Time Restrictions in Natural Resource Management: A Dynamic and Stochastic Analysis¹

by

Amitrajeet A. Batabyal²

and

Hamid Beladi³

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Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. Internet aabgsh@rit.edu

3

Department of Economics and Finance, University of Dayton, 300 College Park, Dayton, OH 45469-2251, USA. Internet beladi@udayton.edu

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Abstract

This paper provides a theoretical analysis of time restrictions in the context of natural resource management in North America. The economic inefficiencies arising from the use of time restrictions to protect natural resources have been well documented by researchers. We first show that in the presence of uncertainty about the evolution of the resource stock, time restrictions can lead to the collapse of the resource that is sought to be protected. Given this finding, in the second part of the paper, we discuss an approach to natural resource management under uncertainty in which time restrictions are used to maximize the likelihood that a particular resource will not collapse in the long run.

Keywords: OR in Natural Resources, Time Restrictions, Uncertainty

JEL Classification: Q20, D81

1. Introduction

The use of time restrictions to regulate the activities of productive units in the western world can be traced back to at least the Fairs and Markets Act of 1448 in England. As Kay and Morris (1987) have noted, most western European nations restrict the number of hours during which shops can remain open. In the United States, many counties and states regulate trading activities with the help of so called “blue laws.” Weninger and Strand (1998), Batabyal and Beladi (2002) and others have noted that production involving the use of noisy machines is typically restricted to normal working hours. In addition to this, there are labor laws that prevent children under certain ages from using their time to work in most kinds of non-agricultural production processes (Krueger, 1996).

These examples point to the fact that the use of time restrictions for regulatory purposes is widespread in modern society. This is particularly true in the realm of natural resource management. Weninger and Strand (1998) and Xu and Batabyal (2002) observe that recreational and commercial hunters for most game are subject to seasonal restrictions. Moreover, such hunters are generally required to hunt during daylight hours. In addition to this, in virtually every state in the USA, sport fishing seasons exist for a whole host of fish species. Commercial fisheries in Canada, the USA, and in western Europe are subject to a variety of time restrictions.⁴

Although time restrictions have been and are frequently used in natural resource management, researchers have clearly documented the distortionary effects of such restrictions. For instance, Karpoff (1985) has noted that larger fishing vessels become less profitable as the length of the fishing season is reduced. Matulich *et al.* (1996) have argued that time restrictions give rise to harvesting

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For a more detailed corroboration of this claim, see Karpoff (1985, 1987), Matulich *et al.* (1996), Homans and Wilen (1997), Hartwick and Olewiler (1998, pp. 152-175), Weninger and Strand (1998), and Xu and Batabyal (2002).

inefficiencies in a fishery. At least in the context of fisheries, researchers agree that time restrictions “convey distributional advantages to politically dominant fishermen at the expense of their *more efficient* competitors” (Karpoff, 1987, p. 192, emphasis added).

Although the inefficiencies associated with time restrictions have been widely recognized by researchers, the same cannot be said about a negative effect that time restrictions are likely to have on the *stock* of a resource that is sought to be managed with such restrictions. As such, this paper has two objectives. First, we show that when there is uncertainty about the evolution of the stock of a resource, time restrictions can lead to the collapse of this resource. Mathematically, this means that the probability that the resource stock will end up in a particularly undesirable state, i.e., in the collapse state, is positive. Given this finding, we next analyze an approach to natural resource management under uncertainty in which time restrictions are used to maximize the likelihood that the resource stock will always remain above a minimum acceptable level.

The rest of this paper is organized as follows: Section 2 first presents a stochastic model of a resource and then formally demonstrates the result regarding the collapse of this resource. Section 3 describes and discusses a model of natural resource management under uncertainty in which time restrictions are optimally used to maximize the probability that the stock of the natural resource under study, will never fall below a minimum acceptable level. Section 4 concludes and offers suggestions for future research.

