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Simulation of an open-loop stepping motor system

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SIMULATION OF AN OPEN-LOOP STEPPING MOTOR SYSTEM

by

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Abstract

This report presents a description and functional model of a hybrid stepping motor drive system. The motor drive methods of fullstepping and backstepping are presented as examples. Test methods for system characterization are described, and response characteristics for the simulated and experimental results are compared to verify the model. This paper shows that an open-loop stepping motor drive system can be accurately simulated to predict real hardware performance.

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Glossary of Terms

V_a,VA	Volts applied to motor phase A, Φ_a	[Volts]
V_b,VB	Volts applied to motor phase B, Φ_b	[Volts]
Φ_a	Motor Phase A	
Φ_b	Motor Phase B	
i_a	Current Circulating in Φ_a	[Amps]
i_b	Current Circulating in Φ_b	[Amps]
L_a	Inductance of Φ_a	[H]
L_b	Inductance of Φ_b	[H]
ω	Rotor Speed	[Rad/sec]
K_o, K	Motor Torque Constant	[Nt-m/A]
NC	Motor Saturation Factor	[Nt-m/A^2]
N	# Motor Teeth	
A	Nominal Phase Inductance	[H]
C	Phase Inductance Parameter	[H]
T_a, TA	Torque from phase A, Φ_a	[Nt-m]
T_b, TB	Torque from phase B, Φ_b	[Nt-m]
T_d, TD	Detent Torque	[Nt-m]
T, TM	Total Motor Torque	[Nt-m]
D	Detent Torque Magnitude	[Nt-m]
J_m, JM	Motor Inertia	[Kg-m^2]
J_L, JL	Load Inertia	[Kg-m^2]
Θ, TH	Rotor Position	[Rad]
$d\Theta/dt$, THD	Rotor Velocity	[rad/sec]
$d^2\Theta/dt^2$, THDD	Rotor Acceleration	[rad/sec^2]
Θ_l, THL	Load Position	[rad]
$d\Theta_l/dt$, THLD	Load Velocity	[rad/sec]
$d^2\Theta_l/dt^2$, THLDD	Load Acceleration	[rad/sec^2]
V_m, VM	Motor Damping Coefficient	[m-s-Nt/rad]
FRM	Motor Internal Friction	[Nt-m]
KCOUP	Coupling Stiffness	[Nt-m/rad]
FR_l, FRL	Load Friction	[Nt-m]

1.0 Introduction

With the advent of increased industrial automation and microprocessor applications, interest in digital motion control systems has also expanded. [1,2] A typical motion control system includes the mechanical load to be driven, an electromechanical actuator (e.g. a motor), and appropriate electronic controls for the motor. Combinations of components and means of implementation are numerous. [2] One example of a motion control system is an open-loop controlled stepping motor system. Such a system is characterized by the use of a stepping motor as the actuator, and drive controls which do not require feedback for proper control. The system is often used in applications requiring stop-start, incremental motion.

There are two common types of stepping motors (steppers): variable reluctance (VR) motors and hybrid stepping motors. This project presents a description and functional model of a hybrid stepping motor, test procedures for quantification of system parameters and system response, and sample comparisons of measured and simulated response. Input to a stepper is current described in the time domain. Loads include inertia, viscous friction, dry friction, and system compliance. The output of the stepping motor system is position, velocity, and acceleration. System response is measured in terms of minimum time to reach the new step position, time to settle within 10% of new position, damped resonant frequency, and the general position profile. Simulations and tests examine single step response and backstepped response of the stepper system and show that backstepping [2] can yield maximized system response. This report models an open-loop stepping motor drive system and verifies that model through testing.

2.0 Stepping Motors

2.1 Description

Overall stepper system response depends on many parameters of the motor driver, the load, and the motor. Each of these components must be characterized before response can be simulated. In order to model the system, it is important first to understand the operation of a two-phase hybrid stepping motor. That operation is the heart of the stepping motor system, interfacing the electronic system to the mechanical one. A stepper can be considered a transfer device. As in Figure 2.1.1, the motor changes electrical signals into mechanical motion. Its electrical and mechanical characteristics are significant in determining the overall system response.

Figure 2.1.1 The Motor as a Transfer Device

A stepping motor, to be more specific, is a positioning device. The energized stepper can be generalized as a single degree-of-freedom, second order system. The lumped components are an inertia, a viscous damper, and a nonlinear spring as shown in Figure 2.1.2. Drive waveforms command the motor to position itself at specific locations. The current in the phase windings determines the positioning and torque characteristics of the motor. Control of the motor is achieved by changing the magnitude and direction of the current in each of the two phases.

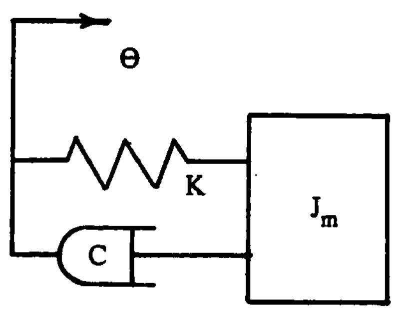


Figure 2.1.2 Simple Model of the Stepping Motor

The inertia term is the actual rotary inertia of the motor. The damping term derives from mechanical and electrical losses within the motor. The spring term is the nonlinear electromagnetic spring developed as a result of current passing through stator windings. The rotor position and stiffness are controlled by the amount of phase current in each winding of the motor, Figure 2.1.3. Adding current to either or both phases increases the magnetic forces. Increased magnetic force causes increased stiffness, and the relative magnitude of the current between the two phases determines the equilibrium position. Motor stiffness is like spring stiffness in the mass-spring system. If the mass is displaced away from equilibrium, there is a reaction force pushing it back. The greater the reaction force for a given displacement, the greater the motor stiffness. A *stiffer* motor has stronger restoring force to hold the rotor in its correct position and can produce faster sytem response. Specifics, such as the nonlinearity of the motor stiffness, are presented in the model description.

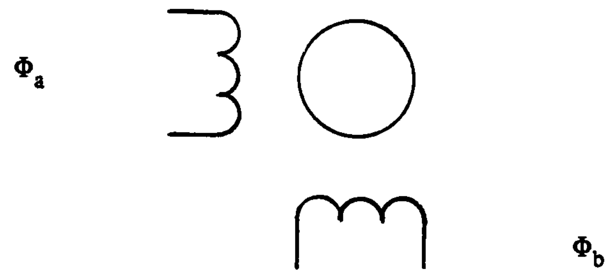


Figure 2.1.3 Motor Winding Schematic

2.2 Single Step Response

When a motor is signalled to execute a single step, the rotor moves to a new position. In a 200 step per revolution motor, one step is 1.8° of rotation. An understanding of this event introduces the general operating characteristics of a stepping motor. Consider a motor with both phases energized (i.e. rated current in both phases). The current causes the electromagnetic spring to be energized, and the motor is locked at a specific equilibrium position. Then, when the current in Φ_a, for example, is switched to the opposite direction, the motor seeks a new equilibrium position. Since the system is lightly damped, the response is a damped oscillation as shown in Figure 2.2.1.

If the current in Φ_b is then switched to the opposite direction, the motor executes another step. When the Φ_a current is switched again, a third step is initiated, and so on. This is a standard way to drive a stepping motor. [3] Change in the direction of current in one phase of the motor, therefore, moves the motor equilibrium position (Θ) forward or backward one step. Sequential switching of current in alternate phases causes continuous motor motion as shown in Figure 2.2.2.

