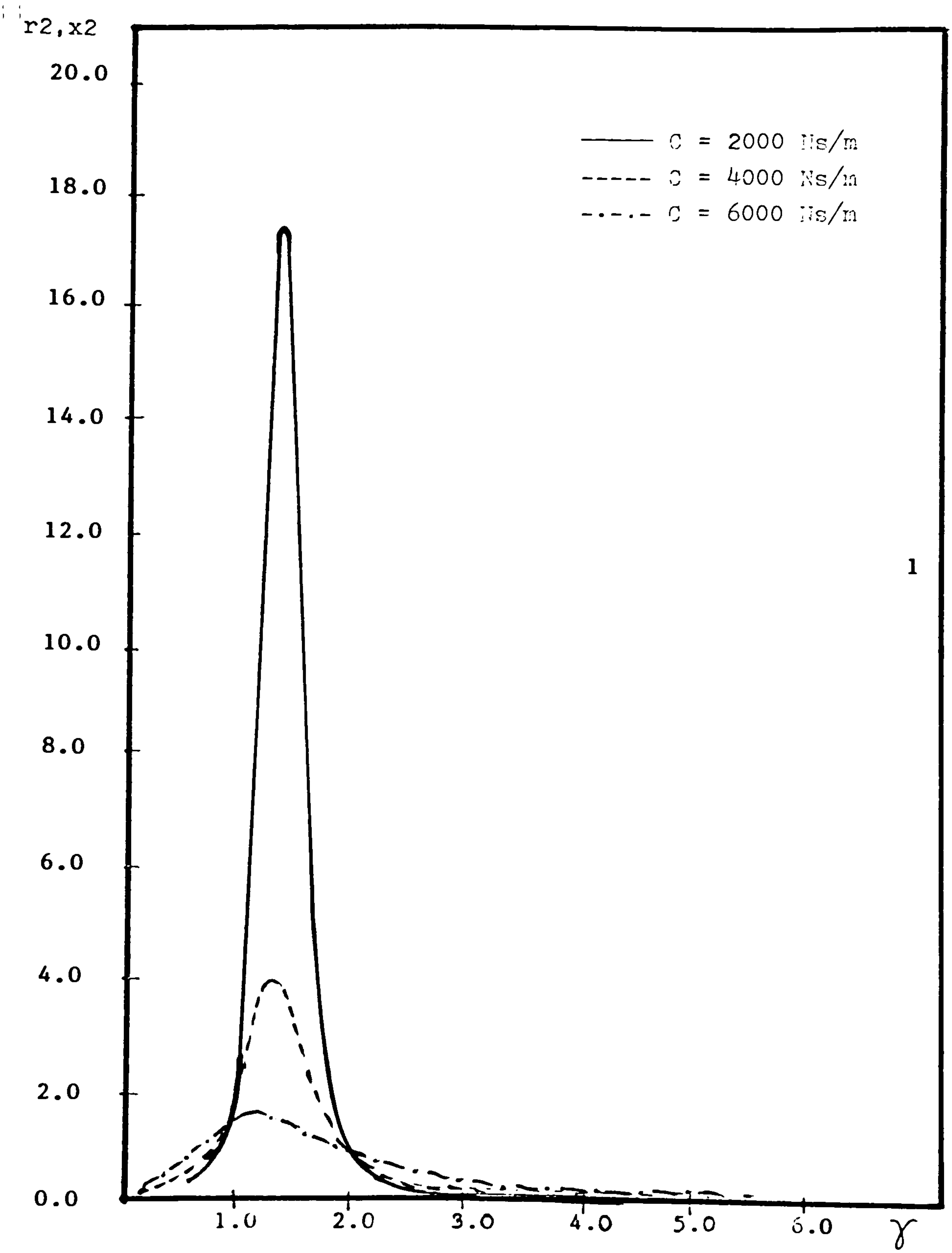

Fig. 8, Spectral Density Of Rattle Space $S_{r2,x2}$

CONCLUSION

The PSD of the acceleration exhibits a fluctuating portion at higher frequencies, which becomes more pronounced with increasing the damping factor C. This fluctuation is due to the delay time between the two correlating inputs at the front and the rear wheel. This phenomenon does not occur in real vehicle vibration, because of the filtering effect of tires. The presence of tires, ingeneral, damps out the amplitude of vibration at higher frequencies.