2. Time Restrictions in a Stochastic Resource

2.1. A Primer on semi-Markov Processes

Consider a semi-Markov process $\{z(t): t \geq 0\}$ made up of two components $\{z(1t), z(2t)\}$. We

suppose that the properties of this process can be described by the parameter ε .⁵ Further, we also suppose that the component $z(1t)$ undergoes transitions slowly, and that the component $z(2t)$ changes state rapidly. Let $z(m)=\{z(1m), z(2m)\}$ denote the state of $z(t)$ after the m th transition. Then it is well known that the sequence $\{z(m):m\geq 0\}$ is the embedded Markov chain of the semi-Markov process $\{z(t):t\geq 0\}$.

We now need to specify the transition probabilities of this embedded Markov chain. To this end, let $z(m)=(i,j)$. Then we say that $z(m+1)=(i,k)$ with probability $\{1-\varepsilon d(i,j)\}b(i,j,k)$, where $d(i,j)\geq 0$, and $b(i,j,k)$ is a probability distribution function for any fixed (i,j) . This means that $b(i,j,k)\geq 0$, and $\sum_k b(i,j,k)=1$. Similarly, from the reference point $z(m)=(i,j)$, we have $z(m+1)=(i^*,k)$ with probability $\varepsilon d(i,j)h(i,j,i^*,k)$, where $h(i,j,i^*,k)$ is also a probability distribution function with $h(i,j,i^*,k)\geq 0$, and $\sum_{i^*,k} h(i,j,i^*,k)=1$. In words, with probability $\{1-\varepsilon d(i,j)\}$, each transition of the semi-Markov process $\{z(t):t\geq 0\}$ involves only the second component, and with probability $\varepsilon d(i,j)$ the transition involves both the first and the second components. Let us now discuss the characteristics of the natural resource that is the subject of this paper.

2.2. The Stochastic Resource

Consider a stochastic resource whose stock can take on a finite number of values $(0,1,2,\dots,S)$. Each of these values corresponds to a particular state of this resource. Note that state here is a proxy for the abundance of the resource stock. In other words, if the resource is a salmon fishery, then its state or stock level represents the number of salmon that are catchable. Similarly, if the resource is an elk hunting ground, then its state or stock level represents the number of elk that can be hunted.

The theory of semi-Markov processes can be used to model this resource in a variety of ways.⁶ We proceed as in Gnedenko and Kovalenko (1989, pp. 136-145) and Kovalenko *et al.* (1997, pp. 176-181). Let us first rank the $S+1$ states of this resource so that 0 is the state of maximal stock abundance and S is the state of minimal stock abundance. Somewhere between 0 and S , lies state n . This state corresponds to the level of the resource stock that is consistent with the total allowable catch of the underlying resource, say, salmon or elk. Under a system of time restrictions (on which more below), if the resource stock is ever in state S , then we shall say that this resource has collapsed. With respect to our earlier salmon fishing and elk hunting examples, S is the state in which the salmon and the elk populations are either actually extinct, or they are practically extinct because their population has fallen below what Clark (1990, p. 17) has called the “minimum viable population level.” From a use perspective, the collapse of the resource means that society will now no longer obtain a flow of services from this resource.

In the absence of time restrictions, as a result of natural factors (*El Nino*, fires), and the continuance of economic activities (fishing, hunting), our resource moves from state 0 to 1, from 1 to 2, and finally from $S-1$ to S with transitions α_0 , α_1 , and α_{S-1} . By assumption, these transitions occur in accordance with a Markov process. The specific nature of the time restrictions (for example, fishing and hunting season length controls) is as follows: In time intervals of length τ , the resource

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We are using a semi-Markov process and not a continuous time Markov chain (CTMC) to model the resource under study because a semi-Markov process is a more general stochastic process than a CTMC. In particular, a CTMC spends an exponentially distributed amount of time in a state before making a transition to some other state. In contrast, a semi-Markov process spends an arbitrarily distributed amount of time in a state before making a transition to some other state. More generally, semi-Markov processes can be used to study terrestrial ecosystems whose behavior is governed by the interactions between small fast-moving systems and large slow-moving systems. For more on these and other related matters, see Rosser (1991) and Perrings (1998).