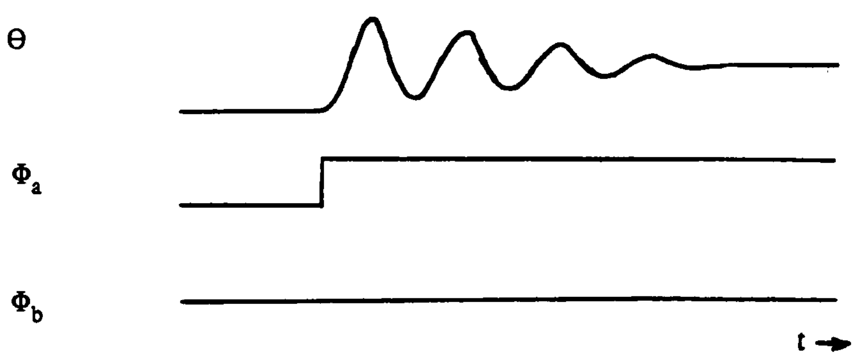


Figure 2.2.1 Single Step Response

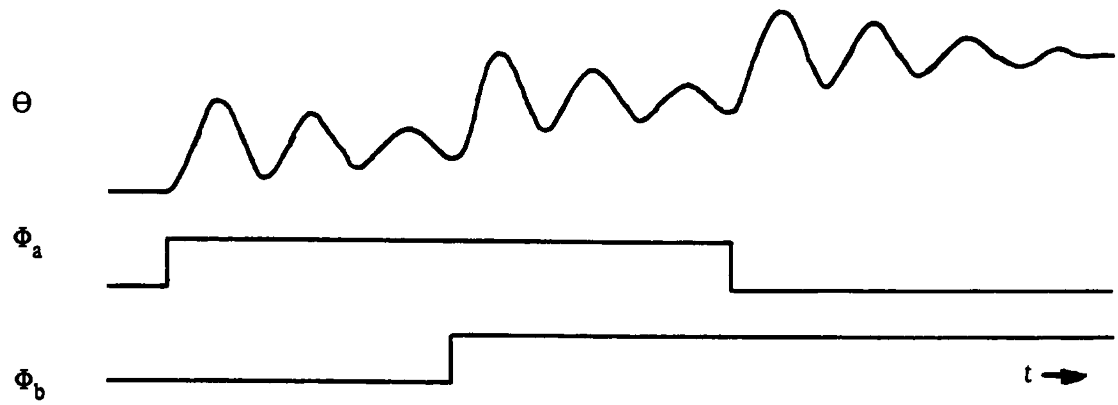


Figure 2.2.2 Multiple Full Steps

Returning to the schematic of the motor model (Figure 2.1.2), we can foresee the effects of added loads. Additional inertia slows down the response of the system and makes the resonant frequency lower. Compliance between the motor inertia and load inertia adds complexity and another degree of freedom.

2.3 Torque Characteristic

As mentioned, angular equilibrium position and torsional stiffness of the motor are determined by the phase current in each winding. The torque characteristics of a two-phase hybrid stepping motor are modeled with the following equations [4]:

$T_a = -(K_o - 1/2\ NC|i_a|)i_a \sin N\Theta$ Torque from phase A (Φ_a) (2-1)

$T_b = (K_o - 1/2\ NC|i_b|)i_b \cos N\Theta$ Torque from phase B (Φ_b) (2-2)

$T_d = -D \sin 4N\Theta$ Detent torque (2-3)

$T_m = T_a + T_b + T_d$ Total motor torque (2-4)

As graphed in Figure 2.3.1, static torque from phase A and phase B are sinusoidal curves. When the two curves are superimposed, the torques combine to result in a new equilibrium position and a stiffer system. Some stepping motors have other higher harmonic torque characteristics, but this paper does not address them.

Detent torque is a sinusoidally varying torque present at all times (with the phase currents on or off). It is caused by permanent magnetic flux acting on the stator poles of the motor. Detent torque amplitude is typically one or two percent of the motor holding torque.

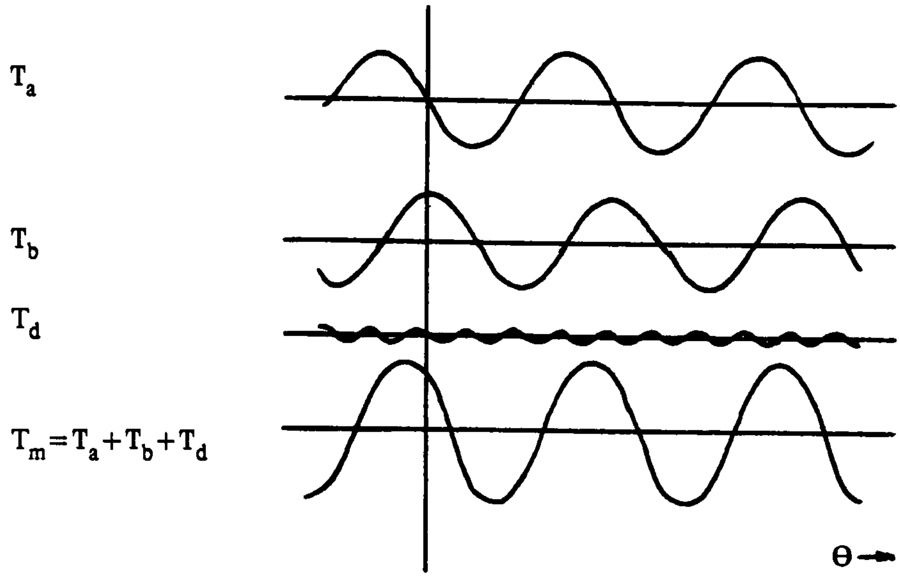


Figure 2.3.2 Motor Torque Components

An increase of current in either phase moves the equilibrium position closer to one phase equilibrium position or the other. By turning one phase off gradually, the opposite phase position is approached.

2.4 Inductance

To this point, the explanation of motor stepping assumed that the direction of current could be switched instantaneously. Actually, the motor has inductance, which limits the rate at which current can change. If the motor response is to be successfully simulated in a model, the electrical characteristics of the motor must be accounted for.

Before the motor can respond mechanically to a command signal, the current in the windings must respond to applied voltage. Torque cannot be generated until the current change takes place. Inductance, therefore, introduces a time delay in the sytem response.

The electrical equations [4] describing the two-phase hybrid stepping motor are:

$V_a = i_a R + L_a di_a/dt - \omega(K_o - NC\,|i_a|)\sin N\Theta)$ Voltage equation for Φ_a (2-5)

$V_b = i_b R + L_b di_b/dt + \omega(K_o - NC\,|i_b|)\cos N\Theta)$ Voltage equation for Φ_b (2-6)

$L_a = A - C(i_a/|i_a|)\cos N\Theta$ Inductance of Φ_a (2-7)

$L_b = A - C(i_b/|i_b|)\sin N\Theta$ Inductance of Φ_b (2-8)

The voltage equations include terms for phase resistance, phase inductance, and back emf (electromotive force). Back emf is dependent on rotor position, rotor speed, and amplitude of the current. Inductance is dependent on direction of the current and rotor position.

2.5 Current Control (PWM)

Since equilibrium position and motor stiffness depend on the current circulating in each phase, the method of current control must be modeled. This project uses a switched or chopping driver.

The chopping motor driver adds power to the desired control waveform. Current control in the motor is achieved through pulse width modulation (PWM). A basic explanation of PWM is shown in Figure 2.5.1. The actual current in the motor is compared with the desired current level as supplied by the motor controller. When the motor current is lower than the reference current, the motor voltage is switched to (+) to pull the current up to the desired level. When the motor current is higher than the desired level, the motor voltage is switched to (−). This voltage switching can be implemented at any frequency. The higher the frequency becomes, the tighter the current control will be. The ability to track the desired current waveform depends on switching frequency, motor power supply voltage, motor inductance, and phase resistance.

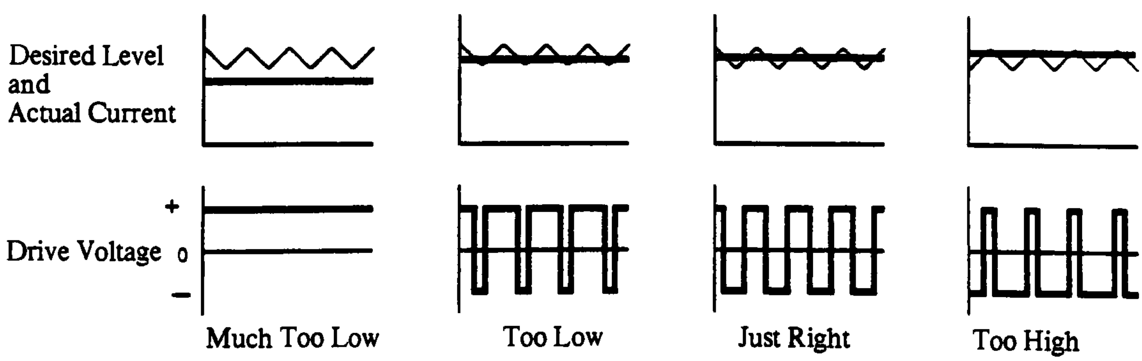


Figure 2.5.1 Current Control Using Pulse Width Modulation

2.6 Drive Waveforms

The type of command signal selected as input to the motor will, to a large extent, determine the response of the overall system. Some of the waveforms that can be input to the motor are reviewed in this section.

Input command signal to the motor is basically the desired current waveform to be seen in the motor windings. The two-phase motor requires two reference waveforms. The actual current in the motor matches the reference waveforms to the extent that the inductance and motor driver capabilities allow.