Also, from the results of the PSD of both acceleration, excluding the fluctuating part, and the rattle space, it is clear that increasing the damping factor C, decreases the peak at the vicinity of the natural frequencies. However, over the whole spectrum (e.g. the area under the curve), increasing C, does not necessarily decreases the acceleration and or rattle space. Ingeneral, there is an optimum value for C, which will result in the best performance, i.e., minimum acceleration and rattle space over the entire spectrum.

Two approaches are presented in this study : Analytical and Numerical. The analytical approach is accurate and requires a thorough understanding of all the details of the problems. However, when dealing with higher degrees of freedom model, the analytical approach is formidable and one has to adopt the numerical technique.

Finally, this study can be extended to more sophisticated models including the effect of tires, inputs and constraints such as higher order systems, nonlinear systems and advanced suspension configuration, e.g., dynamics absorber, an active suspension together with a damped absorber. Also this study can be implemented by optimization, in order to obtain the optimum value of C, for the best performance.

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Appendix I

Laplace Transform of a function f(t) is

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt \qquad (A)$$

Replacing t by t^*/W_{nn}, RHS of (A) becomes,

$$RHS = \frac{1}{W_{nn}} \int_0^\infty e^{-\phi t^*} f(t^*/W_{nn})\,dt^* \qquad (B)$$

where $\phi = s/W_{nn}$

From (A) and (B), we obtain

$$\int_0^\infty e^{-\phi t^*} f(t^*/W_{nn})\,dt^* = W_{nn} F(\phi) \qquad (C)$$

From (1) we have,

$$m\ddot{x} + (c_1+c_2)\dot{x} + (k_1+k_2)x + (c_2 b - c_1 a)\dot{\theta} + (k_2 b - k_1 a)\theta = c_1\dot{x}_1 + c_2\dot{x}_2 + k_1 x_1 + k_2 x_2$$

Using $X = x/x_o$ and $t = t^*/W_{nn}$, we obtain the following non dimensionalized parameters,

$\dot{X} = \dot{x}/x_o W_{nn}$, $\dot{X}_1 = \dot{x}_1/x_o W_{nn}$, $\dot{X}_2 = \dot{x}_2/x_o W_{nn}$, $\ddot{X} = \ddot{x}/x_o W_{nn}^2$

Using above non dimensionalized parameters, equation (1) becomes

$$m x_o W_{nn}^2 \ddot{X} + (c_1 + c_2) x_o W_{nn} \dot{X} + (k_1 + k_2) x_o X + (c_2 b - c_1 a) W_{nn} \dot{\theta} +$$

$$(k_2 b - k_1 a)\theta = (c_1 \dot{X}_1 + c_2 \dot{X}_2) x_o W_{nn} + (k_1 X_1 + k_2 X_2) x_o \qquad (1a)$$

Taking Laplace Transform of (1a), we obtain

$$F(\phi) = (m x_o W_{nn}^2 \phi^2 + (c_1 + c_2) x_o W_{nn} \phi + (k_1 + k_2) x_o) X +$$

$$((c_2 b - c_1 a)\phi + (k_2 b - k_1 a))\theta = (c_1 W_{nn} \phi + k_1) x_o X_1 + (c_2 W_{nn} \phi + k_2) x_o X_2 \qquad (1c)$$

From (1c) and (C), we obtain,

$$\int_0^\infty e^{-\phi t^*} f(t^*/W_{nn}) dt^* + W_{nn} F(\phi) \qquad (1d)$$

where $F(\phi)$ is given by (1c)