manager determines the state (stock level) of the resource.⁷ If the resource is in one of the states $0, 1, 2, \dots, n$, then the total allowable catch limit has not been exceeded. Consequently, in this case, the manager does not impose any time (season closure) restrictions. We shall refer to states $0, \dots, n$ as the acceptable set of states. However, if the manager determines that the resource is in one of the states $n+1, n+2, \dots, S-1$, then (s)he immediately imposes a time restriction. This restriction effectively terminates the fishing or hunting season and it eventually brings the resource back to state 0, the state of maximal stock abundance. Let us denote the states $n+1, n+2, \dots, S-1$ as the unacceptable set of states. It is understood that state S , the state of resource collapse, is also an unacceptable state.

Note that the length of this time restriction, i.e., the length of time during which the resource use season is closed, is itself a random variable. This is because the length of the time restriction is a function of the state of the resource at the time of the manager's determination of the stock. To comprehend this, consider the states $n+1$ and $n+2$ of our resource. As compared to state $n+1$, the stock is lower in state $n+2$. Consequently, the length of time during which the resource use season is closed when the state of the resource is $n+2$, will, *ceteris paribus*, be longer than the corresponding length of time when the state of the resource is $n+1$. Now let the mean length of the season closing be τ_k when the manager determines that the resource is in state k . As indicated previously, under this system of time restrictions, if the resource is ever in state S , then it has collapsed. Our task now is to compute the likelihood that this resource will collapse in a given time interval, say, $(0, t)$. We suppose that at time $t=0$, the resource is in state 0.

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In the case of a fishery, Karpoff (1987, p. 183) notes that "fish catches are typically monitored throughout the fishing season..." This suggests that in practice, τ may actually be a small number. For reasons of analytical tractability, we suppose that this determination of the stock level takes a negligible amount of time.

2.3. Likelihood of Resource Collapse

Denote the probability of collapse in $(0,t)$ by $Prob(t)$. We first need to compute the probability of a state transition from the acceptable set of states to the unacceptable state of states in the time period between managerial determinations of the resource stock. Formally, we want to compute the probabilities $p_{ik}=p_{ik}(\tau)$, $i \in [0,n]$, $k \in [i,S]$. Using our Markovian assumption, repeated integration, and the substitution $t=\tau$, we get

$$p_{ii}(t)=\exp\{-\alpha_i t\}, \quad (1)$$

and

$$p_{ik}(t)=\alpha_{k-1} \int_0^t p_{i,k-1}(y) \exp\{-\alpha_k(t-y)\} dy, \quad k \geq i+1. \quad (2)$$

If we set $\varepsilon=1$, and $d_i=p_{iS}$ for $i \in [0,S-1]$, then—also see section 2.1—we can think of d_i as the probability that our resource will collapse in state i at the beginning of the time interval between managerial determinations of the resource stock. For the moment, let us disallow the possibility that the resource will collapse. Then, as discussed in section 2.1, the behavior of this resource is governed by—see Gnedenko and Kovalenko (1989, p. 137)—an embedded Markov chain with states 0 through n .⁸ We suppose that this chain is recurrent. Let the transition probability matrix of this chain be $Q=[q_{ik}]$. The relevant elements of Q are

$$q_{ik}=\frac{1}{1-d_i} p_{ik}, \quad i \in [0,n], \quad k \in [i,n], \quad (3)$$

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By looking at a semi-Markov process at specific points in time we can discover a discrete-time Markov chain (DTMC). This DTMC is the chain that is said to be embedded in the semi-Markov process. For more on this, see Gnedenko and Kovalenko (1989, p. 137) and Ross (1996, pp. 213-218).