Single step response, as described earlier, gives an initial understanding of the stepper's response. Single step response, Figure 2.6.1, is implemented by switching the direction of current flow through one phase winding.

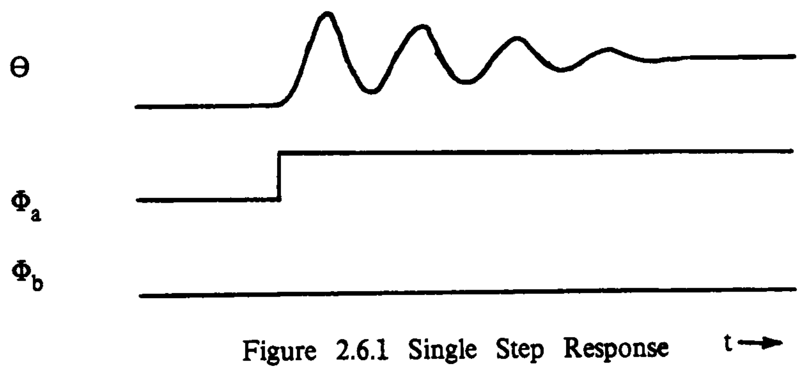


Figure 2.6.1 Single Step Response

Fullstepping is the extension of single steps by stringing them together as shown in Figure 2.6.2. If the steps are spaced far apart in time, then each step overshoots, oscillates, and damps out. The forward motion of the system is stable. As the steps are pushed closer together, though, the interaction between input step frequency and resonant frequency of the system can cause resonance instabilities. Modeling can predict resonance instabilities.

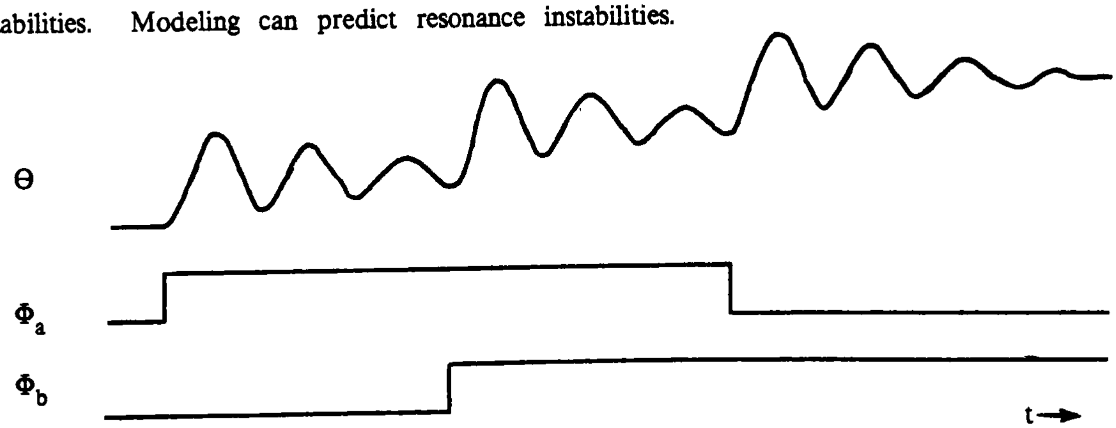
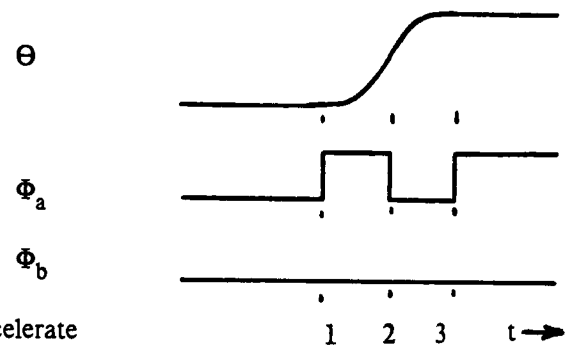


Figure 2.6.2 Example of Fullstepping

Backstepping, sometimes termed electronic damping, may be used in applications where fast response with little or no overshoot is required. The method utilizes full torque capability of the motor to move the rotor to a new position and brake it to stop there. In theory, the implementation of backstepping is straightforward. First, the motor phases are energized, and the rotor sits at its home position. Then, one phase current is reversed (just as in fullstepping). As the rotor accelerates to its new position, the phase current is switched back to pull the rotor back to its home position. The rotor decelerates and as it reaches zero velocity, the same phase current is switched again and restores the new equilibrium position. Backstepping is illustrated in Figure 2.6.3.



1. Accelerate
2. Decelerate
3. Zero Velocity

Figure 2.6.3 Example of Backstepping

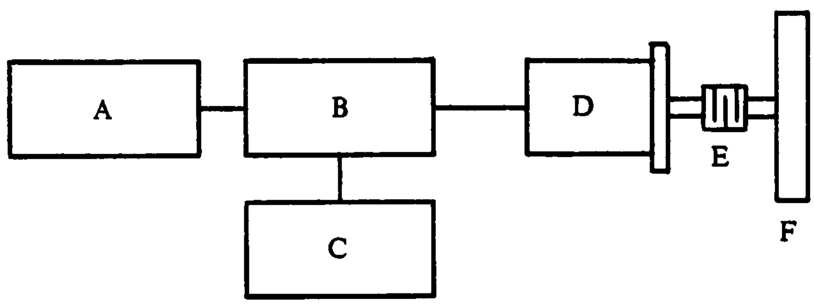
Implementation of backstepping is sensitive to the timing accuracy of the command signals as well as to system damping, friction, inertia, and compliance. Backstepping can be effective in applications where point-to-point, stop-start motion is required.

Sinusoidal current waveforms are sometimes input into steppers in an effort to attain constant velocity out. In each of the previous drive methods, the equilibrium position of the motor was moved abruptly by switching the current abruptly. Using two sinusoidal inputs in quadrature, the equilibrium position of the motor moves smoothly ahead in time. If all other loads and system parameters stay constant, then the motor velocity is constant.

3.0 The Physical System

3.1 Apparatus

The physical system examined in this study is shown in Figure 3.1.1. The mechanical portion of the system consists of a two-phase hybrid stepping motor (Sigma Instruments Inc. model 17-2220D-28456), a flexible coupling, and an inertial load. Each of these components has characteristics which affect the dynamic response of the system. The characteristics are separated into four categories: inertia, compliance, viscous damping, and dry friction. The motor has additional torque characteristics which have already been described. The electrical portion of the system consists of a motor controller (A), which generates the desired drive waveforms, a motor driver (B), which controls the current in the motor windings using pulse width modulation (PWM), and a motor power supply (C).



A. Motor controller
B. Motor driver
C. Power supply
D. Hybrid stepping motor
E. Flexible coupling
F. Inertial load

Figure 3.1.1 Schematic of the Physical System

3.2 The Modelled System

The simulation model is described schematically as shown in figure 3.2.1.

Block 1 represents the motor controller. Here, a desired current waveform is generated. The waveforms for this simulation were designed to synthesize fullstepping and backstepping. Sinusoidal inputs and other drive waveforms can be designed.

Block 2 represents the motor driver. This component works to push the desired reference waveforms into the motor load windings. Current control is implemented using pulse width modulation.

Block 3 contains the equations describing the electrical characteristics of the motor itself. The equations represent the effective motor circuit which converts voltages, as supplied by the driver, into current in the windings.

Block 4 represents the motor transfer device which converts current in the phase windings into shaft torque characteristics. The function includes effects from current in phase A, current in phase B, and detent torque.