Dividing (1d) by $m x_o W_{nn}^3$, we obtain

$$(\phi^2 + 2(\xi_1 + \xi_2)\phi + K + 1) X + (2(\xi_2 B - \xi_1 A) + KB - A)\theta =$$

$$(2\xi_1 \phi + 1) X_1 + (2\xi_2 \phi + K) X_2$$

where

$A = a/x_o$, $B = b/x_o$, $K = k_1/k_2$, $W_{nn} = \sqrt{k_1/m}$

$\xi_1 = c_1/2mW_{nn}$, $\xi_2 = c_2/2mW_{nn}$

Similarly Laplace Transform of

$$J\ddot{\theta} + (c_2 b - c_1 a)\dot{x} + (k_2 b - k_1 a) x + (c_2 b^2 + c_1 a^2)\dot{\theta} + (k_2 b^2 + k_1 a^2)\theta$$

$$= (c_2 \dot{x}_2 + k_2 x_2) b - (c_1 \dot{x}_1 + k_1 x_1) a \quad \text{is}$$

$$(J\phi^2 + 2(\xi_2 B^2 + \xi_1 A^2)\phi + KB^2 + A^2) + (2(B\xi_2 - A\xi_1)\phi + KB - A) X$$

$$= (2\xi_2 \phi + K) B X_2 - (2\xi_1 \phi + 1) A X_1$$

```
*******************************************************************************
C
C       PROGRAM          : SPECTRAL DENSITY
C
C       PROGRAMMER       : SHABBIR LOKHANDWALA
C
C       DATE WRITTEN     : MAY/15/1985
C
C       OBJECTIVE :
C                  TO FIND THE ROOTS OF COMPLEX EQUATION USING
C                  LEQT1C ROUTINE AND THEN FIND THE SPECTRAL
C                  DENSITY OF ACCELERATION OF RANDOM IN-PUT FUNCTION
C
C       DESCRIPTION OF VARIABLES :
C                AA(N,N)  - MASS MATRIX OF COMPLEX VARIABLES
C                CA(N,N)  - FORCE MATRIX
C                C(N,N)   - TRANSFER FUNCTION MATRIX
C                H(N,N)   - COMPLEX CONJUGATE OF C(N,N)
C                X(N)     - COLUMN VECTOR
C                B(N)     - SOLUTION VECTOR OBTAINED FROM LEQT1C
C                N        - ORDER OF MATRIX
C
C
*******************************************************************************
C
      INTEGER*4  N,IA,M,IB,IJOB,IER
      COMPLEX    A(2,2),B(2),C(2,2),BA(2),Q3(2,2),H(2,2),AA(2,2)
      COMPLEX    H1(2,2),S(2,2),Q(2,2),Q2(2,2),CA(2,2),BB(2,2)
      REAL*4     WA(2),P(8),P1(8),V,AM,AK1,AK2,AK,AI,C1(4),C2(4)
      REAL*4     Z1(4),Z2(4),WNN,X0,R,R0,A1,B1,B2,A2,AL,D,F,G
      N   = 2
      IA  = N
      IB  = N
      WRITE(6,100)
 100  FORMAT(1H1////10X,'PROGRAM FOR FINDING '//
     & 10X,'THE VALUES OF VARIABLE X1,X2,X3 & X4 '//
     & 10X,'BY QUASI LINEAR METHOD'///)
C
      V     = 20.
      AM    = 1000.
      C1(1) = 200.
      C1(2) = 2000