$$q_{i0} = \frac{1}{1-d_i}(p_{i,n+1} + \dots + p_{i,S-1}), \quad i \geq 1, \quad (4)$$

$$q_{00} = \frac{1}{1-d_0}(p_{00} + \dots + p_{0,S-1}), \quad (5)$$

and

$$q_{ik} = 0, \quad k \in [1, n]. \quad (6)$$

Having obtained the transition probabilities, we now need to determine the limiting probabilities (π_k) of this Markov chain. As noted in Gnedenko and Kovalenko (1989, p. 139), these limiting probabilities are the solutions to

$$\pi_k = \sum_{i=0}^n \pi_i q_{ik}, \quad k \in [0, n], \quad (7)$$

and

$$\pi_0 + \dots + \pi_n = 1. \quad (8)$$

We now use these limiting probabilities and equation 8.8 in Kovalenko *et al.* (1997, p. 177) to compute the appropriate limiting transition intensity. We get

$$\frac{\sum_{i=0}^n \pi_i d_i}{\sum_{i=0}^n \frac{\pi_i}{1-d_i} \{ \sum_{k=n+1}^S p_{ik} \tau_k \} + \tau}. \quad (9)$$

Using equation (9), we can now write down the expression for the probability of resource collapse in the time interval $(0, t)$. That expression is

$$Prob(t) = 1 - \exp \left[- \left\{ \frac{\sum_{i=0}^n \pi_i d_i}{\sum_{i=0}^n \frac{\pi_i}{1-d_i} \{ \sum_{k=n+1}^S p_{ik} \tau_k \} + \tau} \right\} \cdot t \right]. \quad (10)$$

The key issue now is to determine whether it is possible for this probability to be positive. Inspection of equation (10) tells us that

$$Prob(t) > 0 \Leftrightarrow t > 0. \quad (11)$$

We have just demonstrated

THEOREM 1: *In the presence of uncertainty about the evolution of the stock of a resource, time restrictions will lead to the collapse of this resource with positive probability.*

2.4. Discussion

To the best of our knowledge, Theorem 1 contains a new result in the natural resource management literature. Further, note that this result is *not* an asymptotic one. Specifically, the result in Theorem 1 tells us that the probability of collapse is positive for *any* time interval $(0, t)$, with t positive and finite. The inequality in (11) and Theorem 1 together tell us that this positive probability of resource collapse result depends on the length of a researcher's interval of analysis, i.e., $(0, t)$. The reader will note that the inequality in (11) does not contain an upper bound on t . This means that as the length of a researcher's interval of analysis goes up, the likelihood of resource collapse increases. Indeed, the expressions in (10) and (11) together tell us that in the limit as t tends to infinity, resource collapse will occur with probability one.

In this section we have shown that time restrictions can lead to the collapse of the resource that is sought to be protected. In addition to this and as described in section 1, it is well known that the use of time restrictions results in a number of economic inefficiencies. Despite this, it is unlikely that resource managers will eschew the use of such restrictions in the immediate future. As Karpoff (1987, p. 192) has noted in the context of fisheries, time restrictions are “the [output] of a political system used to redistribute wealth among fishery producers.” Along the same lines, Weninger and

Strand (1998) have remarked that although very recently there has been some movement toward the use of individual transferable quotas (ITQs) in fisheries management in the USA, Congress has mandated that ITQs not be adopted any further until a comprehensive assessment of the pros and cons of input versus time restricted management regimes has been made.