Blocks 5 and 6 are actual lumped inertias of the mechanical system. They are motor and load inertia respectively. The mechanical portion of the system yields two simultaneous second order differential equations. These equations were derived from the lumped parameters of Figure 3.2.1 using standard methods.

$$J_m d^2\Theta/dt^2 = T_m - V_m d\Theta/dt - FR_m(\mathrm{sgn}\, d\Theta/dt) - K_{coup}(\Theta - \Theta_l) \quad (3\text{-}1)$$

$$J_L d^2\Theta_l/dt^2 = -K_{coup}(\Theta_l - \Theta) - FR_l(\mathrm{sgn}\, d\Theta_l/dt) \quad (3\text{-}2)$$

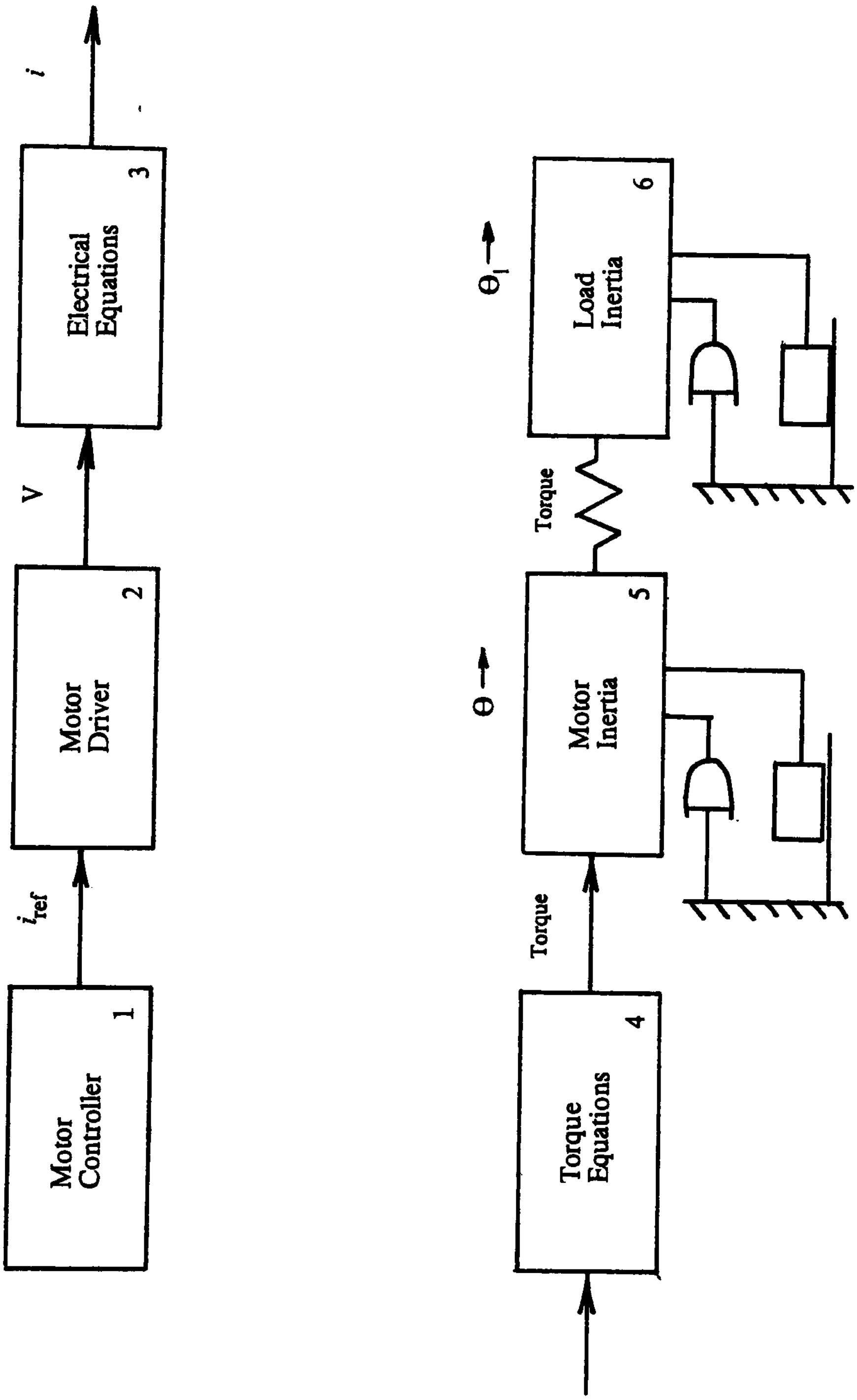


Figure 3.2.1 The Modeled System

The complexity of this system and the nonlinearity of certain components make an analytical solution to the problem difficult. For such problems, numerical techniques can be extremely helpful.

This study uses ACSL (Advanced Continuous Simulation Language) [5] to calculate simulations of the stepping motor drive system. ACSL takes a system of differential equations subject to specific initial conditions and time-dependent forcing functions, and performs numerical integration on the system. The actual program requires its user to take each higher order equation and express it as two or more first-order equations.

The general form of the ACSL program includes an *initial* block where all initial conditions, system parameters, and constants are set; a *derivative* block where the modelled system is described (this is the part of the program which is integrated each iteration.); and a *terminal* block which defines termination conditions for the program.

```
PROGRAM
  INITIAL
      . . .
      . . .
      END INITIAL
  DERIVATIVE
      . . .
      . . .
      END DERIVATIVE
  TERMINAL
      . . .
      . . .
      END TERMINAL
  END PROGRAM
```

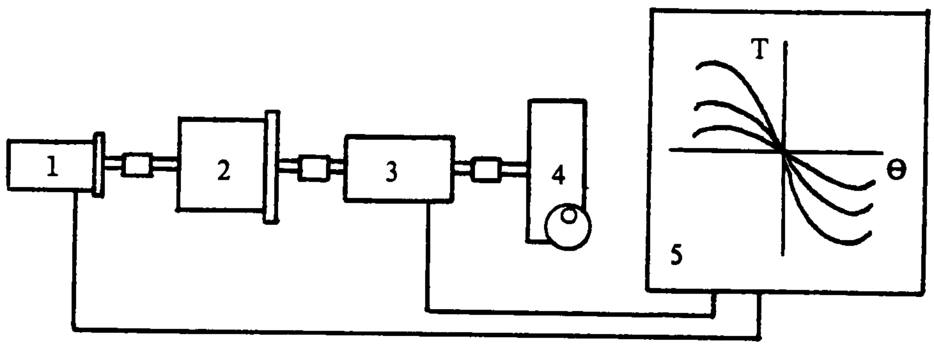
The program listing used for this study is contained in Appendix 1.

3.4 Test Method

Before the dynamic simulations were calculated, all of the system parameters had to be known. Many of the characteristics of the motor, such as motor inertia (J_m), phase resistance (R), detent torque (T_d), and # motor teeth (N) are listed in motor specification catalogues. Load inertia (J_l) was calculated from physical dimensions and density.

The motor torque constant (K) and the motor saturation factor (NC) were determined by measuring the stiffness of the motor at various phase current settings. Motor stiffness is measured with apparatus shown in Figure 4.3.1. The procedure for the measurement is to energize one phase winding of the motor, rotate one side of the torque transducer (Vibrac Model TQ-100), and plot resisting torque of the motor vs. displacement of the motor. This produces plots similar to those seen in Figure 2.3.2. K is the amplitude of the sinusoidal torque characteristic described in section 2.3 and is measured at rated current.

K= Measured Torque @ 1.8° Rotation/Nominal Rated Current [Nt-m/A] (3-3)



1. Angular Displacement Transducer
2. Motor
3. Torque Transducer
4. Rotary Stage
5. X-Y Plotter

Figure 3.4.1 Stiffness Measurement Apparatus

Although current may be increased in the phase windings, motor stiffness does not increase linearly with current. The motor begins to saturate as current is increased. This accounts for the motor saturation factor (NC). This term was determined by plotting a series of stiffness curves at different phase current values, and then solving for NC from the equation.

$$T_a = -(K_o - 1/2\ NC|i_a|)i_a \sin N\Theta \qquad (2\text{-}1)$$

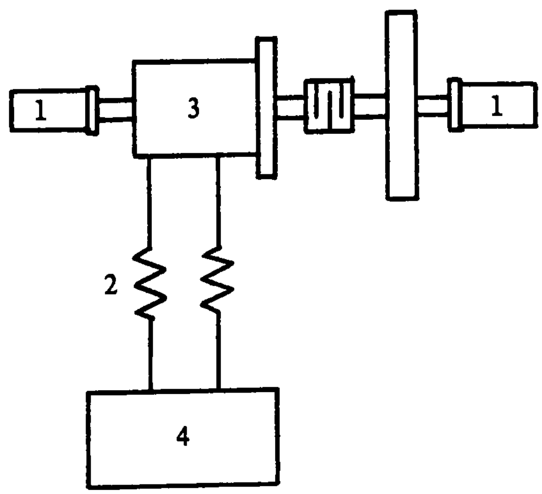
Other parameters such as friction (FRM, FRL) and compliance (KCOUP) were measured from the hardware. Friction torque was measured by wrapping a string around the shaft of interest and measuring the tension required to rotate the shaft. (FR=Tension × Shaft Radius, [Nt-m]) Compliance of the coupling was measured by locking one side of the compliant member, applying a known torque on the free side, and measuring the rotary displacement. This is similar to the procedure described for measuring motor stiffness. Motor viscous damping (V_m) is typically small and is added only for completeness.