      C1(3) = 4000
      C1(4) = 6000
      DO 5736 I = 1,4
 5736 C2(I) = C1(I)
      AK1   = 50000.
      AK2   = AK1
      A1    = 1.3
      B1    = 1.2
      WNN   = (AK1/AM)**.5
      AK    = AK2/AK1
      DO 7465 I = 1,4
      Z1(I) = C1(I)/(2.*AM*WNN)
 7465 Z2(I) = Z1(I)
      X0    = (V/WNN)**0.5
      A2    = A1/X0
      B2    = B1/X0
      AL    = 2.5
      R0    = WNN*AL/V
```

```
      AI   = 2000./(AM*X0**2)
      R    = 1.0
      WRITE(6,110)WNN,(Z1(I),I=1,4),X0,A2,B2,RO,AI
 110  FORMAT(10X,'WNN'2X,'Z1(I)'2X,'X0'2X,'A'2X,'B'2X,
     &        'RO'2X,'AI'//10X,10F15.8//)
C
      DO 7245 II = 1,4
      AA(1,2) = CMPLX(B2*AK-A2,2.*(B2*Z2(II)-A2*Z1(II))*R)
      AA(1,1) = CMPLX(1.+AK-R*R,2.*(Z1(II)+Z2(II))*R)
      AA(2,1) = AA(1,2)
      AA(2,2) = CMPLX(AK*B2*B2+A2*A2-AI*R*R,2.*(Z2(II)*B2*B2+Z1(II)
     &          *A2*A2)*R)
      PRINT*,((AA(I,J),J=1,N),I=1,N)
      WRITE(6,*)'AA'
      WRITE(6,8000)((AA(I,J),J=1,N),I=1,N)
 8000 FORMAT(/10X,4F15.8/)
C
      CA(1,1) = CMPLX(1.,2.*Z1(II)*R)
      CA(1,2) = CMPLX(AK,2.*Z2(II)*R)
      CA(2,1) = -A2*CA(1,1)
      CA(2,2) = B2*CA(1,2)
      PRINT*,((CA(I,J),J=1,N),I=1,N)
      WRITE(6,*)'CA'
      WRITE(6,8001)((CA(I,J),J=1,N),I=1,N)
 8001 FORMAT(/10X,4F15.8/)
C
      M = N
      DO 8002 I = 1,M
      DO 8002 J = 1,M
 8002 BB(I,J) = 0.0
      BB(1,1) = CMPLX(1.0,0.0)
      BB(2,2) = BB(1,1)
      PRINT*,((BB(I,J),J=1,N),I=1,N)
      WRITE(6,*)'BB'
      WRITE(6,8004)((BB(I,J),J=1,N),I=1,N)
 8004 FORMAT(/10X,4F15.8/)
C
      IJOB = 0
      CALL LEQT1C(CA,N,IA,BB,M,IB,IJOB,WA,IER)
      PRINT*,((BB(I,J),J=1,N),I=1,N)
      WRITE(6,*)'BB'
      WRITE(6,8005)((BB(I,J),J=1,N),I=1,N)
 8005 FORMAT(/10X,4F15.8/)
C
      DO 10 I = 1,N
      DO 10 J = 1,N
      Q3(I,J) = 0.0
 10   A(I,J)  = 0.0
      DO 8006 I = 1,N
      DO 8006 K = 1,N
      DO 8006 J = 1,N
      A(I,K) = A(I,K) + BB(I,J)*AA(J,K)
 8006 Q3(I,K) = A(I,K)
      WRITE(6,6)
      WRITE(6,1)((A(I,J),J=1,N),I=1,N)
 6    FORMAT(///10X,'JACOBIAN MATRIX A',/10X,17(1H-)/)
 1    FORMAT(10X,4D12.5/)
C
      M = 1
      B(2) = (1.0,0.0)
```

```
      B(1) = (0.0,0.0)
      PRINT*,((A(I,J),J=1,N),I=1,N),N,IA,IB,M
C
      IJOB = 0.0
      CALL LEQT1C(A,N,IA,B,M,IB,IJOB,WA,IER)