Given this state of affairs and the result contained in Theorem 1, we now address the following question: Suppose that a manager's objective is to maximize the probability that a resource will not collapse. How should (s)he use time restrictions to optimally manage this resource?⁹ The use of this kind of probabilistic objective function is not standard in economics. Consequently, we now briefly explain our rationale for wanting to maximize a probability. As Perrings (1998), Batabyal (1999) and others have noted, because resources such as fisheries, forests, and rangelands are jointly determined, prudent resource management involves paying attention to *both* the ecological and the economic factors that govern the behavior of such resources. Now, the resilience of a resource is an effective indicator of the well-being of this resource. In words, resilience refers to "the amount of disturbance that can be sustained [by a resource] before a change in system control or structure occurs" (Holling *et al.*, 1995, p. 50). Mathematically, resilience can be thought of as a probability (see Krebs (1985, p. 587), Perrings (1998), and Batabyal (1999)). If we think of the collapse state of a resource as an undesirable state, then maximizing the probability that a resource will not collapse is like maximizing the resilience of the desirable states of the resource. This is the reason for using a probability to describe the resource manager's objective function.

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A simple answer to this question is that the manager should impose a permanent time restriction. This means that the evolution of the resource stock is a function of natural factors only. However, this is not a practical way of managing a resource because a permanent time restriction means that the resource is *never used*. Because resource management is all about regulating use levels, if the resource is never used, then the question of "managing" such a resource does not arise.

3. Optimal Resource Management with Time Restrictions

3.1. Preliminaries

Suppose that in the presence of economic activities such as fishing and hunting, and in the absence of time based management, the unacceptably low steady state stock of the resource of interest is S_u .¹⁰ The resource manager would like to raise the level of the resource stock to $S_u + \gamma T$, by controlling the length of time, T , during which the resource use season is closed.¹¹ Here, $\gamma > 0$ is a parameter. As discussed in section 1, the use of time restrictions results in costs to society.¹² Consequently, let us denote the economic cost of closing the resource use season by the cost function $c(T)$, where $c'(T) > 0$ and $c''(T) > 0$. Further, let $V(t) = \{S(t) - S_u - \gamma T\}$ denote the deviation of the resource stock from the steady state stock S_u , where the length of time during which the resource use season is closed is T .

To account for the stochastic nature of the resource stock, we shall model the evolution of the deviation $V(t)$ with a stochastic differential equation.¹³ In particular, we expect the deviations $V(t)$ to display a certain degree of mean reversion over time. Consequently, we suppose that the evolution of $V(t)$ can be described by the Ornstein-Uhlenbeck process.¹⁴ This means that $V(t)$

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In this section, we suppose that all the relevant variables are continuous. This will enable us to work with stochastic differential equations and use calculus.

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Our approach to this resource management problem is related to the literature on the enhancement of fish populations. For more on this literature, see Foerster and Ricker (1941), Mangel (1985, pp. 61-64), and Larkin (1988).

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For more on this, see Karpoff (1985, 1987), Matulich *et al.* (1996), and Homans and Wilen (1997).

¹³

V will denote the random variable and v will denote a particular realization of V .

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For more on the Ornstein-Uhlenbeck process, the reader should consult Karlin and Taylor (1981, pp. 170-173) and Taylor and Karlin (1998, pp. 524-534).

satisfies the linear stochastic differential equation

$$dV = -\eta V dt + \sigma dW, \quad (12)$$

where η is the speed of reversion, σ is the variance parameter, and dW is the increment of a standard Wiener process. We are interested in the steady state behavior of the deviation of the resource stock from the steady state level S_u . From Proposition 5.1 in Karlin and Taylor (1981, p. 219), it follows that the steady state probability distribution function of $V(\cdot)$ is

$$f_\infty(v) = \sqrt{\frac{\eta}{\pi\sigma^2}} \exp\left(-\frac{\eta v^2}{\sigma^2}\right). \quad (13)$$

Suppose that our manager has identified the stock level S_m as the minimum acceptable level of the resource stock. This may be the stock level S of section 2.2 or some other stock level. The manager's task now is to compute the probability that the resource stock will actually fall below S_m , when the length of time during which the resource use season is closed is T .

3.2. The Optimization Problem

To compute the above probability, let $f(s)ds = \text{Prob}\{\text{stationary resource stock} \in (s, s+ds)\}$.