To measure inductance, a high D.C. voltage was applied to the winding, and the rise of current was observed. Phase current was measured by installing 0.1 ohm sense resistors in series with each phase winding. By measuring the voltage drop across the resistor, current was calculated using the equation $i=V/R$. The inductance at any current level is calculated from: $V-iR=L\ di/dt$. The nominal phase inductance (A) and the phase inductance parameter (C) were found by measuring the inductance at various rotor positions and solving for the two terms.

After all of the system characteristics were known, response parameters of the dynamic system could be measured. The primary variables measured were:

- phase current
- motor position
- load position

Phase current was measured as it was in the inductance measurement. Motor and load positions were measured with TRANSTEK angular displacement transducers (ADT). Such transducers offer a linear range of 100 degrees, sensitivity of 200 mV/degree, and 10 mV noise pk-pk. This yields uncertainty of ±3.0 arc minutes or ±0.873 mRad. Test apparatus is shown in Figure 3.4.2.



1. TRANSTEK ADT
2. Current sense resistor
3. Motor
4. Motor driver

Figure 3.4.2 Diagram of Test Apparatus

This instrumentation provides capabilities for measuring phase current, motor position, and load position, with time. These are the parameters that allow comparison between simulated system response and laboratory test results.

4.0 Model Confirmation

For any numerical model to be useful as a predictive tool, its accuracy and reasonableness must be confirmed. The most straightforward method of model confirmation is direct comparison of numerical results with test results. Those comparisons yield the degree of model accuracy. From that point, the model simulations can be extended beyond laboratory capabilities and used to investigate alternate system hardware and parameters. This study examines phase current response, bare motor response, and motor and load response.

4.1 Phase Current Response

As described before, the electrical characteristics of the system impact the rate at which current can rise in the phase windings. System response time strongly depends on the rate at which current can change within the windings. This rate depends on the motor power supply voltage and the phase inductance. For a given motor, the inductance is already fixed by the motor's design. Power supply voltage can be changed, though, to increase or reduce the phase current rise time.

Figure 4.1.1 shows comparisons between simulated current rise profiles and measured current rise profiles for systems driven at 24 and 30 volts (6000 hz chopping frequency). For 24 V, the simulated current rise time is 900 μS and the actual current rise time is 925 μS. For 30 V, the simulated rise time is 750 μS and the actual rise time is 720 μS.

These results show that the electrical description of the motor, as derived from the equations (2-1, 2-2, 2-3, 2-4), is reasonable, and that the modeled response to parameter changes successfully tracks the actual laboratory response. The current rise and PWM control characteristics of the theoretical model are particularly accurate.

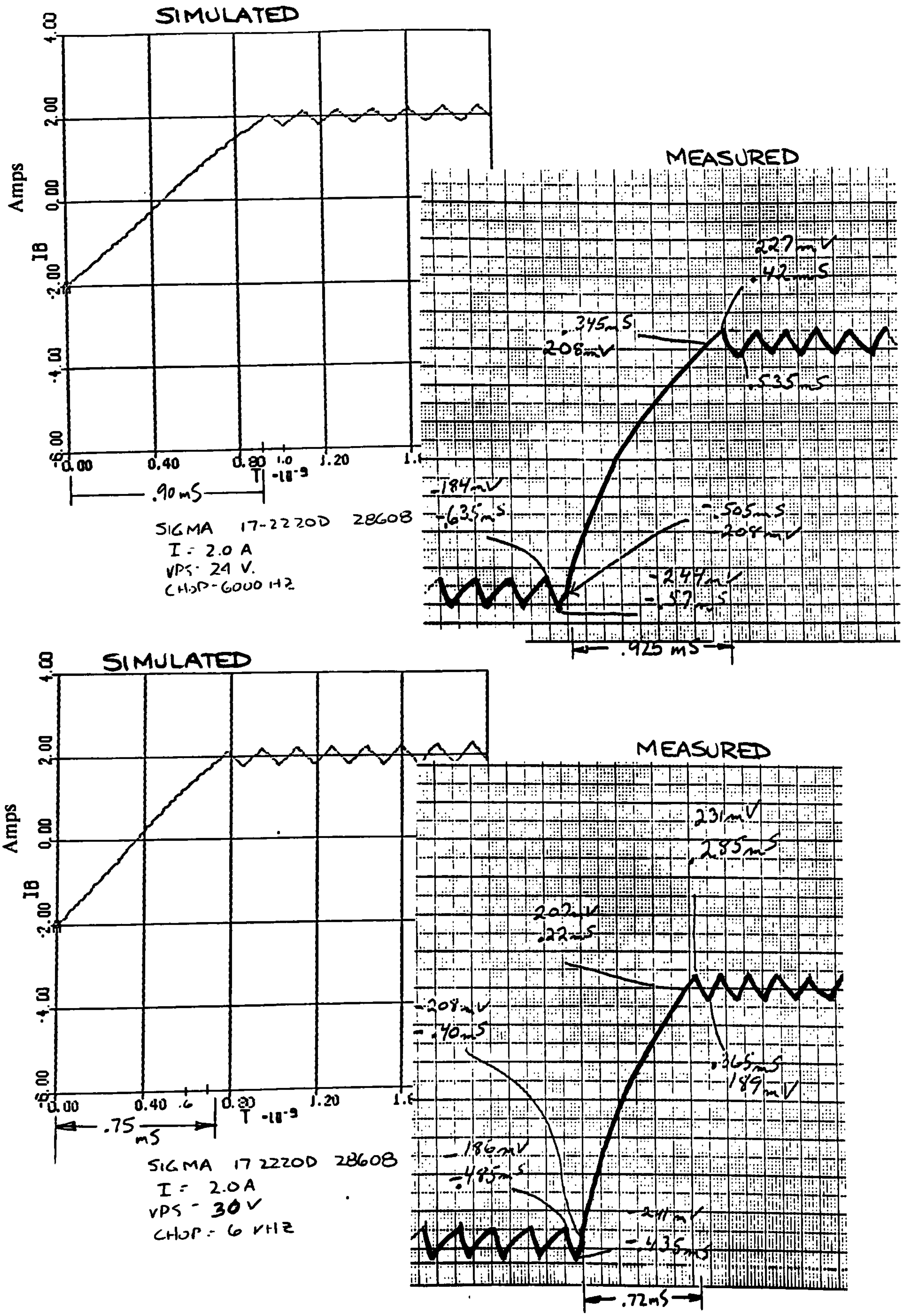


Figure 4.1.1 Phase Current Response

4.2 Bare Motor Response (No Load)

The maximum speed at which the system can respond occurs when the load inertia is minimized. The stepping motor system with no inertial load yields the fastest step times. Comparison of bare motor test results with bare motor simulations is the next step in confirming the accuracy of the dynamic model.

Figure 4.2.1 shows a comparison of the simulated and actual single step response of the bare test motor (driven at rated current of 2.0 amps and voltage of 24 volts). The expanded view of the simulation shows that the calculated initial time to reach the new position is 1.9 mSec, while actual test time was 2.1 mSec. Damping is similar, and damped resonant frequencies are 268 hz and 242 hz for the actual and simulated responses, respectively. This yields a <10% error on the damped resonant frequency between the modeled and measured results.

As described in section 2.6, backstepped waveforms can be implemented to decrease the oscillatory behavior of the system. Figure 4.2.2 shows the backstepped response of the bare motor system. A comparison of the modeled results with the actual results shows good agreement.