      WRITE(6,101)IER
 101  FORMAT(/10X,'B'/10X,'IER          =' I2//)
      C(1,2) = CMPLX(B(1))
      C(2,2) = CMPLX(B(2))
      PRINT*,C(2,2),C(2,1)
C
      M = 1
      BA(1) = (1.0,0.0)
      BA(2) = (0.0,0.0)
      PRINT*,Q3(1,1),Q3(1,2),Q3(2,1),Q3(2,2),N,IA,IB,M
C
      IJOB = 0.0
      CALL LEQT1C(Q3,N,IA,BA,M,IB,IJOB,WA,IER)
      C(1,1) = CMPLX(BA(1))
      C(2,1) = CMPLX(BA(2))
      PRINT*,C(1,1),C(1,2)
      WRITE(6,234)
 234  FORMAT(//10X,'C'//)
      WRITE(6,*)((C(I,J),J=1,2),I=1,2)
C
      P(1) =  REAL(C(1,1))
      P(2) = -AIMAG(C(1,1))
      P(3) =  REAL(C(1,2))
      P(4) = -AIMAG(C(1,2))
      P(5) =  REAL(C(2,1))
      P(6) = -AIMAG(C(2,1))
      P(7) =  REAL(C(2,2))
      P(8) = -AIMAG(C(2,2))
      WRITE(6,*)'P'
      WRITE(6,600) (P(I),I = 1,8)
 600  FORMAT(/10X,8F15.8//)
      PRINT*, (P(I),I = 1,8)
C
      H1(1,1) = CMPLX(P(1),P(2))
      H1(1,2) = CMPLX(P(3),P(4))
      H1(2,1) = CMPLX(P(5),P(6))
      H1(2,2) = CMPLX(P(7),P(8))
      PRINT*, ((H1(I,J),J=1,N),I=1,N)
      WRITE(6,*)'H1'
      WRITE(6,602) ((H1(I,J),J=1,N),I=1,N)
 602  FORMAT(/10X,4F15.8/)
      DO 2001 I = 1,4
      DO 2001 J = 1,4
 2001 H(J,I) = H1(I,J)
      PRINT*,((H(I,J),J=1,N),I=1,N)
      WRITE(6,*)'H'
      WRITE(6,603)((H(I,J),J=1,N),I=1,N)
 603  FORMAT(/10X,4F15.8/)
C
      S(1,1) = CMPLX(1.0,0.0)
      S(1,2) = CMPLX(COS(R*RO),-SIN(R*RO))
      S(2,1) = CMPLX(COS(R*RO),SIN(R*RO))
      S(2,2) = S(1,1)
      PRINT*,((S(I,J),J=1,N),I=1,N)
      WRITE(6,*)'S'
```

```
      WRITE(6,605)((S(I,J),J=1,N),I=1,N)
 605  FORMAT(/10X,4F15.8/)
C
      DO 1005 I = 1,N
      DO 1005 J = 1,N
 1005 Q(I,J) = 0.0
      DO 1008 I = 1,N
      DO 1008 K = 1,N
      DO 1008 J = 1,N
 1008 Q(I,K) = Q(I,K) + S(I,J)*H(J,K)
      PRINT*,((Q(I,J),J=1,N),I=1,N)
      WRITE(6,*)'Q'
      WRITE(6,606)((Q(I,J),J=1,N),I=1,N)
 606  FORMAT(/10X,4F15.8/)
C
      DO 1110 I = 1,N
      DO 1110 J = 1,N
 1110 Q2(I,J) = 0.0
      DO 1111 I = 1,N
      DO 1111 K = 1,N
      DO 1111 J = 1,N
 1111 Q2(I,K) = Q2(I,K) + C(I,J)*Q(J,K)
      WRITE(6,*)'Q2'
      WRITE(6,608)((Q2(I,J),J=1,2),I=1,2)
 608  FORMAT(/10X,4F15.8/)
      PRINT*,((Q2(I,J),J=1,2),I=1,2)
 7245 CONTINUE
      STOP
      END
```