This probability is the same as

$$\text{Prob}\{\text{stationary value of deviation} \in (s - S_u - \gamma T, s - S_u - \gamma T + ds)\}. \quad (14)$$

Using equation (13), the probability in equation (14) can be simplified. This yields

$$f(s) = \sqrt{\frac{\eta}{\pi\sigma^2}} \exp\left\{\left(-\frac{\eta}{\sigma^2}\right)(s - S_u - \gamma T)^2\right\}. \quad (15)$$

We can now state the manager's objective. This manager chooses T , the length of time during which

the resource use season is closed, to maximize the probability that the stock of the resource is above S_m at cost $c(T)$. Formally, our manager solves

$$\max_T \int_{S_m}^{\infty} \left[\sqrt{\frac{\eta}{\pi\sigma^2}} \exp\left\{\left(\frac{-\eta}{\sigma^2}\right)(s-S_u-\gamma T)^2\right\} \right] ds - c(T). \quad (16)$$

The reader should note that equation (16) describes an ecological-economic objective function. In this objective function, the ecological part is given by the term $\int_{S_m}^{\infty} \left[\sqrt{\eta/(\pi\sigma^2)} \exp\left\{(-\eta/\sigma^2)(s-S_u-\gamma T)^2\right\} \right] ds$ and the economic part is given by the term $c(T)$. Stated differently, the manager is choosing T to maximize the resilience of the desirable states of the resource (an ecological criterion), at economic cost $c(T)$. Now making the substitution $k=s-S_u-\gamma T$, the manager's optimization problem can be written as

$$\max_T \int_{S_m-S_u-\gamma T}^{\infty} \left[\sqrt{\frac{\eta}{\pi\sigma^2}} \exp\left\{\left(\frac{-\eta k^2}{\sigma^2}\right)\right\} \right] dk - c(T). \quad (17)$$

The first order necessary condition to this problem is

$$\gamma \left[\sqrt{\frac{\eta}{\pi\sigma^2}} \exp\left\{\left(\frac{-\eta}{\sigma^2}\right)(S_m-S_u-\gamma T)^2\right\} \right] = c'(T), \quad (18)$$

and the second order sufficient condition is

$$\sqrt{\frac{\eta}{\pi\sigma^2}} \exp\left\{\left(\frac{-\eta}{\sigma^2}\right)(S_m-S_u-\gamma T)^2\right\} \left\{ \left(\frac{2\gamma^2\eta}{\sigma^2}\right)(S_m-S_u-\gamma T) \right\} - c''(T) \leq 0. \quad (19)$$

Equation (18) tells us that optimality requires the manager to choose the time restriction (length of time during which the resource use season is closed) so that the marginal economic cost to society of this time restriction (the RHS of equation (18)) is equal to the marginal increase in the likelihood that the resource stock will be above the minimum acceptable level S_m (the LHS of equation (18)). In general, the non-linear equation (18) cannot be solved explicitly for T . However, for some specifications of the cost function $c(T)$, this equation can be solved explicitly. We now discuss such a case.

3.3. An Example

Suppose that the cost function is exponential, i.e., $c(T)=\exp(T)$. Now substituting $c'(T)=\exp(T)$ in equation (18) and then simplifying the resulting expression gives

$$\gamma\sqrt{\frac{\eta}{\pi\sigma^2}}\exp\left\{\left(\frac{2\eta S_u S_m - \eta S_m^2 - \eta S_u^2}{\sigma^2}\right) + \left(\frac{2\gamma\eta S_m T - \gamma^2\eta T^2 - 2\gamma\eta S_u T}{\sigma^2}\right)\right\} = \exp(T). \quad (20)$$

Taking the natural logarithm of both sides of equation (20) and then rewriting the resulting expression yields a quadratic equation in T . That equation is

$$\left[\frac{\gamma^2\eta}{\sigma^2}\right]T^2 + \left[\frac{2\gamma\eta S_u + \sigma^2 - 2\gamma\eta S_m}{\sigma^2}\right]T + \left[\frac{\eta S_m^2 + \eta S_u^2 - 2\eta S_u S_m}{\sigma^2} - \log\left\{\gamma\sqrt{\frac{\eta}{\pi}}\right\}\right] = 0. \quad (21)$$