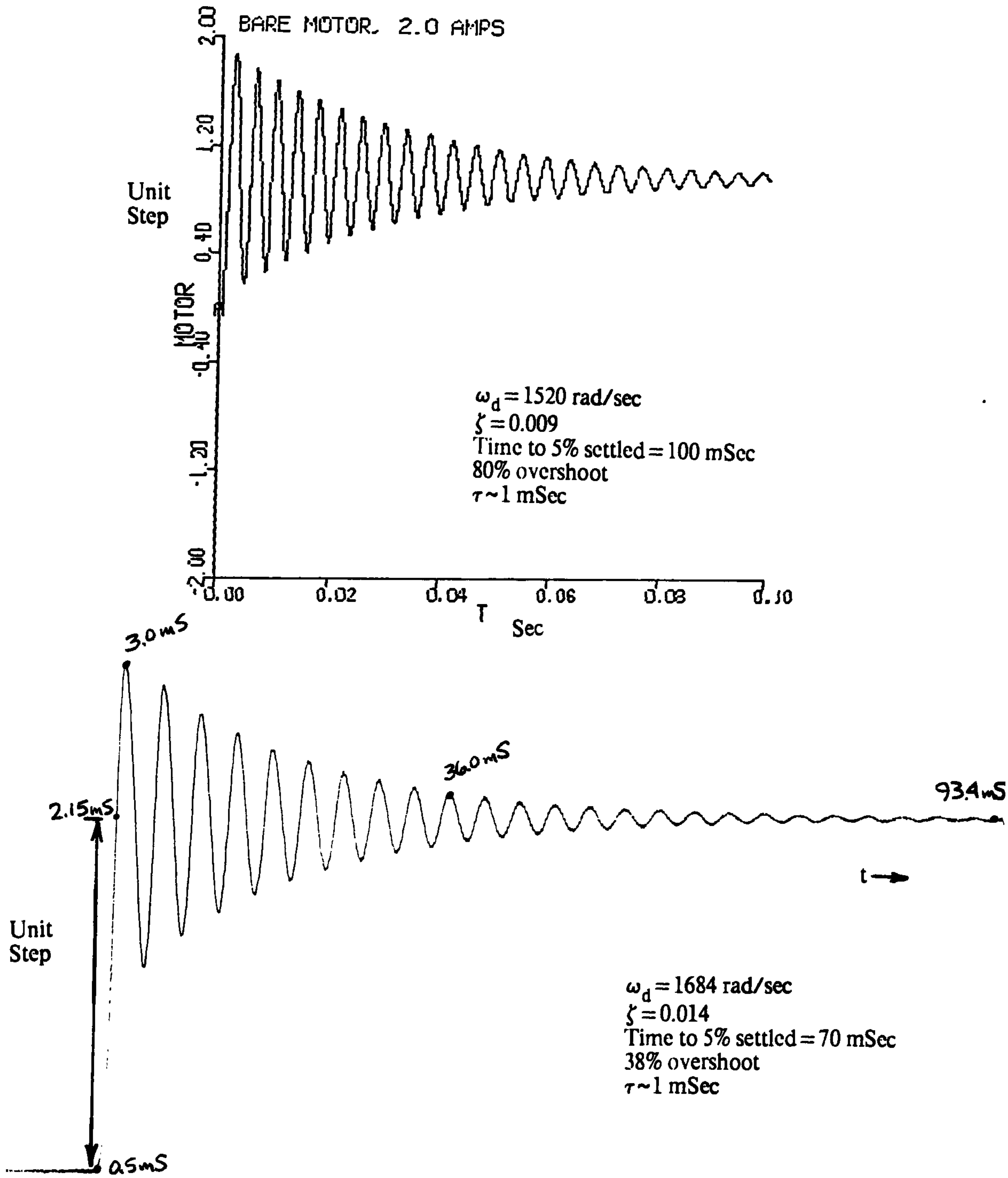


Figure 4.2.1 Single Step Response, Bare Motor

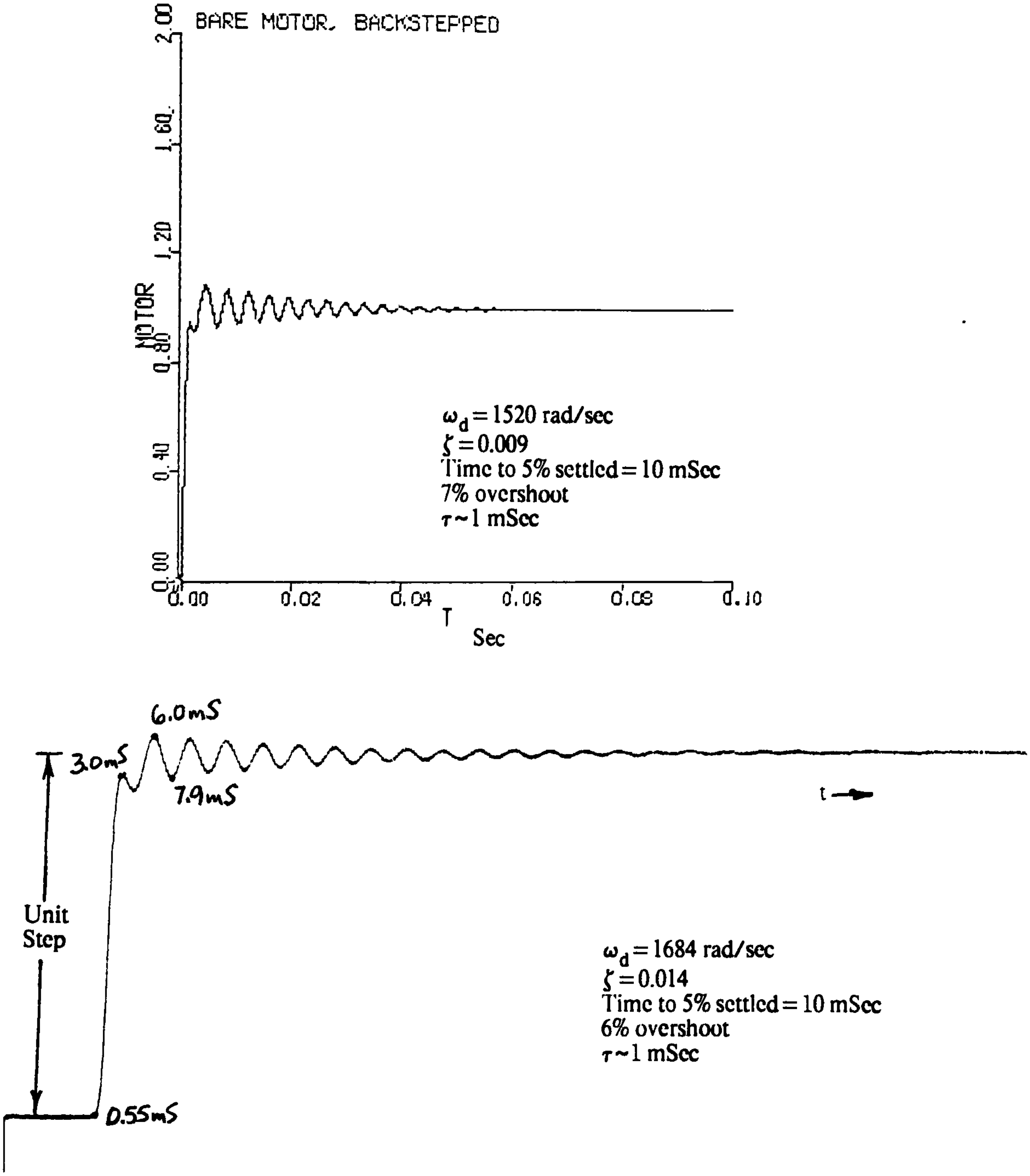


Figure 4.2.2 Backstepped Response, Bare Motor

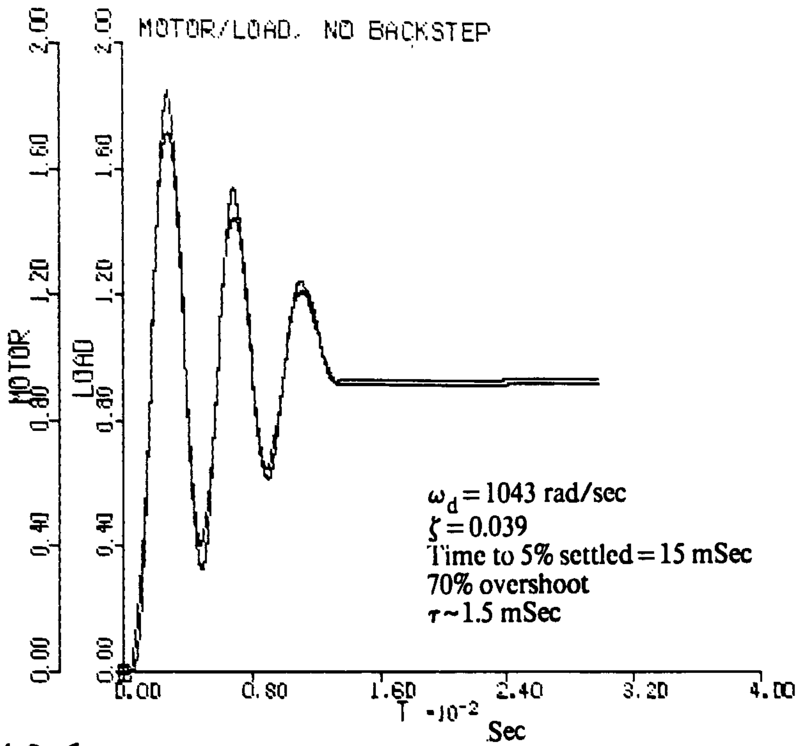
4.3 Motor and Load System Response

To further confirm the simulation model, inertial and frictional loads were added to the motor. Calculated response and actual response were compared. The expected response is slower, due to increased inertia, and has greater damping, due to increased friction. There is also a small phase difference between the motor response and the load response. This is caused by the coupling compliance between the two inertial loads.

Figure 4.3.1 displays the simulated and actual single step response of the loaded system. Resonant frequencies are reduced to 166 hz for the simulated system and 160 hz for the actual system. Damping on both is similar.

Figure 4.3.2 illustrates the loaded, backstepped system. The responses, again, are very similar.

These comparisons between sample simulations and actual tests in the laboratory show in still another situation that the model reasonably represents real life hardware.



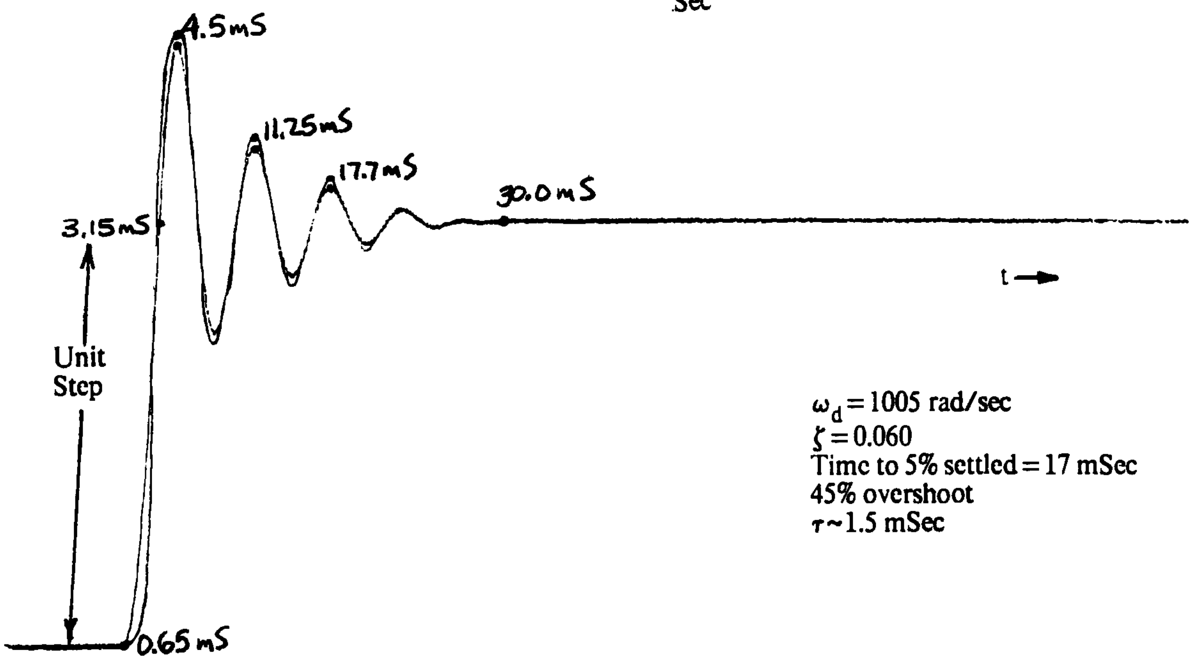


Figure 4.3.1 Single Step Response, Motor and Load

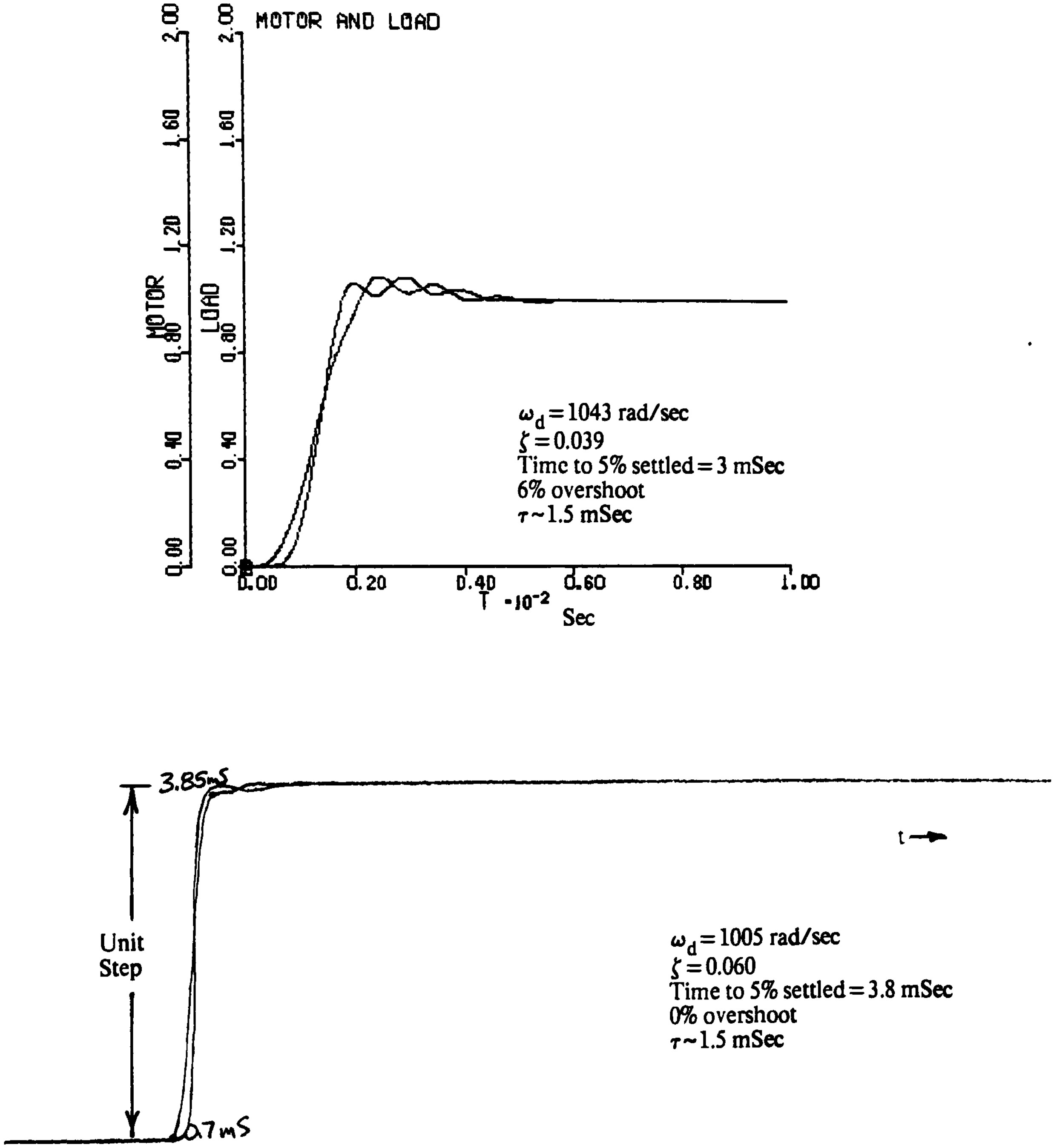


Figure 4.3.2 Backstepped Response, Motor and Load

5.0 Discussion

A stepping motor drive system is a complex combination of electronic controls, electromechanical devices, and mechanical system dynamics. When each of these parts is considered and modeled, a representative simulation of the system can be derived. The model may then be used to predict the performance of a specific stepping motor drive system. Both modeled and test results were used to demonstrate the drive method called backstepping. This method utilizes maximum torque of the motor, and yields the fastest possible system response for a specific motor and load.

This work showed reasonable correlation between simulated and empirical results. Possible sources of discrepancy may be ideal assumptions in various stages of the model. The current control portion, for example, assumes an ideal voltage source and the mechanical model assumes all components are rigid. These assumptions could lead to mismatched results. Another area which requires careful watch is the test apparatus and measurement methods.

Once developed and tested, the model could be expanded in numerous directions. Further expansion of the model can offer potential benefits in better correlation and potential problems in added complexity. In any modeling activity it is good practice to use the simplest model of a specific system which still yields the level of detail required.

6.0 Conclusions

An open-loop hybrid stepping motor system may be described by a set of equations. Model equations may be used to predict the system response of a stepper. If the predicted response accurately simulates an experimental response, then the accuracy of the model is confirmed. In this project, the numerical results of the system single step response and the system backstep response were verified by experiment. Therefore, an open-loop hybrid stepping motor drive system can be accurately simulated to predict real hardware performance and show effects of system parameter changes.