Denote the coefficient of T^2 by A , the coefficient of T by B , and the constant term by Γ . Then the solutions to equation (21) are

$$T_i^* = \frac{-B \pm \sqrt{B^2 - 4A\Gamma}}{2A}, \quad i=1,2, \quad (22)$$

with $B^2 \geq 4A\Gamma$ for obvious reasons. Which of these two values of T —only one value if $B^2 = 4A\Gamma$ —makes most sense for this optimization problem will depend on the parameters of the stochastic differential equation (equation 12) describing the evolution of the deviations $V(t)$, and on the exogenously given resource stock levels S_u and S_m . For instance, it is tedious but straightforward to verify that when $\gamma=5$, $\eta=1$, $\sigma^2=2$, $S_u=15$, and $S_m=20$, $B^2 > 4A\Gamma$ holds. As such, equation (21) has two real roots $T_1^* = 1.0276$ and $T_2^* = 0.8924$. Substituting these values of T_1^* and T_2^* into equation (17) and then performing the necessary calculations¹⁵ tells us that when $\gamma=5$, $\eta=1$, $\sigma^2=2$, $S_u=15$, and $S_m=20$, $T_2^* = 0.8924$ maximizes the manager's objective function. If we measure time in years, then in this example, it is optimal to keep the resource use season open for approximately 39 days of the year.

Suppose that the manager follows this optimal course of action and keeps the resource use season open for 39 days of the year. What is the steady state probability that the resource stock will be above S_m ? To answer this question, substitute $\gamma=5$, $\eta=1$, $\sigma^2=2$, $S_u=15$, $S_m=20$, and $T_2^* = 0.8924$ into the first part of the maximand in equation (17), and then perform the requisite computations. The required steady state probability is 0.2946. Although this specific value is not very high, it is important to remember that this probability is determined in part by the parameters of the problem (γ, η, σ^2) , and by the optimal value of T^* , which is itself a function of these parameters. In other words, even if the resource manager picks the time restriction in accordance with equations (18)-(22), depending on the values of these parameters, (s)he may not have a great deal of success in maintaining the stock of the resource above the minimum acceptable level S_m . In such situations, it may be worthwhile to

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We used the tables in Beyer (1984, pp. 526-529) to perform the required computations.

consider alternate ways of managing stochastic natural resources.

4. Conclusions

We accomplished two tasks in this paper. First, in section 2, we used the theory of semi-Markov processes to demonstrate that when there is uncertainty about the evolution of the stock of a natural resource, the use of time restrictions can lead to the collapse of this resource. Mathematically, this involved showing that the likelihood of finding the resource in a particularly undesirable state, i.e., in the collapse state, is positive. Given this finding, in section 3, we discussed an approach to natural resource management under uncertainty in which time restrictions are used to maximize the likelihood that the stock of a particular resource will always stay above a particular level, say, the collapse level.

The analysis contained in this paper can be extended in a number of different directions. In what follows, we suggest two possible extensions. First, in this paper we modeled the resource under study as a semi-Markov process. Although this is a fairly general stochastic process, it is still Markovian in the sense that a semi-Markov process can be analyzed as a Markov process by appropriately defining the state of the process. Consequently, it would be useful to know whether the result stated in Theorem 1 holds when the resource is modeled with a more general stochastic process.

Second, and on a more practical level, it would be useful to know the range of parameter values (γ, η, σ^2) and stock levels (S_m, S_u) for which equation (21) has complex roots only. If this range is broad, then one would be able to make a much stronger case for not using time restrictions to manage natural resources. Studies of natural resource management that incorporate these aspects of the problem into the analysis will provide additional insights into the management of resources whose

behavior is marked by a great deal of uncertainty.

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