7.0 Recommendations

There are other areas where this work could be expanded as future work. First, using the model as it exists, more comprehensive model confirmation can be done. This would involve testing additional drive waveforms such as multiple fullstep and backstep response or sinusoidal drive inputs. By changing other system parameters, such as inertia, friction, and compliance, further system characterization is possible. These extensions of the work all involve modifying inputs to the model. Other work may include changing the model itself.

The system assumes certain ideal components such as the ideal voltage source. This and the description of other components could be clarified to describe non-linearities and imperfections. The stepper portion of the model can be replaced with another type of motor and a feedback loop can be closed around any of the components.

8.0 References

1. Jacob Tal, "Motion Control by Microprocessors," Galil Motion Control, Inc., 1984.
2. B. C. Kuo, *Proceedings--Annual Symposium Incremental Motion Control Systems and Devices*, 1971-1984.
3. Sigma Instruments Inc., *Sigma Stepping Motor Handbook*, 1972.
4. A. C. Leenhouts, "An Improved Model for the Hybrid Step Motor", *Proceedings of the Thirteenth Annual Symposium on Incremental Motion Control Systems and Devices*, pg. 157-161, 1984.
5. Mitchell and Gauthier, Assoc., Inc., *Advanced Continuous Simulation Language (ACSL) User Guide/Reference Manual*, Concord, Mass., 1981.

```
                    --------------------SYNTHESIZE PWM REFERENCE WAVEFORM"
                     O*(-STEP(BA)+STEP(BA+RE)))
       ISA=IAM
       IRA=ISA+TRI
       IRB=ISB+TRI
       "-----------------------------------PULSE WIDTH MODULATE; SWITCH VOLTAGES
       PROCEDURAL(VA=IA,IRA,VPS)
               VA=-VPS
               IF((IA-IRA).LE.(0.0)) VA=VPS
       END $"OF PROCEDURAL"
       PROCEDURAL(VB=IB,IRB,VPS)
               VB=-VPS
               IF((IB-IRB).LE.(0.0)) VB=VPS
       END $"OF PROCEDURAL"
       "-----------------------------------DESCRIBE PHASE INDUCTANCE"
       LA=A-(C*(IA/SIGN(1.0,IA))*COS(N*TH))
       LB=A-(C*(IB/SIGN(1.0,IB))*SIN(N*TH))
       "-----------------------------------SOLVE FOR PHASE CURRENT"
       IAD=(VA-IA*R+THD*(K-NC*SIGN(1.0,IA))*SIN(N*TH))/LA
       IA=INTEG(IAD,IAIC)
       IBD=(VB-IB*R-THD*(K-NC*SIGN(1.0,IB))*COS(N*TH))/LB
       IB=INTEG(IBD,IBIC)
       "-----------------------------------CALCULATE BACK EMF"
       BEMFA=-THD*(K-NC*SIGN(1.0,IA))*SIN(N*TH)
       BEMFB=THD*(K-NC*SIGN(1.0,IB))*COS(N*TH)
       "-----------------------------------SOLVE FOR MOTOR TORQUE"
       TA=-(K-0.5*NC*SIGN(1.0,IA))*IA*SIN(N*TH)
       TB=(K-0.5*NC*SIGN(1.0,IB))*IB*COS(N*TH)
       TD=-D*SIN(4.0*N*TH)
       TM=TA+TB+TD
       "-----------------------------------SOLVE FOR MOTOR POSITION"
       THDD=(TM-VM*THD-FRM*SIGN(1.0,THD)-KCOUP*(TH-THL))/JM
       THD=INTEG(THDD,THDIC)
       TH=INTEG(THD,THIC)
       MOTOR=(TH+0.0157)/0.0314
       "-----------------------------------SOLVE FOR LOAD POSITION"
       THLDD=(-KCOUP*(THL-TH)-FRL*SIGN(1.0,THLD))/JL
       THLD=INTEG(THLDD,THLDIC)
       THL=INTEG(THLD,THLIC)
       LOAD=(THL+0.0157)/0.0314
END $ "OF DERIVATIVE"
       "-----------------------------------STOP WHEN TIME=TSTP"
       TERMT(T.GE.TSTP)
END $ "OF DYNAMIC"
END $ "OF PROGRAM"
```

```
ROGRAM STEPMOD
      "----------------------------------------------------------------------
      "        THIS PROGRAM SIMULATES A STEPPER MOTOR DRIVE SYSTEM.
      "        SYSTEM FUNCTIONS INCLUDE SYNTHESIS OF MOTOR DRIVE WAVEFORMS,
      "        PULSE WIDTH MODULATION TO CONTROL VOLTAGE TO MOTOR PHASE
      "        WINDINGS, ELECTRICAL RESPONSE OF THE MOTOR TO VOLTAGE CHANGES,
      "        TORQUE PRODUCTION IN RESPONSE TO PHASE CURRENT, AND DYNAMIC
      "        RESPONSE OF THE MECHANICAL SYSTEM TO TORQUE INPUTS.
      "                WRITTEN BY: GREGORY BELL
      "                DATE:    1 JANUARY 1985
      "----------------------------------------------------------------------
NITIAL
      "-------------------------------SET SYSTEM PARAMETERS"
      CONSTANT        K=0.227          $"MOTOR TORQUE CONSTANT"
      CONSTANT        N=50.0           $"# MOTOR TEETH"
      CONSTANT        D=0.076          $"DETENT TORQUE"
      CONSTANT        JM=6.4E-6        $"MOTOR INERTIA"
      CONSTANT        JL=5.1E-6        $"LOAD INERTIA"
      CONSTANT        VM=1.0E-12       $"MOTOR VISCOUS DAMPING"
      CONSTANT        FRM=0.0064       $"MOTOR INTERNAL FRICTION"
      CONSTANT        FRL=0.044        $"LOAD FRICTION"
      CONSTANT        KCOUP=100.0      $"COUPLING STIFFNESS"
      CONSTANT        PI=3.1416        $"PI CONSTANT"
      CONSTANT        THIC=-0.0157     $"ROTOR POSITION IC"
      CONSTANT        THDIC=0.0        $"ROTOR VELOCITY IC"
      CONSTANT        THDD=0.0         $"ROTOR ACCELERATION"
      CONSTANT        THLIC=-0.0157    $"LOAD POSITION IC"
      CONSTANT        THLDIC=0.0       $"LOAD VELOCITY IC"
      CONSTANT        THLDD=0.0        $"LOAD ACCELERATION"
      CONSTANT        IAM=2.00         $"MAX CURRENT PHASE A"
      CONSTANT        IBM=2.00         $"MAX CURRENT PHASE B"
      CONSTANT        IA=2.00          $"CURRENT PHASE A"
      CONSTANT        IB=-2.00         $"CURRENT PHASE B"
      CONSTANT        VPS=24.0         $"POWER SUPPLY VOLTAGE"
      CONSTANT        A=4.97E-3        $"NOMINAL PHASE INDUCTANCE"
      CONSTANT        C=0.99E-3        $"PHASE INDUCTANCE PARAMETER"
      CONSTANT        NC=0.05          $"MOTOR SATURATION FACTOR"
      CONSTANT        R=1.13           $"PHASE RESISTANCE"
      CONSTANT        VA=24.0          $"VOLTAGE ON PHASE A"
      CONSTANT        VB=24.0          $"VOLTAGE ON PHASE B"
      CONSTANT        IAIC=2.00        $"CURRENT PHASE A IC"
      CONSTANT        IBIC=-2.00       $"CURRENT PHASE B IC"
      CONSTANT        IAD=0.0          $"CURRENT CHANGE PHASE A"
      CONSTANT        IBD=0.0          $"CURRENT CHANGE PHASE B"
      CONSTANT        BA=0.0011        $"BACKSTEP TIME"
      CONSTANT        RE=0.0006        $"RESTORE TIME"
      CONSTANT        CHOP=20000.0     $"DRIVER CHOPPING FREQUENCY"
      CONSTANT        TRIIC=0.0        $"TRIANGLE WAVE SYNTHESIS IC"
      CONSTANT        SPS=100.0        $"STEPS PER SECOND"
      CONSTANT        TSTP=0.00999     $"TIME TO TERMINATE SIMULATION"
      CINTERVAL       CINT=0.00001     $"INTEGRATION INTERVAL"
ND $ "OF INITIAL"
      "-----------------------------------------"
YNAMIC
ERIVATIVE
      DRVFQ=SPS/4.0
      PD=1.0/DRVFQ
      "-----------------------------SYNTHESIZE TRIANGLE WAVE"
      CHOPP=1.0/CHOP
      TRID=2.0*(PULSE(0.0,CHOPP,CHOPP/2.0)-0.5)
      TRI=10000.0*(INTEG(TRID,TRIIC)-CHOPP/4.0)
```